Quantifying Uncertainty in the Analysis of Long-Term Social Security Projections

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CBO

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Preface

Analyses of the outlook for Social Security and proposed reforms should take into account the uncertainty in any forecast of Social Security's finances—especially over the 75-year time frame often used for evaluating its prospects. This background paper describes the methods used by the Congressional Budget Office (CBO) to quantify uncertainty in Social Security projections. It updates the December 2001 CBO paper *Uncertainty in Social Security's Long-Term Finances: A Stochastic Analysis*. In keeping with CBO's mandate to provide objective, nonpartisan analysis, this paper makes no recommendations.

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Introduction

In recent studies, the Congressional Budget Office (CBO) has projected that within a few decades, the Social Security system will begin paying out more in benefits than it collects in taxes, if benefits are paid as scheduled.[1] That imbalance is not expected to be temporary; even after the baby-boom generation dies off, a permanent and growing imbalance between system outlays and revenues is expected to remain. The CBO studies show the same basic pattern reported in the latest Social Security Administration's (SSA's) annual *Trustees' Report,*[2] even though CBO employs different techniques than does SSA for various components of the projections. However, any projection of this nature requires assumptions about long-run values for inputs (such as mortality improvement, fertility, immigration, productivity, and other variables) that determine future Social Security taxes and benefits. Because the long-term projections are based on assumptions about uncertain inputs, the projected outcomes also are uncertain.

The technique used by CBO to quantify that uncertainty is known as a Monte Carlo simulation, a description that has its roots in the random (but probability-weighted) outcomes of the gambling activities associated with that well-known Mediterranean enclave. In December 2001, CBO released *Uncertainty in Social Security's Long-Term Finances: A Stochastic Analysis,* which was CBO's first set of stochastic estimates for Social Security.[3] Although the basic Monte Carlo framework developed for that analysis is also used here, improvements in CBO's underlying projection methods have led to changes in the way the stochastic inputs are generated. This paper updates and extends the earlier paper.

The basic tenets of the Monte Carlo simulation are best described by breaking the procedure down into three steps. The first step is to develop a capacity to project outcomes (in this case, Social Security taxes and benefits) using various values for system input assumptions. The second step is to estimate a probability distribution for each input assumption, generally based on the historical values for that variable. The third step in the Monte Carlo simulation is to choose a random value for each input assumption from the estimated probability distribution and solve for the outcomes of interest. The result of repeatedly drawing assumptions and running simulations is a sequence of projected financial outcomes for the system; that sequence of projections is then an estimate of the overall probability distribution for future financial outcomes for the system.

Since its initial 2001 stochastic study on Social Security finances, CBO has improved its methods for making long-term projections. Although the basic population projections are the same as those used in the SSA's *Trustees' Report,* CBO's projections of overall economic activity and the Social Security system's finances are now based on an integrated macroeconomic/microeconomic framework. One notable difference in using that approach is in economywide real wage growth, which is an important determinant of both aggregate system finances and individual taxes and benefits. In the earlier CBO (actuarial) framework, real wage growth is an

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1. See Congressional Budget Office, The Outlook for Social Security (June 2004); and Congressional Budget Office, *Updated Long-Term Projections for Social Security* (March 2005).

2. Social Security Administration Board of Trustees, *2005 Annual Report of the Board of Trustees of the Federal Old-Age and Survivors Insurance and Disability Insurance Trust Funds* (March 2005).

3. In September 2004, the Social Security Administration released an actuarial study describing its stochastic model. The approach described in that study is consistent with CBO's methodology.

exogenous variable (that is, chosen ex ante, outside the modeling framework). In the integrated macroeconomic/microeconomic projections, real wage growth is determined endogenously (that is, within the model itself) by the combination of several assumptions (in this case, most notably total factor productivity, labor force participation, and capital accumulation).

In addition to real wage growth, there are other differences in the list of input assumptions in moving from CBO's 2001 paper to the current approach. Also, because of the integrated macroeconomic/microeconomic approach, a focus on variables of interest requires looking not only at input assumptions (as before) but at some output variables as well. For example, the real 10-year government interest rate (along with other financial asset returns) is now an endogenous variable in CBO's projections. Because of the importance of interest rates in both baseline and policy-change simulations, it is necessary to assess whether the projected stochastic variability in that variable is consistent with observed historical variability.

The innovations in CBO's long-term Social Security projections also affect the scope of the stochastic analysis presented here. In addition to displaying information about aggregate system financial outcomes (such as the balance in the Social Security Trust Fund or revenues and outlays as a share of gross domestic product), the microsimulation component also creates a capacity for studying individual tax and benefit outcomes. As with aggregate finances, the Monte Carlo approach allows analysts to investigate not just expected taxes and benefits, but also uncertainty about various outcomes looking forward. In addition, analysts can assess how that uncertainty changes under proposed Social Security plans.[4]

Monte Carlo Simulation in the CBOLT Integrated Macroeconomic/Microeconomic Framework

Monte Carlo simulation is a technique for studying uncertainty in complex processes where several input assumptions jointly determine some outcome of interest. Projection of long-run Social Security taxes and benefits for individuals and the overall economy is the type of complex process for which the Monte Carlo simulation is well suited. Although there are clear rules about how Social Security taxes and benefits are computed, given an individual's circumstances, the uncertainties about both individual and economywide earnings and demographics interact with those rules, thereby making the outcomes uncertain. The key insight of the Monte Carlo approach is that one cannot gauge uncertainty about the outcomes of interest unless one begins by analyzing uncertainty about the inputs.

Initially, projections under the Congressional Budget Office's long-term model (known as CBOLT) used a Monte Carlo technique in an actuarial modeling framework. The static actuarial model was designed to mimic the Social Security Administration's Office of the Chief Actuary's (OCACT's) methodology. Inputs to the model were exogenous. Since that time, CBO has developed an integrated macroeconomic/microeconomic framework.

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4. See Congressional Budget Office, *Long-Term Analysis of H.R. 3821, the Bipartisan Retirement Security Act of 2004* (July 21, 2004); and Congressional Budget Office, *Long-Term Analysis of Plan 2 of the President's Commission to Strengthen Social Security* (July 21, 2004).

Table 1.

Stochastic Variables in CBO's Long-Term Social Security Projections

Type of Variable	Actuarial Projections	Integrated Macroeconomic/ Microeconomic Projections
Stochastic Demographic Inputs	• Overall fertility rate • Mortality improvement by age and sex • Net annual immigration • Overall disability incidence • Overall disability termination	• Overall fertility rate • Mortality improvement by age and sex • Net annual immigration • Overall disability incidence • Overall disability termination
Stochastic Economic Inputs	• Real wage growth • Inflation • Unemployment • Real 10-year bond rate	• Total factor productivity growth rate • Inflation • Unemployment • Gap between marginal product of capital and real 10-year bond rates • Change in earnings share of compensation • Gap between CPI-W and core price index
Stochastic Intermediate Output Variables	Not applicable	• Real wage growth • Real 10-year bond rate • Real GDP growth rate
Stochastic Asset Return Variables	Not applicable	• Real corporate bond rate • Real equity returns

Source: Congressional Budget Office.

The growth model uses CBO's baseline economic assumptions for the first 10 years of the projection. The macroeconomic model generates endogenous values for the economic determinants of Old-Age, Survivors, and Disability Insurance (OASDI) finances. That mechanism allows the model to estimate some macroeconomic effects of potential policy changes.[5] Some of the differences between the static and growth modules can be illustrated by looking at the differences in the stochastic variables required for the actuarial versus the integrated macroeconomic/microeconomic projections (see Table 1).[6]

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5. CBO's analysis of various Social Security policy proposals have also incorporated results from a different model and methodology to estimate macroeconomic effects. That model uses a stochastic overlapping-generations framework. See Congressional Budget Office, *Long-Term Analysis of H.R. 3821* and *Long-Term Analysis of Plan 2*.

6. For more details on the actuarial, growth, and microeconomic models, see Josh O'Harra, John Sabelhaus, and Michael Simpson, *Overview of the Congressional Budget Office Long-Term (CBOLT) Policy Simulation Model,* CBO Technical Paper 2004-01 (January 2004).

In addition to the growth model, projections are now based on a microsimulation model in which actuarial estimates for crucial variables are replaced with values from a microsimulation of a 1:1,000 representative population sample. Specifically, for each of the 300,000-plus observations in the longitudinal micro sample, CBOLT simulates births, deaths, immigration, marital transitions, labor force participation, hours of work, earnings, Social Security benefits-claiming behavior, and Social Security benefit levels.

An individual's Social Security taxes and benefits are based on his or her earnings history, disability status, age, and the characteristics of the current (or possibly former) spouse. While working, a person pays Social Security taxes at a flat rate, up to a maximum amount. Eligibility for benefits is triggered by age or disability status and, in some cases, individuals receive benefits based on their spouse's earnings history, age, or disability status.[7] After benefits are awarded, they generally continue to be paid for the remainder of the individual's lifetime.

Although the rules are certain, individuals face uncertainty about their Social Security taxes and benefits because of uncertainty about their lifetime earnings and demographic outcomes. The nature of that uncertainty varies over an individual's lifetime. When young, people do not know whether they will be high earners or low earners, whether they will marry, or whether they will even live long enough to receive benefits. As people get older, some uncertainties are resolved—for example, people on the brink of retirement basically know what their Social Security benefit will be because they know their earnings history. However, they still face other uncertainties, such as how long they will live.

The uncertainty facing individuals is compounded when attempting to quantify the uncertainty about overall system finances. When contemplating how much the system will collect in taxes or pay out in benefits in some future year, analysts doing the projections are implicitly dealing with the individual uncertainty described above, aggregated across the entire population. Taxes collected in some future year will depend on the number of workers and the amount each worker earns. The number of workers (if one looks ahead far enough) will depend on overall fertility patterns, and the amount each worker earns will depend on both aggregate economic trends (such as productivity, inflation, and unemployment) and the pattern of those earnings across the population (how much of the earnings in each year is below the maximum taxable threshold). Benefit projections are affected by the same uncertainties, but one also must consider overall disability rates and—probably most important for long-term benefit projections—the rate of improvement in life expectancy.

The list of input assumptions used in CBO's Monte Carlo analysis of Social Security projections speaks directly to the types of uncertainty faced both by individuals and by the overall system (see Figures 1 and 2). The key demographic input assumptions, varied stochastically in the Monte Carlo simulations, are fertility, mortality, net immigration, and rates of disability incidence and termination. The key economic inputs, varied stochastically, are total factor productivity growth, inflation, unemployment, the relationship between interest rates and the return to capital, the share of compensation that shows up as taxable earnings, and the gap between the core gross domestic product (GDP) deflator and growth in the consumer price index for urban wage earners and clerical workers (CPI-W). Those factors together determine

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7. For details about how the current system works, see Congressional Budget Office, *Social Security: A Primer* (September 2001).

Figure 1.

How Stochastic Inputs Affect Individual Tax and Benefit Outcomes

Stochastic Variables	Disability Incidence and Termination	Real Wage Growth	Unemployment	Inflation	Mortality
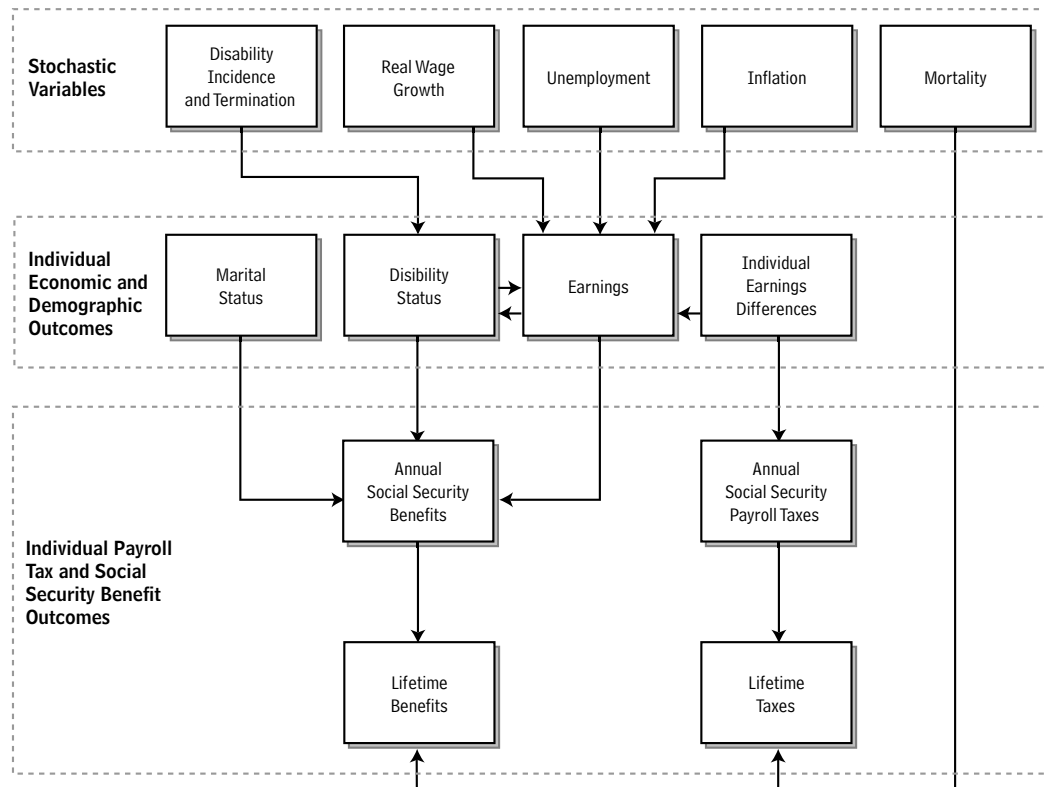
Source: Congressional Budget Office.

the individual and aggregate outcomes of interest in any simulation. When the values for each input are fixed, the projections are called *deterministic.* When the values for inputs are drawn from probability distributions (as in a Monte Carlo simulation), the projections are called *stochastic*.

The first step is to develop methods for projecting the outcomes of interest, using different possible values for the input assumptions. CBO's approach uses a variety of techniques for different pieces of the projection. For example, overall population size and age-and-sex composition (determined by fertility, mortality, and immigration) are projected using techniques adopted directly from SSA. However, CBO's projections for overall economic activity (especially real GDP growth) and Social Security system finances are based on the integrated macroeconomic/microeconomic framework, which involves projecting detailed life outcomes for a sample of the population in each future year.

The economic inputs to the integrated macroeconomic model (total factor productivity growth, inflation, unemployment, the relationship between interest rates and the return to capital, the share of compensation that shows up as taxable earnings, and the gap between the core GDP deflator and the CPI-W) generally operate directly on the aggregate economy, and

Figure 2.

How Stochastic Inputs Affect Social Security System Finances

Stochastic Inputs	Disability Incidence and Termination	Fertility, Mortality, and Immigration	Unemployment	Inflation	TFP[a] and Earnings Share of Compensation	Real Bond Rate Gap

| Intermediate Stochastic Variables | | Population by Age and Sex | Number of Workers | | Real GDP | Interest Rates |

Social Security System Financial Outcomes

- Number of Beneficiaries
- Total Earnings
- Social Security Benefits
- Social Security Taxes
- Social Security Trust Fund

Source: Congressional Budget Office.

a. TFP = total factor productivity.

those outcomes are then distributed across the population in the micro model. In a similar manner, the demographic policy assumptions (such as disability rates) and other demographic events are distributed across the micro sample to create the desired population heterogeneity in each year.

From the perspective of a Monte Carlo simulation, the key to CBO's projection methodology is that the entire sequence of events can be solved for using any feasible combination of the demographic and economic input assumptions. The set of projection outcomes associated with setting each input assumption to its most likely value is often considered the base-case scenario. But any or all of the input assumptions can be varied to study how outcomes of interest (individual taxes and benefits or systemwide finances) are affected by changing that assumption.

Development of the capacity to project outcomes using any feasible combination of inputs is the first half of the preliminary work needed to implement a Monte Carlo simulation. The

other background step involves estimation of probability distributions for each input assumption. As with the projection of expected values for each input, the projection of the probability range for each input is best accomplished by letting history be the guide. The statistical technique most often used in these types of Monte Carlo applications is known as time-series analysis, which focuses on decomposing historical movements in each input into a random component and a predictable component. The random component is projected using computer-generated random outcomes, and the predictable component is solved for in standard fashion.

The implementation of Monte Carlo simulation involves choosing values for each input assumption from the estimated probability distributions and then solving the entire model for outcomes of interest. Any one simulation of this type has limited usefulness—it can be thought of only as a possible outcome. However, repeatedly drawing assumptions and running simulations result in a sequence of projected outcomes that can be characterized as the estimated probability distribution for those outcomes. The more simulations one uses, the better the estimate of the probability distribution for the outcomes of interest.

The Monte Carlo approach is a powerful way to identify uncertainty about the outcomes of interest in a complex process. At the extremes, the estimated probability range can be very narrow, indicating reasonable certainty about its future values, or the range can be very wide, indicating extreme uncertainty. The estimated range for the outcomes is ultimately attributable to two factors: how certain the long-run values for the inputs are, and how much the outcome of interest reacts to changes in those inputs. For example, this study shows that overall Social Security system outlays are very uncertain, largely because overall demographic outcomes (fertility and mortality) have a big effect on the size of taxes and benefits relative to the underlying economy. Conversely, individual benefit-replacement rates (the ratio of benefits to lifetime earnings) are fairly predictable even though many of the determinants of benefits and earnings are uncertain, because the economic factors that cause an individual's earnings to change (such as productivity, inflation, and unemployment) have the same impact on that individual's benefits.

As with any projection technique, Monte Carlo simulation does have some drawbacks. In the CBO approach, one of the limiting factors is computational—a single simulation of CBOLT requires nearly 30 minutes on a top-end personal computer, which limits the number of stochastic simulations that can be reasonably produced and thus limits the statistical accuracy with which the probability distributions for outputs of interest can be identified. Also, any estimates of the future variability of inputs and of the future expected values of inputs are subject to error and, thus, in a sense, there is some uncertainty about the estimated uncertainty. Finally, some of the modeling decisions built into the projections cannot easily be characterized as input assumptions that are subject to stochastic draws in the Monte Carlo simulation. Examples of those types of decisions in CBO's projections range from trends in labor force participation across cohorts to fiscal policy outside Social Security and the saving behavior of future generations. Although all of these input assumptions are uncertain, they are currently treated as deterministic in the model.

Time-Series Modeling of Input Assumptions
The textbook statistical approach to making inferences about future variability in stochastic variables is time-series analysis. The time-series approach starts by breaking down the histori-

cal changes in variables into three main components: annual random shocks (positive or negative but centered around zero), year-to-year correlations in annual values, and random changes in the central tendency of the annual values. Simple inspection of historical patterns shows that most of the input variables under consideration have no random change in central tendency over long periods. Whether other variables—including the fertility rate, immigration, and disability incidence or termination—show changes in central tendency depends on how the history is read.[8]

The time-series modeling approach used in this study focuses on the projection of deviations from CBO's assumed expected values. The following process is repeated for every draw—one for each year, each simulation, and each stochastic variable. First, a normally distributed random number is generated. Second, the random number is used as an input to a variable-specific time-series model to calculate a deviation term. Finally, the deviation term is combined with CBO's expected value for that series for that year to produce the CBOLT input. This process creates an input series with the error distribution centered around the deterministic expected value.

One assumption that underlies the time-series models used in this study is that the probability distribution of annual random shocks can be approximated with the well-known normal pattern. In this standard approach, the values of random shocks have an expected level, or mean—in this case, zero—with a symmetric bell-shaped distribution around that expected level. Thus, a random shock that is close to the mean is much more likely to be drawn than one that is distant.

If the projected outcomes for a variable composed only of a long-run average and of annual random shocks were graphed, all the values would be centered around the average value for the variable because, by definition, the expected value of the random shocks is zero. In addition, the graph would have several features: approximately the same number of high and low values; a greater number of values that are close to the average than those that are far away, because the distribution of the shocks is normal (bell-shaped); and finally—a crucial distinction—no pattern that connects the values over time. (That is, outcomes in each year would be independent of the outcomes in the previous year.)[9]

That description of a variable having only an average value and annual random shocks applies only to total factor productivity and the nonwage, nontax share of compensation and does

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8. The decision about whether to incorporate random changes in central tendency is important because it dramatically affects conclusions about the possible range (and thus the probability distribution) of future values. In particular, if no random change in central tendency occurs, outcomes will vary within a probability range that is constant over time. For example, the range of possible outcomes for a variable such as inflation 100 years from now would probably be the same as the range of outcomes 10 years from now. Allowing random changes in central tendency, by contrast, suggests that the range of possible outcomes will widen over time. For example, the range of outcomes for fertility in 2104 could be much wider than the range in 2014 because changes in central tendency generally occur gradually. In the short run, the fertility rate is likely to vary around a fairly predictable central tendency; in the long run, fundamental social changes can affect average fertility. See Congressional Budget Office, *Uncertainty in Social Security's Long-Term Finances: A Stochastic Analysis* (December 2001).

9. The time-series description of a series made up only of an average value and of annual random shocks is *white noise.*

not appear to fit any other inputs that go into projections of Social Security's finances.[10] The inputs (even those with apparently stable long-run central tendencies) seem to move in one direction or another and then stay there for long periods—implying high correlation between outcomes from year to year—before moving back. For example, inflation was generally high in the 1940s, fairly low through the early 1970s, generally high for the next decade or so, and then generally low again. Clearly, variation occurs from year to year, but the outcomes also seem to be correlated over time.[11]

How much of a particular change is attributable to random shocks and how much to correlation between values over time? Time-series analysis specifies a simple equation for a variable and allows the data to answer that question. In the simplest specifications, the equation relates the current-period value of a variable to three things: a constant term (the central tendency), the value of the variable during the most recent period (in order to capture the correlation over time), and an error term (the random shock).[12] More-complicated versions of time-series equations involve adding more lagged terms (not just for the most recent period but for two, three, or more previous periods) or employing a moving average of error terms, in which random shocks themselves affect outcomes for more than one period.[13]

How can a user tell whether the correct equation was chosen to represent the time series being modeled? The answer is to go back to the premise underlying the equation. If the equation is appropriate, the residuals (error terms) derived from it will have the properties associated with a series of normally distributed random shocks: they will be centered around zero, have about the same number of high and low values, have more realizations close to zero than far away, and show no correlation over time. Thus, the time-series approach involves specifying an equation, using historical data to estimate the parameters, and testing whether the residuals are consistent with a series of random shocks.[14]

In principle, whether an equation passes that test determines whether unexplained changes in central tendency exist for a variable over time. If an equation for a variable generates residuals that appear to be random shocks, then, arguably, no unexplained (random) changes in central

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10. Equity returns, when measured as a white noise process, are an exception to this. However, equity returns are an issue only in proposals that include individual accounts.

11. Inflation is often described as a generalized autoregressive conditional heteroskedasticity (GARCH) process. In that type of process, the variance of errors increases with the level of the variable.

12. The simplest specification for a variable x_t is:

$$x_t = \alpha + \beta x_{t-1} + \varepsilon_t,$$

where t denotes time, α and β are parameters to be estimated, and ε_t is the residual (unexplained error) at time t. As described in the text, α represents the central tendency, and β captures the correlation of values over time. This type of equation can be estimated with standard regression techniques. Note that the derived residual (ε)—which represents implied random shocks—is used to estimate the variance for the random-shock process that feeds into the Monte Carlo simulation described later in this paper.

13. In the language of time-series econometrics, a process is described in terms of its AR and MA properties, with "AR" denoting how many lagged terms are included in the equation and "MA" denoting how long the moving average is for the error terms. The simplest equation is an AR(1), which has only one lagged term and no moving average. The most complicated process in the list of Social Security inputs is an ARMA(4,1), meaning there are four lagged terms and a single-period moving average of errors.

14. The test for random shocks is based on the Durbin-Watson statistic.

tendency exist. All systematic movement in the variable has been captured by the equation, and there is nothing left to explain.[15]

Unfortunately, it is sometimes difficult to tell whether the processes being modeled show random changes in central tendency. The tests used to decide whether derived residuals look like random shocks are not definitive, especially when the time series is short. Assessment of whether changes in central tendency have occurred can involve judgment.[16] If the possibility of a nonsystematic changing central tendency is admitted, the simplest approach is to compute the first difference for the variable in question—that is, to use an equation to describe the change in the variable, rather than the level of the variable.

Modeling change rather than level for a variable may seem like a trivial difference, but it has a profound effect on inferences about the bands of uncertainty around the variable. When change is modeled, any random shock permanently affects the level of the variable—the shock does not disappear by itself after one period, as in the usual specification. Of course, a random shock in the other direction pushes the level of the variable back in the other direction permanently. Thus, in first-differenced models, the level of a variable at any point in time is the result of cumulative shocks up to that point. Because all shocks are random, any accumulation in one direction pushes the level toward a new central tendency. That causes uncertainty bands to grow over time.

After analysts have produced mathematical equations for an input, they can generate probability distributions for annual outcomes. The simplest time-series models imply that annual values depend only on a constant, on the previous period's value (multiplied by a coefficient), and on an annual random shock. Coefficients are generated when historical data are used to estimate the time-series model. The extent to which an input varies around the value predicted by the equation indicates the correct size for annual random shocks. Thus, everything is in place to project future values using computer simulation.

For this analysis, two measures of uncertainty around the expected value for each input are examined. The first measure indicates the 10th and 90th percentiles of the values in each year. Those figures represent the annual variation in a given year. The second measure of uncertainty is the 10th and 90th percentiles for the average value over a specific period. For example, the average growth between 2004 and 2050 can be computed for each simulation, and a distribution of those averages can be computed. Not surprisingly, average values vary less than annual values.

Figures 3 through 18 illustrate uncertainty for the input assumptions and stochastic intermediate variables. Figures 3 through 18 include five lines:

■ The solid line in the center represents the historical actual and future expected values;

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15. A time-series econometrician would describe this as a *stationary series.* The standard test for stationarity is based on the augmented Dickey-Fuller test.

16. Fertility is an example of an input where such judgment is required. The historical data support two different methods for modeling fertility: one with only random changes, and the other with random changes in central tendency. For a more detailed discussion, see Congressional Budget Office, *Uncertainty in Social Security's Long-Term Finances.*

Table 2.

Stochastic Demographic Variables in CBO's Long-Term Social Security Projections

Variable	Description of Input and Time-Series Process
Fertility	• The mean number of children per woman is 2.5 over the historical period, with a standard deviation of 0.6 children. Since the end of the baby boom, the mean has been closer to two children per woman. • ARMA (4,1). The time-series equation includes four lagged values and one moving average term. The series is estimated using data from 1917 to 2002.
Mortality Improvement	• Historically, much volatility has occurred in the overall rate of mortality improvement, averaging at -1 percent, with a standard deviation of 1.5 percent. • There are separate AR(1) equations for 21 age groups and two sex groups, estimated using data from 1900 through 2000. Model draws are correlated across the 42 resulting groups using a multivariate normal distribution.
Immigration	• Total immigration is a very volatile series with some large spikes in the past 15 years. Since 1901, the average number of immigrants per year has been just under 500,000, but the standard deviation has been almost as large. • ARMA(4,1) equation estimated for total immigration using data from 1901 through 2002.
Disability Incidence	• Average disability insurance (DI) incidence from 1975 to 1998 has been just under 5 percent. • AR(1) model for the overall DI incidence rate estimated using data for 1975 through 1998.
Disability Termination	• DI retention rates excluding conversions to Old-Age and Survivors Insurance (OASI) have averaged 93 percent, with a standard deviation of 1.5 percent. • AR(1) model for the DI termination rate, excluding conversion to OASI, estimated using data for 1975 though 1998.

Source: Congressional Budget Office.

■ The outer bands to either side of the expected value show the 10th and 90th percentiles of annual values for the 500 paths generated by the Monte Carlo simulations (suggesting that the outcome in any given year will fall between those bands 80 percent of the time); and

■ The dark shaded bands show the 10th and 90th percentiles of the average values (from 2004 through the year in question).

Details for each time-series equation can be found in the appendix.

Figure 3.

Uncertainty Bands for the Overall Rate of Fertility

(Children per woman)
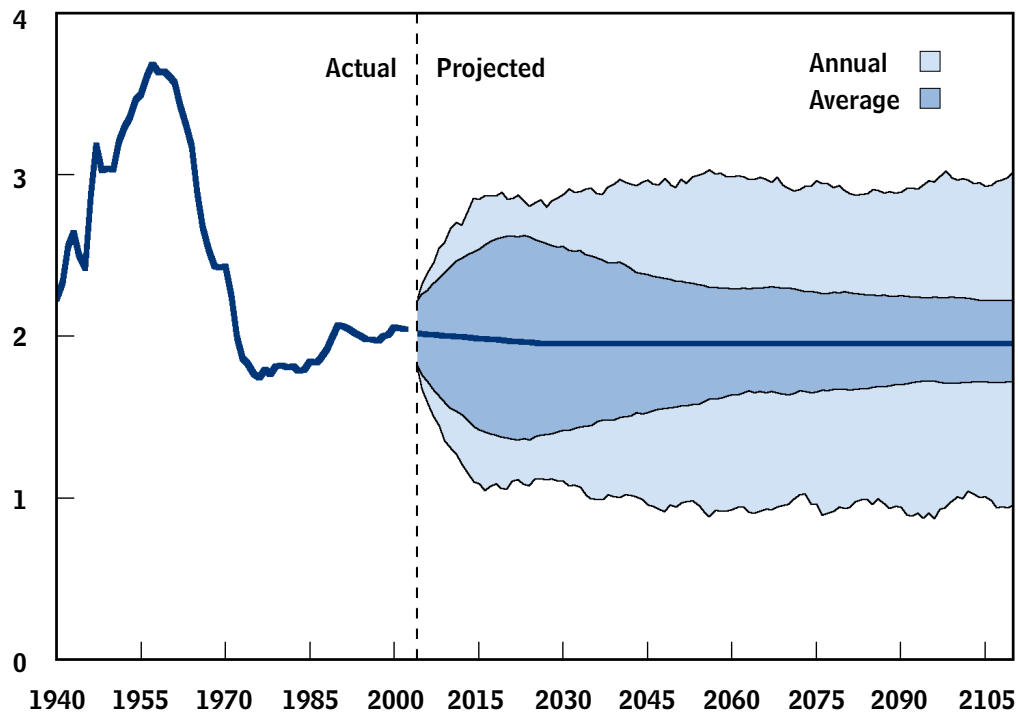
Source: Congressional Budget Office.

Note: Annual uncertainty bands show the 80 percent confidence range for a given year. Average uncertainty bands show the 80 percent confidence range for the average of 2004 through a given year.

Demographic Input Assumptions in CBOLT

The two basic stochastic demographic inputs in CBOLT projections are fertility and mortality improvement. There are also two sets of variables that have an underlying demographic component but are also influenced by policy: immigration and disability. The time-series properties of those stochastic inputs are all very different, and thus the types of models used differ widely (see Table 2).

Fertility. Fertility is modeled using a standard time-series approach that leads to stable error bands (see Figure 3).[17] The estimated equation involves four lags for past fertility rates and a correlated error (moving average) term (see the appendix for details). As suggested, the model explains the baby boom as a combination of annual shocks and highly correlated outcomes. The range for fertility (roughly 1.0 to 3.0 children per woman) contains most of the data points associated with the baby boom and the subsequent baby "bust."

17. Fertility is naturally bounded from below (the rate cannot drop below zero); however, using a bounding transformation requires setting limits in both directions. The uncertainty bands in Figure 3 are based on an arbitrary upper limit of 4.0 children per woman.

Figure 4.

Uncertainty Bands for the Overall Rate of Mortality Improvement

(Percentage change in the mortality rate)
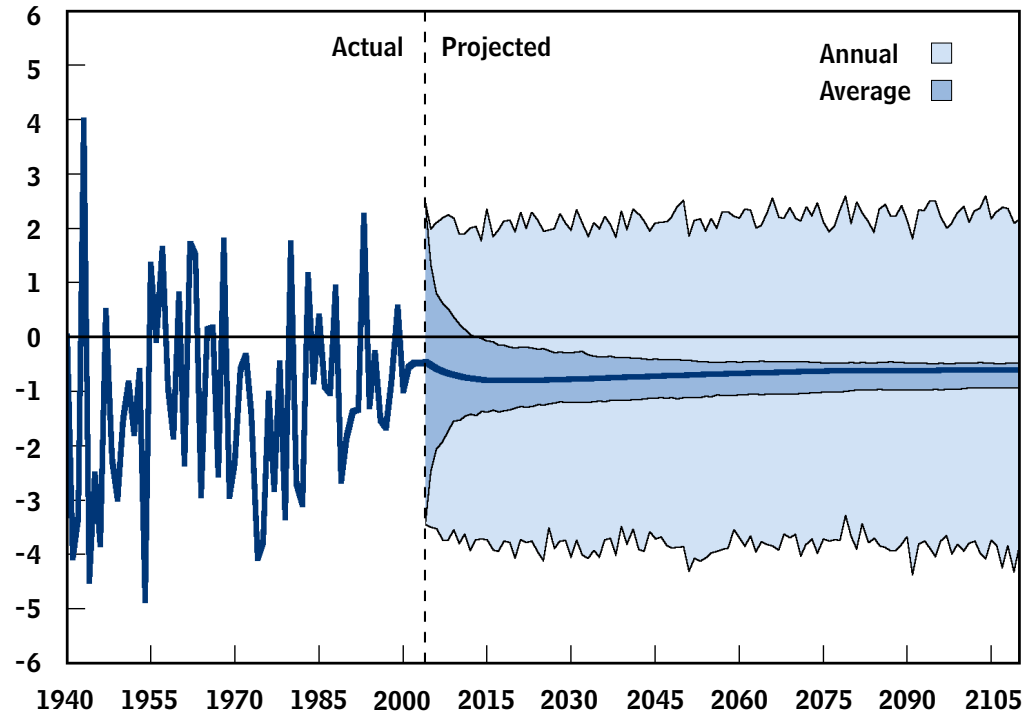
Source: Congressional Budget Office.

Note: Annual uncertainty bands show the 80 percent confidence range for a given year. Average uncertainty bands show the 80 percent confidence range for the average of 2004 through a given year.

Mortality Improvement. CBO continues to use SSA projections of mortality improvement. For both men and women, SSA projects rates of mortality improvement in 21 age groups. Historical data suggest that the rates of improvement for each sex are somewhat correlated between age groups but that differences in central tendency exist within age-and-sex groups and should be accounted for. Thus, separate time-series equations are estimated for mortality improvements for each sex, but the equations are estimated such that correlations in annual random shocks can be accommodated.[18]

As with the overall average rates of mortality improvement, which can be aggregated over age and sex to generate a graph of how mortality is expected to change, uncertainty bands can also be aggregated and graphed (see Figure 4). The range of annual outcomes around the expected rate of improvement is consistent with significant historical variation. As expected, the range of average values is much smaller.

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18. The basic concept is that each variable in the system of equations is unaffected by other variables but that the error terms are potentially correlated between the equations. Correlations between errors are measured after every equation in the system has been estimated.

Figure 5.

Uncertainty Bands for the Level of Immigration

(Thousands of people)
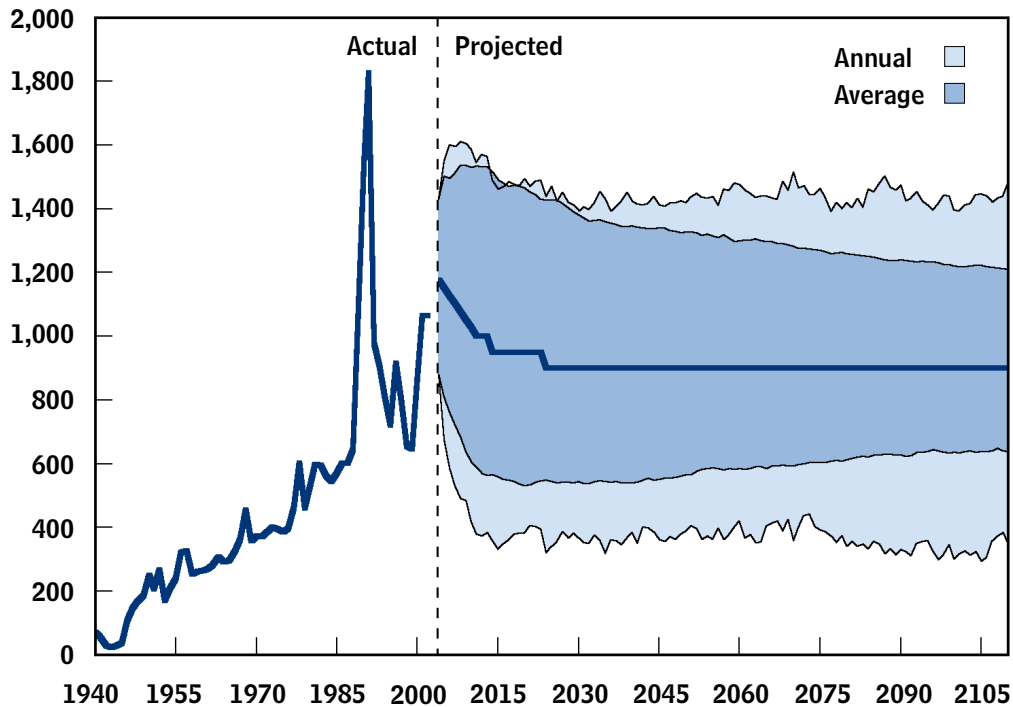
Source: Congressional Budget Office.

Note: Annual uncertainty bands show the 80 percent confidence range for a given year. Average uncertainty bands show the 80 percent confidence range for the average of 2004 through a given year.

As noted earlier, although rates of mortality improvement pass the test for nonrandomness in central tendency, the level of mortality can be thought of as a variable with a random central tendency. In other words, the input assumption being modeled is already the first-differenced version of a variable with expanding uncertainty ranges. Thus, although the uncertainty bands for the rate of mortality improvement are constant over time, a graph of the bands for central death rates would show increasing uncertainty.

Immigration and Disability Incidence and Termination. Annual levels of legal immigration, rates at which people join the Disability Insurance (DI) program, and rates at which people leave the DI program are set in law or are influenced strongly by changes in policy. Historical data for those variables show clear indications of changing central tendencies. However, is it appropriate to think of those changes as random when they are determined to some extent by changes in policy? The answer to that question determines which specification is appropriate for the three variables. The approach used here is to model the processes without random changes in central tendency so that the variation over time is attributed only to random shocks and correlation.

Applying the standard time-series approach to these variables produces significant uncertainty bands (see Figures 5, 6, and 7). The equation for immigration is somewhat more complicated

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Figure 6.

Uncertainty Bands for the Rate of Disability Incidence

(Percent)

Source: Congressional Budget Office.

Notes: Data on rates of disability incidence are only available through 1998.

Annual uncertainty bands show the 80 percent confidence range for a given year. Average uncertainty bands show the 80 percent confidence range for the average of 2004 through a given year.
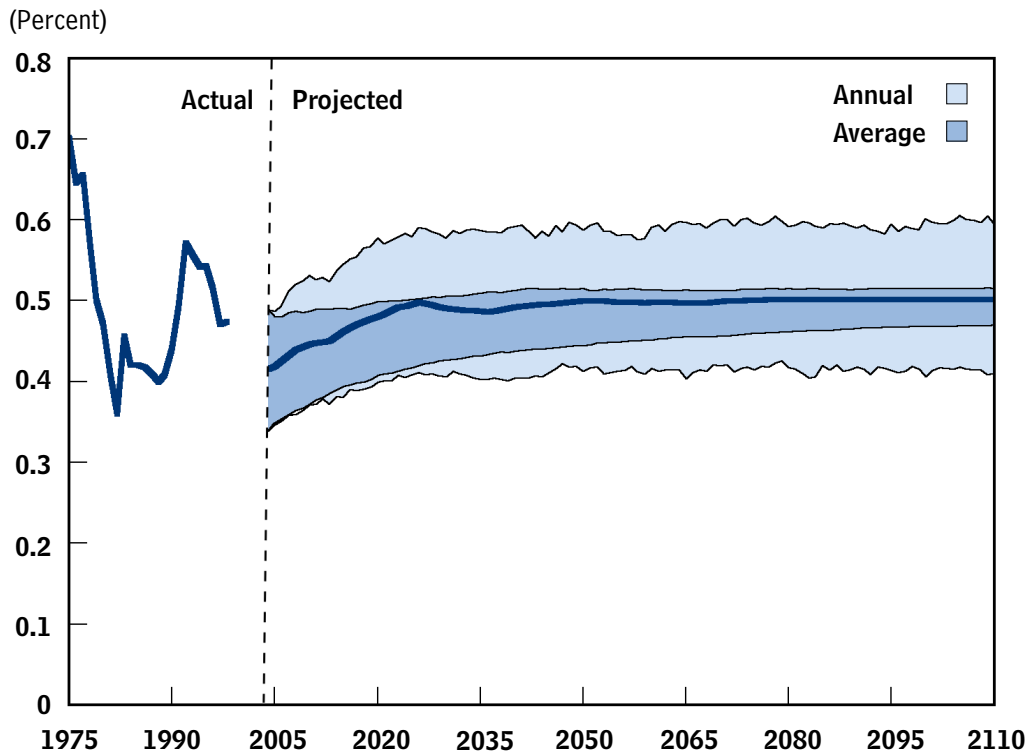
than the standard (one-lag) time-series model because a clear trend in the level of immigration is apparent over time. The wide error bands for both the annual and average values for immigration are consistent with the large autocorrelation, which magnifies shocks over time.

Measuring uncertainty bands for disability incidence and termination is more difficult because of the limited data.[19] Both equations fail the test for a stable time-series variable; however, because their failure is driven by known policy changes and a short data series, it is ignored for this analysis in order to generate fixed error bands. The bands are quite large. Note that these figures look quite different from some other figures because the deterministic value varies over the forecast period. Also note that both rates are naturally restricted to be between zero and one, and so these variables are estimated using a bounding transformation.

Economic Input Assumptions in CBOLT

There are two sorts of economic inputs used in CBOLT projections: the first are modeled independently, and the second are estimated as a system of equations to take advantage of correlations between the variables (see Table 3). Total factor productivity growth, the earnings

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19. No new data are available since CBO's 2001 report, *Uncertainty in Social Security's Long-Term Finances*.

Figure 7.

Uncertainty Bands for the Rate of Disability Termination

(Percent)
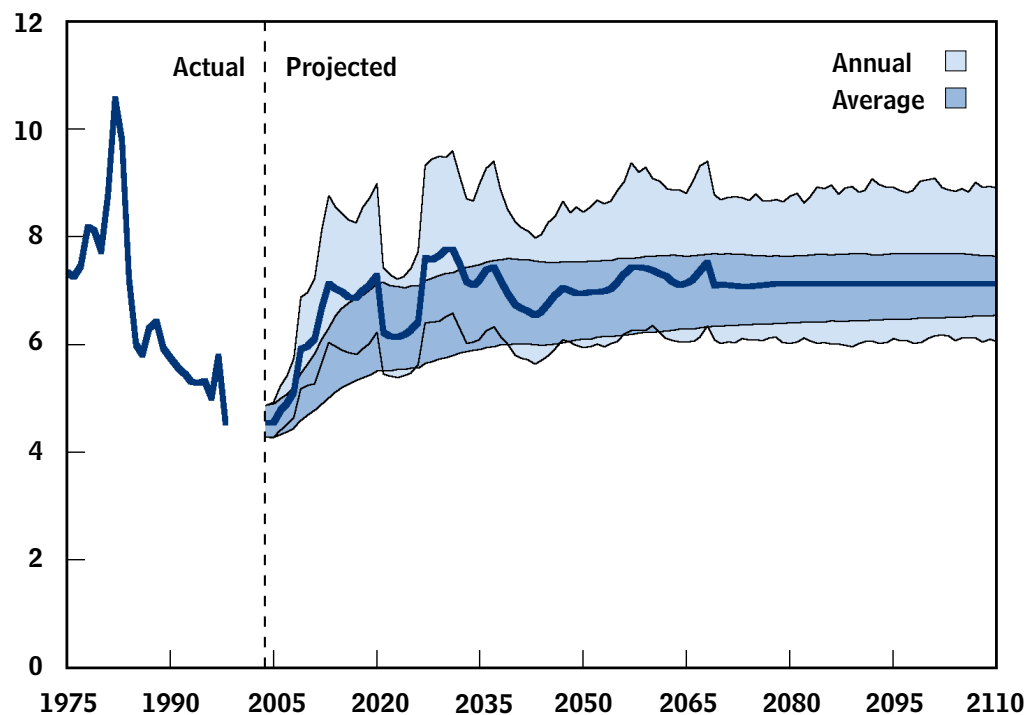
Source: Congressional Budget Office.

Note: Annual uncertainty bands show the 80 percent confidence range for a given year. Average uncertainty
bands show the 80 percent confidence range for the average of 2004 through a given year.

share of compensation, and the gap between the CPI-W and core price index fall into the first
category: the unemployment rate, inflation rate, and real interest rate gap fall into the second.

Total Factor Productivity Growth. Total factor productivity growth is modeled as a white
noise process. In the long run, total factor productivity is assumed to grow at 1.25 percent per
year, abstracting from any random variation. The time-series technique used to model this
growth results in substantial variation around that value (see Figure 8). The 80 percent uncer-
tainty bands for the projection of total factor productivity growth cover a range of more than
2 percentage points in either direction. The uncertainty bands for the average values are con-
siderably narrower.

Earnings Share of Compensation. The earnings share of compensation is not modeled di-
rectly for purposes of estimating the time-series equation. Total compensation is represented
by three general categories: cash earnings; employer-paid payroll taxes; and health, pension,
and other benefits. It is the growth in the share of compensation represented by the last cate-
gory that is modeled. The employer's share of payroll taxes is dictated by tax policy and so
does not vary randomly over time. The growth in the share of nonwage and nontax compen-
sation can vary randomly over time, and it is modeled as such. The earnings share of compen-
sation is assumed to be the residual after accounting for the other two categories.

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Table 3.

Stochastic Economic Variables in CBO's Long-Term Social Security Projections

Variable	Description of Input and Time‑Series Process
Total Factor Productivity Growth	• Total factor productivity (TFP) growth has averaged 1.5 percent since 1950, with a standard deviation of 2.5 percent. • TFP growth is modeled as a white noise process.
Earnings Share of Compensation	• Since 1950, the earnings share of compensation has averaged 87 percent. Its growth has averaged -0.3 percent since 1950, and the growth of the nonwage and nontax share of compensation has grown at 3 percent over that period. In recent years, growth of this share has slowed to less than 1 percent per year. • The growth of the nonwage and nontax share of compensation is estimated using an AR(1) process. Data from 1950 through 2003 are used.
Unemployment Rate	• The unemployment rate has averaged 5.8 percent since 1954, with a standard deviation of 1.4 percent. • VAR model with two lags each for unemployment, inflation, and the real interest rate gap. The model is estimated using data from 1954 through 2003.
Inflation Rate	• The inflation rate has averaged 3.9 percent since 1954, with a standard deviation of 3.0 percent. • VAR model with two lags each for unemployment, inflation, and the real interest rate gap. The model is estimated using data from 1954 through 2003.
Real Interest Rate Gap	• The gap between the average product of capital and the 10-year interest rate has averaged 7.4 percent since 1954, with a standard deviation of 2.5 percent. • VAR model with two lags each for unemployment, inflation, and the real interest rate gap. The model is estimated using data from 1954 through 2003.
Gap Between CPI-W and Core Price Index	• Data begin in 1969. A consistent series is not available prior to that date. From 1969 through 2003, the gap averaged -0.40. • The gap between the CPI-W and core price index is measured as an AR(1) process.

Source: Congressional Budget Office.

Note: VAR = vector autoregression.

Figure 8.

Uncertainty Bands for the Rate of Total Factor Productivity Growth
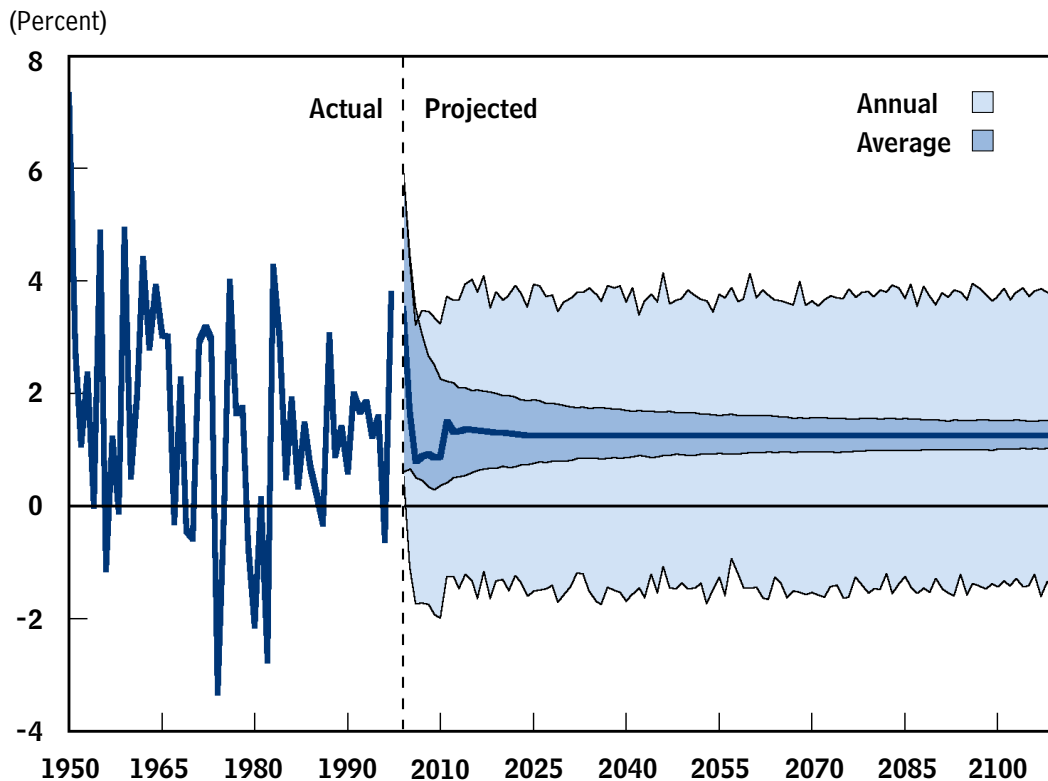
(Percent)

Source: Congressional Budget Office.

Note: Annual uncertainty bands show the 80 percent confidence range for a given year. Average uncertainty bands show the 80 percent confidence range for the average of 2004 through a given year.

The equation used to generate the paths for growth in the earnings share of compensation is the most basic time-series specification. The rate of growth in the nonwage and nontax share of compensation is regressed on a constant only; that is, it is modeled as white noise.

In the absence of random variation, the growth rate in the earnings share of compensation is assumed to be 0.5 percent per year. That allows the share to decline at about 0.1 percent per year, which is consistent with recent history. The uncertainty bands for the earnings share of compensation look quite different than those for other variables (see Figure 9). The uncertainty bands are not symmetric around the expected value. There is an upper bound for earnings as a share of compensation. The share of compensation accounted for by benefits can shrink but, in the absence of a change in law, the share accounted for by payroll taxes cannot. The earnings share of compensation is assumed to be bounded on the lower side and cannot be less than 30 percent of total compensation.[20]

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20. It is assumed that a worker would not accept less than 30 percent of his or her compensation as cash wages.

Figure 9.

Uncertainty Bands for the Earnings Share of Compensation

(Percentage of total compensation)
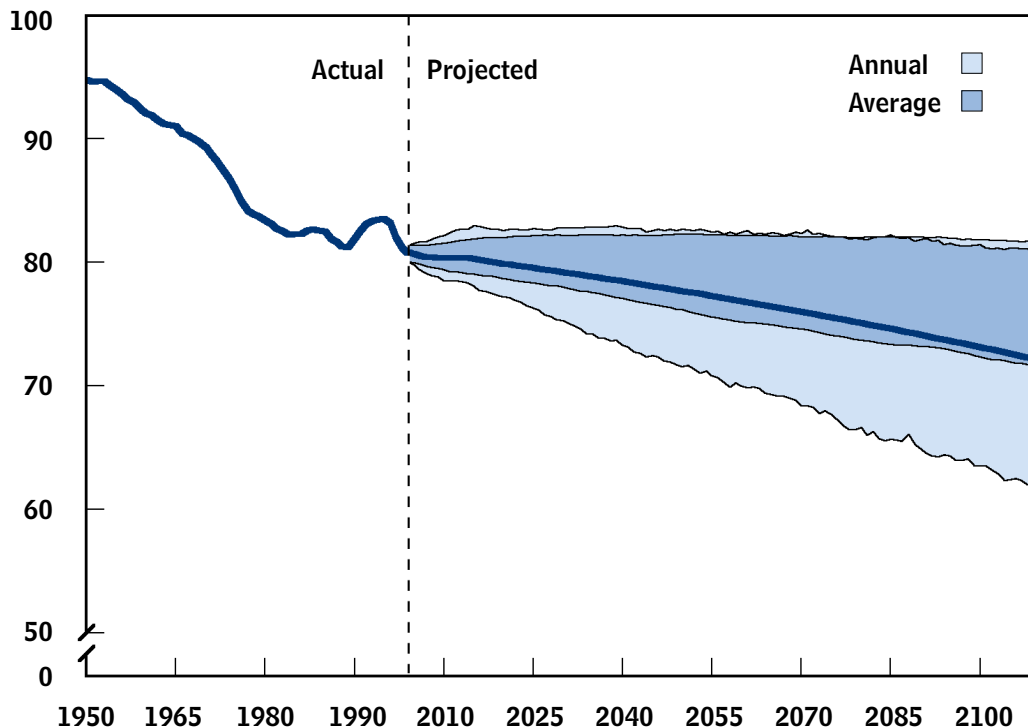
Source: Congressional Budget Office.

Note: Annual uncertainty bands show the 80 percent confidence range for a given year. Average uncertainty bands show the 80 percent confidence range for the average of 2004 through a given year.

Unemployment, Inflation, and the Real Interest Rate Gap. To understand why unemployment, inflation, and interest rates are estimated as a system of equations rather than independently, consider the effect of modeling independent systems. When running model simulations, independent time-series equations for each variable would adequately generate historical variability and correlation between annual outcomes for that variable. However, the fact that outcomes across the three variables are correlated would not be captured. Thus, the technique could violate a basic condition of model simulations and generate combinations of outcomes that have not been measured historically and are not likely to occur.

The technique used to model the three variables simultaneously builds directly on the basic time-series approach. However, rather than regressing a variable only on its own lagged value, each equation includes lagged values for all variables under consideration.[21] Thus, for example, the equation for unemployment includes lagged values for unemployment, the real interest rate gap, and inflation over the prior two years. (See the appendix for details.) Correlations of each variable with its own lagged values are generally positive. The effect of each variable on the other two varies over time but generally reflects well-known properties.

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21. The technique is known as vector autoregression (VAR).

Figure 10.

Uncertainty Bands for the Unemployment Rate

(Percent)

Source: Congressional Budget Office.

Note: Annual uncertainty bands show the 80 percent confidence range for a given year. Average uncertainty bands show the 80 percent confidence range for the average of 2004 through a given year.
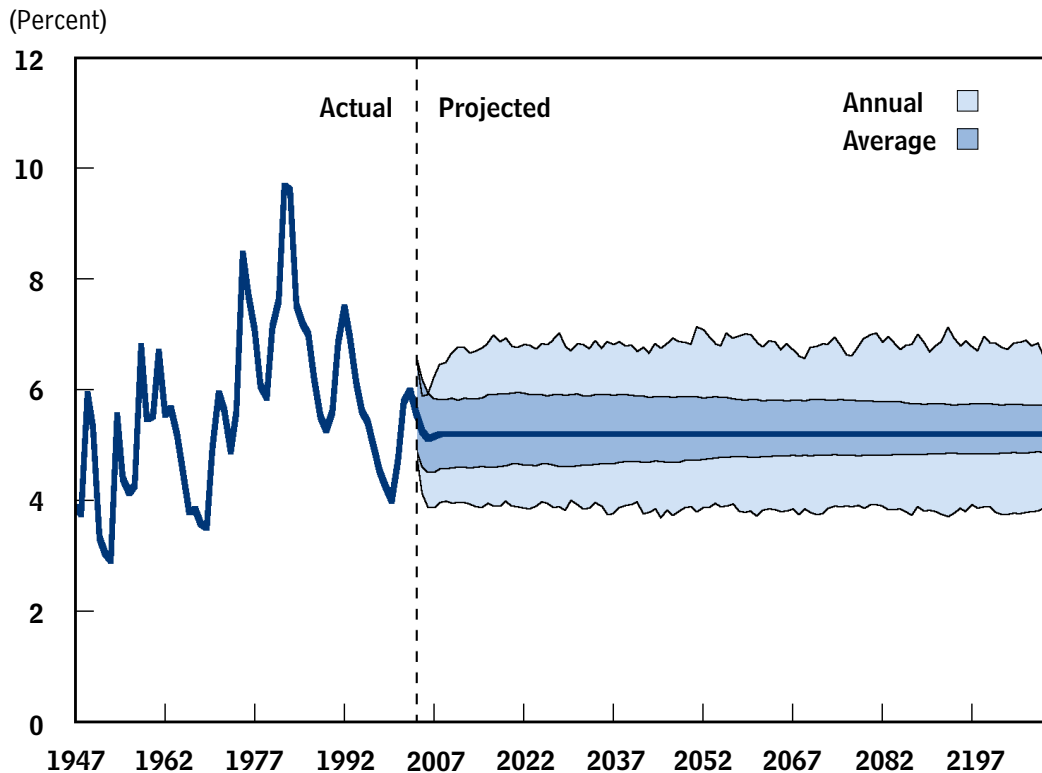
The uncertainty bands for annual values of unemployment, inflation, and the real interest rate gap are again much larger than the uncertainty bands for average values for those variables (see Figures 10, 11, and 12). For annual values, the range for the unemployment rate is 2 or 3 percentage points in each direction, the range for inflation is about 4 percentage points in each direction, and the range for the real interest rate gap is about 3 percentage points in each direction.

The unemployment rate is another example of a *bounded* input—an input that is naturally restricted to a certain range. If the unemployment rate were modeled simply as a level variable, random shocks could be chosen that would lead to negative unemployment—which is, of course, impossible. Thus, estimated equation coefficients (see the appendix) are based on a transformed version of the unemployment rate that is restricted to between zero and one.[22]

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22. The transformation involves taking the log-odds ratio; if u is the unemployment rate, the variable being modeled is $x = \log(u/(1-u))$. No matter what the shocks to x, the outcome of u is bounded between zero and one.

Figure 11.

Uncertainty Bands for the Inflation Rate

(Percent)

Source: Congressional Budget Office.

Note: Annual uncertainty bands show the 80 percent confidence range for a given year. Average uncertainty bands show the 80 percent confidence range for the average of 2004 through a given year.
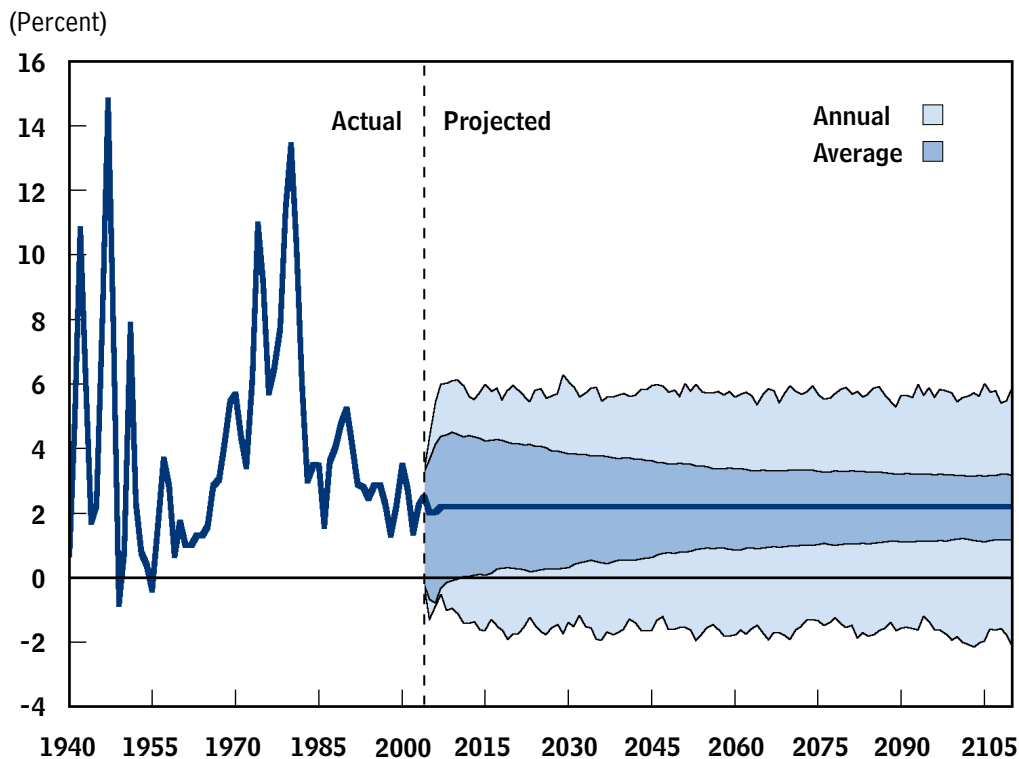
Gap Between Core GDP Deflator and CPI-W Growth. Inflation as measured by the consumer price index for urban wage earners and clerical workers differs from inflation as measured by the change in the GDP deflator. Because the CPI-W is based on a fixed basket of goods and the GDP deflator, by definition, allows substitution, there is a gap between the inflation rates by each measure. CBOLT models a related gap, the gap between the CPI-W and the core GDP deflator,[23] using standard time-series techniques that lead to stable error bands (see Figure 13). The estimated equation involves a single lag of the gap.

Intermediate Stochastic Variables in CBOLT

An additional category of stochastic variables in CBOLT are not inputs themselves but are used in many calculations in the model and, in many cases, are derived from stochastic inputs. Real GDP growth, for example, is affected by growth in total factor productivity. Those variables include real GDP growth, real wage growth, and the real 10-year interest rate (see Table 4 on page 24).[24] Stochastic asset returns include the return on corporate bonds and the real return on equities (see Table 5 on page 25).

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23. The core GDP deflator is a measure of the price level for all output other than for computer equipment, which requires special treatment because prices are falling quickly as the share of output increases quickly.

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Figure 12.

Uncertainty Bands for the Real Interest Rate Gap

(Percent)
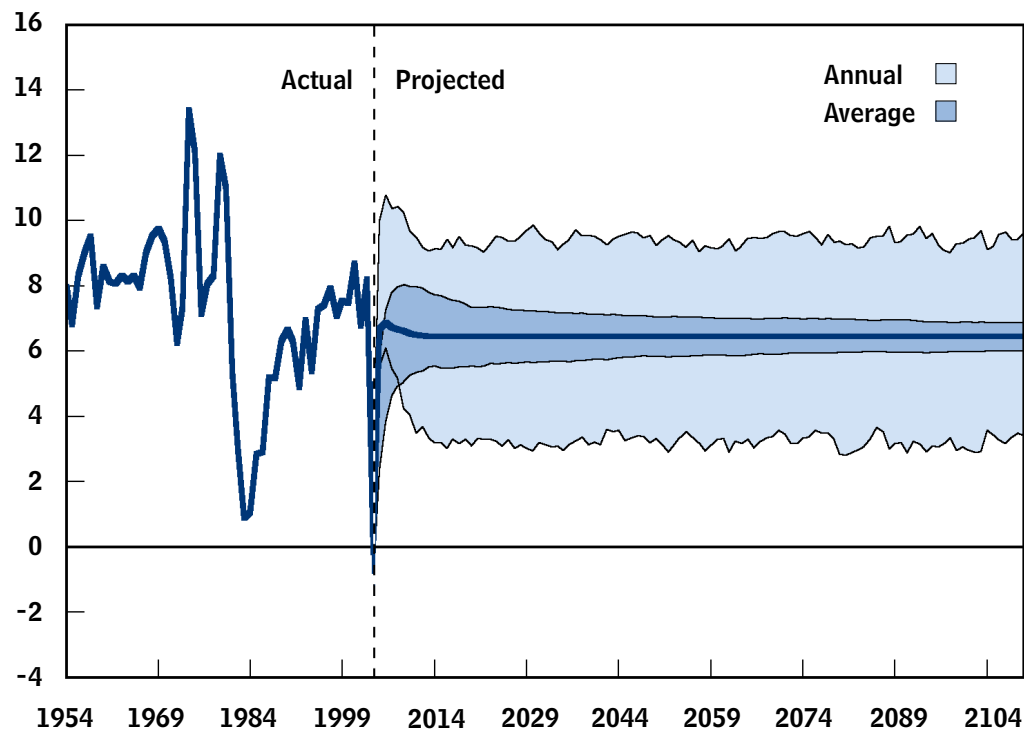
Source: Congressional Budget Office.

Note: Annual uncertainty bands show the 80 percent confidence range for a given year. Average uncertainty bands show the 80 percent confidence range for the average of 2004 through a given year.

Real GDP Growth. Growth in real GDP is determined by the capital stock, labor supply, and growth in total factor productivity and is endogenous to the growth model. Growth in total factor productivity is a stochastic input. The capital stock and labor supply are endogenous and so depend on other inputs, both stochastic and not.

The uncertainty bands vary about 3 percentage points in either direction around an expected value of just under 2 percent in the long run (see Figure 14 on page 26). That range of annual values encompasses most of the historical variation. As expected, the average uncertainty bands are much narrower, varying slightly more above the expected value than below.

Real Wage Growth. Real wage growth is also endogenously determined. The rate of real wage growth is the result of total earnings divided by hours worked, adjusted for earnings as a share of compensation, growth in average hours, and the gap between GDP and inflation. The 80 percent uncertainty bands are relatively stable around the expected value, varying about 3 percentage points on either side of that expected value (see Figure 15 on page 27).

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24. Note that in the earlier version of the model described in Congressional Budget Office, *Uncertainty in Social Security's Long-Term Finances,* real wage growth was a stochastic input into the model. In the growth model, income grows with output, so real wage growth is derived endogenously.

Figure 13.

Uncertainty Bands for the Gap Between the Core GDP Deflator and CPI-W Growth

(Percent)
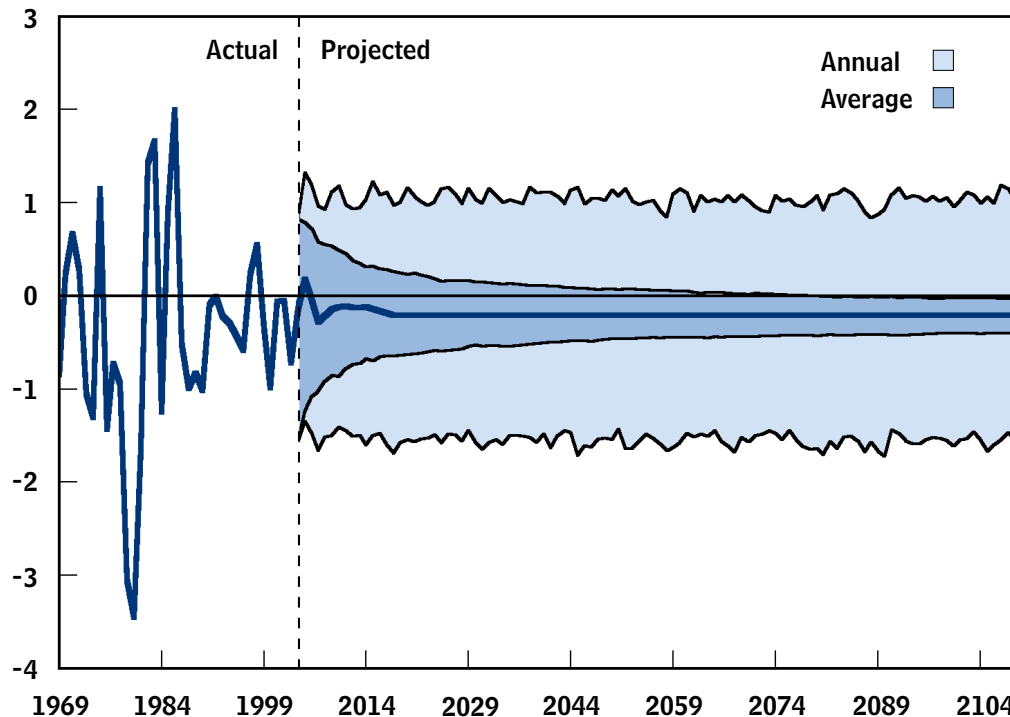
Source: Congressional Budget Office.

Notes: Annual uncertainty bands show the 80 percent confidence range for a given year. Average uncertainty bands show the 80 percent confidence range for the average of 2004 through a given year.

CPI-W = consumer price index for urban wage earners and clerical workers.

Real 10-Year Interest Rate. The return on Treasury securities varies with the rate of return on capital, which is calculated within the growth-model framework, and a gap between the average return to capital and the rate of return on 10-year Treasury securities. It is this gap that is modeled and accounts for the stochastic variation in returns for Treasury securities.

The gap is modeled with unemployment and inflation as part of an economic vector autoregression (VAR) model such that each variable is a function of its own previous values as well as the previous values of the other two variables. The real interest rate gap is described above.

The uncertainty bands vary by about 3 percentage points in each direction around the expected value (see Figure 16 on page 28). This range encompasses most of the historical variation in real interest rates. Again, the uncertainty bands for the average values are much narrower.

Table 4.

Intermediate Stochastic Variables in CBO's Long-Term Social Security Projections

Variable	Description of Input and Time-Series Process
Real GDP Growth	• Real GDP growth has averaged 3.4 percent since 1951, with a standard deviation of 2.3 percent. • No direct time-series process
Real Wage Growth	• Real wage growth has averaged just over 1 percent since 1949, with a standard deviation of almost 2 percent. • No direct time-series process
Real 10-Year Interest Rate	• Since 1954, the real 10-year interest rate has averaged 2.8 percent with a standard deviation of 2.4 percent. • No direct time-series process

Source: Congressional Budget Office.

Stochastic Asset Returns in CBOLT

Adding individual accounts to Social Security introduces the need to model investment returns. Investment in both corporate bonds and equities, rather than just in Treasury bonds, could raise expected benefits, but those increased benefits have an associated cost. The higher expected returns (compared with those from Treasury bonds) also have increased risk. In a deterministic simulation, all assets are assumed to have a market value outcome. In a stochastic simulation, the expected returns differ across assets, as does the associated variance. That is, the price paid for the higher expected return is a higher variance in those returns. Asset returns are estimated using time-series models that allow future projections to be based on historical variation. Equity returns are estimated using a pure random-returns model, with a fixed mean and fixed variance. Corporate bond returns are estimated using a more complicated framework.

Real Return on Equities. Equity returns are estimated using a random returns (white noise) process.[25] Data for modeling equity returns are from Ibbotson Associates (2004) and are available back to 1926.[26] The time period used to measure the average and variance of yields matters. The default is to include 1955 through 2003, but options exist within CBOLT to use all available data or to use data from 1926 through 1990 or from 1926 through 1994. The time periods ending in the 1990s exclude the stock market bubble of that decade. The time period beginning with 1955 follows the Depression and war years. The variability of yields has declined significantly since 1954.

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25. CBOLT allows the user to select a mean-reversion process for estimating the equity rate of return, but the default is the white noise process. For a discussion of which is the more appropriate model, see Joel V. Smith and John Sabelhaus, *Alternative Methods for Projecting Equity Returns: Implications for Evaluating Social Security Reform Proposals,* CBO Technical Paper 2003-8 (May 2003).

26. Ibbotson Associates, *2004 Yearbook: Market Results for 1926-2003* (Chicago, Ill.: Ibbotson Associates, 2004).

Table 5.

Stochastic Asset Returns in CBO's Long-Term Social Security Projections

Variable	Description of Input and Time-Series Process
Real Return on Equities	• Since 1955, the average real return on equities has been 6.8 percent, with a standard deviation of 20 percent.
	• Data are for Ibbotson large-cap returns from 1954 though 2003.
Real Return on Corporate Bonds	• The average of real corporate bond returns from 1954 through 2003 is 3.8 percent, with a standard deviation of 7.8 percent.
	• The level of bond returns, estimated over the 1954-2003 period, is a function of inflation, unemployment, and the interest rate gap.

Source: Congressional Budget Office.

The standard deviation of stock yields from 1955 through 2003 is close to 20 percent. In the simulations involving equity investment, the model assumes a white-noise process with expected real returns of 6.8 percent. The equity return rate for simulations using a white-noise process is based on a time-series equation in which the only dependent variable is an intercept that is equal to the mean for the series.

A white-noise process results in annual uncertainty bands that remain wide throughout the forecast period (see Figure 17 on page 30). The average uncertainty bands approach the expected value, but they approach it asymmetrically.[27]

Return on Corporate Bonds. Real corporate bond yields are integrated into the macroeconomic model, and the equation for corporate bond returns uses the variables in the VAR model described above: unemployment, inflation, and the real interest rate gap. The equation captures the variability in corporate bond returns and the short-run macrodynamics in the rest of the model. Data on corporate bond yields are from Ibbotson Associates and date back to 1926,[28] but because other data used in the equation are available in a consistent format only back to 1954, the equation is estimated using data from 1954 forward.

The resulting uncertainty bands for the annual values are quite wide, varying about 15 percentage points in either direction (see Figure 18 on page 31). The uncertainty bands do not appear to widen as time passes.

Uncertainty About Social Security's Long-Term Finances

Several different statistics can be used to characterize the long-term outlook for Social Security's finances. CBO's June 2004 *Outlook for Social Security* focused on Social Security outlays

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27. The asymmetry in the graphs relates to the way the percentiles are selected from the distribution. The value is selected from the bottom of the percentile group, so the value for the first percentile is equal to the lowest value in the distribution. For certain variables with long tails to their distribution, the 10th and 90th percentiles may appear to be asymmetric to the expected value because of the way the values at those percentiles are selected.

28. Ibbotson Associates, *2004 Yearbook.*

Figure 14.

Uncertainty Bands for Real GDP Growth
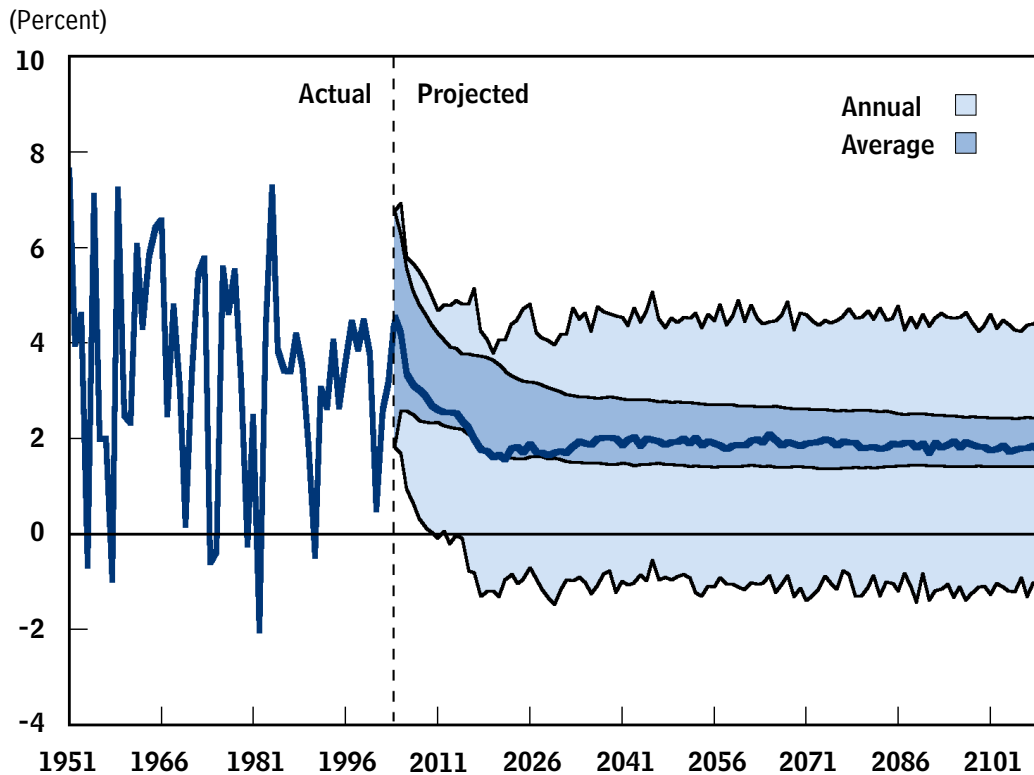
(Percent)

Source: Congressional Budget Office.

Note: Annual uncertainty bands show the 80 percent confidence range for a given year. Average uncertainty bands show the 80 percent confidence range for the average of 2004 through a given year.

and revenues, each expressed as a percentage of GDP, projected forward 100 years.[29] CBO also reported other measures of system finances, including present-value shortfalls over 50- and 100-year horizons and the expected date for exhaustion of the Social Security Trust Fund. Any projection for long-run system finances is uncertain because the input assumptions used to make those projections are uncertain. The Monte Carlo technique is a direct way to estimate that uncertainty.

Base-Case Estimates of Uncertainty About System Finances

In 2004, Social Security revenues are expected to be above Social Security outlays. Not surprisingly, the confidence range (that is, the range of the uncertainty bands) is quite narrow. Projected revenues as a percentage of GDP decrease through 2100, and outlays as a percentage of GDP increase through 2100. The sharpest increase in outlays is between 2004 and 2025, as the baby-boom generation begins to retire (see Table 7 on page 33). The expansion of the confidence range is best illustrated graphically (see Figure 19 on page 33).

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29. Congressional Budget Office, *The Outlook for Social Security.*

Figure 15.

Uncertainty Bands for Real Wage Growth
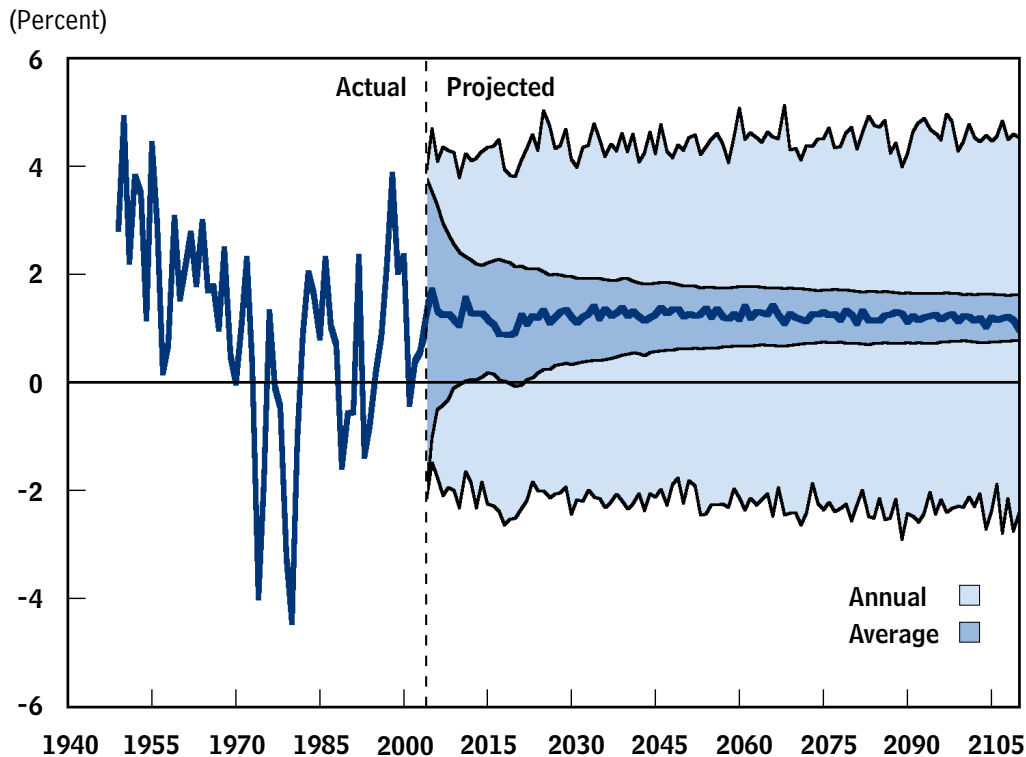
(Percent)

Source: Congressional Budget Office.

Note: Annual uncertainty bands show the 80 percent confidence range for a given year. Average uncertainty bands show the 80 percent confidence range for the average of 2004 through a given year.

The solid lines in Figure 19 show CBO's projection of expected outlays and revenues as a percentage of GDP. The cones around each of those lines show the confidence range, which is by definition bounded by the 10th and 90th percentiles for each outcome. The confidence range for outlays as a percentage of GDP is fairly symmetric around the expected value. By 2100, the confidence range for outlays spans more than 4.5 percent of GDP, ranging from 5.3 percent to 9.8 percent of GDP. Note that the 10th percentile for outlays dips below the expected value for revenues as a percentage of GDP in the latter part of the projection.

The confidence range for revenues as a percentage of GDP is narrower, but it is not symmetric. This asymmetry relates to constraints placed upon earnings as a share of compensation.

Currently, earnings account for more than 80 percent of compensation, but they can fall to as low as 30 percent in any given simulation, to the extent that the employer's share of pension and other benefit contributions increases. If earnings as a share of total compensation fall, there is less payroll to tax, so revenues to the Social Security Trust Fund must decline. Similarly, if earnings represent a relatively small fraction of total payroll, outlays must also be low because benefits are based on workers' earnings histories. By 2100, the confidence range for revenues is 4.3 percent to 5.4 percent, a difference of 1.1 percent of GDP.

27

Figure 16.

Uncertainty Bands for the Real Ten-Year Interest Rate

(Percent)
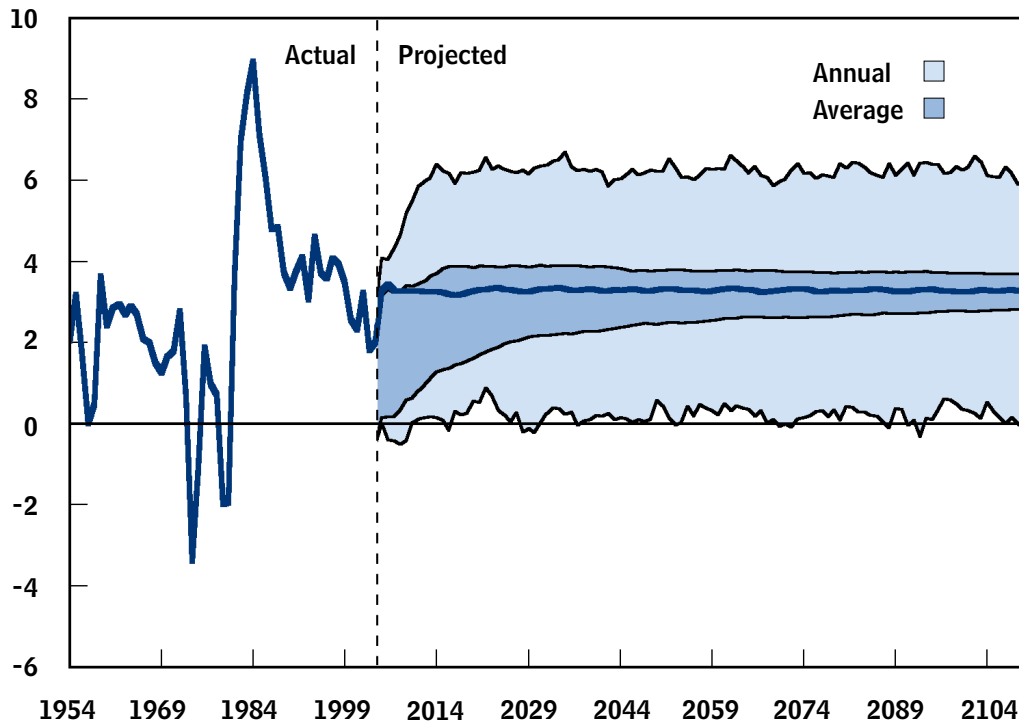
Source: Congressional Budget Office.

Note: Annual uncertainty bands show the 80 percent confidence range for a given year. Average uncertainty bands show the 80 percent confidence range for the average of 2004 through a given year.

The annual balance is another common way to look at Social Security's finances (see Table 6 on page 32). The expected annual balance first becomes negative in 2020 and remains negative thereafter. The 10th and 90th percentiles become negative in 2013 and 2026, respectively. Note that the confidence range for the annual balance is not the same as the difference between the 10th percentile for revenues and the 90th percentile for outlays. The confidence range for the annual balance is narrower than the confidence range described above. It is necessarily narrower because of the correlation of earnings as a share of compensation with both revenues and outlays: when the earnings share is low, revenues in the form of payroll taxes and benefits, which are based on earnings histories, must also be low. The confidence range is more than 5.5 percent.

It is also possible to look at summarized rather than annual statistics (see Table 7 on page 33). Summarized outlays and revenues are the present value of annual outlays and revenues over the relevant period divided by the present value of GDP over that same period. The balance is the present value of revenues minus the present value of outlays, all divided by the present value of GDP over that period. CBO projects respective summarized outlays and revenues at 5.8 percent and 5.3 percent of GDP, resulting in a summarized deficit of 0.6 percent of GDP.

The uncertainty for the balance summarized over 100 years ranges from -1.2 percent of GDP to -0.2 percent of GDP.

Sources of Variability in System Finances

The Monte Carlo simulation technique can be applied to any combination of the 11 input assumptions, including one input at a time. This exercise indicates how much of the overall variability is attributable to each input. Breaking down uncertainty by source shows whether the specification of any given time-series equation significantly affects the results. That process is also helpful in considering policy changes that are designed to lessen the financial risk in the Social Security system.

Holding all other variables fixed at their expected values, the rates of fertility and total factor productivity growth cause the most variation in the system's annual balance and projected outlays as a percentage of GDP in 2100 (see Table 8 on page 34). This is consistent with findings in CBO's 2001 report, where the most variation was attributable to fertility and the rate of real wage growth.[30] Outlays as a percentage of GDP are also affected most by fertility and TFP growth.

The growth in the share of nonwage, nontax compensation causes the most variation in projected revenue as a percentage of GDP in both 2025 and 2100. The other stochastic inputs cause relatively little variation in revenues as a percentage of GDP. As described above, the growth in the share of nonwage, nontax compensation has the potential to significantly affect the earnings share of compensation and, to the extent that the earnings share of compensation falls from its current level of more than 80 percent to the assumed floor of 30 percent, revenues from payroll taxes will fall. Similarly, to the extent that the earnings share of compensation increases, revenues from payroll taxes will increase. This distribution is not symmetric, however, because the upper limit to the earnings share of compensation is closer to historical averages than the lower limit is.

The effect of uncertainty about any variable increases dramatically looking forward in time. In 2025, fertility produces much less variation in the ratio to GDP than do variables that by 2100 produce relatively little variation.

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30. Congressional Budget Office, *Uncertainty in Social Security's Long-Term Finances.*

Figure 17.

Uncertainty Bands for the Real Return on Equities

(Percent)
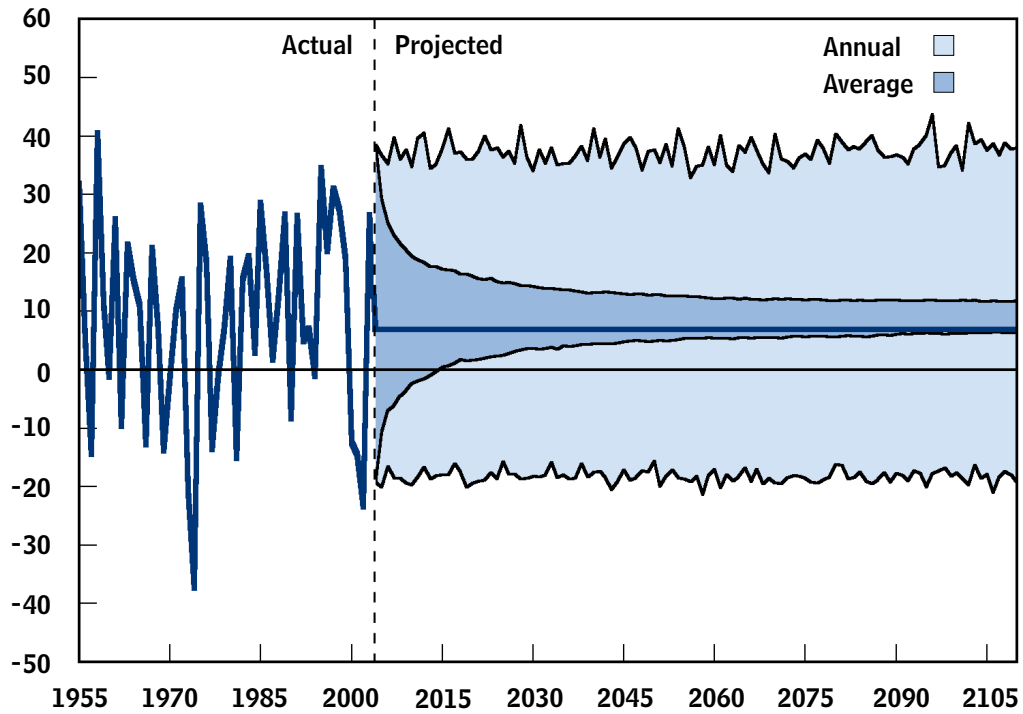
Source: Congressional Budget Office.

Note: Annual uncertainty bands show the 80 percent confidence range for a given year. Average uncertainty bands show the 80 percent confidence range for the average of 2004 through a given year.

Figure 18.

Uncertainty Bands for the Return on Corporate Bonds

(Percent)
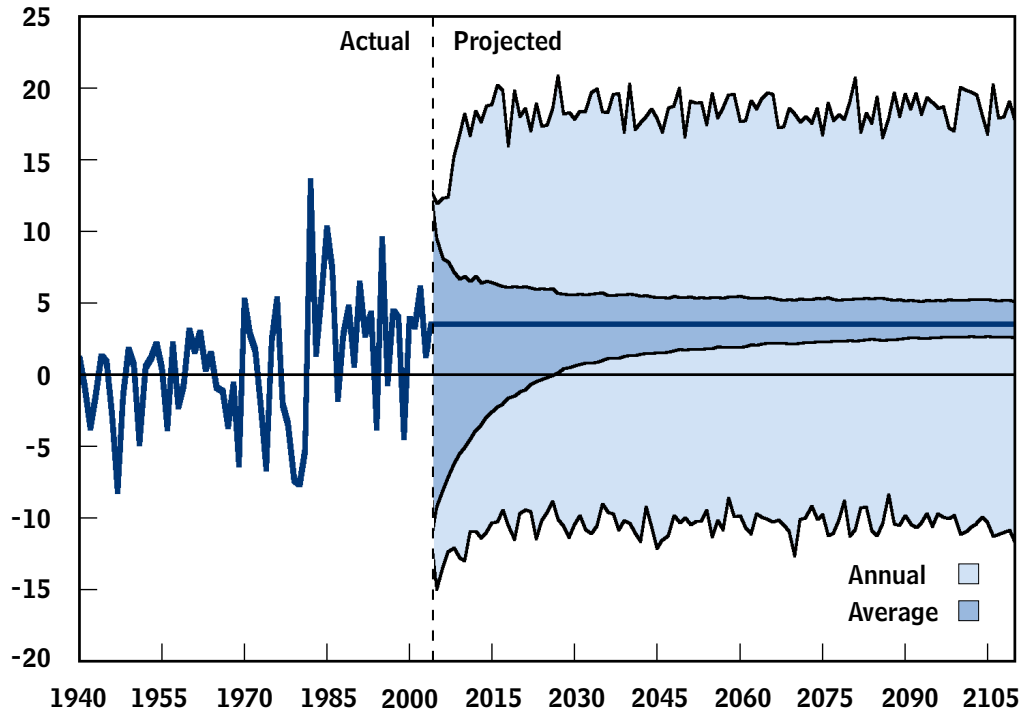
Source: Congressional Budget Office.

Note: Annual uncertainty bands show the 80 percent confidence range for a given year. Average uncertainty bands show the 80 percent confidence range for the average of 2004 through a given year.

Table 6.

Estimated Uncertainty About Social Security's Finances in CBO's Base Case: Annual Measures in Selected Years

(Percentage of GDP)

	2004	2025	2050	2075	2100
	Expected Outcome Under Current Law				
Revenues	4.85	5.11	5.03	4.95	4.81
Outlays	4.27	5.72	6.36	6.70	6.88
Balance	0.57	-0.61	-1.33	-1.75	-2.07
	80 Percent Confidence Range				
Revenues	4.82 to 4.90	4.91 to 5.30	4.68 to 5.33	4.51 to 5.35	4.27 to 5.36
Outlays	4.19 to 4.47	5.02 to 6.63	5.13 to 8.08	5.39 to 9.09	5.30 to 9.82
Balance	0.40 to 0.64	-1.52 to -0.05	-3.09 to -0.28	-4.22 to -0.68	-4.90 to -0.76

Source: Congressional Budget Office.

Figure 19.

Probability Distributions for Social Security Outlays and Revenues as a Share of GDP in CBO's Base Case

(Percentage of GDP)
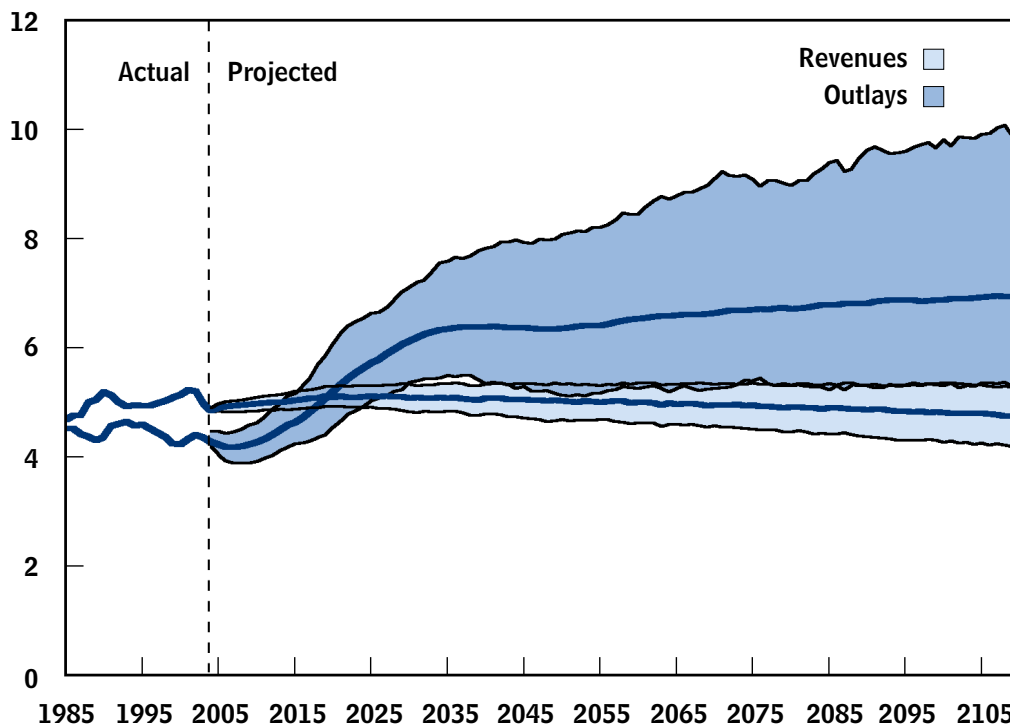
Source: Congressional Budget Office.

Note: Annual uncertainty bands show the 80 percent confidence range for a given year. Average uncertainty bands show the 80 percent confidence range for the average of 2004 through a given year.

Table 7.

Estimated Uncertainty About Social Security's Finances in CBO's Base Case: Summarized Measures

(Percentage of GDP)

	Revenues	Outlays	Balance
Expected Outcome Under Current Law			
50 Years (2004-2053)	5.39	5.50	-0.10
100 Years (2004-2103)	5.25	5.82	-0.56
80 Percent Confidence Range			
50 Years (2004-2053)	5.21 to 5.55	5.05 to 6.07	-0.66 to 0.26
100 Years (2004-2103)	5.04 to 5.45	5.36 to 6.45	-1.18 to -0.22

Source: Congressional Budget Office.

Table 8.

Sources of Uncertainty About Social Security's Finances

	Balance			Outlays			Revenue		
	10th Percentile	Median	90th Percentile	10th Percentile	Median	90th Percentile	10th Percentile	Median	90th Percentile
Ratio to GDP in 2025									
Effect of Changing Individual Inputs									
Fertility	-0.65	-0.61	-0.58	5.69	5.72	5.76	5.10	5.11	5.12
Mortality improvement	-0.75	-0.63	-0.52	5.62	5.74	5.85	5.10	5.11	5.12
Immigration	-0.82	-0.61	-0.44	5.54	5.72	5.91	5.10	5.11	5.12
Total factor productivity growth	-1.31	-0.64	-0.14	5.21	5.71	6.41	5.05	5.11	5.16
Economic VAR	-0.83	-0.63	-0.45	5.44	5.72	6.04	4.99	5.10	5.22
Disability incidence and termination	-0.72	-0.63	-0.53	5.64	5.73	5.83	5.10	5.11	5.11
Growth in share of nonwage, nontax compensation	-0.73	-0.61	-0.52	5.63	5.73	5.81	4.94	5.12	5.27
Gap in Core Price Index and CPI-W	-0.93	-0.62	-0.33	5.40	5.72	6.06	5.07	5.11	5.15
Effect of Changing All Inputs Together	-1.52	-0.68	-0.03	5.02	5.78	6.67	4.91	5.11	5.31
Ratio to GDP in 2100									
Effect of Changing Individual Inputs									
Fertility	-4.23	-2.25	-0.88	5.48	6.92	8.88	4.75	4.81	4.90
Mortality improvement	-2.66	-2.25	-1.85	6.54	6.97	7.40	4.77	4.81	4.84
Immigration	-2.41	-2.15	-1.92	6.63	6.88	7.14	4.78	4.81	4.83
Total factor productivity growth	-3.09	-2.19	-1.49	6.11	6.93	7.87	4.75	4.81	4.88
Economic VAR	-2.45	-2.19	-1.92	6.55	6.91	7.31	4.70	4.81	4.93
Disability incidence and termination	-2.27	-2.16	-2.06	6.78	6.89	7.00	4.80	4.81	4.81
Growth in share of nonwage, nontax compensation	-2.36	-2.18	-1.94	6.22	6.98	7.48	4.21	4.90	5.31
Gap in Core Price Index and CPI-W	-2.57	-2.20	-1.76	6.43	6.90	7.35	4.76	4.81	4.86
Effect of Changing All Inputs Together	-5.18	-2.45	-0.76	5.30	7.18	9.95	4.27	4.88	5.37

Source: Congressional Budget Office.

Note: VAR = vector autoregression; CPI-W = consumer price index for urban wage earners and clerical workers.

Appendix: Estimates of Time-Series Equations for Input Assumptions

In the Congressional Budget Office's (CBO's) analysis, the equations for each input are estimated according to basic techniques of time-series econometrics.[1] For each series, the goal is to find an equation that sufficiently captures the properties of the historical time series and employs a limited number of variables to yield a plausible fit for the input variable. Two sets of tests are performed to guarantee that the historical time series is stationary and that the residuals of the fitted equation are white noise. In this analysis, the only inputs that fail to pass the tests of stationarity are fertility and the variables controlled primarily by law (disability incidence, disability termination, and immigration). For a few age groups, the residuals of the time-series equations for mortality improvement fail to pass the test for white noise.

Currently, the possibility of nonstationarity is recognized only in CBO's estimates of uncertainty in the fertility projection. The other inputs that may contain nonstationarities are estimated as stationary processes, because random changes in their central tendencies are strongly influenced by changes in law. CBO plans to pursue improvements in the way the system of equations for mortality measures that variable; for the present, however, it felt that a "corrected" model would produce results similar to the equations now in place.

Each variable has been estimated using an AR(1) process, a vector autoregression (VAR) model, or an ARMA model involving three or four autoregressive variables with a moving-average representation of the annual fluctuations.[2]

Fertility
To model fertility, the annual level of fertility is transformed logistically so the projected values of the annual total fertility level lie between zero and four, using the formula $x_t = \log(TFR_t/(4 - TFR_t))$, where TFR_t describes the total fertility rate at time t. After the transformed projected values are calculated, they are converted into annual total fertility rates according to the formula $TFR_t = 4 \bullet \exp(x_t)/(1 + \exp(x_t))$.[3] The model does not allow for the possibility that the time series is nonstationary—an approach similar to that used in other projections of fertility, where it has been argued that allowing for nonstationarity may not be appropriate for creating forecasts of the level of fertility. Fertility is estimated according to an ARMA(4,1) process:

$$x_t = 0.652 + 0.560x_{t-1} + 1.814x_{t-2} + 1.204x_{t-3} + 0.812x_{t-4} + 0.449\varepsilon_{t-1},$$

where the standard deviation of the random shock is 0.132, the p value for the unit-root test is 0.190, and the p value for white noise of the residuals is 0.523.

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1. See, for example, Walter Enders, *Applied Econometric Time Series* (New York: John Wiley & Sons, 1995).

2. To preserve the series with an expected value set to the Social Security Administration's intermediate assumption, each result is expressed in deviation form. The deviation is forecast and then added to the deterministic expected value.

3. Ronald D. Lee, "Modeling and Forecasting the Time Series of U.S. Fertility: Age Distribution, Range, and Ultimate Level," *International Journal of Forecasting,* vol. 9 (1993), pp. 187-202.

Table A-1.

Estimated Coefficients for Mortality Reductions

Age Range	Intercept	Beta	Sigma
	Male Mortality		
Under 1	-0.034	-0.233	0.042
1 to 4	-0.050	-0.416	0.081
5 to 9	-0.036	-0.228	0.073
10 to 14	-0.027	-0.236	0.084
15 to 19	-0.011	-0.191	0.134
20 to 24	-0.009	-0.174	0.143
25 to 29	-0.008	-0.213	0.176
30 to 34	-0.011	-0.214	0.155
35 to 39	-0.014	-0.215	0.098
40 to 44	-0.013	-0.139	0.061
45 to 49	-0.012	-0.153	0.043
50 to 54	-0.011	-0.082	0.036
55 to 59	-0.009	-0.031	0.033
60 to 64	-0.009	-0.111	0.032
65 to 69	-0.008	-0.221	0.031
70 to 74	-0.008	-0.219	0.031
75 to 79	-0.007	-0.257	0.032
80 to 84	-0.007	-0.330	0.036
85 to 89	-0.004	-0.231	0.040
90 to 94	-0.003	-0.255	0.041
95 and older	-0.002	-0.248	0.044

Continued

Mortality Improvement

Over a range of 42 age-and-sex groups, the ratio of the percentage reduction between the current year's central death rate (by age and sex) and the previous year's central death rate is estimated according to an AR(1) process. (See Table A-1 for the coefficients and for the p values for the tests of unit root and white noise for the residuals.) The covariance of the annual random shocks is also calculated and used during the simulation process. For instance, the correlation between mortality reductions for males ages 10 to 14 and males ages 15 to 19 is 0.93. Because the deterministic model does not include that estimated covariance, mortality varies far more in that model than in the stochastic model. To simulate the annual random shocks and the covariance among them, a random number generator is used to generate a vector of 21 normal random errors. The 21 normal random errors are then transformed according to the variance-covariance structure of the errors by multiplying the vector of errors by the Cholesky vector, which is the triangular decomposition of the variance-covariance matrix of the random shocks.

Table A-1.

Continued

Age Range	Intercept	Beta	Sigma
		Female Mortality	
Under 1	-0.035	-0.266	0.043
1 to 4	-0.051	-0.427	0.088
5 to 9	-0.038	-0.266	0.089
10 to 14	-0.027	-0.183	0.107
15 to 19	-0.022	-0.167	0.113
20 to 24	-0.023	-0.200	0.144
25 to 29	-0.020	-0.185	0.174
30 to 34	-0.022	-0.222	0.132
35 to 39	-0.022	-0.239	0.078
40 to 44	-0.020	-0.222	0.049
45 to 49	-0.019	-0.248	0.035
50 to 54	-0.017	-0.248	0.032
55 to 59	-0.015	-0.223	0.031
60 to 64	-0.015	-0.342	0.026
65 to 69	-0.012	-0.219	0.028
70 to 74	-0.013	-0.270	0.031
75 to 79	-0.011	-0.279	0.035
80 to 84	-0.011	-0.292	0.039
85 to 89	-0.007	-0.178	0.044
90 to 94	-0.005	-0.218	0.043
95 and older	-0.004	-0.223	0.042

Source: Congressional Budget Office.

Immigration

Immigration fails its test for stationarity. It is estimated without a correction for that property as an ARMA(4,1) model:

$$x_t = 565,029 + 0.802x_{t-1} + 1.703x_{t-2} - 0.961x_{t-3} + 0.390x_{t-4} + \varepsilon_t - 0.147\varepsilon_{t-1},$$

where the standard deviation of the annual random shock is 186,184 people. The tests for stationarity reveal that the series fails to reject the presence of a unit root. The p value is 0.299. The residuals of the model are white noise, based on a p value of 0.739.

Disability Incidence and Retention

Disability incidence and disability retention both fail their tests for stationarity. They are estimated without a correction for that property as an AR(1) model. For the model of disability incidence:

$$x_t = -0.724 + 0.762x_{t-1} + \varepsilon_t,$$

where the standard deviation is equal to 0.090, the p value for the unit-root test is 0.17, and the p value for white noise of the residuals is 0.348. For the model of disability retention:

$$x_t = 0.246 + 0.915x_{t-1} + \varepsilon_t,$$

where the standard deviation of the annual random shock is 0.124, the p value for the unit-root test is 0.821, and the p value for white noise of the residuals is 0.255.[4]

Total Factor Productivity Growth

Total factor productivity growth was estimated as a white noise process, such that

$$x_t = 0.015 + \varepsilon_t,$$

where x_t represents the first difference of the log of total factor productivity, and ε_t represents the annual random shocks to total factor productivity growth. The standard deviation is 0.020. The p value for the Ljung-Box test for white noise of the residuals is 0.836.

Earnings Share of Compensation

It is not the earnings share of compensation that is modeled as a stochastic variable, but rather it is the growth in the share of nonwage and nontax compensation that is stochastic. Payroll taxes are determined by tax policy and should not be stochastic. However, uncertainty surrounds the share of compensation relating to benefits and pensions, so the growth of this share is modeled. The earnings share of compensation is the residual after payroll taxes (which are explicitly modeled in the Congressional Budget Office's long-term model, CBOLT), and the benefits and pension portions are removed.

The growth in the share of nonwage and nontax compensation is modeled as a white-noise process, such that:

$$x_t = 0.030 + \varepsilon_t,$$

where x_t represents the growth in the share of nonwage and nontax compensation, and ε_t represents the random variable that describes the annual random shocks to that growth rate. When using a white-noise process to describe the evolution of the growth of the nonwage and nontax share of compensation, the residuals are not white noise. Allowing for any autocorrelation in this time-series equation leads to an unreasonable number of cases where the bounding condition is binding.

Economic Variables

The economic variables of unemployment, inflation, and the real interest rate gap are estimated together in a VAR model, such that each variable is a function of its own previous

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4. The Congressional Budget Office's long-term model, CBOLT, uses a disability retention rate, which is simply (1 minus the termination rate).

Table A-2.

Estimated Coefficients for CBO's Endogenous Economic Model

Variable	Dependent Variable		
	Unemployment	Inflation	Real Interest Rate Gap
Intercept	-1.110	0.008	0.022
Unemployment$_{t-1}$	0.861	-0.050	-0.036
Unemployment$_{t-2}$	-0.261	0.052	0.034
Inflation$_{t-1}$	5.120	1.148	0.437
Inflation$_{t-2}$	-1.901	-0.307	-0.636
Real Interest Rate Gap$_{t-1}$	-2.046	-0.060	0.410
Real Interest Rate Gap$_{t-2}$	0.194	0.115	0.321
Sigma	0.132	0.014	0.014
p value for Test for Unit Root	0.025	0	0.037
p value for Test for			
White Noise	0.406	0.497	0.916

Sources: Social Security Administration; Congressional Budget Office.

values as well as the previous values of the other two variables.[5] Each series is stationary and its residuals appear to be white noise. As in the mortality projection, the variance and covariance of the random shocks of the three variables are estimated in order to have plausible co-movements between the economic variables (see Table A-2).

Gap Between Core GDP Deflator and CPI-W Growth

This input is modeled as an AR(1) process, such that:

$$x_t = -0.207 + 0.331 x_{t-1} + \varepsilon_t,$$

where x_t represents the gap between the core GDP deflator and growth in the consumer price index for urban wage earners and clerical workers (CPI-W), and ε_t represents the random variable that describes the annual random shocks to this gap and has a standard deviation of 0.958. The p value for the Dickey-Fuller test indicates the rejection of the presence of a unit root. The p value for the Ljung-Box test for white noise of the residuals is 0.088.

Real Wage Growth

Real wage growth was estimated according to an AR(1) process, such that:

$$x_t = 0.004 + 0.548 x_t + \varepsilon_t,$$

where x_t represents real wage growth, and ε_t represents the random variable that describes the annual random shocks to real wage growth and has a standard deviation of 0.016. The

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5. Unemployment rates are expressed as a log-odds ratio in order to bound them between zero and 1.

Table A-3.

Estimated Coefficients for Corporate Bond Return Model

	Corporate Bond Returns
Intercept	0.109
Unemployment$_{t-1}$	0.089
Unemployment$_{t-2}$	-0.112
Inflation$_{t-1}$	-4.585
Inflation$_{t-2}$	4.106
Real Interest Rate Gap$_{t-1}$	1.166
Real Interest Rate Gap$_{t-2}$	-2.875
Sigma	0.078
p value for Test for Unit Root	0.003
p value for Test for White Noise	0.540

Source: Congressional Budget Office.

p value for the Dickey-Fuller test is 0.003, which indicates the rejection of the presence of a unit root. The p value for the Ljung-Box test for white noise of the residuals is 0.54.

Corporate Bond Returns

Corporate bond yields are estimated in a manner similar to the economic vector autoregression as a function of unemployment and its lagged value, inflation and its lagged value, and the real interest rate gap and its lagged value (see Table A-3 for coefficients). The p value for the Dickey-Fuller test is 0.003, indicating the rejection of a unit root. The p value for white noise of the residuals is 0.540, indicating that the residuals are white noise.

Real Equity Returns

Real equity returns are estimated using a white-noise process. The time-series equation is:

$$x_t = 0.068 + \varepsilon_t,$$

where x_t is the natural log of real equity returns, and ε_t represents the random variable that describes the annual random shocks to equity returns and has a standard deviation of 0.203. The period under consideration is 1954 through 2003.

CBOLT also contains an option to estimate real equity returns using a mean-reversion model. When a mean-reversion model is used, the equation includes the lag of the natural log of the dividend to price ratio as an independent variable:

$$x_t = 0.342 + 0.173\gamma_{t-1} + \varepsilon_t,$$

where x_t is the natural log of the real equity return, γ_{t-1} is the natural log of the dividend-to-price ratio in the previous year, and ε_t represents the random variable that describes the annual random shocks to equity returns and has a standard deviation of 0.198. Again, the period of analysis is 1954 to 2003. The p value for the unit root test is 0.003, and the p value for the white noise of the residuals is 0.540.