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Shinichi Nishiyama
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Shinichi Nishiyama
Congressional Budget Office
Washington, D.C. 20515
e-mail: shinichi@cbo.gov

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Abstract

This paper extends a heterogeneous agent overlapping generations model by implementing bequests — both altruistic and accidental — and measures the degrees of time preference and parental altruism through the calibration of the model to the U.S. economy. In this model, a parent household and its adult child households behave consistently and strategically to determine their optimal consumption, working hours, and savings. Based on the obtained parameters, the paper also shows the individual effects of altruistic and accidental bequests on wealth accumulation by examining the impacts of a 100 percent estate tax and a perfect annuity market in the model. To match the economy's capital-output ratio and the relative size of bequests observed in the United States, the parent household would have to discount its children's utility by about 30 percent relative to its own utility, according to the model. The model suggests that total bequests contribute about 14 percent to wealth accumulation in a closed economy and 21 percent in a small open economy. Also, the effect of a perfect annuity market depends on the degree of parental altruism in the economy.

1 Introduction

Most existing macroeconomic models assume either that people care about their descendants as much as they care about themselves (infinite horizon models) or that they don't care about their descendants at all (life-cycle models). But when economists evaluate government policies that involve income redistribution between generations, the effects of these policies critically depend on the degree of the altruism between parents and children. If people are mostly altruistic, infinite horizon models are appropriate. But if they are mostly selfish, overlapping generations models are better.

Previous analyses that evaluate the degree of bequest motives using panel data, however, show several different conclusions, and it is still inconclusive to what extent parents are

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altruistic toward their children. For example, Hurd (1987) and Wilhelm (1996) show that parental altruism is insignificant and that bequests are likely to be accidental. In contrast, Menchik and David (1983) and Bernheim (1991) show that the bequest motives of parents are strong, although these are not necessarily caused by altruism. In fact, Hurd and Bernheim actually used the same data set, the Longitudinal Retirement Household Survey, but reached opposing conclusions using different approaches.

Previous studies also differ greatly with regard to estimating the importance of transfer wealth versus life-cycle wealth. On the one hand, Modigliani (1988) and others estimate that the share of bequeathed wealth among total wealth is at most 20 percent.[1] On the other hand, Kotlikoff and Summers (1981) show that transfer wealth (which includes inter vivos transfers) accounts for more than 80 percent of total wealth. The difference between those two conclusions is significant, although it is due in large part to different definitions of transfer wealth.

This paper constructs a measure of time preference (how much people discount their own future utility) and a measure of parental altruism (how much they discount their children's utility), and it answers the same questions using an extended dynamic general equilibrium model.

More specifically, the paper uses a heterogeneous agent overlapping generations model, extended to include altruistic and accidental bequests.[2] In this model, a parent household and its adult child households play a Cournot-Nash game and decide their optimal consumption, working hours, and savings in each period. The model is calibrated to the U.S. economy. In that calibration, the two main parameters, time preference and parental altruism, are determined simultaneously so that the steady-state equilibrium of the model replicates the U.S. economy in terms of the following two key statistics — the capital-output ratio and the relative size of bequests. Finally, based on the parameters obtained, the paper calculates the individual contribution of altruistic and accidental bequests to wealth accumulation.

For simplicity, the model abstracts from inter vivos transfers as well as education spending by parents. So, in this model, intergenerational transfers occur only at the time of death in the form of bequests.[3] But, in the calibration, both bequests and part of inter vivos transfers in the data are regarded as bequests in the model.[4] Altruism in this model is one-sided, i.e., parents care about their children but children don't care about their parents. In the baseline economy, there are assumed to be no annuity markets other than the Social Security pensions; later, the model is extended to include a private annuity market to analyze the contribution of precautionary savings.

[1]See Modigliani (1988) for several estimates of others.

[2]Intentional bequests are not necessarily motivated by parental altruism but also by risk sharing and by gift exchange. According to the empirical studies, however, those last two motives are not very large. For example, Wilhelm (1996) showed that parents tend to leave an equal amount of bequests to each of their children even if the children's earnings are significantly different. Also, Behrman and Rosenzweig (1998) showed that the relationship between the amount of bequests and the number of visits across children is not significant.

[3]This assumption is partly justified because uncertainty about earnings and lifetime implies incentives for altruistic parents to defer transfers to their children. But, in the presence of borrowing constraints, there are incentives for parents to make inter vivos transfers.

[4]In the calibration, education spending paid by parents and part of inter vivos transfers are not regarded as bequests in this model. Instead, I assume the parent-child working ability correlation, partly due to schooling, using a Markov transition matrix.

The findings in this paper are as follows. With a coefficient of relative risk aversion of 2.0, the annual time preference parameter turns out to be 0.934, and the degree of parental altruism is 0.51 for each child household and 0.69 in total.[5] The last-mentioned number indicates that a parent household cares about its child households 31 percent less than it cares about itself. If a 100 percent estate tax was introduced to eliminate parental bequest motives, national wealth would be reduced by 10 percent in a closed economy and by 15 percent in a small open economy. In addition, if a perfect annuity market was introduced to eliminate precautionary savings and accidental bequests, national wealth would be reduced, in total, by 14.3 percent and 20.5 percent in a closed and a small open economy, respectively. The effect of a perfect annuity market depends critically on whether parents are altruistic. With parental altruism, the introduction of the perfect annuity market would increase national savings slightly.

The remainder of the paper is laid out as follows. The extended overlapping generations model is described in section 2. The model is calibrated to the U.S. economy, and two main parameters are obtained in section 3. The contribution of bequests to wealth accumulation is analyzed in section 4, and section 5 concludes the paper. The algorithm to calculate the steady-state equilibria and an optimal annuity holding are explained in the appendix to this paper.

2 Model

This section describes a four-period heterogeneous agent overlapping generations model with altruistic and accidental bequests. The model considers both parental altruism and lifetime uncertainty, and households in this economy play a Cournot-Nash game to make an optimal decision about their consumption, working hours, and savings.

2.1 Economy

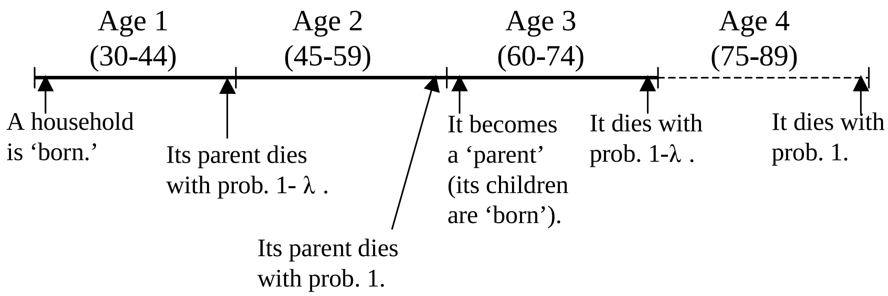
The model is based on a standard growth economy that consists of a large number of households, a perfectly competitive firm, and a government. Each household is assumed to act as a single person. In the calibration of the model, a household is assumed to be a married couple, but all of their decisions are made jointly. Also, there is assumed to be no strategic interaction between siblings.

The Life Cycle of Households. The life of a household is shown in Figure 1. In each period, new households are born without any wealth. The life span of each household is either three or four periods. One period in this model corresponds to 15 years starting from the actual age of 30. So, age 1 corresponds to 30-44 years old, age 2 corresponds to 45-59 years old, age 3 corresponds to 60-74 years old, and age 4 corresponds to 75-89 years old.[6] A household dies either at the end of age 3 or at the end of age 4. The mortality rate is known,

[5] In the calibration, I assume the population growth rate as 1 percent per year and 35 percent per generation. So, each parent household has 1.35 child households.

[6] Because of this setting, households in this model are assumed to retire at the beginning of age 60, contrary to the fact that most people retire at either age 62 or age 65 in the United States.

Figure 1: The Life Cycle of a Household



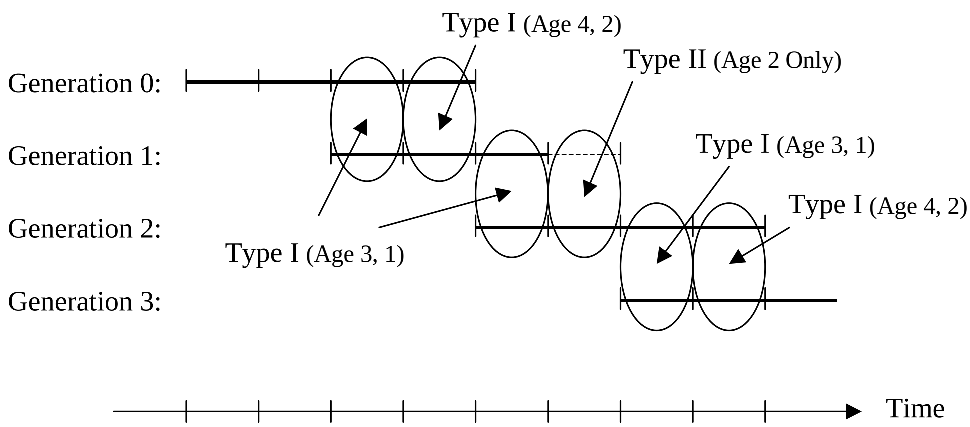
but for each household its own life span is uncertain. When a household reaches age 3, its child households of age 1 are "born," and the former becomes a parent household.

Labor Income and Capital Income. When a household is age 1 or 2, its working ability (labor productivity) at each age is stochastically determined. It receives labor income (earnings) according to the market wage rate, its working hours, and its working ability. A household of age 3 or 4 is assumed to be retired. Though a household can work at home to produce a limited amount of consumption goods and services, its working ability is assumed to be low and deterministic. There are only one kind of assets (which are supposed to be a mixture of bonds, stock, and real estate) a household can hold. It receives capital income according to its wealth level and the market interest rate. There is assumed to be a borrowing constraint, and the wealth of each household must be nonnegative.

Taxes and Social Security Benefits. A household pays federal income tax according to its total income (the sum of labor income and capital income). A household that inherits any wealth from its parent also pays federal and state estate taxes. In addition, a household of age 1 or 2 pays payroll tax for Social Security and Medicare based on its labor income. A household of age 3 or 4 receives Social Security benefits. The Social Security system is assumed to be one of defined benefit type. Every household of age 3 or 4 is eligible for Social Security benefits and, for simplicity, the size of the benefit is assumed to be the same for all households.

Dynasty and Altruism. Since each household lives either three or four periods, at any period of time there are two types of dynasties — the dynasties with both a parent household and its child households (Type I), and the dynasties with age 2 households only (Type II). Figure 2 shows the two types of dynasties in this economy. Every parent household cares about its child households and is assumed to be equally altruistic. (Since a parent also

Figure 2: Two Types of Dynasties



knows its children are altruistic toward its grandchildren, it actually cares about all its descendants indirectly.) Thus, a parent household chooses end-of-period wealth, which will be bequeathed to its child households if it dies. The wealth choice is made so as to maximize the weighted sum of its own utility and its children's utility.

Strategy of a Parent and a Child. Beginning-of-period wealth of a parent household and its child households, the working ability of the child households, and the mortality of the parent household (at the end of age 3) are known to each other. A parent and its children choose, simultaneously, their own optimal consumption, working hours, and end-of-period wealth. Since a parent household is altruistic toward its child households, and the children know their parents are altruistic, the decisions of a parent and its children are dependent on each other. For example, if a parent knew its children's wealth was going to be higher in the next period, it would reduce the amount of bequest since the marginal value of the bequest would be smaller. Also, if a child knew the bequest of its parent was going to be higher, it would reduce its own savings and consume more.[7]

2.2 The Households' Problem

2.2.1 The Preference of a Household

Consider a household that lives either three or four periods and call it a Generation 0 household. If this household is selfish, the household's problem is shown as

$$\max_{\{c_i^0, h_i^0\}_{i=1}^4} u^0 = E\left[\sum_{i=1}^{3} \beta^{i-1} u\left(c_i^0, h_i^0\right) + \lambda \beta^3 u\left(c_4^0, h_4^0\right)\right]$$

[7] In this model, a parent household and its child households behave strategically. But, this model doesn't consider so-called strategic bequests (or gift exchanges) in which the child offers service to the parent in exchange for future bequests.

subject to some budget constraint, where u^0 is the lifetime utility of Generation 0; $u(.,.)$ is an instantaneous utility function; c_i^0 and h_i^0 are consumption and working hours, respectively, at age i; β is a time preference factor; and λ is the survival rate at the end of age 3. This household chooses its optimal consumption and working hours to maximize its lifetime utility u^0.

Suppose that Generation 0 considers not only the utility from its own consumption and leisure (say, $h_{\max} - h_i^0$) but also the utility of Generation 1, and that Generation 1 considers the utility of Generation 2 as well, and so on. Then the total utility of Generation 0 (including the utility from its descendants) U^0 is shown as

$$\begin{aligned} U^0 &= \max_{\left\{c_i^0, h_i^0\right\}_{i=1}^4} E\left[u^0 + \varphi U^1\right] \\ &= \max_{\left\{c_i^0, h_i^0\right\}_{i=1}^4} E\left[u^0 + \varphi \left\{ \max_{\left\{c_i^1, h_i^1\right\}_{i=1}^4} u^1 + \varphi \left\{ \max_{\left\{c_i^2, h_i^2\right\}_{i=1}^4} u^2 + \ldots \right\}\right\}\right], \end{aligned}$$

where φ denotes the discount factor by the parent household on the utility of its child households. This paper assumes that $\varphi = \beta^2 \eta\, n < 1$, where n is the number of child households per parent household and η is the degree of parental altruism. Since the consumption and leisure of the child household occur two periods later, the utility is discounted by β^2. The parent cares about its children proportionally to the number n of its child households. The degree of parental altruism η shows how much the parent household cares about each of its adult child households relative to how much it cares about itself in the same period.

This paper decomposes φ and measures the degree of time preference β and the degree of parental altruism η (per child) or $\eta\, n$ (in total) through the calibration using the aggregate statistics of national wealth and intergenerational transfers.

2.2.2 The State of a Dynasty

Since the utility maximization problem of a household is nested as shown above, in the following sections the preference of households is described by value functions.

For Type I dynasties, the state of each dynasty is shown by the ages of parent and child households $\{(3,1), (4,2)\}$, the beginning-of-period wealth of the parent $a_p \in A = [0, a_{\max}]$ and that of its children $a_k \in A$, and the labor productivity (which determines hourly wage) of the children $e_k \in E = [E_{\min}, E_{\max}]$. In the calibration, $e_{k,i}$ is a member of $\{e_{k,i}^1, e_{k,i}^2, e_{k,i}^3\}$ for age i =1 or 2, and it follows a Markov process. For Type II dynasties, the state of each dynasty is simply shown by the age $\{2\}$, the beginning-of-period wealth, and the working ability of the age 2 households (a, e).

For notational simplicity, let s_I and s_{II} denote the states of a Type I dynasty and a Type II dynasty, respectively, where

$$\mathrm{s}_I = (a_p, a_k, e_k) \quad \text{and} \quad \mathrm{s}_{II} = (a, e).$$

Then the value function of a Type I household of age i is denoted as $v_{I,i}(\mathrm{s}_I)$, and the value function of a Type II household of age 2 is denoted as $v_{II,2}(\mathrm{s}_{II})$.

2.2.3 Type I Households

An Age 3 Parent and Its Age 1 Children. The value function of an age 3 parent household is shown as

$$\begin{aligned} v_{I,3}(\mathbf{s}_I;\mathbf{\Phi}_1) &= \max_{c_p,h_p,a'_p} \{u(c_p,h_p) + \eta\, n\, u(c_k,h_k) \\ &\quad + \beta\, E\left[\lambda\, v_{I,4}(\mathbf{s}'_I) + (1-\lambda)\,\eta\, n\, v_{II,2}(\mathbf{s}'_{II}) \,|\, e_k\right]\} \end{aligned} \tag{1}$$

subject to

$$a'_p = \frac{1}{1+\mu}\{w\, e_p h_p + (1+r)\, a_p + tr_{SS} - \tau_F(r\, a_p) - c_p\}, \tag{2}$$

where $\mathbf{s}_I$ is the state of this dynasty,

$$\mathbf{s}_I = (a_p, 0, e_k)\,,$$

$\mathbf{\Phi}_1$ is the parent's conjecture of its age 1 child households' decision,

$$\mathbf{\Phi}_1(\mathbf{s}_I) = (c_k, h_k, a'_k),$$

and the law of motion of the state of this dynasty is

$$\begin{aligned} \mathbf{s}'_I &= \left(a'_p, a'_k, e'_k\right), \\ \mathbf{s}'_{II} &= \left(a'_k + a'_p/n - \tau_E\left(a'_p/n\right), e'\right). \end{aligned} \tag{3}$$

The parent household chooses its optimal consumption c_p, working hours (housework only) h_p, and end-of-period wealth level (normalized by the economic growth) a'_p, taking the decision of its child households $\mathbf{\Phi}_1$ as given. It discounts the utility of each of n child households by η. At the end of age 3, the parent household dies with probability $(1-\lambda)$. The value of this household at the beginning of the next period is the weighted average of its own future value $v_{I,4}$ (when this household is alive) and its n children's future value $n\, v_{II,2}$ discounted by η (when this household is deceased). The term $E\left[\,.\,|\, e_k\right]$ denotes a conditional expectation given that the current working ability of an age 1 child household is e_k, i.e.,

$$E\left[v_{I,4}(\mathbf{s}'_I)\,|\, e_k\right] = \int_E v_{I,4}(\mathbf{s}'_I)\, \pi_{1,2}(e'_k\,|\, e_k)\, \mathrm{d}e'_k,$$

where $\pi_{1,2}(e'_k\,|\, e_k)$ is a conditional probability of the working ability being e'_k in the next period. The equation (2) is a budget constraint of this parent household, where μ is the growth rate of the economy, w is the wage per efficiency unit of labor, r is the rate of return on capital, tr_{SS} denotes Social Security benefits, and $\tau_F(.,.)$ is a federal income tax function. When the parent household dies, its end-of-period wealth a'_p is split equally and bequeathed to each of n child households. In the law of motion (3), $\tau_E\,(\,.\,)$ denotes an estate tax (both federal and local) function.

Similarly, the value function of an age 1 child household is shown as

$$v_{I,1}(\mathbf{s}_I;\mathbf{\Phi}_3) = \max_{c_k,h_k,a'_k} \{u(c_k,h_k) + \beta\, E\left[\lambda\, v_{I,2}(\mathbf{s}'_I) + (1-\lambda)\, v_{II,2}(\mathbf{s}'_{II})\,|\, e_k\right]\} \tag{4}$$

subject to

$$a'_k = \frac{1}{1+\mu}\{w\, e_k h_k + (1+r)a_k - \tau_F(w\, e_k h_k, r\, a_k) - \tau_S(w\, e_k h_k) - c_k\}, \tag{5}$$

where $\mathbf{\Phi}_3$ is the child's conjecture of its age 3 parent household's decision,

$$\mathbf{\Phi}_3(\mathbf{s}_I) = (c_p, h_p, a'_p),$$

and the law of motion of the state is (3).

The child household chooses its optimal consumption c_k, working hours h_k, and end-of-period wealth level a'_k, taking the decision of its parent household $\mathbf{\Phi}_3$ as given. The value of the child household at the beginning of the next period is the weighted average of its own future value when its parent is alive, $v_{I,2}$, and its future value when its parent is deceased, $v_{II,2}$. The equation (5) is a budget constraint of this child household, and $\tau_S(.)$ is a Social Security and Medicare tax (payroll tax) function.

An Age 4 Parent and Its Age 2 Children. An age 4 parent household is assumed to die at the end of this period, and each of its child households becomes a parent at the beginning of the next period. So, the value function of an age 4 parent household is shown as

$$v_{I,4}(\mathbf{s}_I; \mathbf{\Phi}_2) = \max_{c_p, h_p, a'_p} \{u(c_p, h_p) + \eta\, n\, u(c_k, h_k) + \beta\, \eta\, n\, E\left[v_{I,3}(\mathbf{s}'_I) \mid e_k\right]\} \tag{6}$$

subject to (2), where the law of motion of the state is

$$\mathbf{s}'_I = (a'_k + a'_p/n - \tau_E(a'_p/n), 0, e'_k). \tag{7}$$

The parent household considers its children's value at the beginning of the next period $n\, v_{I,3}$ discounted by η. Similarly, the value function of an age 2 child household is shown as

$$v_{I,2}(\mathbf{s}_I; \mathbf{\Phi}_4) = \max_{c_k, h_k, a'_k} \{u(c_k, h_k) + \beta\, E\left[v_{I,3}(\mathbf{s}'_I) \mid e_k\right]\} \tag{8}$$

subject to (5), where the law of motion of the state is (7).

The Strategy of a Parent and Its Children. Let $\mathbf{d}_p$ and $\mathbf{d}_k$ be the set of decisions of a parent household and each of its child households, respectively, i.e.,

$$\mathbf{d}_p = (c_p, h_p, a'_p), \quad \mathbf{d}_k = (c_k, h_k, a'_k).$$

The best response functions of an age 3 parent and an age 1 child are derived from the value functions (1) and (4) as follows:

$$\begin{aligned}\mathbf{R}_3(\mathbf{d}_k; \mathbf{s}_I) \;&=\; \arg\max_{c_p, h_p, a'_p} \{u(c_p, h_p) + \eta\, n\, u(c_k, h_k) \\ &\quad + \beta\, E\left[\lambda\, v_{I,4}(\mathbf{s}'_I) + (1-\lambda)\, \eta\, n\, v_{II,2}(\mathbf{s}'_{II}) \mid e_k\right]\}\end{aligned}$$

subject to (2), and

$$\mathbf{R}_1(\mathbf{d}_p; \mathbf{s}_I) = \arg\max_{c_k, h_k, a'_k} \{u(c_k, h_k) + \beta\, E\left[\lambda\, v_{I,2}(\mathbf{s}'_I) + (1-\lambda)\, v_{II,2}(\mathbf{s}'_{II}) \mid e_k\right]\}$$

subject to (5), where the law of motion of the state is (3).

Similarly, the best response functions of an age 4 parent and an age 2 child are derived from the value functions (6) and (8) as follows:

$$\mathbf{R}_4(\mathbf{d}_k;\mathbf{s}_I) = \arg\max_{c_p,h_p,a'_p} \left\{ u(c_p,h_p) + \eta\, n\, u(c_k,h_k) + \beta\,\eta\, n\, E\left[v_{I,3}(\mathbf{s}'_I)\,|\,e_k\right]\right\}$$

subject to (2), and

$$\mathbf{R}_2(\mathbf{d}_p;\mathbf{s}_I) = \arg\max_{c_k,h_k,a'_k} \left\{ u(c_k,h_k) + \beta\, E\left[v_{I,3}(\mathbf{s}'_I)\,|\,e_k\right]\right\}$$

subject to (5), where the law of motion of the state is (7).

Solving these best response functions, Nash equilibrium decision rules are obtained as

$$\mathbf{d}_{I,i}(\mathbf{s}_I) = \left(c_{I,i}(\mathbf{s}_I), h_{I,i}(\mathbf{s}_I), a'_{I,i}(\mathbf{s}_I)\right)$$

for $i \in \{1,2,3,4\}$. The consistency condition is $\mathbf{\Phi}_i(\mathbf{s}_I) = \mathbf{d}_{I,i}(\mathbf{s}_I)$ for all $\mathbf{s}_I \in A^2 \times E$ and $i \in \{1,2,3,4\}$.

2.2.4 Type II Households

The value function of an age 2 household is simply

$$v_{II,2}(\mathbf{s}_{II}) = \max_{c,h,a'} \left\{ u(c,h) + \beta\, E\left[v_{I,3}(\mathbf{s}'_I)\,|\,e\right]\right\} \tag{9}$$

subject to

$$a' = \frac{1}{1+\mu}\left\{w\, e\, h + (1+r)\, a - \tau_F(w\, e\, h, r\, a) - \tau_S(w\, e\, h) - c\right\}, \tag{10}$$

where the law of motion of the state is

$$\mathbf{s}'_I = (a', 0, e'_k).$$

The household chooses its optimal consumption c, working hours h, and end-of-period wealth level a'. The household's decision rules are obtained as

$$\mathbf{d}_{II,2}(\mathbf{s}_{II}) = \left(c_{II,2}(\mathbf{s}_{II}), h_{II,2}(\mathbf{s}_{II}), a'_{II,2}(\mathbf{s}_{II})\right).$$

2.3 The Measure of Households

Let $x_{I,i}(\mathbf{s}_I)$ denote the measure of Type I households of age $i \in \{1,2,3,4\}$, and let $x_{II,2}(\mathbf{s}_{II})$ denote that of Type II households of age 2. Also, let $X_{I,i}(\mathbf{s}_I)$ and $X_{II,2}(\mathbf{s}_{II})$ be the corresponding cumulative measures.

2.3.1 Population of Households

The population of age 1 child households is normalized to be unity, i.e.,

$$\int_{A^2\times E} \mathrm{d}X_{I,1}(\mathbf{s}_I) = 1.$$

Then, the population of Type I child households of age 2 is

$$\int_{A^2\times E} \mathrm{d}X_{I,2}(\mathbf{s}_I) = \frac{\lambda}{1+\nu},$$

where ν is a population growth rate and λ is the survival rate of the parent household. The population of Type II households of age 2 is

$$\int_{A\times E} \mathrm{d}X_{II,2}(\mathbf{s}_{II}) = \frac{1-\lambda}{1+\nu}.$$

Since every Type I parent household has $n = (1+\nu)^2$ child households, the population of Type I parent households of age 3 is

$$\int_{A^2\times E} \mathrm{d}X_{I,3}(\mathbf{s}_I) = \frac{1}{n},$$

and the population of Type I parent households of age 4 is

$$\int_{A^2\times E} \mathrm{d}X_{I,4}(\mathbf{s}_I) = \frac{\lambda}{n\ (1+\nu)}.$$

Since we don't have to use $x_{I,3}(\mathbf{s}_I)$ and $x_{I,4}(\mathbf{s}_I)$ because

$$x_{I,3}(\mathbf{s}_I) = \frac{1}{n}x_{I,1}(\mathbf{s}_I) \quad \text{and} \quad x_{I,4}(\mathbf{s}_I) = \frac{1}{n}x_{I,2}(\mathbf{s}_I)$$

for all $\mathbf{s}_I \in A^2 \times E$, from now on only $\{x_{I,1}(\mathbf{s}_I), x_{I,2}(\mathbf{s}_I), x_{II,2}(\mathbf{s}_{II})\}$ is used as the measure of dynasties.

2.3.2 The Law of Motion of the Measures

Let $\mathbf{1}_{[a'=y]}$ be an indicator function that returns 1 if $a' = y$ and 0 if $a' \neq y$. Then, the law of motion of the measure of Type I dynasties is

$$x'_{I,2}(\mathbf{s}'_I) = \frac{\lambda}{1+\nu}\int_{A^2\times E} \mathbf{1}_{\left[a'_p=a'_{p,3}(\mathbf{s}_I)\right]}\,\mathbf{1}_{\left[a'_k=a'_{k,1}(\mathbf{s}_I)\right]}\,\pi_{1,2}(e'_k \mid e_k)\,\mathrm{d}X_{I,1}(\mathbf{s}_I), \tag{11}$$

and

$$\begin{aligned} x'_{I,1}(\mathbf{s}'_I) \quad = \quad (1+\nu)\Bigg\{ &\int_{A^2\times E} \mathbf{1}_{\left[a'_p=a'_{k,2}(\mathbf{s}_I)+a'_{p,4}(\mathbf{s}_I)/n-\tau_E\left(a'_{p,4}(\mathbf{s}_I)/n\right)\right]} \\ &\qquad\qquad\qquad\qquad \times \pi_{2,1}(e'_k \mid e_k)\,\mathrm{d}X_{I,2}(\mathbf{s}_I) \\ &+\int_{A\times E} \mathbf{1}_{\left[a'_p=a'_2(\mathbf{s}_{II})\right]}\,\pi_{2,1}(e'_k \mid e)\,\mathrm{d}X_{II,2}(\mathbf{s}_{II})\Bigg\}, \end{aligned} \tag{12}$$

and the law of motion of the measure of Type II households is

$$x'_{II,2}(\mathbf{s}'_{II}) \;=\; \frac{1-\lambda}{1+\nu}\left\{\int_{A^2\times E}\mathbf{1}_{\left[a'=a'_{k,1}(\mathbf{s}_I)+a'_{p,3}(\mathbf{s}_I)/n-\tau_E\left(a'_{p,3}(\mathbf{s}_I)/n\right)\right]}\right.$$
$$\left.\times\,\pi_{1,2}(e'\,|\,e_k)\,\mathrm{d}X_{I,1}(\mathbf{s}_I)\right\}. \tag{13}$$

The steady-state condition is

$$x'_{I,i}(\mathbf{s}_I) = x_{I,i}(\mathbf{s}_I) \quad \text{and} \quad x'_{II,2}(\mathbf{s}_{II}) = x_{II,2}(\mathbf{s}_{II}) \tag{14}$$

for all $\mathbf{s}_I \in A^2 \times E,\ \mathbf{s}_{II} \in A \times E$, and $i \in \{1,2\}$.

2.4 The Firm's Problem

There is only one perfectly competitive firm in this economy. In a closed economy, the stock of fixed capital K is equal to the sum of total private wealth and the government net wealth W_g. Total labor demand L is equal to total labor supply of households in efficiency units.

$$K = \sum_{i=1}^{2}\int_{A^2\times E}(a_p/n + a_k)\,\mathrm{d}X_{I,i}(\mathbf{s}_I) + \int_{A\times E} a\,\mathrm{d}X_{II,2}(\mathbf{s}_{II}) + W_g, \tag{15}$$

$$L \;=\; \sum_{i=1}^{2}\int_{A^2\times E}\left(e_p\,h_{I,i+2}(\mathbf{s}_I)/n + e_k\,h_{I,i}(\mathbf{s}_I)\right)\,\mathrm{d}X_{I,i}(\mathbf{s}_I)$$
$$+\int_{A\times E} e\,h_{II,2}(\mathbf{s}_{II})\,\mathrm{d}X_{II,2}(\mathbf{s}_{II}). \tag{16}$$

In a closed economy, the total output Y is determined by a production function,

$$Y = F(K, AL).$$

The profit-maximizing condition of the firm is

$$r + \delta = F_K(K, AL), \qquad w\,(1+\tau'_S) = F_L(K, AL), \tag{17}$$

where δ is the depreciation rate of capital and τ'_S is the marginal Social Security tax rate.

2.5 The Government's Policy Rule

Government tax revenue consists of federal income tax T_F, Social Security and Medicare taxes T_S, and federal and state estate taxes in the next period T'_E. These revenues are calculated as follows:

$$T_F \;=\; \sum_{i=1}^{2}\int_{A^2\times E}\tau_F\left(w\,e_p\,h_{I,i+2}(\mathbf{s}_I),\, r\,a_p\right)/n\,\mathrm{d}X_{I,i}(\mathbf{s}_I)$$
$$+\sum_{i=1}^{2}\int_{A^2\times E}\tau_F\left(w\,e_k\,h_{I,i}(\mathbf{s}_I),\, r\,a_k\right)\,\mathrm{d}X_{I,i}(\mathbf{s}_I)$$
$$+\int_{A\times E}\tau_F\left(w\,e\,h_{II,2}(\mathbf{s}_{II}),\, r\,a\right)\,\mathrm{d}X_{II,2}(\mathbf{s}_{II}), \tag{18}$$

$$
\begin{aligned}
T_S &= \sum_{i=1}^{2} \int_{A^2 \times E} \tau_S \left(w\, e_k\, h_{I,i}(\mathbf{s}_I)\right) \, \mathrm{d}X_{I,i}(\mathbf{s}_I) \\
&\quad + \int_{A \times E} \tau_S \left(w\, e\, h_{II,2}(\mathbf{s}_{II})\right) \, \mathrm{d}X_{II,2}(\mathbf{s}_{II}), \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
T'_E &= (1-\lambda) \int_{A^2 \times E} \tau_E \left(a'_{I,3}\left(\mathbf{s}_I\right)/n\right) \, \mathrm{d}X_{I,1}(\mathbf{s}_I) \\
&\quad + \int_{A^2 \times E} \tau_E \left(a'_{I,4}\left(\mathbf{s}_I\right)/n\right) \, \mathrm{d}X_{I,2}(\mathbf{s}_I). \qquad (20)
\end{aligned}
$$

Total tax revenue is the sum of these three tax revenues and Social Security tax from employers, i.e.,

$$T = T_F + 2\, T_S + T_E.$$

The law of motion of the government wealth (debt if it is negative) is

$$W'_g = \frac{1}{1+\mu+\nu} \left\{ (1+r)\, W_g + T - C_g - tr_{SS}\, N_{OLD} \right\}, \qquad (21)$$

where N_{OLD} is the population of households of age 3 or 4, i.e.,

$$N_{OLD} = \frac{1}{n} \left(1 + \frac{\lambda}{1+\nu} \right).$$

2.6 Recursive Competitive Equilibrium

This paper considers the steady state of the economy only, because the main purpose of the model is to measure the degrees of time preference and parental altruism and to estimate the contribution of bequests to wealth accumulation. The transition analyses will be presented in other papers. The definition of a steady-state recursive competitive equilibrium (which is also a Markov Perfect Equilibrium) for this model is as follows:

Definition 1 ***Steady-State Recursive Competitive Equilibrium:*** *Let $\mathbf{s}_I$ and $\mathbf{s}_{II}$ be the state of a Type I dynasty and that of a Type II dynasty, respectively, where*

$$\mathbf{s}_I = (a_p, a_k, e_k), \quad \mathbf{s}_{II} = (a, e).$$

Given the time invariant government policy rules,

$$\mathbf{\Psi} = \left\{ \tau_F(.), \tau_{\mathbf{S}}(.), \tau_E(.), tr_{SS}, C_g, W_g \right\};$$

factor prices, r and w; the value functions of households,

$$\left\{ v_{I,i}\left(\mathbf{s}_I\right) \right\}_{i=1}^{4} \text{ and } v_{II,2}\left(\mathbf{s}_{II}\right);$$

the decision rules of households,

$$\left\{ c_{I,i}\left(\mathbf{s}_I\right), h_{I,i}\left(\mathbf{s}_I\right), a'_{I,i}\left(\mathbf{s}_I\right) \right\}_{i=1}^{4} \text{ and } \left\{ c_{II,2}\left(\mathbf{s}_{II}\right), h_{II,2}\left(\mathbf{s}_{II}\right), a'_{II,2}\left(\mathbf{s}_{II}\right) \right\};$$

the measures of dynasties,

$$\left\{ x_{I,i}\left(\mathbf{s}_I\right) \right\}_{i=1}^{2} \text{ and } x_{II,2}\left(\mathbf{s}_{II}\right),$$

are in a steady-state recursive competitive equilibrium if, in every period,

1. each household solves the utility-maximization problem, (1) – (9), taking its counterpart's (either its parent's or child's) decision as given,
2. the firm solves the profit-maximization problem, and the capital and labor markets clear, i.e., (15) – (17) hold,
3. the government policy rules satisfy (18) – (21),
4. the goods market clears, and
5. the measures of dynasties are constant, i.e., (14) holds.

3 Calibration

The two main parameters, the degree of time preference β and that of parental altruism η, are determined simultaneously so that the steady state of the model replicates the U.S. economy in terms of two key statistics — the capital-output ratio and the relative size of bequests including a part of inter vivos transfers. The functional forms and other parameters are chosen so as to be consistent with macroeconomic and cross-section data in the United States.

As is explained below, a Cobb-Douglas-CRRA utility function and a Cobb-Douglas production function are used for the calibration. Table 1 summarizes the choice of these parameters.

Table 1: Parameters

Share Parameter for Consumption	α	0.765
Coefficient of Relative Risk Aversion	γ	2.0
Capital Share of Output	θ	0.32
Depreciation Rate of Capital Stock	δ	0.046*
Long-Term Real Growth Rate	μ	0.011*
Population Growth Rate	ν	0.010*
Survival Rate at the End of Age 3	λ	0.546

* annual rate

The following subsections describe the choice of functional forms and parameter values, the choice of two target variables and values, and, finally, the result of the calibration — the degrees of time preference and parental altruism.

3.1 Households

There is assumed to be a large number of households, and the population of age 1 households is normalized to be unity.

Table 2: The Working Hours of a Household (Head and Wife, including Housework)

	Ages 1 and 2 (30-59 years old)
Mean	4,810
Standard Deviation	1,353
Skewness	−0.121
Kurtosis	3.835
90th percentile	6,430

Source: PSID 1993 Family Data.

Utility Function. According to Cooley and Prescott (1995), the elasticity of substitution of consumption for leisure is not far from unity. So, the model uses the following Cobb-Douglas utility function with constant relative risk aversion (CRRA),

$$u(c_i, h_i) = \frac{\{c_i^{\alpha}(h_i^{\max} - h_i)^{1-\alpha}\}^{1-\gamma} - 1}{1-\gamma}.$$

Here γ is the coefficient of relative risk aversion, and it is set to be 2.0 in the main calibration. Later, I also show the results when the coefficient is toggled from 1.0 to 4.0. The maximum working hours $h_i^{\max}$ depend on the age of the household and are explained below.

Working Hours. Table 2 shows the working hours of a household in the Panel Study of Income Dynamics (PSID) 1993 Family Data. The annual working hours are the sum of the working hours of a husband (Head) and a wife (Wife), including housework.

$$\begin{aligned}\text{Total Work Hours} \quad &= \quad \text{Head's Market Work Hours} + \text{Head's Housework Hours} \\ &\quad\ +\text{Wife's Market Work Hours} + \text{Wife's Housework Hours.}\end{aligned}$$

Suppose the 90th percentile (6,430 hours) is regarded as the maximum working hours $h_i^{\max}$ ($i = 1$ or 2). Then, the share of average working hours (4,810 hours) is 0.75, and the share of leisure is 0.25.[8] So, the share parameter for consumption in the Cobb-Douglas utility function should not be very far from 0.75. In this calibration, the parameter α is chosen to be 0.765 so that average working hours of age 1 and age 2 become approximately 4,810 hours in the steady state.

Market Work and Housework. According to the PSID data, for married couples between ages 30 and 59, about 68 percent of total working hours are declared as market work and the remaining hours are declared as housework. This ratio is used to compute the taxable earnings of each household in the model. Also, one hour of housework is assumed to produce

[8]The maximum working hours in the PSID data are actually 11,400 hours in 1992. But, I use the 90th percentile instead because the average of the maximum in 15 years must be significantly smaller.

the same value of consumption goods or services as the after-tax hourly market wage of this household.

For simplicity, there is no retirement decision in this model, and all of the age 3 and age 4 households are retired. The maximum working hours of age 1 and age 2 are assumed to be 6,430 hours, in which 68 percent are assumed to be market working hours. Subtracting this amount, the maximum working hours of age 3 and age 4, $h_i^{\max}$ ($i = 3$ or 4), are set to be 2,058 hours.

Working Ability. The working ability in this model corresponds to the hourly wage of each household. According to the PSID data, the average hourly wage of a married couple has the distribution shown in Table 3.[9] Data are calculated by using the family weight in PSID and the following formula,

$$\text{Hourly Wage} = \frac{\text{Head's Labor Income} + \text{Wife's Labor Income}}{\text{Head's Market Work Hours} + \text{Wife's Market Work Hours}}.$$

Table 3: The Average Hourly Wage of a Married Couple (in U.S. dollars)

	Age 1 (30-44 years old)	Age 2 (45-59 years old)
Mean	18.181	18.663
Standard Deviation	19.475	17.403
Skewness	6.708	4.583
Kurtosis	66.332	35.280
Gini Index	0.380	0.385

Source: PSID 1993 Family Data.

Three discrete levels of working abilities in Table 4 are chosen based on the statistics of the hourly wage. For each age, three levels of ability e^1, e^2, and e^3 and their probabilities p^1 and p^2 are computed so that these five numbers attain the five statistics in Table 3.

According to Bureau of Labor Statistics data, average hourly earnings of production workers have increased by 16.2 percent since 1992. Multiplying by 1.162, the numbers e^1, e^2, and e^3 are converted into those corresponding to 1997 U.S. dollars.

Markov Transition Matrix. Since one period in this model corresponds to 15 years, to compute a transition matrix of working ability from the data, we need to have 30 years of longitudinal hourly wage data. But the PSID has at most 27 years of longitudinal series (1968-1994). So, the transition matrix from age 1 to age 2 and the matrix from an age 2 parent to an age 1 child were constructed based on the statistics of hourly wages. The steady-state distribution of each age is consistent with the five statistics in Table 3. Also, the correlation of hourly wages of age 1 and age 2 is assumed to be 0.80, and that of an age 2

[9] If we consider households that didn't work at all in the market in 1992, the distribution of actual working ability will be slightly different from that of hourly wages.

Table 4: Working Abilities of a Household (in 1992 U.S. dollars per hour)

	Age 1 (30-44 years old)		Age 2 (45-59 years old)	
e^1	6.94	8.06*	7.51	8.73*
e^2	27.87	32.38*	29.45	34.22*
e^3	208.16	241.88*	170.19	197.76*
p^1	0.535		0.542	
p^2	0.457		0.450	
$1-p^1-p^2$	0.008		0.008	

Note: p^1 and p^2 are the shares of e^1 and e^2 households, respectively. $1-p^1-p^2$ is the share of e^3 households.
* in 1997 U.S. dollars.

parent and an age 1 child is assumed to be 0.40.[10] The Markov transition matrices used in this calibration are

$$\Gamma^{12}=\begin{pmatrix}0.879 & 0.121 & 0\\ 0.157 & 0.840 & 0.003\\ 0 & 0.171 & 0.829\end{pmatrix}, \qquad \Gamma^{21}=\begin{pmatrix}0.622 & 0.378 & 0\\ 0.440 & 0.550 & 0.010\\ 0 & 0.562 & 0.438\end{pmatrix}$$

where $\Gamma^{12}(i,j)=\pi(e_2=e_2^j\,|\,e_1=e_1^i)$ and $\Gamma^{21}(i,j)=\pi(e_1=e_1^j\,|\,e_2=e_2^i)$.

Population Growth and Mortality. The population growth rate is assumed to be annually 1.0 percent and 16.1 percent per period (15 years). Since new households are born to the dynasty every 30 years, the number of child households per parent household n is 1.348 ($=1.01^{30}$). In the United States, the life expectancy of a 60-year-old male is 80.68 and that of a 60-year-old female is 85.71.[11] Taking the simple average of male and female, the life expectancy becomes 83.20 years. The survival rate at the end of age 3 (75 years old) λ is set at 0.546 so that the life expectancy in this model is also 83.20.

3.2 The Firm

There is only one perfectly competitive firm in this economy.

Production Function. The model uses the Cobb-Douglas production function,

$$F(K_t, A_tL_t)=K_t^{\theta}\left(A_tL_t\right)^{1-\theta},$$

where

$$A_t = e^{\mu t}A, \qquad L_t=e^{\nu t}L,$$

[10] See Solon (1992) for the father-son correlations in hourly wages.
[11] Source: http://www.retireweb.com/, which is based on the Group Annuity Mortality Table (GAM83).

μ is the growth rate of labor productivity, and ν is the population growth rate. The capital share of output θ is chosen by

$$\theta = 1 - \frac{\text{Compensation of Employees } + (1-\theta) \times \text{Proprietors' Income}}{\text{National Income } + \text{ Consumption of Fixed Capital}}.$$

The average of θ in 1996-1998 is 0.32. The annual growth rate of total factor productivity is assumed to be 0.74 percent. This number is the average in 1980-1995 computed from the data. When the labor share of output, $1-\theta$, is equal to 0.68, the growth rate of labor productivity is 1.1 percent. The growth rate of the economy μ is also assumed to be 1.1 percent. The annual population growth rate ν between 1970-1995 is around 1.0 percent. The labor productivity A is chosen to be 0.974 so that the wage per unit of efficient labor is normalized to be unity.

Fixed Capital and Private Wealth. To compute the GNP and factor prices in the model, the fixed capital K is computed by the following formula:

$$\begin{aligned}\text{Fixed Capital} \quad &\equiv \quad \text{Fixed Reproducible Tangible Wealth} \\ &\quad\; -\text{Durable Goods Owned by Consumers.}\end{aligned}$$

These data are taken from the Survey of Current Business (1997). In 1990-1996, fixed capital accounted for 89.7 percent of fixed reproducible tangible wealth, and durable goods owned by consumers accounted for the remainder.

To connect the total private wealth with the fixed capital, it is assumed that all of the private capital is owned by households and that part of the government-owned fixed capital is effectively owned by households in the form of government bonds.

$$\begin{aligned}\text{Fixed Capital} \quad &= \quad \text{Private Wealth } + \text{ Government-Owned Fixed Capital} \\ &\quad\; - \text{ Government Bonds Owned by Private Sector} \\ &\quad\; - \text{ Durable Goods Owned by Consumers.}\end{aligned}$$

Based on the data in 1990-1996, I use an approximate relationship,

$$\text{Fixed Capital} = 0.956 \times \text{Private Wealth},$$

in the model.

The Depreciation Rate of Fixed Capital. The depreciation rate of fixed capital δ is chosen by

$$\delta = \frac{\text{Total Gross Investment}}{\text{Fixed Capital}} - \text{Productivity Growth} - \text{Population Growth.}$$

In 1998, gross private domestic investment accounted, on average, for 16.1 percent of GDP, and gross government investment (federal and state) accounted for 2.8 percent of GDP. Based on these numbers and the capital-output ratio of 2.81, the gross investment - fixed capital ratio is 6.7 percent. Subtracting the productivity and population growth rates, the annual depreciation rate is assumed to be 4.6 percent.

3.3 Taxes and Transfers

Federal Income Taxes. For federal income tax, the model uses the following tax function estimated by Gouveia and Strauss (1994),

$$\text{Federal Income Tax } = \phi_0 \left(y - (y^{-\phi_1} + \phi_2)^{-1/\phi_1} \right),$$

where y is the total income of a household after the standard deduction and exemption for four people. In 1997, the sum of this deduction and exemption was $17,900. The parameters $\phi_1 = 0.768$ and $\phi_2 = 0.031$ are taken from Gouveia and Strauss's estimation of 1989 federal income tax function. The parameter ϕ_0 corresponds to the limit of the marginal tax rate. To replicate the statutory income tax, the parameter must be around 0.4 (40 percent). But, because of itemized deductions, the effective tax rate of high-income households is regarded as much lower.[12] Since in 1997 the ratio of total private income tax to nominal GDP was 0.091, ϕ_0 is assumed to be 0.25 so that the income tax - GDP ratio is around 9 percent in the steady-state equilibrium.

Payroll Tax. The Social Security tax rate is 6.2 percent and the Medicare tax rate is 1.45 percent. These are assumed to be taxed on labor income. In 1997, employee compensation above $68,400 was not taxable for Social Security. But, for simplicity, this ceiling is not considered in this analysis, i.e., it is assumed to be a 7.65 percent flat payroll tax in total. The same amount of taxes is also paid by employers. So the firm's profit-maximization problem becomes

$$w \times (1 + \text{Marginal Social Security Tax Rate}) = AF_L(K, AL),$$

where the marginal Social Security (and Medicare) tax rate is 0.0765.

Estate Taxes. For federal estate tax, the model uses the same function as federal income tax, i.e.,

$$\text{Federal Estate Tax per Capita} = \psi_0 \left(b - (b^{-\psi_1} + \psi_2)^{-1/\psi_1} \right).$$

In this function, b is the number of bequests per capita minus the exemption of $600,000. The parameters $\psi_0 = 0.55$, $\psi_1 = 0.265$, and $\psi_2 = 0.049$ are chosen so that the function replicates the federal estate tax schedule. It is simply assumed that each household consists of a married couple (two persons) and that a husband and a wife receive bequests from his or her own parents separately. In total, a household can receive at most $1.2 million without paying federal tax. The federal estate tax that each household pays is twice the number in the above equation.

In addition, each household pays state estate taxes. Since the estate tax rate differs from one state to another, in this calibration it is simply assumed to be a 6 percent flat tax.[13]

[12] See Gouveia and Strauss (1994) for effective federal tax rates.

[13] For example, the estate tax in Pennsylvania is a 6 percent flat tax and interspousal transfers are not taxable. The estate tax in New York State is a progressive tax, ranging from 2 percent on amounts below $50,000 to 21 percent (marginal) on amounts above $10.1 million. The average tax rate when $1 million is bequeathed is 5.4 percent.

Social Security Benefits. Social Security benefits depend on average indexed monthly earnings and the corresponding replacement rate. To calculate the benefit, we need to have an additional state variable, the working history of the parent, for each dynasty. But, in this calibration, for simplicity, the annual benefits per married couple are assumed to be the same for all eligible households. Table 5 shows the example of benefits for four types of married couples.[14] If we take an average of these four couples, the annual benefit per couple is $13,161. If this number is adjusted using the consumer price index, the estimated benefit per couple for 1997 is $18,600. Thus, the annual benefit is assumed to be $18,600 in the steady-state equilibrium.

Table 5: Monthly Social Security Benefits for Retiring Couples in 1987 (in U.S. dollars)

	Couple A	Couple B	Couple C	Couple D
Earnings				
Husband	24,000	12,000	16,000	24,000
Wife	0	12,000	8,000	8,000
Monthly Benefits				
Husband	797	499	606	797
Wife	398	499	393	398
Total Benefits	1,195	998	999	1,195

Source: Table 4-4 in Schulz (1995)

3.4 Target Variables

Capital-Output Ratio. The target value of the capital-output ratio is 2.81. The capital stock used here is measured by 'fixed reproducible tangible wealth' minus 'durable goods owned by consumers.' These data are taken from the Survey of Current Business (1997). For the output data, the nominal gross domestic product is used. So, the average capital-output ratio for 1990-1996 is 2.81.

The Relative Size of Bequests: Gale and Scholz (1994). For the relative size of bequests, the model uses the flow data from Gale and Scholz (1994) based on the 1986 Survey of Consumer Finance (SCF) in which each head of household was asked if he or she contributed $3,000 or more to other households during 1983-1985. Gale and Scholz computed the flow of intergenerational transfers to estimate the stock of transfer wealth. Table 6 shows the annual flows of intergenerational transfers and their relative sizes as a percentage of net wealth.[15]

Table 7 shows the target values used in this calibration. First, I include both trusts and life insurance in bequests from parents to children. Then, the relative size of bequests becomes 1.06 percent of total private wealth. Second, since no inter vivos transfers are assumed in

[14] Excerpted from Table 4-4 in Schulz (1995). Original source: Technical Committee on Earnings Sharing, *Earnings Sharing in Social Security: A Model for Reform* (Washington, D.C.: Center for Women Policy Studies, 1988), Table A-1.3.1.2.3.

[15] Excerpted from Table 4, pp.152, Gale and Scholz (1994), and rearranged.

Table 6: The Annual Flows of Intergenerational Transfers — Gale and Scholz (1994)

Transfer Category	Annual Flow In Billions of Dollars	As a Percentage of Net Wealth*
Support Given to:		
Children or Grandchildren	37.74	0.32
Parents or Grandparents	3.44	0.03
Trusts	14.17	0.12
Life Insurance	7.84	0.07
Bequests	105.00	0.88
College Payments	35.29	0.29

*Aggregate net wealth in the SCF in 1986 was $11,976 billion.

this model, I include a half of gifts to children or grandchildren in bequests from parents to children as disguised bequests. The relative size of the total transfer becomes 1.22 percent.

According to the estimate by Gale and Scholz, the stock of inter vivos transfers as a percentage of net wealth goes down from 20.8 percent to 17 percent if they don't include the supplemental high-income subsample of SCF. To avoid the influence of very wealthy households, I simply apply this rate of reduction to the flow data. The second column of the table shows the target variables used in this calibration.

Table 7: Target Variables and Values on Bequests

Transfer Category	As a Percentage of Net Wealth	Adjusted* ($\times$17/20.8)
The Annual Flow of Bequests, Trusts, Life Insurance, and a Part of Gifts	1.22	1.00

*Adjusted to remove the effects of the supplemental high-income subsample.

The Transfer Wealth Ratio: Kotlikoff and Summers (1981). Alternatively, we can use the transfer wealth measure defined by Kotlikoff and Summers (1981) after making some adjustments. In their definition, the capital return on transfer wealth is also regarded as transfer wealth. In this model, the interest rate at the steady-state equilibrium is approximately 6.8 percent, and the after-tax interest rate is 5.44 percent if the tax rate on interest is around 20 percent. Based on the table in their paper, at this interest rate the share of transfer wealth is around 90 percent.

One of the difficulties in applying their measure to this calibration is that their transfer wealth includes all transfers from parents to children above age 18. To distinguish inter vivos transfers when children are below age 30 (such as college tuition and other expenses) from others, the same ratio used for the SCF data is applied. Then, 73 percent of the total flow is transferred as bequests and inter vivos transfers when children are above age 30. If the timing of these transfers is considered, 54 percent of transfer wealth can be regarded as bequeathed assets.[16] Then, the share of transfer wealth (bequests and disguised bequests) that corresponds to this model becomes 49 percent. The target value of the transfer wealth in this model is set to be 49 percent of total private wealth.

3.5 Obtained Main Parameters

Table 8 shows the key parameters obtained through the calibration when the target variable is the annual flow of bequests (= 1.0 percent). Since each parent household in a Type I dynasty is assumed to have $n = 1.35$ child households, for the altruistic parameter η the first column shows the parameter per recipient and the second column shows the parameter per donor.

Table 8: When the Annual Flow of Bequests is 1.0 Percent of Total Private Wealth

	Per Recipient		Per Donor	
Annual Time Preference	β	0.934		
Parental Altruism	η	0.509	ηn	0.686
Preference on the Next Generation	$\beta^{30}\eta$	0.066	$\beta^{30}\eta n$	0.089

Note: γ (the coefficient of relative risk aversion) $= 2.0$.

The annual time preference parameter β turned out to be 0.934. In other words, the annual discount rate of a household's future utility is 6.6 percent.

The degree of parental altruism η is 0.509. This number shows the relative importance of the current (future) utility of each child household to the current (future) utility of the parent's own. Since a parent household is assumed to have 1.35 child households in this calibration, the degree of parental altruism toward its child households is 0.686 in total. This means that a parent household cares about its child households 31 percent less than it cares about itself.

Table 9 shows the obtained parameters when the target variable is the transfer wealth ratio (= 49 percent). The degree of time preference is almost equal to the figure in the previous calibration, but the degree of parental altruism is a little higher.

Table 9: When the Transfer Wealth Is 49 Percent of Total Private Wealth

	Per Recipient		Per Donor	
Annual Time Preference	β	0.934		
Parental Altruism	η	0.580	ηn	0.782
Preference on the Next Generation	$\beta^{30}\eta$	0.074	$\beta^{30}\eta n$	0.099

Note: γ (the coefficient of relative risk aversion) $= 2.0$.

[16] Here, I assumed the difference of timing is 25 years, and the gap between the after-tax interest rate and the productivity and population growth rate is 3.34 percent.

In the main calibration, the coefficient of relative risk aversion γ is assumed to be 2.0. Table 10 shows the results under different assumptions of γ from 1.0 to 4.0.

Table 10: Obtained Parameters Under Different Assumptions of γ

		Coefficient of Relative Risk Aversion γ			
		1.0	2.0	3.0	4.0
Annual Time Preference	β	0.945	0.934	0.921	0.906
Parental Altruism	η	0.697	0.509	0.393	0.328

If γ were higher (lower), both the parameter of time preference and the degree of altruism would be lower (higher) to keep the capital-output ratio and the relative sizes of bequests and inter vivos transfers at the same level.

4 Policy Experiments

Policy experiments in this paper focus on the effect of altruistic and accidental bequests on wealth accumulation. If there were no parental altruism nor lifetime uncertainty, there would be no bequests in the economy.[17] The wealth of parent households would be reduced. But, at the same time, the savings of child households might be increased because those households could not expect any bequests from their parents any more. The purpose of the following policy experiments is to evaluate the overall effect of bequests on national wealth.

First, a 100 percent estate tax is introduced to the baseline economy. If the estate tax rate was raised to 100 percent, all of the altruistic bequests would be eliminated. Thus, from the donor's point of view, the remaining bequests would be accidental because of lifetime uncertainty. From the recipient's point of view, accidental bequests would also disappear because of the 100 percent tax. It is assumed that the increase in tax revenue from the estate tax on these accidental bequests is transferred to all households in a lump-sum manner. Thus, the change in national wealth can be regarded as the contribution of altruistic bequests.

Next, a perfect annuity market is introduced to the baseline economy to evaluate the contribution of accidental bequests to wealth accumulation. The perfect annuity market has two effects on private savings — (1) to increase the marginal value of savings, and (2) to decrease the precautionary savings caused by lifetime uncertainty. It will be shown that, in the presence of parental altruism, the net effect of the perfect annuity market on wealth accumulation is small and may be opposite to the effect in the absence of altruism.

Finally, both the 100 percent estate tax and the perfect annuity market are introduced at the same time, eliminating all of the bequests from the economy. In the absence of altruistic bequests, the effect of the perfect annuity market is significantly large. In total, national wealth would be reduced by 14.3 percent in a closed economy and 20.5 percent in a small open economy.

[17] This is not necessarily true if bequests are motivated by risk sharing or by gift exchange.

4.1 A 100 Percent Estate Tax

If a 100 percent estate tax was introduced, there would be no incentive for parent households to leave any bequests, but accidental bequests due to lifetime uncertainty would remain in the absence of perfect annuity markets. Here, the tax increase from the estate tax on accidental bequests is assumed to be transferred to all households in a lump-sum manner so that the government wealth level as well as expenditure would not change. The result of this policy experiment is shown in Table 11.

Table 11: Changes from the Baseline Economy When a 100 Percent Estate Tax Is Added

	Closed Economy	Small Open Economy
$\%\Delta$ Capital Stock	-10.0	-15.0
$\%\Delta$ Labor	0.0	0.3
$\%\Delta$ GNP	-3.3	-4.6
$\%\Delta$ Bequests	-33.6	-42.1
$\%\Delta$ Interest Rate$^{(1)}$	0.8	no change
$\%\Delta$ Wage Rate	-3.3	no change
Δ Annual Lump-Sum Transfers$^{(2)}$	1.010	0.807

(1) Change in Percentage Points

(2) $1,000 per household

The capital stock, which corresponds to national wealth, would be reduced by 10.0 percent in a closed economy and 15.0 percent in a small open economy. Thus, altruistic bequests account for about 10 percent to 15 percent of national wealth, and life-cycle savings and accidental bequests (precautionary savings) account for the remainder. The reduction in wealth is smaller in a closed economy because of the general equilibrium effect. In a closed economy, the rate of return on capital would rise by 0.8 percentage points and encourage household savings. The wage rate would fall by 3.3 percent in a closed economy.

The gross national product would be reduced by 3.3 percent in a closed economy and by 4.6 percent in a small open economy. Bequests would be reduced by about 33.6 percent and 42.1 percent, respectively, in these two economies.[18] Because of the increase in the tax revenue on accidental bequests, the annual lump-sum transfer would be increased by $1,010 in a closed economy and $807 in a small open economy.

4.2 A Perfect Annuity Market

In the previous policy experiment, even if a 100 percent estate tax was introduced, about 58 percent to 66 percent of the original level of bequests would remain as accidental bequests. What would happen if a perfect annuity market was introduced to the economy to elimi-

[18] We cannot conclude that altruistic bequests account for 34 percent to 42 percent of total bequests and accidental bequests account for the remainder because these two types of bequests are not additively separable.

nate all accidental bequests? The extension of the model for the perfect annuity market is described in the appendix to this paper.

First, only the effect of a perfect annuity market is evaluated. In this case, precautionary savings for lifetime uncertainty would be reduced in the presence of annuity markets, but the marginal value of savings would be increased. The total effect is shown in Table 12.

Table 12: Changes from the Baseline Economy When a Perfect Annuity Market Is Added

	Closed Economy	Small Open Economy
$\%\Delta$ Capital Stock	0.5	0.5
$\%\Delta$ Labor	0.5	0.5
$\%\Delta$ GNP	0.5	0.5
$\%\Delta$ Bequests	-43.6	-43.6
$\%\Delta$ Interest Rate$^{(1)}$	0.0	no change
$\%\Delta$ Wage Rate	0.0	no change
Δ Annual Lump-Sum Transfers$^{(2)}$	0.205	0.205

(1) Change in Percentage Points

(2) $1,000 per household

The effect of a perfect annuity market on wealth accumulation is relatively small. The capital stock (national wealth) would be increased by 0.5 percent in both a closed economy and a small open economy. Because the marginal value of savings would increase, households are encouraged to work longer, and the gross national product would be increased by 0.5 percent both in a closed economy and in a small open economy. When accidental bequests were eliminated by the perfect annuity market, the level of bequests was reduced by roughly 44 percent in both economies.

4.3 A Perfect Annuity Market with a 100 Percent Estate Tax

What is the total contribution of bequests, both altruistic and accidental, to the wealth accumulation in the United States? Table 13 shows the result.

If there were no bequests at all, national wealth would be reduced by 14.3 percent in a closed economy and by 20.5 percent in a small open economy. The remainder can be considered as life-cycle savings. Though labor supply would be increased slightly, the gross national product would be reduced by 4.7 percent and 6.2 percent, respectively, in closed and small open economies.

Two things should be noted about this result. First, in the calibration, the bequest in this model includes altruistic and accidental bequests and half of inter vivos financial transfers only, and it doesn't include another half of inter vivos transfers as well as educational spending paid by parents. This is the main reason why the reduction of national wealth is not very large compared with the result of Kotlikoff and Summers (1981). Second, in the general equilibrium analysis, forward-looking households would increase their life-cycle savings

Table 13: Changes from the Baseline Economy When a Perfect Annuity Market and a 100 Percent Estate Tax Are Added

	Closed Economy	Small Open Economy
$\%\Delta$ Capital Stock	-14.3	-20.5
$\%\Delta$ Labor	0.1	0.5
$\%\Delta$ GNP	-4.7	-6.2
$\%\Delta$ Bequests	-100.0	-100.0
$\%\Delta$ Interest Rate$^{(1)}$	1.3	no change
$\%\Delta$ Wage Rate	-4.8	no change
Δ Annual Lump-Sum Transfers$^{(2)}$	-0.534	-0.555

(1) Change in Percentage Points

(2) $1,000 per household

when they could not expect to receive future bequests from their parents. This reaction of the recipient households keeps the national wealth at a relatively high level.

In the previous subsection, the introduction of a perfect annuity market actually increased national savings. But, is the effect of a perfect annuity market the same even if altruistic bequests don't exist? Table 14 shows how the economy with only a 100 percent estate tax would change in a perfect annuity market. In an economy without any altruistic bequests, or equivalently, in the absence of parental altruism, the introduction of a perfect annuity market would reduce national wealth by 4.8 percent in a closed economy and by 6.5 percent in a small open economy. The gross national product would be reduced by 1.5 percent and 1.7 percent, respectively.

Table 14: Changes from the Economy with a 100 Percent Estate Tax When a Perfect Annuity Market Is Added

	Closed Economy	Small Open Economy
$\%\Delta$ Capital Stock	-4.8	-6.5
$\%\Delta$ Labor	0.1	0.2
$\%\Delta$ GNP	-1.5	-1.7
$\%\Delta$ Bequests	-100.0	-100.0
$\%\Delta$ Interest Rate$^{(1)}$	0.4	no change
$\%\Delta$ Wage Rate	-1.6	no change
Δ Annual Lump-Sum Transfers$^{(2)}$	-1.544	-1.362

(1) Change in Percentage Points

(2) $1,000 per household

5 Concluding Remarks

This paper constructed a measure of time preference and one of parental altruism by extending a heterogeneous agent overlapping generations model. Then, the model was calibrated to the U.S. economy, and the degree of time preference and that of altruism were obtained. Those parameters depend on the choice of target variable of the relative size of bequests and the variable's value. When the statistics from the Survey of Consumer Finance summarized by Gale and Scholz (1994) were used, the degree of parental altruism turned out to be 0.51 (per child household) and 0.69 (in total). This means, on average, a parent household cares about its adult child households about 31 percent less than it cares about itself. When the transfer wealth measure defined by Kotlikoff and Summers (1981) was used as the target variable after proper adjustments, the degree of altruism was 0.58 and 0.78, respectively. Though the obtained numbers are slightly different, these results show that the altruistic model of bequests presented in this paper appears to be in harmony with the U.S. economy.

One of the main features of the model is that it captures the behavior of a parent household (a donor of bequests) and that of its child household (a receiver of bequests) consistently in a dynamic context. This is the similar feature of the model by Abel (1985). He showed the closed form solution for the decision of a household in the economy with lifetime uncertainty and imperfect annuity markets. The present paper also considered parental altruism and showed the decision of households numerically. Regarding the mechanism to determine the optimal level of consumption, savings, and working hours, the model assumed the Cournot-Nash game between a parent household and its child household, and it calculated Markov Perfect Equilibria.

For future research projects, the following extensions of the model are planned. The first is to introduce inter vivos transfers because, in the presence of borrowing constraints, the effect of inter vivos transfers is different from that of bequests even if the present values of those transfers are the same. The second is to measure the degree of a child's altruism toward its parents. If two-way intergenerational transfers are properly allowed, the model will actually contain both an infinite horizon model and a pure life-cycle model as special cases. Then, the Ricardian equivalence proposition can be re-evaluated in the presence of intergenerational altruism and transfers. Other related topics include the education spending by parents, the retirement decision of elderly households, and the working decision of wives.

6 Appendix

6.1 The Computation of Equilibria

The equilibria of the model were obtained numerically in the following way. This section describes, first, how the state space of dynasties is discretized for the computation; next, the algorithm to find steady-state equilibria of this model; and, finally, the algorithm to find the decision rules of households. For a variety of procedures used in the computation of equilibria of heterogeneous agent economies, see Ríos-Rull (1995, 1997).

6.1.1 The Discretization of the State Space

In this model, the state of a dynasty is shown as $\mathbf{s}_I = (a_p, a_k, e_k) \in A^2 \times E$ for Type I dynasties, or $\mathbf{s}_{II} = (a, e) \in A \times E$ for Type II dynasties, where a_p and a_k are the beginning-of-period wealth of a parent and a child, respectively, and e_k is the working ability of a child, and $A = [0, a_{\max}]$ and $E = [e_{\min}, e_{\max}]$. To compute an equilibrium, the state space of a dynasty is discretized as $\widehat{\mathbf{s}}_I \in \widehat{A}^2 \times \widehat{E}$ and $\widehat{\mathbf{s}}_{II} \in \widehat{A} \times \widehat{E}$, where $\widehat{A} = \{a^1, a^2, ..., a^{N_a}\}$ and $\widehat{E} = \{e^1, e^2, ..., e^{N_e}\}$.

For all these discrete points, compute

1. the optimal decision of households, $\{\mathbf{d}_{I,i}(\widehat{\mathbf{s}}_I)\}_{i=1}^4$ and $\mathbf{d}_{II,2}(\widehat{\mathbf{s}}_{II})$, where $\mathbf{d}_{I,i}(\widehat{\mathbf{s}}_I)$ or $\mathbf{d}_{II,2}(\widehat{\mathbf{s}}_{II}) \in (0, c_{\max}] \times [0, h_{i\max}] \times A$,
2. the marginal values, $\mathbf{v}'_{I,i}(\widehat{\mathbf{s}}_I) = (\frac{\partial}{\partial a_p} v_{I,i}(\widehat{\mathbf{s}}_I), \frac{\partial}{\partial a_k} v_{I,i}(\widehat{\mathbf{s}}_I))$ and $v'_{II,2}(\widehat{\mathbf{s}}_{II}) = \frac{\partial}{\partial a} v_{II,2}(\widehat{\mathbf{s}}_{II})$, given the government policy rule and factor prices.

Note that $a'_{I,i}(\widehat{\mathbf{s}}_I)$ and $a'_{II,2}(\widehat{\mathbf{s}}_{II})$ belong to $A = [0, a_{\max}]$ instead of $\widehat{A} = \{a^1, a^2, ..., a^{N_a}\}$. To find the optimal end-of-period wealth, the model uses the Euler equation method and bilinear (for Type I households) or linear (for Type II households) interpolation of marginal value functions in the next period.[19]

In this paper, N_a is set at 51 and N_e is set at 3. The total number of discrete states for Type I dynasties is 7,803 in each age; for Type II dynasties, the total number is 153.

Table 15: The Choice of Discrete Wealth Levels for the Computation

The Wealth of a Household			The Interval of	The Number of
Above or Equal		Below	Discrete Points	Discrete Points
\$0	∼	\$30,000	\$5,000	6
\$30,000	∼	\$100,000	\$10,000	7
\$100,000	∼	\$200,000	\$20,000	5
\$200,000	∼	\$500,000	\$50,000	6
\$500,000	∼	\$1,000,000	\$100,000	5
\$1,000,000	∼	\$2,000,000	\$200,000	5
\$2,000,000	∼	\$5,000,000	\$500,000	6
\$5,000,000	∼	\$10,000,000	\$1,000,000	5
\$10,000,000	∼	\$20,000,000	\$2,000,000	5
\$20,000,000				1
Total				51

Table 15 shows the discrete wealth levels (1997 U.S. \$1,000) used for the calibration. Note that the wealth space is not divided equally. We need to have a smaller number of discrete points as the wealth level increases because the curvature of the marginal value decreases as the level rises.

[19]Because of this bilinear interpolation of marginal value functions, any equilibrium in this model is shown as a mixed strategy equilibrium.

The discretized working ability $\{e^1, e^2, e^3\}$ for age 1 households and age 2 households is shown in Table 4 in section 3.

6.1.2 Steady-State Equilibria

The algorithm to compute a steady-state equilibrium is as follows. Let $\mathbf{\Psi}$ denote the government policy rules $\mathbf{\Psi} = \{\tau_F(.), \tau_S(.), \tau_E(.), tr_{SS}, C_g, W'_g\}$.

1. Set the initial values of factor prices (r^0, w^0) and the government policy variables $(tr^0_{SS}, C^0_g, W'^0_g)$ if these are determined endogenously.

2. Find the decision rule of households given factor prices and the government policy variables, $\{\mathbf{d}^0_{I,i}(\widehat{\mathbf{s}}_I)\}^4_{i=1}$ and $\mathbf{d}^0_2(\widehat{\mathbf{s}}_{II})$, for all $\widehat{\mathbf{s}}_I \in \widehat{A}^2 \times \widehat{E}$ and $\widehat{\mathbf{s}}_{II} \in \widehat{A} \times \widehat{E}$.

3. Find the steady-state measure of dynasties, $\{x^0_{I,i}(\widehat{\mathbf{s}}_I)\}^2_{i=1}$ and $x^0_{II,2}(\widehat{\mathbf{s}}_{II})$, using the decision rule obtained in step 2 and the Markov transition matrix on the working ability of households.

4. Compute the aggregate capital stock and labor supply (K, L) and government aggregate variables and find factor prices (r^1, w^1) and the government policy variables $(tr^1_{SS}, C^1_g, W'^1_g)$.

5. Compare $(r^1, w^1, tr^1_{SS}, C^1_g, W'^1_g)$ with $(r^0, w^0, tr^0_{SS}, C^0_g, W'^0_g)$. If the difference is sufficiently small then stop. Otherwise, replace $(r^0, w^0, tr^0_{SS}, C^0_g, W'^0_g)$ with $(r^1, w^1, tr^1_{SS}, C^1_g, W'^1_g)$ and return to step 2.

6.1.3 The Decision Rule of Households

The algorithm to find the decision rule of Type I households is as follows. For simplicity, the explanation is abstracted from population growth, productivity growth, and lifetime uncertainty.

1. Set the initial numbers of marginal values $\{\mathbf{v}'^0_{I,i}(\widehat{\mathbf{s}}_I)\}^4_{i=2}$.

2. For each $(\hat{\mathbf{s}}_I, i) \in \widehat{A}^2 \times \hat{E} \times \{1, 2, 3, 4\}$ find the decision rules of all households, $\mathbf{d}_{I,i}(\widehat{\mathbf{s}}_I) = \mathbf{d}_p$ or $\mathbf{d}_k$, taking government policy rules $\mathbf{\Psi} = \{\tau_F(.), \tau_{\mathbf{S}}(.), \tau_E(.), tr_{SS}, C_g, W_g\}$, factor prices (r^0, w^0), and the marginal values as given.

   (a) Set the initial values on the decision of the child household $\mathbf{d}^0_k = (c^0_k, h^0_k, a'^0_k)$.

   (b) Given the decision of the child household $\mathbf{d}^0_k$, find the optimal decision of its parent household $\mathbf{d}^0_p = (c^0_p, h^0_p, a'^0_p)$.

      i. Set the initial value of the parent's end-of-period wealth $a'^0_p(\mathbf{d}^0_k)$.

      ii. Find the level of consumption and working hours, $c^0_p(a'^0_p, \mathbf{d}^0_k)$ and $h^0_p(a'^0_p, \mathbf{d}^0_k)$, using the marginal rate of substitution of c^0_p for h^0_p and after-tax marginal wage rate.

iii. Check the Euler equation of the parent household. If

$$\frac{\partial}{\partial c_p} u(c_p^0, h_p^0) \geq \begin{cases} \beta E \frac{\partial}{\partial a_p'} v_{I,i+1}(\widehat{\mathbf{s}}_I') & (\text{if } i = 3) \\ \beta E \eta \frac{\partial}{\partial a_p'} v_{I,i-2}(\widehat{\mathbf{s}}_I') & (\text{if } i = 4) \end{cases}$$

with equality holds when $a_p'^0 > 0$, go to step (c). Otherwise, replace $a_p'^0$ with $a_p'^1$ where

$$a_p'^1 = \begin{cases} \arg\min \left| \beta E \frac{\partial}{\partial a_p'} v_{I,i+1}(\widehat{\mathbf{s}}_I') - \frac{\partial}{\partial c_p}(c_p^0, h_p^0) \right| & (\text{if } i = 3) \\ \arg\min \left| \beta E \eta \frac{\partial}{\partial a_p'} v_{I,i-2}(\widehat{\mathbf{s}}_I') - \frac{\partial}{\partial c_p}(c_p^0, h_p^0) \right| & (\text{if } i = 4) \end{cases}$$

subject to $a_p'^1 \geq 0$, and return to step (ii).

(c) Similarly, given the decision of the parent household $\mathbf{d}_p^0$ obtained in step (b), find the optimal decision of its child household $\mathbf{d}_k^1 = (c_k^1, h_k^1, a_k'^1)$.

(d) Compare the new decision of the child household, $\mathbf{d}_k^1$, with the old one, $\mathbf{d}_k^0$. If the difference is sufficiently small, then go to step (e). Otherwise, replace $\mathbf{d}_k^0$ with $\mathbf{d}_k^1$ and return to step (b).

(e) Compute the marginal values $(\mathbf{v}_{I,4}'^1(\widehat{\mathbf{s}}_I), \mathbf{v}_{I,2}'^1(\widehat{\mathbf{s}}_I))$ or $\mathbf{v}_{I,3}'^1(\widehat{\mathbf{s}}_I)$ using $(\mathbf{d}_p^0, \mathbf{d}_k^0)$.

3. Compare the new marginal values $\{\mathbf{v}_{I,i}'^1(\widehat{\mathbf{s}}_I)\}_{i=2}^4$ with $\{\mathbf{v}_{I,i}'^0(\widehat{\mathbf{s}}_I)\}_{i=2}^4$. If the difference is sufficiently small, then stop. Otherwise, replace $\{\mathbf{v}_{I,i}'^0(\widehat{\mathbf{s}}_I)\}_{i=2}^4$ with $\{\mathbf{v}_{I,i}'^1(\widehat{\mathbf{s}}_I)\}_{i=2}^4$ and return to step 2.

6.2 Optimal Annuity Holdings of a Household

In this model, death is uncertain at the end of age 3. In the presence of perfect annuity markets, age 3 households choose the optimal level of end-of-period annuity holdings, $q\, b_{p,3}'(\mathbf{s}_I)$, where q is the price of annuity and $q = \lambda$. Clearly, it will not exceed the end-of-period wealth level, i.e., $0 \leq q_3\, b_{p,3}'(\mathbf{s}_I) \leq a_{p,3}'(\mathbf{s}_I)$. If an age 3 household has $q\, b_p'$ of its wealth in the form of annuity at the end of this period and if it is alive in the next period, its wealth at the beginning of age 4 is $(a_p' - q\, b_p') + b_p' = a_p' + (1-\lambda)\, b_p'$. If it dies at the end of age 3, the wealth inherited by its child is simply $(a_p' - \lambda\, b_p')\,/\,n$.

The best response functions of an age 3 parent and an age 1 child are written as follows:

$$\begin{aligned} \tilde{\mathbf{R}}_3(c_k, h_k, a_k'; \mathbf{s}_I) \;\; &= \;\; \arg \max_{c_p, h_p, a_p', b_p'} \Big\{ u(c_p, h_p) + \eta\, n\, u(c_k, h_k) \\ &\qquad + \beta\, E \left[\lambda\, v_{I,4}\left(\mathbf{s}_I'\right) + (1-\lambda)\, \eta\, n\, v_{II,2}\left(\mathbf{s}_{II}'\right) \;|\; e_k \right] \Big\}, \end{aligned}$$

$$\begin{aligned} \tilde{\mathbf{R}}_1(c_p, h_p, a_p', b_p'; \mathbf{s}_I) \;\; &= \;\; \arg \max_{c_k, h_k, a_k'} \Big\{ u(c_k, h_k) \\ &\qquad + \beta\, E \left[\lambda\, v_{I,2}\left(\mathbf{s}_I'\right) + (1-\lambda)\, v_{II,2}\left(\mathbf{s}_{II}'\right) \;|\; e_k \right] \Big\}, \end{aligned}$$

where the law of motion of the state is

$$\mathbf{s}_I' = \left(a_p' + (1-\lambda)\, b_p', a_k', e_k' \right),$$

$$\mathbf{s}'_{II} = \left(a'_k + \left(a'_p - \lambda\, b'_p \right)/n - \tau_E \left(\left(a'_p - \lambda\, b'_p \right)/n \right), e' \right).$$

Solving these two functions for (c_p, h_p, a'_p, b'_p) and (c_k, h_k, a'_k), we get a Nash equilibrium end-of-period wealth combination and the optimal annuity holding,

$$\begin{aligned} \tilde{\mathbf{d}}_{I,3}(\mathbf{s}_I) &= \left(c_{I,3}(\mathbf{s}_I), h_{I,3}(\mathbf{s}_I), a'_{I,3}(\mathbf{s}_I), b'_{I,3}(\mathbf{s}_I) \right), \\ \tilde{\mathbf{d}}_{I,1}(\mathbf{s}_I) &= \left(c_{I,1}(\mathbf{s}_I), h_{I,1}(\mathbf{s}_I), a'_{I,1}(\mathbf{s}_I) \right), \end{aligned}$$

for each state $\mathbf{s} \in A^2 \times E$.

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