CURRENT POPULATION SURVEY STATE-LEVEL VARIANCE ESTIMATION

Richard Griffiths and Khandaker Mansur

U.S. Bureau of the Census

Abstract

The problem of estimation for small area means and totals is well-documented in the literature. Not so, the problem of estimating variances for small areas. Yet, small area variance estimators suffer from the same general problem as small area estimators of means and totals – a small sample size on which to base the estimates. Current Population Survey state-level variance estimators are an example of variance estimation with relatively small sample sizes. In this paper we examine two modeling approaches to address this small area estimation problem.

Key words: variance, Current Population Survey, small area estimation, modeling

Introduction

The Current Population Survey (CPS) sample design is a two-stage stratified, cluster design for each state and the District of Columbia. Within each state primary sampling units (PSUs), which are groups of counties, are stratified. A single PSU is selected into the sample in each stratum and a systematic sample of clusters of housing units is then drawn from each sampled PSU. Sampling is done independently in each state.

There are two types of strata in the CPS sample design: self-representing (SR) and non-self-representing (NSR). Each SR stratum contains a single PSU, which is selected into the sample with probability one. Each NSR stratum contains at least two PSUs, one of which is selected into the sample. The variances of CPS estimators thus have two components in NSR strata: a between-PSU component and a within-PSU component. In SR strata the estimators have only a within-PSU component of variance.

The U.S. Census Bureau currently calculates monthly estimates of variances for CPS state labor force estimators. Both successive difference replication and modified half-sample replication methods are used to calculate these state-level variance estimates. (See Fay and Train, 1995, and U.S. Census Bureau, 2000.) The method of successive difference replication is used to estimate within-PSU variances in SR strata and the half-sample replication method is used to estimate total variances in NSR strata. These variance estimates suffer from two known problems: They are based on relatively small sample sizes and they are subject to a bias induced by the procedure of collapsing NSR strata to estimate between-PSU variances. In this paper we address the first of these problems by discussing methods for modeling the variance estimates to improve their precision. Griffiths and Mansur (forthcoming) relate a method for reducing the bias of the variance estimators.

This paper reports the results of research and analysis undertaken by Census Bureau staff. It has undergone a Census Bureau review more limited in scope than that given to official Census Bureau publications. This report is released to inform interested parties of ongoing research and to encourage discussion of work in progress.

The approach we take to modeling is colored by the purposes for which the state-level variance estimates are (or can be) used. One use is as vital signs of the CPS. Since the U.S. Census Bureau is responsible for the statistical methodology of the CPS, it is important for us to monitor the quality (the “health,” if you will) of the estimators that come from the CPS. The state-level variance estimates are important quantities in diagnosing the health of the CPS methodology – they are vital signs. A second use is for research purposes: accurate state-level variance estimates are important in designing changes to the CPS methodology. Another use of the variances is in models based on CPS data. In particular, state-level small area estimation models need good estimates of variance. Beyond that, small area models often require covariance estimates. (See, for example, Tiller, 1992.)

From these uses we can see a need for modeling both variances and covariances. Our primary concern is in modeling the variances, as they are most useful to us as vital signs of the CPS. However, we recognize the need for estimating covariances – a need that will probably grow as small area estimation proliferates. The models we describe in this paper are, in one form or another, capable of modeling both variances and covariances.

In the following sections, we describe a model introduced by Otto and Bell (1995) for modeling variance-covariance matrices for the Census Bureau’s Small Area Income and Poverty Estimates program state-level model. We then proceed to examine a generalized linear model (GLiM) fit by maximum partial likelihood estimation. This model explicitly models only variances, but by making some assumptions about the relationship between variances and covariances, could be used to estimate covariances. After introducing these models, we examine the results of fitting them to several years of monthly state-level variance and covariance estimates.

The Otto/Bell Model

Otto and Bell (1995) proposed a model for improving estimates of state-level variance-covariance matrices for March CPS income and poverty estimators. This model is based on the principle that the mean of the variance-covariance matrix estimator is a function of several components: state effects, characteristic estimates, and sample sizes. Furthermore, the model assumes a structural relationship between variance and covariance estimates based on the autocorrelated nature of CPS sampling errors. Otto and Bell (1995) assumed that the variance-covariance matrix estimator follows a Wishart distribution; inference about the mean of this distribution can be made either through maximum likelihood estimation or in a Bayesian framework by assuming a prior distribution on the model parameters.

The Otto/Bell model assumes that $n_s\mathbf{C_s} \sim Wishart(n_s, \mathbf{V_s})$, where $\mathbf{C_s}$ is the sample-based variance-covariance matrix estimator for state s, n_s is the degrees of freedom for $\mathbf{C_s}$, and $\mathbf{V_s}$ is the mean variance-covariance matrix for state s. $\mathbf{C_s}$ is a MxM matrix, with M being the number of months for which we have estimated variances and covariances. We denote the (i,j)th element of $\mathbf{C_s}$ by C_{sij}, where C_{sij} is the covariance of the estimators from months i and j in state s.

We assume $\mathbf{V_s}$ has the form

$$\mathbf{V_s}(\underset{\sim}{h}) = \mathbf{w_s} \cdot \mathbf{GVF_s} \cdot \mathbf{R} \cdot \mathbf{GVF_s} \cdot \mathbf{w_s} \qquad (1)$$

where $\underset{\sim}{h}$ is a vector of unknown parameters contained in the terms on the right-hand side of (1); $\mathbf{w_s}$ is a diagonal matrix, with each diagonal element being the square root of the state effect for state s; $\mathbf{GVF_s}$ is a diagonal matrix containing the square roots of the generalized variance function (GVF) estimates divided by the sample size for state s; and **R** is a matrix which accounts for the autocorrelated nature of the sampling errors. In this paper we assume the tth diagonal element of $\mathbf{GVF_s}$ is $\sqrt{a \cdot x_{st}^2 / n_{st} + b \cdot x_{st} / n_{st}}$, *t*=1,2,…,M, where *a* and *b* are GVF coefficients, x_{st} is the estimated characteristic total in state s for the tth month, and n_{st} is the state sample size in month t. This is the form of the official CPS GVF, divided by the state sample sizes. (See U.S. Census Bureau, 2000.)

The **R** matrix has the following form

$$\mathbf{R} = \begin{pmatrix} 1 & Corr(e_{s1}, e_{s2}) & Corr(e_{s1}, e_{s3}) & \dots & Corr(e_{s1}, e_{sM}) \\ Corr(e_{s2}, e_{s1}) & 1 & Corr(e_{s2}, e_{s3}) & \dots & Corr(e_{s2}, e_{sM}) \\ \dots & & & & \\ Corr(e_{sM}, e_{s1}) & Corr(e_{sM}, e_{s2}) & Corr(e_{sM}, e_{s3}) & \dots & 1 \end{pmatrix}$$

where e_{st} is the sampling error for the estimator from state s in month t. So, $Corr(e_{st}, e_{s,t-k})$ is the lag k sampling error autocorrelation. We assume the sampling errors represent a stationary stochastic process. The process assumed then determines the form of the elements of **R**. As an example, if we assume the sampling errors follow an ARMA(1,1) process, then $e_{st} = f e_{s,t-1} + e_{st} - q e_{s,t-1}$, where $\{e_{st}\}_{t=1,\dots,M}$ is a white noise process, and the (i,j)th element of **R** has the form: $\dfrac{(1 - fq)(f - q)}{1 + q^2 - 2fq} f^{|j-i|-1}$, i…j. (See Vandaele, 1984, pp. 46-47.)

We thus see that the Otto/Bell model assumes the mean of the variance estimator is the product of the state effect and the GVF divided by the sample size: $E(C_{stt}) = w_s(ax_{st}^2 / n_{st} + bx_{st} / n_{st})$, where w_s is the state effect for state s. And it assumes that the mean of the covariance estimator C_{stu}, *t*…*u*, is $w_s \sqrt{(ax_{st}^2 / n_{st} + bx_{st} / n_{st})(ax_{su}^2 / n_{su} + bx_{su} / n_{su})} \cdot Corr(e_{st}, e_{su})$.

Parameter Estimation

The vector of unknown parameters in (1) may be written as

$$\underset{\sim}{h} = (w_1, w_2, \dots, w_{51}, f_1, f_2, \dots, f_p, q_1, q_2, \dots, q_q, a, b, df)$$

where the f and q parameters are from the ARMA process used to describe the sampling error autocorrelations, which will include seasonal terms, and *df* is a parameter used to estimate the degrees of freedom in each state ($n_s = df - h_s$, where h_s is the number of strata in state s). In this paper, we examine using estimated monthly state-level variance-covariance matrices $\mathbf{C_s}$ to calculate maximum likelihood estimates of $\underset{\sim}{h}$ and thus of the $\mathbf{V_s}$. We note that Otto and Bell (1995) treated the w_s as random effects in their work, because they had only five annual variance estimates from each state for model fitting. We treat the w_s as fixed effects since we have more observations, owing to the fact that we fit the model using monthly estimates.

Since we have assumed a Wishart distribution for $n_s\mathbf{C_s}$, the likelihood function we work with is

$$L(\underset{\sim}{h}|\mathbf{C_s}) = \frac{[\det(n_s\mathbf{C_s})]^{(n_s - M - 1)}\exp\left[-\frac{1}{2}tr(\mathbf{V_s}^{-1}n_s\mathbf{C_s})\right]}{2^{n_s M/2}p^{\,M(M-1)/4}[\det(\mathbf{V_s})]^{n_s/2}\prod_{i=1}^{M}\Gamma[(n_s - i + 1)/2]}$$

Since CPS sampling is done independently in each state, we assume the state-level variance estimators are independent and the likelihood function using data from all states is then given by

$$L(\underset{\sim}{h}|\mathbf{C}) = \prod_{s=1}^{51} L(\underset{\sim}{h}|\mathbf{C_s}), \qquad (2)$$

where $\mathbf{C}=(\mathbf{C_1},\mathbf{C_2},\ldots,\mathbf{C_{51}})$.

Maximizing (2), or the log of (2), for $\underset{\sim}{h}$ gives us $\underset{\sim}{\hat{h}}$, the MLE. The MLE of $\mathbf{V_s}$ is then obtained by substituting $\underset{\sim}{\hat{h}}$ into (1).

Partial Likelihood Generalized Linear Model

As an alternative to the Otto/Bell model for improving state-level variance estimates, we consider a generalized linear model (GLiM) fit through partial likelihood (PL) estimation. (See Kedem and Fokianos, forthcoming, and Fokianos and Kedem, 1998, for more on PL estimation in the context of GLiMs.) We examine this alternative model for two reasons:

- The Otto/Bell model fits both variances and covariances. For some purposes (e.g., variances as CPS vital signs), we are more concerned with the estimation of variances than covariances. (In fact, in this paper, we emphasize variances over covariances.) For these purposes, a properly-specified model that fits only variances will be more efficient than a variance-covariance model.
- This alternative model will help us evaluate the fit of the Otto/Bell model for variances.

The basis of this method is a decomposition of the joint density of the C_{stt} and x_{st}. We may write the joint probability density function for state s as

$$f_s(C_{s11},C_{s22},\ldots,C_{sMM},x_{s1},x_{s2},\ldots,x_{sM},\mathbf{A_s}) =$$
$$f_s(x_{s1}|\mathbf{A_s})\prod_{t=2}^{M} f_s(x_{st}|C_{s11},x_{s1},\ldots,C_{s,t-1,t-1},x_{s,t-1},\mathbf{A_s})\prod_{t=1}^{M} f_s(C_{stt}|C_{s11},\ldots,C_{s,t-1,t-1},x_{s1},\ldots x_{st},\mathbf{A_s}) \qquad (3)$$

where $\mathbf{A_s}$ is the fixed auxiliary information for state s. Here $\mathbf{A_s}=(n_{s1},\ldots,n_{sM})$. Rather than basing inference on the likelihood, we will base it on the partial likelihood. The last product in (3) is the partial likelihood.

Since CPS sampling is done independently in each state, we take the partial likelihood over all states to be

$$\prod_{s=1}^{51}\prod_{t=1}^{M} f_s(C_{stt}|C_{s11},\ldots,C_{s,t-1,t-1},x_{s1},\ldots,x_{st},\mathbf{A_s}) \qquad (4)$$

To develop the model for state-level variances, we assume that $f_s(C_{stt}|C_{s11},\ldots,C_{s,t-1,t-1},x_{s1},\ldots x_{st},\mathbf{A_s})$ is a gamma density, s=1, …, 51; that V_{stt} is the conditional mean of the state-level variance estimator for month t: $V_{stt} = E(C_{stt}|C_{s11},\ldots,C_{s,t-1,t-1},x_{s1},\ldots,x_{st})$; and that

$$g(V_{stt}) = w_s + b_1 C_{s,t-1,t-1} + b_2 x_{st} / n_{st} + b_3 x_{st}^2 / n_{st},$$

for some link function g. We include the most recent lagged value $C_{s,t-1,t-1}$ in the formulation based on evidence that state-level variance estimates follow an AR(1) process. (See Mansur and Griffiths, 2001.) Since V_{stt} is a parameter in the density, (4) is a function of $\underset{\sim}{b} = (w_1,..,w_{51},b_1,b_2,b_3)$. Specifying a form for g allows us to estimate $\underset{\sim}{b}$, and thus V_{stt}, by maximizing (4) for $\underset{\sim}{b}$. This estimator is the maximum partial likelihood estimator (MPLE) of $\underset{\sim}{b}$. (See Wong, 1986, and Fokianos and Kedem, 1998, for discussions of the asymptotic properties of the MPLE.)

Choosing the link function as $g(V_{stt}) = \ln(V_{stt})$ gives a model with multiplicative state effects:

$$V_{stt} = e^{w_s} e^{b_1 C_{s,t-1,t-1} + b_2 x_{st}/n_{st} + b_3 x_{st}^2/n_{st}}.$$

Choosing the link function as $g(V_{stt}) = V_{stt}$ (the identity link) gives a model with the mean as a linear function of the covariates:

$$V_{stt} = w_s + b_1 C_{s,t-1,t-1} + b_2 x_{st} / n_{st} + b_3 x_{st}^2 / n_{st}.$$

Both forms of link function exhibit commonalities with the specification of the mean variance in the Otto/Bell model and thus will be used in comparing the PL GLiM to the Otto/Bell model in this paper. Note that the PL GLiM gives us a model very similar to a GVF and provides a theoretical justification for regressing on random covariates and past variance estimates.

Results from Fitting the Models

In this section we discuss results obtained by fitting the models to state-level total variance and covariance estimates. We used estimated variances and covariances for the (uncomposited) estimates of number of people unemployed from January 1996 to September 2000 for this fitting. We made no attempt to account for the bias due to collapsing NSR strata.

We first examined the fit of several versions of both the Otto/Bell model and PL GLiMs. After determining which of these models were most appropriate to use in a further evaluation, we compared their fits for the estimated variances.

Fit of the Otto/Bell Model

We examined the fit of the full model (1) and several reduced models. The **R** matrix we used in fitting these models is for an ARMA(1,1)x$(0,1)_{12}$ process. This is the form that corresponds to sampling error autocorrelation patterns for estimated number of people unemployed. (See Griffiths and Mansur, 2000.) We looked at the following reduced models:

- Model (1) with $\Theta_{12} = 0$ (i.e. no seasonal MA term)
- Model (1) with $w_s = 1$ for all s (i.e. no state effects)

Table 1 Results of Fitting Full and Reduced Otto/Bell Models

Model	Number of Parameters Estimated[2]	AIC
Full Model	48	241,322
Model with no seasonal MA term	47	241,976
Model with no state effects	6	305,254

Table 1 gives the AIC values for the fit of each of these models, along with the number of parameters estimated for each model. The AIC is calculated as $-2\sum_{s\in\mathbf{S}} \ln L(\underset{\sim}{h}|\mathbf{C_s}) + 2k$, where k is the number of parameters in $\underset{\sim}{h}$, which depends on the version of the model being fit, and **S** indexes the states used in the fitting (see footnote).

From this table we conclude that the state effects and seasonal MA term are important in the model. We thus used the full version of model (1) in the subsequent evaluation.

Fit of the Partial Likelihood GLiMs

We performed a similar analysis on the PL GLiMs with log link and identity link functions. Tables 2 and 3 give the AIC values and number of parameters estimated for each model. For these models we calculated the AIC as $-2\ln(PL) + 2k$, where PL is the partial likelihood given by (4) conditioned on an observation at time t=0 and k is the number of parameters estimated for the model being fit; thus,

$$\text{AIC} = -2\sum_{s\in\mathbf{S}} \sum_{t=1}^{M} \ln f_s(C_{stt}|C_{s00},\ldots,C_{s,t-1,t-1},x_{s1},\ldots,x_{st},\mathbf{A_s}) + 2k$$

where C_{s00} is the estimated variance from the month prior to month t=1. From the results shown in these tables, we conclude that the full models are the appropriate models to use in evaluating the fits of the PL GLiMs.

Table 2 Fit of Full and Reduced Versions of PL GLiM with Identity Link

Model with identity link	Number of parameters Estimated[1]	AIC
Full Model	45	25,709
Model with no lagged value of C_{stt}	44	25,736
Model with no state effects	3	26,281

Table 3 Fit of Full and Reduced Versions of PL GLiM with Log Link

Model with log link	Number of parameters Estimated[1]	AIC
Full Model	45	25,951
Model with no lagged value of C_{stt}	44	25,959
Model with no state effects	3	33,755

[2] The number of parameters excludes state effects for states with no NSR PSUs and Hawaii. These states were not included in the model fitting.

Comparison of the Models for Variances

Comparing the fit of the models for improving variance estimates is in some sense unfair since the Otto/Bell model was designed to model both variances and covariances and the PL GLiMs were designed to model only variances. On the other hand, the form of the GLiMs studied in this paper was restricted to make them somewhat comparable to the Otto/Bell model, at least in terms of link functions used. However, the comparison is made to help determine if we need different models for the different purposes mentioned in the opening section of this paper, or if one model will work for all our purposes. Below, we first compare the models on deviance. We then examine the models in terms of the similarity of their fits. To do this we look at the degree of smoothing each model provides and at graphs of the observed and modeled variances.

Deviance

The deviance is a measure of the discrepancy between the modeled and observed variances – a goodness-of-fit statistic. To determine the appropriate form of the deviance to use in comparing the models, we note that we assumed a gamma distribution for the C_{stt} under the PL GLiMs.

Under the Otto/Bell model $n_s\mathbf{C_s} \sim Wishart(n_s, \mathbf{V_s})$; thus, $n_sC_{stt}/V_{stt} \sim c^2(n_s) = gamma(\frac{n_s}{2}, 2)$, where V_{stt} is the t[th] diagonal element of $\mathbf{V_s}$. So, we examined the deviance of the model fits under the assumption of a gamma distribution on the C_{stt}.

The model deviance for a gamma distribution may be written as

$$2\sum_{t=1}^{M}\left[-\ln(C_{stt}/\hat{V}_{stt}) + (C_{stt} - \hat{V}_{stt})/\hat{V}_{stt}\right],$$

where $\hat{V}_{stt}$ is the modeled variance for state s, month t. (See McCullagh and Nelder, 1983.) We calculated the deviance of each of the models for all states. We found that the overall deviance for the Otto/Bell model (230.3) was similar to that of the PL GLiM with identity link (233.8), while that of the PL GLiM with log link (259.1) was quite a bit larger. A detailed look at the deviances by state also indicated that the Otto/Bell model and PL GLiM with identity link had generally similar deviances, while the PL GLiM with log link tended to have consistently larger deviances. Based on the deviance criteria, then, we might conclude that the Otto/Bell model and the PL GLiM with identity link fit the observed state-level variance data about equally well.

Similarity of the Fitted Variances

Graphs of the time series of observed state-level variances and modeled variances under each of the models are given in Figure 1 for several states. Two things stand out in these graphs: the modeled variances are similar under all three models and they generally describe smoother time series than do the observed variances. The similarity of modeled variances, especially those of the Otto/Bell model and the PL GLiM with identity link, is a result of the commonalities among the models, notably in the form of the hypothesized means of the variance estimators.

As for the smoothing, under the assumption that the variance estimates are somewhat unstable (i.e., the small sample size problem), we would like to see time series of modeled variances that

Table 4 Degree of Smoothing

Characteristic of the Distribution	Otto/Bell Model	Partial Likelihood GLiM with identity link	Partial Likelihood GLiM with log link
Mean	.466	.477	.389
Median	.434	.473	.334
Standard Deviation	.096	.136	.215

are less oscillatory than those of observed variances. To measure the smoothness of each variance time series, we looked at the total variation in the observed and modeled variance time series. We defined the total variation in a time series $\{V_t\}_{t=1,\ldots,M}$ as $\sum_{t=2}^{M} |V_t - V_{t-1}|$.

We used the ratio of the total variation in the modeled variances to the total variation in the observed variances to assess the degree of smoothing. We calculated this ratio for each state and each model. Table 4 contains some characteristics of the distribution of this ratio over all states for the models.

Before proceeding, we note the informal nature of the total variation in assessing the smoothness of the fits. We have no absolute criterion for the degree of smoothing that is best. It is certainly possible to smooth the estimates too much. In other words, there could well be a good deal of oscillation in a time series of true variances (e.g., seasonality) and smoothing the variances too much might mask this true oscillation. However, we feel this measure of smoothness provides a nice descriptive tool for understanding the graphs in Figure 1, plus it gives us another way of showing the similarities among the modeled variances.

Table 4 shows that the degree of smoothing attained by the PL GLiM with log link was, on average, greater than that of the other two models, as well as more variable over the states. Overall, the smoothing for the PL GLiM with identity link was similar to that of the Otto/Bell model, though a little more variable over the states. This is another indication of the similarity of the fitted variances from the Otto/Bell and PL GLiM with identity link models.

Discussion

In this paper we have examined models for improving CPS state-level variance and covariance estimates, though we have been primarily concerned with the variance estimates. We have seen that fitting the Otto/Bell model to variance-covariance matrices and a PL GLiM to variance estimates resulted in substantially similar modeled variances.

While the Otto/Bell model and the PL GLiM are aimed at somewhat different goals, their structures, along with some of the assumptions we've made, will allow us to substitute one for the other. We have seen that the Otto/Bell model when examined solely for its fit to variance estimates performed at least as well as the PL GLiMs examined in this paper. It would thus seem that, unless we can specify a much better form for the PL GLiM, the Otto/Bell model might be preferred for modeling both variances and covariances, purely on the basis of quality of fit.

With this in mind, though, we also note that given a set of modeled variances from the Otto/Bell model, the modeled covariances will be completely determined by the estimated ARMA parameters in the **R** matrix. Thus, if we assume the same structural relationship between variance and covariance estimates as the Otto/Bell model, we can use the PL GLiM variances to calculate covariance estimates. Since we have seen that the Otto/Bell model and PL GLiM produced similar variance estimates, applying the same estimated ARMA parameters to the PL GLiM variance estimates would give modeled covariances similar to those of the Otto/Bell model. Note, though, that without first fitting the Otto/Bell model, the ARMA parameters would have to be estimated outside of the PL GLiM model fitting. They would have to be based on an analysis of sampling error autocorrelations similar to that of Griffiths and Mansur (2000).

Finally, we note that we need to do more work to determine an appropriate link function for the PL GLiM. The links studied in this paper were used more for comparability with the Otto/Bell model than for quality of fit. However, due to the relative computational simplicity of implementing the PL GLiM over the Otto/Bell model, we believe that a form of PL GLiM should be used to produce modeled CPS state-level variance and covariance estimates.

Acknowledgments

The authors would like to thank Bill Bell and Harland Shoemaker of the Census Bureau for their helpful comments.

References

Fay, R.E. and G.F. Train (1995), "Aspects of Survey and Model-based Postcensal Estimation of Income and Poverty Characteristics for States and Counties," Proceedings of the Section on Government Statistics, American Statistical Association, 154-159.

Fokianos, K. and B. Kedem (1998), "Prediction and Classification of Non-stationary Categorical Time Series," Journal of Multivariate Analysis, 277-296.

Griffiths, R. and K. Mansur (2000), "Preliminary Analysis of State Variance Data: Sampling Error Autocorrelations (VAR90-36)," Internal U.S. Census Bureau memorandum.

Griffiths, R. and K. Mansur (forthcoming), "The CPS State Variance Estimation Story (VAR90-38)," Internal U.S. Census Bureau memorandum.

Kedem, B. and K. Fokianos (forthcoming), Regression Models for Time Series Analysis, John Wiley & Sons, Inc.

Mansur, K. and R. Griffiths (2001), "Analysis of the Current Population Survey State Variance Estimates," paper presented at the 2001 Joint Statistical Meetings of the American Statistical Association, Section on Survey Research Methods.

McCullagh, P. and J.A. Nelder (1983), Generalized Linear Models, Chapman and Hall.

Otto, M.C. and W.R. Bell (1995), "Sampling Error Modelling of Poverty and Income Statistics for States," Proceedings of the Section on Government Statistics, American Statistical Association, 160-165.

Tiller, R.B. (1992), "Time Series Modeling of Sample Survey Data from the U.S. Current Population Survey," Journal of Official Statistics, 149-166.

U.S. Census Bureau, Bureau of Labor Statistics (2000), Current Population Survey: Design and Methodology, Technical Paper 63,Washington, DC.

Vandaele, W. (1983), Applied Time Series and Box-Jenkins Models, Academic Press, Inc.

Wong, W.H. (1986), "Theory of Partial Likelihood," The Annals of Statistics, 88-123.

Figure 1 Time Series of Observed and Modeled Variances