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**DOES SOCIAL SECURITY PRIVATIZATION PRODUCE
EFFICIENCY GAINS?**

Shinichi Nishiyama
Congressional Budget Office
Washington, DC
E-mail: shinichi.nishiyama@cbo.gov

Kent Smetters
University of Pennsylvania & NBER
Philadelphia, PA
E-mail: smetters@wharton.upenn.edu

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Abstract

The economic literature shows that privatizing Social Security can improve labor supply incentives, but it can also reduce risk sharing when households face uninsurable risks. We simulate a stylized 50-percent privatization with transaction costs financed by consumption taxes and examined its impact on macroeconomic variables as well as on the welfare across generations and income classes. Our overlapping-generations model includes heterogeneous agents with elastic labor supply who face idiosyncratic earnings shocks and longevity uncertainty. The transition path is calculated, which allows us to rigorously calculate the overall *efficiency* gain or loss from privatization in general equilibrium.

We find that this partial privatization can have a powerful effect on labor supply incentives: When wage shocks are insurable, privatization produces about \$21,900 of new resources for each future household (growth adjusted over time) after all households have been fully compensated for their possible transitional losses. However, when wages are not insurable, privatization *reduces* efficiency by about \$5,600 per future household despite improved labor supply incentives.

We check the robustness of these results to different model specifications and arrive at several surprising conclusions. First, privatization actually performs relatively better in a *closed* economy, where interest rates decline with capital accumulation, than in an open economy where capital can be accumulated without reducing interest rates. Second, privatization also performs relatively better when an actuarially-fair private annuity market does *not* exist than when it does exist. Third, introducing progressivity into the privatized system to restore risk sharing must be done carefully. In particular, having the government match private contributions on a progressive basis is *not* very effective at restoring risk sharing; in fact, too much matching actually harms efficiency. However, increasing the progressivity of the remaining traditional system *is* very effective at restoring risk sharing, thereby allowing partial privatization to produce efficiency *gains* of \$2,700 per future household.

Journal of Economic Literature Classification Numbers: H0, H2, H3

Key Words: overlapping generations, idiosyncratic risks

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1 Introduction

It has been known for some time that shutting down (“privatizing”) a pay-as-you-go social security system would simply reallocate resources between generations when all economic variables are deterministic and labor supply is inelastic. In particular, no new resources would be created in present value once the “winners” have fully compensated the “losers.”¹ Allowing for elastic labor supply as well as various risks that are difficult to insure in the private market, however, changes things considerably.

With elastic labor supply, privatization could produce large efficiency gains by reducing the effective tax rate on labor supply, even if the government’s set of policy instruments is second best (Smetters, 2005). Social Security’s payroll taxes distort labor supply decisions because the program redistributes resources both *across* and *within* generations.

Across generations, Social Security benefits are financed on a pay-as-you-go basis with taxes on younger workers. When the economy grows slower than the interest rate (i.e., is dynamically efficient), workers with average earnings receive less than one dollar in present value in future benefits for each dollar they contribute. This loss services the implicit debt inherited from past generations who received *more* from Social Security than they paid.

Within generations, the U.S. Social Security system is progressive by giving a household with a lower average index of monthly wages (AIME) a larger Social Security benefit *relative* to their AIME (i.e., a higher “replacement rate”). Contributions by households with higher-than average lifetime earnings are effectively used to subsidize the contributions by households with lower-than average lifetime earnings. This redistribution increases effective marginal tax rates on households with AIME’s above the economy-wide average while reducing effective marginal tax rates on households with smaller AIME’s.

The U.S. Social Security system also provides two sources of risk sharing that could be reduced by privatization. First, the progressive benefit formula shares wage shocks among participants that are difficult to insure in the private market. Privatization could reduce this insurance unless it were complemented with some form of redistribution. Second, Social Security pays benefits until the beneficiary and spouse die rather than over a fixed number of

¹See, e.g., Breyer (1989), Feldstein (1995), Geanakoplos, Mitchell and Zeldes (1998), Murphy and Welch (1998), Mariger (1999), Shiller (1999), and Diamond and Orszag (2003).

years. To the extent that longevity uncertainty is also difficult to insure privately, privatization could also reduce annuity protection.

Overview of Our Approach. Determining the overall change in efficiency from privatization requires simulation analysis. We use a heterogeneous overlapping-generations model in which agents with elastic labor supply face idiosyncratic earnings shocks and longevity uncertainty. The economy's entire transition path after privatization is calculated, allowing us to determine the efficiency gain or loss from this reform.² All households across generations and income classes receive rebates or are taxed by exactly enough to return their expected remaining lifetime utility to their pre-reform levels. Privatization produces an efficiency gain if the *net* amount of new resources remaining after these rebates and taxes is positive, and produces an efficiency loss if negative. Following Auerbach and Kotlikoff (1987), new net resources (positive or negative) are distributed to future households in equal amounts (growth adjusted over time).

In our particular policy experiment, we consider a stylized partial privatization where households immediately begin to redirect 50 percent of their payroll tax into private accounts. Traditional Social Security benefits are reduced slowly across cohorts and eventually reach 50 percent of their original scheduled value for the youngest workers alive at the time of reform. Cash flow deficits, therefore, emerge during the transition which are financed with a consumption tax; Kotlikoff, Smetters, and Walliser (2001) used a deterministic model to show that consumption tax financing of the transition creates fewer distortions to labor supply and capital accumulation than income or wage tax financing.³

Summary of Our Findings. We find that partial privatization can substantially improve labor supply incentives. When wage shocks are assumed to be insurable in the private market, our stylized partial privatization produces new net resources equal to \$21,900 per future

²The insurance value of pay-as-you-go Social Security has also been investigated by İmrohoroğlu, İmrohoroğlu, and Joines (1995) and Conesa and Krueger (1999). İmrohoroğlu *et al.* focus on steady states and find that the value of risk sharing is outweighed by the reduction in capital. Conesa and Krueger consider whether a majority of the population would support privatization. Our analysis finds that while privatization typically raises long-run welfare it often fails to increase efficiency due to larger losses to transitional generations.

³Nishiyama and Smetters (2004) show that a consumption tax, though, might be less efficient than a progressive income tax in the presence of idiosyncratic shocks. Still, using a proportional increase in the income tax, as examined in KSW, would not overturn our key results.

household in our benchmark model. However, when, more realistically, wages are not insurable, privatization *reduces* efficiency by about \$5,600 per future household despite improving labor supply incentives. This loss occurs even though privatization substantially increases the welfare of those born in the *long run* by increasing the capital stock.

The efficiency loss that we calculate when wages are not insurable makes three key assumptions appear at first to be biased against privatization. Several surprising insights emerge as we investigate these assumptions more closely.

First, our benchmark economy is closed to international capital flows. As a result, capital accumulation after privatization reduces interest rates, discouraging more accumulation. If, instead, capital could flow across borders then more capital could be accumulated with no reduction in the rate of return to saving.

We find that the efficiency *losses* are actually *larger* (equal to about \$6,600 per future household) in a small open economy version of our model that allows for perfect international capital flows. To be sure, privatization leads to substantially more capital accumulation in this case. For the purpose of determining *efficiency* gains, however, the higher interest rate in the open economy also means that it is more costly to borrow against the long-run gains from privatization in order to compensate households alive during the transition that would otherwise lose from privatization. This finding emphasizes the fact that gains to macroeconomic variables alone are not necessarily good metrics for inferring efficiency gains.

Second, our benchmark calculations assume that a private annuity market does not exist, and so the pre-reform Social Security system not only provides insurance against wage uncertainty but against longevity as well. Households must guard against outliving their resources after privatization by relying more on precautionary saving. Whereas insurance efficiently shares idiosyncratic shocks that are uncorrelated across households, precautionary saving instead tries to self-insure against living “too long” using a capital buffer, which is less effective at smoothing consumption across states.

Rather surprisingly, we find that allowing for an actuarially-fair private annuity market also *increases* efficiency *losses* (about \$7,200 per future household). This result can be traced to several factors including the relative larger amount of precautionary saving after privatization when private annuity markets do not exist. The smaller interest rate in this

case lowers the cost at which compensation can be made across generations. More savings and labor supply also expand the tax bases, thereby reducing the income tax rates that are required to fund the rest of government.

Third, our benchmark partial privatization does not explicitly include any mechanism that shares idiosyncratic wage shocks that were previously partially insured under Social Security. Since private markets generally do not exist for insuring wages, households must again rely on more precautionary saving, which is less efficient than insurance.

We also simulate privatization where the government matches contributions made by poorer households; the match is financed by increasing general-revenue income taxes proportionally. This match is permanent across time but phased out linearly across income classes so that a household with median income receives no match. A match equal to 10 percent of income at a low level of income is indeed effective at reducing the efficiency loss to about \$4,400 per future household, but a 20-percent match actually *increases* the *loss* to \$9,900. This non-monotonic behavior is due to the trade-off between risk sharing and labor supply distortions: *some* match is beneficial but is dominated by distortions at higher tax rates. In fact, there is no match rate that allows privatization to produced efficiency gains; alternative designs of contribution matching performed even worse.

Finally, we show that simply increasing the progressivity of the smaller Social Security program that remains after partial privatization performs quite well at restoring risk sharing and can even lead to overall efficiency *gains* after privatization (about \$2,700 per future household). Compared to contribution matching, this alternative produces smaller *marginal* tax rates because Social Security benefits are computed based on lifetime earnings whereas the match is based on contemporaneous earnings.

The rest of the paper is as follows: Section 2 describes the model, Section 3 explains the calibration of the model, Section 4 results from various privatization simulations, and Section 5 concludes the paper. The Appendices explain the computational algorithm in more detail and contain various calibration information.

2 Model

Our model has three sectors: heterogeneous households with elastic labor supply; a competitive representative firm with constant-returns-to-scale production technology; and a government with a full commitment technology. Like most previous analyses of Social Security reform, our model's pre-reform neoclassical economy is stationary by construction, and so we don't capture the effects of projected demographic changes.⁴ We, however, are only interested in comparing the efficiency of gains from privatization against the baseline, not examining the implications of demographics. We used a similar model in Nishiyama and Smetters (2004) to analyze potential reforms of the federal income tax; the model in the current paper adds a Social Security system which requires an additional state variable in order to track a household's historical average earnings.

2.1 The Household Sector

Households are heterogeneous with respect to the following variables: age, i ; working ability, e_i (measured by their hourly wages); beginning-of-period wealth holdings, a_i ; and, average historical earnings, b_i , that determine their Social Security benefits. Each year, a large number (normalized to unity) of new households of age 20 enter the economy. Population grows at a constant rate, ν . A household of age i observes an idiosyncratic working ability shock, e_i , at the beginning of each year and chooses its optimal consumption c_i , working hours h_i , and end-of-period wealth holding a_{i+1} , taking as given the government's policy schedule and future factor prices.⁵ At the end of each year, a fraction of households die according to standard mortality rates; no one lives beyond age 109. For simplicity, all households represent two-earner married couples of the same age.

Let \mathbf{s}_i denote the state of an age i household,

$$\mathbf{s}_i = (i, e_i, a_i, b_i),$$

⁴We are aware of only a few papers, including De Nardi, İmrohoroğlu and Sargent (1999), Kotlikoff, Smetters and Walliser (2001), and Nishiyama (2004), that attempt to capture the effect of non-stationary demographics on baseline factor prices.

⁵Because there are no aggregate shocks in the present model, households can perfectly foresee these factor prices and policy variables, using the current distribution of households and the current policy variables. Yet, their own future working ability and mortality are uncertain.

where $i \in I = \{20, \dots, 109\}$ is the household's age, $e_i \in E = [e^{\min}, e^{\max}]$ is its working ability (the hourly wage), $a_i \in A = [a^{\min}, a^{\max}]$ is its beginning-of-period wealth, and $b_i \in B = [b^{\min}, b^{\max}]$ is its average historical earnings for Social Security purposes.⁶

Let \mathbf{S}_t denote the state of the economy at the beginning of year t ,

$$\mathbf{S}_t = (x_t(\mathbf{s}_i), W_{LS,t}, W_{G,t}),$$

where $x_t(\mathbf{s}_i)$ is the joint distribution of households where $\mathbf{s}_i \in I \times E \times A \times B$. $W_{LS,t}$ is the beginning-of-period net wealth held by the Lump-Sum Redistribution Authority (LSRA), which as described below, is used to determine the efficiency gain or loss from privatization. $W_{G,t}$ is the net wealth of the rest of the government.

Let Ψ_t denote the government policy schedule known at the beginning of year t ,

$$\Psi_t = \{W_{LS,s+1}, W_{G,s+1}, C_{G,s}, \tau_{I,s}(\cdot), \tau_{P,s}(\cdot), \tau_{C,s}, tr_{SS,s}(\mathbf{s}_i), tr_{LS,s}(\mathbf{s}_i)\}_{s=t}^{\infty},$$

where $C_{G,s}$ is government consumption, $\tau_{I,s}(\cdot)$ is an income tax function, $\tau_{P,s}(\cdot)$ is a payroll tax function for Social Security (OASDI), $\tau_{C,s}$ is a consumption tax rate, $tr_{SS,s}(\mathbf{s}_i)$ is a Social Security benefit function, and $tr_{LS,s}(\mathbf{s}_i)$ is an LSRA wealth redistribution function. The specifications of these functions are described below.

The household's problem is

$$v(\mathbf{s}_i, \mathbf{S}_t; \Psi_t) = \max_{c_i, h_i} u_i(c_i, h_i) + \beta(1 + \mu)^{\alpha(1-\gamma)} \phi_i E[v(\mathbf{s}_{i+1}, \mathbf{S}_{t+1}; \Psi_{t+1}) | e_i] \quad (1)$$

subject to

$$\begin{aligned} a_{i+1} &= \frac{1}{1 + \mu} \{w_t e_i h_i + (1 + r_t)(a_i + tr_{LS,t}(\mathbf{s}_i)) \\ &\quad - \tau_{I,t}(w_t e_i h_i, r_t(a_i + tr_{LS,t}(\mathbf{s}_i)), tr_{SS,t}(\mathbf{s}_i)) \\ &\quad - \tau_{P,t}(w_t e_i h_i) + tr_{SS,t}(\mathbf{s}_i) - (1 + \tau_{C,t})c_i\} \geq a_{i+1,t}^{\min}(\mathbf{s}_i), \\ a_{20} &= 0, \quad a_{i \in \{65, \dots, 110\}} \geq 0, \end{aligned} \quad (2)$$

where the utility function, $u_i(\cdot)$, takes the Cobb-Douglas form nested within a time-separable

⁶The average historical earnings are used to calculate the Social Security benefits of each household. The variable b_i approximates the average indexed monthly earnings (AIME) multiplied by 12 as of age i .

isoelastic specification,

$$u(c_i, h_i) = \frac{\left\{ ((1 + n_i/2)^{-\zeta} c_i)^\alpha (h_i^{\max} - h_i)^{1-\alpha} \right\}^{1-\gamma}}{1 - \gamma}; \quad (3)$$

γ is the coefficient of relative risk aversion; n_i is the number of dependent children at the parents' age i ; ζ is the "adult equivalency scale" that converts the consumption by children into their adult equivalent amounts; and, h_i^{\max} is the maximum working hours.

The constant β is the rate of time preference; ϕ_i is the survival rate at the end of age i ; w_t is the wage rate per efficiency unit of labor (accordingly, $w_t e_i h_i$ is total labor compensation at age i in time t); and, r_t is the rate of return to capital. Individual variables of the model are normalized by the exogenous rate of labor augmenting technological change, μ . Our choice for $u_i(\cdot)$ is consistent with the conditions that are necessary for the existence of a long-run steady state in the presence of constant population growth. Hence, μ is also equal to the per-capita growth rate of output and capital in steady state. The term $\beta(1 + \mu)^{\alpha(1-\gamma)}$ is the *growth-adjusted* rate of time preference.

$a_{i+1,t}^{\min}(\mathbf{s}_i)$ is the state-contingent minimum level of end-of-period wealth that is sustainable, that is, even if the household receives the worst possible shocks in future working abilities.⁷ At the beginning of the next period, the state of this household when private annuity markets do not exist becomes

$$\mathbf{s}_{i+1} = (i + 1, e_{i+1}, a_{i+1} + q_t, b_{i+1}), \quad (4)$$

where q_t denotes accidental bequests that a household receives at the end of the period. In the presence of perfect annuity markets, the household's state in the next period is instead

$$\mathbf{s}_{i+1} = (i + 1, e_{i+1}, a_{i+1}/\phi_i, b_{i+1}). \quad (5)$$

⁷In particular, $a_{i+1,t}^{\min}(\mathbf{s}_i)$ is allowed to be negative but cannot exceed in magnitude the present value of the worst possible future labor income stream at maximum working hours. Although not shown explicitly in equation (2) in order to save on notation, any borrowing (i.e., $a_{i+1} < 0$) by an agent age i at time t must be done at rate r_t/ϕ_i in order cover the chance that they will die before repaying their loan.

The average historical earnings b_i follows

$$b_{i+1} = \begin{cases} 0 & \text{if } i \leq 24 \\ \frac{1}{i-24} \left\{ (i-25)b_i \frac{w_t}{w_{t-1}} + \min(w_t e_i h_i / 2, weh_t^{\max}) \right\} & \text{if } 25 \leq i \leq 59 \\ b_i / (1 + \mu) & \text{if } i \geq 60 \end{cases}, \quad (6)$$

where weh_t^{\max} is the Old-Age, Survivors, and Disability Insurance (OASDI) tax cap, which is \$80,400 in 2001. For simplicity, the model assumes that the highest 35 years of earnings correspond to between ages 25 and 59.⁸

Let $x_t(\mathbf{s}_i)$ denote the measure of households, and let $X_t(\mathbf{s}_i)$ be the corresponding cumulative measure. The measure of households is adjusted by the steady-state population growth rate, ν . The population of age 20 households is normalized to unity in the baseline economy along the balanced growth path, that is,

$$\int_E dX_t(20, e_{20}, 0, 0) = 1.$$

Let $\mathbf{1}_{[a=y]}$ be an indicator function that returns 1 if $a = y$ and 0 if $a \neq y$. Then, the law of motion of the measure of households is

$$x_{t+1}(\mathbf{s}_{i+1}) = \frac{\phi_i}{1 + \nu} \int_{E \times A \times B} \mathbf{1}_{[a_{i+1}=a_{i+1}(\mathbf{s}_i, \mathbf{S}_t; \Psi_t) + q_i]} \\ \times \mathbf{1}_{[b_{i+1}=b_{i+1}(w_t e_i h_i(\mathbf{s}_i, \mathbf{S}_t; \Psi), b_i)]} \pi_{i,i+1}(e_{i+1}|e_i) dX_t(\mathbf{s}_i),$$

where $\pi_{i,i+1}$ denotes the transition probability of working ability from age i to age $i + 1$.

The *aggregate* value of accidental bequests each period is deterministic in our model because all risks are idiosyncratic and uncorrelated across households. Accidental bequests could be simply distributed equally and deterministically across all surviving households, as in some other papers. That approach suffers from two shortcomings. First, households would anticipate receiving a bequest with certainty, thereby artificially crowding out their pre-bequest savings. This savings reduction would be mitigated if bequests were random. Second, empirically, the inequality of bequests is important in generating a realistic measure of wealth inequality.

⁸U.S. Social Security benefits are computed on the basis of the highest 35 years of earnings, up to age 60. We assume that these years occur between ages 25 and 59 to avoid 34 additional state variables.

Our alternative strategy distributes bequests randomly to surviving working-age households. Each household receives a bequest q_t with constant probability η :

$$q_t = \frac{\sum_{i=20}^{109} (1 - \phi_i) \int_{E \times A \times B} a_{i+1}(\mathbf{s}_i, \mathbf{S}_t; \Psi_t) dX_t(\mathbf{s}_i)}{\sum_{i=20}^{109} (1 - \phi_i) \int_{E \times A \times B} dX_t(\mathbf{s}_i)},$$

$$\eta = \frac{\sum_{i=20}^{109} (1 - \phi_i) \int_{E \times A \times B} dX_t(\mathbf{s}_i)}{\sum_{i=20}^{64} \phi_i \int_{E \times A \times B} dX_t(\mathbf{s}_i)}, \quad (7)$$

where q_t is the average wealth left by deceased households and η is the ratio of deceased household to the surviving working-age households.

2.2 Government

Government tax revenue consists of federal income tax $T_{I,t}$, payroll tax for Social Security (OASDI) $T_{P,t}$, and consumption tax $T_{C,t}$. These revenues are

$$T_{I,t} = \sum_{i=20}^{109} \int_{E \times A \times B} \tau_{I,t}(w_t e_i h_i(\mathbf{s}_i, \mathbf{S}_t; \Psi_t), r_t(a_i + tr_{LS,t}(\mathbf{s}_i)), tr_{SS,t}(\mathbf{s}_i)) dX_t(\mathbf{s}_i), \quad (8)$$

$$T_{P,t} = \sum_{i=20}^{109} \int_{E \times A \times B} \tau_{P,t}(w_t e_i h_i(\mathbf{s}_i, \mathbf{S}_t; \Psi_t)) dX_t(\mathbf{s}_i), \quad (9)$$

$$T_{C,t} = \sum_{i=20}^{109} \int_{E \times A \times B} \tau_{C,t} c_i(\mathbf{s}_i, \mathbf{S}_t; \Psi_t) dX_t(\mathbf{s}_i). \quad (10)$$

Social Security (OASDI) benefit expenditure $Tr_{SS,t}$ is

$$Tr_{SS,t} = \sum_{i=20}^{109} \int_{E \times A \times B} tr_{SS,t}(\mathbf{s}_i) dX_t(\mathbf{s}_i). \quad (11)$$

The law of motion of the government wealth (normalized by productivity growth and population growth) is

$$W_{G,t+1} = \frac{1}{(1 + \mu)(1 + \nu)} \{(1 + r_t)W_{G,t} + T_{I,t} + T_{P,t} + T_{C,t} - Tr_{SS,t} - C_{G,t}\}, \quad (12)$$

where $C_{G,t}$ is government consumption.

2.3 Measuring Efficiency Gains

We follow Auerbach and Kotlikoff (1987) by measuring efficiency gains from Social Security privatization using a Lump-Sum Redistribution Authority that compensates households who would otherwise lose from reform.⁹ To be clear, the LSRA is not being proposed as an actual government institution that attempts to produce Pareto-improving policy changes. Instead, it is simply a hypothetical mechanism that allows us to measure the standard Hicksian efficiency gains in general equilibrium associated with privatization.

To see how the LSRA works, suppose that a new policy is announced at the beginning of period 1. First, the LSRA makes a lump-sum compensating variation transfer or tax, $tr_{CV,1}(\mathbf{s}_i)$, to *each living* household of age i to return its expected remaining lifetime utility at state \mathbf{s}_i to its pre-reform level in the baseline (pre-reform) economy. Next, the LSRA makes a lump-sum transfer or tax, $tr_{CV,t}(\mathbf{s}_{20})$, to *each future* household (born in periods 2, 3, ...) to make it as well off as in the baseline economy, conditional on its initial state at age 20. Thus far, the net present value of these taxes and transfers across living and future households will generally not sum to zero. So, finally, the LSRA makes an additional lump-sum transfer (tax), Δtr , to each future household so that the net present value across all transfers is zero. For illustrative purposes, we assume, like Auerbach and Kotlikoff, that these additional transfers are uniform across future generations on a growth-adjusted basis. The lump-sum transfers made by the LSRA, therefore, are

$$tr_{LS,t}(\mathbf{s}_i) = \begin{cases} tr_{CV,t}(\mathbf{s}_i) & \text{if } t = 1 \\ tr_{CV,t}(\mathbf{s}_i) + \Delta tr & \text{if } t > 1 \text{ and } i = 20 \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

If $\Delta tr > 0$ then privatization has produced net new resources and so we say that this reform “increases efficiency.” Conversely, if $\Delta tr < 0$ then privatization “reduces efficiency.”

The aggregate net lump-sum transfers / taxes to living households at time t , $Tr_{LS,t}$, is

$$Tr_{LS,t} = \sum_{i=20}^{109} \int_{E \times A \times B} tr_{LS,t}(\mathbf{s}_i) dX_t(\mathbf{s}_i), \quad (14)$$

⁹We, however, extend the Auerbach and Kotlikoff approach to a heterogeneous-agent environment.

and the law of motion of the LSRA wealth (normalized by productivity growth and population growth) is

$$W_{LS,t+1} = \frac{1}{(1+\mu)(1+\nu)}(1+r_t)(W_{LS,t} - Tr_{LS,t}). \quad (15)$$

2.4 Aggregation and Production

National wealth W_t is the sum of total private wealth, government net wealth $W_{G,t}$, and LSRA net wealth $W_{LS,t}$, and total labor supply L_t is measured in efficiency units:

$$W_t = \sum_{i=20}^{109} \int_{E \times A \times B} a_i dX_t(\mathbf{s}_i) + W_{LS,t} + W_{G,t}, \quad (16)$$

$$L_t = \sum_{i=20}^{109} \int_{E \times A \times B} e_i h_i(\mathbf{s}_i, \mathbf{S}_t; \Psi_t) dX_t(\mathbf{s}_i). \quad (17)$$

In a closed economy, capital stock is equal to national wealth, that is, $K_t = W_t$, and gross national product Y_t is determined by a constant-returns-to-scale production function,

$$Y_t = F(K_t, L_t).$$

The profit-maximizing condition for this competitive firm is

$$F_K(K_t, L_t) = r_t + \delta, \quad (18)$$

$$F_L(K_t, L_t) = w_t, \quad (19)$$

where δ is the depreciation rate of capital.

In a small open economy, factor prices, r_t^* and w_t^* are fixed at international levels, and domestic capital stock $K_{D,t}$ and labor supply L_t are determined so that the firm's profit maximizing condition satisfies

$$F_K(K_{D,t}, L_t) = r_t^* + \delta, \quad (20)$$

$$F_L(K_{D,t}, L_t) = w_t^*. \quad (21)$$

Gross domestic product $Y_{D,t}$ is determined by the production function,

$$Y_{D,t} = F(K_{D,t}, L_t),$$

and gross national product Y_t is determined by

$$Y_t = (r_t^* + \delta) W_t + w_t^* L_t.$$

Net foreign investment is shown by the difference between national wealth and domestic capital stock, that is, $W_t - K_{D,t}$.

2.5 Recursive Competitive Equilibrium

Definition Recursive Competitive Equilibrium: Let $\mathbf{s}_i = (i, e_i, a_i, b_i)$ be the individual state of households, let $\mathbf{S}_t = (x_t(\mathbf{s}_i), W_{LS,t}, W_{G,t})$ be the state of the economy, and let Ψ_t be the government policy schedule known at the beginning of year t ,

$$\Psi_t = \{W_{LS,s+1}, W_{G,s+1}, C_{G,s}, \tau_{I,s}(\cdot), \tau_{P,s}(\cdot), \tau_{C,s}, tr_{SS,s}(\mathbf{s}_i), tr_{LS,s}(\mathbf{s}_i)\}_{s=t}^{\infty}.$$

A series of factor prices $\{r_s, w_s\}_{s=t}^{\infty}$, accidental bequests $\{q_s\}_{s=t}^{\infty}$, the policy variables $\{W_{LS,s+1}, W_{G,s+1}, C_{G,s}, \tau_{C,s}, tr_{LS,s}(\mathbf{s}_i)\}_{s=t}^{\infty}$, the parameters of policy functions $\{\varphi_s\}_{s=t}^{\infty}$, the value function of households $\{v(\mathbf{s}_i, \mathbf{S}_s; \Psi_s)\}_{s=t}^{\infty}$, the decision rule of households

$$\{\mathbf{d}(\mathbf{s}_i, \mathbf{S}_s; \Psi_s)\}_{s=t}^{\infty} = \{c_i(\mathbf{s}_i, \mathbf{S}_s; \Psi_s), h_i(\mathbf{s}_i, \mathbf{S}_s; \Psi_s), a_{i+1}(\mathbf{s}_i, \mathbf{S}_s; \Psi_s)\}_{s=t}^{\infty},$$

and the measure of households $\{x_s(\mathbf{s}_i)\}_{s=t}^{\infty}$, are in a recursive competitive equilibrium if, in every period $s = t, \dots, \infty$, each household solves the utility maximization problem (1)–(6) taking Ψ_t as given; the firm solves its profit maximization problem, the capital and labor market conditions (16)–(21) clear, and the government policy schedule satisfies (13)–(15).

In steady-state,

$$\mathbf{S}_{t+1} = \mathbf{S}_t \tag{22}$$

for all t and $\mathbf{s}_i \in I \times E \times A \times B$.

3 Calibration

Tables 1 and 2 summarize the model's key parameters, as discussed below.

Table 1: Common Parameters

Coefficient of relative risk aversion	γ	2.0
Capital share of output	θ	0.30
Depreciation rate of capital stock	δ	0.047
Long-term real growth rate	μ	0.018
Population growth rate	ν	0.010
Probability of receiving bequests	η	0.0161
Total factor productivity ¹	A	0.949

¹Total factor productivity is chosen so that w equals 1.0.

Table 2: Other Main Parameters

		Representative- Agent Model	Heterogeneous-Agent Economy with Wage Shocks		
		without Wage Shocks	Lower Transi- tory Shocks to		
			1/2	1/5	
Time preference parameter ¹	β	1.004	0.985	0.992	1.000
Share parameter for consumption ²	α	0.436	0.466	0.456	0.450
Income tax adjustment factor ³	φ_I	1.000	0.818	0.847	0.874
OASDI benefit adjustment factor ⁴	φ_{SS}	1.232	1.385	1.388	1.388

¹The capital-GDP ratio is targeted to be 2.74 ($r = 6.25$ percent) without annuity markets.

²The average annual working hours are 3414 per married couple when $h_{\max} = 8760$.

³In a heterogeneous-agent economy, the ratio of income tax revenue to GDP is 0.123.

⁴The OASDI budget is assumed to be balanced.

Table 3: Number of People Under Age 18 Living in a Married Household

Age cohorts	Number of children	Age cohorts	Number of children
20-24	0.642	50-54	0.908
25-29	1.167	55-59	0.562
30-34	1.451	60-64	0.231
35-39	1.755	65-69	0.156
40-44	1.753	70-74	0.055
45-49	1.439	75-plus	0.000

Source: Authors' calculations from the 2001 Survey of Consumer Finances (SCF).

3.1 Households

Utility Function Parameters. The coefficient of relative risk aversion, γ , is assumed to be 2.0. The number of dependent children at the parents' age i , n_i , is calculated using the 2001 Survey of Consumer Finances (SCF) as shown in Table 3. The "adult equivalency scale," ζ , is set at 0.6.¹⁰ As discussed later, β is chosen to hit a target capital-output ratio, producing an interest rate of 6.2 percent in the initial steady state. The maximum working hours of husband and wife, h_i^{\max} , is set at 8,760, equal to 12 hours per day per person \times 365 days \times two persons.¹¹ A smaller value for h_i^{\max} would reduce the effective labor supply elasticity, and tend to reduce the gains from privatization. The parameter α is chosen so that the average working hours of households between ages 20 and 64 equals 3,414 hours in the initial steady-state economy, the average number of hours supplied by married households in the 2001 SCF.

Working Ability. The working ability in this calibration corresponds to the hourly wage (labor income per hour) of each household in the 2001 SCF.¹² The average hourly wage of a married couple (family members #1 and #2 in SCF) used in the calibration is calculated by

$$\text{Hourly Wage} = \frac{\text{Regular and Additional Salaries (\#1 + \#2) + Payroll Taxes} / 2}{\max\{\text{Working Hours (\#1 + \#2)}, 2080\}}.$$

We adjusted the salaries in the numerator by adding imputed payroll taxes paid by their employers, which allows us to levy the entire payroll tax on employees in order to incorporate the payroll tax ceiling. The max operator in the denominator adjusts the hourly wage for a small fraction of households in the SCF with large reported salaries but few reported working hours such as the self-employed.

Table 4 shows the eight discrete levels of working abilities of five-year age cohorts. We use a shape-preserving cubic spline interpolation between each five-year age cohort to obtain

¹⁰Hence, a married couple with two dependent children must consume about 52 percent (i.e., $2^{0.6} = 1.517$) more than a married couple with no children to attain the same level of utility, *ceteris paribus*.

¹¹The 95th and 99th percentiles of the working hours per married couple of aged 20-64 in the 2001 SCF are 5,280 and 6,375, respectively.

¹²According to Bureau of Labor Statistics data, the average hourly earnings of production workers have increased by 3.8 percent from 2000 to 2001. Since the 2001 SCF wages correspond to year 2000 while our tax function introduced below is calibrated to the year 2001, we multiply the SCF wages shown in Table 4 by 1.038 to convert the hourly wages in 2000 into growth-adjusted wages in 2001.

Table 4: Working Abilities of a Household (in U.S. Dollars per Hour)

Percentile		Age Cohorts					
		20-24	25-29	30-34	35-39	40-44	45-49
e^1	0-20th	3.89	5.47	6.86	6.01	7.43	5.73
e^2	20-40th	8.35	10.11	12.38	12.27	13.90	13.14
e^3	40-60th	10.28	14.04	16.46	16.96	18.76	18.47
e^4	60-80th	12.31	17.30	21.87	22.57	25.79	25.71
e^5	80-90th	17.47	21.58	29.37	30.19	35.37	35.56
e^6	90-95th	22.17	27.21	33.96	46.92	48.30	54.59
e^7	95-99th	29.43	36.60	43.76	81.75	96.44	97.48
e^8	99-100th	42.31	62.29	182.78	327.65	262.03	284.00

Percentile		Age cohorts					
		50-54	55-59	60-64	65-69	70-74	75-79
e^1	0-20th	5.00	2.42	0.00	0.00	0.00	0.00
e^2	20-40th	13.99	10.65	3.36	0.00	0.00	0.00
e^3	40-60th	20.95	15.60	11.00	1.92	0.00	0.00
e^4	60-80th	29.13	24.60	18.33	11.14	1.77	1.30
e^5	80-90th	40.89	34.75	29.08	19.14	10.93	10.21
e^6	90-95th	54.11	51.62	44.41	29.99	20.66	20.88
e^7	95-99th	91.67	99.24	91.12	56.19	38.26	26.41
e^8	99-100th	282.18	333.58	555.90	244.71	193.00	86.76

Source: Authors' calculations from the 2001 SCF.

the working ability for each age cohort.¹³ In the version of our model where we “turn off” the idiosyncratic wage shocks, the hourly wages of the representative household are assumed to be those of the 40–60th percentile households.

Table 4 only shows the different potential “wage buckets” by age as well as the proportion of households in each bucket. It does not itself capture the uncertainty over wages. Using the Panel Study of Income Dynamics (PSID), we estimate Markov transition matrixes that specify the probabilities that a household’s wage will move from one level to a different level the next year. Separate transition matrixes were constructed for four age ranges—20-29, 30-39, 40-49, and 50-59—in order to capture the possibility that the probabilities themselves might change over the lifecycle. For households aged 60 or older, we used the matrix for ages 50-59. The Appendix reports the matrixes in detail. We check the sensitivity of our

¹³An alternative approach of estimating eight different wage rates for each age would have relied on too few observations.

simulation results to this specification later in the paper.

Population Growth and Mortality. The population growth rate ν is set to one percent per year, consistent with Social Security Administration (2001) long-run estimates. The survival rate ϕ_i at the end of age $i = \{20, \dots, 109\}$ are the weighted averages of the male and female survival rates as calculated by SSA. The survival rates at the end of age 109 are replaced by zero, thereby capping the maximum length of life. See the Appendix for more details.

3.2 Production

Capital and Private Wealth. Capital K is the sum of private fixed assets and government fixed assets. In 2000, private fixed assets were \$21,165 billion, government fixed assets were \$5,743 billion, and the public held about \$3,410 billion of government debt.¹⁴ Government net wealth is set equal to 9.5 percent of total private wealth in the initial steady-state economy. The time preference parameter β is chosen in each variant of our model explored below so that the capital-GDP ratio in the initial steady state economy is 2.74, the empirical value in 2000.¹⁵

Production Technology. Production takes the Cobb-Douglas form,

$$F(K_t, L_t) = A_t K_t^\theta L_t^{1-\theta},$$

where, recall, L_t is the sum of working hours in efficiency units. The capital share of output is given by

$$\theta = 1 - \frac{\text{Compensation of Employees} + (1 - \theta) \times \text{Proprietors' Income}}{\text{National Income} + \text{Consumption of Fixed Capital}}.$$

The value of θ in 2000 was 0.30.¹⁶ The annual per-capita growth rate μ is assumed to be 1.8 percent, the average rate between 1869 to 1996 (Barro, 1997). Total factor productivity A is set at 0.949, which normalizes the wage (per efficient labor unit) to unity.

¹⁴Source: Department of Commerce, Bureau of Economic Analysis.

¹⁵*Ibid.*

¹⁶Source: Department of Commerce, Bureau of Economic Analysis. The average of θ in years between 1996 and 2000 is 0.31.

The Depreciation Rate of Fixed Capital. The depreciation rate of fixed capital δ is chosen by the following steady-state condition,

$$\delta = \frac{\text{Total Gross Investment}}{\text{Fixed Capital}} - \mu - \nu.$$

In 2000, private gross fixed investment accounted for 17.2 percent of GDP, and government (federal and state) gross investment accounted for 3.3 percent of GDP.¹⁷ With a capital-output ratio of 2.74, the ratio of gross investment to fixed capital is 7.5 percent. Subtracting productivity and population growth rates, the annual depreciation rate is 4.7 percent.

3.3 The Government

Income Taxes. Federal income tax and state and local taxes are assumed to be at the level in year 2001 before the passage of the “Economic Growth and Tax Relief Reconciliation Act of 2001” (EGTRRA). Since households in our model are assumed to be married, we use a standard deduction of \$7,600. Following Altig *et al.* (2001), we allow higher income households to itemize deductions when it is more valuable to do so, and we assume that the value of the itemized deduction increases linearly in the Adjusted Gross Income.¹⁸ The additional exemption per dependent person is \$2,900 where the number of dependent children is consistent with Table 3. Table 5 shows the statutory marginal tax rates before EGTRRA.¹⁹ As noted earlier, a household’s labor income in this calibration includes the imputed payroll tax paid by its employer. Thus, taxable income is obtained by subtracting the employer portion of payroll tax from labor income.

The standard deduction, the personal exemption, and all tax brackets grow with productivity over time so that there is no real bracket creep; this indexing is also needed for the initial economy to be in steady state. Also, since the effective tax rate on capital income is reduced by investment tax incentives, accelerated depreciation and other factors (Auerbach, 1996), the tax function is further adjusted so that the cross-sectional average tax rate

¹⁷*Ibid.*

¹⁸In particular, the deduction taken by a household is the greater of the standard deduction and $0.0755 \times \text{AGI}$, or $\max\{\$7600, 0.0755 \times \text{AGI}\}$.

¹⁹The key qualitative results reported herein are unaffected if the tax function were instead modeled as net taxes, that is, after subtracting transfers indicated in the Statistics of Income.

Table 5: Marginal Individual Income Tax Rates in 2001 (Married Household, Filed Jointly)

Taxable Income		Marginal Income Tax Rate (%)
\$0	– \$45,200	$15.0 \times \varphi_I$
\$45,200	– \$109,250	$28.0 \times \varphi_I$
\$109,250	– \$166,500	$31.0 \times \varphi_I$
\$166,500	– \$297,350	$36.0 \times \varphi_I$
\$297,350	–	$39.6 \times \varphi_I$

Table 6: Marginal Payroll Tax Rates in 2001

Taxable Labor		Marginal Tax Rate (%)	
Income per Worker		OASDI	HI
\$0	– \$80,400	$12.4 \times \varphi_P$	2.9
\$80,400	–	$0.0 \times \varphi_P$	2.9

Note: The payroll tax adjustment factor φ_P equals 1.0 in the baseline economy.

on capital income is about 25 percent lower than the average tax on labor income.²⁰ In 2000, the ratio of total individual federal income tax revenue (not including Social Security and Medicare taxes) to GDP was 0.102 and the ratio of corporate income tax to GDP was 0.021. Each statutory federal income tax rate shown in Table 5 is multiplied by φ_I so that tax revenue (including corporate income tax) totals 12.3 percent of GDP in the initial steady state. The adjustment factor is between 0.82 and 0.87 for heterogeneous-agent economies with idiosyncratic wage shocks and 1.0 for the representative-agent economy without wage shocks. State and local income taxes are modeled parsimoniously with a 4.0 percent flat tax on income above the deduction and exemption levels used at the federal level.

Social Security. The tax rate levied on employees for Old-Age, Survivors, and Disability Insurance (OASDI) is 12.4 percent, and the tax rate for Medicare (HI) is 2.9 percent. In 2001, employee compensation above \$80,400 was not taxable for OASDI. (See Table 6.)

Social Security benefits are based on each worker’s Average Indexed Monthly Earnings (AIME), $b_i/12$, and the replacement rate schedule in the United States. The replacement

²⁰This relative reduction to the tax rate on capital is commonly used by CBO, and it balances the legal tax preferences given to capital versus the legal tax benefits given to labor, including tax-preferred fringe benefits.

Table 7: OASDI Replacement Rates in 2001

AIME (b/12)	Marginal Replacement Rate (%)
\$0 – \$561	$90.0 \times \varphi_{SS}$
\$561 – \$3,381	$32.0 \times \varphi_{SS}$
\$3,381 –	$15.0 \times \varphi_{SS}$

Note: The OASDI benefit adjustment factor φ_{SS} is set so that the OASDI is pay-as-you-go in the baseline economies.

rates are 90 percent for the first \$561, 32 percent for amounts between \$561 and \$3,381, and 15 percent for amounts above \$3,381. Social Security, therefore, is progressive in that a worker’s benefit relative to AIME (the “replacement rate”) is decreasing in the AIME.

The U.S. OASDI also pays spousal, survivors’ and disability benefits in addition to the standard retirement benefit described above. Indeed, retiree benefits accounted for only 69.1 percent of total OASDI benefits in December 2000.²¹ OASDI benefits are adjusted upward by the proportional adjustment factor φ_{SS} so that total benefit payments equal total payroll tax revenue. The adjustment factor φ_{SS} equals about 1.39 in our model with wage shocks and 1.23 in our model without wage shocks. This adjustment proportionally distributes non-retiree OASDI payments across retirees.

4 Policy Experiments

We simulate a stylized phased-in partial privatization of Social Security that begins in year 1. Workers immediately start redirecting 50 percent of their payroll tax payments into private saving accounts. As is implicit in most previous work on privatization, assets in the new private accounts are assumed to be perfect substitutes with other private assets, including earning the same market rate of return and being subject to the same income tax schedule, as outlined above. As a result, the new private accounts do not have to be explicitly modeled; the redirected payroll simply increases regular household saving.

Social Security benefits are reduced linearly over time. Households age 66 and older in year 1 receive the current-law (baseline) benefits throughout the rest of their lifetime; households of age 65 in year 1 receive benefits that are 1.25 percent lower than the current-

²¹See Table 5.A1 in Social Security Administration (2001).

law level throughout the rest of lifetime; households of age 64 in year 1 receive benefits 2.5 percent lower than the current law-level, and so on. Households age 25 or younger in year 1 receive one half of their traditional Social Security benefits when they turn 65.

Since benefit payments are reduced more slowly than payroll taxes during the transitional period, Social Security faces a cash-flow deficit that we finance with a consumption tax; work by Kotlikoff, Smetters and Walliser (2001) also focused on consumption tax financing as a method of producing the largest macroeconomic gains from privatization. Since changes to the Social Security system will also influence the size of the capital stock and, hence, non-Social Security government receipts, the rest of the government budget is balanced by proportional changes in marginal income tax rates.

We first consider the representative-agent economy without wage shocks (equivalently, with *insurable* wage shocks) where all households have the wage profile of the 40–60th percentile in Table 4, i.e., lifetime income group e^3 . We then turn to a heterogeneous-agent economy with *uninsurable* wage shocks. We initially assume that both economies are closed to international capital flows and that a private annuity market does not exist.

4.1 Representative-Agent Economy without Wage Shocks

As shown in Run 1 in Table 8, partial privatization of Social Security in the representative-agent economy increases national wealth by 20.7 percent in the long run compared to the baseline economy. GNP increases by 9.3 percent in the long run, while labor supply increases by 4.8 percent. These large gains are driven by pre-funding a portion of Social Security’s liabilities that were previously financed on a pay-as-you-go (unfunded) basis.

Moreover, national wealth and GNP increase throughout the entire transition path. Labor supply initially increases by 3.4 percent in the first year as households reoptimize their lifecycle choices after the policy change. Then, it declines slightly between years 1 and 20 after the reform before eventually increasing. The reason for this non-monotonic behavior is two-fold. First, as shown in Table 8, labor supply increases faster than capital during part of the first decade after reform, briefly decreasing the wage rate by about 1 percent. Second, the new temporary consumption tax, which starts at 6.5 percent and comes down over time as Social Security benefits are gradually reduced, discourages labor supply during the

Table 8: Percent Change in Selected Macro Variables Relative to Baseline

Run #	Year t	GNP	National Wealth	Labor Supply	Interest Rate	Wage Rate	Income Tax Rate* ²	Consumption Tax Rate
1 Representative Agent without Wage Shocks* ¹	1	2.4	0.0	3.4	4.2	-1.0	-13.4	6.5
	10	3.8	6.4	2.7	-4.3	1.1	-14.6	5.7
	20	5.4	10.9	3.1	-8.7	2.2	-16.6	4.3
	40	8.3	17.6	4.6	-13.8	3.6	-20.1	1.1
	Long Run	9.3	20.7	4.8	-16.4	4.3	-21.2	-0.2
2 Heterogenous Agents with Wage Shocks* ¹	1	1.3	0.0	1.8	2.3	-0.5	-5.7	5.5
	10	2.5	4.3	1.7	-3.0	0.7	-6.6	4.8
	20	4.0	8.1	2.2	-6.7	1.7	-8.1	3.6
	40	6.7	15.1	3.4	-12.7	3.3	-10.6	0.9
	Long Run	7.8	18.7	3.5	-16.0	4.2	-11.3	-0.2

*1. Closed economy, no private annuity markets, and LSRA is off.

*2. The proportional change in marginal tax rates across all households.

transition.²²

Despite these positive gains to economic variables, not everyone wins from privatization. As shown for Run 1 in Table 9 (“Without LSRA”), all households alive at the time of the reform (that is, aged 20 or older) are worse off. For example, the aged-20 household at the time of policy change *loses* \$16,900, as measured by the equivalent variation of an one-time wealth transfer made at the time of the reform. The aged-40 household loses \$60,000. These losses are mainly caused by the transitional consumption tax, which especially harms middle-aged households that hold a large amount of non-Social Security wealth.²³ In a closed economy, these households are also hurt by the fall in the interest rate shown in Table 8.²⁴

Future households, who pay very little of the transition costs gain substantially from privatization. For example, newborns in Year 1 gain about \$24,600 per couple while generations born 20 years later gain about \$47,100. These gains arise mainly from higher wages and from the reduced distortions from payroll taxes. Federal income taxes also decline by 21.2 percent in the long run due to an expanded economy.

²²The consumption tax does not return exactly to zero in the long run due to a larger labor supply interacting with the progressive benefit schedule within the smaller Social Security system that remains after privatization.

²³Social Security wealth under law and in our model is protected by inflation, including that caused by consumption taxes.

²⁴For example, the welfare loss of a 40-year old is reduced by half in a small open economy (not shown), where the interest rate does not change.

Table 9: Change in Welfare per Household (1,000 dollars in 2001)

Run #	Age in Year 1	Without LSRA * ¹				With LSRA * ²
		Select Productivities				For all Productivities
		e^1	e^3	e^5	e^8	
1	79	-	-7.5	-	-	0.0
Representative	60	-	-47.4	-	-	0.0
Agent without	40	-	-60.0	-	-	0.0
Wage Shocks	20	-	-16.9	-	-	0.0
	0	-	24.6	-	-	21.9
	-20	-	47.1	-	-	21.9
2	79	-4.8	-5.7	-14.7	-79.3	0.0
Heterogenous	60	-27.6	-43.5	-64.4	-361.8	0.0
Agents with	40	-18.7	-46.7	-76.4	-368.4	0.0
Wage Shocks	20	2.2	-1.5	-5.2	-15.5	0.0
	0	32.8	33.7	36.1	43.4	-5.6
	-20	52.4	56.7	63.5	84.3	-5.6

*1. Standard equivalent variations measures.

*2. Value of Δtr .

Does privatization actually produce overall *efficiency gains*, that is, after the “winners” compensate the “losers?” To make this determination, we simulate the same experiment but with an operative Lump-Sum Redistribution Authority (LSRA).²⁵ Recall that the LSRA transfers exactly enough wealth to would-be losing households alive at the time of the reform so that their remaining expected lifetime utilities return to their pre-reform levels. These transfers must be financed with borrowing that is financed with the gains to future generations. All net new resources (positive or negative) are then distributed equally to future households (growth adjusted over time). As shown in Table 9, privatization produces about \$21,900 (in 2001 growth-adjusted dollars) in additional net resources per each future household that enters the economy in year 2 and later. In other words, the 50-percent phased-in privatization does indeed increase efficiency in our deterministic economy setting.

4.2 Heterogeneous-Agent Economy with Wage Shocks

Run 2 in Table 8 shows the effects of the same stylized privatization experiment in a more realistic economy with *uninsurable* wage shocks. Unlike in the deterministic economy,

²⁵To save on space, we don't report the change in macroeconomic variables for the experiments with the LSRA. These tables are available upon request.

Social Security's progressive benefit formula now provides some risk sharing that is unattainable in the private market. National wealth now increases by 18.7 percent in the long run, but by less than in the representative-agent economy (Run 1) because a portion of private saving is now for precautionary motives, which is less sensitive to changes in Social Security policy. Labor supply increases by 3.5 percent in the long run and GNP is 7.8 percent higher.

Similar to the representative-agent economy, most households alive at the time of the reform are worse off because they have to pay higher taxes to finance the transition. Run 2 in Table 9 shows that relatively high wage (and, hence, wealthier) households tend to be hit the hardest by the consumption tax. For example, a 40-year old in the top one percent of the wage distribution (e^8) at the time of privatization loses \$368,400. As with Run 1, future households gain considerably from the lower payroll and income tax rates as well as higher wages. Even households in the lowest 20 percent of the wage distribution (e^1) born 20 years later gain \$52,400 (in 2001 growth-adjusted dollars). Overall, privatization, though, no longer improves efficiency. After the LSRA returns the welfare of all households to their pre-reform levels, it distributes a *negative* \$5,600 to each future household. This loss contrasts sharply with the gain of \$21,900 in the deterministic economy discussed above.

4.3 Alternative Experiments in the Heterogeneous-Agent Economy

In a Small Open Economy. Run 3 shows the effect of privatization in the setting of a small open economy where any changes in the capital-labor ratio are nullified immediately by international capital flows. Since interest rates do not decline, national wealth increases by more than in the closed economy setting. The increase in labor supply is also smaller since the wage rate does not rise. Table 10 shows that GNP increases gradually and by 11.5 percent in the long run.

Table 11 shows that the welfare losses of households alive at the time of the reform tend to be smaller relative to the closed economy (Run 2, Table 9), mostly due to the fixed interest rate. Furthermore, the gains to future households tend to be a little larger in the small open economy, especially for wealthier households who see the reduction in the rate of return to capital in the closed economy but not in the small open economy. On the whole, it might appear at first that privatization in an open economy setting would produce a smaller

Table 10: Alternative Experiments in the Heterogenous Agent Economy with Wage Shocks
Percent Change in Selected Macro Variables Relative to Baseline

Run #	Year t	GNP	National Wealth	Labor Supply	Interest Rate	Wage Rate	Income Tax Rate*2	Consump- tion Tax Rate
3 Small Open Economy*1	1	2.1	0.0	3.0	0.0	0.0	-7.3	5.4
	10	3.6	7.3	2.0	0.0	0.0	-7.5	4.9
	20	5.6	14.5	1.8	0.0	0.0	-8.6	3.7
	40	9.3	27.1	1.7	0.0	0.0	-10.4	1.0
	Long Run	11.5	36.5	0.8	0.0	0.0	-10.8	-0.1
4 Perfect Annuity Markets*1	1	1.2	0.0	1.8	2.1	-0.5	-5.8	5.5
	10	2.3	4.2	1.5	-3.1	0.8	-6.4	4.8
	20	3.5	7.4	1.9	-6.0	1.6	-7.6	3.7
	40	5.7	12.4	2.9	-10.0	2.7	-9.5	0.9
	Long Run	6.4	14.4	3.2	-11.7	3.1	-10.1	-0.2
5 Contribution Matching Starting at 10%*1	1	0.0	0.0	-0.1	-0.1	0.0	0.5	5.6
	10	0.7	2.2	0.1	-2.5	0.6	-0.4	5.0
	20	2.0	5.1	0.8	-5.1	1.3	-2.0	3.8
	40	4.7	11.5	2.0	-10.6	2.7	-4.6	1.0
	Long Run	5.9	15.1	2.1	-14.0	3.6	-5.6	-0.2
6 Contribution Matching Starting at 20%*1	1	-1.9	0.0	-2.7	-3.4	0.8	9.0	5.8
	10	-1.5	-0.5	-2.0	-1.8	0.4	8.0	5.3
	20	-0.5	1.2	-1.2	-3.0	0.8	6.5	4.0
	40	2.2	7.2	0.1	-8.2	2.1	3.3	1.0
	Long Run	3.4	11.0	0.3	-12.0	3.1	2.1	-0.1
7 More Progressive S.S. Bend Points 120 / 32 / 10%*1	1	0.5	0.0	0.7	0.9	-0.2	-4.2	7.1
	10	1.3	2.2	0.8	-1.7	0.4	-5.1	6.4
	20	2.6	5.3	1.5	-4.5	1.1	-6.6	5.0
	40	5.5	12.2	2.7	-10.5	2.7	-9.2	1.8
	Long Run	6.7	16.1	2.9	-14.2	3.7	-10.1	0.4
8 More Progressive S.S. Bend Points 150 / 32 / 10%*1	1	-0.2	0.0	-0.3	-0.3	0.1	-3.0	8.9
	10	0.1	0.3	0.1	-0.3	0.1	-3.6	8.3
	20	1.3	2.6	0.7	-2.2	0.6	-5.1	6.6
	40	4.3	9.4	2.1	-8.2	2.1	-8.0	2.8
	Long Run	5.5	13.4	2.3	-12.1	3.1	-8.9	1.2

*1. Each Run represents one change in assumption relative to Run 2, i.e., the changes are not cumulative.

*2. The proportional change in marginal tax rates across all households.

Table 11: Change in Welfare per Household (1,000 dollars in 2001)

Run #	Age in Year 1	Without LSRA * ¹ Select Productivities				With LSRA * ² For all
		e^1	e^3	e^5	e^8	Productivities
3 Small Open Economy* ³	79	-4.8	-5.6	-14.4	-75.7	0.0
	60	-25.8	-40.1	-54.6	-184.8	0.0
	40	-15.3	-37.3	-57.1	-155.5	0.0
	20	1.1	1.6	3.5	15.9	0.0
	0	23.6	29.9	39.2	73.0	-6.6
	-20	39.8	50.1	64.2	112.5	-6.6
4 Perfect Annuity Markets* ³	79	-4.3	-5.1	-12.7	-63.1	0.0
	60	-21.5	-33.4	-47.9	-296.9	0.0
	40	-13.1	-32.4	-51.5	-288.5	0.0
	20	4.9	3.3	2.5	1.1	0.0
	0	24.6	26.6	30.2	43.5	-7.2
	-20	36.2	40.7	47.7	72.3	-7.2
5 Contribution Matching Starting at 10%* ³	79	-5.3	-6.3	-16.5	-91.2	0.0
	60	-28.3	-40.8	-68.3	-558.4	0.0
	40	-9.3	-48.8	-86.8	-471.4	0.0
	20	11.6	7.1	-4.3	-32.1	0.0
	0	40.4	40.4	34.9	24.1	-4.4
	-20	60.5	63.7	62.7	65.7	-4.4
6 Contribution Matching Starting at 20%* ³	79	-6.0	-7.1	-19.0	-107.6	0.0
	60	-29.4	-38.5	-74.0	-825.8	0.0
	40	-0.2	-53.1	-103.8	-614.9	0.0
	20	19.3	13.4	-6.5	-56.8	0.0
	0	45.9	44.6	30.3	-4.3	-9.9
	-20	66.6	68.5	58.7	38.8	-9.9
7 More Progressive S.S. Bend Points 120 / 32 / 10%* ³	79	34.2	32.9	20.7	-69.3	0.0
	60	7.1	6.1	-25.2	-469.9	0.0
	40	-13.2	-41.1	-76.4	-407.2	0.0
	20	-4.9	-9.8	-15.3	-32.6	0.0
	0	25.8	26.0	26.9	28.3	-0.1
	-20	47.9	51.6	57.6	74.2	-0.1
8 More Progressive S.S. Bend Points 150 / 32 / 10%* ³	79	76.5	75.2	61.9	-45.6	0.0
	60	47.8	59.0	25.7	-548.5	0.0
	40	-9.8	-37.7	-75.5	-442.6	0.0
	20	-13.4	-19.4	-26.9	-50.5	0.0
	0	17.6	16.8	16.3	12.0	2.7
	-20	41.7	45.1	50.1	63.0	2.7

*1. Standard equivalent variations measures.

*2. Value of Δtr .

*3. Each Run represents one change in assumption relative to Run 2, i.e., the changes are not cumulative.

efficiency loss than in the closed economy. This hunch is incorrect. When the LSRA is operative, Table 11 shows that the efficiency losses are actually slightly *larger* in the small open economy, equal to \$6,600 per each future household. The reason is that the LSRA's cost of borrowing is higher in the small open economy setting since the interest rate does not decrease over time.

Perfect Annuity Markets. Thus far, we have assumed private annuity markets do not exist and so, in addition to sharing wage uncertainty, the Social Security system shares *longevity* uncertainty in a way the private market cannot. It would appear at first glance that privatization has a better chance of producing efficiency gains if we instead assumed that a private annuity market is available. This intuition also turns out to be incorrect.

Run 4 in Table 11 shows that the efficiency losses are actually *larger* with perfect private annuity markets than without (Run 2). In particular, each household loses \$7,200, compared to \$5,600 shown earlier (Run 2, Table 9) without an annuity market. As shown for Run 2 in Table 8, privatization *without* a private annuity market leads to a 18.7 percent increase in national wealth in the long run, which is larger than the 14.4 percent increase for Run 4 shown in Table 10. The reason is that households increase their precautionary savings in Run 2 as the annuity insurance provided by Social Security is reduced; in contrast, households can rely more on the private annuity market rather than precautionary savings in Run 4. The smaller amount of precautionary savings in Run 4 produces larger efficiency losses for three reasons: (i) the LSRA must borrow at a higher interest rate; (ii) income taxes are higher since there is less capital and labor income; and (iii) the interest elasticity of saving is higher, increasing the role that falling interest rates have on discouraging additional saving.

Contribution Matching. The pre-reform Social Security system is progressive by giving households with a lower average index of pre-retirement wages a larger Social Security benefit relative to their pre-retirement wages, that is, a higher “replacement rate.” To be sure, the privatized experiment considered above preserves some progressivity by reducing benefits proportionally only by half in the long run—the poor still get a larger replacement rate. Yet, a proportional reduction in benefits still reduces progressivity; for example, reducing all

benefits by 100 percent would eliminate progressivity.

Run 5 considers privatization with a progressive contribution match. In particular, working households with low levels of labor income receive a fairly generous match equal to 10 percent of their *earnings*. This matching rate declines linearly to zero as labor income approaches \$60,000, which is slightly above the median household income in the model economy.²⁶ While ensuing Social Security deficits continue to be financed with a consumption tax, the contribution match is financed each year by increasing the marginal income tax rates proportionally across all households; the key qualitative results would be unchanged if the consumption tax were used instead. As before, income tax rates are changed so that the rest of government budget is balanced throughout the transition path.

As shown in Run 5 in Table 10, privatization with contribution matching increases national wealth, GNP and labor supply throughout the transition path compared to the pre-reform baseline economy. However, the long-run increases in national wealth, labor supply, and GNP are smaller relative to the case without contribution matching (Run 2 in Table 8) since the match must be financed with a distorting income tax. In the long run, GNP increases by only 5.9 percent, compared to 7.8 percent without the match.

The welfare gains for Run 5 reported in Table 11 show that contribution matching tends to improve the welfare of poorer households relative to Run 2 without the match. Whereas the poorest household born in the future gains \$52,400 without the match, they gain \$60,500 with the match. Not surprisingly, the richest households are worse off since they don't receive any of the match but must help finance it; they gained \$84,300 without the match in the long run but only \$65,700 with the match. With the LSRA, privatization still leads to efficiency losses, equal to about \$4,400 (in 2001 growth-adjusted dollars) per future household. Yet, this loss is smaller than the \$5,600 loss without the match.

Run 6 doubles the starting match rate from 10 percent at zero wage income to 20 percent. As before, this match declines linearly to zero as labor income approaches \$60,000. Table 11 shows that, with the LSRA, this more generous match produced efficiency *losses* equal to \$9,900 per future household, which is larger in magnitude relative to the 10 percent match

²⁶This matching schedule is equivalent with the marginal labor income tax of -10 percent at \$0 of labor income, 0 percent at \$30,000, 10 percent at \$60,000, and 0 percent for labor income above \$60,000.

and even Run 2 without *any* match. This non-monotonic behavior is due to the trade-off between risk sharing and labor supply distortions: *some* match is beneficial but is dominated by distortions at higher tax rates.²⁷ In fact, there is no match rate that allows privatization to produce efficiency gains. Moreover, several alternative designs of contribution matching performed even worse.²⁸

Progressive Benefit Schedule. Run 7 takes a different approach to maintaining some progressivity after privatization. It immediately increases the progressivity of the Social Security benefit that remains after privatization by raising the replacement rate of the lowest wage income bracket from 90 percent to 120 percent while reducing the replacement rate of the highest wage income bracket from 15 percent to 10 percent. Run 8 is even more aggressive in redistribution by raising the replacement rate of the lowest wage income bracket to 150 percent. Table 11 shows that this approach is especially effective at protecting the welfare of the poor at the time of reform as well as reducing efficiency losses. Now, privatization reduces efficiency by only \$100 per future household under the LSRA in Run 8. However, privatization actually increases efficiency by \$2,700 per future household in Run 8. Increasing the progressivity of the smaller Social Security system that remains after privatization is more efficient than contribution matching because a considerable amount of redistribution can be accomplished with less distortion to labor supply, that is, with smaller effective tax rates at the margin. In particular, whereas contribution matching is based on the labor income in any given year, Social Security's progressive benefit is based on a household's lifetime earnings, which is harder to change.

²⁷These distortions arise from both the income tax used to finance the match as well as the effective marginal tax rates caused by the match itself.

²⁸For example, a proportional match for *all* households performed worse because, while eliminating implicit marginal tax rates in the phase-out range, enhanced the income tax distortions since more revenue is needed. We also considered financing the phased-out match with a *negative* match on those with above-average incomes. Although potentially more efficient at redistribution than an income tax, as the poor are not financing their own match, it also performed worse. Labor supply tends to be fairly elastic in our model whereas the savings elasticity is relatively low with precautionary saving.

Table 12: Alternative Transitory Shocks
Percent Change in Selected Macro Variables Relative to Baseline

Run #	Year t	GNP	National Wealth	Labor Supply	Interest Rate	Wage Rate	Income Tax Rate* ²	Consumption Tax Rate
9 $\frac{1}{2}$ Transitory Shocks* ¹	1	1.5	0.0	2.2	2.7	-0.7	-6.3	5.6
	10	3.0	5.0	2.1	-3.4	0.8	-7.5	4.9
	20	4.6	9.4	2.6	-7.7	1.9	-9.2	3.7
	40	7.5	16.6	3.8	-13.7	3.6	-11.7	0.9
	Long Run	8.7	20.4	4.0	-17.1	4.5	-12.6	-0.3
10 $\frac{1}{5}$ Transitory Shocks* ¹	1	1.6	0.0	2.3	2.8	-0.7	-6.4	5.7
	10	3.1	5.4	2.2	-3.8	0.9	-7.8	5.0
	20	4.9	10.1	2.7	-8.3	2.1	-9.6	3.7
	40	7.9	17.4	4.0	-14.3	3.7	-12.4	0.9
	Long Run	9.1	21.4	4.2	-17.8	4.7	-13.4	-0.3

*1. Each Run represents one change in assumption relative to Run 2, i.e., the changes are not cumulative.

*2. The proportional change in marginal tax rates across all households.

4.4 Alternative Assumptions of Transitory Shocks and Persistence

A key assumption in our model is the size of the transitory working ability shocks and their persistence. Recall that we constructed the age-working ability schedule from the 2001 Survey of Consumer Finances (SCF) and the transition matrices from the 1989-92 Panel Study of Income Dynamics (PSID). To deal with possible measurement errors, our benchmark Markov transition matrixes are calculated after taking three-year moving averages of workers' hourly wages. This *ad hoc* treatment reduces the size of transitory wage shocks in the original data by about one-third. Floden and Lindé (2001) argue that measurement error in the PSID might be as large as the size of the real fluctuation. Thus, although we have already "smoothed away" some of that error by focusing on three-year moving averages, the transitory shocks in our model might still be too large.

Run 9 shown in Tables 12 and 13 show the economic and welfare effects, respectively, of privatization when the transitory shocks are reduced to only half of their previous values we used in the main calibration. Run 10 goes even further: it reduces the shocks to only *one-fifth* of the values under the main calibration. National wealth increases by 20.4 percent in the long run in Run 9, compared to 18.7 percent for Run 2. However, the efficiency losses actually *increase* to \$8,200 per future household (relative to a \$5,600 loss in Run 2) under the

Table 13: Change in Welfare per Household (3) (1,000 dollars in 2001)

Run #	Age in Year 1	Without LSRA * ²				With LSRA * ³
		Select Productivities				For all Productivities
		e^1	e^3	e^5	e^8	
9	79	-4.2	-5.1	-13.8	-116.5	0.0
$\frac{1}{2}$ Transitory Shocks* ¹	60	-25.6	-44.4	-65.4	-388.0	0.0
	40	-28.0	-58.6	-81.4	-419.1	0.0
	20	-7.2	-12.9	-15.4	-13.1	0.0
	0	18.7	21.6	31.6	73.0	-8.2
	-20	35.2	43.6	61.1	127.4	-8.2
10	79	-4.8	-6.1	-13.1	-171.5	0.0
$\frac{1}{5}$ Transitory Shocks* ¹	60	-26.3	-48.4	-68.6	-411.4	0.0
	40	-40.5	-73.7	-84.3	-408.6	0.0
	20	-19.5	-26.9	-22.6	16.9	0.0
	0	0.6	8.6	35.8	161.5	-5.8
	-20	13.6	30.3	69.7	241.0	-5.8

*1. Each Run represents one change in assumption relative to Run 2.

*2. Standard equivalent variations measures.

*3. Value of Δtr .

LSRA. This counter-intuitive result can be explained by the fact that a reduction in transitory shocks also increases the *persistence* of any shock. As a result, the effect of any negative shock becomes more permanent, potentially increasing the value of the risk sharing in the former Social Security system.²⁹

In the limit, the model collapses to one with no wage uncertainty if the transitory shocks are eliminated. In Run 10, the transitory shocks are reduced to one-fifth of their benchmark levels and in this case, privatization produces a smaller efficiency loss, compared to Run 9, equal to about \$5,800 per future household. These two policy experiments show that changes in the amount of wage uncertainty have a *non-monotonic* impact on efficiency gains.

5 Concluding Remarks

This paper investigated whether a stylized Social Security privatization generates efficiency gains or losses in the presence of an overlapping-generations economy with elastic labor supply and idiosyncratic wage shocks and longevity uncertainty. We found that partial privatization of Social Security produces efficiency gains in a representative-agent economy

²⁹We benefited from a helpful conversation with Dirk Krueger on this point.

without wage shocks (or, equivalently, if these shocks are insurable). In a heterogeneous-agent economy with idiosyncratic and uninsurable wage shocks, however, the overall efficiency of the economy is reduced by our stylized privatization since the existing Social Security system provides a valuable source of risk sharing through its progressive benefit formula. This result was fairly robust to a wide range of model considerations: (i) the degree of openness of the economy; (ii) allowing the availability of actuarially-fair private annuities; (iii) the introduction of various risk-sharing mechanisms after privatization; and (iv) reducing by half the size of transitory wage shocks. Only in the case in which the benefit replacement rate of the lowest bracket was increased to 150 percent from 90 percent did we find efficiency gains. Rather surprisingly, privatization performs *relatively* better in a closed economy where the rate of return falls as capital is accumulated and even when private annuity markets do *not* exist. Furthermore, matching the contribution of poorer workers can actually do more harm than good, whereas increasing the progressivity of the Social Security system that remains works well.

One of the possible limitations of our model is that it does not distinguish by various demographic groups, including race and gender. There is some evidence, for example, that black Americans do not live as long as non-blacks, even after controlling for differences in earnings. Blacks are also less likely to be married at the point of retirement and, therefore, less likely to qualify for a spousal benefit.³⁰ In contrast, women have a higher life expectancy than males, and they might also face a higher effective marginal tax rate on their contributions if they are the household's secondary earner. Incorporating these additional sources of heterogeneity would possibly change the welfare implications of this paper.

³⁰See, for example, Gustman and Steinmeier (2001).

Appendices

A Transition Matrixes and Survival Rates

Markov Transition Matrixes. The Markov transition matrixes of working ability are constructed for four age groups—20-29, 30-39, 40-49, and 50-59—from the hourly wages in the PSID individual data 1990, 91, 92, and 93. To reduce the size of transitory shocks, possibly caused by measurement errors, the transition matrix of each age group is calculated with transition probabilities from the average wages between 1989 and 91 to the average wages between of 1990 and 92. For households aged between 60 and 78, we used the matrix for ages 50-59.

$$\Gamma_{i \in \{20, \dots, 29\}} = \begin{pmatrix} 0.7601 & 0.2101 & 0.0289 & 0.0000 & 0.0009 & 0.0000 & 0.0000 & 0.0000 \\ 0.1919 & 0.6171 & 0.1635 & 0.0221 & 0.0054 & 0.0000 & 0.0000 & 0.0000 \\ 0.0406 & 0.1546 & 0.5976 & 0.1794 & 0.0153 & 0.0068 & 0.0028 & 0.0029 \\ 0.0063 & 0.0181 & 0.1787 & 0.6723 & 0.1128 & 0.0118 & 0.0000 & 0.0000 \\ 0.0018 & 0.0000 & 0.0506 & 0.2524 & 0.5792 & 0.0995 & 0.0165 & 0.0000 \\ 0.0000 & 0.0000 & 0.0244 & 0.0000 & 0.2744 & 0.5719 & 0.1293 & 0.0000 \\ 0.0010 & 0.0000 & 0.0000 & 0.0000 & 0.0372 & 0.1937 & 0.6646 & 0.1035 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.4727 & 0.5273 \end{pmatrix},$$

$$\Gamma_{i \in \{30, \dots, 39\}} = \begin{pmatrix} 0.8537 & 0.1332 & 0.0082 & 0.0049 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1283 & 0.6837 & 0.1744 & 0.0129 & 0.0007 & 0.0000 & 0.0000 & 0.0000 \\ 0.0175 & 0.1723 & 0.6788 & 0.1269 & 0.0000 & 0.0045 & 0.0000 & 0.0000 \\ 0.0005 & 0.0109 & 0.1314 & 0.7374 & 0.1014 & 0.0144 & 0.0040 & 0.0000 \\ 0.0000 & 0.0000 & 0.0144 & 0.2008 & 0.7016 & 0.0728 & 0.0104 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0460 & 0.1874 & 0.6182 & 0.1484 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0298 & 0.0011 & 0.2008 & 0.7147 & 0.0536 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.2145 & 0.7855 \end{pmatrix},$$

$$\Gamma_{i \in \{40, \dots, 49\}} = \begin{pmatrix} 0.8561 & 0.1371 & 0.0068 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1301 & 0.7275 & 0.1222 & 0.0145 & 0.0057 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1111 & 0.7145 & 0.1588 & 0.0000 & 0.0156 & 0.0000 & 0.0000 \\ 0.0000 & 0.0179 & 0.1484 & 0.6992 & 0.1221 & 0.0124 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0162 & 0.2548 & 0.6594 & 0.0696 & 0.0000 & 0.0000 \\ 0.0252 & 0.0257 & 0.0000 & 0.0000 & 0.1702 & 0.6744 & 0.1045 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0933 & 0.8366 & 0.0701 \\ 0.1496 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1308 & 0.7196 \end{pmatrix},$$

$$\Gamma_{i \in \{50, \dots, 78\}} = \begin{pmatrix} 0.8249 & 0.1629 & 0.0000 & 0.0122 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1552 & 0.6754 & 0.1694 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0200 & 0.0997 & 0.7643 & 0.1052 & 0.0108 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0620 & 0.0663 & 0.8079 & 0.0559 & 0.0079 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.1492 & 0.7194 & 0.1314 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.2942 & 0.6239 & 0.0819 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1024 & 0.8573 & 0.0403 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1613 & 0.8387 \end{pmatrix},$$

where $\Gamma_i(j, k) = \pi(e_{i+1} = e_{i+1}^k | e_i = e_i^j)$.

Survival Rates of Households. The survival rates ϕ_i at the end of age $i = \{20, \dots, 109\}$ are the weighted averages of males and females and calculated from the period life table (Table 4.C6) in Social Security Administration (2001). The survival rates at the end of age 109 are replaced by zero.

Table 14: Survival Rates in 1998 in the United States (Weighted Average of Males and Females)

Age	Survival Rate	Age	Survival Rate	Age	Survival Rate	Age	Survival Rate	Age	Survival Rate
20	0.999104	40	0.997966	60	0.989315	80	0.937788	100	0.676630
21	0.999057	41	0.997807	61	0.988305	81	0.931527	101	0.658554
22	0.999027	42	0.997640	62	0.987134	82	0.924684	102	0.639355
23	0.999018	43	0.997451	63	0.985773	83	0.917252	103	0.618962
24	0.999023	44	0.997252	64	0.984249	84	0.909150	104	0.597297
25	0.999034	45	0.997027	65	0.982548	85	0.900275	105	0.574281
26	0.999040	46	0.996778	66	0.980759	86	0.890541	106	0.549828
27	0.999033	47	0.996514	67	0.979000	87	0.879882	107	0.523850
28	0.999006	48	0.996237	68	0.977325	88	0.868264	108	0.496251
29	0.998962	49	0.995938	69	0.975647	89	0.855676	109	0.000000
30	0.998911	50	0.995603	70	0.973769	90	0.842119		
31	0.998857	51	0.995222	71	0.971613	91	0.827606		
32	0.998796	52	0.994797	72	0.969264	92	0.812154		
33	0.998727	53	0.994324	73	0.966703	93	0.795784		
34	0.998651	54	0.993795	74	0.963868	94	0.778522		
35	0.998564	55	0.993198	75	0.960661	95	0.761075		
36	0.998466	56	0.992534	76	0.957027	96	0.743640		
37	0.998358	57	0.991818	77	0.952967	97	0.726432		
38	0.998240	58	0.991051	78	0.948449	98	0.709688		
39	0.998111	59	0.990216	79	0.943423	99	0.693653		

Source: Authors' calculations from the Table 4.C6, Social Security Administration (2001).

B The Computation of Equilibria

The algorithm to solve the model for a steady-state equilibrium and an equilibrium transition path is similar to those in Conesa and Krueger (1999), Nishiyama (2002), and Nishiyama and Smetters (2004).³¹

B.1 The Discretization of the State Space

The state of a household is $s_i = (i, e_i, a_i, b_i) \in I \times E \times A \times B$, where $I = \{20, \dots, 109\}$, $E = [e^{\min}, e^{\max}]$, $A = [a^{\min}, a^{\max}]$, and $B = [b^{\min}, b^{\max}]$. To compute an equilibrium, the state space of a household is discretized as $\hat{s}_i \in I \times \hat{E}_i \times \hat{A} \times \hat{B}$, where $\hat{E}_i = \{e_i^1, e_i^2, \dots, e_i^{N_e}\}$,

³¹The authors are grateful to José Víctor Ríos-Rull for his teaching of the computational procedure of heterogeneous agent models.

$\hat{A} = \{a^1, a^2, \dots, a^{N_a}\}$, and $\hat{B} = \{b^1, b^2, \dots, b^{N_b}\}$. For all these discrete points, the model computes the optimal decision of households, $\mathbf{d}(\hat{\mathbf{s}}_i, \mathbf{S}_t; \Psi_t) = (c_i(\cdot), h_i(\cdot), a_{i+1}(\cdot)) \in (0, c^{\max}] \times [0, h_i^{\max}] \times A$, the marginal values, $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}_i, \mathbf{S}_t; \Psi_t)$ and $\frac{\partial}{\partial b} v(\hat{\mathbf{s}}_i, \mathbf{S}_t; \Psi_t)$, and the values $v(\hat{\mathbf{s}}_i, \mathbf{S}_t; \Psi_t)$, given the expected factor prices and policy variables.³²

To find the optimal end-of-period wealth, the model uses the Euler equation and bilinear interpolation (with respect to a and b) of marginal values at the beginning of the next period.³³ In a heterogeneous-agent economy, N_e , N_a , and N_b are 8, 57, and 8, respectively. In a representative-agent economy, the numbers of grid points are 1, 61, and 6, respectively.³⁴

B.2 A Steady-State Equilibrium

The algorithm to compute a steady-state equilibrium is as follows. Let Ψ denote the time-invariant government policy rule $\Psi = (W_{LS}, W_G, C_G, \tau_I(\cdot), \tau_P(\cdot), \tau_C, tr_{SS}(\hat{\mathbf{s}}_i), tr_{LS}(\hat{\mathbf{s}}_i))$.

1. Set the initial values of factor prices (r^0, w^0) , accidental bequests q^0 , the policy variables $(W_{LS}^0, C_G^0, \tau_C^0)$, and the parameters $(\varphi_I^0, \varphi_{SS}^0)$ of policy functions $(\tau_I(\cdot), tr_{SS}(\hat{\mathbf{s}}_i))$ if these are determined endogenously.³⁵
2. Given $\Omega^0 = (r^0, w^0, q^0, W_{LS}^0, C_G^0, \tau_C^0, \varphi_I^0, \varphi_{SS}^0)$, find the decision rule of a household $\mathbf{d}(\hat{\mathbf{s}}_i; \Psi, \Omega^0)$ for all $\hat{\mathbf{s}}_i \in I \times \hat{E}_i \times \hat{A} \times \hat{B}$.³⁶
 - (a) For age $i = 109$, find the decision rule $\mathbf{d}(\hat{\mathbf{s}}_{109}; \Psi, \Omega^0)$. Since the survival rate $\phi_{109} = 0$, the end-of-period wealth $a_{i+1}(\hat{\mathbf{s}}_{109}; \cdot) = 0$ for all $\hat{\mathbf{s}}_{109}$. Compute consumption and working hours $(c_i(\hat{\mathbf{s}}_{109}; \cdot), h_i(\hat{\mathbf{s}}_{109}; \cdot))$ and, then, marginal values $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}_{109}; \Psi, \Omega^0)$ and values $v(\hat{\mathbf{s}}_{109}; \Psi, \Omega^0)$ for all $\hat{\mathbf{s}}_{109}$.³⁷
 - (b) For age $i = 108, \dots, 20$, find the decision rule $\mathbf{d}(\hat{\mathbf{s}}_i; \Psi, \Omega^0)$, marginal values $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}_i; \Psi, \Omega^0)$, and values $v(\hat{\mathbf{s}}_i; \Psi, \Omega^0)$ for all $\hat{\mathbf{s}}_i$, using $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}_{i+1}; \Psi, \Omega^0)$ and $\frac{\partial}{\partial b} v(\hat{\mathbf{s}}_{i+1}; \Psi, \Omega^0)$ recursively.
 - i. Set the initial guess of $a_{i+1}^0(\hat{\mathbf{s}}_i; \cdot)$.

³²Because the marginal value with respect to historical earnings, $\frac{\partial}{\partial b} v(\hat{\mathbf{s}}_i, \mathbf{S}_t; \Psi_t)$, is difficult to obtain analytically, it is approximated by $(v(\cdot, b^{j+1}, \mathbf{S}_t; \Psi_t) - v(\cdot, b^j, \mathbf{S}_t; \Psi_t)) / (b^{j+1} - b^j)$ where $j = 1, 2, \dots, N_b$.

³³The marginal values with respect to wealth, $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}_i, \mathbf{S}_t; \Psi_t)$, are used in the Euler equation to obtain optimal savings, the marginal values with respect to historical earnings, $\frac{\partial}{\partial b} v(\hat{\mathbf{s}}_i, \mathbf{S}_t; \Psi_t)$, are used in the marginal rate of substitution condition of consumption for leisure to obtain optimal working hours, and the values, $v(\hat{\mathbf{s}}_i, \mathbf{S}_t; \Psi_t)$, are used to calculate welfare changes measured by compensating and equivalent variations in wealth.

³⁴The grid points on A and B are not equally spaced. In a heterogeneous-agent economy, \hat{A} ranges from -\$271,000 to \$33,825,000 (in 2001 growth-adjusted dollars) and \hat{B} ranges from \$8,000 to \$80,400. In a representative-agent economy, \hat{A} and \hat{B} range from -\$333,000 to \$1,573,000 and from \$8,000 to \$80,400, respectively.

³⁵If we find the capital-labor ratio, both r and w are calculated from the given production function and depreciation rate. In this paper, the endogenous policy variables are C_G and φ_{SS} in baseline economies, and τ_C , φ_I , $tr_{LS}(\hat{\mathbf{s}}_i)$, and W_{LS} in policy experiments.

³⁶In the steady-state economy, the decision rule of a household $\mathbf{d}(\hat{\mathbf{s}}_i; \Psi, \Omega^0)$ is not a function of the aggregate state of economy $\hat{\mathbf{S}} = (x(\hat{\mathbf{s}}_i), W_{LS}, W_G)$. The measure of household $x(\hat{\mathbf{s}}_i)$ is determined uniquely by the steady-state condition, and the government's wealth W_G is determined by the policy rule Ψ .

³⁷The marginal value with respect to historical earnings, $\frac{\partial}{\partial b} v(\hat{\mathbf{s}}_i; \Psi, \Omega^0)$, is zero when $i > 60$ in this paper.

- ii. Given $a_{i+1}^0(\hat{s}_i; \cdot)$, compute $(c_i(\hat{s}_i; \cdot), h_i(\hat{s}_i; \cdot))$, using $\frac{\partial}{\partial b}v(\hat{s}_{i+1}; \Psi, \Omega^0)$. Plug these into the Euler equation with $\frac{\partial}{\partial a}v(\hat{s}_{i+1}; \Psi, \Omega^0)$.
 - iii. If the Euler error is sufficiently small, then stop. Otherwise, update $a_{i+1}^0(\hat{s}_i; \cdot)$ and return to Step ii.
3. Find the steady-state measure of households $x(\hat{s}_i; \Omega^0)$ using the decision rule obtained in Step 2. This computation is done forward from age 20 to age 109. Repeat this step to iterate q for q^1 .
 4. Compute new factor prices (r^1, w^1) , accidental bequests q^1 , the policy variables $(W_{LS}^1, C_G^1, \tau_C^1)$, and the parameters $(\varphi_I^1, \varphi_{SS}^1)$ of policy functions.
 5. Compare $\Omega^1 = (r^1, w^1, q^1, W_{LS}^1, C_G^1, \tau_C^1, \varphi_I^1, \varphi_{SS}^1)$ with Ω^0 . If the difference is sufficiently small, then stop. Otherwise, update Ω^0 and return to Step 2.

B.3 An Equilibrium Transition Path

Assume that the economy is in the initial steady state in period 0, and that the new policy schedule Ψ_1 , which was not expected in period 0, is announced at the beginning of period 1, where $\Psi_1 = \{W_{LS,t+1}, W_{G,t+1}, C_{G,t}, \tau_{I,t}(\cdot), \tau_{P,t}(\cdot), \tau_{C,t}, tr_{SS,t}(\hat{s}_i), tr_{LS,t}(\hat{s}_i)\}_{t=1}^\infty$. Let $\hat{S}_1 = (x_1(\hat{s}_i), W_{LS,1}, W_{G,1})$ be the state of the economy at the beginning of period 1. The state of the economy \hat{S}_1 is usually equal to that of the initial steady state. The algorithm to compute a transition path to a new steady-state equilibrium (thereafter, final steady-state equilibrium) is as follows.

1. Choose a sufficiently large number, T , such that the economy is said to reach the new steady state within T periods. Set the initial guess, $\Omega_1^0 = \{r_s^0, w_s^0, q_s^0, W_{LS,s}^0, \tau_{C,s}^0, \varphi_{I,s}^0\}_{s=1}^T$, on factor prices, accidental bequests, and the policy variables. Because there are no aggregate productivity shocks in this model, a time series $\Omega_t = \{r_s^0, w_s^0, q_s^0, W_{LS,s}^0, \tau_{C,s}^0, \varphi_{I,s}^0\}_{s=t}^T$ is deterministic, and each household perfectly foresees Ω_t based on the information \hat{S}_t in an equilibrium. Since \hat{S}_t is in a household decision rule only to make the household expect Ω_t rationally, in the computation, we can avoid the ‘‘curse of dimensionality’’ by replacing $\mathbf{d}(\hat{s}_i, \hat{S}_t; \Psi_t)$ with $\mathbf{d}(\hat{s}_i; \Psi_t, \Omega_t)$.
2. Given $W_{LS,T}$, find the final steady-state decision rule $\mathbf{d}(\hat{s}_i; \Psi_T, \Omega_T^0)$, marginal values, $\frac{\partial}{\partial a}v(\hat{s}_i; \Psi_T, \Omega_T^0)$, and values $v(\hat{s}_i; \Psi_T, \Omega_T^0)$ for all $\hat{s}_i \in I \times \hat{E}_i \times \hat{A} \times \hat{B}$. (See the algorithm for a steady-state equilibrium.)
3. For period $t = T - 1, T - 2, \dots, 1$, based on the guess, Ω_t^0 , find backward the decision rule $\mathbf{d}(\hat{s}_i; \Psi_t, \Omega_t^0)$, marginal values $\frac{\partial}{\partial a}v(\hat{s}_i; \Psi_t, \Omega_t^0)$, and values $v(\hat{s}_i; \Psi_t, \Omega_t^0)$ for all $\hat{s}_i \in I \times \hat{E}_i \times \hat{A} \times \hat{B}$, using the next period marginal values $\frac{\partial}{\partial a}v(\hat{s}_{i+1}; \Psi_{t+1}, \Omega_{t+1}^0)$ and values $v(\hat{s}_{i+1}; \Psi_{t+1}, \Omega_{t+1}^0)$ recursively.
4. For period $t = 1, 2, \dots, T - 1$, compute forward $(r_t^0, w_t^0, q_t^0, W_{LS,t}^0, \tau_{C,t}^0, \varphi_{I,t}^0)$ and the measure of households $x_{t+1}(\hat{s}_i)$, using the decision rule $\mathbf{d}(\hat{s}_i; \Psi_t, \Omega_t^0)$ obtained in Step 3 and using the state of economy $\hat{S}_t = (x_t(\hat{s}_i), W_{LS,t}, W_{G,t})$ recursively.

5. Compare Ω_t^1 with Ω_t^0 . If the difference is sufficiently small, then stop. Otherwise, update Ω_t^0 and return to Step 2.

B.4 The Lump-Sum Redistribution Authority

When the Lump-Sum Redistribution Authority (LSRA) is assumed, the following computation is added to the iteration process.

1. For period $t = T, T - 1, \dots, 2$, compute the lump-sum transfers to newborn households $tr_{CV}(\hat{s}_{20}; \Psi_t, \Omega_t^0)$ to make those households as well off as under the pre-reform economy.
 - (a) Set the initial value of lump-sum transfers $tr_{CV}(\hat{s}_{20}; \Psi_t, \Omega_t^0)$ to newborn households.
 - (b) Given $tr_{CV}(\hat{s}_{20}; \Psi_t, \Omega_t^0)$, find the decision rule of newborn households $\mathbf{d}(\hat{s}_{20}; \Psi_t, \Omega_t^0)$ and values $v(\hat{s}_{20}; \Psi_t, \Omega_t^0)$.
 - (c) Find the compensating variation in wealth $\Delta tr_{CV}(\hat{s}_{20}; \Psi_t, \Omega_t^0)$ to make those households indifferent from the baseline economy. (The initial wealth of newborn households is assumed to be zero since they do not receive any bequests.) If the absolute value of $\Delta tr_{CV}(\hat{s}_{20}; \Psi_t, \Omega_t^0)$ is sufficiently small, then go to Step (d). Otherwise, update $tr_{CV}(\hat{s}_{20}; \Psi_t, \Omega_t^0)$ by adding $\Delta tr_{CV}(\hat{s}_{20}; \Psi_t, \Omega_t^0)$ and return to Step (b).
 - (d) Set the lump-sum transfers $tr_{LS,t}(\hat{s}_{20}) = tr_{CV}(\hat{s}_{20}; \Psi_t, \Omega_t^0) + \Delta tr$ where an additional lump-sum transfer Δtr is precalculated, and find the decision rule of newborn households $\mathbf{d}(\hat{s}_{20}; \Psi_t, \Omega_t^0)$.
2. For period $t = 1$, compute the lump-sum transfers to all current households $tr_{CV}(\hat{s}_i; \Psi_t, \Omega_t^0)$ to make those households as much better off as the pre-reform economy. The procedure is similar to Step 1. Set the lump-sum transfers $tr_{LS,1}(\hat{s}_i) = tr_{CV}(\hat{s}_i; \Psi_t, \Omega_t^0)$.
3. Compute an additional lump-sum transfer Δtr to newborn households so that the net present value of all transfers becomes zero. Compute the LSRA wealth, $\{W_{LS,t}^1\}_{t=1}^T$, which will be used to calculate national wealth. Recompute Δtr and $\{W_{LS,t}^1\}_{t=1}^T$ using new interest rates $\{r_t\}_{t=1}^T$.

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