APPENDIX G: MOMENT-BALANCING THE FAULT RUPTURE MODELS

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Introduction

As described in **Chapter 3**, the fault characterization sub-groups assigned preliminary relative likelihood to the various rupture sources by assembling *fault rupture models* for each system. A fault rupture model consists of combinations of *rupture scenarios* that define the complete rupture of the fault system (e.g., **Figure 3.2**). Each scenario is assigned a weight, or relative frequency, which specifies the amount that that mode of failure contributes to the long-term seismic behavior of the fault. One to five fault rupture models were developed for each fault system; where more than one, competing models were weighted collectively by expert opinion (see below). The variation in scenario frequencies between models (e.g., across a row of **Table 3.4**) reflects the degree of certainty that exists in the community about the strength and persistence of segmentation points on each fault. In each realization of the model, a single fault rupture model (a set of relative scenario rates, e.g., a column of **Table 3.4**) is selected for each of the seven fault systems.

In general, the relative scenario frequencies within a given fault rupture model will not result in a moment-rate-balanced model (i.e., will not satisfy $\dot{M}_0$ on each fault segment) because the rupture sources within each rupture scenario have different moments, and those moments vary with the choices of *L*, *W* and *R* made in a given realization of the SFBR model. The problem of moment-rate balancing the model is underdetermined because there are generally more rupture source rates than segment moment rates (or slip rates *v*) to constrain them. Therefore, we use least-squares regression to obtain a set of revised relative rates that are the best fit to the relative rates supplied by the subgroups. This approach in detail in this Appendix, along with an example case worked out in detail, a table of WG02's results, and commentary on the procedure's success.

The need to balance moment

As part of the fault characterization process, groups of experts developed fault rupture models that gave the relative rate of each rupture scenario, r_j. They were asked to consider that if the complete fault system ruptured many (say, 100) times, what would be the percentages of the different rupture scenarios that would be observed. For example, given a two-segment fault, what percentage of the time would the fault generate two-segment ruptures versus pairs of single-segment ruptures.

Since the area of different rupture sources are not all equal in any given rupture scenario, this approach does not necessarily lead to models that are moment-balanced. Again consider the example of the two-segment fault. The seismic moment of the characteristic earthquake scales exponentially with segment area, (e.g. $M_o \propto 10^{1.5A}$) and the moment–rate scales linearly with segment area ($\dot{M}_o \propto A$). If one segment is longer than the other, the two segments will not have the magnitudes or rates of single segment rupture earthquakes.[1] But in the development of the rupture models, the rates of the two segments are assumed to be the same. Therefore, the rupture models do not lead to moment-balanced models.

Since the rupture models given by the source characterization groups (SCG's) will not produce moment-balanced models, the problem is to find a set of moment-balanced weights for each permissible rupture scenario that lead to relative rates that are as close as possible to those specified in the rupture models by the geologists. A mathematical process for finding the best set of moment-balanced weights is given below.

Moment balancing procedure

We wish to compute the mean rate of occurrence of earthquakes produced by every rupture source. Within a given fault rupture model, multiple rupture scenarios are considered, each with a relative frequency determined by a source characterization sub-group. Using these scenario frequencies, we compute relative rates for each rupture source. These relative rupture rates are not likely to balance the moment rate (long-term slip rate) of each fault segment; therefore, we first adjust the scenario frequencies such that moment rate is balanced.

The problem is to find a set of moment-balanced frequency for each permissible scenario, and additionally to make these as close as possible to those specified by the source characterization subgroups. To do this, we define *relative* rupture rates in terms of the available moment rate and the mean moments of each earthquake that occurs on the fault. In this case the relative rates satisfy the moment rate $\dot{M}_0$ of each fault segment. Then, we minimize the difference between these two sets of relative rupture rates (the one dictated by moment rates and the one provided by the source subgroups), and in so doing, solve for a set of constants that distribute each segment's moment rate onto the various ruptures that involve that segment. It is that distribution of moment rate that allows us to calculate absolute rates for each rupture source. The following paragraphs explain these steps in more detail.

First we compute relative rupture source rates R_G using the scenario weights f supplied by the sub-groups. We make the interpretation that the weights apply to all events in the rupture scenario. That is, if there are two rupture sources in a rupture scenario, then that scenario produces two times as many earthquakes as a scenario with only a single rupture source.

For example, consider a fault with two segments. The two segments can rupture separately (scenario A), or they can rupture together (scenario B). If these scenarios are given equal weight, then there are two single segment ruptures in scenario A for each multi-segment rupture in scenario B. More generally, the relative rate of the i^{th} rupture sources is found as the ratio of sums*:*

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[1] Rupture source areas depend on values of length, width and seismic scaling (*R*) factor for the component fault segments. Each of these, and therefore rupture source moments, vary between realizations of the SFBR earthquake model depending on which logic tree branches are selected.

$$R_{G_i} = \frac{\sum_{j=1}^{Nscen} f_j E_{i,j}}{\sum_{j=1}^{Nscen} f_j N_j} \qquad (4.9)$$

where f_j is the relative frequency of rupture scenario *j*, and N_j is the number of earthquakes in fault rupture scenario *j*. The summation in the numerator is across only those scenarios that include rupture source *i* (we employ the dummy variable *E*(*i*,*j*) which has value of 1 if rupture source *i* is included in scenario *j*, and value 0 otherwise). A floating earthquake counts as one earthquake. The relative rates R_{G_i} for each fault system sum to 1.0, but do not necessarily balance segment moment rates. Values of R_{G_i} are listed in **Table G.4**.

Next, we define an analogous set of relative rates, R_{MB_i}, that *do* balance the moment, and solve for a set of constants C_j that minimize the difference $R_{G_i} - R_{MB_i}$.In a moment-balanced model, the moment is partitioned into each rupture. The relative rate of the individual ruptures sources is given by:

$$R_{MB_i} = \frac{\sum_{j=1}^{Nscen} \frac{\dot{M}_{0i}}{\overline{M}_{0i}} C_j}{\sum_{k=1}^{Nrup} \sum_{j=1}^{Nscen} \frac{\dot{M}_{0k}}{\overline{M}_{0i}} C_j} \qquad (4.10)$$

The $\dot{M}_0$'s are the maximum moment rates for rupture sources (product of μ, seismogenic area, and mean slip rate, see (4.8)), the $\overline{M}_0$'s are the mean rupture source moments calculated earlier, and the *C*'s are constants that partition the segment moment rate onto the various rupture sources. (Again, the summations in *j* are across those scenarios that include the rupture source of interest.) Note that the ratios of moment rates to mean moments define the maximum rate of a rupture source (its rate if it accommodates the full slip rate of all its component fault segments), and the *C*'s temper this rate to recognize that the slip rate may be accommodated by earthquakes on other sources as well.

What remains is only to solve for the values of the partition factors C_j, which we do using least-squares regression, to minimize $R_{G_i} - R_{MB_i}$ across the suite of *i* rupture sources. The result is a set of partition factors that balance the fault segment slip rates while retaining the character of the fault rupture model to the extent possible.

Example of a two-segment fault

As a simple example, consider a fault with a single segmentation point and the following segment characteristics:

Segment 1:
Length = 100 km

width = 15 km
slip-rate = 10 mm/yr
mu = 3×10^{11} dyne/cm^2
moment-rate$_1$ = 4.50×10^{24} dyne-cm/yr

Segment 2:
Length = 50 km
width = 15 km
slip-rate = 10 mm/yr
mu = 3×10^{11} dyne/cm^2
moment-rate$_2$ = 2.25×10^{24} dyne-cm/yr

There are two rupture scenarios (we will not allow a floating rupture source):
Scenario 1: segments 1 and 2 rupture independently
Scenario 2: segments 1 and 2 rupture together

Assume a magnitude-area relation of M=4.0 + log*A* and use a delta function for the magnitude pdf. The resulting magnitudes and moment per earthquake (Mo/eqk) for each rupture source are listed in **Table G.1**.

Table G.1.

Rupture source	M	Mo/eqk	Rate
segment 1	7.18	6.6E+26	0.0068 C_1
segment 2	6.89	2.4E+26	0.0094 C_1
segment 1 + 2	7.35	1.2E+27	0.0056 C_2

Let C_1 and C_2 be factors that partition the moment-rate into scenarios 1 and 2, respectively. (C_1+C_2=1.0)

Then it is straight forward to develop a moment-balanced model for the rate of each rupture source. We just divide the available moment-rate by the Mo/eqk for each rupture source:

Rate(seg$_1$) = C_1*moment-rate$_1$/(Mo/eqk)$_1$

Rate(seg$_2$) = C_1*moment-rate$_2$/(Mo/eqk)$_2$

Rate(seg$_1$) = C_2 (moment-rate$_1$+ moment-rate$_1$)/(Mo/eqk)$_{1+2}$

The resulting rates are listed in the last column of **Table G.1**. This is what most people understand by the term moment-balancing.

The problem is that the weights given to the rupture scenarios are based on the relative rates of the different rupture sources, not on partitioning the moment rate. Call the weights from the SCG's w_1 and w_2 for rupture scenarios 1 and 2, respectively.

The relative rate of a rupture source is computed by dividing the weighted number of occurrences of rupture source by the total weighted number of rupture sources. The "weighted number" of rupture source j is computed by adding the P_i values for each rupture scenario that includes rupture source j.

In this example, the weighted numbers for the three rupture sources are listed in **Table G.2** and the total weighted number of ruptures is $2w_1+w_2$. The relative weights from the geologists weights are then computed (Column 3). The relative rate can also be computed from the moment-balanced rates given in Table 1. These are listed in the last column of **Table G.2**.

Table G.2.

Rutpure source	Weighted number	Relative Rate from geologists	Relative Rate (from moment-balance)
segment 1	w_1	$w_1/(2w_1+w_2)$	$\frac{0.0068C_1}{(0.0068C_1+0.0094C_1+0.0056C_2)}$
segment 2	w_1	$w_1/(2w_1+w_2)$	$\frac{0.0094C_1}{(0.0068C_1+0.0094C_1+0.0056C_2)}$
segment 1+2	w_2	$w_2/(2w_1+w_2)$	$\frac{0.0056C_2}{(0.0068C_1+0.0094C_1+0.0056C_2)}$

The moment-balancing done in the WG02 code finds the values of C_i that minimize the difference in the rates computed with the geologists weights and the moment-balanced weights. This is done using least-squares. In this example, we have 3 equations (rates for 3 ruptures sources) and 2 unknowns (C_1 and C_2).

In this example, assume that $w_1=0.8$ and $w_2=0.2$. The least-squares solution is

$C_1 = 0.72$

$C_2 = 0.28$

The resulting rates are given in **Table G.3**.

Table G.3.

rupture source	Relative Rate from geologists	Relative Rate (from moment-balance)
segment 1	0.444	0.370
segment 2	0.444	0.511
segment 1+2	0.111	0.119

WG02 results

Resulting values of R_{MB_i} are listed in **Table G.4**. Comparison of R_{MB_i} to R_{G_i} reveals that, for most fault rupture models on most fault systems, only minor adjustments of a few percent or less were required. Larger adjustments were driven by high weight on floating-earthquake scenarios and by large contrasts in moment rate (L, R, and/or v) between fault segments. The Calaveras Fault suffers in both regards, and substantial adjustments were required to obtain fault rupture models that satisfy the moment rates of its three segments. In all four fault rupture models for the Calaveras, the relative rate of rupture of the CN segment was reduced due to its low slip rate (6 mm/yr) relative to CC and CS (15 mm/yr). Nonetheless, the basic character of each fault rupture model (e.g., the relative frequency of single- versus combined-segment ruptures) is preserved.

Comments on the moment-balancing process

The moment-balancing process described below was developed anew by WG99 as a means of handling the problem of determining rates for multiple, overlapping rupture sources. The process turned out to be cumbersome—both to perform and to describe. For example, the meaning of scenario weights was confusing to sub-group members. Also, the process obscures the link between R_{MB_i} and the specific observations, interpretations, and opinions that underlie them. The process for developing moment-balanced models is cumbersome and was confusing to many of the SCG members. As an alternative, we could have solicited a set of moment-balanced models to start with. In fact, we tried that approach but found that some SCG's had difficulty partitioning moment into the different rupture scenarios. Many experts were more comfortable giving the relative rates for each rupture scenario, so we accepted those results and converted those relative rates to moment-balanced models.

We recommend that future working groups consider alternative approaches. One promising alternative is to describe the relative strength (rupture-stopping potential) of segmentation points, rather than relative scenario frequency. This approach was used by the Hayward/Rodgers Creek SCG in developing their estimates of the relative scenario rates. While this approach has the advantage that is seems simpler, it has its own difficulties. First, there is the issue of correlation. For example, if we have a three-segment fault, does the probability that that rupture goes through segmentation point 2 depend on if the rupture went through segmentation point 1? In addition, for the models to be moment-balanced, the strength of the segmentation point (e.g. probability of rupturing through a segmentation point) may need to be different if the rupture starts on one side or the other. In the end, some sort of method for developing moment-balanced models still needs to be applied to the input from the SCG's. This is a topic that would benefit from additional research before the next working group analysis.

Table G.4. Relative rates of SFBR rupture sources, initial and moment-balanced.

	Relative rates from sub-group weights f, R_{Gi}					Relative rates in moment-balanced model, R_{MBi}				
Fault, rupture source	Model 1	Model 2	Model 3	Model 4	Model 5	Model 1	Model 2	Model 3	Model 4	Model 5
San Andreas										
SAS	0.105	–	0.147	0.092	0.081	0.118	–	0.164	0.104	0.089
SAP	0.105	–	0.088	0.092	0.081	0.100	–	0.088	0.089	0.076
SAN	–	–	–	0.012	0.074	–	–	–	0.005	0.067
SAO	–	–	–	0.023	0.094	–	–	–	0.026	0.101
SAS+SAP	0.038	0.310	0.029	0.218	0.074	0.057	0.319	0.050	0.238	0.079
SAN+SAO	0.143	0.310	0.118	0.299	0.081	0.117	0.299	0.102	0.275	0.077
SAS+SAP+SAN	–	–	–	0.012	0.020	–	–	–	0.008	0.010
SAP+SAN+SAO	–	–	0.059	–	–	–	–	0.028	–	–
SAS+SAP+SAN+SAO	0.571	0.310	0.412	0.138	0.329	0.570	0.311	0.416	0.139	0.331
floating	0.038	0.069	0.147	0.115	0.168	0.038	0.072	0.152	0.116	0.170
Hayward/RC										
HS	0.147	0.217	0.313	0.294		0.144	0.213	0.311	0.280	
HN	0.119	0.174	0.229	0.267		0.136	0.196	0.252	0.299	
HS+HN	0.261	0.174	0.042	0.057		0.294	0.215	0.090	0.110	
RC	0.379	0.348	0.271	0.324		0.337	0.296	0.212	0.255	
HN+RC	0.028	0.044	0.083	0.027		0.027	0.042	0.076	0.036	
HS+HN+RC	0.043	0.022	0.029	0.012		0.041	0.020	0.027	0.007	
floating	0.024	0.022	0.033	0.019		0.022	0.020	0.032	0.014	
Calaveras										
CS	0.190	0.095	0.196	0.265		0.209	0.169	0.213	0.273	
CC	0.177	0.095	0.174	0.245		0.189	0.155	0.190	0.242	
CS+CC	0.111	–	0.022	0.020		0.207	–	0.120	0.112	
CN	0.376	0.429	0.370	0.327		0.116	0.108	0.110	0.099	
CC+CN	0.013	–	0.022	0.020		0.012	–	0.014	0.016	
CS+CS+CN	0.022	–	0.022	0.020		0.051	–	0.051	0.051	
Floating	0.022	0.048	0.022	0.041		0.052	0.113	0.053	0.073	
Floating CS+CC	0.089	0.333	0.174	0.061		0.164	0.455	0.249	0.134	
Concord/GV										
CON	0.222	0.083	0.083			0.214	0.081	0.080		
GVS	0.111	0.042	0.042			0.102	0.040	0.039		
CON+GVS	0.111	0.042	0.042			0.087	0.032	0.032		
GVN	0.222	0.083	0.083			0.240	0.090	0.090		
GVS+GVN	0.111	0.042	0.042			0.122	0.045	0.046		
CON+GVS+GVN	0.111	0.667	0.042			0.118	0.669	0.044		
Floating	0.111	0.042	0.667			0.118	0.044	0.669		
San Gregorio										
SGS	–	0.259	0.412			–	0.176	0.284		
SGN	–	0.259	0.412			–	0.319	0.515		
SGS+SGN	0.700	0.259	–			0.700	0.271	–		
Floating	0.300	0.222	0.177			0.300	0.234	0.202		
Greenville										
GS	0.375					0.373				
GN	0.375					0.374				
GS+GN	0.188					0.189				
Floating	0.063					0.064				
Mt Diablo										
MTD	1.000					1.000				

A dash "–" indicates that that rupture source does not exist for that fault rupture model; for example, in fault rupture model 2 for the San Andreas fault, no single-segment ruptures are permitted.