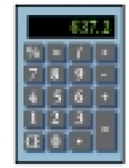
Mathematics Lesson: Are Things Getting Better?
A Data-Analysis Activity

IV. E.



Concepts

- Data Analysis—Students organize, display, and analyze data in order to make valid decisions, inferences, arguments, and predictions; interpret, evaluate, and communicate information obtained from an authentic source and communicate reasoning used.

- Algebra and Patterns—Students explore, represent, model, analyze, and communicate mathematical real world situations involving patterns and functional relationships with and without the use of technology.

- Transplant waiting lists

- Need for more donors

Overview

Students analyze data on the number of deaths over a 10-year period for patients on the heart-transplant waiting list by calculating and plotting the annual death rate. Students then use their graphs to further analyze these data in order to answer questions and predict future trends.

Instructional Objectives

By completing the student packet, students will show an understanding of—

- line-of-best-fit.

- slope and *y*-intercept.

- extrapolation.

Introduction

This activity examines data that show a declining annual death rate over a 10-year period for patients on the heart-transplant waiting list. Students will analyze the data by first calculating the death rate per 1,000 Patient-Years, and then by plotting that death rate. (Patient-Year is a number that reflects the amount of time that each patient actually spent on the waiting list. See note following the death-rate equation in the Procedure section in the Student Guide). Placing a line-of-best-fit allows students to determine the linear relationship between the numbers. Developing an equation for that line allows students to extrapolate 5 years in the future and discuss the implications. (Graphing calculators may be used in this activity.)

Materials

Black-line Master

- Student Packet (4.14)

NOTE: Students should first work individually on this problem and then be allowed to work with a partner to complete it.

Answers to Questions 1-4

1-3a. Below are data for the table that students will complete, the completed graph with the best-fit line that they plot, and sample answers to questions students will answer.

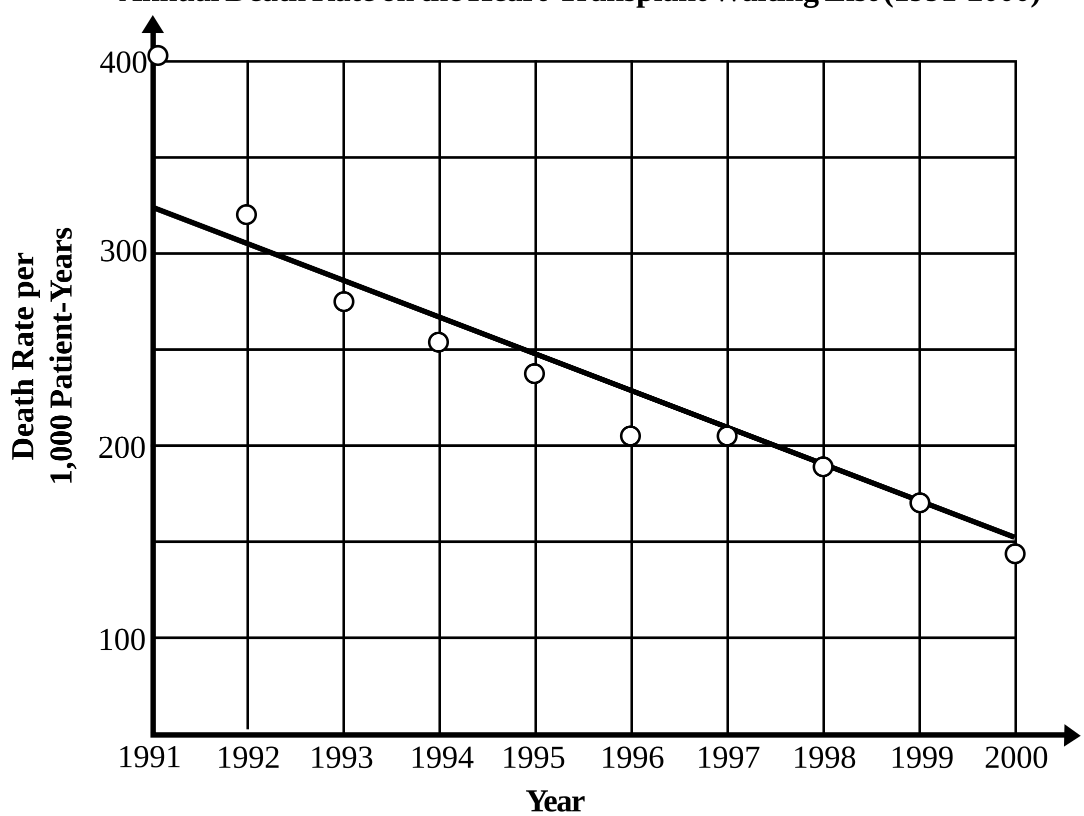
Teacher Table 1

Deaths on the Heart-Transplant Waiting List (1991-2000)

Year	Patients	Patient-Years	Deaths	Death Rate per 1,000 Patient-Years
1991	5,401	1,955	790	404
1992	5,946	2,461	781	317
1993	6,288	2,788	763	274
1994	6,380	2,882	731	254
1995	6,971	3,211	772	240
1996	7,165	3,591	743	207
1997	7,291	3,769	782	207
1998	7,658	4,027	770	191
1999	7,540	4,187	714	171
2000	7,336	4,083	592	145

Data and commentary are from the *2001 Annual Report of the U.S. Organ Procurement and Transplantation Network and the Scientific Registry of Transplant Recipients*, U.S. Department of Health and Human Services, Washington, DC

Teacher Table 2
Annual Death Rate on the Heart-Transplant Waiting List (1991-2000)



3.b *Develop an equation for the best-fit-line using y = mx + b.*

This answer will vary, but the best answer will ignore the first data point and be close to the following:

$$y = -19.2\ x + 318$$

4a. *Use the equation you developed under 3b to predict the death rate in the year 2005. Be sure to use 14 for x in the equation.*

$$y = -19.2 \times 14 + 318$$

$$y = 49.2$$

4b. *Give three reasons why you think the death rate of patients on the heart-transplant waiting list may be falling even though the number of patients on the list is rising.**

A. Doctors may be able to keep alive more patients who are awaiting a transplant.

B. There may be more donor hearts available each year.

C. A combination of the two.

*Accept all reasonable answers.

4c. *Do you think that the death rate will ever fall to zero? Explain.*

It is unlikely that the rate will fall to zero.*

*Accept all reasonable answers.

NOTE: Use of this equation is complicated by the fact that there is no zero point. For it to work, you must create a year scale that begins with zero (1991) and goes up 1 year at a time ending with 9 (2000). This does not affect the *y*-axis.

Student Objective

- Prepare a graph to analyze data.
- Use algebra to analyze heart-transplant waiting-list data.

Background

You need a heart transplant. You have been placed on a waiting list.

There are two ways you will be removed from this list. One is good news, one bad. If you are lucky, a donor will be found, a transplant performed, and you are removed from the list. Now for the bad news. If a donor is not found in time and your condition worsens, you might die. Death while on the waiting list will, of course, remove you from the list.

Every year, patients are added and patients are removed, but the waiting list almost always grows. Does that mean that things are getting worse, or better? As a statistician, it's your job to figure that out.

Procedure

Table 1 provides real data on the number of deaths that have occurred on the heart-transplant waiting list from 1991-2000. The data indicate a rising number of patients on the waiting list and a steady-to-falling number of deaths on the list. This sounds good, but is it? And, if it's good, just how good is it?

As a statistician, you have decided to calculate a new measure that will join together two of the raw measures. You plan to calculate the death rate per 1,000 Patient-Years by using the following equation:

$$\text{Death Rate} = \text{Deaths} \times 1{,}000/\text{Patient-Years}^{*}$$

> For example: In a year when waiting list deaths total 800 and the number of Patient-Years is 2000, what would be the Death Rate per 1,000 Patient-Years?
>
> $$\text{Death Rate} = 800 \times 1{,}000/2{,}000 = 800{,}000/2{,}000 = 400$$

* Patient-Year is a number that reflects the amount of time that each patient actually spent on the waiting list. For example, patient A is on the list for 6 months; patient B is on the list for 3 months; and patient C is on the list for the entire year. Patient A contributes 0.5 Patient-Years to the calculation; patient B contributes 0.25 Patient-Years; and patient C contributes 1 Patient-Year.

4.14 Math Lesson: Are Things Getting Better? Student Packet

1. Complete Table 1 by calculating Death Rate per 1,000 Patient-Years for each year from 1991-2000.

Table 1

Deaths on the Heart Waiting List (1991-2000)				
Year	Patients	Patient-Years	Deaths	Death Rate per 1,000 Patient-Years
1991	5,401	1,955	790	______
1992	5,946	2,461	781	______
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1998	7,658	4,027	770	______
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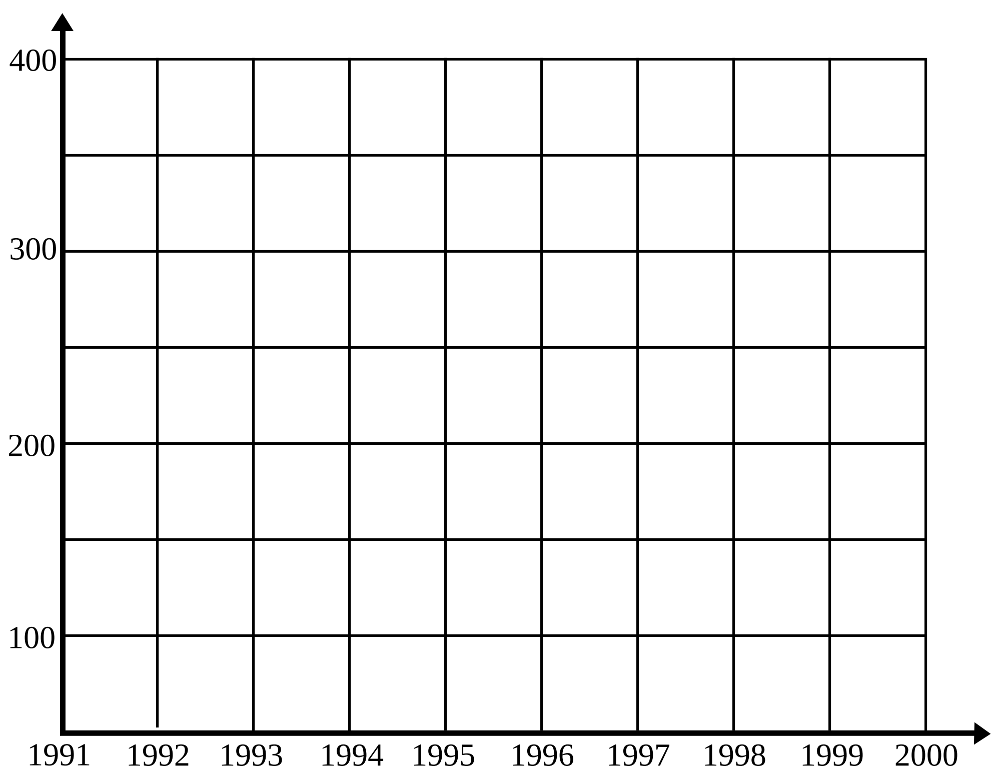
2. Using the data from your completed Table 1, plot the death-rate points on the grid provided for you. Give the completed graph an appropriate title, and label the *x*-axis and *y*-axis.



3. Use your graph to complete the following steps:

a. Place a line-of-best-fit on the graph. (You should ignore any single point that lies far outside the rest of the data.)

b. Develop an equation for the best-fit line using $y = mx + b$.

4. Use the space provided to answer the following questions:

a. Use the equation you developed under 3b to predict the death rate in the year 2005.

b. Give three reasons why you think the death rate of patients on the heart-transplant waiting list may be falling even though the number of patients on the list is rising.

c. Do you think that this death rate will ever fall to zero? Explain.