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Valuing the Student Loan Consolidation Option

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Preface

Federal student loans include a complex consolidation option that gives borrowers the opportunity to combine several loans into a single loan with a longer term to maturity and, for loans originated before July 2006, to convert from a variable- to a fixed-rate loan. This Congressional Budget Office (CBO) paper presents a method for evaluating the cost of that complex future contingent claim using a Hull-White trinomial tree interest rate model. The estimates obtained using that method were reported in a May 2006 CBO paper, *The Cost of the Consolidation Option for Direct and Guaranteed Student Loans*. In that report, CBO estimated historical and prospective costs for several alternative forms of the consolidation option, with those costs being measured at the time that the borrower leaves school. This background paper explains the model used to produce those estimates. The method may also be useful for valuing complex options in other federal credit programs.

Steven Weinberg prepared this paper under the supervision of Robert Dennis and Marvin Phaup. Christine Bogusz edited the paper, and Kate Kelly proofread it. Allan Keaton prepared the report for publication, Lenny Skutnik printed copies of the report, and Simone Thomas produced the electronic version for CBO's Web site (www.cbo.gov).

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Overview

The consolidation option for federal student loans allows borrowers to consolidate one or more loans into a single loan and to extend the time to repay that debt. For loans originated before July 2006, borrowers also have the right to consolidate a variable-rate loan into a fixed-rate loan at the short-term interest rate. The option adds substantial costs to the federal student loan programs.

Accurately estimating the cost of the consolidation option to the government presents a number of technical challenges. To begin with, market prices cannot be used to estimate that value because the student loan consolidation option differs substantially from private options. For example, holders of variable-rate student loans have the option to convert them to a fixed-rate loan at either the prevailing short-term interest rate or the rate for the next year. There is uncertainty as to when a borrower will consolidate and at what interest rate, because some borrowers do not consolidate when it is most financially advantageous to do so, and others do not consolidate at all. Borrowers' failure to take full advantage of consolidation reduces the cost of the option to the government.

This Congressional Budget Office (CBO) background paper describes the valuation of a typical student loan consolidation option, one in which a variable-rate 10-year loan is converted into a fixed-rate loan with a term to maturity of 20 years. That case was one of many that CBO analyzed to find the expected cost of the option, which was reported in an earlier CBO paper.[1]

The Federal Student Loan Programs and the Consolidation Option

Several types of student loans are eligible for consolidation.[2] The most prevalent are Stafford loans, which are available to students, and PLUS loans, which are offered to parents of undergraduate students. If originated prior to July 1, 2006, Stafford loans carry a variable interest rate that is adjusted each July. That rate is the three-month Treasury bill rate (determined at the last Treasury auction in May) plus a markup of 2.3 percentage points. For Stafford loans, the variable interest rate is capped at 8.25 percent. For PLUS loans, the variable interest rate is also based on the three-month Treasury bill rate, with a markup of 3.1 percentage points and a cap of 9.0 percent.

Consolidation is available under the student loan program in which the federal government is the direct lender to students (FDSLP) as well as under the program in which private lenders make loans that are guaranteed by the government (FFELP).

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1. See Congressional Budget Office, *The Cost of the Consolidation Option for Direct and Guaranteed Student Loans,* CBO Paper (May 2006).

2. This description and analysis applies to program terms in effect from July 1998 through June 2006. Prior to that, interest rates on Stafford student loans in repayment were set at 2.5 percentage points above the three-month Treasury bill rate. Before July 1995, the markup was 3.1 percentage points. Under the Deficit Reduction Act of 2005, the interest rate on loans originated after June 2006 will be fixed at 6.8 percent.

The interest rate charged for the consolidated loan is fixed for the life of the loan and is based on the weighted-average interest rate on the loans consolidated. For Stafford loans originated between July 1, 1998, and July 1, 2006, that interest rate on loans being consolidated is the rate on the three-month Treasury bill from the last auction in May plus a markup of 2.3 percentage points and is rounded up to the nearest 1/8 percent.

The Deficit Reduction Act of 2005 changed the terms on loans originated after July 1, 2006.[3] However, the consolidation option was not changed under that legislation, meaning that variable-rate loans can still be consolidated after June 2006 under the same rules. The cost of consolidation to the government (and the benefit to borrowers) is measured as the difference in the present value of cash flows (discounted at market rates) between the new consolidated loan and the original loan(s) it replaces.[4]

Consolidation has substantial value for borrowers when short-term interest rates are low relative to long-term rates. This is because the fixed rate for a consolidated loan is tied to a short-term interest rate rather than a longer-term rate appropriate for the term to maturity of the loan. A second benefit, available only to borrowers who consolidate during the grace period (the six months between leaving school and the start of scheduled repayments), is a fixed rate 0.6 percentage points below the rate offered to consolidators already in repayment. Third, benefits increase with the term to maturity of the consolidated loan, which borrowers can extend at consolidation.

For most borrowers in the federal student loan programs, the first occasion to consider loan consolidation occurs after graduation (or other termination of student status).[5] The consolidation option grants borrowers the right, but not the obligation, to convert existing variable-rate loans into a fixed-rate loan, and borrowers can choose to consolidate when interest rates are most advantageous.

How Consolidation Compares with an Interest Rate Swap

When borrowers exercise their option to consolidate, they combine one or more variable-rate loans into a single fixed-rate loan. The resulting change in interest rate terms is similar to an interest rate swap.[6] In an interest rate swap, a stream of fixed-rate payments is exchanged for a stream of variable-rate payments for a given period. In financial markets, the variable interest rate is based on a short-term interest rate,

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3. Loans originated after June 2006 carry a fixed rate of 6.875 percent after consolidation. That interest rate does not vary with market rates.

4. Appendix B provides a stylized example of this computation, showing how the value of a fixed-rate loan compares with the value of a variable-rate loan.

5. Some consolidations have taken place while borrowers are in school. The Deficit Reduction Act of 2005 eliminated in-school consolidations.

6. Swaps are the most widely used interest rate derivative, and their use continues to grow rapidly. According to the 2004 BIS triennial survey, average daily turnover in interest rate swaps exceeded $600 billion, an increase of more than 100 percent from the level reported in the 2001 survey.

such as the three-month LIBOR (London Inter-Bank Offer Rate) or the three-month Treasury bill rate, and the fixed rate is based on market rates for instruments with maturities of the length of the swap. For a 10-year swap, fixed- and variable-interest-rate payments are exchanged for 10 years. The fixed interest rate is based on the yield on a 10-year instrument, such as a Treasury note, that has 10 years to maturity. By using the long-term market rate to set the fixed rate of the swap, the swap is set to have an initial value of zero, which means that the present values of the fixed and floating payments are equal.

In contrast, the fixed rate for a consolidated student loan is based on a short-term interest rate, which is usually lower than the observed rate on the longer-maturity loan. This feature means that the present values of the fixed- and variable-rate payments will typically not be equal. In recent years, the present value of the gain to a borrower from exercising the option has been as high as $350 per $1,000 in principal consolidated. (Appendix B treats consolidation in a similar manner to an interest rate swap and demonstrates the effect on value of assigning a short-term interest rate to a long-term loan.)

Another way in which consolidation differs from a standard interest rate swap is that the variable rate students pay is capped. An interest rate cap is always valuable to the borrower and costly to the lender.

How the Consolidation Option Compares with a Swaption

The option to consolidate also has characteristics similar to a swaption, an option to swap interest rate payments at some point in the future. In both cases, the holder has the right, but not the obligation, to exchange floating-rate payments for fixed-rate payments. A swaption has initial value, even while the underlying interest rate swap has no initial value, because as interest rates move over time, the value of the fixed stream of payments changes relative to the value of the variable-interest-rate payments. Because the option gives the holder the right (but not the obligation) to engage in a swap, the holder will exercise the option only if interest rates have moved to his or her benefit.

Just as there are several ways in which student loan consolidation differs from a standard interest rate swap, there are features particular to the consolidation option that distinguish it from a standard swaption. First, in the student loan program, repayments are amortized, which means that the principal is gradually paid off over time. In contrast, in a standard interest rate swap or swaption, no payments of principal are made. Second, because of the terms of the consolidation option, borrowers can know both the current interest rate and the rate to be offered in the following year. By waiting to decide whether to consolidate until after the following year's interest rate has been announced, borrowers can choose the more favorable of the two rates. That aspect of the program is valuable to borrowers and gives the consolidation option a feature similar to a look-back option (in which the holder of the option receives the most favorable terms that occurred during the lifetime of the option).

Each year, the interest rate on the consolidated loan is determined by the rate on the three-month Treasury bill from the final auction each May. The new rate takes effect on July 1 of that year. After the May auction and before July 1, borrowers can consolidate at the interest rate for the current year. Hence, they can choose the lower of the current and upcoming interest rates. Supposing that the upcoming year offers a lower interest rate, borrowers can delay consolidation until the following June and wait to see if rates fall even farther. That prospect of obtaining a lower interest rate by waiting, without giving up the opportunity to consolidate at the current rate, is a valuable feature of the option.

The value of the look-back feature of the option can be maximized by the following simple decision rule:

1. Compare the current year's interest rate with the rate available in the following program year.

2. If the current year's interest rate is lower, consolidate immediately.

3. If the following year's interest rate is lower, delay consolidation. Then, after one year, repeat Step 1.

That rule does not require any forecast of interest rates, as the relevant interest rates are already known.[7]

Interest Rate Forecasts and the Cost of the Consolidation Option

That simple decision rule is useful for modeling how borrowers ought to act when trying to take full advantage of the terms of the student loan program. Such a rule by itself is not sufficient to estimate the cost of the consolidation option, however, because the government will not know what interest rates will be when borrowers consolidate in the future. To determine the cost of the option, therefore, a model of future interest rates is required. This analysis uses a trinomial Hull-White interest rate model.[8] In the Hull-White model, a single factor—the short-term interest rate—

7. For the heuristic decision rule to be considered optimal, borrowers' decisions should minimize the expected cost of the loan using the information available to them when they decide to wait or to consolidate. A potential decrease in the interest rate has to be weighed against the value of money over time and the lower principal consolidated as a result of amortization. A decision to consolidate immediately may not always result in the lowest realized costs if interest rates were to rise or stay the same in the first year but then decline sufficiently in the following year to offset the decline in principal and the time value of money.

8. The Hull-White model was first published by John Hull and Alan White as "Pricing Interest Rate Derivative Securities," *Review of Financial Studies*, vol. 3 (1990). Hull and White published a more straightforward numerical procedure for implementing the model in "Numerical Procedures for Implementing Term Structure Models I: Single-Factor Models," *Journal of Derivatives* (Fall 1996).

drives changes in the term structure of interest rates. The model is arbitrage-free, meaning that it initially values fixed-income securities across the maturity spectrum to match their market prices. This is because the expected values of short-term interest rates are constructed to match forward interest rates derived from the term structure of interest rates. (Appendix B shows how the forward interest rates can be obtained from spot interest rates.) The Hull-White model incorporates mean reversion in the interest process, which keeps the maximum interest rate within the range of historical experience.[9]

By construction, the Hull-White model accurately prices existing fixed-income securities. To estimate costs as they change with interest rates, such as in the consolidation option, the volatility of interest rates needs to be modeled. The Hull-White model has two parameters that control changes in interest rates: the volatility of the short-term interest rate and the speed at which the short-term interest rate reverts to the forward rate observed in the market. The parameter values used here were taken from Hull and White's model published in 1996.

A Model of the Cost of Student Loan Consolidation

CBO's analysis proceeds by first estimating the value of the consolidation option at the time when a borrower leaves school, under the assumption that the borrower chooses optimally when to consolidate to minimize the cost of the student loan. Then, in the second stage, the assumption of optimal behavior is relaxed, because experience suggests that many borrowers consolidate either sooner or later than would be optimal to minimize the overall cost of their loan, and some borrowers do not consolidate at all.

Stage 1: Valuing the Option When the Borrower Leaves School

The value of consolidation at any point in time is the difference between the value of the fixed-rate loan and the value of the variable-rate loan(s) it replaces. The value of consolidating immediately is the intrinsic value of the option. In the case of student loans, the intrinsic value represents a lower bound on the potential value of the consolidation option. Because the option allows a borrower to choose when to consolidate, and interest rate movements may create a more favorable opportunity, he or she can wait to consolidate. That additional contingent value from the opportunity to wait is the time value of the option.

Figure 1 illustrates the use of the trinomial interest rate tree structure to model the future movement of interest rates and the value of the option. At each node in Figure 1, there is an interest rate and a value of consolidation. The term structure of interest

9. Other single-factor models, such as Black-Derman-Toy, that do not incorporate mean reversion or other techniques to keep interest rates within reasonable bounds allow for the possibility of extremely high interest rates.

rates at the time the borrower leaves school is the source of the interest rates across the center nodes.

The value of the consolidation option is the sum of the estimated contributions to its value from possible lower interest rate movements in future years. The starting point, the intrinsic value, is the value if the loans were consolidated immediately upon the borrower's leaving school. Added to that value is the expected positive contribution from consolidating one year later if the short-term interest rate was to fall. (The nodes at which the interest rate either moves higher or remains the same make no contribution to the value of the option, because a borrower behaving optimally would only exercise the option at an interest rate that was lower than that in the previous program year.)[10]

For example, assume that there is a one-in-six chance that the interest rate will decline and that the marginal contribution from that fall in rates would be $50, which is the difference between $110 and the value of $60 if the borrower had instead consolidated in the first year. Therefore, the expected value from the chance of a decline in interest rates over the first year is $8.33, which is a major portion of the time value of the option. For the following year, assume there is again a one-in-six chance that the interest rate will decline, and the probability that rates will fall in both the first and second years is one-in-36. Contributions from subsequent years add additional, but smaller, amounts to the value of the option, because the probability today (at Time 0) of three or more consecutive declines in interest rates is very low.

Stage 2: Modeling Borrowers' Actual Behavior

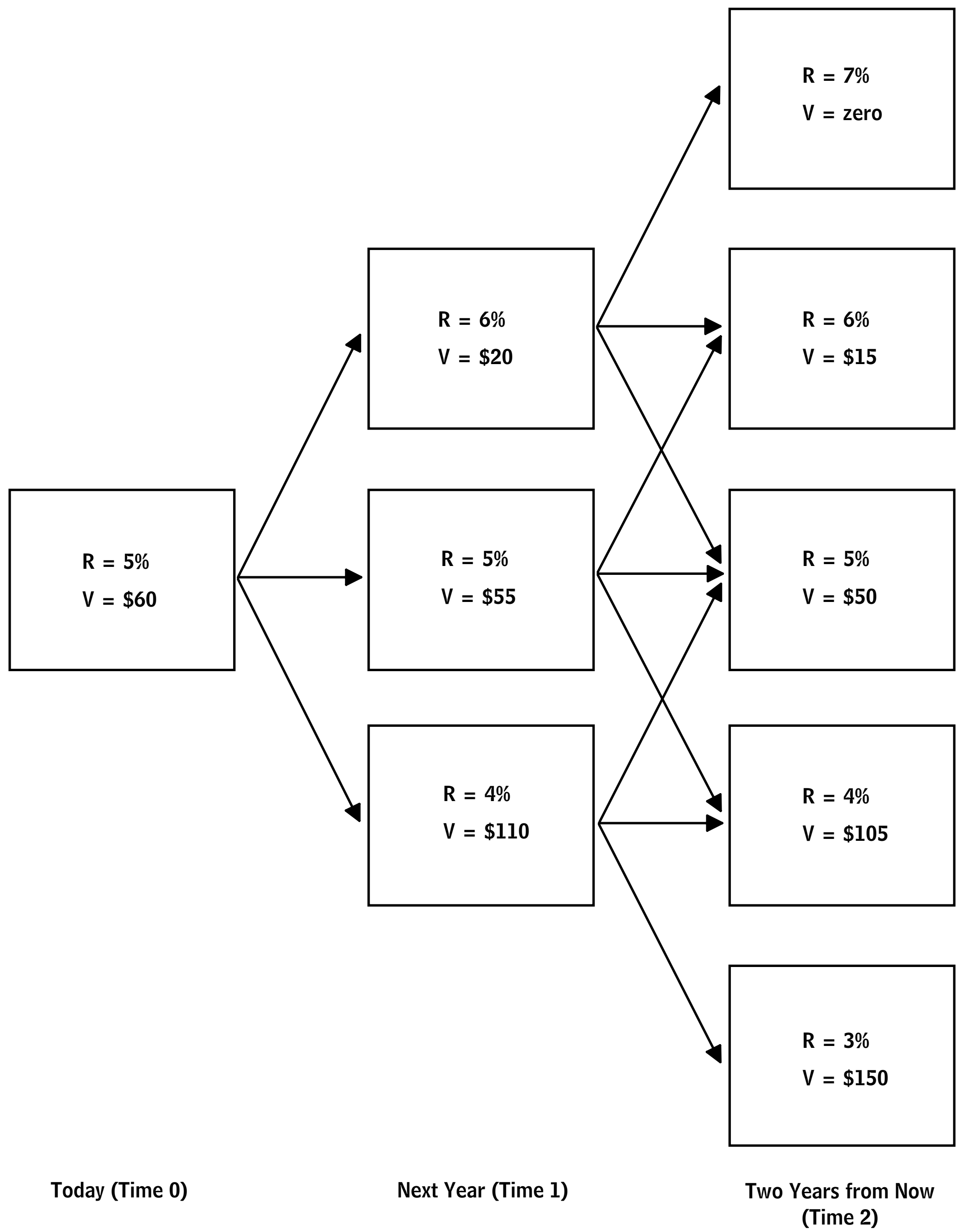
Many borrowers do not consolidate when it is to their financial advantage, and some borrowers consolidate when they could reduce the costs of their loan by waiting. Thus, the value of the consolidation option under the assumption that it is exercised at the optimal time overstates the realized costs of the option. To accurately model the costs of the option to the government, therefore, it is necessary to incorporate a model of borrowers' actual behavior.

The likelihood that a borrower will consolidate at any node of the decision tree depends on the value of consolidation at that point. That estimated relationship is included in the analysis by adding an additional variable to the interest rate tree, as shown in Figure 2. That parameter—shown in the third line in each box—represents the cumulative probability of consolidation. The other variables (the interest rate and the value of exercising the consolidation option) are the same as in Figure 1.

10. This treatment is analogous to that of out-of-the money nodes in a binomial tree pricing of stock options. For a description of the binomial model for stock options, see Congressional Budget Office, *Estimating the Value of Subsidies for Federal Loans and Loan Guarantees* (August 2004).

Figure 1.

The Hull-White Trinomial Tree

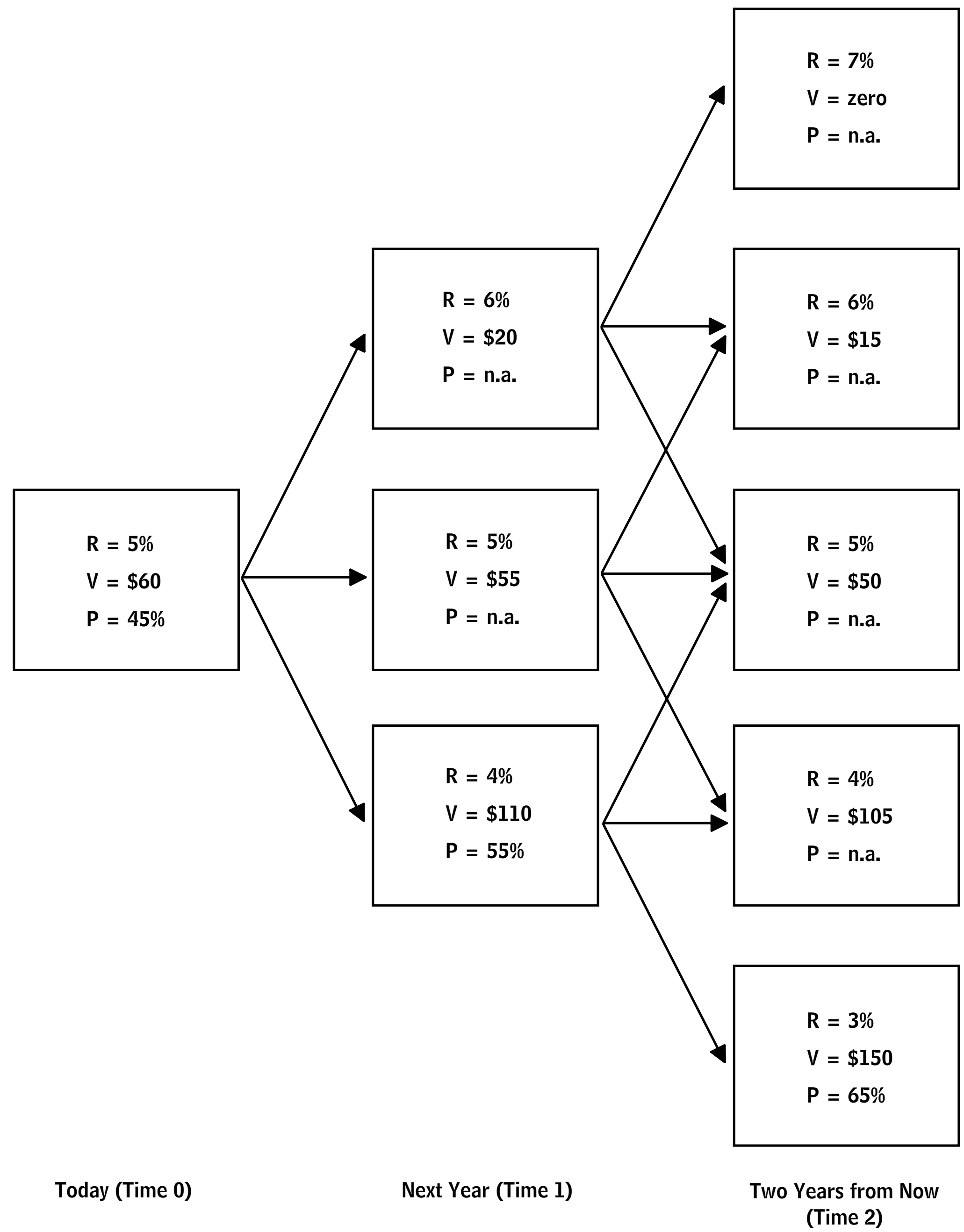


Source: Congressional Budget Office.

Note: The first line in each node is the interest rate (R), and the second line is the value of consolidation at that node (V).

Figure 2.

The Augmented Hull-White Trinomial Tree



Source: Congressional Budget Office.

Notes: The first line in each node is the interest rate (R), the second line is the value of consolidation (V), and the third line is the cumulative probability of consolidation (P), which depends on the value of consolidation at that point.

n.a. = not applicable.

In this example, with an initial value of consolidation of $60, it is estimated that 45 percent of borrowers from this cohort would exercise the consolidation option at Time 0, when the interest rate was 5 percent (unless rates fell enough to induce them to wait). If interest rates increased or stayed the same, it would not be beneficial for borrowers to postpone consolidation, so no percentages are given for those nodes.[11] Hence, the $60 per $1,000 value of consolidation at Time 0 would be the maximum value. If the interest rate declined at Time 1 (the next year) to 4 percent, such as in the bottom node, the value of consolidation would rise to $110. At that value, 55 percent of borrowers are assumed to consolidate. If the interest rate fell to 3 percent at Time 2 (two years from now), the value of consolidation would be still higher, at $150, and the consolidation rate would climb to 65 percent in that year, CBO estimates.

From the tree in Figure 2, the value of the consolidation option can be determined. Assume that the probability of moving to either a higher or lower node is one-in-six and that the probability of staying at the same level is two-in-three. This means that the probability that the interest rate would fall to 4 percent at Time 1 is one-in-six and the probability (at Time 0) that the interest rate would be 3 percent at Time 2 would then be one-in-36. The intrinsic value of the option is its value at Time 0 of $60. However, if only 45 percent of borrowers, by loan volume, are willing to exercise the option, the realized cost will be $27. The time value of the option is the incremental value based on the chance of lower interest rates. If the interest rate moves to 4 percent at Time 1, the option will be worth $110, and 55 percent of borrowers will be willing to exercise it. However, there is only a one-in-six chance that this node will occur. The incremental value is 1/6*(110-60)*55%, which equals $4.60. When Time 2 is included, the total expected cost of the option is $33.21, meaning that the time value of the option is $6.21.[12]

While the financial value of the option at each node in the tree is readily computed, a robust estimate of the participation rate at each node is more difficult to obtain. The Department of Education reports the volume of student loans consolidated each year and the volume of loans originated each year. CBO estimates the participation rate as

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11. For the situation in which rates remain at 5 percent in Time 1, the borrower faces a trade-off between consolidating immediately and waiting until Time 2 to see if rates fall to 4 percent. There are two sources of costs that the borrower incurs while waiting, the time value of money and the declining principal from amortization. The options-pricing model compares the expected value of waiting with the value of consolidating immediately. By construction, in this case the value of consolidating immediately is greater than the expected value of waiting.

    Anecdotal evidence suggests that some borrowers consolidate after rates have already risen or before rates fall. If more detailed data on borrowers' consolidation behavior became available, CBO could estimate the probabilities in those suboptimal nodes. The effect of the failure of borrowers to consolidate at all is reflected in the magnitude of the probabilities at the optimal nodes. CBO scaled down the probabilities at those nodes to compensate for borrowers who do consolidate, but not when it is optimal.

12. When all borrowers exercise the option to consolidate optimally, the time value of the option is $10.83.

the share of loans consolidated among those that are eligible. The size of the pool of loans eligible for consolidation is approximated by the average volume of loans originated over the preceding five years.[13]

In fact, the annual participation rate in those data is highly correlated with the value of consolidation for that year. However, the consolidation option has been available under the rules being modeled in this paper for only a few years. Because of the small size of the sample, CBO used a linear regression to estimate the consolidation rate from the financial value of consolidation (the middle values in the nodes on the tree in Figure 2). Using that model to estimate the likelihood of borrowers' consolidating, CBO finds that the cost of the option (Stage 2) is half what it would be if borrowers instead took full advantage of the option (Stage 1).

Alternative Consolidation Policies

The method described in this paper for valuing the option to consolidate a variable-rate loan into a fixed-rate loan also applies to fixed-to-fixed and variable-to-variable consolidation loans. Under current terms, new Stafford loans are offered at a fixed interest rate of 6.8 percent. Consolidation is still available, allowing borrowers to extend the term to maturity of their loan, but the interest rate at which loans originated under those terms are consolidated is fixed at 6.875 percent. Under those terms, borrowers will no longer have incentives to delay consolidation, as the interest rate does not change. This will make estimates of the cost of consolidation less dependent on estimating when a borrower might consolidate. The participation rate of consolidation can be easily estimated after the program has operated for a few years under those terms.

To model the cost of the fixed-to-fixed consolidation option under those terms, the same approach is used. In the first stage, the value of the option is calculated under the assumption that all borrowers exercise it if it lowers their costs. With the interest rates for both the original and consolidation loans fixed and constant, the cost to the government of those loans depends on how that fixed rate compares with long-term market rates, which are provided by the term structure of interest rates at the time of consolidation. In the second stage, that cost is adjusted downward using an estimate of the number of borrowers who will actually consolidate.

13. For unsubsidized Stafford loans, interest charges accrue to loan principal until the borrower begins repayment. That increase in principal is included in both the numerator and the denominator when calculating the participation rate.

Appendix A: Treatment of Amortization

Amortization is one feature of the federal student loan program that complicates valuation of the consolidation option. Computing the value of an amortizing fixed-rate loan presents no particular difficulties, but computing the value of a variable-rate loan can be more complicated. Because the interest rate on a variable-rate loan adjusts annually, the amortized payment schedule changes as well.

The primary difficulty in valuing the variable-rate loan is its path dependency. What that means in the context of an interest rate tree is that the outstanding principal at a point in time depends not only on the interest rate at that point, but also on the path that interest rates took in reaching it. In order to value the consolidation option using the Hull-White trinomial tree, the value of the variable-rate loan and the amount of the outstanding principal must be found for each node of the tree.

Because of the many possible combinations of paths, the Congressional Budget Office used Monte Carlo simulation within the framework of the Hull-White tree to obtain the expected value of the loan principal and the value of the variable-rate loan at each node of the tree. The Monte Carlo simulation ran 10,000 independent trials, each creating a path for interest rates from the beginning of repayment until the loan principal was fully repaid. For each year in the simulation, the rate on the variable-rate loan adjusts to the interest rate at that point in the path. The outstanding principal depends on the path interest rates have followed, and the value of the loan for each trial is found by continuing to track the path of interest rates until the principal is fully repaid. The Monte Carlo simulation also provides an efficient way to handle the 8.25 percent cap on the interest rate, allowing the discount rate to increase with market rates while the rate the borrower pays is capped.

Appendix B: Using the Term Structure of Interest Rates to Value the Consolidation Option

Under a standard interest rate swap contract, two parties exchange interest payments, with one party paying a variable rate and the other party paying a fixed rate. When a borrower exercises the consolidation option on his student loans, he effectively swaps a variable interest rate for a fixed rate. The benefit to the borrower (and the cost to the government) of the consolidation option comes from the difference in value between the rate the borrower is charged under the program and the market rate.

The interest rate the market would charge for a fixed-rate loan that has been offered in exchange for a variable-rate loan is called the swap rate. A standard interest rate swap contract is designed to have a value of zero at origination, meaning that the present value of the fixed payments equals the present value of the variable-rate payments. The example in Table B-1 uses the yield curve to show how the correct fixed interest rate is determined for a 10-year loan of $1,000 with annual interest payments. This example is for an interest rate swap with annual payments, and, as is typically the case with swaps, the principal is not amortized. The spot (zero-coupon) rates, from June 2004, are listed in column 1. They show an upward-sloping spot yield curve, starting at an interest rate of 1.83 percent for a one-year loan and rising to 4.66 percent for 10 years. The interest rate applicable to borrowers—including a markup of 2.3 percentage points—is shown in column 2.

Implied forward interest rates based on the borrower's interest rate are shown in column 3. A forward interest rate contract is an agreement between parties to lend or borrow a specific amount of funds in the future at a particular interest rate. For example, the forward interest rate for the second year is 5.55 percent. Forward interest rates are determined from spot rates. The present value of a stream of payments based on the spot rates equals the value of those payments from forward interest rates. The forward interest rate for the second year, f_2, is found by equating the value of a two-year loan at the zero-coupon rate, z_2, with a one-year forward loan at the beginning of the first year, then a one-year forward loan at the beginning of the second year: $(1+z_2)^2=(1+f_1)(1+f_2)$. The forward interest rate at the beginning of the first year, f_1, is the same as the one-year spot rate, z_1.

Forward interest rates are used by market participants in valuing variable-rate loans as they represent the contractual rates for the future. To borrow $1,000 for 10 years, one could enter into 10 consecutive forward contracts of $1,000 each. The payments, which are determined at the inception of the contracts, are shown in column 4. At the end of the first year, the borrower pays $41.30 in interest and would also repay the $1,000 principal of the first loan. The borrower has previously contracted to borrow $1,000 at that time, so the net cash outflow is the $41.30. The borrower would pay $55.50 in interest payments in the second year, as the forward interest rate is higher because of the upward-sloping yield curve. Column 5 shows that the discounted present value of the variable-rate loan repayments is $1,000, using the borrower's spot rate as the discount factor for each year. Because the effective interest rate from a series of forward interest rate contracts is equal to the spot rate for the same period, the discounted value of those payments equals the amount borrowed. That is why market participants value a variable-rate loan at par independent of the term structure of interest rates.

For the interest rate swap to have no value, the present value of the fixed payments must also be $1,000. Using the discount factors in column 2, a fixed rate of 6.737 percent is computed. The fixed payments are shown in column 6. Discounting those payments gives a present value of $1,000, as is shown in column 7. Hence, the fixed- and floating-rate legs of the swap are of equal value. If, as in the current federal student loan program, the fixed interest rate had been 3.5 percent rather than 6.737 percent, the present value of the fixed payments would be lower than the present value of the variable-rate payments. The final column shows that the discounted value of fixed payments of $35 is $764.68. The value of the swap to the party who pays a 3.5 percent fixed rate and receives the floating rate is $235.30, the difference between the value of the floating-rate loan, which is $1,000, and the value of the fixed-rate loan.

This stylized example provides some guidance for how the intrinsic value of the student loan consolidation option is computed. Because student loans are amortized and the maximum variable rate that a borrower pays is capped at 8.25 percent, the computations for actual loans are more complex, although they essentially follow the same procedure.

Table B-1.

Determining the Value of a Fixed-Rate Loan

Year	Treasury Spot Yield Curve[a] (Percent)	Borrower's Spot Yield Curve[b] (Percent)	Implied Borrower's Forward Interest Rate (Percent)	Expected Floating Payments (Dollars)	PV of Expected Floating Payments (Dollars)	Market Fixed Payments (Dollars)	PV of Market Fixed Payments (Dollars)	Consolidation Fixed Payments (Dollars)	PV of Consolidation Fixed Payments (Dollars)
1	1.83	4.13	4.13	41.30	39.66	67.37	64.70	35.00	33.61
2	2.54	4.84	5.55	55.55	50.54	67.37	61.29	35.00	31.84
3	3.10	5.40	6.53	65.29	55.76	67.37	57.54	35.00	29.89
4	3.46	5.76	6.83	68.27	54.58	67.37	53.86	35.00	27.98
5	3.81	6.11	7.54	75.42	56.07	67.37	50.08	35.00	26.02
6	4.04	6.34	7.47	74.67	51.65	67.37	46.60	35.00	24.21
7	4.26	6.56	7.92	79.20	50.77	67.37	43.18	35.00	22.43
8	4.31	6.61	6.93	69.34	41.56	67.37	40.38	35.00	20.98
9	4.35	6.65	7.03	70.27	39.36	67.37	37.73	35.00	19.60
10	4.66	6.96	9.76	1,097.60	560.05	1,067.37	544.63	1,035.00	528.11
Total	n.a.	n.a.	n.a.	n.a.	1,000.00	n.a.	1,000.00	n.a.	764.68

Source: Congressional Budget Office.

Notes: This table uses the yield curve to show how the correct fixed interest rate is determined for a 10-year loan of $1,000 with annual interest payments.

PV = present value; n.a. = not applicable.

a. Rates are from June 2004.

b. Includes a markup of 2.3 percentage points.