TAX AND ECONOMIC DEPRECIATION OF MACHINERY AND EQUIPMENT

A Theoretical and Empirical Appraisal

Phase II Report

Economic Depreciation of the U.S. Capital Stock: A First Step

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*Note: This is a preliminary report and may not be quoted or referenced without consent of the authors and the Office of Tax Analysis.

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ECONOMIC DEPRECIATION OF THE U.S. CAPITAL STOCK: A FIRST STEP

I. INTRODUCTION AND SUMMARY: CONTRACT ROLE AND OBJECTIVES

A. The Role of this Report in OTA's Research Plan

This paper consists of the second phase of a three-phase research contract let by the Office of Tax Analysis (OTA). OTA, within the office of the Secretary of the Treasury, let this contract for two central purposes: first, to provide a publicly defensible *set of initial estimates of the actual rates* of economic depreciation of the major assets which comprise the U.S. capital stock. As was made clear in the contract statement, and which will be reemphasized below, many of the estimates which will be provided in this report *are based more on judgment than on analysis.* However, two major analytic contributions, which will provide the foundations for future analysis, are contained in this report: first, we provide a detailed methodology for estimating economic depreciation from data on used asset prices. Second, we implement this methodology for a number of specific assets which represent a rather large proportion of the total stock, so that a definite starting point is provided for subsequent measurement efforts.

This contract also makes progress toward another major advancement for the Office of Tax Analysis. For many years, OTA has analyzed tax laws and proposals on a nearly case by case basis. While utilizing a data-tax-model, OTA has not had available to it a single major analytic model from which it could draw definitive quantitative conclusions. *However*, under the leadership of Emil Sunley, David Bradford, Harvey Galper, Gary Robbins and others at OTA, this situation began to change in 1976 and 1977. It became evident to these analysts that policy recommendations from the government should be based upon a coherent analytic model of the tax system so that the proposals presented and evaluations undertaken over a wide range of topics and over a long period of time would be internally consistent with one another. The conviction that such a coherent and consistent framework could be built was timely indeed for a number of major research breakthroughs provided just the model needed to meet this OTA objective. Two major strains of research were brought together in the late 1970's to provide OTA with the analytic and quantitative material necessary to develop this model. The first strain was launched by Arnold C. Harberger in (1962). Harberger developed a model in which one could determine the incidence of a corporate income tax imposed on one industry in an economy containing two industry groups. Herbert Scarf in (1969) then developed a converging computer algorithm for quantitatively measuring the set of general equilibrium prices for an economy with, at least conceptually, any number of industries based upon their supply and demand schedules. Applying Harberger's tax incidence analysis to Scarf's computer algorithm one could obtain quantitative measures of the impact by industry of a change in the tax code. While such analysis is still to some extent in its infancy, several of Scarf's students, especially John Shoven and John Whalley, have actually developed a general equilibrium model with the Scarf computer algorithm for a large set of industries and for a variety of types of taxes. A number of scholars have since been working on this type of computer algorithm in order to evaluate taxes. However, a major difficulty with these models from a practical point of view has been the poor data base available for the analysis. It is this second problem to which the second strain of research in recent years has been addressed.

This second body of research began with the famous studies in the early 1960's by Dale Jorgenson. Jorgenson was one of the first economists to fully appreciate the ability of economics to integrate its conceptual ideas with the powerful data base provided by the U.S. government. Jorgenson, with a number of collaborators, provided empirical estimates of U.S. investment demand for a number of industries. Jorgenson showed that one could provide reliable estimates of investment requirements using actual data on U.S. capital goods prices and quantities when employing a neoclassical capital demand framework. Central to Jorgenson's approach to investment was the notion that neoclassical economics provided the analytic basis for investment demand. Three major components to Jorgenson's investment model are essential: (1) a flexible accelerator represents the demand for investment, (2) an aggregate production function represents the underlying demand for capital and (3) a user-cost-of-capital measure represents the price of capital goods. Throughout the 1960's and 1970's Jorgenson and his collaborators consistently improved their measures of the quantities and prices of capital goods.

Nevertheless, the central problem in Jorgenson's work, from a measurement point of view, continued to be the difficulty of measuring the quantity of capital in place. Jorgenson was one the first economists to appreciate the importance of maintaining an internally consistent depreciation model. In an important summary work, "The Economic Theory of Replacement and Depreciation," in (1973) Jorgenson applied this concept of internal consistency to show that it is necessary in a coherent model to utilize a method of depreciation which is consistent with the method one uses for the replacement of capital.

Recognizing the value of providing a data base which is as coherent and consistent as the theoretical model itself, OTA turned to Jorgenson to provide for the quantitative basis with sufficient quality to provide the advanced conceptual framework and computer algorithm being used by Shoven.

Employing the concepts developed by Hall in (1968) and by Jorgenson in his earlier investment studies, Wykoff in (1970) developed a user-cost based study of capital depreciation. Wykoff employed the theory of depreciation and replacement to actual empirical estimates of the depreciatio of automobiles in the United States. Later, under the auspices of OTA, Charles Hulten and Frank Wykoff in (1975) and in (1977) extended the methodology developed by Jorgenson, Hall and Wykoff and applied this new methodology to the study of economic depreciation of commercial and industrial structures It became evident to OTA at this time that Hulten and Wykoff could provide estimates of economic depreciation which could be used, in turn, by Jorgenson to develop measures of capital and investment flows by industry. This would provide the kind of measurement base needed by Shoven so that OTA could implement his model.

Thus, in 1975 and 1976 OTA began to develop a model which brought together these two major branches of research. The ultimate objective will be a computer algorithm, based upon actual estimates of the U.S. capital stock, for evaluating various types of business taxes and for estimating the impacts by industry of various proposals to change the tax laws. With this capabilit OTA will have an internally consistent conceptual model with the highest quality data base available. Furthermore, OTA will be able to continually upgrade both the data base and the conceptual framework as new breakthroughs are made in the economics profession. In other words, with the culmination of this major research effort on the part of OTA, it will have developed both a model for analyzing all major tax questions and a foundation for building its research capability into the foreseeable future.

We turn now to a discussion of the specific contributions of this report to the requirements of OTA. It will be recalled that this project addresses several specific tax issues in its own right which are quite important in light of some of the major controversies concerning today's tax code.

The accurate definition and measurement of the tax base is an important consideration in the administration of any tax. Distortions in the base of a tax can lead to violations of the standard canons of equity and efficiency, and to popular dissatisfaction with the tax. Unfortunately, most tax bases present some difficulty in this direction, but few present more problems than the taxation of income from capital.

The difficulty in defining the base of the tax on capital income lies primarily in the distinction between accrual and realization. Many components of capital income—capital gains, depreciation, inventory revaluation—accrue during a tax period but are not realized in any market transaction. Consequently, no direct test of the size on these accruals is available, and indirect methods are required. In this study we focus on one particularly troublesome component of capital income—economic depreciation.

Economic depreciation is the amount of money which must be replaced in order to keep the original capital investment intact. It arises from the fact that some forms of capital—notably plant and equipment—are used up or become obsolete in the course of generating income. The Federal Income Tax Code has, since its inception in 1913, recognized the principle of allowing a deduction for depreciation of capital assets. Major difficulties have, however, arisen in the attempt to implement this principle. Many approaches have been tried and rejected, and the recent collapse of the Asset Depreciation Range vintage reporting system signals yet another period of controversy over depreciation procedures. This controversy is likely to center on the issue of whether the Treasury and Congress should continue in their attempt to base depreciation allowances on actual taxpayer experience, somehow measured, or whether the Treasury should recognize the near impossibility of measuring this component of economic income and provide more or less arbitrary, but administratively feasible, guidelines for depreciation allowances.

The revaluation of assets for depreciation purposes is another controversial area of tax reform. See Aaron (1976). The tax code currently allows depreciation deductions to be based on the original cost of an asset. The inflation of recent years has, however, caused the prices of new and used capital assets to increase. Rising asset prices lead to rising replacement costs which should be taken into account when defining taxable income.

B. Primary Objectives of this Report

Recognizing the above policy problems and planning its new analytic tax model, OTA decided to determine the feasibility of developing empirical depreciation estimates for a variety of asset classes with special emphasis on producer durable equipment. The Contract Work Statement clearly states one of the first objectives of this study:

> Employing the multiple asset model of economic depreciation and the econometric models of estimation outlined (in the Work Statement) above, average relative productive efficiencies and average economic depreciation rates for the various classes of assets will be estimated within several broad asset categories: (A) Machine Tools, (B) Vehicles, (C) Heavy Duty Construction Equipment, and (D) possibly additional asset classes specified in Tables 2 and 3 (of the Work Statement).

The first purpose of this report, then, is to measure the actual depreciation and revaluation of some, but by no means all, types of plant and equipment. Our approach is based on the analysis of the market prices of used capital goods. The observed market prices of used (or "vintage") capital should decline in value as it ages precisely because the capital asset is used up in production or because it becomes obsolete. By measuring and correctly interpreting the vintage price effects, insight can be obtained about the reasonableness of depreciation policy. The use of vintage prices as a means of assessing depreciation policy is hardly new, but this approach has only slowly been gaining widespread acceptance among economists because of the long held view that used asset markets do not exist for most assets, and that the markets that do exist are too thin to provide meaningful data. (A discussion of existing studies appears in the Phase I report.) There has been, furthermore, skepticism about whether assets which do appear in used good markets are representative of those which never enter the market place.

In Phase I of this contract we confronted these arguments and reached the following conclusion: The market data for used capital are considerably richer than the conventional wisdom suggests. Used buildings, autos, trucks, machine tools, office equipment, electrical equipment, and construction equipment are all transacted in reasonably active resale markets. While this list hardly encompasses all fixed capital assets, it does account for a surprisingly large fraction of total fixed investment. Equipment categories for which we have found vintage price data account for 55% of 1977 investment expenditures in producer durable equipment, and structure categories for which data exists account for 42% of 1977 investment expenditures for nonresidential structures.

Secondly, we argued that while some vintage prices may be biased downward, the direction of the bias favors the taxpayer at the expense of the Treasury. This is not necessarily inappropriate, since recent tax practice generally requires that the Treasury not disturb depreciation claims without good reason, and any bias in favor of the taxpayer provides a margin of error for the Treasury.

Having concluded from our Phase I Report that vintage asset prices are a meaningful source of information, we now, in this report consider the econometric problem of obtaining estimates of the depreciation process and of converting these estimates into estimates of the relative productive efficiencies of specific assets. In the conceptual sections of this report we discuss difficulties associated with inflation, asset retirement, obsolescence and the endogeneity of depreciation. Several explicit econometric models are outlined and discussed in some detail. A new econometric model is also developed in this conceptual section. These models are than applied to thirty specific asset groups. These thirty types of assets represent seven classes of producer durable equipment, two classes of private nonresidential structures and one class of consumer durable assets. These ten asset categories contain nearly 50% of the entire stock of fixed capital in the United States. The econometric addendum of this Phase II Report contains in extensive detail the analysis of these thirty specific assets organized by the relevant asset classes (needed by Jorgenson and Shoven). This econometric addendum, consisting of some 1200 pages, thus represents an attempt to provide a defensible set of estimates of the depreciation process for the entire stock of U.S. capital assets, which embodies information obtained from the market for these assets.

One major result of policy significance that follows from this analysis is that the pattern of economic depreciation of machinery and equipment appears to be accelerated relative to the straight line pattern. This result suggests that accelerated forms of depreciation such as those now allowed in the U.S. Tax Code--declining balance and sum of years digits--are warrented. We also found in an earlier study undertaken for OTA, TOS-74-27, that accelerated forms of depreciation are warrented for structures as well. However, it appears that the available tax deductions permitted on both private nonresidential structures and producer durable equipment may well have been overly generous given past rates of inflation

The second central purpose of this report is also clearly enunciated in the Work Statement of the contract.

> For asset classes in which data is insufficient for full econometric estimation, other methods, with supporting justification, will be employed for making the required estimates. ... In cases where data is insufficient, the best professional judgment will be used for making the required estimates of depreciation.

While we feel that the data we have is reasonably useful for 6 producer durable equipment classes (hereafter referred to as PDE) and 2 private non-residential structure classes (PNS) and 2 consumer durables classes (CD), we have only partial information on 2 PDE classes, 2 PNS classes and 2 CD classes. Furthermore, we have no actual data on the remaining asset categories--namely 14 PDE classes and 9 PNS classes and 5 CD classes. In terms of the volume of capital represented we have reasonably good estimates representing approximately 47% of the U.S. capital stock and only partial information for the remaining 53% of the stock. Consequently, the second purpose of this report will be to convert the detailed estimates we have for specific assets into depreciation estimates and productive efficiency estimates for the 22 PDE classes and the 10 PNS classes. As indicated in the Work Statement and again in the Phase I Report of this contract, the depreciation rates and efficiency estimates for the asset classes for which we did not have detailed data are based upon judgment. Perhaps the next step in continuing study of the depreciation problem should be to try to provide both a methodology and some actual estimates of depreciation for those classes not covered in detail by this study.

In addition to the two objectives outlined above for this report, namely the detailed study of specific assets and the extension to estimates for the major PDE and PNS classes, verbal requests on the part of Treasury officials indicated a desire to also obtain estimates for consumer durable assets. This problem is somewhat more difficult than the earlier two problems because while the Treasury has long had some basis for estimating depreciation on PDE and PNS classes which are taxed, no factual basis whatever exists for providing estimates for consumer durables (nor for non-taxable PNS assets). Nevertheless, we shall provide some judgmental estimates on both the non-taxed PNS assets and the 7 consumer durable asset categories.

The contract Work Statement contains two convenient summary comments which clarify the purpose of this report. First, from page 20 of the Work Statement, "The econometric methods to be used are discussed in detail in The Economic Depreciation of Non-Residential Structures, by Hulten and Wykoff." This paper was reproduced for the Treasury as a part of the Phase I Report of this contract. Consequently, we shall only briefly summarize the general econometric methodology to be used here. The major exception is that we shall discuss in detail a new method we have developed for dealing with asset retirements. Finally, the overall statement of objectives for this Phase II Report, as contained in the Work Statement, is:

> Construction of economic depreciation and efficiency function estimates for (A) Machine Tools, (B) Vehicles, (C) Heavy Duty Construction Equipment and (D) providing estimates based on the best professional judgment for all other asset classes.

The outline of this Phase II Report will be as follows: this introductory section contains two more parts. Part C contains a brief summary statement of the Phase I Report of this contract. Part D of the introduction contains a summary and overview of the major results which follow from this Phase II study. (This summary statement may be read by those who only wish to obtain the basic results and a very brief overview of this report.) The second section of this Phase II Report titled "Econometric Analysis of Specific PDE and CD Assets" contains three sections: Section A contains the theory and methodology of the study. Section B illustrates the major econometric results which appear in greater detail in the Appendix. And Section

C summarizes the efficiency figures for each asset studied in detail as well as the depreciation rate estimates which provide the basis for the final set of depreciation and efficiency measures suggested to the Treasury for its overall study. The third and final section of this report, entitled "Judgmental Estimates of Depreciation and Efficiencies for U.S. Depreciable Capital Stocks," contains a discussion of the decision rules and problems encountered in converting the specific asset by asset depreciation estimates into judgments of depreciation for large asset classes.

C. A Brief Overview of the Phase I Report: Assessment of the Quality and Availability of Data on Vintage Prices of Machinery and Equipment

Phase I was a report of a major data search undertaken for this contract. The outcome of the Phase I Report was a body of data, to be studied here, on specific assets. To assist in our summary of Phase I, Table I lists the major asset classes for which estimates are required in the Jorgenson-Shoven analysis. From Table I depreciable assets are seen to fall into three broad categories: (A) Producer Durable Equipment (PDE), (B) Private Nonresidential Structures (PNS), and (C) Consumer Durables (CD). PDE contains twenty-two classes, PNS and CD have 10 and 7 respectively. The search undertaken in the Phase I Report consisted of studying three types of sources: (1) existing library sources or bibliography in economics, business and engineering, (2) commercial and industrial sources, or published price series used in various industries and (3) government agency sources (especially the General Services Administration and the Treasury Department itself). On the basis of this data search, the 22 PDE classes, 10 PNS classes and 7 CD classes were partitioned into three types of asset classes based upon the availability of data for research. These asset categories are referred to as Type A, Type B, and

Table 1

MAJOR ASSET CLASSES

(A) Producer Durable Equipment
  1. Furniture and fixtures
  2. Fabricated metal products
  3. Engines and turbines
  4. Tractors
  5. Agricultural machinery (except tractors)
  6. Construction machinery (except tractors)
  7. Mining and oilfield machinery
  8. Metalworking machinery
  9. Special industry machinery (not elsewhere classified)
  10. General industrial equipment
  11. Office, computing and accounting machinery
  12. Service industry machinery
  13. Electrical transmission, distribution and industrial apparatus
  14. Communications equipment
  15. Electrical equipment (not elsewhere classified)
  16. Trucks, buses and truck trailers
  17. Autos
  18. Aircraft
  19. Ships and boats
  20. Railroad equipment
  21. Instruments
  22. Other

(B) Private Nonresidential Structures
  1. Industrial
  2. Commercial
  3. Religious
  4. Educational
  5. Hospital and institutional
  6. Other[1]
  7. Public utilities
  8. Farm
  9. Mining exploration, shafts and wells
  10. Other[2]

(C) Consumer Durables
  1. Motor vehicles and parts
  2. Furniture
  3. Kitchen and household appliances
  4. Radio and television receivers, recorders, musical instruments
  5. Wheel goods, durable toys, sports equipment
  6. Residential structures
  7. Other

(D) Inventories
  1. Farm
  2. Non-farm

(E) Land

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[1] Consists of buildings used primarily for social and recreational activities and buildings not elsewhere classified.

[2] Consists of streets, dams and reservoirs, sewer and water facilities

Type C asset classes. Type A asset classes are those for which we have extensive data and with which we apply our methodology to provide what we consider to be reasonably reliable estimates of economic depreciation for those classes. Type B asset categories are those for which we have found some existing studies or for which we have some data but which we do not consider to be either sufficiently reliable nor sufficiently extensive to warrent defensible estimates based solely on the data. Type C asset categories are those for which we have no data whatever. Table 2 contains the partitioning of asset classes from Table 1 into the three types of asset groups. Within the Type A categories asset classes fall into three subgroupings. The first subgrouping consists of PDE classes 4, 6, 8, 10, 16 and 17. These asset classes' estimates are based upon the analysis reported in this Phase II Report, Section 2. The consumer durable class 1, Autos, is also studied in detail in this report. The two PNS class estimates are based upon the extensive study undertaken in Contract TOS 74-27. The remaining asset category labeled as a Type A asset is the consumer durable class of residential structures. We believe the two studies undertaken of depreciation of residential structures by Weston and Leigh are reliable enough to include this as Type A assets.

While of thirty-nine possible asset classes listed in Table 1, we are only placing ten into the Type A category, the proportion of U.S. depreciable capital which falls into the Type A category is quite large. Based on total U.S. private purchases of new assets in 1976, the six classes of PDE for which we shall provide reasonably good estimates on some assets contained nearly 50% of the total producer durable equipment sales. Type A assets cover 42% of the total PNS purchases, and 66% of total 1976 CD purchases.

A caveat was mentioned in the Phase I Report which bears repeating here. Some of the PDE, PNS and CD asset categories are quite broadly defined and our

Table 2

TYPES OF ASSET CLASSES BY AVAILABILITY OF DATA AND RELIABILITY OF ESTIMATES

Type A	Type B	Type C
PDE 4	PDE 11	PDE 1-3
6	19	5
8		7
	PNS 5	
10		9
	6	
16		12-15
17	CD 2	18
	4	20-22
PNS 1		
2		PNS 3
		4
CD 1		
		7-10
6		
		CD 3
		5
		7

data applies to only a portion of the assets in these broad categories. It is unrealistic to think that our estimates represent comprehensive coverage of the millions of types of specific machinery employed in the U.S. and this is true even for the asset categories which are listed under Type A.

We turn now to illustrate the data contained in Phase I which forms the basis for the analysis in this report. We illustrate the detail of the data provided in the Phase I Report by using as an example one subclass of assets--the D-7 Tractor.

Our tractor data consists of the prices of used tractors reported in Blue Book of Current Market Prices of Used Heavy Construction Equipment, Forke Brothers Blue Book Co., Lincoln, Nebraska, 1968-1977. These prices reflect actual transaction prices of individual units sold on open auctions in the U.S. In some instances, prices may reflect units which are not actually sold to a new owner but are paid back at a pre-arranged price to the original owner. These "paybacks" reflect the in-use value to the existing owner. Most prices, according to Forke Brothers and industry sources, do reflect actual sales. Units sold at auction are thought by industry sources to be representative of tractors in place. Tractors are usually bought and sold at auctions often by dealers who acquire tractors, new or used, for specific projects. When projects are completed, dealers sell off their capital to other users in order to liquidate until they arrange a new project. Used tractors are also sold by various agricultural companies much as used automobiles are sold by households. However, perhaps unlike automobile buyers, used tractor purchasers appear to be rather sophisticated dealers with some knowledge about machinery. Consequently auction prices do not appear to suffer from the type of lemon bias suggested by Ackerlof in (1970).

Tractors come in many shapes and sizes and may be used for a variety of purposes from farming to road construction to dam building. Tractors are often sold with ancillary equipment including winches, rippers, cable control units, canopies and the like. Furthermore, within a general size-class of tractor, say D-7, indicating a large, heavy (25,000 to 35,000 lbs.) tractor usually, though not exclusively, used on agricultural jobs, each unit often has unique characteristics. Major distinctions are indicated by engine letter types, but other distinctions are also indicated by different engine serial numbers. We standardized tractor prices by pricing ancillary equipment and by determining the relative prices of various engine types. Asset prices were modified so that each price represented the price of a D-7 tractor with a straight dozer and a ROPS canopy (after 1971), having standard equipment only. Thus ancillary equipment prices were deducted from sale prices. Table 3 illustrates the ratios used to standardize prices.

Table 3

RATIO OF STANDARDIZED TRACTOR TO
TRACTORS WITH ANCILLARY EQUIPMENT*

Ancillary Equipment	Type of Engine		
	E	F	G
(1) Type of Bulldozer			
Bare	1.133	--	--
Straight	1.000	1.000	1.000
U	.986	.986	.988
Angle	.997	1.002	1.017
Cable	1.017	--	--
Ripper	--	.926	--
(2) Winch	--	.876	.895
(3) D-7 Ripper	.902	.899	.912
(4) Kelly Ripper	.965	--	--
(5) #29 C.U.C.	.980	--	--
(6) ROPS Canopy	--	1.148	--

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*The prices of ancillary equipment were found in various issues of Green Guide, Vol. I: The Handbook of New and Used Construction Equipment Values, Equipment Guide Book Co., Mountain View, CA., and in Sale Kit II, Caterpillar Tractor Co., Peoria, Illinois.

The actual prices used in the analysis are summarized in Table 4. The sample contains 582 observations, and covers years from 1968 to 1977, and ages one to thirty-five. Figure 1 portrays the average age-price pattern for the sample as a whole. Each observation is deflated by a price index of a new asset.

The retirement distribution used in this study to weight the observed prices is taken from the Iowa Engineering Studies undertaken in the 1930s predominently by Robley Winfrey as reported in Marsten, et.al.(1953) These retirement distributions report the percent surviving of an original cohort of assets according to a given probability distribution about the average class life. The Winfrey L5 distribution was chosen for this study. After conferring with industry sources, we selected 25 years as the average retirement age for tractors—few tractors are retired before 20 years, about 10%, then by 25 years only 47% of the original cohort remain.

The Phase I Report contains data analysis of a total of 26 specific assets. Table 5 contains a list of these specific assets organized by asset class.

D. Summary of Major Results from Phase II

(1) Results for Specific Assets:

Part II of this report includes a description of four different methods for estimating economic depreciation from vintage asset prices. In this report these four different methodologies were applied to estimating depreciation for 26 different specific types of assets. In addition, this same methodology was applied earlier for the Treasury to a study of a dozen different types of commercial and industrial structures. All combined, then, we have studied the economic depreciation process of over 30 different assets ranging from machine tools, trucks and construction equipment to commercial

Table 4
D-7 TRACTOR PRICE ARRAY

YEAR	1977	1976	1975	1974	1973	1972	1971	1970	1969	1968	Row Total
1	0.00 0 0.00	56000.00 2 0.00	55300.00 1 0.00	0.00 0 0.00	0.00 0 0.00	0.00 0 0.00	41350.00 1 0.00	30300.00 1 0.00	18437.00 1 0.00	0.00 0 0.00	42847.8 6 15808.4
2	0.00 0 0.00	57500.00 2 7071.07	53125.00 4 7375.35	0.00 0 0.00	0.00 0 0.00	0.00 0 0.00	30001.50 2 1084.00	29077.25 4 1694.35	23750.00 1 0.00	0.00 0 0.00	40594.0 13 14311.9
3	0.00 0 0.00	47358.00 3 12722.68	40979.00 3 1626.23	41990.00 2 5546.55	41560.50 2 6019.60	31565.00 1 0.00	30443.00 1 0.00	23000.00 1 0.00	19932.00 1 0.00	0.00 0 0.00	38360.1 14 10342.9
4	47000.00 1 0.00	30040.00 4 6140.55	36500.00 1 0.00	42199.00 2 787.73	40906.25 4 7192.40	25500.00 1 0.00	0.00 0 0.00	18333.50 6 1715.29	20983.33 3 4617.99	21676.00 2 4580.64	30241.9 24 11297.3
5	0.00 0 0.00	39465.33 3 5352.28	43468.00 1 0.00	38779.20 5 5944.67	32922.17 6 7384.57	26805.00 3 1154.42	20211.00 1 0.00	18335.75 8 4160.59	19322.50 4 2513.30	18036.80 5 2573.28	26897.7 36 10093.7
6	0.00 0 0.00	36501.25 4 4119.64	32527.50 2 2322.85	30195.83 6 3925.83	33658.50 2 1190.07	26028.00 1 0.00	17431.00 4 5367.07	16852.46 13 2944.85	16110.13 8 4705.68	19000.00 1 0.00	22494.2 41 8578.9
7	36784.00 2 1108.74	30549.00 3 1280.56	30916.67 6 5268.18	29766.43 7 5198.87	29913.80 5 4070.46	15777.33 3 3176.62	15488.38 8 3252.74	14099.89 9 3176.46	15866.43 7 2666.03	16319.50 2 1586.04	22124.8 52 8637.3
8	32015.50 2 9689.48	28695.00 1 0.00	30334.86 7 2304.35	33426.80 5 5223.33	22299.00 7 7304.01	18674.29 7 2859.15	13571.00 3 3656.07	12860.67 6 1427.72	7176.00 1 0.00	0.00 0 0.00	22668.4 3 8915.1
9	0.00 0 0.00	18750.00 1 0.00	29165.60 5 2216.39	22397.60 5 4005.34	20569.75 4 5432.16	21118.33 3 10433.88	11405.00 6 2724.75	8598.50 10 2165.53	9461.50 4 2347.41	7574.00 1 0.00	15949.4 3 8340.3
10	39000.00 1 0.00	25252.00 1 0.00	23282.40 5 2982.40	29843.00 2 5680.90	16985.67 3 1290.95	13155.00 1 0.00	9821.00 4 2376.10	8600.00 1 0.00	7788.14 7 1311.80	7393.67 3 440.04	15323.0 2 9152.
11	0.00 0 0.00	26404.50 2 135.06	27834.00 1 0.00	16031.00 2 1677.26	15377.20 5 7259.86	0.00 0 0.00	8400.00 1 0.00	8365.00 3 712.14	7433.33 3 3043.57	8326.80 5 3373.19	13046. 7535.
12	22048.50 2 5811.71	22200.00 2 554.37	11154.00 1 0.00	15811.56 9 2988.97	9997.67 3 459.45	10308.67 3 1402.11	7290.25 4 3274.62	6239.00 2 338.00	6267.00 2 2147.99	5491.50 2 860.55	12267.4 3 5910.
13	18000.00 1 0.00	19012.67 3 913.07	17676.00 1 0.00	11134.00 1 0.00	10295.00 1 0.00	5395.00 1 0.00	7699.60 5 1433.74	5233.88 8 1081.21	6374.67 3 988.81	7221.00 5 1503.57	8797. 4864.
14	17640.00 1 0.00	16300.80 5 1389.02	8895.00 1 0.00	12874.33 3 2034.00	7989.00 2 2776.10	0.00 0 0.00	4840.00 4 2053.46	6078.00 5 1420.65	5383.75 8 2287.62	6279.00 1 0.00	8726. 4883.
15	18935.00 1 0.00	10409.00 1 0.00	11153.00 2 1213.40	10214.00 2 647.71	6813.20 5 582.61	9023.00 2 1397.24	5359.83 6 1716.84	5355.89 9 1357.46	4700.00 2 424.26	3059.33 3 638.78	6731. 3324.
16	0.00 0 0.00	8306.00 2 1033.79	9790.00 2 220.62	8457.75 4 1382.20	8029.67 3 1889.74	7662.33 3 573.44	5537.57 7 1280.36	3189.00 1 0.00	4653.60 5 1329.27	5753.50 2 1338.55	6732. 2182.
17	0.00 0 0.00	0.00 0 0.00	10998.50 2 5547.25	8494.00 1 0.00	8055.50 4 2764.09	7390.25 4 250.43	2746.00 2 500.63	2630.00 3 831.08	0.00 0 0.00	4385.00 2 1832.82	6357. 3411.
18	7500.00 2 707.11	7443.67 3 1420.98	5839.67 3 906.51	0.00 0 0.00	9167.00 2 103.05	0.00 0 0.00	4421.50 4 458.88	0.00 0 0.00	4276.00 6 518.36	2890.00 1 0.00	5908. 2219.
19	0.00 0 0.00	9794.67 3 477.66	10682.67 3 2981.60	0.00 0 0.00	6200.00 2 5727.56	4805.00 1 0.00	3145.00 2 219.20	3157.80 5 893.51	3526.00 3 1000.15	3971.50 2 1209.06	5685. 3535.
20	7500.00 1 0.00	0.00 0 0.00	7550.00 1 0.00	0.00 0 0.00	6795.00 2 1390.17	0.00 0 0.00	3900.00 1 0.00	3833.33 3 2130.54	2400.00 1 0.00	2981.50 2 450.43	4763 2237.
21	5100.00 1 0.00	5177.33 6 2070.85	5097.75 8 1652.48	4602.00 8 1030.15	4805.00 1 0.00	3264.00 2 1207.74	2226.20 5 1430.20	3067.00 8 1425.24	2087.00 6 1261.56	1502.00 1 0.00	3796 1842.
Total	23591.40 15 13352.17	25416.27 51 16352.73	23715.02 60 14979.70	22098.66 64 12432.07	19460.03 63 12462.15	14704.39 34 8352.63	9930.47 71 7791.04	10909.92 106 7310.41	9496.00 76 6560.20	8917.10 40 6208.05	15804. 12315.

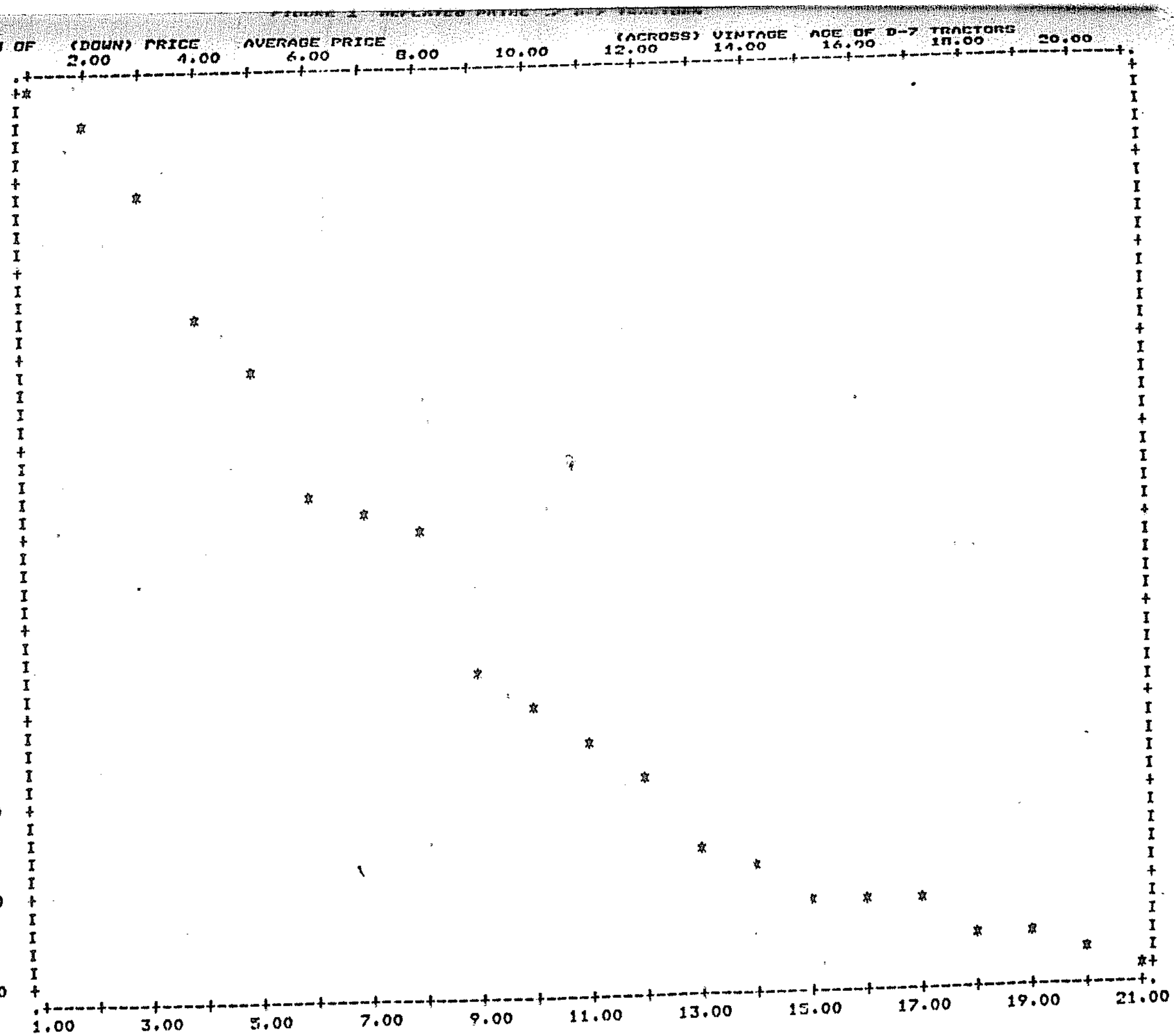
OF (DOWN) PRICE AVERAGE PRICE
(ACROSS) VINTAGE AGE OF D-7 TRACTORS
2.00
4.00
6.00
8.00
10.00
12.00
14.00
16.00
18.00
20.00
1.00
3.00
5.00
7.00
9.00
11.00
13.00
15.00
17.00
19.00
21.00

Table 5

ASSETS STUDIED IN DETAIL BY ASSET CLASS

Producer Durable Equipment

4: Tractors:
- D-4 Tractor
- D-6 Tractor
- D-7 Tractor
- D-8 Tractor
- D-9 Tractor

6: Construction Machinery (except tractors):
- Air Compressor
- Motor Grader
- Rubber Tired Loader

8, 10: Metalworking Machinery and General Industrial Equipment:
- MPG 9—Milling, drilling and boring machines, small
- MPG 12—Drilling machines and boring machines, large
- MPG 19—All other tools

11: Office, Computing and Accounting Machinery:
- Remington Typewriters (electric) (GSA)

16: Trucks, Buses and Truck Trailers:
- GMC Pickup Truck (half-ton)
- Ford Pickup Truck (three-quarter ton)
- Tandem Truck Tractor (6-wheeled rig)
- Tandem Dump Truck (ten ton)

17: Autos:
- GSA Chevrolet
- GSA Ford
- GSA Plymouth

Consumer Durables

1: Motor Vehicles and Parts:
- Buick
- Cadillac (DeVille)
- Chevrolet (Nova)
- Chevrolet (Stationwagen, Standard)
- GMC Pickup Truck
- Ford Pickup Truck
- Plymouth
- Volkswagen

and industrial buildings and to consumer automobiles. Central to our study was a test to determine whether geometric or straightline depreciation is an appropriate form. Our statistical finding was to reject both geometric and straight line depreciation process. However, in general, the analysis of depreciation and of the productive efficiency sequences indicates an accelerated pattern relative to straight line. In other words, the age-price patterns tend to be distinctly convex. While this convex pattern could possibly be the result of biases in vintage asset prices, as discussed in the addendum to the Phase I report, convexity appears for asset classes which are not subject to severe secondary market problems. Consequently it is unreasonable to ascribe the convex pattern to biases in the data. We conclude that depreciation appears to be very generally one of convexity.

The four basic methodologies employed in our study were: (1) the polynomial regression, (2) the Box-Cox power transformation, (3) the Box-Cox model on retired prices, and (4) the Box-Cox model with a truncated distribution. Each of these methods is discussed either in the appendix to the Phase I Report or in Part II of this report. The effect of <u>retiring</u> assets and then estimating the depreciation process seems to be to lower the depreciation rate for the early years but to significantly raise the average rate of economic depreciation over asset life. In other words, the average annual rate of economic depreciation when one accounts for the retirement of assets, as well as for in-place loss in value, is greater when one fails to account for retirement. Depreciation rates typically increased from say around 9% to around 11%, or from 14% to 18%. In some cases the percent increases from retirement were dramatic. For example, in the structure classes, the average depreciation rates were increased by retirement from about 1 1/2% to about 2 1/2%. Usually, however, allowance for retirement has only modest effects, because retirement takes place late in an asset's life when little is left in the productive process anyway. In-place loss in value plays a greater role in the depreciation process than retirement itself. While this result, which is borne out by the empirical evidence, seems intuitively obvious, it has not been generally accepted by the economics profession. Many economists persist in arguing that assets tend to be one-hoss shay in nature and that the retirement process is the major force for depreciation. This study suggests that this conventional wisdom is not correct.

In addition to retiring assets according to a predetermined retirement distribution, we also tried a method developed for dealing with censored-sample problems. The procedure is to treat retirement as a stochastic process. Used assets are randomly dropped from the sample population of the original cohort only if their prices fall below some minimum level. The result is a truncated distribution. While we have no strict test procedure for choosing between depreciation estimates with retired data and those derived from the truncated distribution, we can compare the two approaches. Truncating the distribution at some low price increases the average depreciation rate for older assets but has only negligible effect on newer assets. The reason for this result is obvious, the distribution of new asset prices rarely dips as low as the truncated level. The effect of retiring assets is more substantial and in some cases even reverses the pattern of depreciation from accelerated to decelerated.

The choice of retired or truncated depreciation must depend upon one's assessment of the theoretical plausibility of the two stories. The retirement distribution approach has the advantage that it can be, as shown by Hulten and Wykoff in (1976), fully integrated into the Hotelling-Hall-Jorgenson model of replacement and depreciation. Furthermore, the retirement distribution is an extension of the perfect foresight assumption utilized by these authors. The principal advantage of the truncation approach is that the retirement process is seen to be a stochastic process which depends on the remaining value of the asset at the time of retirement and not simply on its age. Furthermore, it is not necessary to assume, under the truncation approach, that owners of assets have some prior knowledge of when their particular asset may be retired. Nor is it necessary to assume that retired assets and unretired assets all have the same characteristics while in place, an assumption which is probably unrealistic but which is necessary under the retirement distribution approach. For purposes of this Phase II Report, we choose to use the predetermined retirement distribution approach. The truncation approach is relatively new and has only been applied to a sample of the assets studied. We do consider this approach to be promising and shall pursue it in the future.

Before summarizing the depreciation results derived from the regression equations, it is useful to assess the regressions in terms of goodness-of-fit. While statistical detail will be presented subsequently, we present at this point in Figure 2 a comparison of the actual prices, when retired, to the fitted prices from the Box-Cox method on retired prices. Figure 2 illustrates the actual price-age pattern of the D-7 Tractor, using retired prices, from ages 0 to 35 compared to the price-age pattern predicted by the Box-Cox procedure. In both cases, prices are normalized by setting the price of a one-year-old asset to one. The actual prices presented here are the average prices from the entire body of data which are deflated according to the average year for each age group obtained from the data. The Box-Cox fitted values are not deflated but are predicted at the same average year for which the actual prices have been deflated. (In other words the Box-Cox procedure automatically deflates as well as depreciates assets.) It is evident from visual inspection of Figure 2 that the Box-Cox procedure tracks the actual prices extremely well.

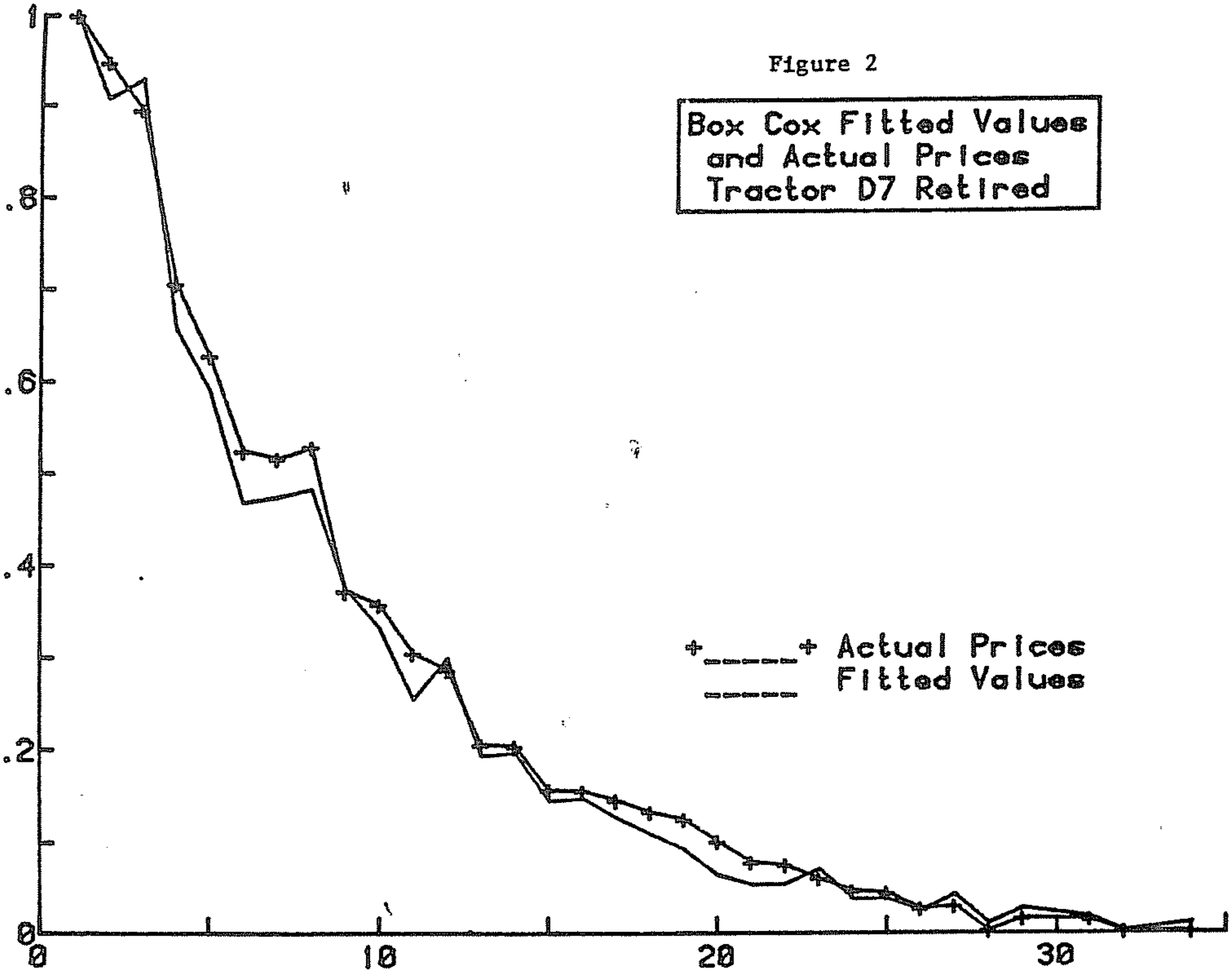

Figure 2

Box Cox Fitted Values
and Actual Prices
Tractor D7 Retired

Selected rates of depreciation for a sample of assets are shown in Tables 6A, B and C. Table 6A contains depreciation rates for producer durable equipment; Table 6B contains rates for private non-residential structures, and the rates pertaining to consumer automobiles and some figures for residential structures appear in Table 6C. The general thrust of these results are not implausible. Structures depreciate far slower than other assets with commercial and industrial structures depreciating at around 2-3% per year compared to residential structures which depreciate at around 1-1 1/2% per year. Consumer autos depreciate at around 20-25% per year whereas the producers' autos depreciate at a more rapid rate of around 30%. Tractors and trucks appear to be depreciating at around a 10-15% rate on average and office equipment and metal working machinery appear to have depreciation rates in the same range.

As noted above, retirement tends to increase the average annual rate of depreciation for all but very young assets. The depreciation processes are quite accelerated for structures, perhaps even more so than geometric whereas for automobiles and some types of producer durable equipment the depreciation rates appear to be flat or slightly decelerated vis-a-vis the geometric rate. Because there is some ambiguity as to the degree of acceleration of depreciation, we employed the Box-Cox method to calculate declines in efficiency. These efficiency functions are not illustrated until later in this report, however generally the efficiency functions are more accelerated than those produced by a semi-log (geometric) price-age pattern. In other words, our procedures indicate that a flexible functional form produces a depreciation process which is more accelerated than that produced by a direct geometric pattern.

It is helpful at this this point to draw a brief comparison between our estimates for the various asset classes and existing studies. Our new

Table 6A

SELECTED RATES OF DEPRECIATION FOR PRODUCER DURABLE EQUIPMENT BY AGE (UNCONSTRAINED BOX-COX MODEL)

	TRACTOR (D-7)			METALWORKING MACHINERY (MPG 12)	
Age	Unretired	Retired	Truncated	Unretired	Retired
1	15.3%	8.8%	12.2%	13.1%	19.8%
5	11.0	9.5	9.9	7.0	11.1
10	9.9	10.7	9.8	5.6	9.3
15	9.6	12.2	10.5	5.2	9.1
20	9.7	14.2	11.8	5.1	9.7
25	10.0	17.1	13.9	5.2	11.0

	CONSTRUCTION MACHINERY (LOADER)			OFFICE EQUIPMENT (TYPEWRITERS)	
Age	Unretired	Retired	Truncated	Unretired*	Retired
1	13.0%	4.8%	14.3%	—	—
5	11.5	8.3	10.1	3.8%	20.4%
10	10.8	11.3	10.8	5.8	25.6
15	10.4	13.9	14.8	7.8	27.4
20	10.2	16.7	30.6	10.2	27.4
25	10.0	19.7	—	13.0	26.6

*Sample did not contain observations on new assets.

	TRUCKS, BUSES, ETC. (TANDEM DUMP)			AUTOS (GSA FORD)	
Age	Unretired	Retired	Truncated	Unretired	Retired
1	22.8%	8.9%	21.7%	26.1%	34.0%
2	18.8	11.1	18.4	--	--
3	16.6	13.0	16.6	--	--
5	14.3	16.6	14.7	18.8	30.1
10	12.1	28.2	12.9	19.9	39.3
15	11.3	54.2	12.5	25.6	75.0

Table 3B

SELECTED RATES OF DEPRECIATION FOR PRIVATE NONRESIDENTIAL STRUCTURES (BOX-COX MODEL)[a]

	Class							
	Retail	Office	Warehouse	Factory	Retail	Office	Warehouse	Factory
	Box-Cox (Transformed)				Box-Cox (Untransformed)			
1[b]	3.54	4.32	5.57	3.02	5.39	5.72	6.81	3.00
5	2.77	2.85	3.68	2.99	2.41	2.66	3.23	2.02
10	2.47	2.64	3.05	3.01	1.63	1.84	2.26	1.68
15	2.32	2.43	2.74	3.04	1.29	1.48	1.83	1.50
20	2.22	2.30	2.55	3.07	1.09	1.27	1.57	1.39
30	2.10	2.15	2.32	3.15	0.86	1.02	1.27	1.25
40	2.03	2.08	2.19	3.24	0.73	0.88	1.10	1.17
50	1.99	2.04	2.11	3.34	0.64	0.79	0.98	1.11
60	1.96	2.02	2.05	3.45	0.57	0.72	0.90	1.06
70	1.94	2.02	2.01	3.57	0.53	0.66	0.83	1.03

[a] Percentage decline

[b] 1 denotes the age of a new asset.

Table 6C

SELECTED RATES OF DEPRECIATION FOR CONSUMER DURABLES

Automobiles

	BUICK			CHEVROLET (NOVA)		
Age	Unretired	Retired	Truncated	Unretired	Retired	Truncated
0	16.5%	15.8%	15.1%	19.0%	18.9%	15.5%
1	17.0	18.1	14.3	15.5	17.3	15.1
2	17.8	19.6	15.0	15.1	17.4	15.2
3	18.6	21.2	16.3	15.2	18.0	15.5
5	20.7	24.7	20.4	16.0	20.0	16.8
7	23.5	29.2	28.4	17.6	22.9	18.8

	PLYMOUTH			CHEVROLET STATION WAGON		
Age	Unretired	Retired	Truncated	Unretired	Retired	Truncated
0	23.2%	23.2%	23.4%	19.0%	18.4%	19.0%
1	20.9	20.9	19.4	18.3	19.6	18.3
2	21.1	21.1	19.0	18.6	20.7	18.6
3	21.8	21.8	19.2	19.1	21.8	19.1
5	24.2	24.2	20.8	20.5	24.5	20.5
7	28.0	28.0	23.4	22.4	27.8	22.4

Residential Structures[a]

	Rafael Weston		Wilhelmina Leigh (1950-1970)	
	Owner-Occupied	Tenant-Occ'd.	Unadjusted Starts	Adjusted Starts
Average	1.6%	1.5%	1.06%	.95%

[a] Leigh, Wilhelmina A., "Economic Depreciation for the Residential Housing Stock of the U.S., 1950-1970," Harvard University, Dept. of City and Regional Planning, March, 1979.

studies of automobiles and tractors follow very closely the results of existing studies by Wykoff, Ramm, Ackerman and Griliches. However, one important difference should be stressed: the retirement process has a substantial effect on the depreciation pattern for older assets. The retirement pattern used for automobiles was based upon actual registration figures from R.L. Polk & Co. Constructing an actual retirement pattern from these registration figures, and retiring the vintage prices accordingly, produced depreciation patterns which were larger on average than the depreciation rates reported in the literature.

Table 6B illustrates the depreciation rates produced by use of the Treasury study of commercial and industrial structures. Again, the structure depreciation patterns do not appear surprising and they are consistent across a wide variety of methodologies and a wide variety of assets. These results differ sharply, however, from earlier and rather weak econometric analysis by Taubman and Rasche in (1969). We have explained elsewhere why we disagree with the Taubman and Rasche results. Specifically, they dealt with only five data points and employed a methodology which imposed more stringent assumptions about the workings of the economy than were imposed here. Furthermore, their econometric testing procedure was of more limited flexibility than ours.

Finally, we also undertook analysis on machine tool data made available by the Office of Industrial Economics through the work of Professor Carl Beidleman. Our results are consistent with Beidleman's except for the fact that once one allows for retirement and once one uses a flexible estimation procedure such as the Box-Cox, the depreciation patterns on average are somewhat more rapid.

(2) Summary Results of Average Depreciation Rates for All Asset Classes

While the results presented for specific assets were based upon careful procedures and formal statistical hypothesis tests, the method of deriving average rates and representative efficiencies for broad classes of assets such as those listed in the BEA Statistical Tables involved considerable judgment and ad hoc method. We shall briefly summarize these ad hoc methods and judgmental procedures here and then present the estimates by asset class.

Our analysis begins with the Type A classes, from Table 2 above: PDE classes 4, 6, 8, 10, 16 and 17; PNS classes 1 and 2, and CD classes 1 and 6. All classes, except CD 6, were studied in detail either in Report TOS-74-27 or here. The average rates presented here were obtained by calculating the best geometric approximation to the predicted Box-Cox prices on retired data. These best geometric approximations (hereafter BGA rates) are the minimum variance averages to those presented in Table 6. These average rates are in Table 7. These rates are averages over the BGA rates of the specific assets in each class. The specific assets listed by class appeared in Table 5. The average BGA rates are our best judgments as to the average rates for these classes. As mentioned above, these estimates are based on considerable econometric research and they apply to depreciation of assets which comprise approximately 55% of the total stock of producer durable equipment, 42% of the total stock of private nonresidential structures and 66% of the total stock of consumer durable goods.

The remainder of the U.S. capital stock falls into either Type B or Type C asset categories. For assets in these latter categories, our estimates of the average rates are based more on judgment than on analysis. Neverthe-

Table 7

BGA DEPRECIATION RATES TYPE A ASSET CLASSES

Asset Class	BGA Rate
Producer Durable Equipment	
4 Tractors	16.3%
6 Construction Machinery	17.2
8 Metalworking Machinery	12.3
10 General industrial equipment	12.3
16 Trucks, bases and truck trailers	25.4
17 Automobiles	33.3
Private Nonresidential Structures	
1 Industrial	3.6
2 Commercial	2.5
Consumer Durables	
1 Motor vehicles and parts	27.3
6 Residential structures	1.3

less, we believe that the conventional treatment of these remaining assets should be modified. This conclusion follows from the relationship between our estimates of the Type A assets and the conventional treatment of the Type A assets and from existing studies of economic depreciation of other assets. In the case of Class B assets, we analyzed each asset case by case and brought into our judgment (1) ancillary studies undertaken by others, (2) the treatment of depreciation by BEA, Dale Jorgenson, BLS and Jack Faucett Associates, as well as (3) some judgmental analysis on our part. For the Type C assets in which we had no specific data available, we drew our inferences from similar assets within the Type A categories and from adjustments implied by our analysis to the conventional wisdom. These procedures and judgments are described in detail later in this report. The average BGA and judgmental depreciation rates for the Type B and Type C asset classes are presented in Table 8.

In order to appreciate the implications of our study for depreciation estimation, we present Tables 9A and B in which our depreciation estimates for each asset class are compared to four alternative treatments of depreciation. The first three alternatives are based upon asset lives used by a number of research institutions including Jack Faucett Associates, the Bureau of Labor Statistics, and Dale Jorgenson, Inc. The rates presented in Tables 9A and B are calculated by applying a double declining balance scheme in the first row, a 1.5 declining balance scheme in the second row, and in the third row a straight declining balance scheme, each applied to the lives given to us by Professor Jorgenson. Thus the first yardstick for comparison are rates based on <u>Bulletin F</u> lives. The fourth columns of Tables 9A and B represent rates implicit in the published figures on investment flows and capital stocks of the Bureau of Economic Analysis (BEA). We calculated these BEA implicit average rates, and we will discuss our procedures later. The final column of Tables 9A and B contains our estimates.

Table 8

BGA AND JUDGMENTAL DEPRECIATION RATES BY ASSET CLASS

TYPE B AND C ASSET CLASSES

Asset Class	Rate	Asset Class	Rate
Producer Durable Equipment		Private Nonresidential Structures	
1	11.00%	3	1.88%
2	9.17	4	1.88
3	7.86	5	1.88
5	9.71	6	2.90
7	16.50	7	3.16
9	10.31	8	2.37
11	27.29	9	5.63
12	16.50	10	2.90
13	11.79		
14	11.79	Consumer Durables	
15	11.79	2	10.00
18	18.33	3	15.00
19	7.50	4	15.00
20	6.60	5	15.00
21	14.73	7	15.00
22	14.73		

Table 9A

DEPRECIATION RATES BY ASSET CLASS

COMPARISON OF RESULTS

Class	Jorgenson Lives DDB	1.5 DB	DB	Implicit BEA (BGA)	Hulten-Wykoff (BGA)
PDE 1	.1333	.1000	.0667	.1092	.1100
2	.1111	.0833	.0556	.0803	.0917
3	.0952	.0714	.0476	.0646	.0786
4	.2500	.1875	.1250	.2564	.1633
5	.1176	.0882	.0588	.1516	.0971
6	.2222	.1667	.1111	.3388	.1722
7	.2000	.1500	.1000	.2118	.1650
8	.1250	.0938	.0625	.1300	.1225
9	.1250	.0938	.0625	.1424	.1031
10	.1429	.1071	.0714	.1676	.1225
11	.2500	.1875	.1250	.0330	.2729
12	.2000	.1500	.1000	.1311	.1650
13	.1429	.1071	.0714	.1565	.1179
14	.1429	.1071	.0714	.1565	.1179
15	.1429	.1071	.0714	.1565	.1179
16	.2941	.2206	.1471	.1298	.2537
17	.2941	.2206	.1471	.4057	.3333
18	.2222	.1667	.1111	.2276	.1833
19	.0909	.0682	.0455	.1078	.0750
20	.0800	.0600	.0400	.1362	.0660
21	.1818	.1364	.0909	.1282	.1473
22	.1818	.1364	.0909	.1748	.1473

Table 9B

COMPARISON OF RESULTS, Continued

Class		Jorgenson Lives			Implicit BEA	Hulten-Wykoff
		DDB	1.5 DB	DB	(BGA)	(BGA)
PNS	1	.0741	.0556	.0370	.0835	.0361
	2	.0556	.0417	.0278	.0409	.0247
	3	.0417	.0313	.0208	.0430	.0188
	4	.0417	.0313	.0208	.0430	.0188
	5	.0417	.0313	.0208	.0430	.0233
	6	.0645	.0484	.0323	.0640	.0454
	7	.0741 .0667	.0556 .0500	.0370 .0333	.1016 Chic. .0567 RR	.0316
	8	.0526	.0395	.0263	—	.0237
	9	.1250	.0938	.0625	—	.0563
	10	.0645	.0484	.0323	.0590	.0290

		Goldsmith		Flow of Funds		Hulten-Wykoff
		DDB	DB	DDB	DB	(BGA)
CD	1	.1333	.0667	.2500	.1250	.2725
	2	.1333	.0667	.2000	.1000	.1000
	3	.1667	.0833	.2500	.1250	.1500
	4	.2000	.1000	.2500	.2500	.1500
	5	.2000	.1000	.2500	.2500	.1500
	6	(.0110)				.0128
	7					.1500

With a few exceptions, BEA rates are more rapid compared to the corresponding rates of our analysis. Also, double declining balance, which has been so popular in econometric research, is too rapid. Thus, even though the depreciation patterns which we observed for all of our specific assets are accelerated, the rates are considerably less than the double-declining balance scheme popularly used in the tax code and in economic research. Unfortunately, the recent adjustments made by BEA actually tend to operate in the wrong direction. Specifically, BEA capital stock figures imply depreciation rates which are even more rapid than Jorgenson's double declining balance rates. The analysis of this report and our earlier research implies that economic depreciation is not as rapid as double declining balance of the Jorgenson lives. At the same time, the depreciation process for producer durable equipment is more rapid than 1.5 declining balance. We settled as our best estimate on a 1.65 declining balance scheme applied to the Bulletin F lives for the asset classes for which we had no independent information. In general, then, our analysis suggests that the appropriate average depreciation rate would be obtained by calculating a 1.65 declining balance method on the lives provided by Jorgenson.

In the case of structures our results are somewhat different. Again, the double declining balance method is too rapid and again the BEA estimates are even larger than the double declining balance method. Our study of industrial and commercial structures indicates that depreciation should be quite a bit slower than double declining balance. On average, our estimates for private nonresidential structures imply, if one were to use the Jorgenson lives, a declining balance scheme calculated at .9 rather than 2 times the declining balance rate. For the additional structure classes then, we imposed a depreciation method which was .9 declining balance on the Jorgenson asset lives.

Our treatment of consumer durables is based on the Flow of Funds Account of the Federal Reserve and by early capital stock studies of Raymond Goldsmith. We reproduce these as the conventional wisdom in Table 9B. (These sources were discussed in the Phase I Report.) In the case of automobiles our estimates are more rapid than either the Flow of Funds Accounts, calculated at a double declining balance method, or the Goldsmith rates. Since our estimates are based on far more actual information, it is evident that they should be employed. The central cause of these relatively rapid depreciation rates appears to be that the retirement process is combined with vintage price data.

In the case of private residential structures, we based our best professional judgment rate of 1.3% upon an average of four rates, two obtained by a study of Rafael Weston as part of his PhD. thesis at Harvard under Dale Jorgenson and two provided by Professor Wilhemina Leigh from her PhD. thesis studied under Professor Charles R. Hulten at Johns Hopkins University. These estimates are reasonably close to the only alternative estimate available, the earlier study by Goldsmith.

Later in this report we will discuss in more detail the methods used to derive the estimates of depreciation which appear in Table 9. Furthermore, we shall present the relative efficiency sequences for assets in addition to the BGA estimates of the average rates.

This concludes our summary discussion of the results of this Phase II study. In the next section, Part II of this report, we present in detail our theoretical, econometric, and empirical analysis of the specific assets.

In the following section, Part III, we discuss the development of the actual depreciation and efficiency function estimates based upon the detailed analysis described in Part II of this report.

II. THEORETICAL AND ECONOMETRIC ANALYSIS OF THE VINTAGE PRICES OF INDIVIDUAL ASSETS

In the pages which follow we present an overview of the general theoretical framework employed in the analysis of economic depreciation and asset efficiency. In the addendum to Phase I we presented the general theoretical model employed here and discussed extensively the econometric problems involved. Here we will briefly sketch those theoretical and econometric issues. Several new issues are brought to the fore in this report. First, we discuss at some length the role of capital taxes in the derivation of productive efficiencies from vintage prices. This analysis includes a discussion of the incidence of taxes implicit in the analysis of Harberger (1962), Jorgenson (1967), Stiglitz (1972), and Feldstein and Rothchild (1974). The objective of this discussion is to place in perspective our treatment of tax incidence in the construction of economic depreciation and relative efficiencies from vintage asset prices. We show that our treatment of taxes is consistent with that employed by Jorgenson and that our procedure for estimating depreciation from vintage prices rests on the basic notion of duality commonly employed in microeconomic theory.

Second, a number of very thorny and subtle econometric problems will be discussed. We comment briefly on the choice of flexible functional forms which were discussed in Phase I. We then introduce a new method for dealing with asset retirements. Because vintage prices represent only assets which have survived to a particular age, we employed an asset retirement pattern suggested by early studies of Robley Winfrey. However here we apply a new method based on the idea that the retirement process is stochastic, and that retirement strikes those assets which are least valuable (lowest in price) at any particular age. This "truncation" approach is discussed in detail in this section. We shall also comment on several other problems such as the method used to deflate vintage prices in calculating depreciation. The final conceptual contribution in this section is to illustrate the calculation of depreciation and asset inflation from the econometrically estimated vintage asset prices.

The final section of Part II consists of an example of the actual empirical analysis of individual assets. The full details comprise the appendix to this report. The appendix itself consists of over 1200 pages of econometric analysis of individual assets, organized by asset class. Those who wish to examine the econometric analysis in detail may do so with the use of this appendix. We turn now to discussion of the theoretical basis for estimating economic depreciation from vintage prices and for utilizing these estimates for purposes of tax analysis.

A. Taxes and the Relative Efficiency Function

In this section, we develop the model of capital prices with special emphasis on taxes and relative efficiencies. Beginning with the capital price theory of Hotelling (1925), Hall (1968) and Jorgenson (1973), we assume that in a world with no taxes, an optimizing capital user, operating with perfect certainty in an efficient and competitive capital market, will equate the purchase price (or acquisition cost) of a capital asset to the present discounted value of the future flow of user-costs (or service prices) on the asset:

$$1. \quad q(0,t) = \sum_{s=0}^{L} \frac{c(s,t+s)}{(1+r)^{s+1}}$$

where q(s,t) and c(s,t) denote the respective purchase price and user cost of an asset age-s in year-t, and where r is the constant discount rate and L is the asset's life.

Now assuming that the capital user is taxed on his income from production, eq. 1 must be extended to allow for the tax structure. Following Hall and Jorgenson in (1967) and in Fromm (1971), Feldstein and Rothschild (1974) and our own analysis in Hulten and Wykoff (1977), we shall assume that the tax falls on the capital user and that the rate of return is exogenously determined.

Since these assumptions are by no means widely accepted, and since a number of different models have been constructed to suggest that the incidence of a tax on the use of a particular type of capital may be shifted elsewhere, we shall discuss this assumption. Harberger in (1962) and Fromm in (1971) argued that a tax on one type of capital will lower its return relative to other rates, thus inducing resource re-allocation away from the taxed capital. This resource flow lowers returns on untaxed capital and raises returns on taxed capital, which diffuses the tax burden. Ballentine and Eris (1975) provide empirical support for the position that all capital bears the full burden of the tax.

Along a completely different line of analysis, Stiglitz in (1973) contends that capital users evade the tax burden altogether by resorting to debt finance. Debt finance, which generally enjoys tax deductible interest payments, avoids the tax on capital. This theoretical argument is by no means resolved however. King in (1974) shows, under assumptions slightly different from those of Stiglitz, that the cost of capital will change under some financing methods (use of retained earnings or new stock issues) but not others (debt finance). Both Stiglitz and King analyze incidence in a certain, partial equilibrium analysis setting. King points out, in (1973), that: "To discuss the incidence of the tax requires, in general, a fully-fledged general equilibrium model,..."[1] Furthermore, since, as King also notes, the "raison d'etre of incorporation and the advantages of limited liability, however, are to be seen only in a world of uncertainty,"[2] it would seem premature to accept an incidence conclusion based on debt finance in a certain world. The assumption that the capital user pays the tax would rule out his ability to shift the tax either by resource re-allocation or by debt finance.

The tax structure may be represented by the following symbols:

u: marginal tax rate

T_i: tax life for depreciation purposes under rule i

i = 1,2,3 where 1: Bulletin F lives
2: Revenue Procedure 62.21 lives
3: A.D.R. lives

$D_{ij}(s)$: tax depreciation deduction at age-s on an asset valued at $1.00 when new, given tax life T_i and depreciation method j,

j = 1,2,3 where 1: straight line
2: double declining balance
3: sum of years digits

Z_{ij}: present value of tax depreciation deductions on a new $1.00 asset given life T_i and method j.

$$Z_{ij} = \sum_{s=0}^{T_i} \frac{D_{ij}(s)}{[1+r(s)]^{s+1}}$$

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1. King (1974) p. 277.
2. King (1975) p. 279.

k: investment tax credit

α: proportion of k deducted in calculating depreciable basis.

For a capital user subject to tax rate u, given deductions initially valued at Z_{ij} and tax credit k, eq. 1 is:[3]

$$2.\quad q(0,t) = \frac{(1-u)}{[1-k-(1-\alpha k)\, uz_{ij}]} \sum_{s=0}^{L} \frac{c(s,t+s)}{(1+r)^{s+1}} .$$

The many changes which the tax code has undergone since 1954,[4] are summarized, for machinery and equipment, in Table 10. (See Hulten and Wykoff (1977) for corresponding rules on structures.)

Table 10

TAX PARAMETER VALUES 1952-1979

	u	i	j	k	α
Pre-1954	.52	1	1	0	-
1954-61	.52	1	3	0	-
1962-63	.52	2	3	.07	1
1964	.50	2	3	.07	0
1965-70	.48	2	3	.07	0
1971	.48	3	3	0	0
1972-77	.48	3	3	.10	0
1978	.46	3	3	.10	0
1979	.45	3	3	.10	0

Source: Sc. 1250, U.S. Tax Code.

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[3] See Hall and Jorgenson (1968)

[4] See Prentice-Hall (1972)

Setting $\alpha=0$ for convenience, eq. 2 may be written as:

$$3.\quad q(0,t) = \frac{(1-u)c(0,t)}{(1+r)} + \left\{k + \frac{uD_{ij}(0)}{1+r}\right\} q\,(0,t) + + \frac{(1-u)}{(1+r)} \sum_{s=1}^{L} \frac{c(s,t+s)}{(1+r)^{s}} + uZ_{ij}(1)\frac{q(0,t-1)}{(1+r)}$$

where $Z_{ij}(1) = \sum_{s=1}^{T_i} D_{ij}(s)/(1+r)^{s}$

In eq. 3 the price of new capital is decomposed into two parts. The first two terms on the right hand side of eq. 3 comprise the user-cost and tax liability on a new asset, and the second two terms, therefore, consist of the period-t present value of a one year old asset in the next period. Thus, we have:

$$4.\quad q(0,t) = \frac{(1-u)c(0,t) + \tau(0)q(0,t) + q(1,t+1)}{(1+r)}$$

where $\tau(0) = (1+r)k+uD_{ij}(0)$. Solving 4 for the user-cost of capital yields:

$$5.\quad c(0,t) = \frac{1}{(1-u)}\left\{rq(0,t) + q(0,t) - q(1,t+1) - \tau(0)q(0,t)\right\}$$

Eq. 5 depicts the user cost of new capital which is seen to depend upon the asset acquisition price when new $q(0,t)$ the rate of return, r, the price after the first period, $q(1,t+1)$ and the parameters u, k, $D_{ij}(0)$, of the tax structure. The user-cost can be estimated from data on $q(s,t)$ and r and details of tax law: u, k and $D_{ij}(s)$. A similar expression to eq. 5 for $c(s,t)$ may be derived as well.

The optimizer equates the marginal rate of substitution between various vintages of capital to the ratio of their user-costs. Letting $\phi(s,t)$ for $s=0,1,2,\ldots,L$, depict the marginal rates of substitution for age-s to new assets at time-t, we have:

$$6. \quad \phi(s,t) = \frac{c(s,t)}{c(0,t)} \qquad s=0,1,2,\ldots,L$$

or, using 5 and the corresponding expression for $c(s,t)$:

$$7. \quad \phi(s,t) = \frac{rq(s,t) + q(s,t) - q(s+1,t+1) - \tau(s)q(0,t-s)}{rq(0,t) + q(0,t) - q(1,t+1) - \tau(0)q(0,t)}$$

where $\tau(s) = uD_{ij}(s)$.

Jorgenson in (1973) calls $\phi(s,t)$ the asset <u>relative efficiency function</u>, because it represents the in-use productivity of an age-s asset relative to that of a new asset in period-t. Jorgenson's econometric analysis contains the assumption that $\phi(s,t)$ is stationary and geometric, i.e.,

$$8. \quad \phi(s,t) = \phi(s) = e^{-\delta s} \qquad s = 0,1,2,\ldots,\infty$$

where δ is the constant (geometric) rate of loss in productive efficiency.

In (1974) Feldstein and Rothschild argue that relative asset efficiency is neither geometric nor stationary. They present two analytic cases in which an optimizing durable goods <u>producer</u> is seen to alter his asset technology on the basis of changes in tax rules or in rates of return. In one case, producers alter the lives of one-hoss-shay assets and in another they alter the in-use productivity of perpetuities. In each case, they optimize the present value of the future stream of after-tax user-costs. If Feldstein and Rothschild are correct, then the capital-user pays the after-tax user-cost, $(1-u)c(0,t) + \tau(0)q(0,t)$, not $c(0,t)$. Eq. 7, employed by Hall and Jorgenson, assumes that the capital user pays $c(0,t)$. The questions of whether the relative efficiency function, $\phi(s,t)$ is stationary and geometric, within the context of Jorgenson's analytic framework, must be addressed empirically with the use of eq. 7; not by an alternative theoretical model.

Some have argued that the Jorgenson framework is hopelessly rigid in its assumptions. Robert Hall in (1977) argues that of all the controversial assumptions in Jorgenson's theory of optimal capital accumulation, however, only the one that $\phi(s,t)$ is geometric is essential. Hall's argument is as follows: a rational, well-informed decision to acquire a long-lifed asset requires that the producer act as if he knows the asset's relative productivity in the future as well as in the current period. But the productivity of the asset in the future, when say 3 years old, relative to that of a new asset 3 years hence will depend, in general, on the quantities of capital acquired in the intervening years as well as on the current level of acquisitions. Consequently, current and (all) future investment decisions to be made rationally must be made simultareously. To Hall the problem so framed "appears hopelessly complex."

The assumption that he perceives $\phi(s,t)$ to equal $e^{-\delta s}$ allows Jorgenson's optimizer, Hall argues, to ignore future investment levels in scheduling current investment as long as a positive level of gross investment is planned in each period. Thus, in Jorgenson's world, the choice of scheduling an investment either now or in the future depends only on the relation between the marginal value product of new capital and its user-cost. In other words, any errors made in terms of flows beyond the current period can be corrected in the future provided that future marginal rates of substitution are believed to be known <u>ex ante</u>.

Hall argues that future components of asset price can be ignored if $\phi(s,t)$ is geometric in the future. In fact, $\phi(s,t)$ need only be stationary, and any stationary schedule will do. Of course, $e^{-\delta s}$ is very simple analytically and greatly facilitates aggregation. It is not however essential. The Jorgenson investment story does depend though on $\phi(s,t)$ being perceived to be stationary.

Hall also argues that models not using <u>ex ante</u> stationarity of $\phi(s,t)$ are intractable. William Schwarm (1977), following Treadway (1969) and Brechling (1975), developed a model in which relative asset efficiencies are endogenously determined. Schworm's producer simultaneously sets maintenance requirements, utilization rates and the level of net new investment. Schworm's producer must be able to forecast prices far into the future and makes all present and future investment decisions simultaneously. Although possible, it seems somewhat implausible that producers take into account long forecasts of all future investment decisions in determining current acquisitions. Thus we agree with Hall's conclusion that, "as a practical matter, then, a model that assumes a simple predetermined relation between the future marginal values of different vintages seems a good guide for investment." We conclude this section by noting that the central assumption of Jorgenson's model, that $\phi(s,t) = \phi(s)$ for all t, requires examination within the context of Jorgenson's conceptual framework. The duality relation between the physical loss in productivity $\phi(s,t)$ and relative asset user-costs $c(s,t)/c(0,t)$, expressed in eq. 7 allows the study of $\phi(s,t)$ from vintage price data, however these assumptions are not needed in the calculation of economic depreciation.

B. Econometrics of Estimating Vintage Acquisition Prices

From a complete sample of asset vintage prices on a homogeneous class of assets, one can arrange a rectangular array by age and date of the prices.

From a rate of return and the tax structure as well, one can form an array $s \times t$ of user costs $c(s,t)$ and estimate relative efficiencies directly from the array. See for example Wykoff in (1970). Often, however, we do not have complete price arrays nor perfectly homogeneous assets. Producer durable equipment is quite specialized and various prices represent slightly different types of equipment. We must, therefore, rely on statistical inference to estimate the average pattern of vintage asset prices. That is, we can fit a regression plane to prices to obtain a set of fitted prices by age and date. These fitted prices may be combined with after-tax rates of return to produce relative asset efficiencies.

To avoid imposing *a priori* a specific form on the price-age pattern of assets, we employ flexible functional forms in our regression analysis. Two forms are used: The Box-Cox power transformation and the polynomial regression.

The Box-Cox power transformation, an intrinsically nonlinear procedure discussed at some length by Zarembka in (1974) and in Treasury Contract TOS-74-27, permits joint estimation of (a) parameters which determine a specific functional form within the Box-Cox class and (b) parameters which determine the slope and intercept. Since certain restrictions on the unknown form parameters produce one-hoss-shay, linear and geometric forms, one may employ classical hypothesis testing procedures to evaluate the suitability of these patterns. Estimation of the Box-Cox parameter is undertaken using a non-linear maximum likelihood procedure. Asymptotic likelihood ratio tests at 95% levels of significance are used on joint restrictions and asymptotic normal tests are used on individual parameter restrictions.

The polynomial regressions are intrinsically linear and include one-hoss-shay, linear and accelerated patterns as special cases. Linear estimation methods permit the addition of distinguishing characteristic variables in a straightforward way (i.e. by using multiple regression). The degree of polynomial, and hence the functional form, is determined by starting with fourth degree polynomials and deleting successive powers by age and year until the residual regression variance is minimized. This procedure produces the appropriate specification within the polynomial class. Attempts to compare the best polynomial form with semi-log forms, implying geometric decay, were undertaken as well.

(1) A Truncated Distribution to Allow for Scrappage

In section A of Part II above, the model of capital prices and user costs refers to the prices of individual assets. For several reasons, our interest is in groups, or cohorts, of assets, not in individual units. First, investment decisions ordinarily involve acquisitions of cohorts of assets and firms must consider the productive prospects of the average asset in the cohort not merely of one unit. Even if assets are homogeneous in terms of their built-in productivity, different units last for different lengths of time. Second, on a more pragmatic level, as we deal with vintage price data we have only prices of those vintage assets not yet retired. The average used asset price of the original cohort should reflect the prices of retired units as well as of the survivors. Thus to reflect the average price performance of the original cohort, used prices must be modified to allow for retirements.

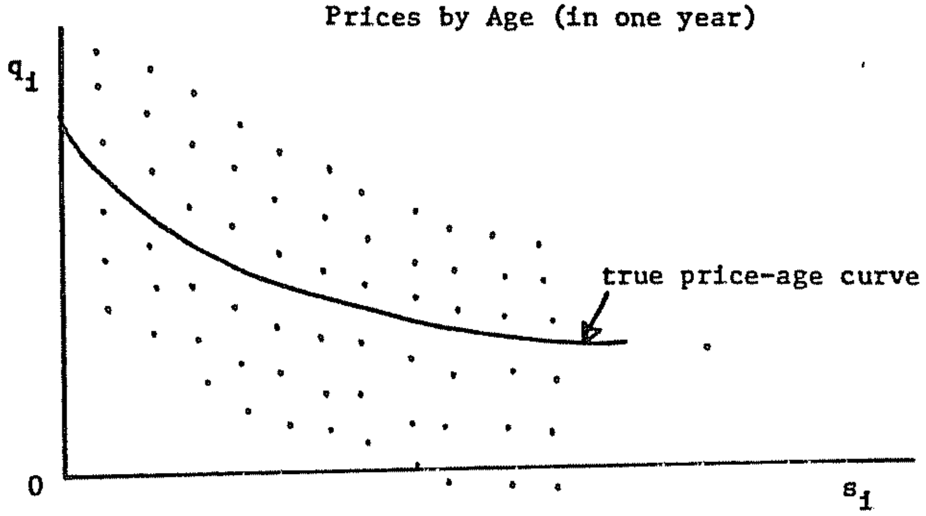
In (1977) we studied the retirement problem as an extension of the perfect certainty assumption of the Hotelling-Hall-Jorgenson model outlined above. Each asset in the cohort is assumed to be identical while in place, but each has a different, yet certain, retirement date. Thus all assets of a given cohort will have the same relative efficiency sequence $\phi(s,t)$ while in place, so that assets retired early are not assumed to deteriorate more rapidly than long-lifed assets. Furthermore, since all retirements are anticipated with perfect foresight, unintentional casualty losses and errors are ruled out. Under these assumptions, calculations of the average cohort used price merely corresponds to premultiplying observed vintage prices by their probability of having survived to that age. Given prices, adjusted for the retirement process, one can proceed to estimate the price-age performance of a cohort of assets using classical testing procedures. We implemented this method in TOS-74-27 using a retirement distribution from Marston et.al. (1952) on structures. The same approach is employed in this study of PDE and CD assets.

Here we introduce a new analysis of the retirement problem from a quite different perspective. Instead of viewing retirements of individual units as known with certainty, we assume the retirement process to be stochastic. This approach is suggested by the work on censored samples of Amemiya (1973) and Tobin (1958) and by the work on truncated samples of Berndt, Hall, Hall and Hausman (1974) and Hausman and Wise (1977).

We think of asset vintage prices as behaving as follows. (For convenience of exposition we think in terms of a specific asset: tractors.) Tractor prices fall with age because of wear and tear and obsolescence. At each age, however, the prices of individual tractor units will vary about the average price of the cohort due to differences in intensity of use, the variety of tasks performed, differences in policies with respect to maintenance and repair and so forth. A typical scatter diagram for a given class of tractors might look something like Figure 3 following.

Figure 3

Hypothetical Scatter of Tractor
Prices by Age (in one year)



In this simple example suppose the solid line depicts the true decline, on average, of tractor prices with age. The specification which is assumed to generate the scatter in Figure 3 is:

$$9. \quad q_i = Ae^{-\alpha S_i} e^{u_i}$$

where A and α are positive unknown constants, q_i is the price of tractor i, S_i the age of tractor i and u_i are independently distributed random variables assumed to be normal with mean zero and constant variance σ^2. (We assume this simple form for ease of exposition. Later, we will introduce flexible functional forms.) We would like to estimate $-\alpha$ from the scatter. $-\alpha$ is the true unknown parameter representing the percentage rate of price decline per year.

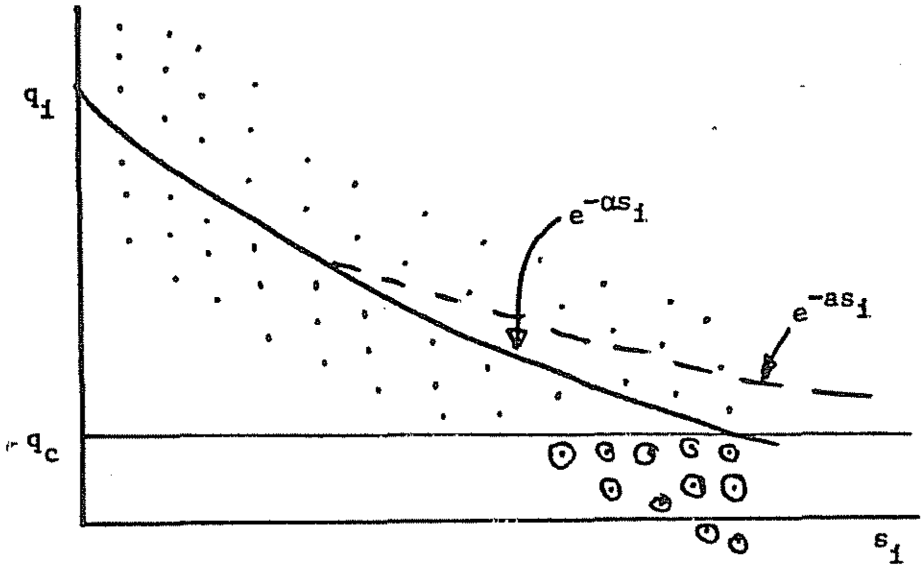
An actual scatter of points like those in Figure 3 is generated from a sample of vintage asset prices. In the case at hand, tractors, the prices are taken from public auctions of used equipment. Not all tractors are included in the sample—only used tractors available for resale enter the data base. We may ask how this sampling procedure might fail to represent all assets of a given vintage? One possible problem is that assets up for resale may differ systematically from those still in service but not re-sold. In (1970) Akerlof argues that vintage prices of automobiles taken from published Blue Books represent lemons which are not typical of the average used car. Peter Chinloy deals at length with the lemon problem in (1976). However, we agree with Heal (1976) that the lemon argument fails to account for the ability of buyers to assess the value of assets as well as sellers, and, thus, that this asymmetry in information flows is analytically inadequate. Furthermore, industry sources tell us that used producers assets are sold quite often for a variety of reasons quite unrelated to any problem associated with the units themselves so that the selection seems relatively neutral with respect to assets in service at the time.

However, another sampling problem of vintage prices does concern us. Assets which have been scrapped or withdrawn from service will have no chance of entering the sample. Once an asset's price falls below some critical level, it will cease to be of service--it is most likely scrapped for parts at this point. Thus a sample of vintage tractor prices, for example, will not include all the points shown in Figure 3. This exclusion of observations below some level of the dependent variable is analogous to a problem studied by Hausman and Wise (1977).

Figure 4 illustrates a situation in which assets (e.g., tractors) are assumed to have been scrapped when their price falls below some critical level q_c. The scatter points circled are not observed by the researcher, because these units have all been scrapped. If the researcher ignores the fact that some assets have been retired from service, then his estimator $-a$ of $-\alpha$ will be biased downward. He will understate the percentage rate of price decline on the average asset in the original cohort. His estimator is based only on prices of the best assets in the original cohort.

Figure 4

Hypothetical Scatter of Tractor
Prices by Age (in one year)
Effects on Data of Scrappage



The magnitude of the bias in $-a$ of $-\alpha$ can be shown to depend upon the scrappage level price q_c the true slope α, the variance of u_i, σ^2, and the age distribution of the sample S_i. See Hausman and Wise (1977) for details. The solution to the problem of dealing with a truncated sample is to view the stochastic terms u_i as having a truncated normal distribution. Before solving the problem we should point out that our specification of the true line is more complex than the simple semi-log form used for illustrative purposes above. Therefore we turn now to the flexible Box-Cox power transformation form.

Following our work in (1977), we assume that the form of the true price-age curve falls within the class of Box-Cox power functions:*

$$10. \quad q^*_i = \beta s^*_i + u_i$$

where $q^*_i = \dfrac{q_i^{\theta_1}-1}{\theta_1}$ and $s^*_i = \dfrac{s_i^{\theta_2}-1}{\theta_2}$

---

*We assume here for simplicity that all variables are variations from the mean.

where q_i and s_i are the price and age of tractor i respectively and where β, θ_1 and θ_2 are unknown parameters, β may be thought of as the "slope" parameter and (θ_1,θ_2) as the form parameters. If $(\theta_1,\theta_2) = (1,1)$ then the price-age curve is linear; and if $(\theta_1,\theta_2) \to (0,0)$, then the price-age curve becomes log-linear. Finally, $(\theta_1,\theta_2) \to (0,1)$ implies the semi-log form used in the above example.

We are now prepared to deal with the exclusion of non-survivors in the sample. We assume s_i to be non-stochastic and u_i to be a truncated normal distribution. The normal distribution of the u_i has a mean of zero and a constant variance σ^2. Thus the distribution of q^*_i is normal mean βs^*_i variance σ^2 and is truncated at price q^*_{ci} or, visually:

Figure 5

Truncated Distribution of q^*_i

$q^*_i \sim N(\beta s^*_i, \sigma^2)$

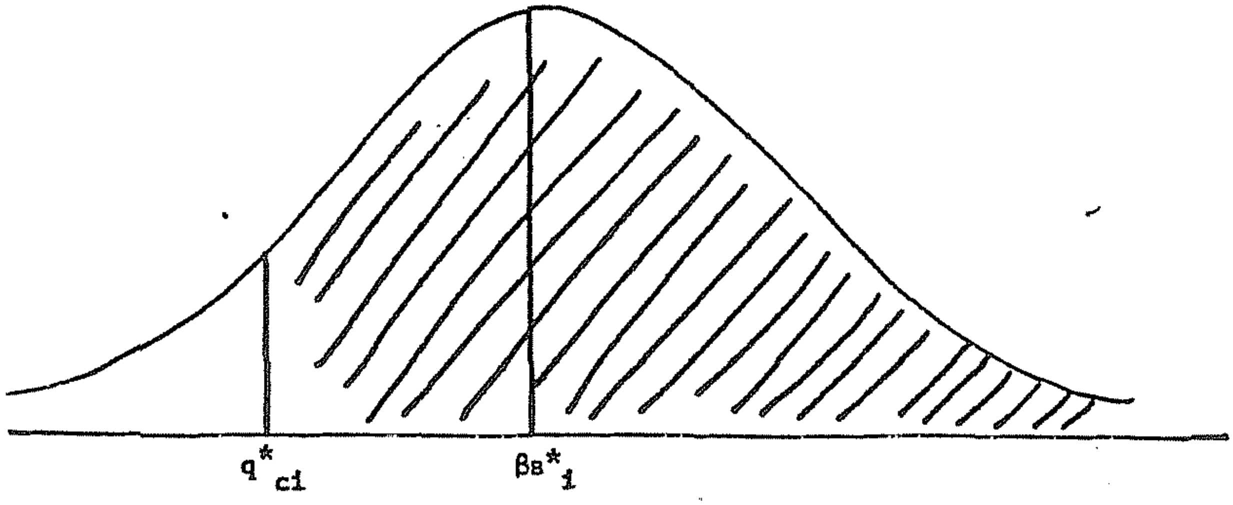


Figure 5 implies that at any given age s_i, tractor prices will be observed to be distributed normally about a value βs^*_i but that prices below some critical value Qc will not be observed because these tractors will have been scrapped. The truncation value in Figure 5 is $q^*_c = (q_c^{\theta_1}-1)/\theta_1$. Recall that the q_i are deviations from the mean, so that $q_c = (Q_c - \bar{Q})$ where $\bar{Q}$ is the average tractor price, therefore

$$11. \quad q^*_{ci} = \frac{(Q_c - \bar{Q})^{\theta_1} - 1}{\theta_1}$$

For estimation purposes we must study the distribution of the u_i. Since $u_i = q^*_i - \beta s^*_i$ from eq. 10, u_i is truncated at $u_{ci} = q^*_{ci} - \beta s^*_i$, therefore,

$$12. \quad u_{ci} = \frac{(Q_c - \bar{Q}) \quad -1}{\theta_1} - \frac{(S_i - \bar{S}) \quad -1}{\theta_2}$$

From $u_i = q^*_i - \beta s^*_i$, we may calculate the change in u_i from a given a change in q_i as:

$$13. \quad \frac{du_i}{dq_i} = q_i^{\theta_1 - 1}$$

The likelihood function for the sample of observations $q_1, q_2, \ldots, q_n$ is:

$$14. \quad L(q_1, q_2, \ldots, q_n) = \prod_{i=1}^{n} f(q_i) \text{ where}$$

$$f(q_i) = \left| \frac{du_i}{dq_i} \right| f(u_i)$$

or following Zarembka (1974):

$$15. \quad L(\cdot) = \prod_{i=1}^{n} q_i^{\theta_1 - 1} f(u_i)$$

Our next problem is to determine the frequency function of the truncated normal u_i. Let $F(u_i)$ be defined as the cumulative distribution function for each value of u_i. That is, $F(u_i)$ is the area to the left of u_i remaining in the truncated distribution:

16. $$F(u_i) = \begin{cases} Pr[U_i \leq u_i : U_i > u_{ci}] & \text{for } u_i > u_{ci} \\ 0 & \text{for } u_i \leq u_{ci} \end{cases}$$

or

17. $$F(u_i) = \begin{cases} Pr[u_{ci} < U_i \leq u_i] / Pr[U_i > u_{ci}] & \text{for } u_i > u_{ci} \\ 0 & \text{otherwise} \end{cases}$$

The conditional density function $f(u_i)$ can be calculated from F because $f(u_i) = F'(u_i)$:

18. $$f(u_i) = \begin{cases} g(u_i) / \int_{u_{ci}}^{\infty} g(u_i)du_i & \text{for } u_i > uc_i \\ 0 & \text{otherwise} \end{cases}$$

Where $g(u_i)$ is a normal density function with mean 0 and variance σ^2:

19. $$g(u_i) = \frac{1}{\sqrt{2\pi\sigma}} \text{ exp. } -1/2[u_i/\sigma]^2$$

where exp. is the exponential, $u_i = q^*_i - \beta s^*_i$, and where $\int_{u_{ci}}^{\infty} g(u_i)du_i$ is a standardized unit normal distribution function $G[u_{ci}/\sigma]$ and u_{ci} is given in eq. 12 above.

Eq. 15, the likelihood function, may be written as:

20. $$L(\cdot) = \prod_{i=1}^{n} q_i^{\theta i-1} \frac{g(u_i)}{G(\cdot)}$$

The corresponding log-likelihood function becomes:

21. $$\mathcal{L} = \ln L(\cdot) = (\theta_i - 1) \sum_{i=1}^{n} \ln q_i - \eta \ln \sigma \sqrt{2\Pi}$$

$$-1/2 \sum_{i=1}^{n} [(q^*_i - \beta s^*_i)\sigma]^2$$

$$- \sum_{i=1}^{n} \ln G[u_{ci}/\sigma]$$

Maximum likelihood estimators of θ_1, θ_2, and σ^2 may be obtained from maximization of eq. 21 using procedures set out in Berndt, Hall, Hall and Hausman (1974). Similar log likelihood functions can be constructed for different specifications of the true price-age curve than the Box-Cox forms in eq. 10. Before presenting the data and empirical results, we shall now briefly summarize our treatment of three econometric problems: (1) the choice of functional forms, (2) the treatment of asset retirements, and (3) the treatment of capital gains:

(2) Econometric Problems Summarized:

The first problem in estimating the price-age pattern of capital is to specify a model flexible enough to determine the patterns from the evidence rather than to use a predetermined functional form which restricts the shape of the price-age curve *a priori*. In (1977), we applied two flexible functional forms, the polynomial regression and the Box-Cox power transformation, as well as two more conventional functional forms, linear and semi-log, to vintage prices of structures. The latter two specifications each represent a commonly assumed price-age pattern, straight-line price-age curve against a higher order polynomial alternative using a straight forward f-test.

The semi-log form can also be compared to the polynomial using a test suggested by Theil (1971). However, the results of the Theil test are usually ambiguous. As noted above, the Box-Cox power transformation includes both the semi-log and linear forms as special cases and again one can test these restrictions using classical hypothesis testing procedures.

Table 11 depicts the four specifications to be studied. In (1977) we show that the Box-Cox and polynomial forms themselves are members of a more general class of Box-Cox functional forms. However, as a practical matter, the general form is too complex for conventional non-linear estimation procedures, so that we are unable to discriminate statistically between the Box-Cox and polynomial forms.

Table 11

Specifications for Empirical Work

LINEAR:

$$q_i = \alpha+\beta S_i+\gamma t_i+u_i \qquad i=1,2,\ldots,n$$

SEMI-LOG:

$$\ln q_i = \alpha+\beta S_i+\gamma t_i+u_i \qquad i=1,2,\ldots,n$$

POLYNOMIAL:

$$q_i = \alpha+\beta_1 S_i+\beta_2 S_i^2+\beta_3 S_i^3+\beta_4 S_i^4+\gamma_1 t_i+\gamma_2 t_i^2+\gamma_3 t_i^3+\gamma_4 t_i^4+u_i$$

$$i=1,2,\ldots,n$$

BOX-COX:

$$q^*_i = \alpha+\beta S^*_i+\gamma t^*_i+u_i \qquad i=1,2,\ldots,n$$

where

$$q^*_i = \frac{q_i^{\theta_1}-1}{\theta_1}, \quad S^*_i = \frac{S_i^{\theta_2}-1}{\theta_2}, \quad t^*_i = \frac{t_i^{\theta_3}-1}{\theta_3}$$

q_i: auction price of asset i (either adjusted or non-adjusted for retirements

S_i: age of asset i at auction

t_i: year of auction on asset i

u_i: random disturbance term assumed to be $N(o,\sigma^2 I)$

$\alpha,\beta,\gamma,\beta_j,\gamma_j,\theta_j$: unknown parameters

A second problem in estimating price-age curves of used assets is to allow for asset retirements. As noted above, one approach is to treat retirements as non-stochastic. Observed used-asset prices are pre-multiplied by a survivor probability based on a retirement distribution for the particular class of assets. The result is essentially a new set of vintage prices—each price in the old set pre-multiplied by the probability of having survived to that age. This approach as shown in (1977) implies that scrapped assets were worth the same as survivors when scrapped. This implication is not terribly plausible for some assets such as tractors. Those scrapped are likely to be worth considerably less than survivors. At one extreme scrapped assets are worth zero. If scrapped assets are worth nil, then, as shown in (1977), estimation can proceed on observed prices without explicit regard for scrappage, i.e., the appropriate procedure is to use unretired data. Here we report on estimation results using unadjusted prices, retirement adjusted prices, and the truncated distribution approach outlined here.

A third problem warranting comment involves the treatment of inflation. The equations in Table 11 imply that asset prices depend upon ages and upon date. The inclusion of date reflects the fact that, given age, asset prices vary from year to year as a result of inflation and possible intertemporal shifts in supply or demand. However, the forms in Table 11 may not be an entirely satisfactory way of treating these capital gains and losses. See, for example, Taubman's remarks on Hulten and Wykoff in (1977). Consequently, we select a price-deflator relevant to producers durable equipment to deflate asset prices _a priori_ in order to capture the general inflation aspect of the capital gain phenomenon and present results for deflated and undeflated data.

(3) Calculation of Depreciation and Revaluation from Estimated Vintage Prices:

The year-to-year changes in the present value of an asset can be decomposed into an effect due to increasing age and an effect due to the passing of time. Formally,

$$22. \quad q(s,t) - q(s+1, t+1) = [q(s,t) - q(s+1,t)] + [q(s+1,t) - q(s+1,t+1)]$$

Equation 22 indicates that the difference between the precent value of an s year old asset in year t and its value in the following year, when it is an s+1 year old asset, can be thought of as (a) the difference between the present value q(s,t) and the value of s+1 year old asset in the same year, and (b) the difference between the price of an s+1 year old asset in year t and t+1 respectively. The effect (a) is <u>economic depreciation</u>, the decline in asset price due to age. The effect (b) is <u>revaluation</u>, the capital gain or loss due to other inflationary factors influencing the trend of asset prices.

Figure 6

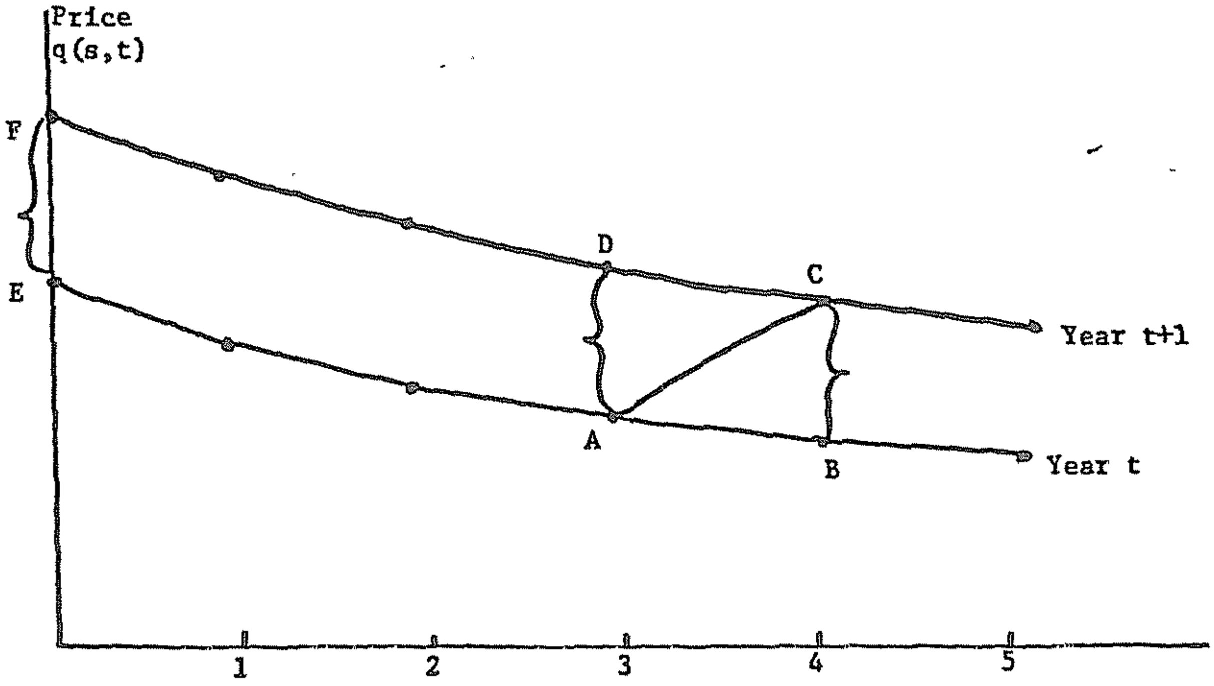


Figure 6 portrays the effects described by eq. 22. The average value of vintage assets in year t is described by the curve EAB, while the average value of assets in year t+1 is FDC. The curve FDC is drawn above EAB to indicate that asset inflation has occurred between t and t+1, and is skewed to indicate that age-price relationship may not be uniformly affected by the general revaluation of assets. The value of a three year old asset in year t is denoted by the point A; in year t+1, this same asset is located at point C. Equation 22 indicates that the change in asset price is composed of economic depreciation, the movement along the curve EAB from A to B, and asset revaluation, the shift in the curve from B to C. As drawn, Figure 6 shows that revaluation outweighed depreciation, and that the price of the asset actually increased despite the downward effect due to depreciation.

What factors determine the shape of the curves EAB and FDC? This can be determined by substituting the present value eq. 2 into the price decomposition eq. 22. Changes in asset values can then be seen to depend on changes in the expected quasi-rent, the expected life, the expected rate of discount, and the expected tax treatment of the asset. The implicit rent will, in general, decline with age because of deterioration in the form of "output decay" and "input decay", to use the terminology of Feldstein and Rothschild (1974). Output decay results when the machine generates less output due to deteriorated condition or more "down time," and input decay refers to the need to use more inputs of labor, materials, and maintenance to maintain the same flow of output. Deterioration and impending retirement will, in general, cause the curves EAB and FDC to slope downward from left to right. Obsolescence, less generous tax benefits, and increased uncertainty about future income and costs can also contribute to the downward sloping pattern. The year-to-year shift in age-price curves results from inflation, changes in expectations, changes in the discount rate and tax treatment, and changes in optimal utilization and economic life.

Whatever the factors influencing the age-price patterns of assets, the important point is that these patterns can be observed for certain types of assets. Curves like EAB and FDC can in principle be constructed for each year and used to evaluate the actual experience of that class of assets. Average rates of depreciation can then be calculated which can be compared with the corresponding tax treatment, and issues like the reasonableness of accelerated depreciation can be evaluated. The observed age-price patterns can also be used as a framework for judging the reasonableness of various methods of revaluing assets for depreciation purposes. And, since the age-price patterns are based on direct observations of vintage asset prices, this approach does not depend on assumptions about how the vintage prices are actually determined (for example, whether the present value formulation of eq. 1 is in fact the way used prices are actually formed).

C. Illustrative Results for D-7 Tractor

We have applied the methodology outlined in the preceding sections to a wide variety of assets. In addition to several classes of commercial and industrial structures studied in an earlier report, we present here new analysis applied to twenty-six assets which fall in the classes of producer durable equipment and consumer durables. The full statistical and econometric detail appears in the appendix where the evidence is listed by asset categories beginning with Producer Durable Equipment class 4—Tractors, and ending with Consumer Durable class 6--Residential Structures.

The bulk of the analysis applies to the Type A asset classes (referred to in the introduction to this report). In this section we provide a comparatively detailed description of the analysis of one subclass of assets, the D-7 Tractor.

The nature of the D-7 Tractor data was outlined in the introductory section earlier. This section contains only the econometric results for estimation of depreciation and calculations of efficiency sequences for the D-7 Tractor.

Table 12 gives parameter estimates for the case in which the sample prices are weighted by the probability of survival, i.e., retired prices. The first two lines show the results of the linear and semi-log regression, while the next three lines give the maximum likelihood estimates for the Box-Cox model, with the most general form of the model appearing on line 5. The polynomial results are shown on the bottom line. The standard errors, given in parentheses, indicate that all estimates are significant at conventional levels. The estimates of α, β and γ also have the expected sign: negative for the depreciation parameter, β.

Table 13 provides test statistics for determining the most likely functional form. $\lambda = -2[\text{Log}\ (\hat{\omega}) - \text{Log}\ (\hat{\Omega})]$ is approximately chi-square for large N when the null hypothesis is true, the results of Table 13 indicate that all constraints failed to be accepted at the 95% level of significance. This implies that neither the linear nor geometric forms are likely to have generated the observed sample. The case against these two depreciation patterns is strengthened by the fact that the asymptotic normal test of the unconstrained estimate of θ_1 cannot accept the hypothesis that $\theta_1 = 1$ or that $\theta_1 \to 0$. This implies that the dependent variable is neither linear nor logarithmic.

Table 12

TRACTOR

(Model D-7)

Undeflated and Retired Price Data

(n=582)

	θ_1	θ_2	θ_3	α	β	γ	Log L
Linear	1	1	1	21907.6 (685.6)	-1543.4 (39.4)	2305.3 (92.6)	-5865.4
Semi-Log	0	1	1	10.203 (.040)	-.141 (.002)	.140 (.005)	-5592.3
Box-Cox							
2 Constraints	.455 (.004)	.455 (.004)	.455 (.004)	247.83 (8.917)	-32.406 (1.142)	23.219 (1.161)	-5567.6
1 Constraint	.212 (.004)	1.077 (.044)	1.077 (.044)	35.102 (1.094)	-.781 (.098)	.952 (.085)	-5486.0
0 Constraint	.232 (.004)	.998 (.046)	1.296 (.131)	41.278 (1.397)	-1.147 (.148)	.809 (.179)	-5483.2

Polynomial Equation

α	β_1	β_2	β_3	γ_1	γ_2	γ_3
45594.3	-4737.0	186.17	-2.818	-5097.3	1475.2	-86.95

Table 13

TRACTOR (D-7)

Table : Box-Cox Hypothesis Tests

(Likelihood Ratio Tests)

$\lambda(\omega) = -2\{L^*(\omega) - L^*(\Omega)\}$[1]

Constraints		Log Likelihoods				
	$\omega \subset \Omega$	$L^*(\omega)$	$L^*(\Omega)$	λ	n	$\chi^2(n)$
I	$\theta_2 = \theta_3$	-5486.0	-5483.2	5.6	1	3.84
II	$\theta_1 = \theta_2 = \theta_3$	-5567.6	-5483.2	168.8	2	5.99
III	$\theta_1 \rightarrow 0 \quad \theta_2 = \theta_3 = 1$	-5592.3	-5483.2	218.2	3	7.81
IV	$\theta_1 = \theta_2 = \theta_3 = 1$	-5865.4	-5483.2	764.4	3	7.81

---

[1] The terms are defined in the Table for D-4 Tractors.

What shape, then, do the unconstrained Box-Cox estimates imply? This can be determined by investigating the first and second order partial derivatives of the non-stochastic part of the Box-Cox functional form (eqs. 1 and 2): The partial derivatives are, respectively:

$$\frac{\partial q}{\partial s} = \beta q^{1-\theta_1} s^{\theta_2 - 1} = -1.147\, q^{0.768} s^{-.002} < 0$$

and

$$\frac{\partial^2 q}{\partial s^2} = \frac{\theta_2 - 1}{s} \frac{\partial q}{\partial s} + \frac{1-\theta_1}{q} \left[\frac{\partial q}{\partial s}\right]^2 = -\frac{0.002}{s} \frac{\partial q}{\partial s} + \frac{0.768}{q} \left[\frac{\partial q}{\partial s}\right]^2 > 0$$

A uniformly negative first order partial derivative and a uniformly positive second order partial derivative indicates that Box-Cox age-price pattern is strictly convex. Thus, while the age-price pattern is apparently not geometric, it does have the same general shape. This result is eminently reasonable in view of the actual age-price pattern shown in Figures 1 and 2 of the introduction.

Table 14 compares the parameters of the unconstrained Box-Cox model estimated under different assumptions about the retirement process. The estimates vary across retirement assumptions. It is plausible for the unretired case to differ from the other cases, but the divergence between the truncated and survival-weighting approaches is relatively modest.

Finally, the test of the polynomial regression against the semi-log regression proved ambiguous. Since, however, the unconstrained Box-Cox likelihood was always larger than either the geometric or the polynomial, the Box-Cox model appears preferable for this class of assets.

We have applied the methods described in the preceding sections to a wide variety of assets: commercial and industrial buildings, automobiles,

Table 14

TRACTOR

(Model D-7)

Comparison of Transformed Prices to Truncation Form

Unconstrained Box-Cox (Undeflated Prices)

	Form Parameters			Slope Parameters			
Form	θ_1	θ_2	θ_3	α	β	γ	Log L
Unretired	.217 (.004)	.640 (.046)	1.266 (.128)	38.714 (1.31)	-2.082 (.254)	.758 (.163)	-5519.2
Retired	.232 (.004)	.998 (.046)	1.296 (.131)	41.278 (1.397)	-1.147 (.148)	.809 (.179)	-5483.2
Truncated*	.344 (.004)	.713 (.048)	1.242 (.124)	96.404 (3.994)	-6.213 (.812)	2.799 (.579)	-5473.4

*q_{ci} = \$2047 (Taken from min $\{q_i: i = 1, \ldots, 582\}$)

trucks, machine tools, construction equipment, and typewriters. Our general findings are roughly consistent with the results reported for D-7 tractors. Geometric and straight-line are almost uniformly rejected, and the estimated age-price patterns are almost always accelerated relative to straight-line.

The final step in studying depreciation and productive efficiency of an individual asset is to construct the relative efficiency function from the estimated vintage asset prices. It will be recalled that productive efficiencies of physical assets are derived from vintage prices by employing the concepts of the user cost of capital and of duality. Duality establishes the linkage between the marginal rates of substitution of two pieces of capital and the ratio of their respective user costs. We constructed the user cost formulas utilizing the theoretical model outlined earlier in Section A of Part II of this report. We normalized the efficiency functions on the user cost of a new asset. In order to compare the efficiency sequences produced by the Box-Cox prices, we plotted the efficiency function derived from the Box-Cox approach and in the same graph an efficiency function derived from a semi-log equation. These graphs appear in the appendix. Figure 7 produces this figure for the D-7 tractor utilizing retired data.

The actual efficiency sequences calculated for every year in which data exists for each asset we studied appear in the appendix as well. Table 15 contains the annual Box-Cox efficiency sequences based on retired data for the D-7 Tractor from 1968 through 1977. Because it may be necessary eventually to construct capital stock estimates outside the sample period, we have produced the average Box-Cox efficiency sequency over the years. Table 15 also contains the average Box-Cox efficiencies ages new to 32. By comparison, the efficiency functions one would obtain by assuming a constant geometric rate derived from the semi-log equations (see Table 12

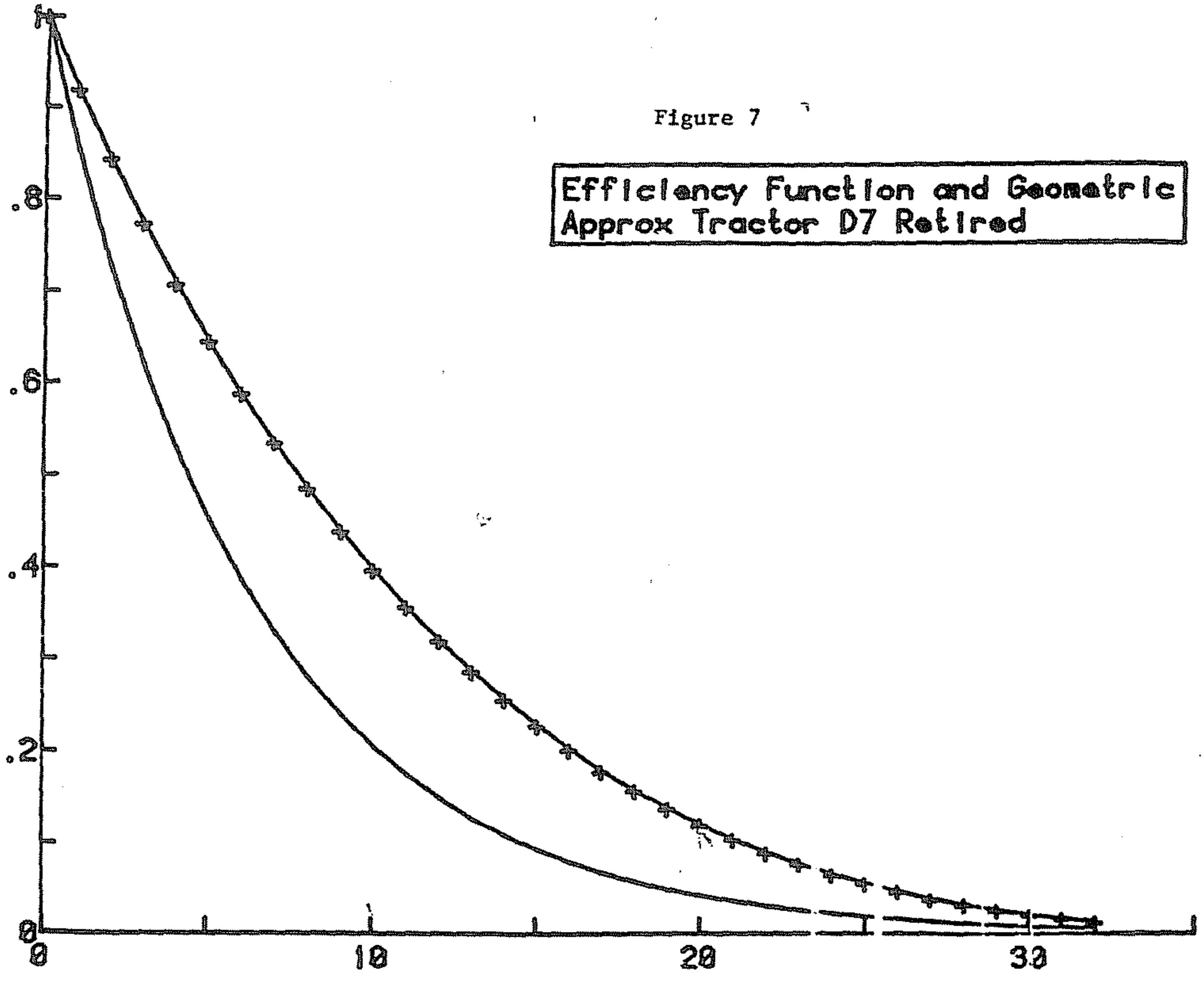


Figure 7

Efficiency Function and Geometric Approx Tractor D7 Retired

Table 15

BOX-COX EFFICIENCIES

Asset: Tractor D-7 Years: 1968-1977 (1975-1977)
Retired Prices

Age	1975	1976	1977	Box-Cox	Semi-Log
0	1	1	1	1	1
1	0.92322	0.92573	0.92738	0.91841	0.853
2	0.85199	0.85676	0.85985	0.84299	0.72761
3	0.78518	0.79193	0.79631	0.77256	0.62065
4	0.72249	0.73095	0.73645	0.70675	0.52941
5	0.66368	0.67361	0.68008	0.64531	0.45159
6	0.60856	0.61972	0.62704	0.58801	0.38521
7	0.55696	0.56914	0.57717	0.53464	0.32858
8	0.50872	0.52172	0.53034	0.48501	0.28028
9	0.46367	0.47731	0.48641	0.43893	0.23908
0	0.42168	0.43578	0.44526	0.39622	0.20393
1	0.3826	0.39701	0.40676	0.35671	0.17396
2	0.34628	0.36085	0.3708	0.32023	0.14838
3	0.31259	0.3272	0.33726	0.28662	0.12657
4	0.2814	0.29594	0.30603	0.25573	0.10797
5	0.25258	0.26694	0.277	0.2274	0.092095
6	0.22602	0.2401	0.25006	0.20148	0.078557
7	0.20158	0.2153	0.22511	0.17784	0.067009
8	0.17915	0.19245	0.20205	0.15634	0.057159
9	0.15863	0.17144	0.18079	0.13685	0.048756
20	0.1399	0.15217	0.16122	0.11923	0.041589
21	0.12285	0.13454	0.14326	0.10336	0.035476
22	0.10738	0.11846	0.12682	0.089131	0.030261
23	0.093399	0.10383	0.11182	0.076418	0.025812
24	0.080803	0.090577	0.098155	0.065113	0.022018
25	0.069501	0.078604	0.085762	0.055108	0.018781
26	0.059403	0.067832	0.074556	0.046302	0.01602
27	0.050424	0.05818	0.064462	0.038596	0.013665
28	0.042481	0.04957	0.055406	0.031895	0.011657
29	0.035492	0.041928	0.047316	0.02611	0.0099431
30	0.029382	0.035181	0.040124	0.021154	0.0084815
31	0.024075	0.029259	0.033763	0.016945	0.0072347
32	0.019503	0.024096	0.02817	0.013405	0.0061712

Table 15

BOX-COX EFFICIENCIES

Asset: Tractor D-7 (Con't.) Years: 1968-1974
Retired Prices

1968	1969	1970	1971	1972	1973	1974
1	1	1	1	1	1	1
0.9108	0.91106	0.91287	0.91651	0.91872	0.91834	0.91949
0.82879	0.8292	0.83259	0.83947	0.84363	0.84278	0.84486
0.75266	0.75322	0.75795	0.7676	0.77345	0.77221	0.7751
0.68196	0.68267	0.68854	0.70052	0.70781	0.70627	0.70988
0.61639	0.61726	0.62406	0.63795	0.64645	0.64472	0.64894
0.55565	0.55669	0.56425	0.57966	0.58917	0.58732	0.59206
0.4995	0.50069	0.50885	0.52544	0.53575	0.53386	0.53906
0.44768	0.44903	0.45764	0.47509	0.48602	0.48416	0.48972
0.39996	0.40147	0.41039	0.4284	0.43979	0.43801	0.44387
0.35612	0.35777	0.36688	0.3852	0.3969	0.39525	0.40134
0.31594	0.31772	0.32691	0.34531	0.35718	0.3557	0.36195
0.27921	0.28111	0.29028	0.30856	0.32046	0.31919	0.32555
0.24573	0.24774	0.2568	0.27477	0.28661	0.28555	0.29197
0.21531	0.2174	0.22627	0.24378	0.25546	0.25463	0.26106
0.18775	0.18991	0.19853	0.21545	0.22687	0.22629	0.23267
0.16288	0.16508	0.17339	0.18961	0.2007	0.20036	0.20665
0.14051	0.14274	0.15069	0.16611	0.17681	0.17671	0.18288
0.12048	0.12272	0.13026	0.14483	0.15507	0.1552	0.16122
0.10262	0.10485	0.11196	0.12561	0.13535	0.1357	0.14153
0.086782	0.088979	0.095628	0.10832	0.11752	0.11807	0.1237
0.072804	0.074948	0.081122	0.092835	0.10147	0.10221	0.10759
0.060542	0.062616	0.068303	0.07903	0.087083	0.087975	0.093104
0.049856	0.051842	0.057038	0.066782	0.074236	0.075264	0.080119
0.040611	0.042493	0.047199	0.055975	0.062823	0.063961	0.06853
0.032677	0.034441	0.038665	0.046497	0.05274	0.053962	0.058234
0.02593	0.027564	0.031317	0.038239	0.043883	0.045163	0.04913
0.020253	0.021747	0.025045	0.031098	0.036155	0.037467	0.041125
0.015532	0.016879	0.019742	0.024974	0.029461	0.03078	0.034128
0.01166	0.012855	0.015306	0.019771	0.02371	0.025011	0.02805
0.0085354	0.0095763	0.011643	0.015398	0.018814	0.020076	0.02281
0.0060624	0.0069496	0.0086606	0.011767	0.014691	0.015893	0.018327
0.0041499	0.0048872	0.0062738	0.0087955	0.01126	0.012383	0.014528

for the semi-log coefficients) also appear in Table 15. This completes our discussion of the econometric analysis of the D-7 Tractor.

A similar set of procedures was undertaken for each of the 26 assets studied in detail. All of these results appear in the appendix to this report. Each asset group is organized by its asset classification. Preceding each asset class's actual econometric results is a brief outline of the information available on that particular asset. This should make it more convenient to study the actual analysis of the assets. We turn now to the problem of deriving from these individual asset studies estimates of depreciation for the entire U.S. capital stock.

III. DEPRECIATION RATES AND EFFICIENCY FUNCTIONS FOR THE U.S. CAPITAL STOCK: METHODOLOGY AND RESULTS

The objective of this section is to produce our best professional estimates of economic depreciation and relative productive efficiencies for the U.S. capital stock organized into 22 PDE, 10 PNR and [illegible] CD classes. The final estimates are built upon three types of information: (1) our detailed econometric investigation of the 30 specific capital assets. The econometric results appear in two sources: (a) The results for PDE classes and for the consumer automobile appear in the appendix. (b) The evidence for private non-residential structures appears in the Phase II Report of Contract TOS-74-29. (2) The existing literature on economic depreciation: the bibliography search, in the Phase I report, produced over 180 studies with direct bearing on the theory, measurement and policy issues involving economic depreciation. Several dozen of these studies actually produced depreciation estimates of specific types of capital equipment. (3) the conventional treatment of depreciation in existing literature: the point of departure for measuring economic depreciation for producer durable equipment and private non-residential structures must begin with the seminal work of Dale Jorgenson and his followers as well as with the recent capital stock studies of the Bureau of Economic Analysis (BEA). For consumer durables two sources have been located which suggest the conventional treatment for these assets: (a) the well-known work of Raymond Goldsmith and (b) the Flow of Funds

Accounts of the U.S. Federal Reserve System.

The problem confronted in this section is to convert these three sources of information into specific depreciation and efficiency function estimates for each of the 39 classes of assets which comprise the U.S. capital stock. This conversion is accomplished in three stages as now reported. In section A below we convert our estimates, the literature search and the conventional wisdom into more useable forms. This primarily consists of simplifying the econometric results obtained for the individual assets as discussed above. The second step, reported in section B, is to produce average depreciation and efficiency function estimates for the asset classes in which we have considerable data, Type A asset classes. The third and final step involves inferring our best estimates of depreciation and productive efficiency for the Class B and C assets for which we have only scattered or no data. The final outcome will consist of two tables containing our best professional judgment (BPJ) average rates of depreciation for each of the 39 asset classes along with two sets of efficiency sequence estimates for each asset class. One set of efficiency sequences is derived directly from the Box-Cox power transformation and the other from the best geometric approximation (BGA) to that functional form.

A. Conversion of Micro Estimates into Useable Form for Macro Approximations:

The first specific problem to be solved in the construction of depreciation estimates by asset class is to convert the detailed econometric analysis of individual assets into forms which can be easily averaged for purposes of constructing macro estimates. The first step in accomplishing this conversion is to consider the following question: If the depreciation pattern produced by a Box-Cox equation were to be approximated by one smooth, constant geometric pattern, what would be the geometric rate and how close would be the fit of the geometric pattern which approximates the Box-Cox to the latter form? To answer this question, we employ the following equation:

$$\ln \hat{q} = \alpha + \beta s + \gamma t$$

The left hand side of the above equation consists of the logs of the predicted asset prices, by age and date, from the Box-Cox power transformation. Based on our hypothesis tests of various functional forms we selected the unconstrained Box-Cox power transformation form. It is the maximum likelihood set of parameter estimates from among the four Box-Cox power transformations tried. While the resultant predicted prices are our best guesses as to the prices of assets by age and date, the functional form is extremely complex and impossible to easily aggregate. To resolve this problem, we utilize the predicted Box-Cox prices in the equation above to estimate the approximate average rate of decline. The coefficient β in the equation above will represent the average rate of price decline with age according to the Box-Cox power transformation. We label this coefficient the best geometric approximation to the Box-Cox depreciation process. (These rates are hereafter referred to as BGA rates.)

The BGA depreciation and revaluation rates from the unconstrained Box-Cox form appear in Table 16. Briefly, these depreciation and revaluation rates are derived by estimating the above geometric equation using as a dependent variable the predicted prices from the unconstrained Box-Cox power transformation estimated on retired asset prices. As an indication of the closeness of the BGA rates to the Box-Cox predicted prices we include in Table 16 the coefficient of determination or R^2 for the above equation.

For each asset class, the depreciation rate estimates fall within

Table 16

BGA Depreciation and Revaluation Rates
and R^2-Values for Specific Assets
(Unconstrained Box-Cox on Retired Prices)

Asset	β	γ	R^2	Asset	β	γ	R^2
	PDE 4 (-16.33%)				PDE 11 (-27.37%)		
Tractor D-4	-12.04	17.92	.954	Typewriter	-24.58	--	.923
D-6	-18.05	25.23	.911		PDE 17 (-33.33%)		
D-7	-16.22	19.52	.957	Chevrolet	-39.76	--	.978
D-8	-17.32	20.76	.966	Plymouth	-31.03	--	.945
D-9	-18.05	18.37	.978	Pickup	-29.19	--	.948
	PDE 6 (-17.22%)				PNS 1 (-3.61%)		
Compressor	-16.76	2.55	.833	Factory	- 3.61	3.08	.997
Grader	-19.69	15.68	.962		PNS 2 (-2.47%)		
Loader	-15.22	14.67	.966	Office	-2.47	3.84	.989
	PDE 8, 10 (-12.25%)			Retail	-2.20	4.17	.993
MPG 9	-14.08	--[1]	.991	Warehouse	-2.73	2.99	.995
MPG 12	-10.51	--	.977		CD 1 (-27.25%)		
MPG 19	-12.02	--	.958	Buick	-27.54	6.06	.970
	PDE 16 (-25.37%)			Cadillac	-29.54	2.86	.975
GMC Truck	-22.25	5.12	.952	Chevrolet	-27.75	2.60	.962
Ford Truck	-23.70	5.06	.967	Plymouth	-30.50	3.00	.981
Truck Tractor	-31.34	-7.61	.779	Wagon	-29.48	0.00	.994
Dump Truck	-24.18	12.03	.922				

[1] No γ coefficient appears for data based on one or two years only.

a comparatively narrow range. For example, the range of depreciation estimates for the five types of tractors is from -12% to -18%. The range for consumer automobiles is from -27% to -30%. The range for PDE class 16 is from 22% to 31%. The range for private non-residential structures commercial is from 2.2% to 2.7%. As we shall see later these ranges are quite narrow in comparison to the total range of depreciation values reported in the conventional treatment of assets by other analysts. The R^2 values indicate that the BGA approach is very close to the underlying, unconstrained Box-Cox estimation procedure. Of the 29 assets studied and reported in Table 16, 26 have R^2 values above .9. (We only report here the BGA rates for assets which are used in the subsequent analysis. In particular we do not report estimates of the remaining structure classes which do not fit into the PNS categories, nor evidence for a number of MPG classes which were studied in some detail, nor evidence for a few asset classes for which the Box-Cox power transformation failed to converge.)

We conclude from Table 16 that the best geometric approximation to the Box-Cox power transformation produces a set of estimates of depreciation and revaluation which are quite close to the true Box-Cox rates. Furthermore, within each asset class, the range of estimates is very narrow, so that average rates by class should be reasonably reliable. Referring again to Table 16, the number in parentheses next to the title of each asset class is the average BGA rate which will be employed to represent the average depreciation rate for that class. For example, for PDE Class 4 the average is 16.33%. For PDE Class 16 the average rate is 25.37% and for the PNS Class 2 the average rate is 2.47%. These average BGA rates for each class will be used in subsequent analysis.

The literature search reported in the Phase I Report has served a number of purposes in this study. In addition to acquainting us with the state of the art in depreciation research, several studies which reported actual depreciation estimates gave us some notion of the reasonableness of our results. In particular, the studies of automobiles by Wykoff, Ramm and Ackerman confirm the reasonableness of our automobile depreciation estimates. Our estimates are somewhat higher than these earlier studies, because we have introduced the retirement of automobiles, as well as the loss of in-place value. Additional confirmations were derived from the earlier study of tractors by Griliches and from the study of trucks by Robert Hall. These studies confirm the rank ordering of depreciation rates across these various asset classes as well as the general magnitudes. The only major study which appears to be quite far out of line with our estimates is the work of Taubman and Rasche on office buildings. As we noted earlier, we have reason to disagree with the Taubman and Rasche results. Finally, we note that Robert Coen reports similar patterns of depreciation but disagrees as to the appropriate rate.

In addition to providing an independent basis for judgment of the quality of our estimates, the literature search produced several studies which report rates of depreciation for assets not covered in our own econometric analysis. Two of these studies are by Rafael Weston (1972) and Wilhemina Leigh (1977) are of a very important consumer asset class—6, residential structures. In addition, the literature search produced two other studies which we will employ in the development of our best guesses as to depreciation rates by asset class. Table 17 contains the depreciation rate values we have derived from these other studies in addition to the rates we have derived from Weston and Leigh. The two remaining studies consist of a study of ships by Bun Song Lee and of furniture and radios by Garcia dos Santos. We feel that these latter studies are somewhat less reliable than the others primarily because they do not deal with data from the U.S. capital stock.

As we noted above, depreciation of the U.S. capital stock must begin with the work undertaken by Dale W. Jorgenson, BEA, BLS, Raymond Goldsmith, and the Federal Reserve System. Just as our econometric analysis had to be modified to make it conformable to the construction of depreciaiton estimates by asset class, some modificiation is necessary for the evidence provided by these conventional sources. We begin first with the asset lifetimes provided to us by Dale W. Jorgenson. Tables 18A and 18B contain the lifetimes provided to us by Professor Jorgenson for 22 PDE classes and for 19 non-residential structure classes. It is our understanding that these service lives are essentially those used in the BEA capital stock study. However, there are some modest exceptions based on work undertaken in recent months at BLS and by Jack Faucett Associates. We used these lives to construct depreciation estimates for each of the PDE and PNS classes required in this study by calculating the double declining balance, 1.5 declining balance and straight declining balance methods from them. These declining balance rates appeared earlier in this report in Tables 9A and 9B. It should be recalled that the methodology employed by Jorgenson and others in constructing capital stock estimates has involved utilizing the double declining balance method applied to asset lives. Thus the double declining balance column of Tables 9A and 9B employed with the Jorgenson lives is the point of departure for our estimation procedure.

Table 17

Selected Estimates by Asset Class from Literature Search

	Assets Studied	Evidence	Average	Authors
PDE 19	Steel Hull (50-500 ton)	-13.4 to -14.3	-13.8	Lee
	Wood Hull (50-500 ton)			
	(Japanese)			
CD 2	Furniture		- 3.8	Garcia dos Santos
CD 4	Radio		- 7.1	Garcia dos Santos
CD 6	Owner Occupied	-1.6	- 1.55	Weston
	Tenant Occupied	-1.5		
	Census (Unadjusted)	-1.06	-1.01	Leigh
	Census (Adjusted)	.95		

Table 18A

Producers Durable Equipment

(see NIPA Table 5.6)

		lifetime
1.	Furniture and fixtures	15
2.	Fabricated metal products	18
3.	Engines and turbines	21
4.	Tractors	8
5.	Agricultural machinery, except tractors	17
6.	Construction machinery, except tractors	9
7.	Mining and oilfield machinery	10
8.	Metalworking machinery	16
9.	Special industry machinery, N.E.C.	16
10.	General industrial, including materials handling, equipment	14
11.	Office, computing, and accounting machinery	8
12.	Service industry machinery	10
13.	Electrical transmission, distribution and industrial apparatus	14
14.	Communication equipment	14
15.	Electrical equipment, N.E.C.	14
16.	Trucks, buses and truck trailers	6.8
17.	Autos	6.8
18.	Aircraft	9
19.	Ships and boats	22
20.	Railroad equipment	25
21.	Instruments	11
22.	Other equipment	11

Table 18B

Non-residential Structures

(see Interindustry Transactions in New Structures and Equipment, 1963 and 1967, Volume I p. v, vi)

		lifetime
1.	Industrial buildings	27
2.	Office buildings	36
3.	Warehouses	36
4.	Garages and service stations	36
5.	Stores and restaurants	36
6.	Religious buildings	48
7.	Education buildings	48
8.	Hospital buildings	48
9.	Other nonfarm buildings	31
10.	Telephone and telegraph facilities	27
11.	Railroads	51
12.	Electric utility facilities	30
13.	Gas utility facilities	30
14.	Petroleum pipelines	20
15.	Farm residential buildings	50
16.	Farm service facilities	38
17.	Oil and gas wells	16
18.	Oil and gas exploration	16
19.	Other nonbuilding facilities	31

An alternative source of benchmark estimates for depreciation rates for the U.S. capital stock comes from capital stock studies undertaken by BEA. BEA has produced capital stock studies for some time, and we have used the estimates reported in the April 1970 issue of the Survey of Current Business. The BEA procedures for estimating depreciation were described in the October 1966 issue of the Survey of Current Business and involve utilizing both straight line and double declining balance depreciation methods combined with the Winfrey-3 retirement distribution. However, BEA emphasizes the estimates based on their straight line variant. By employing a perpetual inventory equation and utilizing the gross investment data from the national accounts and the capital stock data produced by the BEA study, we were able to derive average annual rates of depreciation implicit in the BEA studies. The formula employed is as follows:

$$\delta_t = \frac{I_t - K_t + K_{t-1}}{K_{t-1}}$$

As with the rates from Jorgenson's lives, the implicit BEA rates are in Tables 9A and 9B. With the exception of the office equipment class, the depreciation rates produced from the BEA capital stock studies are not completely unreasonable. However, as we shall see later, these rates are in general higher than those of our study.

For purposes of comparison, we turned to two studies which employ depreciation rates for consumer durables. The first is a study of housing by Raymond Goldsmith based on data produced in a 1937 Financial Survey of Urban Housing. The second source for consumer durable benchmark estimates are service lives assumed by the Federal Reserve System in the Flow of Funds Accounts which is discussed in Phase I. The Federal Reserve also reports service lives derived from the Goldsmith studies in the 1962 NBER Reports (also discussed in Phase I). Employing the same method for constructing rates from Jorgenson's service lives, we constructed double declining balance and declining balance depreciation rates from the Goldsmith and Flow of Funds service lives. These depreciation rates also appear in Table 9B.

B. Derivation of Depreciation Rates and Efficiency Functions by Asset Class for the U.S. Capital Stock

As noted in the introduction to this Phase II Report, the 22 PDE, 10 PNS and 7 CD asset classes are partitioned into three types. The type A asset classes are those for which we have done extensive research in this project and in previous Treasury work or for which we have reliable estimates from other studies. The type B asset classes are those for which we have partial evidence and type C asset classes are those for which we have no micro studies. The asset classes are listed by type earlier in the report in Table 2. For the type A asset classes, 10 altogether, we utilize the average BGA depreciation rates reported in Table 16. These rates are based upon the extensive analysis undertaken in this study and in the study presented to the Treasury under Contract TOS-74-27. The consumer durable class 6--residential structures rate employed is an average of the rates estimated by Weston and Leigh. These rates are listed in the last column of Tables 9A and B opposite the appropriate asset class number. We consider our estimates in Tables 9A and B to be more reliable than any estimates made to date on these asset classes. Consequently, it is instructive to observe how these estimates, carefully constructed by historical standards, compare to the conventional treatment of assets by Jorgenson and BEA. These comparisons will serve as a basis for inferences about the appropriate depreciation patterns we should use for both type B and type C classes.

Inspection of Tables 9A and B indicates that of the 8 asset classes for which we have reliable estimates, 7 of our estimates produce rates smaller than the BEA rates and 7 produce rates smaller than the Jorgenson rates. The BEA truck rates are small and the Jorgenson auto rate is a bit small, but these can both be attributed probably to our allowance for retirements. Consequently our estimation process implies that both the BEA and the Jorgenson depreciation method, double declining balance applied to the lives reported in Tables 18, are too large.

Taking the 6 PDE classes first, we are interested in drawing a general inference about the Jorgenson methodology of deriving a depreciation rate from asset service lives. If we can establish a general pattern of the relationship between our depreciation estimates and those derived from the Jorgenson service lives, then we can apply the same procedure to deriving new depreciation estimates from the service lives for the remaining asset classes. In other words, we hope to infer a method of deriving a rate from the Jorgenson lives by comparing our methodology to the rates produced by Jorgenson. We proceed by using the following equation

$$\hat{\delta} = X/L$$

This formula is used for deriving a depreciation rate $\hat{\delta}$ from a service life L. If the double declining balance method is used then the X is replaced by 2. If a 1.5 declining balance scheme is used then

X becomes 1.5. We solve for the unknown on the right hand side of the equation by employing our depreciation rates for each of the six classes for which we had evidence and by using Jorgenson's service lives. The result is the appropriate number to which one will then apply a declining balance method with Jorgenson's service lives and derive a new depreciation rate for other asset classes.

For Producer Durable Equipment, the appropriate declining balance pattern, based on our new depreciation estimates are invariably larger than 1.5 but less than double declining balance. In the six cases studied, with the exception of the auto class, the declining balance rates ranged from 1.3 to 1.9. The auto rate we believe can be explained by the fact that our auto estimate is based on automobile prices derived from GSA data. These automobile prices reflect the depreciation of autos used by industry rather than by households. Jorgenson's rates are probably based on depreciation studies of consumer autos. (This conclusion is supported by the fact that our consumer auto rates are somewhat lower than Jorgenson's double declining rates and therefore consistent with the trend observed in these other PDE classes.) The average declining balance method implied by our estimates, then, is 1.65 declining balance. In other words, if one had no information other than the service lives on each asset class, then the appropriate depreciation method to use for each of the additional classes would be to apply a 1.65 declining balance scheme to the Jorgenson service lives.

For Private Non-residential Structures, as we have discussed at some length in (1977) and in the addendum to the Phase I Report, the implicit declining balance method applied to Jorgenson's lives for structures would be less than a straight declining balance rate. In particular, for the four largest structures studied, factories, offices, retail stores, and warehouses, the implicit declining balance method ranged from .79 to .98. The average rate was .91. Again, if one had no other information for the other structure classes than the Jorganson service lives, one would impose a .9 declining balance scheme to the lives in order to derive the appropriate depreciation rate.

Because automobiles and residential structures are really unique consumer durable goods, it is unwarrented to derive inferences for the other consumer durable classes such as furniture, radios, toys and the like. We shall have to use other methods in drawing estimates for these assets.

We are now prepared to produce our judgmental estimates for type B asset classes. These classes are: PDE 11--Office Computing and Accounting Machinery, PDE 19--Ships and Boats, PNS 5--Hospital and Institutional Buildings, PNS 6--Other Buildings (mainly social and recreational), CD 2--Furniture and CD 4--Radio, TV, Recorder and Musical Instruments. We shall discuss each of these asset classes case by case. We employ the following pieces of information in deriving these judgmental estimates: (a) the evidence from type A assets, (b) the conventional treatment of these assets by Jorgenson, BEA, Goldsmith or the Federal Reserve, and (c) any general information we feel should have bearing on these particular asset classes.

The evidence we have for PDE 11, Office Computing and Accounting Machinery, consists of our econometric estimates of the depreciation rate implicit in vintage price data on Royal Typewriters made available by GSA. The typewriter depreciation rate is 27.29%. Jorgenson's double declining balance scheme implies a rate of 25%. The BEA capital figures evidently contain a typographical error because their rate suggests 3%. The 1.65 declining balance scheme applied to the Jorgenson service lives, suggested by the type A asset information, suggests a depreciation rate of 20.6%.

Thus the range of values which we might consider runs from 20.6% to 27.3%. Even though the typewriter obviously does not represent the largest category of office computing and accounting machinery, we believe that it is closer to the true depreciation value than any of the other possibilities. The reason for this belief is that the major types of assets in this category are probably computers and accounting machinery. These types of assets have undergone substantial technological changes over the past several decades and there are indications, from the computer industry especially, that such changes are likely to continue for some time. Since obsolescence has played such a major role in depreciating these types of assets, we feel that the typewriter estimate is not out of line and therefore we have settled on a rate of 27.3% for Class PDE 11.

The evidence for Class PDE 19—Ships and Boats comes from the Lee study of the Japanese fishing fleet. The average rate we derived from Lee's study is 13.8%. The conventional estimates for PDE 19 depreciation are 9.1% by Jorgenson and 10.8% by BEA. The inference from our type A assets, 1.65 declining balance applied to the Jorgenson service lives, suggests a depreciation rate of 7.5%. Thus the range of values is from 7.5% to 13.8%. Because the Lee study deals with Japanese vessels and because the U.S. commercial fleet tends to be quite a bit older than the Japanese commercial fleet we believe that the 7.5% rate, based on the other PDE type A asset classes is closer to correct and we employ this rate here.

The two PNS classes listed as Class B asset classes, PNS 5—Hospital and Institutional Buildings and PNS 6—Other Buildings (Social and Recreational especially) are listed as type B asset categories because we did study some somewhat similar buildings in Contract TOS-74-27. We employ the BGA rates to the unconstrained Box-Cox estimates for these two asset classes. For medical buildings the depreciation rate is 2.33% and for recreational buildings the BGA rate was 4.54%. These rates will be used here.

We turn now to our judgmental estimates for Consumer Durable Classes 2 and 4. These suggested depreciation rates are based on even sketchier information than the ones above. The reason for this is that we have no solid factual evidence for U.S. consumer durable classes 2 and 4. The only study we have available was undertaken by Garcia dos Santos from which we obtained estimates of 3.8% for CD 2 and 7.1% for CD 4. These rates compare respectively with Goldsmith's 13.3% and 20% rates and the Federal Reserve System's 20% and 25% rates. While the rank orderings are the same, the Garcia dos Santos estimates, from British data, seem quite far out of line when compared to the conventional wisdom. The Garcia dos Santos rates are quite a bit lower. Nevertheless, our general evidence suggests that the conventional treatment has been to depreciate assets too rapidly. Consequently, we have decided to lower our estimates of the depreciation of these two classes of consumer durables. We employ a 10% depreciation rate for CD 2 (furniture) and a 15% rate for CD 4 (radios, etc.).

The final step in developing average depreciation rates for the asset classes required in this study is to provide estimates for the type C asset classes. Because we have no information other than the inferences we can draw from the asset classes which we have studied intensely, we employ the suggested declining balance scheme to the Jorgenson service lives. For Producer Durable Equipment classes we employ a 1.65 declining balance scheme to Jorgenson's service lives. For Private Non-residential Structure classes, we impose a .9 declining balance scheme to the Jorgenson service lives. For the Consumer Durable classes 3, 5 and 7, we maintain the rank ordering suggested by the Federal Reserve System but lower the average rate to be commensurate with the rates we have already developed for type B assets: [illegible]. The average depreciation rates which we suggest all appear in Table 9, the last column.

C. Best Professional Estimates of Depreciation and Relative Efficiencies of the U.S. Capital Stock by Asset Class

The estimates of depreciation and relative efficiencies which follow involve a high degree of judgment as well as econometric analysis. There is little doubt in our minds that eventually improved estimates will be derived. At the same time, however, it is our conviction that this study represents an attempt to provide a comprehensive econometric base for the derivation of depreciation estimates for the entire U.S. capital stock.

The preceding section discussed the derivation of the average depreciation rates to be used for each class. Essentially, two methodologies were employed in deriving these average rates. One involved approximating average rates from the more sophisticated non-constant unconstrained Box-Cox power transformation applied to retired asset prices. The second involved using constant rates inferred from other studies. These two procedures suggest that we have available two possible sources for the derivation of efficiency sequences. Relative asset efficiencies, it will be recalled, may be derived from vintage asset prices when one employs the user-cost-of-capital model and the principal of price-quantity duality implied by neoclassical economic theory. We have two possible sets of vintage asset prices, both based on estimation procedures, from which efficiency sequences may be derived. We shall make available here both sets of efficiency sequences. (However, we have the specialized Box-Cox "variable rate" efficiency sequences only for the type A classes. It would seem to us unreasonable to attempt to derive non-constant depreciation patterns in any detail for the other types of asset classes.)

Tables 19 and 20 contain our best professional judgment estimates of average depreciation by asset class and of the efficiency sequences corresponding to those asset classes. In Table 19 we present the BGA depreciation rates, and the average Box-Cox efficiency sequences. These Box-Cox efficiency functions are averages over the data. The statistical appendix to this report contains the Box-Cox efficiency sequences for each year for each of the individual assets studied. The efficiency sequences produced in Table 19 for the type A asset classes are those which were derived for the specific asset for which the BGA rate was as close to the class average. These selections are as follows:

Asset Class	Specific Asset
PDE 4	D-7
PDE 6	Loader
PDE 8, 10	MPG-19
PDE 16	Ford Pickup
PDE 17	GSA Plymouth
PNS 1	Factory
PNS 2	Offices
CD 1	Buick
CD 6	BGA

While the efficiency sequences presented in Table 19 vary across ages and while they reflect the general nature of the Box-Cox patterns better than

Table 19

Depreciation (BGA) and Efficiencies (Box-Cox)

Producer Durable Equipment
(Type A Asset Classes Only)

	Class					
Item	4	6	8	10	16	17
BGA Rate	-.1633	-.1722	-.1225	-.1225	-.2537	-.3333
			Efficiency Sequences $\phi(s)$			
0	1	1	1	1	1	1
1	0.91841	1.1719	1.1075	1.1075	1.3361	1.3959
2	0.84299	1.2303	1.1331	1.1331	1.4265	1.5456
3	0.77256	1.2419	1.1304	1.1304	1.3978	1.5554
4	0.70675	1.2239	1.111	1.111	1.295	1.4694
5	0.64531	1.1856	1.0805	1.0805	1.1476	1.321
6	0.58801	1.133	1.0423	1.0423	0.97792	1.1377
7	0.53464	1.0704	0.99845	0.99845	0.8029	0.94178
8	0.48501	1.0012	0.95059	0.95059	0.63517	0.75046
9	0.43893	0.92804	0.89996	0.89996	0.48346	0.57571
10	0.39622	0.85311	0.84755	0.84755	0.35304	0.42481
11	0.35671	0.77813	0.79415	0.79415	0.24626	0.30096
12	0.32023	0.70447	0.74043	0.74043	0.16308	0.20415
13	0.28662	0.63322	0.68695	0.68695	0.10165	0.1321
14	0.25573	0.5652	0.63417	0.63417		
15	0.2274	0.50102	0.58251	0.58251		
16	0.20148	0.44112	0.5323	0.5323		
17	0.17784	0.38574	0.48381	0.48381		
18	0.15634	0.33503	0.43728	0.43728		
19	0.13685	0.28899	0.39291	0.39291		
20	0.11923	0.24755	0.35084	0.35084		
21	0.10336	0.21056	0.31121	0.31121		
22	0.089131	0.1778	0.2741	0.2741		
23	0.076418	0.14904	0.23956	0.23956		
24	0.065113	0.12398	0.20763	0.20763		
25	0.055108	0.10234	0.17832	0.17832		
26	0.046302	0.083785	0.15161	0.15161		
27	0.038596	0.068023	0.12746	0.12746		
28	0.031895	0.054745	0.10583	0.10583		
29	0.02611	0.043659				
30	0.021154					
31	0.016945					
32	0.013405					
33						
34						
35						
36						
37						

Table 19 (Con't.)

Depreciation Rates (BGA) and Efficiency Sequences
(Box-Cox)
(Type A Asset Classes)

	Private Non-residential Structures		Consumer Durables	
Item	Industrial	Commercial	Automobiles	Residential Structures
BGA rate	-.0361	-.0247	-.2725	-.0128
		Efficiency Sequences $\phi(s)$		
0	1.0	1.0	1.0	1.0
1	.9595	.7234	.9150	.9872
2	.9274	.6416	.7853	.9746
3	.8979	.5896	.6559	.9621
4	.8699	.5509	.5352	.9498
5	.8431	.5198	.4264	.9376
6	.8174	.4937	.3308	.9256
7	.7926	.4711	.2490	.9138
8	.7688	.4511	.1806	.9021
9	.7455	.4331	.1251	.8905
10	.7231	.4167	.0817	.8791
11	.7013	.4017	.0494	.8679
12	.6803	.3878	.0268	.8568
13	.6598	.3748	.0126	.8458
14	.6400	.3627	.0049	.8350
15	.6207	.3513		.8243
16	.6020	.3406		.8137
17	.5838	.3304		.8033
18	.5662	.3208		.7930
19	.5490	.3116		.7829
20	.5324	.3028		.7729
21	.5162	.2949		.7630
22	.5005	.2865		.7532
23	.4853	.2788		.7436
24	.4705	.2714		.7341
25	.4561	.2644		.7247
26	.4421	.2576		.7154
27	.4285	.2510		.7062
28	.4153	.2447		.6972
29	.4024	.2386		.6883
30	.3900	.2327		.6795
31	.3779	.2270		.6708
32	.3661	.2214		.6623
33	.3547	.2161		.6537
34	.3436	.2109		.6453
35	.3328	.2059		.6371
36	.3224	.2010		.6289
37	.3122	.1963		.6209
38	.3023	.1917		.6129
39	.2928	.1873		.6051
40	.2834	.1829		.5973

Depreciation (BGA) and Efficiencies (BGA)

Producer Durable Equipment

Item	1	2	3	4	5	6
BGA Rate	.1100	.0917	.0786	.1633	.0971	.1722

Efficiency Sequences $\phi(s)$

0	1	1	1	1	1	1
1	0.89	0.9083	0.9214	0.8367	0.9029	0.8278
2	0.7921	0.82501	0.84898	0.70007	0.81523	0.68525
3	0.70497	0.74936	0.78225	0.58575	0.73607	0.56725
4	0.62742	0.68064	0.72076	0.49009	0.6646	0.46957
5	0.55841	0.61823	0.66411	0.41006	0.60006	0.38871
6	0.49698	0.56153	0.61191	0.3431	0.5418	0.32178
7	0.44231	0.51004	0.56382	0.28707	0.48919	0.26637
8	0.39366	0.46327	0.5195	0.24019	0.44169	0.2205
9	0.35036	0.42079	0.47867	0.20097	0.3988	0.18253
10	0.31182	0.3822	0.44104	0.16815	0.36008	0.1511
11	0.27752	0.34715	0.40638	0.14069	0.32511	0.12508
12	0.24699	0.31532	0.37444	0.11772	0.29355	0.10354
13	0.21982	0.28641	0.34501	0.09849	0.26504	0.08571
14	0.19564	0.26014	0.31789	0.08240	0.23931	0.07095
15	0.17412	0.23629	0.2929	0.06895	0.21607	0.05873
16	0.15497	0.21462	0.26988	0.05769	0.19509	0.04861
17	0.13792	0.19494	0.24867	0.04827	0.17615	0.04024
18	0.12275	0.17706	0.22912	0.04038	0.15904	0.03331
19	0.10925	0.16083	0.21111	0.03379	0.1436	0.02757
20	0.09723	0.14608	0.19452	0.02827	0.12966	0.02283
21	0.08653	0.13268	0.17923		0.11707	
22	0.07701	0.12052	0.16514		0.1057	
23	0.06854	0.10946	0.15216		0.09543	
24	0.06100	0.09942	0.1402		0.08616	
25	0.05429	0.09030	0.12918		0.07780	
26		0.08202	0.11903		0.07024	
27		0.07450	0.10967		0.06342	
28		0.06767	0.10105		0.05726	
29		0.06146	0.09311		0.05170	
30		0.05583	0.08579		0.04668	

Table 20 (Con't.)

Producer Durable Equipment

Item	7	8	9	10	11	12
BGA Rate	.1650	.1225	.1031	.1225	.2729	.1650

Efficiency Sequences ϕ(s)

0	1	1	1	1	1	1
1	0.835	0.8775	0.8969	0.8775	0.7271	0.835
2	0.69723	0.77001	0.80443	0.77001	0.52867	0.69723
3	0.58218	0.67568	0.72149	0.67568	0.3844	0.58218
4	0.48612	0.59291	0.64711	0.59291	0.2795	0.48612
5	0.40591	0.52028	0.58039	0.52028	0.24322	0.40591
6	0.33894	0.45654	0.52055	0.45654	0.14776	0.33894
7	0.28301	0.40062	0.46688	0.40062	0.10744	0.28301
8	0.23632	0.35154	0.41875	0.35154	0.078118	0.23632
9	0.19732	0.30848	0.37557	0.30848	0.0568	0.19732
10	0.16476	0.27069	0.33685	0.27069	0.041299	0.16476
11	0.13758	0.23753	0.30212	0.23753	0.030029	0.13758
12	0.11488	0.20843	0.27097	0.20843	0.021834	0.11488
13	0.09592	0.1829	0.24304	0.1829	0.015875	0.095923
14	0.08009	0.16049	0.21798	0.16049	0.011543	0.080096
15	0.06688	0.14083	0.19551	0.14083	0.002392	0.06688
16	0.05584	0.12358	0.17535	0.12358		0.055845
17	0.04663	0.10844	0.15727	0.10844		0.046631
18	0.03893	0.09515	0.14106	0.09515		0.038936
19	0.03251	0.08350	0.12651	0.08350		0.032512
20	0.02714	0.07327	0.11347	0.07327		0.027147
21		0.06429	0.10177	0.06429		
22		0.05642	0.09127	0.05642		
23		0.04950	0.08186	0.04950		
24		0.04344	0.07342	0.04344		
25		0.03812	0.06585	0.03812		

Table 20 (Con't.)

Producer Durable Equipment

Item	13	14	15	16	17	18
BGA Rate	.1179	.1179	.1179	.2537	.3333	.1833

Efficiency Sequences $\phi(s)$

0	1	1	1	1	1	1
1	0.8821	0.8821	0.8821	0.7463	0.6667	0.8167
2	0.7781	0.7781	0.7781	0.55696	0.44449	0.667
3	0.68636	0.68636	0.68636	0.41566	0.29634	0.54474
4	0.60544	0.60544	0.60544	0.31021	0.19757	0.44489
5	0.53406	0.53406	0.53406	0.23151	0.13172	0.36334
6	0.47109	0.47109	0.47109	0.17277	0.087818	0.29674
7	0.41555	0.41555	0.41555	0.12894	0.058548	0.24235
8	0.36656	0.36656	0.36656	0.09622	0.039034	0.19792
9	0.32334	0.32334	0.32334	0.07181	0.026024	0.16165
10	0.28522	0.28522	0.28522	0.05359	0.01735	0.13202
11	0.25159	0.25159	0.25159	0.03999	0.011567	0.10782
12	0.22193	0.22193	0.22193	0.02985	0.007712	0.088054
13	0.19576	0.19576	0.19576	0.02227	0.005141	0.071914
14	0.17268	0.17268	0.17268	0.01662	0.003427	0.058732
15	0.15232	0.15232	0.15232	0.01240	0.002285	0.047967
16	0.13436	0.13436	0.13436			0.039174
17	0.11852	0.11852	0.11852			0.031994
18	0.10455	0.10455	0.10455			0.026129
19	0.09222	0.09222	0.09222			0.02134
20	0.08135	0.08135	0.08135			0.017428
21	0.07175	0.07175	0.07175			
22	0.06329	0.06329	0.06329			
23	0.05583	0.05583	0.05583			
24	0.04925	0.04925	0.04925			
25	0.04344	0.04344	0.04344			
26						
27						
28						
29						
30						

Table 20 (Con't.)

Producer Durable Equipment

Item	19	20	21	22
BGA Rate	.0750	.0660	.1473	.1473
0	1	1	1	1
1	0.925	0.934	0.8527	0.8527
2	0.85562	0.87236	0.7271	0.7271
3	0.79145	0.81478	0.62	0.62
4	0.73209	0.761	0.52867	0.52867
5	0.67719	0.71078	0.4508	0.4508
6	0.6264	0.66387	0.38439	0.38439
7	0.57942	0.62005	0.32777	0.32777
8	0.53596	0.57913	0.27949	0.27949
9	0.49576	0.54091	0.23832	0.23832
10	0.45858	0.50521	0.20322	0.20322
11	0.42419	0.47186	0.17328	0.17328
12	0.39237	0.44072	0.14776	0.14776
13	0.36295	0.41163	0.12599	0.12599
14	0.33573	0.38446	0.10744	0.10744
15	0.31055	0.35909	0.09161	0.09161
16	0.28726	0.33539	0.078116	0.078116
17	0.26571	0.31325	0.06661	0.06661
18	0.24578	0.29258	0.056798	0.056798
19	0.22735	0.27327	0.048432	0.048432
20	0.2103	0.25523	0.041298	0.041298
21	0.19453	0.23839	0.035214	0.035214
22	0.17994	0.22265	0.030027	0.030027
23	0.16644	0.20796	0.025604	0.025604
24	0.15396	0.19423	0.021833	0.021833
25	0.14241	0.18141	0.018617	0.018617
26	0.13173	0.16944		
27	0.12185	0.15826		
28	0.11271	0.14781		
29	0.10426	0.13806		
30	0.096439	0.12895		

Table 20 (Con't.)

Private Non-Residential Structures

Item	1	2	3	4	5
BGA Rate	.0361	.0247	.0188	.0188	.0188

Efficiency Sequences Ø(s)

0	1	1	1	1	1
1	0.9639	0.9753	0.9812	0.9812	0.9812
2	0.9291	0.95121	0.96275	0.96275	0.96275
3	0.89556	0.92772	0.94465	0.94465	0.94465
4	0.86323	0.9048	0.92689	0.92689	0.92689
5	0.83207	0.88245	0.90947	0.90947	0.90947
6	0.80203	0.86066	0.89237	0.89237	0.89237
7	0.77308	0.8394	0.87559	0.87559	0.87559
8	0.74517	0.81866	0.85913	0.85913	0.85913
9	0.71827	0.79844	0.84298	0.84298	0.84298
10	0.69234	0.77872	0.82713	0.82713	0.82713
11	0.66735	0.75949	0.81158	0.81158	0.81158
12	0.64326	0.74073	0.79633	0.79633	0.79633
13	0.62003	0.72243	0.78135	0.78135	0.78135
14	0.59765	0.70459	0.76666	0.76666	0.76666
15	0.57608	0.68718	0.75225	0.75225	0.75225
16	0.55528	0.67021	0.73811	0.73811	0.73811
17	0.53523	0.65366	0.72423	0.72423	0.72423
18	0.51591	0.63751	0.71062	0.71062	0.71062
19	0.49729	0.62176	0.69726	0.69726	0.69726
20	0.47934	0.60641	0.68415	0.68415	0.68415
21	0.46203	0.59143	0.67129	0.67129	0.67129
22	0.44535	0.57682	0.65867	0.65867	0.65867
23	0.42927	0.56257	0.64628	0.64628	0.64628
24	0.41378	0.54868	0.63413	0.63413	0.63413
25	0.39884	0.53513	0.62221	0.62221	0.62221
26	0.38444	0.52191	0.61051	0.61051	0.61051
27	0.37056	0.50902	0.59904	0.59904	0.59904
28	0.35719	0.49644	0.58777	0.58777	0.58777
29	0.34429	0.48418	0.57672	0.57672	0.57672
30	0.33186	0.47222	0.56588	0.56588	0.56588

Table 20 (Con't.)

Private Non-Residential Structures

Item	1	2	3	4	5 Con't.
			Efficiency Sequences Ø(s)		
31	0.31988	0.46056	0.55524	0.55524	0.55524
32	0.30834	0.44918	0.54481	0.54481	0.54481
33	0.2972	0.43809	0.53456	0.53456	0.53456
34	0.28648	0.42727	0.52451	0.52451	0.52451
35	0.27613	0.41671	0.51465	0.51465	0.51465
36	0.26617	0.40642	0.50498	0.50498	0.50498
37	0.25656	0.39638	0.49548	0.49548	0.49548
38	0.24729	0.38659	0.48617	0.48617	0.48617
39	0.23837	0.37704	0.47703	0.47703	0.47703
40	0.22976	0.36773	0.46806	0.46806	0.46806
41	0.22147	0.35865	0.45926	0.45926	0.45926
42	0.21347	0.34979	0.45063	0.45063	0.45063
43	0.20577	0.34115	0.44215	0.44215	0.44215
44	0.19834	0.33272	0.43384	0.43384	0.43384
45	0.19118	0.3245	0.42569	0.42569	0.42569
46	0.18428	0.31649	0.41768	0.41768	0.41768
47	0.17762	0.30867	0.40983	0.40983	0.40983
48	0.17121	0.30105	0.40213	0.40213	0.40213
49	0.16503	0.29361	0.39457	0.39457	0.39457
50	0.15907	0.28636	0.38715	0.38715	0.38715

Table 20 (Con't.)

Private Non-Residential Structures

Item	6	7	8	9	10
BGA Rate	.0290*	.0316	.0237	.0563	.0290

Efficiency Sequences $\phi(s)$

0	1	1	1	1	1
1	0.971	0.9684	0.9763	0.9437	0.971
2	0.94284	0.9378	0.95316	0.89057	0.94284
3	0.9155	0.90816	0.93057	0.84043	0.9155
4	0.88895	0.87947	0.90852	0.79311	0.88895
5	0.86317	0.85168	0.88699	0.74846	0.86317
6	0.83814	0.82476	0.86596	0.70632	0.83814
7	0.81383	0.7987	0.84544	0.66656	0.81383
8	0.79023	0.77346	0.8254	0.62903	0.79023
9	0.76731	0.74902	0.80584	0.59362	0.76731
10	0.74506	0.72535	0.78674	0.5602	0.74506
11	0.72346	0.70243	0.7681	0.52866	0.72346
12	0.70247	0.68023	0.74989	0.49889	0.70247
13	0.6821	0.65874	0.73212	0.47081	0.6821
14	0.66232	0.63792	0.71477	0.4443	0.66232
15	0.64311	0.61776	0.69783	0.41929	0.64311
16	0.62446	0.59824	0.68129	0.39568	0.62446
17	0.60635	0.57934	0.66514	0.3734	0.60635
18	0.58877	0.56103	0.64938	0.35238	0.58877
19	0.5717	0.5433	0.63399	0.33254	0.5717
20	0.55512	0.52613	0.61896	0.31382	0.55512
21	0.53902	0.50951	0.6043	0.29615	0.53902
22	0.52339	0.49341	0.58997	0.27948	0.52339
23	0.50821	0.47782	0.57599	0.26374	0.50821
24	0.49347	0.46272	0.56234	0.24889	0.49347
25	0.47916	0.44809	0.54901	0.23488	0.47916
26	0.46526	0.43393	0.536	0.22166	0.46526
27	0.45177	0.42022	0.5233	0.20918	0.45177
28	0.43867	0.40694	0.5109	0.1974	0.43867
29	0.42595	0.39408	0.49879	0.18629	0.42595
30	0.4136	0.38163	0.48697	0.1758	0.4136

Table 20 (Con't.)

Private Non-Residential Structures

Item	6	7	8	9	10 Continued
			Efficiency Sequences $\phi(s)$		
31	0.4016	0.36957	0.47542	0.1659	0.4016
32	0.38996	0.35789	0.46416	0.15656	0.38996
33	0.37865	0.34658	0.45316	0.14775	0.37865
34	0.36767	0.33563	0.44242	0.13943	0.36767
35	0.357	0.32503	0.43193	0.13158	0.357
36	0.34665	0.31475	0.42169	0.12417	0.34665
37	0.3366	0.30481	0.4117	0.11718	0.3366
38	0.32684	0.29518	0.40194	0.11058	0.32684
39	0.31736	0.28585	0.39242	0.10436	0.31736
40	0.30815	0.27682	0.38312	0.098482	0.30815
41	0.29922	0.26807	0.37404	0.092938	0.29922
42	0.29054	0.2596	0.36517	0.087705	0.29054
43	0.28212	0.25139	0.35652	0.082768	0.28212
44	0.27393	0.24345	0.34807	0.078108	0.27393
45	0.26599	0.23576	0.33982	0.07371	0.26599
46	0.25828	0.22831	0.33177	0.06956	0.25828
47	0.25079	0.22109	0.3239	0.065644	0.25079
48	0.24351	0.21411	0.31623	0.061948	0.24351
49	0.23645	0.20734	0.30873	0.058461	0.23645
50	0.22959	0.20079	0.30141	0.055169	0.22959

Table 20 (Con't.)

Consumer Durables

Item	1	2	3	4
BGA Rate	.2725	.1000	.1500	.1500

Efficiency Sequences Ø(s)

0	1	1	1	1
1	0.7275	0.9	0.85	0.85
2	0.52926	0.81	0.7225	0.7225
3	0.38503	0.729	0.61413	0.61413
4	0.28011	0.6561	0.52201	0.52201
5	0.20378	0.59049	0.44371	0.44371
6	0.14825	0.53144	0.37715	0.37715
7	0.10785	0.4783	0.32058	0.32058
8	0.078463	0.43047	0.27249	0.27249
9	0.057082	0.38742	0.23162	0.23162
10	0.041527	0.34868	0.19687	0.19687
11	0.030211	0.31381	0.16734	0.16734
12	0.021978	0.28243	0.14224	0.14224
13	0.015989	0.25419	0.12091	0.12091
14	0.011632	0.22877	0.10277	0.10277
15	0.0084624	0.20589	0.087354	0.087354
16		0.1853	0.074251	0.074251
17		0.16677	0.063113	0.063113
18		0.15009	0.053646	0.053646
19		0.13509	0.045599	0.045599
20		0.12158	0.03876	0.03876
21		0.10942	0.032946	0.032946
22		0.098477	0.028004	0.028004
23		0.088629	0.023803	0.023803
24		0.079766	0.020233	0.020233
25		0.07179	0.017198	0.017198

Table 20 (Con't.)

Consumer Durables

Item	5	6	7
BGA Rate	.1500	.0128	.1500
	Efficiency Sequences Ø(s)		
0	1	1	1
1	0.85	0.9872	0.85
2	0.7225	0.97456	0.7225
3	0.61413	0.96209	0.61413
4	0.52201	0.94977	0.52201
5	0.44371	0.93762	0.44371
6	0.37715	0.92562	0.37715
7	0.32058	0.91377	0.32058
8	0.27249	0.90207	0.27249
9	0.23162	0.89053	0.23162
10	0.19687	0.87913	0.19687
11	0.16734	0.86787	0.16734
12	0.14224	0.85677	0.14224
13	0.12091	0.8458	0.12091
14	0.10277	0.83497	0.10277
15	0.087354	0.82428	0.087354
16	0.074251	0.81373	0.074251
17	0.063113	0.80332	0.063113
18	0.053646	0.79304	0.053646
19	0.045599	0.78288	0.045599
20	0.03876	0.77286	0.03876
21	0.032946	0.76297	0.032946
22	0.028004	0.75321	0.028004
23	0.023803	0.74356	0.023803
24	0.020233	0.73405	0.020233
25	0.017198	0.72465	0.017198
26		0.71538	
27		0.70622	
28		0.69718	
29		0.68825	
30		0.67945	
31		0.67075	
32		0.66216	
33		0.65369	
34		0.64532	
35		0.63706	
36		0.62891	
37		0.62086	
38		0.61291	
39		0.60506	
40		0.59732	
41		0.58967	
42		0.58212	
43		0.57467	
44		0.56732	
45		0.56006	
46		0.55289	
47		0.54581	
48		0.53882	
49		0.53193	
50		0.52512	

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a geometric approximation, it is our view that they may be too detailed for reasonable application to large asset studies. It appears to us that the BGA rates perform rather well in approximating the Box-Cox forms and consequently little should be lost in terms of accuracy in using efficiency functions implied by a constant rate of depreciation rather than the non-constant Box-Cox rates. Since the constant depreciation base efficiency functions should be far easier to deal with, it is our recommendation that they be used in subsequent research. Table 20 contains the depreciation estimates and efficiency estimates based on the best geometric approximation to the unconstrained Box-Cox estimates imposed on retired prices. It is Table 20 which contains what we consider to be the outcome of this study for purposes of Jorgenson's capital stock study and Shoven's study of tax impacts by industry.

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*All other articles referred to in this Report are listed in the Phase I Report either in the "Bibliography" section or in the references to "Theory" section.