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Using the Treasury Nominal and Inflation-Indexed Spread to Estimate Expected Long-Run Changes in the CPI-U under Inflation Uncertainty, Risk Aversion, and the Probability of Deflation

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Abstract

Yield spreads between rates on Treasury nominal and inflation-indexed securities are thought to be distorted measures of expected inflation because of various biases. Three sources of bias are investigated in this paper: risk aversion, inflation uncertainty, and failure to account for the explicit option on the redemption value of the Treasury's inflation-indexed security when there is a probability of deflation. The analysis produces estimates of all three biases. It finds that significant bias could arise from the second and third sources, while any bias from the first source appears negligible.

Introduction

Yields calculated from prices of Treasury's inflation-indexed securities provide a market-based measure of inflation-adjusted, or real, interest rates. That information, together with yields on Treasury's conventional, or nominal securities, can be used to provide prompt information on the bond markets' consensus of medium-term to long-term expectations of inflation. In addition, such information can help to determine how unfolding events are influencing changes in those expectations. In contrast, surveys of professional forecasters, which are often used to measure inflation expectations, are available much less promptly and may be less useful for analyzing the effects on inflation expectations of specific events.

But as a measure of expected inflation, the spread between nominal and indexed Treasuries could be distorted by inflation uncertainty, risk aversion, and the probability of deflation.[1] Inflation uncertainty imparts a downward bias to the measure of expected inflation that varies directly with the degree of inflation uncertainty. Risk aversion imparts an upward bias that varies directly with the degree of risk aversion. The probability of deflation matters because of the option on the redemption value of Treasury's indexed bond as explained later. That probability imparts an upward bias to the measure of the real yield on indexed bonds that varies directly with the probability of deflation; thus, this source imparts a downward bias to the spread-estimate of expected inflation. A fourth distortion, not examined here, could arise from the different tax treatment of indexed and nominal Treasuries. The inflation accrual is taxable when earned so that the indexed-bond investor must finance tax payments from other sources of income, whereas the nominal-bond investor can finance the payments directly from interest income. This distortion has been examined elsewhere, however, and does not appear to be significant.[2]

This study develops and applies an easily understandable framework for using market and econometric information to extract a measure of long-term expected inflation adjusted for the three potential distortions just described. Briefly, the adjustments produce an estimate of long-term inflation expectations that is higher than the unadjusted spread, but, for ten-year Treasuries, still somewhat lower than that reported in a survey of professional economists. However, longer-term Treasuries yield expected inflation measures that are very close to the survey measures, possibly suggesting that survey respondents also might have in mind a longer horizon than ten years.

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[1] The theoretical basis for the roles of inflation variability and risk aversion were first demonstrated by Stanley Fischer (1975). The application of options pricing analysis to the Treasury Inflation Indexed bond's terminal option is new.

[2] See Scott Hein and Jeffery Mercer (2003).

Treasury's Inflation-indexed Securities

Treasury inflation-indexed securities (TIIS) have been issued regularly since January 1997. Outstanding volume has risen from about $15 billion in 1997 to about $155 billion by mid-2003 and trading among primary securities dealers has risen from a daily average of $1 billion in early 1998 to about $3 billion by late 2002.[3] Interest payments are calculated as the product of the semiannual coupon rate that is set at origination and the inflation adjusted par value. The inflation measure is the seasonally unadjusted CPI-U (consumer price index, all urban consumers). The inflation adjustment for any given month augments par value cumulatively as the ratio of the CPI three months earlier to the reference CPI (the value of the CPI-U three months prior to the auction, where the indexed value for days within a month is obtained through daily, linear interpolation between the first and last day of the month).[4] The value for redemption at maturity is the larger of either the original issue par value or the cumulative, inflation-adjusted par value, a feature that will be shown to have prominence when investors give more than negligible probability to the chance of deflation. TIIS are sold at auction in maturities thus far of 10 and 30 years, in multiples of $1,000 original issue par value, with interest payable semi-annually. Informal estimates of bid-ask spreads, a measure of trading liquidity, have tended to be higher for TIIS than for nominal Treasuries.[5]

Basic mechanics of price and yield

The yield-to-maturity calculation for an indexed bond follows from a present value equation in much the same way as that for a conventional bond with the additions of mechanisms for inflation adjustment and redemption valuation at maturity. A present value equation for an indexed security is shown here:

$$B_t = \frac{cV_{t+\delta}}{(1+R_t)^{\delta}} + \frac{cV_{t+\delta+1}}{(1+R_t)^{\delta+1}} + \ldots\ldots\ldots\ldots\ldots + \frac{cV_{t+\delta+M}}{(1+R_t)^{\delta+M}} + \frac{E_t\, Max\{1,\ V_{t+\delta+M}\}}{(1+R_t)^{\delta+M}} \quad (1)$$

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3 These estimates of volume and trading are from the Treasury Department (2003).

4 This lag feature of the inflation adjustment stems from the timing of the CPI release and the decision by the Treasury Department that an investor would always want to be able to ascertain the value of the inflation adjusted par value at the time of sale of purchase.

5 See Brian Sack and Robert Elsasser (2002). Their estimate, obtained from an informal survey in early 2002, found, for maturities greater than five years, TIIS spreads of 2/ 32nds to 6/32nds compared to off-the-run nominal Treasury spreads of .25/32nds to 1/32nd. In contrast, quotes published daily, for example, in the *Wall Street Journal* (which gives eSpeed/Cantor Fitzgerald as source), tend to show TIIS spreads of 1/32nd.

where: B_t = current (settlement date) price; c=coupon rate at origination; δ=fractional time from settlement date to next coupon payment period; E_t = current expectation of maximum function, $V_{t+\delta+k}$=cumulative inflation adjustment from the settlement date to the k-th coupon date; R_t=nominal interest rate, or yield to maturity; M=maturity period.[6]

The index bond's cumulative inflation adjustment from the settlement date to any future coupon payment period can be written as:

$$V_{t+\delta+k} = (1+\pi_t)^{\delta+k} V_t, \quad k = 0,1,.....,M \tag{2}$$

where π_t is average inflation from settlement date, "t," to future date, "t+δ+k," and V_t is the value of the adjustment factor from the original issue date to the current settlement date. The inflation term for future periods should also be thought of as the inflation prediction embedded in a nominal bond of the same maturity, under the assumption that the TIIS and nominal Treasury markets are sufficiently integrated.

Using the inflation adjustment (Equation 2) in the bond price (Equation 1) results in a price equation involving both an inflation-adjusted rate and a nominal interest rate, where "r_t" is the inflation-adjusted interest rate, or real yield, to remaining maturity:

$$B_t = \frac{cV_t}{(1+r_t)^{\delta}} + \frac{cV_t}{(1+r_t)^{\delta+1}} + ...+ \frac{cV_t}{(1+r_t)^{\delta+M}} + \frac{E_t Max\{1, V_t(1+\pi_t)^{\delta+M}\}}{(1+R_t)^{\delta+M}} \tag{3}$$

This last equation indicates three features in computing the yield to maturity of inflation-indexed bonds. First, it shows how inflation indexation converts an original coupon rate into an inflation-adjusted rate through the product term, cV_t; this term scales the original coupon rate "c," by the cumulative adjustment for inflation. Second, except for the final term, it shows how indexation converts nominal yield to maturity R_t into an inflation-adjusted yield to maturity r_t. Third, it shows that computing the inflation-adjusted yield requires solving the maximum function. If deflation has not occurred and is not expected, the function would be evaluated as its rightmost term which will exceed unity. In that case, the standard solution based on the market price, B_t, is the inflation-adjusted yield to maturity, r_t. But if a deflation of sufficient magnitude is expected, the rightmost term could become less than unity and the function would evaluate as unity. In that case, the terminal value would be discounted at a nominal rate before solving for r_t. The true value of the maximum

[6] The presentation in the text simplifies notation. Since bond interest intervals are semiannual, the rates of interest and inflation should be understood as semiannual, or as annual with implicit division by two, and the period numbers, 1,...,M, as semiannual.

function will be shown later to be the value of an option on the inflation-indexed bond's value at maturity.

Some Properties of Calculated Yields to Maturity

The six-year history of TIIS has revealed several patterns (see Figures 1 and 2):

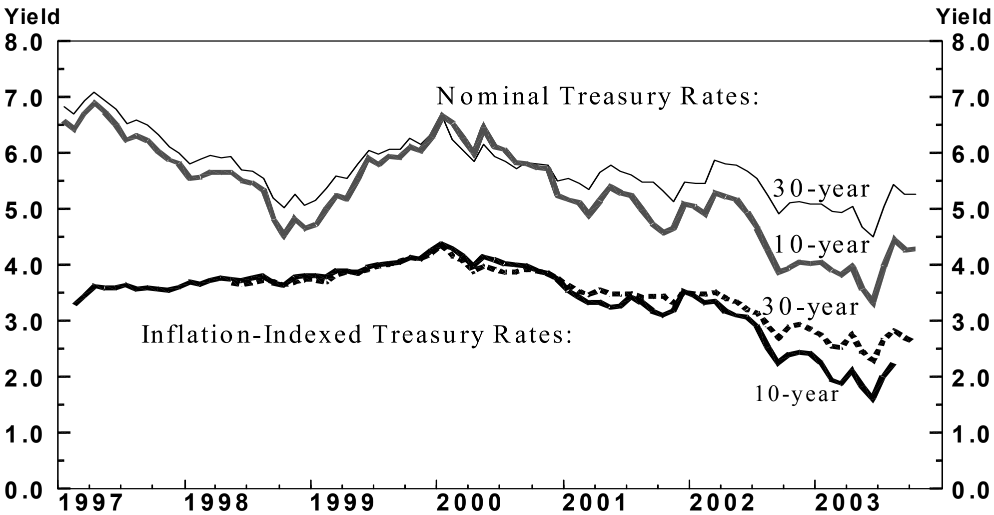
- First, the calculated yields to maturity have been lower for TIIS than for nominal Treasuries of comparable maturities, presumably because medium- to long-term expected inflation has been positive.

- Second, yields on TIIS display a term-structure similar to that for nominal Treasuries, suggesting that both types of Treasuries may have comparable cyclical behavior.

- Third, while yields on TIIS moved in a narrower range than yields on nominal Treasuries during the financial turmoil of 1997-1998, they moved more in tandem during the turmoil that set in after early 2000. This could imply close alignment between the yield on TIIS and the real component of the yield on nominal Treasuries for the latter period.

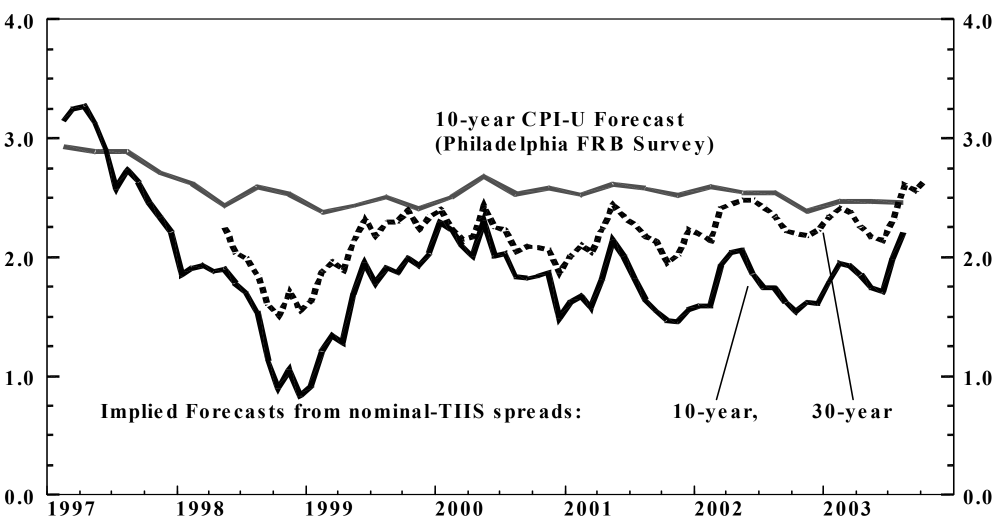
- Fourth, as a measure of expected inflation, the yield-spread between nominal Treasuries and TIIS has tended to be lower than the survey measure reported by the Federal Reserve Bank of Philadelphia (see Figure 2). That difference between the survey and bond measures has been much greater for the 10-year bond measure than for the 30-year bond measure, except for the period near mid-2000 when nominal interest rates were at a peak. This closer correspondence of the survey to the TIIS 30-year measure reflects the fact that the rate on the 30-year bond has varied much less than that on the 10-year bond over the recent economic cycle.

Figure 1. Treasury Nominal and Inflation-Indexed Rates, February 1997 to August 2003.



Source: Haver Analytics.
Note to Figure 1: *The 10-year inflation-indexed Treasury rate consists of the rates for on-the-run issues (i.e., the most recent issue for each month in the chart).*

Figure 2. Yield Spread between Nominal and TIIS vs the Philadelphia Federal Reserve (quarterly) Survey of 10-year Inflation Forecasts, 1997Q1 to 2003Q3.



Sources: Haver Analytics, Federal Reserve Bank of Philadelphia.

Distortions to the Bond-Based Measure of Inflation

The fact that the survey measure of long-term inflation expectations has almost always been higher than that computed from the spread between nominal and TIIS yields suggests that simply using the nominal-TIIS spread may give a distorted estimate of inflation expectations. The presence of distortions due to inflation uncertainty, risk aversion, and possibly mis-measurement because of the maximum function explain a portion of the difference between the survey and bond measures of expected inflation.

The Distortions Due to Inflation Uncertainty and Risk Aversion

These two distortions are examined separately. The distortion due to inflation uncertainty is examined first. When future inflation is uncertain and investors are risk neutral, they will tend to pay more for a nominal bond and less for an indexed bond than if they were risk averse; this, in turn, will cause the yield spread between nominal and inflation-indexed Treasuries to understate expected inflation. A simple example illustrates this result.

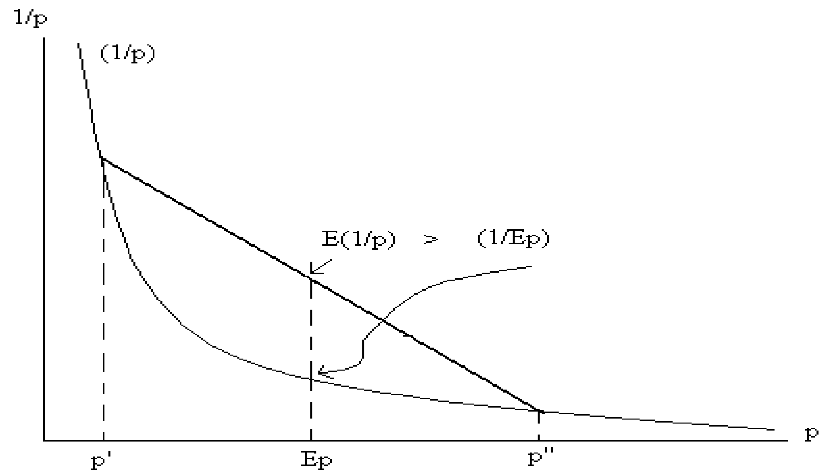
Consider two investment possibilities for an investor. First, suppose the investor could sell one unit of purchasing power today, invest the dollar proceeds of P(0) in a conventional bond that returns nominal purchasing power of P(0)[1+R] tomorrow and real purchasing power of P(0)[1+R]/P(1), where "R" is the nominal rate, P(0) and P(1) are today's and tomorrow's respective price levels, and the expression effectively is purchasing-power yield or real return. Alternatively, suppose the investor could directly invest one unit of purchasing power today in an inflation-indexed bond that returns inflation protected purchasing power of [1+r] tomorrow. Given the two possibilities, a risk neutral investor could demand a real rate of "r" that equates purchasing power from the inflation-protected bond to the expected purchasing power of the conventional bond. If so, then,

$$(1+r) = E\{\frac{P_0(1+R)}{P_1}\} = (1+R)P_0E\{\frac{1}{P_1}\} > (1+R)P_0(\frac{1}{E\{P_1\}}) = \frac{(1+R)}{(1+E\pi})) \qquad (4)$$

where EP(1)=P(0)(1+Eπ), defines expected inflation. The inequality follows from Jensen's inequality, which implies that E{1/(1+π)}>{1/(1+Eπ)} (see Figure 3).

As a result, (1+R)/(1+ r) <(1+ Eπ), which implies that uncertainty about inflation causes the spread between the nominal and inflation-indexed rates to be less than expected inflation.

Figure 3. Jensen's Inequality for $E(1/(1+\pi)) > 1/(1+E\pi)$



To examine the distortion due to risk aversion, I first exploit the definition of risk aversion. This leads directly to an equation with an adjustment factor that is needed to correct the nominal-TIIS spread for this distortion. It is then shown that the specification of the adjustment factor in that equation will differ according to assumptions about investor behavior.

When investors are risk averse instead of risk neutral, they exhibit diminishing, incremental satisfaction from each increment to resources available for consumption and are willing to pay to avoid risky outcomes. As before, they can hold a nominal bond offering risky purchasing power in the future or an index bond offering certain purchasing power in the future. (This is a simplification. As seen later, it does not mean investors will hold only one type of bond.) To avoid risk, they will pay an additional amount for the inflation-indexed bond compared to the risk-neutral case, and receive a smaller, safe return. The difference in the return on the inflation-protected bond under risk aversion versus risk neutrality is accounted for by the risk premium. With risk aversion, the expected satisfaction, or utility, from the real return on the risky bond is less than the satisfaction from the expected real return on the risky bond as shown here (see Figure 4).

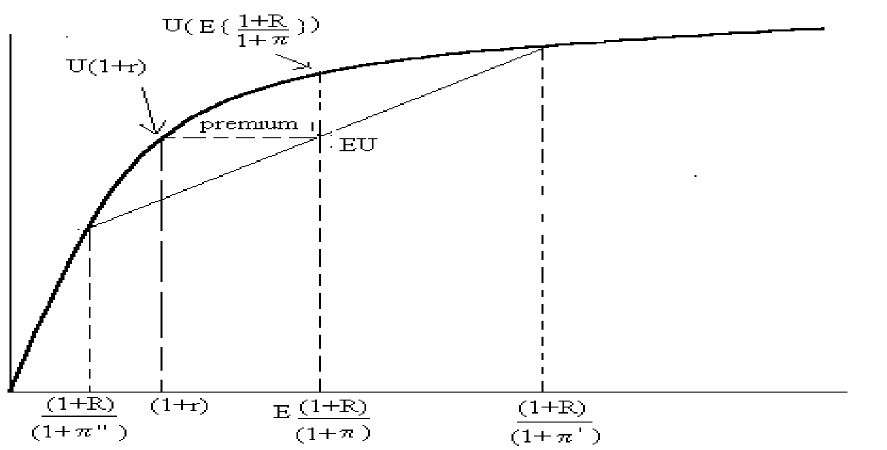
$$EU\left(\frac{1+R}{1+\pi}\right) < U\left(E\left\{\frac{1+R}{1+\pi}\right\}\right) \tag{5}$$

One way to eliminate risk could be for the investor to give up an amount such that the sure satisfaction from the safe return is the same as the expected satisfaction from the risky return. The risk premium, denoted by ρ, would then be as defined in proportional form in Equation 6:

$$U(1+r) \;=\; U(E\{\frac{1+R}{1+\pi}\}\{1-\rho\}) = EU(\frac{1+R}{1+\pi}) \tag{6}$$

A graphical representation of the risk premium is shown in Figure 4:

Figure 4. Risk Aversion and the Risk Premium



With risk aversion and the risk premium, the real return on the indexed bond can then be viewed as equivalent to the safe return, or:

$$1+\mathrm{r} \;=\; \mathrm{E}(\frac{1+\mathrm{R}}{1+\pi})(1-\rho) \tag{7}$$

The interpretation of Equation 7 is that under risk aversion the indexed bond will command a higher price and lower real yield than under risk neutrality, and this makes the estimate of expected inflation based on the spread between TIIS and nominal Treasury rates higher than otherwise. Thus, in contrast to the downwardly biased estimate for expected inflation because of inflation uncertainty, risk aversion creates an upward bias.

The two sources of bias, inflation uncertainty and risk aversion, can be written out directly from Equation 7 if inflation follows a normal process.[7] In that case,

[7] That is, if inflation is normally distributed with dispersion, σ_π, then: $Ee^{-\pi} \approx (1+E\pi)^{-1}(1+\sigma_\pi^2)$. For additional discussion, see A.G. Malliaris and W.A. Brock (1982).

$$1+r = \left(\frac{1+R}{1+E\pi}\right)(1+\sigma_{\pi}^{2})(1-\rho) \tag{8}$$

where, σ_{π}, is the standard deviation of the inflation process.

If Equation 8 is solved for the spread, using the approximation that $(1-\rho) \approx 1/(1+\rho)$, then the net bias will be seen to depend on the relative strengths of the biases from inflation uncertainty and risk aversion as shown here:

$$\left(\frac{1+R}{1+r}\right) = (1+E\pi)\left(\frac{1+\rho}{1+\sigma_{\pi}^{2}}\right). \tag{9a}$$

Rewriting Equation 9a to solve for expected inflation shows that expected inflation will exceed, equal, or fall short of the spread depending on the relative strengths of the inflation-uncertainty and risk-aversion biases.

$$(1+E\pi) = \left(\frac{1+R}{1+r}\right)\frac{(1+\sigma_{\pi}^{2})}{(1+\rho)} \tag{9b}$$

Obviously, based on Equation 9b, a time-series analysis using observations on the spread between the nominal Treasury and TIIS yields has two unobserved (though not unobservable) components to reckon with before extracting an estimate of expected inflation, $E\pi$. Estimates of risk aversion and inflation uncertainty are therefore required. Estimates of the inflation uncertainty parameter are described in the next section since they also play a key role in the option on the terminal value of TIIS.

Having exploited the definition of risk aversion to derive an equation for adjusting the nominal-TIIS spread still leaves open the specification of the adjustment factor. Any specification of the premium for risk aversion, ρ, will depend on the utility function (as suggested by Equation 6) and the assumptions about investor behavior. For utility, it is convenient to assume constant relative risk aversion. For investor behavior, assuming the investor only attempts to equalize the level of satisfaction between the risky and inflation-protected bond leads to an expeditious specification that follows directly from the definition of risk aversion and requires only the variance of the TIIS rate of interest (as suggested by Equation 7). If the utility function has constant relative risk aversion, the risk premium has the form (see Appendix 1):

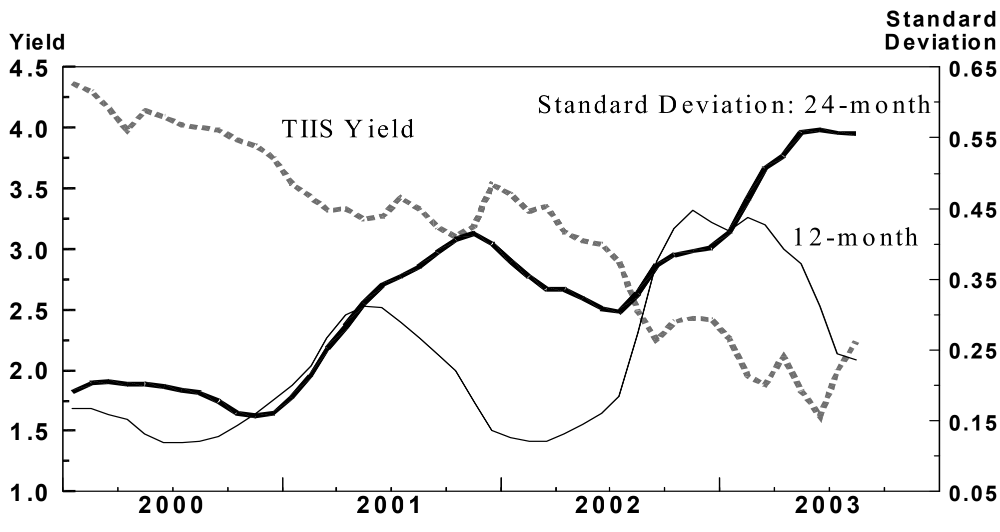
$$\rho = \tfrac{1}{2}\sigma_r^2\,\gamma,\ when\ U(C) = \tfrac{1}{1-\gamma}C^{1-\gamma} \qquad (10a)$$

Alternatively, we could adopt the assumption that investors behave according to the consumption-capital asset pricing model. In that framework, a risk-averse investor takes into account the correlation between inflation and consumption, choosing the portfolio of nominal and inflation-indexed bonds that provides the maximum amount of current and future satisfaction from consumption. Taking this approach results in an alternative expression for the risk premium as shown here (see Appendix 1)[8]:

$$\rho^* = -(1+R)Cov(m, \frac{1}{1+\pi}) > 0, where\ m = \frac{U'(C_{t+1})}{U'(C_t)} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \qquad (10b)$$

Estimates of the utility parameter "γ" are available from the finance literature and, for the Equation 10a form of risk aversion, estimates of the variance of the real rate can readily be obtained from data on TIIS yields. For example, using on-the run TIIS yields, we could select a 12- or 24-month moving-standard deviation of monthly yield levels (see Figure 5). Various estimates of "γ" are found in the literature. Those reported in Prescott (1996) range between 1 and 2, but other estimates have ranged even higher such as 50 or beyond. For the alternative form of risk aversion shown in Equation 10b, estimates of the covariance expression are similarly computable.

Figure 5. Level and Moving-Standard Deviation of on-the-run, 10-year TIIS Yields, January 2000 to August 2003.



For both measures of the risk aversion factor, however, the computed estimates turn out to be quite small. For example, using the Equation 10a definition and an average

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8 I am indebted to Albert Metz of CBO for suggesting this alternative expression for the risk premium and for providing key insights for the derivation of equation 10b.

estimate of the standard deviation of about 0.4 percentage points, the risk-aversion factor ranges from $1.2x10^{-5}$ for γ=1.5, to $40x10^{-5}$ for γ=50. For Equation 10b, the same range of values of γ, and the estimate of the covariance term over the period since the early 1980s, the estimated range for the risk-aversion factor again is quite small, from 5.6×10^{-6} to about 2.0×10^{-4}. Such small factors suggest that risk aversion plays little role in distorting the nominal TIIS spread.[9]

<u>Deflation and Distortion of the Indexed-Yield to Maturity</u>

Finally, the third source of bias to be examined is the upward bias to the calculated yield on the indexed bond in the presence of a positive probability of deflation. As already indicated, published yields on TIIS impose the assumption that the value of the maximum function in Equation 3 is its last argument which, in turn, implies an assumption of no probability of deflation over the remaining maturity of the bond. The expected value of the maximum is examined to determine the consequence of this assumption.

The expected value of the maximum function is the value of an option on the indexed bond's principal at maturity. In real terms, the option's benefit to holders is the promise of an increase in real purchasing power when there is deflation, the same benefit received by holders of conventional Treasury securities. In contrast, with inflation, the purchasing power of principal at maturity is preserved for TIIS while it is eroded for conventional Treasuries. In nominal terms, as a result of the option, deflation is not allowed to erode nominal principal for TIIS below original par value, thus promising holders at maturity no less than they would receive from a conventional Treasury at maturity.

By itself, the maximum function in Equation 3 is the option's "intrinsic value." That value will depend on the accumulated inflation adjustment that already has occurred since the bond was issued, V_t, and the critical value of deflation, π^*, that would erode those accumulated adjustments over the period remaining to maturity, $(\delta + M)$. Taking these factors together into the composite term, $V_t(1+ \pi^*)^{\delta + M}$, determines the "strike price" of the option. Because the strike price has a value of unity, the critical rate of deflation can be determined immediately for any TIIS, as $V_t^{-1/(\delta + M)}-1$. For example, greater accumulated adjustment from past inflation requires even greater deflation before the guaranteed minimum becomes effective. A shorter remaining maturity has the same interpretation (see Figure 6). Critical values for currently outstanding TIIS with original maturities of 10 years are shown in Table 1.

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9 It is well known from the literature on the equity premium puzzle that abnormally high values of γ are needed to explain the spread between returns on equity and Treasury bills. On this see, Rajnish Mehra and Edward C. Prescott (February 2003).

Figure 6. Option-Value of the Indexed Bond's Principal at Maturity

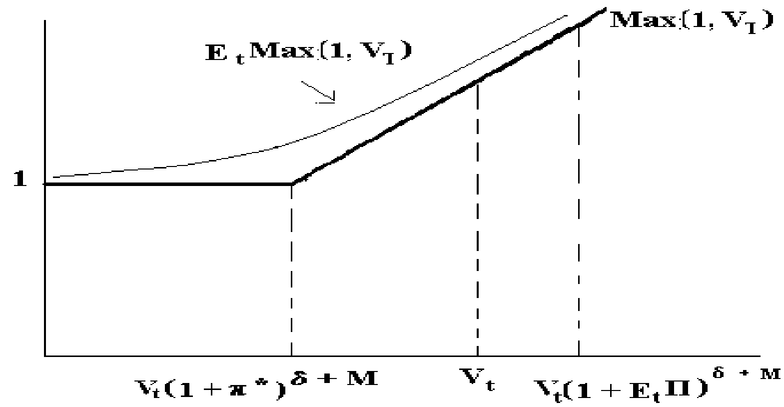


Table 1. Inflation Indexed Bonds: Accumulated Inflation Adjustments, Critical Deflation Rates, and Spreads to Nominal Treasuries.

Bond's Maturity	15-Jan 2007	15-Jan 2008	15-Jan 2009	15-Jan 2010	15-Jan 2011	15-Jan 2012	15-Jul 2012
Inflation-Adjustment Factor, V_t	1.16	1.14	1.12	1.09	1.06	1.04	1.02
Critical Deflation Factor, π*	-3.59	-2.52	-1.87	-1.25	-0.68	-0.39	-0.24
Spread to Nominal Treasury	1.20	1.26	1.36	1.41	1.48	1.58	1.71

Note: Author's calculations using bond ask prices for May 27, 2003 as published in *The Wall Street Journal,* May 28, 2003.

While the option's intrinsic value can be determined without reference to uncertainty, finding the expected value of the option requires additional information about the stochastic process governing inflation as measured by the consumer price index. Using the same assumption as before, that the inflation rate follows a normal process (which is equivalent to assuming that the price level, and thus the index adjustment, $V_{t,}$ follows a log-normal process), the option's expected value, or the expected value of the maximum function, will additionally depend on the volatility of inflation forecasts over the time remaining until maturity. The greater the volatility and the longer is remaining maturity, the greater will be the option's expected value. An illustration of the option's value is shown by the curved line in Figure 6.

Using standard methods in pricing options, the resulting equation for the inflation-indexed bond, inclusive of the expected value of the embedded option is shown here (see Appendix 2 for derivation):

$$B_t = \sum_{j=0}^{j=M} \frac{cV_t}{(1+r_t)^{\delta+j}} + \frac{V_t}{(1+r_t)^{\delta+M}} N_w\left(\frac{L}{\sigma_v} + \sigma_v\right) + \frac{1}{(1+R_t)^{\delta+M}}\left[1 - N_y\left(\frac{L}{\sigma_v}\right)\right], \quad (11)$$

where:

$N_w(..)$, is the cumulative normal distribution with mean, $-\sigma_v$, and unit variance;
$N_y(..)$, is cumulative standard normal (zero mean and unit variance);
$\sigma_v^2 = \sigma_\pi^2 M$;
$L = E_t \ln(V_T) = \ln(V_t) + [E_t\pi - ½(\sigma_\pi)^2]M = \ln(V_t) + (R_t - r_t)M - ½(\sigma_\pi)^2 M$.

According to Equation 11, given price B_t, the calculated yield to maturity involves additional terms in r_t that appear in the two cumulative normal distributions. The first of these two distributions, $N_w(..)$, gives the probability that inflation will be higher than the critical value of deflation, π^*, shown in Figure 5; the second distribution, $1-N_y(..)$, gives the probability that deflation outcomes will be at or below the critical value, π^*. As deflation becomes less and less probable the weights of the two distributions shift increasingly in favor of the term, $V_t/(1+r_t)^{\delta+M}$.

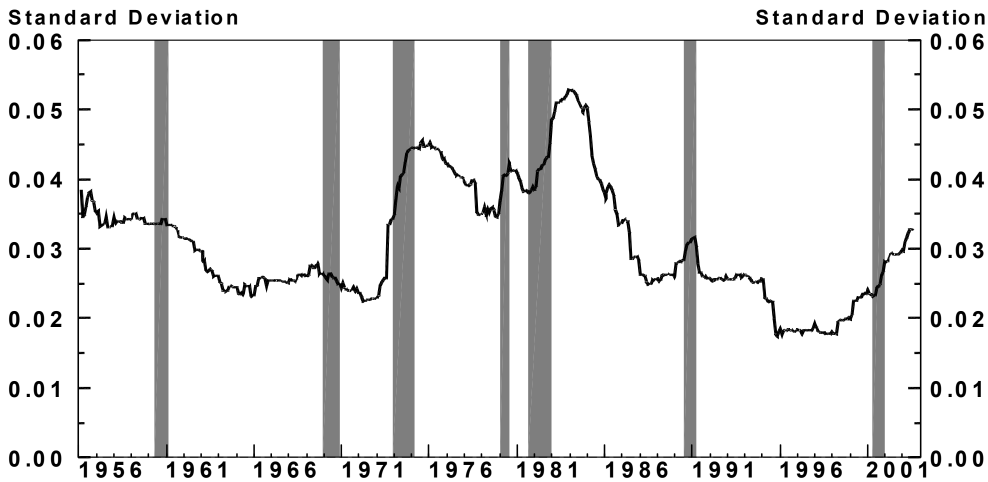
Because investors are aware of the contingent nature of payment at maturity, they presumably price the securities and produce some consensus likelihood for each of the two states of inflation and deflation. The existence of a positive deflation probability implies, according to Figure 5 and Equation 11, that the real rate could be much lower than that reported in published calculations.

Estimates of the bias due to ignoring the option on the bond's redemption value at maturity require an estimate of the standard deviation, or volatility, of inflation. One way to estimate volatility is to use the standard deviation of semiannual inflation, the same frequency at which interest payments are made until the bond reaches maturity. Using this measure, the standard deviation is estimated to be about 0.016. A second way, using monthly inflation in the CPIU, where inflation is calculated as 12 times the logarithmic first difference, finds the estimated standard deviation over the period since 1951 (annualized), is higher at about 0.04. Over 60-month spans, however, the annualized, monthly standard deviation has varied from just below 0.02 to just above 0.05 (see Figure 7). A third way is to use the errors in CBO's 2-year ahead forecast of the CPI which have an annualized standard deviation of 0.008.[10] A fourth way, not

[10] Congressional Budget Office (July 2000).

explored here, would be to base volatility on the estimate of standard deviations taken from econometric price equations.

Figure 7. CPIU(NS), 60-Month Moving Standard Deviation of Annualized Monthly Logarithmic Change, February 1956 to August 2003.



Sources: Haver Analytics and Author's calculations.

An illustration of the potential bias to the real rate has been constructed, based on Equation 11 and using, as alternative values of volatility, multiples of a standard deviation equal to 0.016. Table 2 shows the results using market data in late May 2003 when deflation concerns were prevalent. For a given maturity, the bias is measured as the difference between the yield ignoring volatility (volatility = 0 x standard deviation) and that including volatility (volatility = (1 or 2) x standard deviation)

Table 2. Illustrative Estimates of TIIS Rate Adjusted for the Option at Maturity

volatility: multiples of Std. Dev.	Inflation-adjusted yields per maturity using multiples of standard deviation in eq. 11 (Calculations use ask prices as of May 27, 2003)						
	Maturity of TIIS Issue						
	15-Jan-07	15-Jan-08	15-Jan-09	15-Jan-10	15-Jan-11	15-Jan-12	15-Jul-12
0	0.698	0.984	1.254	1.445	1.597	1.674	1.611
1	0.698	0.984	1.253	1.439	1.575	1.648	1.589
2	0.530	0.741	0.991	1.125	1.202	1.294	1.259

Source: Author's calculations using bond price data from *The Wall Street Journal*, Wednesday, May 28, 2003 (p. B4).

It appears from Table 2 that a greater distortion to real-rate estimates occurs for TIIS with longer remaining maturities and for larger estimates of the volatility of inflation. Both results are consistent with the theory of options pricing. For bonds with longer maturities, their options have longer lives and offer greater expected value to the holder for any given level of volatility; with greater volatility, a wider range of outcomes is possible, which offers another source of value to the holder. Bonds with longer maturities also have less accumulation from past inflation and therefore require less deflation in the future as indicated in Table 1.

The results shown in Table 2 offer a range of estimates for adjusting published TIIS yields under the current low inflation environment. They suggest that the distortion could range from being negligibly small, as in the case on the TIIS maturing in January 2007 using estimated volatility = .016, to being possibly as large as 35 basis points, as in the case of the TIIS maturing in July 2012 and using estimated volatility = .032.

Combined estimate of bias to expected inflation

A framework that combines the three separate sources of bias to the nominal-TIIS spread estimate of long-term inflation expectations is Equation 12. The adjusted estimate of expected inflation consists of three adjustments to the spread between the nominal rate and the observed rate on TIIS: an upward adjustment to offset the effect of ignoring the option on terminal value; an upward adjustment to offset the effect of inflation uncertainty; and a downward adjustment to offset the effect of risk aversion.

$$(1+\hat{\pi}) = \frac{(1+R)}{(1+r*)}\frac{(1+\sigma_{\pi}^{2})}{(1+\frac{1}{2}\gamma\sigma_{r}^{2})} \quad , \qquad (12)$$

where r^* represents the option-adjusted yield on TIIS.

Supplying values to the parameters needed for computing the adjusted measure of expected inflation is likely to be as much art as science. It is unlikely the parameters are constant.[11] An example compares the unadjusted spread between nominal Treasuries and TIIS, the adjusted spread, and the ten-year ahead forecast of the Consumer price index that is published quarterly by the Federal Reserve Bank of Philadelphia. For the example, we use the matched sequence of on-the-run nominal Treasury and TIIS yields with original maturities of ten years, a 15 basis point adjustment for the terminal option, a five-year moving variance of CPIU inflation, a 24-month moving variance of the observed yield on TIIS, and a utility parameter of 1.5 (see Figure 8).

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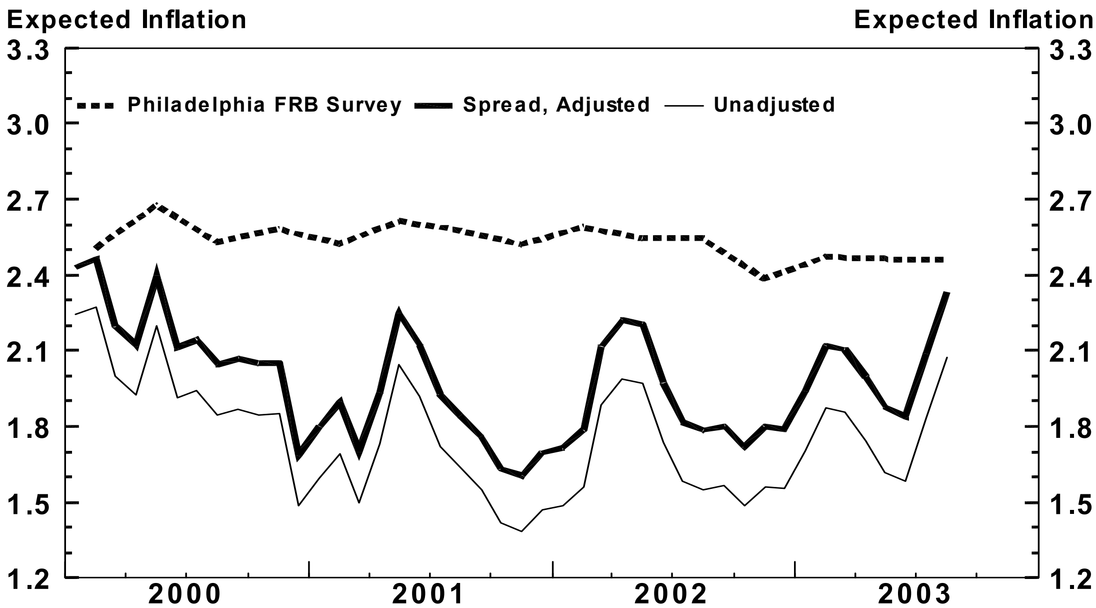
[11] Arguments for treating the parameters as variable can be found in Robert C. Merton (1990).

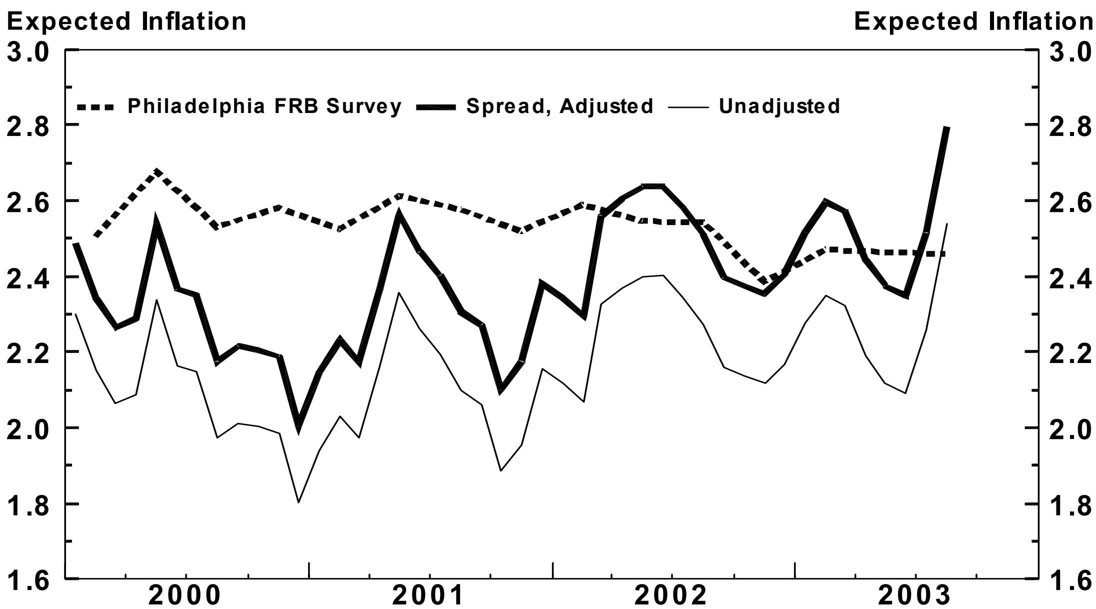
Figure 8. Comparing Measures of 10-year Expected Inflation: Survey average from Philadelphia Federal Reserve vs. Spread Between Nominal and Inflation-Index Treasury, and Spread Adjusted for Distortions.



Sources: Philadelphia Federal Reserve, Haver Analytics, Author's calculations.

According to the example of Figure 8, the adjustments imply a higher level of expected inflation than is otherwise found in the spread between yields on nominal and inflation-indexed Treasury securities. The adjustments for inflation uncertainty and the terminal option account for virtually all of the increase, as the adjustment for risk aversion is negligible. Nevertheless, the adjusted measure of expected inflation is still significantly below the survey measure of long-term inflation. It may be that bond-market participants, who implicitly determine the expected inflation in the spread, have taken a greater hold on the possibility of substantially low inflation than have the economists participating in surveys on the long-term inflation outlook. However, if the survey measure is compared to the longer-term spread, the difference is greatly diminished (see Figure 9):

Figure 9. **Comparing Measures: Implied Inflation from 30-year, Original Maturity Spread Between Nominal and Inflation-Index Treasury, Spread Adjusted for Distortions, and Survey Average of 10-year Expected Inflation from Philadelphia Federal Reserve.**



Sources: Philadelphia Federal Reserve, Haver Analytics, Author's calculations.

This last figure reinforces the earlier suggestion that the survey measure may encompass a longer horizon, one that also may be more consistent with expectations embedded on the longer-maturity Treasuries. Without the adjustments, however, even the longer-term spread has been everywhere lower than the survey estimate. But the adjustments for distortions also tend to reduce the downward bias in the spread between nominal and inflation-indexed bonds.

Appendix 1. Derivation of Equations 10a and 10b.

This appendix expands upon Equation 6 to show the derivation of the risk premium as defined in Equations 10a and 10b. First, in the case of Equation 10a, the following simplified notation is used:[12]

$$\frac{1+R}{1+\pi} = 1+\tilde{r} \text{ , with mean, 1+Er, and variance about the mean, } \sigma_{\mathrm{r}}^{2} . \qquad \text{(A1.1)}$$

Using A1.1, rewrite the second and third terms of Equation 6 as shown here:

$$U(E\{1+\tilde{r}\}\{1-\rho\}) = EU(1+\tilde{r}). \qquad \text{(A1.2)}$$

The derivation proceeds by taking a second order Taylor's expansion of the terms on each side of the equality in A1.2 around (1+Er).

Expanding the right-hand side of the equality in A1.2 yields:

$$U(1+\tilde{r}) = U(1+Er) + \frac{\partial U}{\partial(1+r)}\{(1+\tilde{r})-(1+Er)\} + \frac{1}{2}\frac{\partial^2 U}{\partial(1+r)^2}\{(1+\tilde{r})-(1+Er)\}^2 \qquad \text{(A1.3)}$$

Then, taking the expected value yields:

$$EU(1+\tilde{r}) = U(1+Er) + \tfrac{1}{2}\frac{\partial^2 U}{\partial(1+r)^2}\sigma_{1+r}^2 \ . \qquad \text{(A1.4)}$$

Taking the Taylor expansion of the left-hand side of the equality in A1.2 yields:

$$U(\{1+Er\}\{1-\rho\}) = U(1+Er) - \frac{\partial U}{\partial(1+Er)}(1+Er)\rho + \tfrac{1}{2}\frac{\partial^2 U}{\partial(1+Er)^2}(1+Er)^2\rho^2 \qquad \text{(A1.5)}$$

If the risk premium is not too large, this can be simplified further as:

$$\mathrm{U}(\{1+\mathrm{Er}\}\{1-\rho\}) = \mathrm{U}(1+\mathrm{Er}) - \frac{\partial \mathrm{U}}{\partial(1+\mathrm{Er})}(1+\mathrm{Er})\rho \qquad \text{(A1.6)}$$

Finally, equating the right-hand sides of Equations A1.4 and A1.6, eliminating terms common to both sides of the resulting equality, and solving for the risk premium yields:

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[12] This derivation follows that in Laffont (1989).

$$\rho = -\tfrac{1}{2}\frac{\dfrac{\partial^2 \mathrm{U}}{\partial(1+\mathrm{Er})^2}}{\dfrac{\partial \mathrm{U}}{\partial(1+\mathrm{Er})}}\frac{\sigma_{\mathrm{r}}^2}{(1+\mathrm{Er})} \quad . \tag{A1.7}$$

Using the Utility function in the text, the definition of the coefficient of relative risk aversion as,

$$\gamma = -\frac{\dfrac{\partial^2 U}{\partial(1+Er)^2}}{\dfrac{\partial U}{\partial(1+Er)}}(1+Er), \tag{A1.8}$$

and assuming that Er is sufficiently small gives the approximation that is shown in Equation 10a of the text.

For Equation 10b, the investor's decision problem is the following:

$$\max\{U(c_t)+\beta U(c_{t+1})\}$$
$$\textit{subject to}\ \ c_t + d_t + b_t = a_t\ , c_{t+1} = \frac{(1+R)}{(1+\pi)}d_t\ +\ (1+r)b_t$$

where "d, b, and a" respectively represent the real values of nominal bonds, inflation-indexed bonds, and initial endowment, and β is time preference. This problem is the basis for the "consumption capital-asset pricing model," that has become standard in the financial economics literature.[13] The solution to the problem results in,

$$E\left(\frac{1}{1+\pi}\right) = \left(\frac{1+r}{1+R}\right)\left(1-(1+R)\operatorname{cov}\left(\frac{U'(c_{t+1})}{U'(c_t)},\frac{1}{1+\pi}\right)\right) \tag{A1.9}$$

Finally, using the expression,

$$E(1+\pi) = \frac{1+\sigma_\pi^2}{E\left(\dfrac{1}{1+\pi}\right)} \tag{A1.10}$$

and substituting for A1.9 yields Equation 10b in the text..

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13 See, for example, Sumru Altuğ and Pamela Labadie (1994).

Appendix 2. Deriving the Value of the Option on the Redemption Value of TIIS.

This appendix shows how the expected terminal value expression in Equation 3 of the text is equal to the option on terminal value that appears in Equation 11 of the text.[14] It is easiest to begin by first noting the following equivalence from adding and subtracting one in the numerator of the expression to the left of the equality:

$$\frac{\mathrm{E_t}\max\{1, \mathrm{V_t}(1+\pi)^{\delta+\mathrm{M}}\}}{(1+\mathrm{R_t})^{\delta+\mathrm{M}}} = \frac{1}{(1+\mathrm{R_t})^{\delta+\mathrm{M}}} + \frac{\mathrm{E_t}\max\{0, \mathrm{V_t}(1+\pi)^{\delta+\mathrm{M}} - 1\}}{(1+\mathrm{R_t})^{\delta+\mathrm{M}}} \quad \text{(A2.1)}$$

In this way the maximum function is scaled to zero, instead of one, when π is at or below the critical value, π^*, as pictured in Figure 6 of the text.

We can now use standard methods to evaluate the discounted expected maximum function as written on the right-hand side of Equation A2.1. In doing so, it is convenient to proceed in continuous time and afterwards convert back into discrete time. The first step is to assume the dynamics governing the behavior of the inflation index, V_t, are log normal and define the following quantities:

$$L \equiv \ln(V_T);\ \bar{L} \equiv E_t(L);\ \sigma_L^2 \equiv E_t(L-\bar{L})^2 = \sigma_\pi^2(\delta + M) \quad \text{(A2.2)}$$

Log normality implies that the probability density of V_T has the following form:

$$f(V_T)\,dL = \frac{1}{\sigma_L\sqrt{2\Pi}} e^{-\frac{(L-\bar{L})^2}{2\sigma_L^2}} dL = \frac{1}{\sigma_L\sqrt{2\Pi}} e^{-\frac{(L-\bar{L})^2}{2\sigma_L^2}} \frac{dV_T}{V_T} \quad \text{(A2.3)}$$

Now we can define the value of the call option on terminal value as:

$$C = e^{-R(\delta+M)} E_t \max\{0, V_T - 1\} = e^{-R(\delta+M)} \int_{V_T=1}^{+\infty} (V_T - 1) f(V_T)\,dL \quad \text{(A2.4)}$$

The integral begins at one, instead of at -∞, because the maximum function has a zero value up to the value $V_T = 1$ (see Figure 6).

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[14] This derivation follows that in Stephen Figlewski, William L. Silber, and Marti G. Subrahmanyam (1990).

We can now value the two parts of the integral separately, taking first that part involving the strike price of one. The procedure involves the change-of-variable method to standardize the normal distribution, using the following definitions:

$$y = \frac{L - \overline{L}}{\sigma_L};\ dy = \frac{1}{\sigma_L} dL = \frac{1}{\sigma_\pi \sqrt{\delta + M}} \frac{dV_T}{V_T} \tag{A2.5}$$

Using the definitions from Equation A2.5, the following results obtain for the part of the integral in Equation A2.4 that involves only the strike price "1":

$$e^{-R(\delta+M)} \int f(V_T) dL = e^{-R(\delta+M)} \int_{\frac{L\{V_T=1\}-\overline{L}}{\sigma_L}}^{+\infty} \frac{1}{\sqrt{2\Pi}} e^{-y^2} dy = e^{-R(\delta+M)} [1 - N_y(\frac{-\overline{L}}{\sigma_L})] \tag{A2.6}$$

Using the equivalences, N(-Z) = 1 - N(Z), results in this portion of the call having the value:

$$e^{-R(\delta+M)} N_y(\frac{\overline{L}}{\sigma_L}) \approx \frac{1}{(1+R)^{\delta+M}} N_y(\frac{\overline{L}}{\sigma_L}). \tag{A2.7}$$

Because a nearly identical procedure applies to that part of the integral in Equation A2.4 involving V_T, we will not show all the steps here. (They are identical to that developed in Figlewski, et. al (1990), pp.130-132.) They result in the remaining part of the call option in Equation A2.4 having the value,

$$V_t e^{-(R-E\pi)(\delta+M)} N_w(\frac{\overline{L}}{\sigma_L} + \sigma_L) \approx \frac{V_t}{(1+r)^{\delta+M}} N_w(\frac{\overline{L}}{\sigma_L} + \sigma_L) \tag{A2.8}$$

Substituting terms from Equations A2.7 and A2.8 into Equation A2.4 and then into Equation A2.1 results in Equation 11 of the text.

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