SUBSIDENCE PREDICTION USING A LAMINATED, BOUNDARY-ELEMENT PROGRAM

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ABSTRACT

Historically, the surface subsidence over underground coal mines has been predicted using empirical profile or influence functions which have little or no connection to the actual mechanics of the subsidence. Without a mechanistic basis, establishing the exact site-specific parameters to use in these empirical methods has been problematic. A practical subsidence predictive method based on mechanics has the appealing capability of allowing the determination of site-specific parameters from fundamental properties of the overburden with minimal field calibration work. This paper presents a case study where a mechanics-based, boundary-element program is used to calculate the surface subsidence associated with several panels of a Northern Appalachian longwall coal mine. The program used in this case study is called LAMODEL, and it incorporates a frictionless, laminated overburden into a general purpose displacement-discontinuity code primarily designed for calculating the stresses and displacements in coal mines or other thin seam or vein type deposits. In this paper, the program is used to calculate both the underground convergence and the resulting surface subsidence at three longwalls. The subsidence results from the model are compared with field measurements and analyzed. The results from the case study in this paper demonstrate that the laminated model with calibrated properties can easily provide fairly accurate subsidence predictions and is fairly flexible for fitting measured subsidence. However, additional subsidence predictive case studies are recommended in order to ultimately evaluate the potential of the laminated overburden model for practical subsidence prediction

KEYWORDS

Case History, Coal, Computer Program, Design, Discontinuities, Mining, Numerical Analysis, Stress, Subsidence

INTRODUCTION

Historically, the surface subsidence over underground coal mines has been predicted using profile or influence functions which have little or no connection to the actual mechanics of the ground movement (Heasley, 1988; Adamek *et al.*, 1987; Kratzch, 1983). Without a mechanistic basis, establishing the exact seam convergence and function parameters to use in these empirical methods has typically required extensive and expensive field measurements in order to calibrate the function parameters to a specific mining area. Recently, a laminated overburden model which is derived from plate mechanics has been used to predict surface subsidence with fairly good results (Salamon, 1991, 1989a, 1989b; Yang, 1992), and has shown the capability of fitting a generic, empirically-derived subsidence curve for Northern Appalachia (Heasley and Salamon, 1996). This same laminated overburden model has currently been coded into a full-featured displacement-discontinuity program, LAMODEL, for analyzing coal mine stresses and displacements, as well as surface subsidence (Heasley, 1998). In this program, the various properties of the seam and gob materials are mechanically combined with the laminated overburden properties to realistically calculate seam stresses and convergence. This calculated seam convergence can then be projected to surface subsidence using the laminated overburden mechanics. Since the laminated overburden model has been shown to accurately calculate both underground displacements and stress (Heasley and Salamon, 1996), and surface subsidence, in separate instances, the combination of these capabilities in the LAMODEL program gives it the potential to accurately calculate both underground stresses and displacement, and the associated surface subsidence with the same mechanical model. Using an accurate mechanical basis, surface subsidence in different mining areas could conceivable be calculated from fundamental properties of the overburden, and would require minimal field calibration work. The following paper relates the application of the laminated overburden in LAMODEL to subsidence prediction at several longwall panels in Northern Appalachia and provides an initial evaluation of the program's accuracy and utility for subsidence prediction.

THE LAMODEL PROGRAM

LAMODEL is a PC-based program for calculating the stresses and displacements in coal mines or other thin seam or vein type deposits. It is primarily designed to be utilized by mining engineers for investigating and optimizing pillar sizes and layouts in relation to overburden, abutment and multiple-seam stresses (Heasley, 1998). The program use a displacement-discontinuity variation of the boundary-element method and a Successive Over-Relaxation (SOR) iterative technique for solving the elastic equations of equilibrium around the mine openings. LAMODEL simulates the overburden as a stack of homogeneous isotropic layers with frictionless interfaces, and with each layer having the identical elastic modulus, Poisson's Ratio, and thickness. This "homogeneous stratification" formulation does not require specific material properties for each individual layer, and yet it still provides a realistic suppleness to the overburden that is not possible with the classic, homogeneous isotropic elastic overburden. LAMODEL was originally derived in 1994 and is written in C++.

The LAMODEL program has numerous features including: single and multiple seam simulations; numerous individual excavation steps; seam level convergence and stress calculations; off-seam convergence and stress calculations; user-defined overburden properties; up to 26 different in-seam materials from 5 different material models (elastic, elastic-plastic, strain-softening, bilinear strain-hardening and exponential strain-hardening); user-defined convergence criteria; and up to a 300 by 300 grid size. Also, LAMODEL can include effects from a traction-free surface for shallow seams and can include the effects from a variable surface topography.

THE MINE

The location of this subsidence case study is a longwall mine in Barbour county in the northwest corner of West Virginia. This mine started production in 1975 with continuous miners in room-and-pillar sections. In 1982, the first longwall was installed and by the time of the final subsidence monitoring in this study (1985) the mine had successfully completed 5 longwall panels (Heasley, 1988). The mine operates in the Lower Kittanning seam which averages 1.8 m in thickness and which has an overburden between 120 and 420 m across the property. The immediate roof of the seam consists of a thinly laminated, sandy-shale overlain with a main roof of interbedded sandstones, shales and limestones. The mine area is also noted for high horizontal, in situ stresses. At this mine, the subsidence over the first, second and fifth panels was monitored with surface survey stations.

THE V-1 PANEL

The first panel at which the subsidence was investigated using LAMODEL is called the V-1 panel, and it is actually the fifth longwall panel to be extracted at the mine (see Figure 1). The panel is 285 m wide and 640 m long with a coal thickness of 1.8 m and overburden that averages 120 m. The panel was started on June 15, 1984 and finished on October 19, 1984 in a total of 126 days at an advance rate of 7 m per work day. The panel advanced from the northwest towards the southeast, and as shown in Figure 1, there were two transverse lines and one longitudinal line of subsidence monitoring stations over the later half of the panel.

For this initial subsidence fitting exercise, the entire panel was discretized into a LAMODEL grid 150 elements wide by 270 elements high using 3 m elements and rigid boundary conditions on all four sides. The overburden was set at a constant 120 m with a pressure gradient of 0.025 MPa/m. The elastic modulus of the rock mass was set at 20 GPA and the modulus of the coal was set at 2 GPa. The coal thickness was set at a constant 1.8 m, and the strength of the coal in the seam was set such that the pillar strengths essentially followed the Mark-Bieniawski formula (Mark and Chase, 1997) using an elastic, perfectly-plastic material model (Heasley, 1998). For the subsidence calculation, the off-seam capability of LAMODEL was used to calculate the displacement at the surface every 6 m along lines coincident with the surface survey lines. The first-order, free-surface effects were included in this model by simply doubling the surface subsidence as calculated in the infinite media, identical to the process used in Yang's (1992) influence-function derivation.

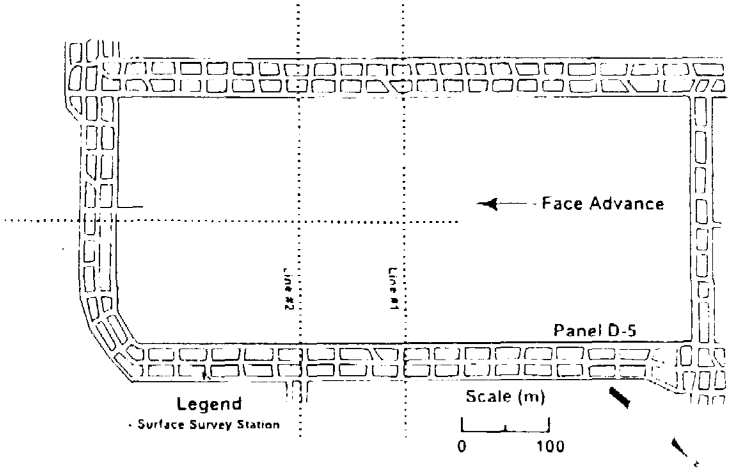

Figure 1: Map of the V-1 panel.

The two primary factors which influence the shape and magnitude of the subsidence (particularly in LAMODEL) are the gob compaction stiffness and the overburden flexural stiffness. Therefore, the primary parameters which are adjusted in LAMODEL for fitting the measured subsidence are the final gob modulus (E_f) which is used to control the gob stiffness, and the lamination thickness (t) which is used to control the overburden stiffness (as discussed in Heasley (1998), the lamination thickness and the overburden modulus affect the overburden behavior in essentially the same manner; therefore, in order to simplify the calibration process, the overburden modulus was held constant while only the lamination thickness was varied). The gob material model used in the following subsidence calculation is the strain-hardening gob as described in Zipf (1992a and 1992b). In all of the LAMODEL subsidence runs, the initial gob modulus (E_i) is held constant at 0.7 MPa; the virgin vertical stress (σ_v) is set at 27.6 MPa; and the gob height factor (n) is set at unity. With these three parameters fixed, raising or lowering the value of the final gob modulus effectively raises or lowers the modulus of the gob.

In this process of analyzing LAMODEL for surface subsidence calculation, the first panel was used to "calibrate" the model. This calibration process consisted of an interactive trial-and-error process where: the critical model parameters (in this case the lamination thickness and the final gob modulus) were initially estimated, the program was run to calculate the surface subsidence, the calculated subsidence was compared to the measured subsidence, the model parameters were adjusted to improve the fit, the program was run again, etc. This cycle continues until the calculated subsidence fits the measured subsidence as close as desired.

For this first calibration process on the V-1 panel, it was found that a wide range of lamination thicknesses and final gob moduli combinations could be fit equally well to the measured subsidence. A distributed sample of these parameter combinations is listed in Table 1 and shown in Figures 2 and 3. The range of parameters shown in Table 1 covers the complete spectrum of reasonable behavior for this panel. For the thinnest laminations (1.5 m), the peak gob load is essentially equal to the overburden load (see Table 1); therefore, at this lamination thickness, the gob is supporting the total overburden load at the middle of the panel and the flexural stiffness of the laminations is not effectively supporting any overburden load. On the other end for the spectrum, for the thickest lamination (7.5 m), the peak gob load is only about 1/6 of the overburden load, and the flexural stiffness of the laminations is supporting the other 5/6 of the overburden load. Thus, for a given maximum subsidence, the thinnest, most flexible laminations are associated with the stiffest gob, while the thickest, stiffest laminations are associated with the softest gob.

Looking at Figures 2 and 3, a number of observations can be made. First, the nature of the measured subsidence is worth noting. In the two measured cross-sections (see Figure 2), the subsidence flattens considerable in the middle of the trough and appears to be super-critical as suggested by the panel dimensions. Although, the

TABLE 1
CALIBRATED LAMODEL PARAMETERS FOR PANEL V-1

Lamination Thickness (m)	Final Gob Modulus (MPa)	Peak Gob Stress (MPa)	Average Gob Stress (MPa)
1.5	124	3.03	2.05
3.0	104	2.52	1.56
4.5	100	2.07	1.23
6.0	69.0	1.31	0.83
7.5	1.38	0.48	0.37

subsidence trough is not perfectly flat in the middle as an idealistic super-critical model might suggest (Kratzch, 1983). Also, in all three measured curves, the subsidence shows anomalies that are not consistent with a homogeneous overburden, horizontal seam, constant depth, and constant seam thickness. In particular, the number 2 transverse line and the longitudinal line show considerable variability along the bottom of the subsidence trough. In a theoretically perfect situation, the subsidence curves for the two transverse lines should be identical, when in reality, they show considerable differences. In contrast, the calculated subsidence curves in Figures 2 and 3 are very smooth and consistent. Obviously, in the real world, natural variations in geology, depth, seam thickness, etc., will cause anomalous variations in the subsidence. However, because LAMODEL assumes a constant seam thickness, a constant depth (for subsidence prediction), a level seam and a homogeneous overburden, the subsidence curves it produces will generally be very smooth and consistent, and will not exactly match every anomalous variation in the measured subsidence curves. In general, any idealized subsidence prediction technique will produce smooth, consistent subsidence and will not exactly match the actual anomalous measured subsidence. (However, in a previous subsidence prediction study at this site (Heasley, 1988; Heasley and Saperstein, 1987, 1986), the use of more realistic, seam thickness, overburden depth and seam tilt was shown to significantly improve the fit of the calculated subsidence.)

Next, some general comparisons between the measured and calculated (or fitted) subsidence can be made. In the center of the subsidence trough, the maximum subsidence for the calculated curves was calibrated to approximately 1.2 m. Thus, all of the calculated curves were forced to have the same value of maximum subsidence, 1.2 m, which appears to be a reasonable compromise between the maximum subsidence values of the two measured cross-sections. Amongst the various lamination thicknesses, the thinnest 1.5 m laminations appear to give the best shape for the bottom of the subsidence trough in the transverse profiles (see Figure 2) and provide the most reasonable gob stress (see Table 1), while the thicker laminations, 6.0 and 7.5 m, appear to provide the best shape for the bottom of the longitudinal profile (see Figure 3). At the edges of the subsidence troughs, the

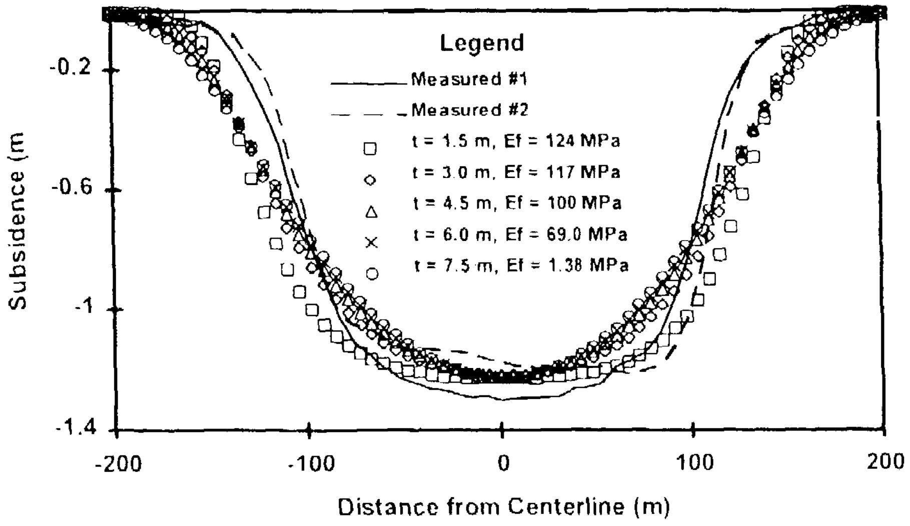

Figure 2: The measured and fitted transverse subsidence for the V-1 panel.

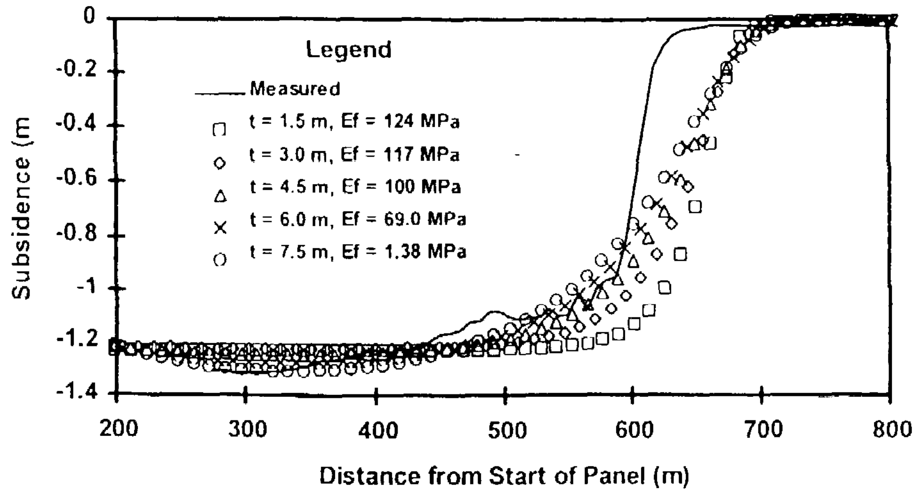

Figure 3: The measured and fitted longitudinal subsidence for the V-1 panel.

calculated curves invariably show the subsidence starting some 20 to 50 m wider that the measured subsidence with the 1.5 m laminations consistently providing the narrowest profile and closest fit to the measured subsidence.

This problem, that the calculated subsidence tends to be wider than the measured subsidence, has been found throughout the history of subsidence prediction (Heasley, 1988; Kratzsch, 1983). Generally, when using the empirical, profile or influence-function methods of subsidence prediction, the subsidence trough is narrowed by functionally limiting the seam convergence at the edge of the panel using various artificial or empirically-derived parameters. In LAMODEL, where the input material properties are intended to accurately determine the seam convergence, it is not reasonable to apply some artificial adjustment to the panel edge in order to correct the width of the subsidence trough. Therefore, in the subsidence calculations presented in this paper, only the given in-mine geometry was used in the subsidence calculations and only the material properties were adjusted (as described above) in order to fit the observed subsidence.

If the only goal in this investigation of the subsidence prediction capabilities of LAMODEL was the best fit to the subsidence profiles in Figure 2, then the 3.0 or 4.5 m laminations would probable provide the best numerical fit. Also, if the previously determined value (Yang, 1992) of the overburden constant (ω – 6.9) was utilized, the recommended lamination thickness would be 4.2 m. However, the shape of the subsidence trough produced with the 1.5 m laminations seems to fundamentally fit the measured subsidence better. Specifically, the 1.5 m thickness provides a flat bottomed trough with steep sides similar to the measured subsidence. Also, for a super-critical panel, the peak gob stress should essentially be equal to the total overburden load as with the 1.5 m laminations (Table 1). If the width of the panel in the LAMODEL simulation were artificially narrowed, it is fairly apparent in Figure 2 that the 1.5 m lamination thickness would provide the best fit to the measured data. But, this empirical panel narrowing would essentially corrupt the primary in-seam modeling capabilities of LAMODEL.

THE D-3 AND D-5 PANELS

The next two panels at which the subsidence was investigated using LAMODEL are known as the D-3 and D-5 panels, and they are the first and second panels to be extracted at the mine (see Figure 4). The D-3 panel is 180 m wide and 1250 m long with a coal thickness of 1.8 m and overburden that averages 230 m. The panel was started on January 22, 1982 and finished on February 26, 1983 in a total of 400 days at an advance rate of 4 m per work day. The D-5 panel is directly adjacent to the D-3 panel. It is 168 m wide and 1050 m long with a similar coal thickness of 1.8 m and overburden of 230 m. The D-5 panel was started on February 28, 1983 and finished on August 7, 1983 in a total of 160 days at an advance rate of 7 m per work day. Both of these panels advanced from the northwest towards the southeast, and each panel had its own longitudinal line of subsidence monitoring stations and a shared transverse line which extends over both the panels and the intervening gate road (see Figure 4). Since these two panels share a gate road, they allow investigating the utility of using LAMODEL to calculate the interactive subsidence from adjacent panels. Also, because the panels are considerably narrower (< 180 m) and deeper (> 230 m) than the V-1 panel, the surface subsidence is expected to be sub-critical.

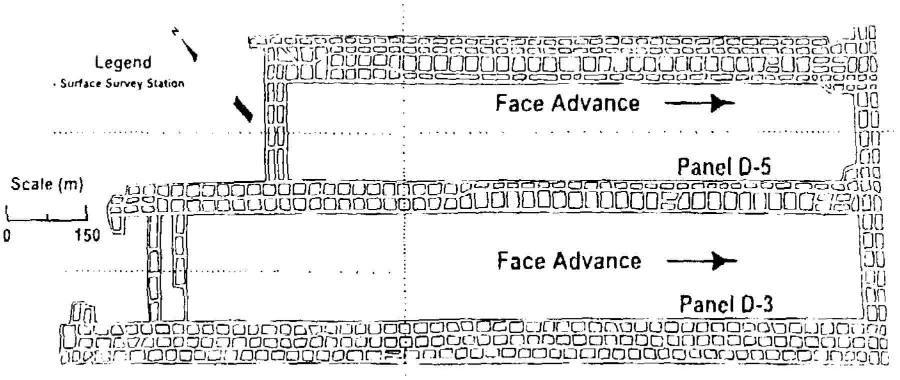

Figure 4: Map of the D-3 and D-5 panels.

For the subsidence calculation at these two panels, a single LAMODEL grid 300 elements wide by 300 elements high with 3 m elements was created. This grid covered the initial half of both panels. In the model, the overburden was set at a constant 230 m with a pressure gradient of 0.025 MPa/m. The elastic modulus of the rock mass was set at 20 GPA and the modulus of the coal was set at 2 GPa. Also, the coal thickness was set at a constant 1.8 m, and the strength of the coal in the seam was set such that the pillar strengths essentially followed the Mark-Bieniawski formula (Mark and Chase, 1997) using an elastic, perfectly-plastic material model (Heasley, 1998). For the subsidence calculation, the off-seam grid capability of LAMODEL was used to calculate the displacement on the surface every 6 m along lines coincident with the surface survey lines. Once again, the first-order, free-surface effects were included by simply doubling the surface subsidence as calculated in the infinite media.

In the first attempt at calculating the subsidence for the D-3 panel, the optimized parameters from the previous fitted subsidence at the V-1 panel (Table 1) were used. The results of this initial subsidence calculation are shown in Figure 5, and basically, the V-1 parameters greatly over-predict the amount of subsidence. This result is a little disturbing. Since these panels are only a few miles apart, it was hoped that the same set of material parameters would provide reasonable approximations to the subsidence at all of the panels. The observation that the optimized parameters from the V-1 panel do not provide a reasonable fit to the subsidence from the D-3 panel evokes a couple of possible explanations. First, the geology may have changed between the panels (a reasonable conclusion since the depth of overburden is so different). And indeed, further investigation indicates that there is approximately 10% competent rock (sandstone and limestone) over the V-1 panel in contrast to 37% competent rock over the D-3 and D-5 panels (Mark, 1987; Jeran and Barton, 1985). This increase in the percentage of competent rock may result in an increase in gob bulking factor and/or overburden stiffness and account for the reduced subsidence at panel D-3. Also, it must certainly be considered that the mechanical overburden model used in LAMODEL is not sufficiently realistic for accurate subsidence prediction at sites with widely varying geometries (widths and depths), but that is the purpose of this case study.

Next, in order to optimize the fit of the calculated subsidence to the measured subsidence, the final gob modulus was increased for three of the lamination thicknesses. The results of this optimization process are shown in Figures 6 and 7, and the optimized combinations of lamination thickness and final gob modulus are listed in Table 2. In general, the final gob modulus was increased approximately threefold in order to limit the maximum calculated subsidence and provide the best fit to the measured data.

TABLE 2
CALIBRATED LAMODEL PARAMETERS FOR PANEL D-3

Lamination Thickness (m)	Final Gob Modulus (MPa)	Peak Gob Stress (MPa)	Average Gob Stress (MPa)
1.5	383	5.7	4.5
4.5	372	5.4	3.6
7.5	324	4.3	2.3

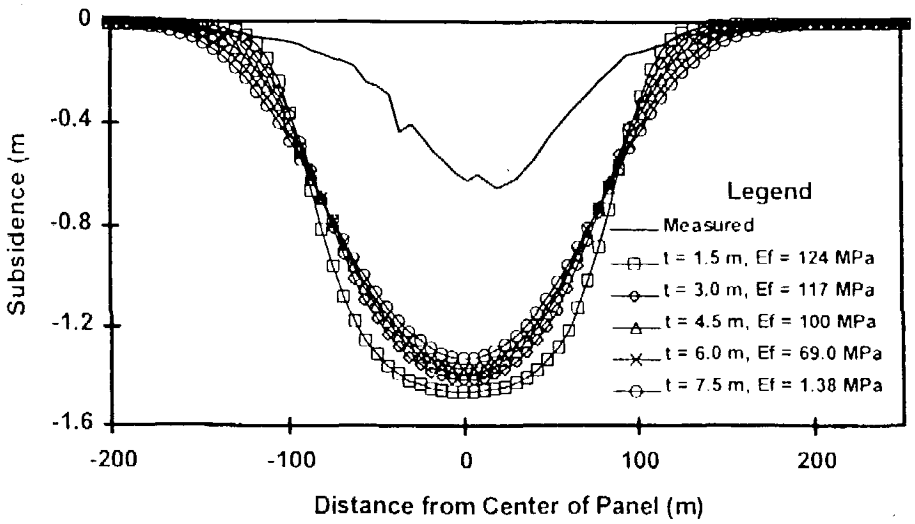

Figure 5: The measured and predicted transverse subsidence for the D-3 panel.

A close examination of Figures 6 and 7 reveals a number of observations. First, the measured subsidence appears to be sub-critical as suggested by the panel dimensions and confirmed by the narrow, V-shaped subsidence trough and the limited amount of subsidence (0.6 m) as compared to the supercritical V-1 panel (1.2 m). Also, as in the previous measured curves, the subsidence shows various anomalies that are not consistent with a homogeneous overburden, horizontal seam, constant depth, and constant seam thickness. In particular, the transverse subsidence in Figure 6 is unsymmetric about the center of the panel and the longitudinal subsidence in Figure 7 shows considerable variability along the bottom of the subsidence trough. In contrast, the calculated subsidence curves are symmetric about the panel centerline and very smooth.

In comparing the calculated (or fitted) subsidence with the measured subsidence, a number of observations can be made. First, it is evident that the maximum subsidence for the calculated curves was calibrated to approximately 0.63 m. Therefore, all of the calculated curves converge at the center of the subsidence trough at the same point on the measured subsidence curve. In general, the thickest 7.5 m lamination appears to provide the best fit to the measured subsidence in both the transverse and longitudinal cross-sections (although it is doubtful that any simplistic subsidence prediction technique can provide a good fit to the highly variable subsidence along the bottom of the longitudinal profile). In contrast to the measured subsidence, the thinnest 1.5 m laminations generate a transverse subsidence trough which is flat along the bottom and appears super-critical. At the edges of the subsidence troughs, the calculated curves show the subsidence starting some 20 to 50 m wider than the measured subsidence as was the case with the V-1 panel.

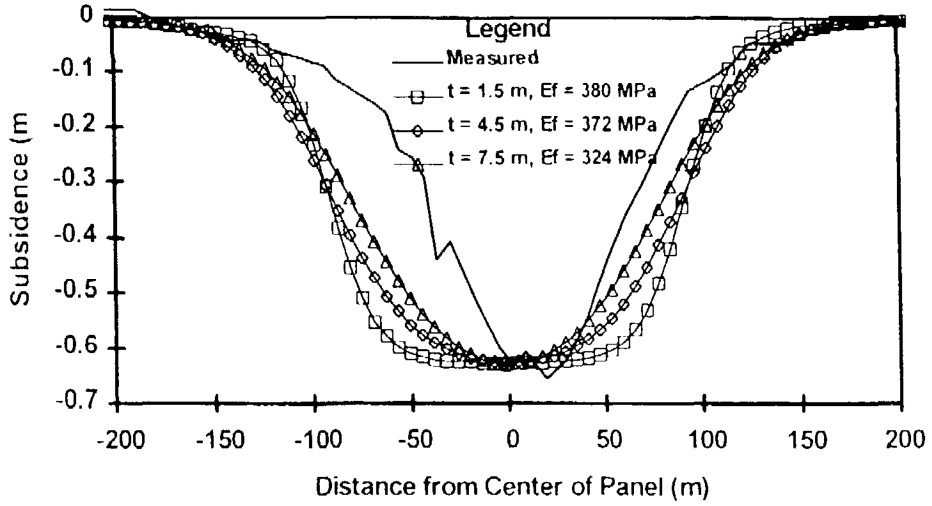

Figure 6: The measured and fitted transverse subsidence for the D-3 panel.

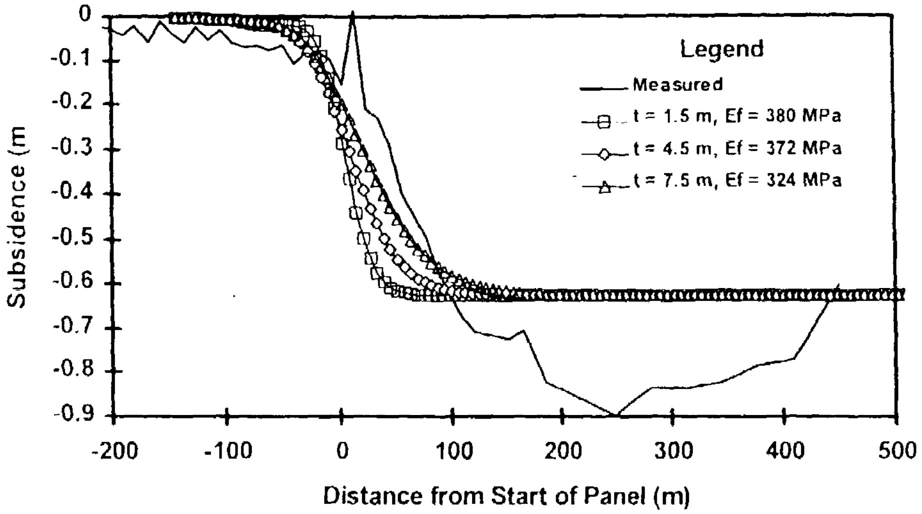

Figure 7: The measured and fitted longitudinal subsidence for the D-3 panel.

Next, the calibrated parameters from the D-3 panel were used to predict the subsidence at the D-5 panel. In this final subsidence prediction process with adjacent panels, the strength of the coal in the intervening gate road was varied in order to help fit the subsidence over the gate road, even though the gob moduli and lamination thicknesses were held consistent with the calibrated values from the previous D-3 panel. Ultimately, the best fit was found with coal that yielded at 60% of the nominal Mark-Bieniawski coal strength (Mark and Chase, 1997).

The results of the subsidence calculation for the D-5 panel are shown in Figures 8 and 9, and in general, the calculated subsidence matches the measured subsidence fairly well. In particular, the calculated subsidence in the transverse profile in Figure 8 has about the same maximum subsidence as the measured profile, and the sides of the calculated subsidence trough align well with the measured subsidence. In fact, the calculated subsidence for the left side of the transverse profile in the D-5 panel fits better than the original calibrated curves in Figure 6. Similarly, for the longitudinal profile in Figure 9, the maximum calculated subsidence approximates the maximum measured subsidence, and the calculated subsidence at the end of the panel aligns well with the measured subsidence at the end of the panel.

One of the main reasons that the calculated subsidence fits so well is that the measured transverse subsidence profile is "wider" than the previous measured profiles. On the right side of the measured profile in Figure 8, the additional width, or spread, of the trough is attributed to convergence in the gateroad between panel D-3 and D-5. In LAMODEL, this spread of the subsidence profile is realistically modeled by convergence of the gate road through failure and yielding of the gate road pillars. And indeed, the modeled gateroad failure improves the fit

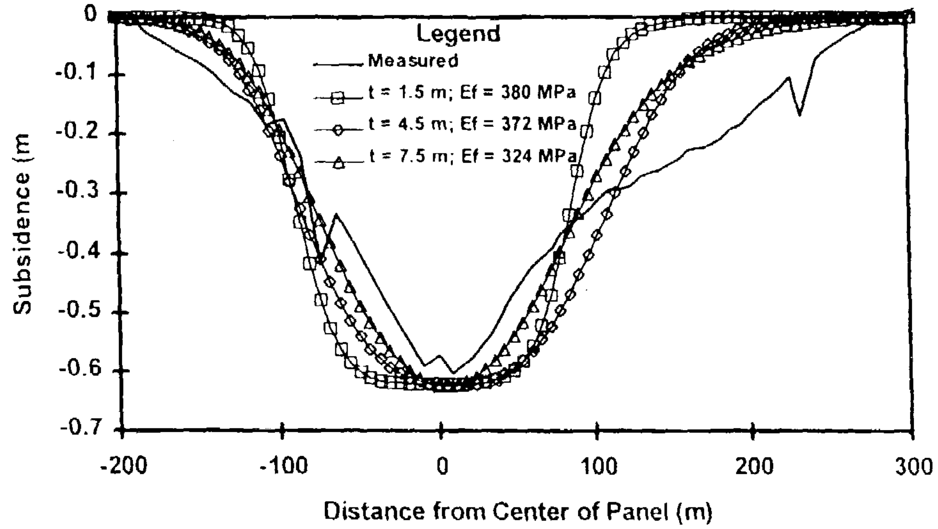

Figure 8: The measured and fitted transverse subsidence for the D-5 panel.

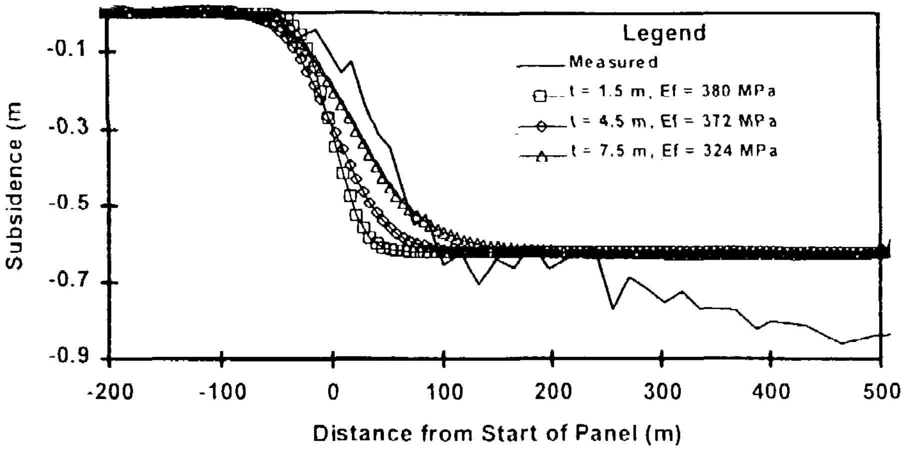

Figure 9: The measured and fitted longitudinal subsidence for the D-5 panel.

of the calculated subsidence as shown in Figure 8, particulary for the 4.5 and 7.5 m laminations. The left side of the measured subsidence trough also exhibits a wider profile which is closer to the natural profile calculated by LAMODEL. The reason that the subsidence at the left side of the measured profile is wider than the previous two panels is not evident from the available data.

In both figures 8 and 9, as in the figures for panel D-3, it appears that the 7.5 m laminations provide the best fit to the measured subsidence in comparison to the thinner laminations. This lamination thickness of 7.5 m corresponds to a value of 7.2 for the inherent overburden constant, ω, proposed by Yang (1992). This value agrees very well with the average value of 6.9 previously determined by Yang (1992), and supports the premise that the optimum value of ω for subsidence prediction is a constant (or narrow range). By asserting that ω is a constant, it is implied that the best lamination thickness to use for subsidence prediction must increase with increasing depth. Considering that the laminated overburden model essentially provides a simplified approximation to the actual bedded behavior of the overburden stratigraphy and that the lamination thickness parameter essentially functions as a crude measure of the vertical distance between the functional horizontal slip planes in the overburden, it seems reasonable that as the overburden, or area-of-interest, increases that the effective slip plane distance and hence the lamination thickness should increase. The same result, that the optimum lamination thickness is proportional to the area-of-interest, was found for stress modeling where thinner laminations were best for modeling the small scale inter-seam stresses but thicker laminations were optimum for modeling wide area longwall abutment stresses (Heasley, 1998). This hypothesis that the lamination thickness needs to increase with increasing depth or scale seems physically reasonable; however, it is contradictory to the initial hypothesis at the beginning of this study that the lamination thickness should be an inherent constant of the overburden.

CONCLUSIONS

In this paper, the laminated overburden model with calibrated properties was demonstrated to easily provide fairly accurate subsidence predictions over several longwall panels. Also, the laminated model demonstrated a considerable amount of flexibility for subsidence fitting through varying only two mechanical parameters, the lamination thickness and the gob modulus. And, since the laminated overburden model in this case study was implemented as part of a full-featured, displacement-discontinuity program (LAMODEL), the asymmetrical subsidence associated with multiple panels and yielding gateroads, which is traditionally difficult to predict, was easily simulated through realistic failure and yielding of the gate road pillars. This demonstration of the capabilities of the laminated overburden model is a major step towards the goal of developing a practical, mechanics-based, subsidence predictive method which would allow the determination of site-specific parameters from the fundamental properties of the overburden and thereby minimize the amount of required calibration work.

However, even though the capabilities of LAMODEL are promising, a number of items, or areas, need to be investigated before practical subsidence prediction with the program is a reality. The question of whether the lamination thickness should be an overburden constant or proportional to the depth needs to be further examined. Also, the possibility of a correlation between the amount of competent rock in the overburden and the optimum gob modulus and/or lamination thickness should be researched. More realism could be added to the model by including the actual variable seam thickness, seam tilt and overburden in the subsidence calculation. In previous work (Heasley, 1988), this type of realistic detail was shown to significantly increase the accuracy of subsidence prediction. Also, the tendency of the program to systematically produce subsidence troughs which are wider than observed needs to be addressed. In the short term, some type of "edge adjustment" might be added to the program in order to compensate for the tendency towards overly wide subsidence troughs. Finally, additional subsidence predictive case studies at various sites need to be performed in order to investigate the above items and to ultimately evaluate the potential of LAMODEL for practical subsidence prediction.

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