A time-dependent hybrid vertical coordinate for the WRF model

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Outline

- Coordinate definition and model equations
- Details of implementation
- Tests and validation
- Outlook

Reference: Zängl, 2007, MWR 135, 228-239

Coordinate definition and model equations

1. Introduce a three-dimensional field for the vertical coordinate

$$p(i,j,k) = (p_{sfc}(i,j) - p_{top})\eta_{3D}(i,j,k) + p_{top}$$

$p_{sfc}(i,j)$ **: surface pressure**

p_{top} **: pressure at model top**

The coordinate field may also be time-dependent. It has the same "unit" as the original η variable but varies along coordinate surfaces.

To derive the generalized model equations resulting from this definition, one may introduce a (hypothetical) variable s defining the position of the coordinate surfaces. This variable may be associated with the k index of the η (or η_{3D}) field. The metric term arising from the transformation is $\partial\eta/\partial s$.

Coordinate definition and model equations

Let $\mu = p_{sfc} - p_{top}$

Then, the hydrostatic equation transforms from $\dfrac{\partial \phi}{\partial \eta} = -\mu\alpha$

into $\dfrac{\partial \phi}{\partial s} = -\mu \dfrac{\partial \eta}{\partial s}\alpha$ **.**

Define $\tilde{\mu} = -\mu \dfrac{\partial \eta}{\partial s}, U = \tilde{\mu}u, W = \tilde{\mu}w, \Theta = \tilde{\mu}\theta$

In 2D, the WRF equation system (without diabatic, Coriolis and frictional terms) then reads:

Coordinate definition and model equations

$$\frac{\partial \tilde{\mu}}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial \tilde{\mu}\dot{s}}{\partial s} = 0$$

$$\frac{\partial U}{\partial t} + \frac{\partial uU}{\partial x} + \frac{\partial \dot{s}U}{\partial s} = -\tilde{\mu}\alpha \frac{\partial p^*}{\partial x} - \frac{\partial \phi}{\partial x}\frac{\partial p^*}{\partial s}$$

$$\frac{\partial W}{\partial t} + \frac{\partial uW}{\partial x} + \frac{\partial \dot{s}W}{\partial s} = g\left(\frac{\partial p^*}{\partial s} - \tilde{\mu}\right)$$

$$\frac{\partial \Theta}{\partial t} + \frac{\partial u\Theta}{\partial x} + \frac{\partial \dot{s}\Theta}{\partial s} = 0$$

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} + \dot{s}\frac{\partial \phi}{\partial s} = gw$$

…, which is formally equivalent to the original set of equations. The differences are that horizontal derivatives are now evaluated along s **surfaces, and that the mass coupling with** μ **changes into a coupling with the layer thickness** $\mu\, \partial \eta / \partial s$.

Coordinate definition and model equations

A possible time dependence of the coordinate variable enters the equations through the generalized vertical velocity, which is related to the original one via

$$\frac{\partial\eta}{\partial s}\dot{s} = \dot{\eta} - \left.\frac{\partial\eta}{\partial t}\right|_{x,s} - u\left.\frac{\partial\eta}{\partial x}\right|_{t,s}$$

Relating the s **variable to the** k **index of the** η_{3D} **field turns** $\partial\eta/\partial s$ **into the layer thickness between two adjacent model levels, which may be denoted as** $\Delta\eta_{3D}$**. In discretized form:**

$$\Delta\eta_{3D}(i,j,k+1/2) = \eta_{3D}(i,j,k+1) - \eta_{3D}(i,j,k)$$

Moreover, $\dot{s}\frac{\partial\eta}{\partial s} = \dot{\eta}_{3D}$; $\dot{s}\frac{\partial}{\partial s} = \dot{\eta}_{3D}\frac{\partial}{\partial\eta_{3D}}$

(this can be viewed as a definition because $\dot{\eta}_{3D}$ **is defined accordingly in the model code).**

Implementation

Because the structure of the model equations remains the same, implementing the generalized coordinate is relatively straightforward (although there is a lot of typing work in some subroutines). Specifically:

- **The coordinate field (znw in WRF notation) and its derivatives (layer thickness, vertical interpolation coefficients) have to be changed from 1D into 3D; this frequently involves horizontal interpolation due to C-grid staggering**
- **In the context of horizontal derivatives, the mass coupling with μ changes into a coupling with the mass represented by a specific grid point ($\mu\,\Delta\eta_{3D}$)**
- **However, vertical derivatives do not change compared to the original code because the switch from η to η_{3D} does involve any rescaling in the vertical.**

Coordinate specification

The equation system allows for specifying an (in principle) arbitrary prognostic equation for η_{3D} . Here, it will be used to emulate a hybrid-isentropic system in which the coordinate surfaces are relaxed towards isentropic surfaces.

General structure of the prognostic equation for the coordinate field:

$$\frac{\partial \eta_{3D}}{\partial t}(i,j,k) = \frac{\eta_{3DT}(i,j,k) - \eta_{3D}(i,j,k)}{\tau_{rel}} + K_v(\eta_{3D}(i,j,k+1) + \eta_{3D}(i,j,k-1) - 2\eta_{3D}(i,j,k))$$
$$+ K_h(\mu(i+1,j)\eta_{3D\theta}(i+1,j,k) + \mu(i-1,j)\eta_{3D\theta}(i-1,j,k) + \mu(i,j+1)\eta_{3D\theta}(i,j+1,k) +$$
$$\mu(i,j-1)\eta_{3D\theta}(i,j-1,k) - 4\mu(i,j)\eta_{3D\theta}(i,j,k))/\mu(i,j)$$

$\eta_{3DT}(i,j,k)$ **: Target field for relaxation**

K_h, K_v **: Diffusion coefficients (diffusion is needed to suppress noise in the coordinate field and to prevent model surfaces from approaching each other too closely)**

Coordinate specification

The vertical diffusion coefficient is specified to be zero above a certain threshold of vertical layer spacing and linearly increases with decreasing layer spacing below the threshold (see Zängl, 2007, for details). The horizontal diffusion coefficient is constant.

Specification of target field η_{3DT} :

$$\eta_{3DT}(i,j,k) = \eta_{3Dp}(i,j,k)(1-w(k)) + \eta_{3D\theta}(i,j,k)w(k)$$

$\eta_{3Dp}(i,j,k)$ **: static three-dimensional sigma field defined such that the topographic structures in the coordinate surfaces decay rapidly with height**

$\eta_{3D\theta}(i,j,k)$ **: time-dependent field carrying the coordinate values of selected isentropic surfaces; the potential-temperature field used for computing $\eta_{3D\theta}$ may differ from the prognostic model variable, which allows for heavy smoothing in order to remove breaking gravity waves etc.**

Coordinate specification

w(k): **Weighting factor; the coordinates are specified to be terrain-following close to the ground, quasi-isentropic between the middle troposphere and the lower stratosphere and isobaric near the model top (because the upper b.c. has constant pressure)**

Specific form of η_{3Dp}:

$$\eta_{3Dp}(i,j,k) = \frac{\mu_{00}\eta(k) + [\mu_{ref,smt}(i,j,k) - \mu_{00}]\eta(k)^{\alpha} + [\mu_{ref}(i,j,k) - \mu_{ref,smt}(i,j,k)]\eta(k)^{\beta}}{\mu_{ref}(i,j,k)}$$

μ_{00} **: constant**

μ_{ref} **: reference μ field based on a horizontally homogeneous reference atmosphere (as in MM5)**

$\mu_{ref,smt}$**: Same as μ_{ref}, but for a heavily smoothed topography**

α, β **: Exponents controlling the vertical decay of the topography signal**

Details of implementation

1. Preprocessing

The vertical interpolation needs to be adjusted:

- **Calculation of** η_{3Dp} **field and interpolation of** θ **field to model levels**
- **(1st analysis time) Computation of horizontally averaged** θ **for each model level; these values are saved and will afterwards constitute the target** θ **values for the model levels**
- **Computation of** $\eta_{3D\theta}$ **field**
- **Definition of weighting factor** *w(k)*
- **Computation of** η_{3D} **field**
- **Interpolation of atmospheric variables to** η_{3D} **levels**

Details of implementation

1. Preprocessing

- **Additional namelist variables for (a) output of 3D coordinate fields, (b) generalized vertical coordinate (η_{3Dp}) and (c) hybrid coordinate**
- **Additional output fields: weighting factor (*wgtfac_t*), target θ field (*th8znw*), coordinate field (*znw3d*), smooth topography (*ht_smt*)**

Details of implementation

2. WRF

General changes:

- *znw* **and derived fields (layer thicknesses, interpolation coefficients between full and half levels) become 3D**
- **For velocity points, horizontal interpolation of these fields is needed (this accounts for a substantial fraction of the additional computational cost)**
- *rdn*/*rdnw* **are removed because accurate horizontal interpolation in the denominator requires computing** *1/0.5*(dnw(i,j,k)+dnw(i-1,j,k))* **rather than averaging** *rdnw*
- **The generalized code is backward-compatible but computationally more expensive; work on analyzing and reducing the overhead is in progress**

Details of implementation

2. WRF

Advection:

- To keep the code changes localized, μ is not redefined to $\mu\,\Delta\eta_{3D}$. Rather, auxiliary fields for $u\Delta\eta_{3D}$ and $v\Delta\eta_{3D}$ are defined and passed to the advection routines.
- At the end of the calculation of advection, the advective tendencies are divided by $\Delta\eta_{3D}$ to return to the original „dimension“ of the tendencies.
- For vertical advection, the generalized velocity *ww* is redefined to become $\dot{\eta}_{3D}$ and treated in the same way as before
- An exception holds for the basic-state geopotential, which is rediagnosed after each time step instead of being advected with $\partial\eta_{3D} / \partial t$

Details of implementation

2. WRF

Coordinate management:

- **1st step: Computing $\eta_{3D\theta}$: This starts with interpolating θ to full model levels, followed by some smoothing in the horizontal and the vertical. Vertical smoothing in particular removes super-adiabatic layers**
- **$\eta_{3D\theta}$ is then calculated through linear vertical interpolation between *th8znw* and *znw3d*; it holds the η_{3D} values corresponding to isentropic surfaces**
- **Option to impose upper and lower limits to the vertical distance between $\eta_{3D\theta}$ levels; this reliably suppresses collapsing of model layers and avoids vertical resolution becoming too coarse in deep adiabatic layers**

Details of implementation

2. WRF

Coordinate management:

- **2nd step: Compute the temporal tendency for η_{3D}:**

$$\frac{\partial \eta_{3D}}{\partial t}(i,j,k) = \frac{\eta_{3DT}(i,j,k) - \eta_{3D}(i,j,k)}{\tau_{rel}} + K_v(\eta_{3D}(i,j,k+1) + \eta_{3D}(i,j,k-1) - 2\eta_{3D}(i,j,k))$$
$$+ K_h(\mu(i+1,j)\eta_{3D\theta}(i+1,j,k) + \mu(i-1,j)\eta_{3D\theta}(i-1,j,k) + \mu(i,j+1)\eta_{3D\theta}(i,j+1,k) +$$
$$\mu(i,j-1)\eta_{3D\theta}(i,j-1,k) - 4\mu(i,j)\eta_{3D\theta}(i,j,k)) / \mu(i,j)$$

- **The horizontal diffusion is needed to suppress grid-scale noise in the coordinate field in long-term integrations; note that the noise is already damped before it enters η_{3D}; moreover, the diffusion does not introduce topography signals into the model levels**
- **3rd step: Update *znw3d* and recalculate derived fields and basic state variables**

Validation tests

- Oscillating coordinate in a uniform stratified flow over flat terrain
- Oscillating coordinate in uniform neutral flow over a mountain (conservation test for mass and temperature)
- Linear and nonlinear gravity-wave flow over a mountain with standard, oscillating and hybrid-isentropic coordinate
- Real-case simulations over one month (August 2005) for central Europe

Oscillating coordinate in uniform stratified flow over flat topography

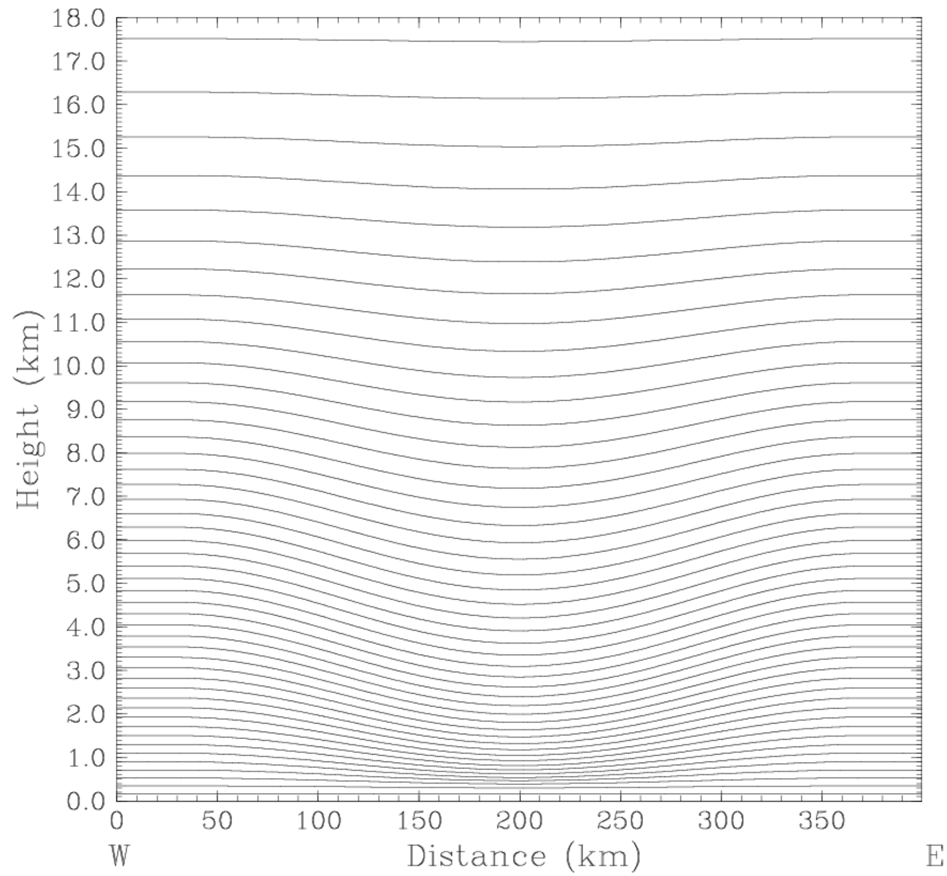
2D with radiative lateral boundaries, mesh size 2 km

51 levels, top at ~25 km

Uniform wind (10 m/s) and static stability (10^{-2} s^{-1})

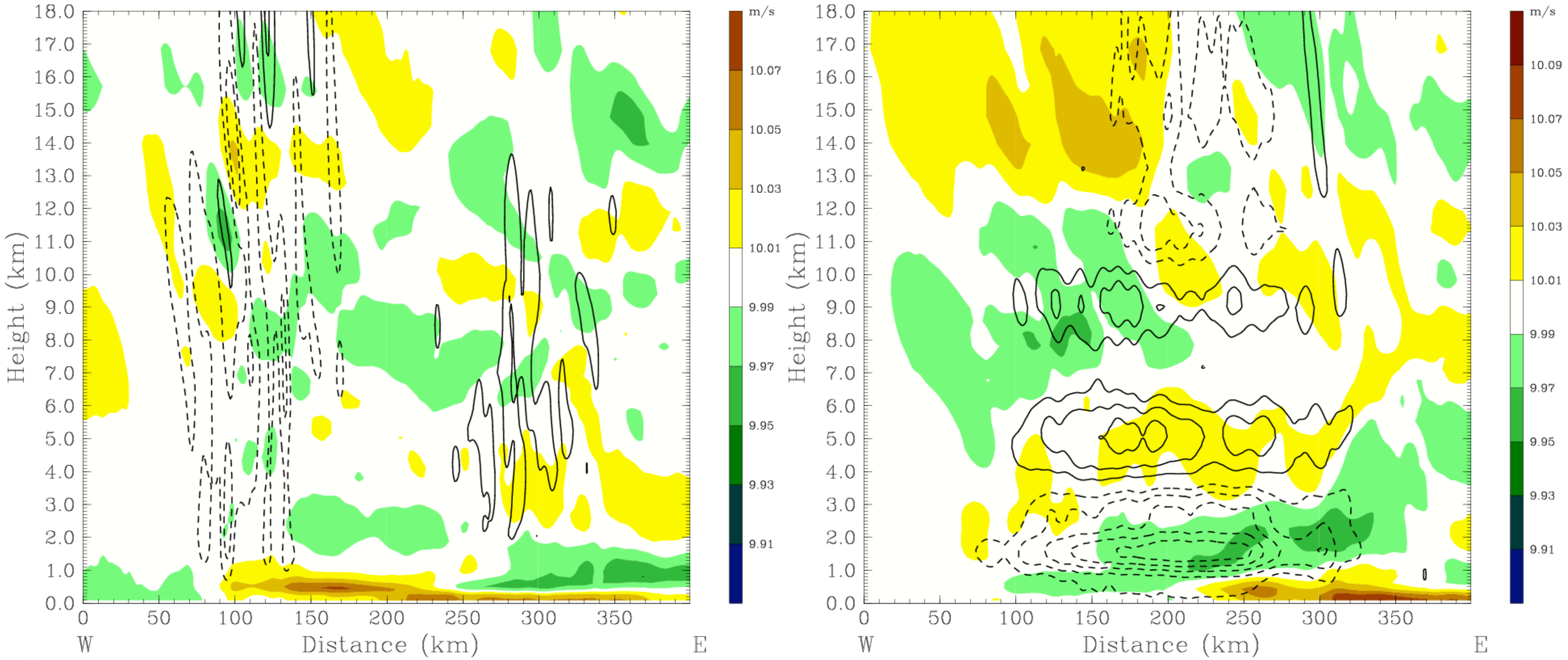
Sinusoidal oscillation of coordinate surfaces with a period of 6h and a maximum peak-to-peak amplitude of about 3.5 km (corresponding to maximum vertical motion of about 50 cm s^{-1})

Figure: coordinate surfaces at t=13.5h



Oscillating coordinate in uniform stratified flow over flat topography

Colors: horizontal wind speed (c.i. 0.02 m/s)
Isolines: vertical wind (c.i. 0.25 cm/s)



t = 13.5h
(maximum coordinate displacement)

t = 15h
(maximum coordinate motion)

Conservation test

2D with periodic boundaries, mesh size 2 km

51 levels, top at ~25 km

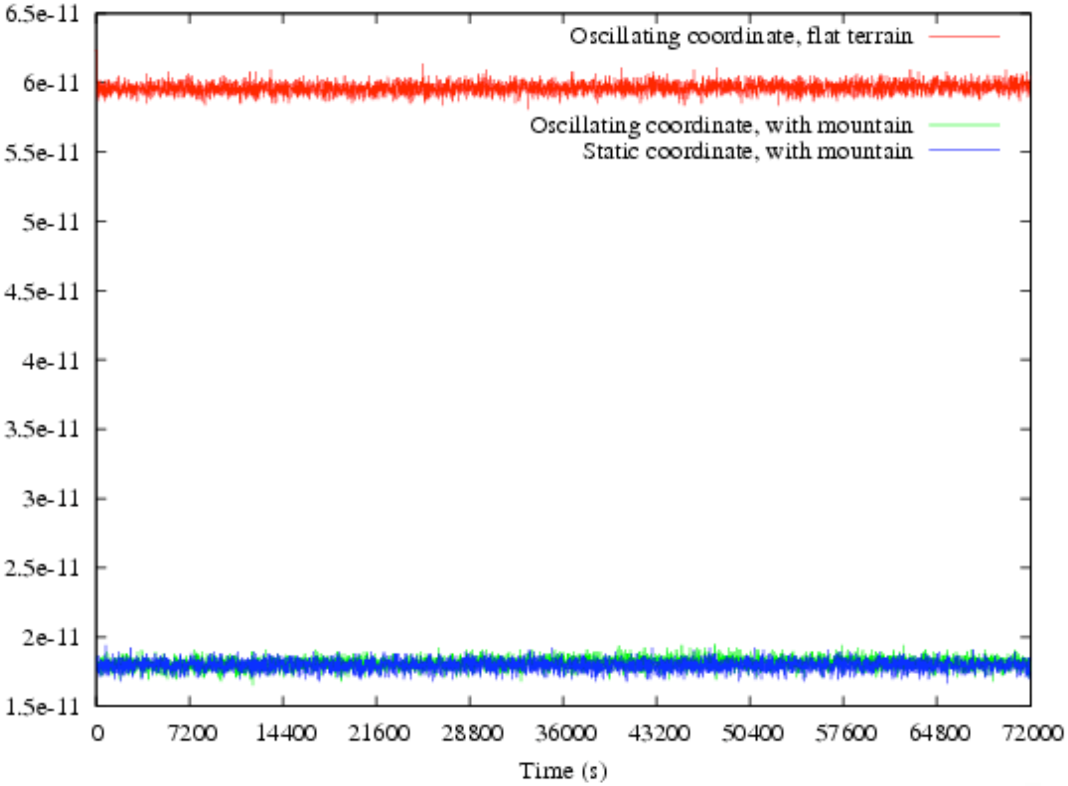
Mountain height 0 m / 500 m

Uniform wind (10 m/s) and potential temperature (312 K)

QV = 10 g/kg

Oscillating coordinates

Domain-average mass; deviation from initial value (Pa)

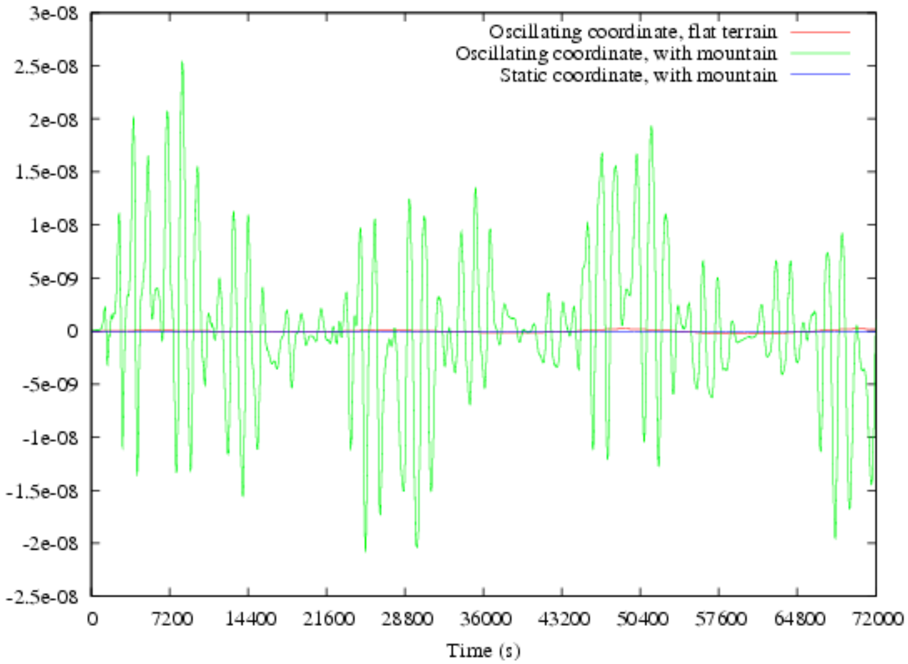


Relative error: ~10^{-16}

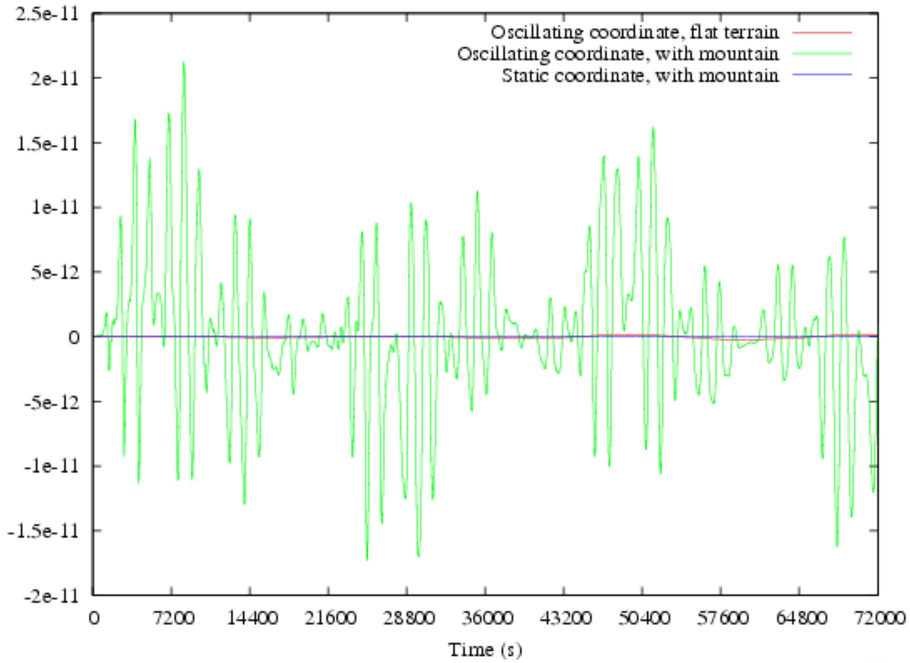
Conservation test

Time series of domain-averaged potential temperature and water vapor mixing ratio (deviation from initial value)

Potential temperature (K)



Mixing ratio

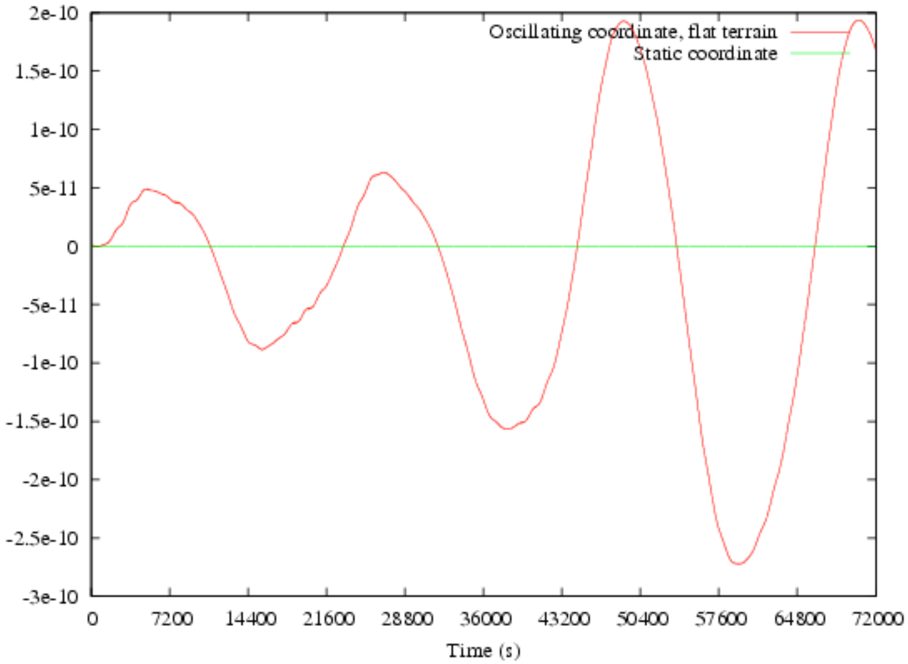


The relative amplitude of the perturbations is 10^{-9} (basic values 12 K and 10 g/kg)

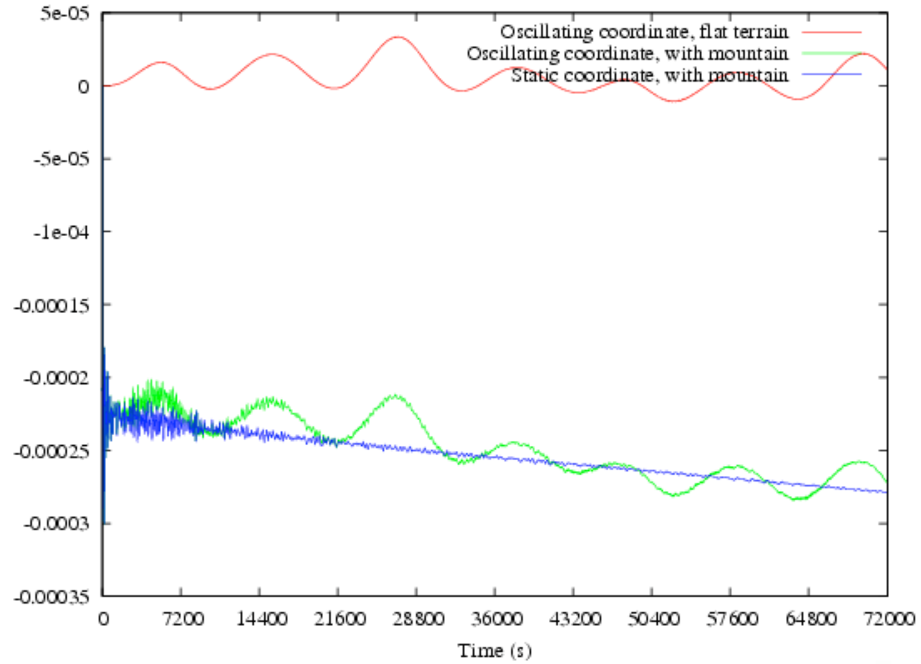
Conservation test

Time series of domain-averaged potential temperature (without mountain) and wind speed (deviation from initial value)

Potential temperature (K)



Wind speed (m/s)



Linear and nonlinear orographic gravity waves

2D with radiative lateral boundaries, mesh size 2 km

51 levels, top at ~25 km, diffusive damping layer in upper 10 km

Mountain height 250 m or 1000 m

Uniform wind (10 m/s) and Brunt-Väisälä-frequency (10^{-2} s^{-1})

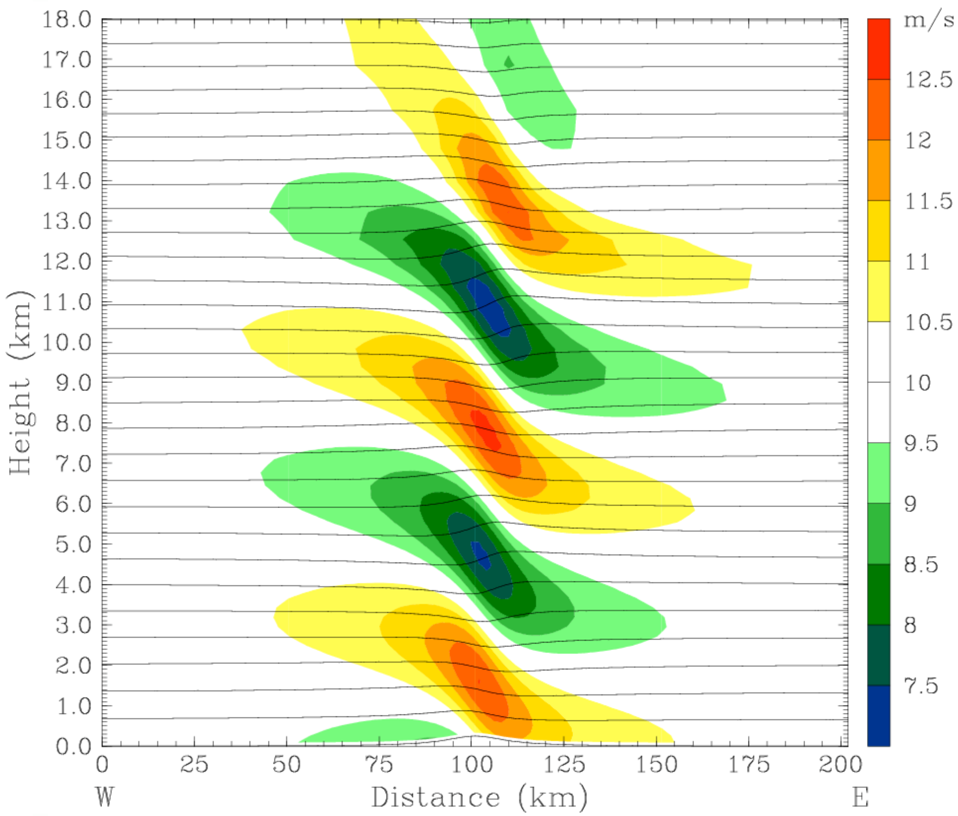
Smagorinsky-type horizontal diffusion (background value 250 m^2 s^{-1})

Tests with static, oscillating and hybrid-isentropic coordinates

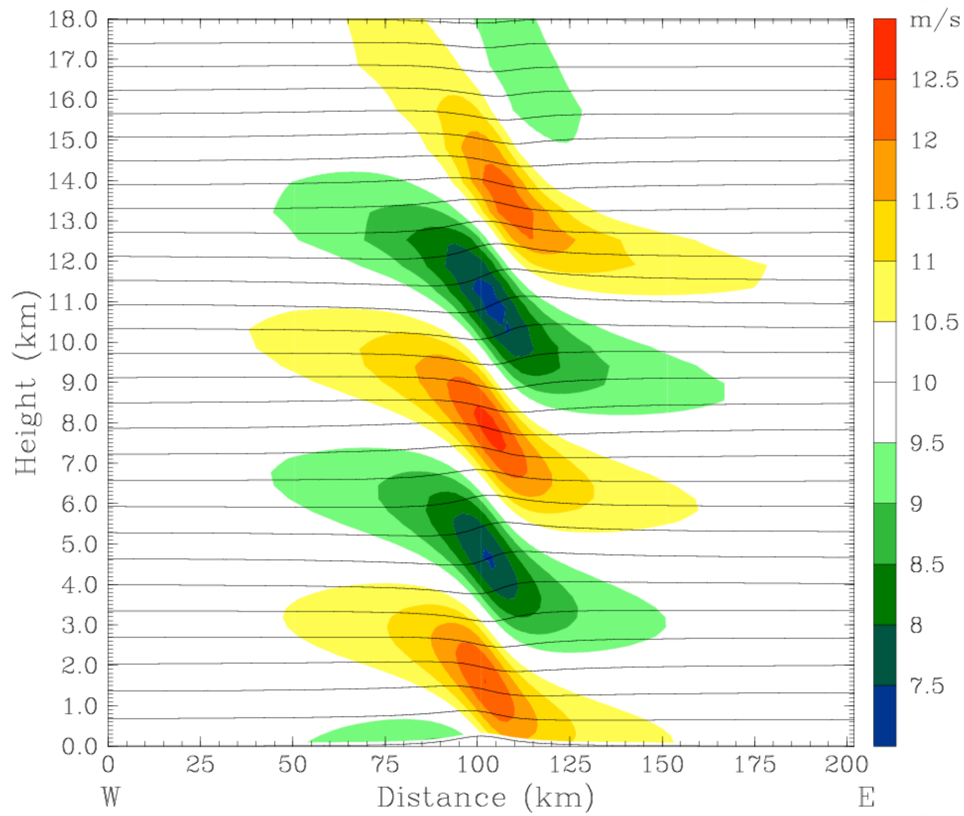
No upper limit on vertical layer spacing is imposed

Horizontal wind speed for 250-m mountain, t = 20 h

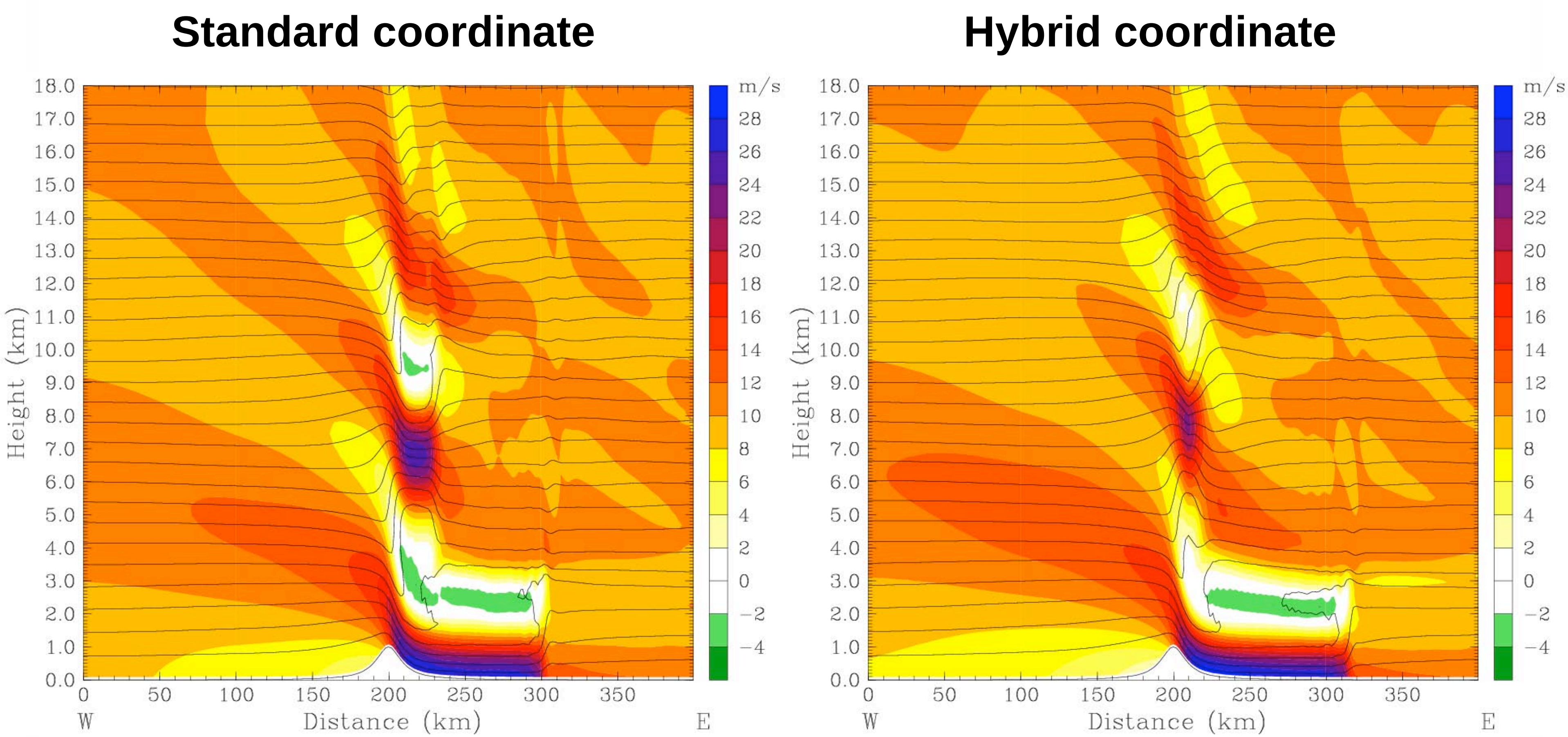
Standard coordinate



Hybrid coordinate

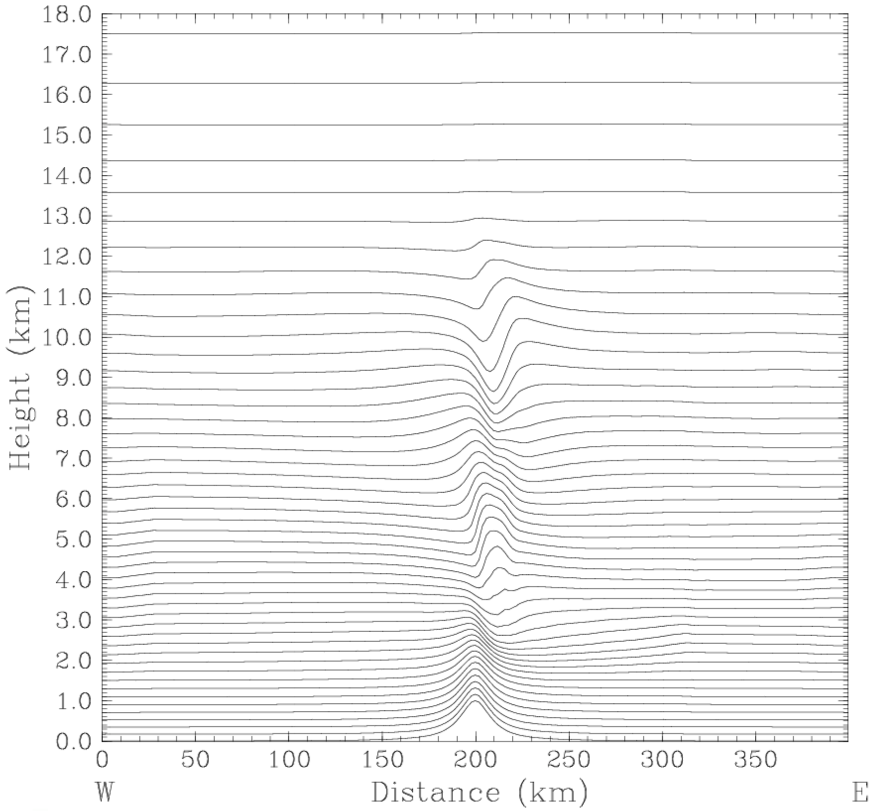


Horizontal wind speed for 1000-m mountain, t = 10 h

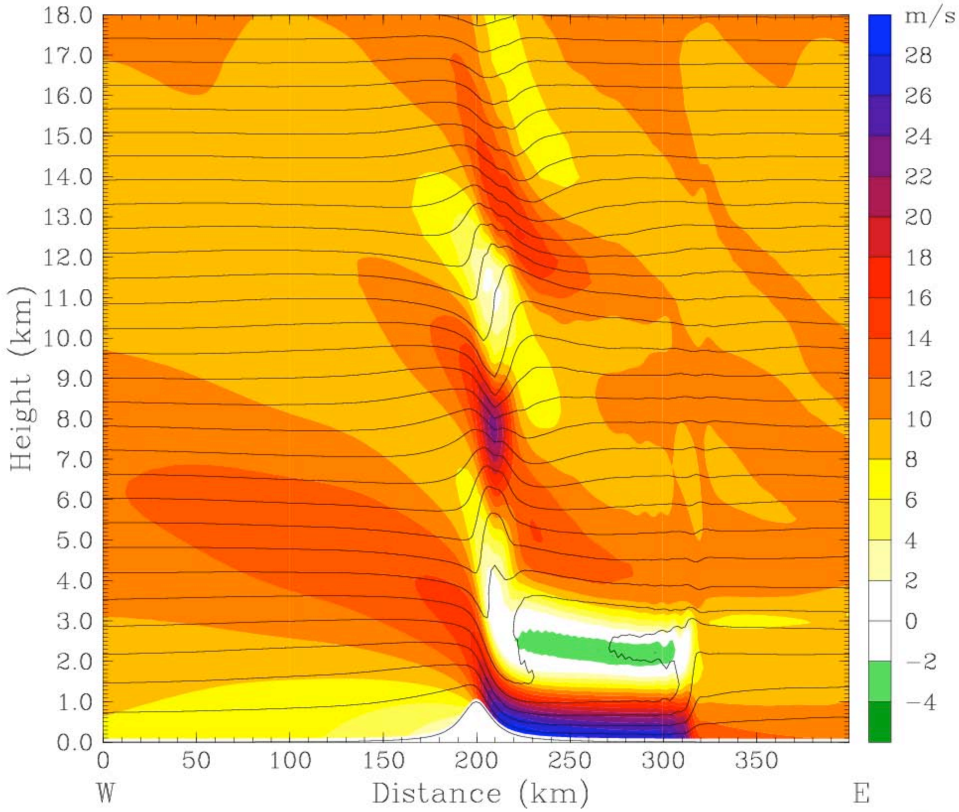


Results for 1000-m mountain, t = 10 h

Coordinate levels

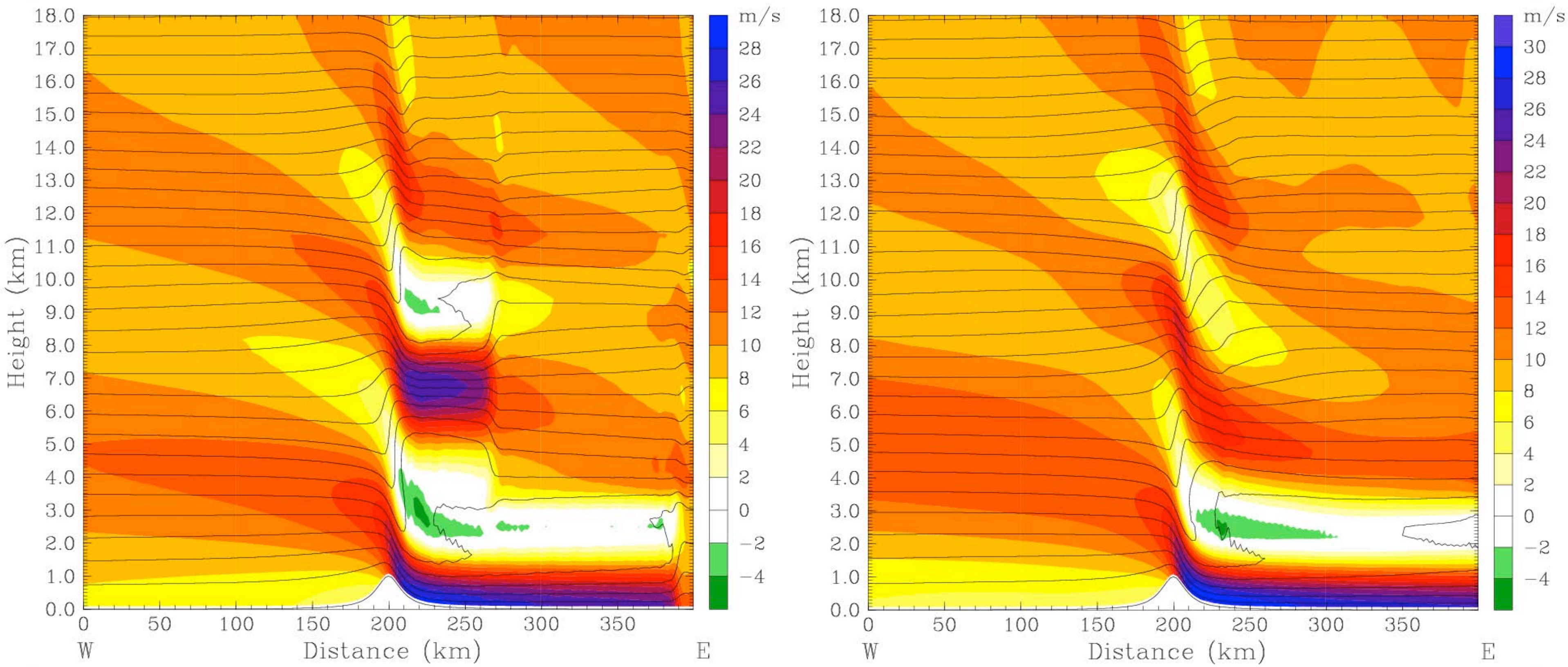


Wind speed



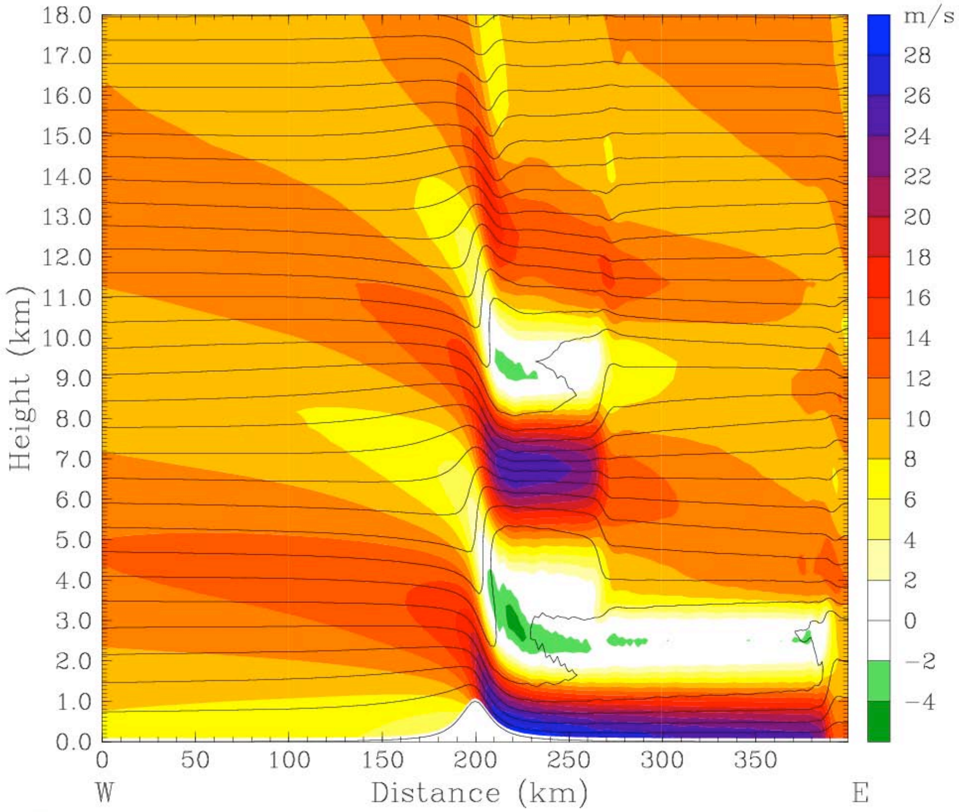
Horizontal wind speed for 1000-m mountain, t = 20 h

Standard coordinate **Hybrid coordinate**

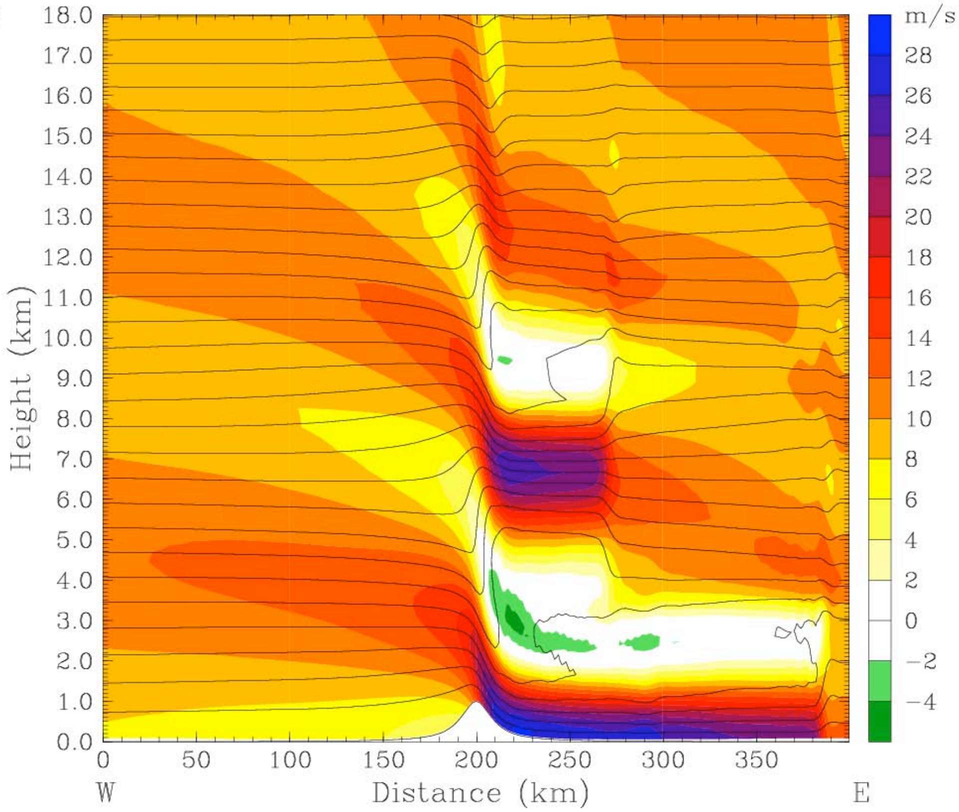


Horizontal wind speed for 1000-m mountain, t = 20 h

Standard coordinate



Oscillating coordinate



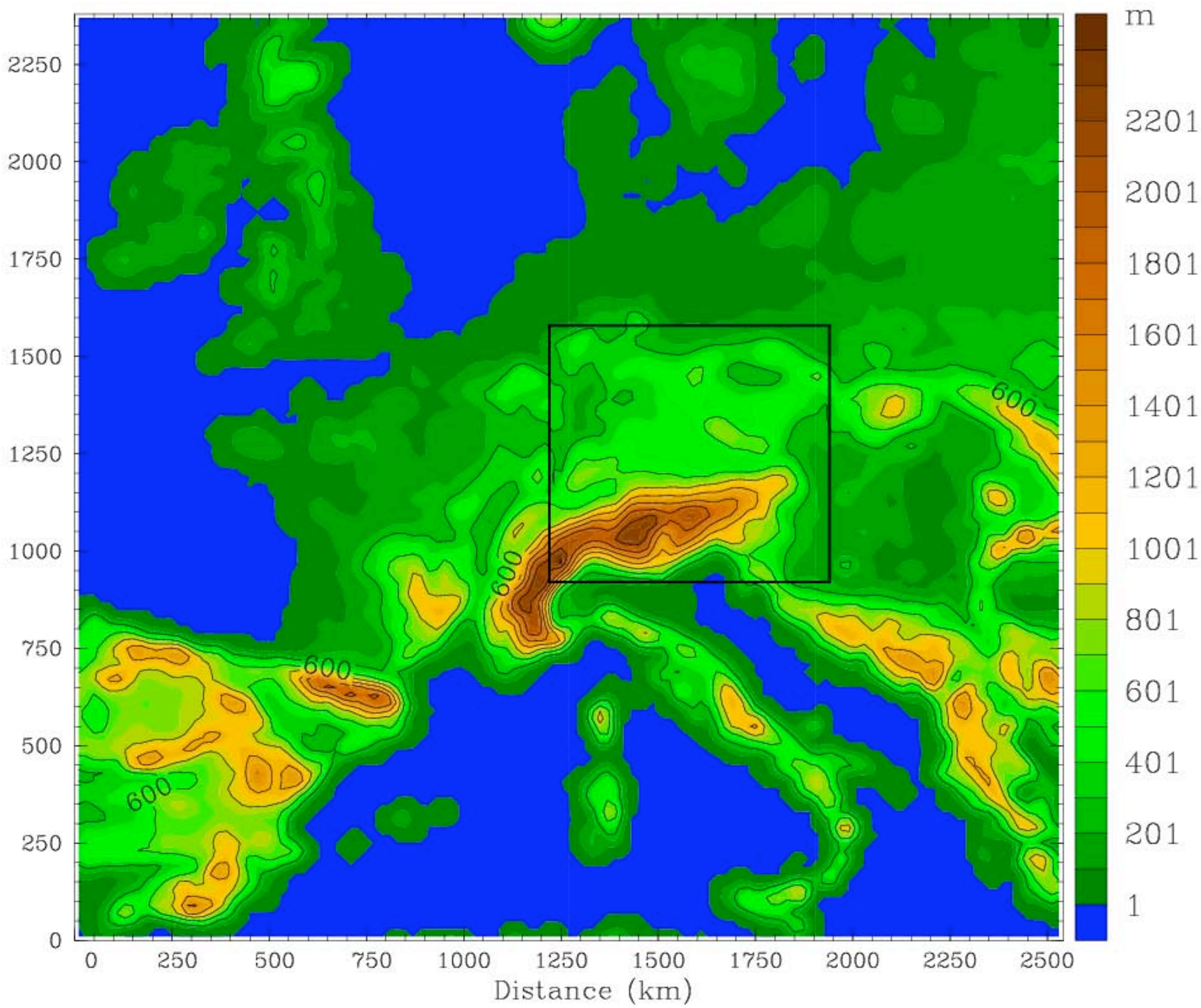
Summary for idealized validation tests

- The adaptive sigma coordinate retains the conservation properties of the WRF equation system
- The perturbations caused by the coordinate motion are much smaller than the vertical motions occurring in synoptic systems
- The gravity-wave tests indicate that the hybrid-isentropic system is not well suited for simulating steady gravity-wave breaking unless imposing restrictive upper limits on the vertical model layer spacing

Real-case test simulations for hybrid coordinate

- Continuous (climate-mode) simulations for August 2005
- Two nested domains, 20 and 6.67 km mesh size
- Initial and boundary conditions from operational ECMWF data, either on standard-pressure levels or on model levels
- Simulations with original WRF code and with hybrid coordinate
- Physics parameterizations: WSM6 for microphysics, Kain-Fritsch for cumulus convection, RRTM/Dudhia for radiation, YSU for boundary layer

Model orography and location of nested domain

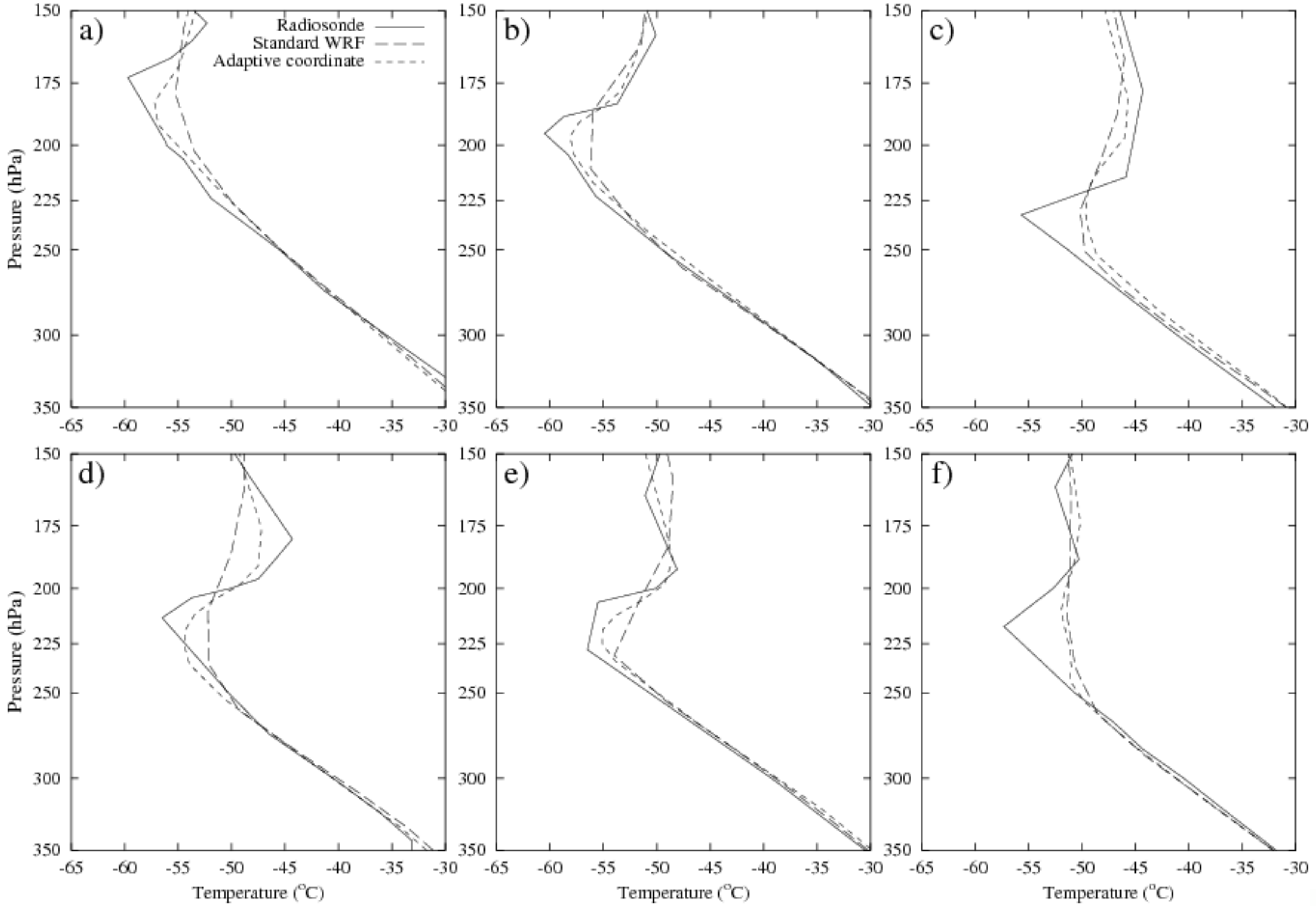


Radiosonde validation of tropopause parameters for central Europe, August 2005

Database: 42 radiosonde stations over central Europe, 2 ascents per day for most of the stations. Model data are interpolated to the station locations

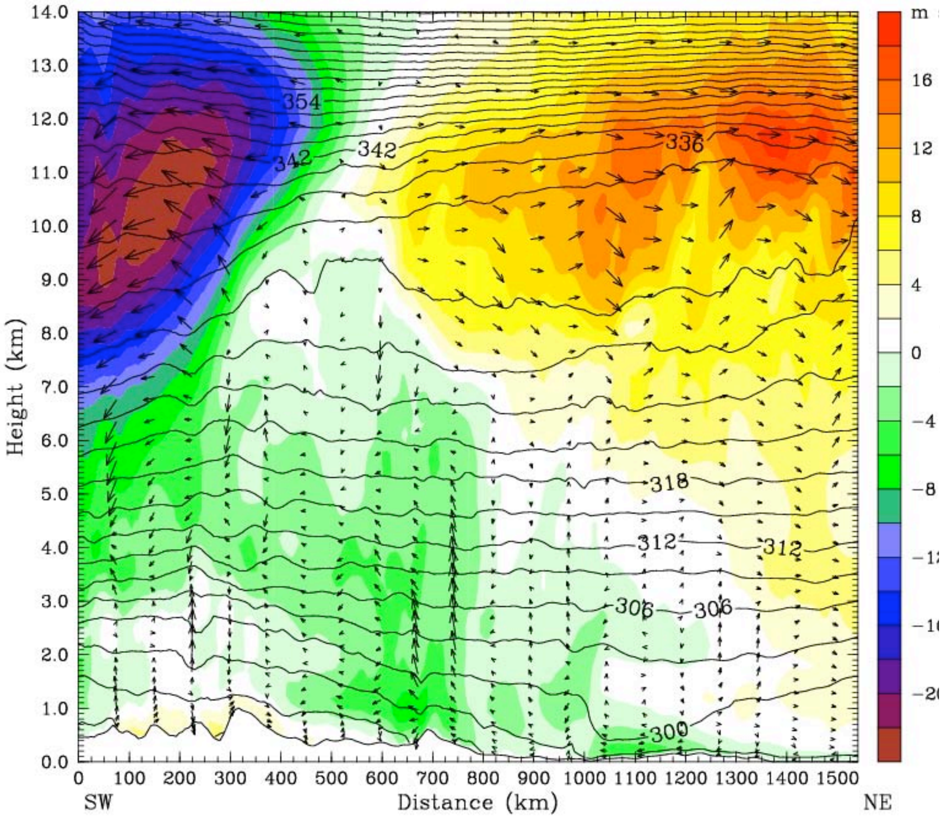
Values for	sigma-PL	hybrid-PL	sigma-ML	hybrid-ML
Bias tropopause temp (K):	1.83	1.52	1.42	1.13
Bias TP pressure (hPa) :	-8.45	-6.62	-6.92	-2.65
Bias dT/dz above TP (K/km):	-3.28	-2.84	-3.25	-2.64
MAE dT/dz above TP (K/km):	3.42	3.04	3.37	2.90

Selected radiosonde ascents (from simulation with pressure-level initialization)

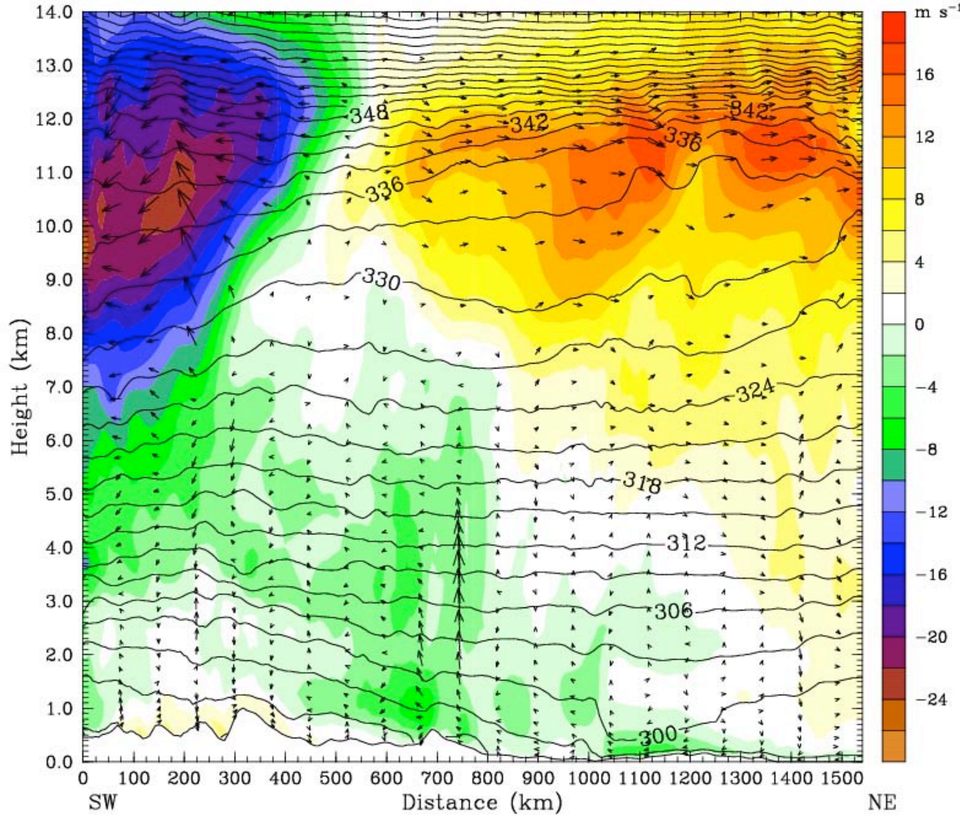


Exemplary vertical cross-sections: Wind component along cross-section and potential temperature, 12 Aug 00 UTC

standard coordinate

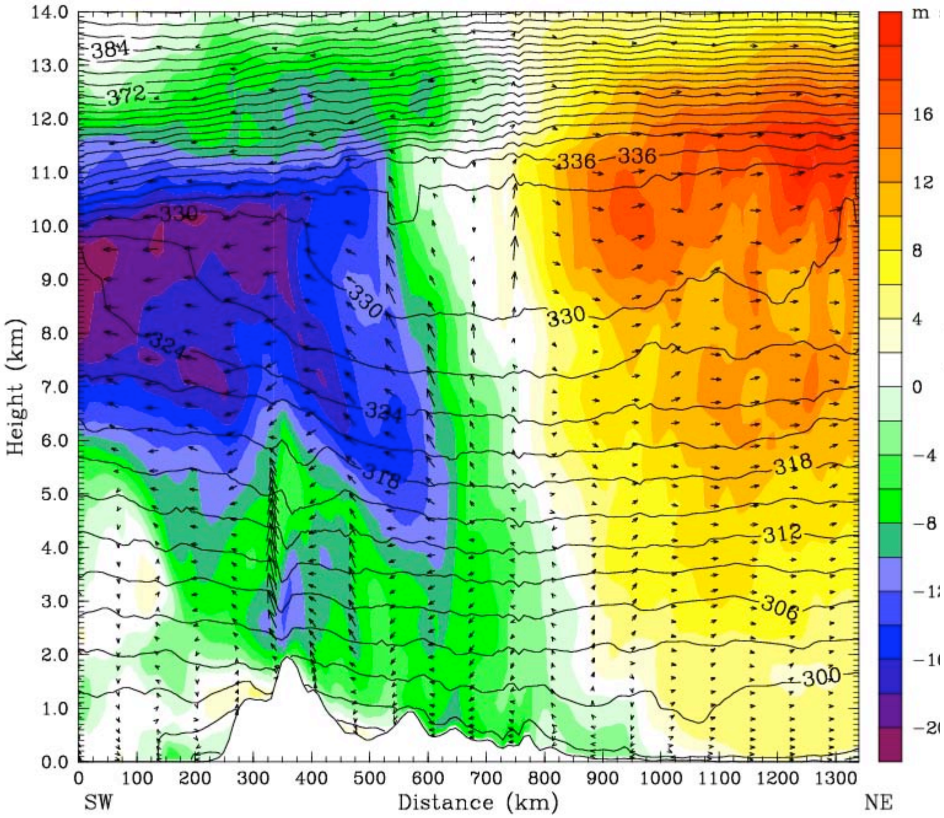


adaptive coordinate

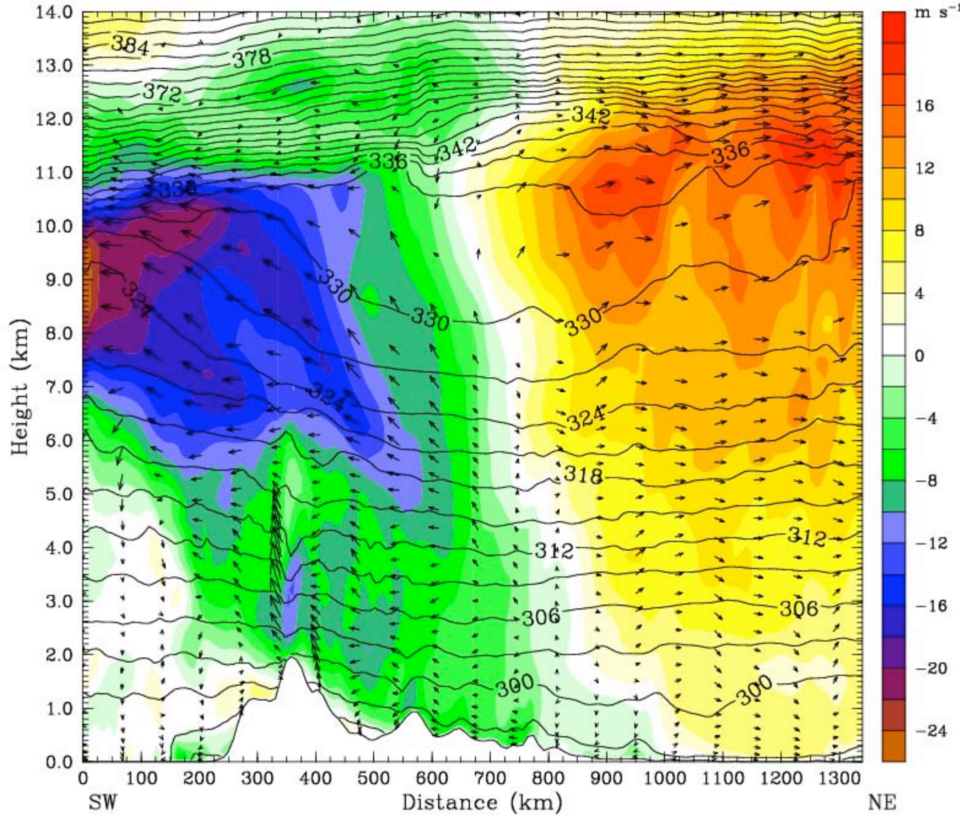


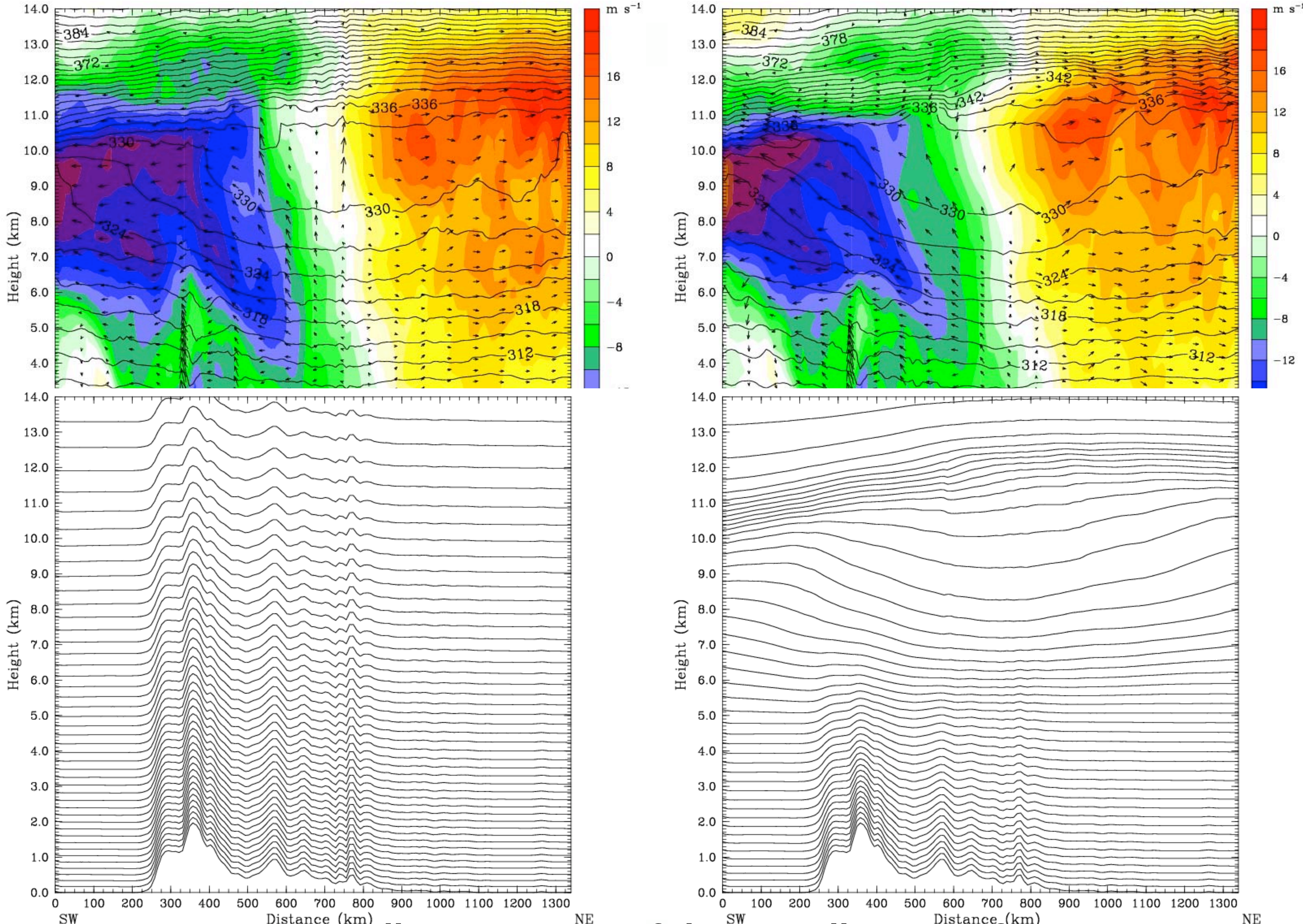
Exemplary vertical cross-sections: Wind component along cross-section and potential temperature, 12 Aug 00 UTC

standard coordinate



adaptive coordinate

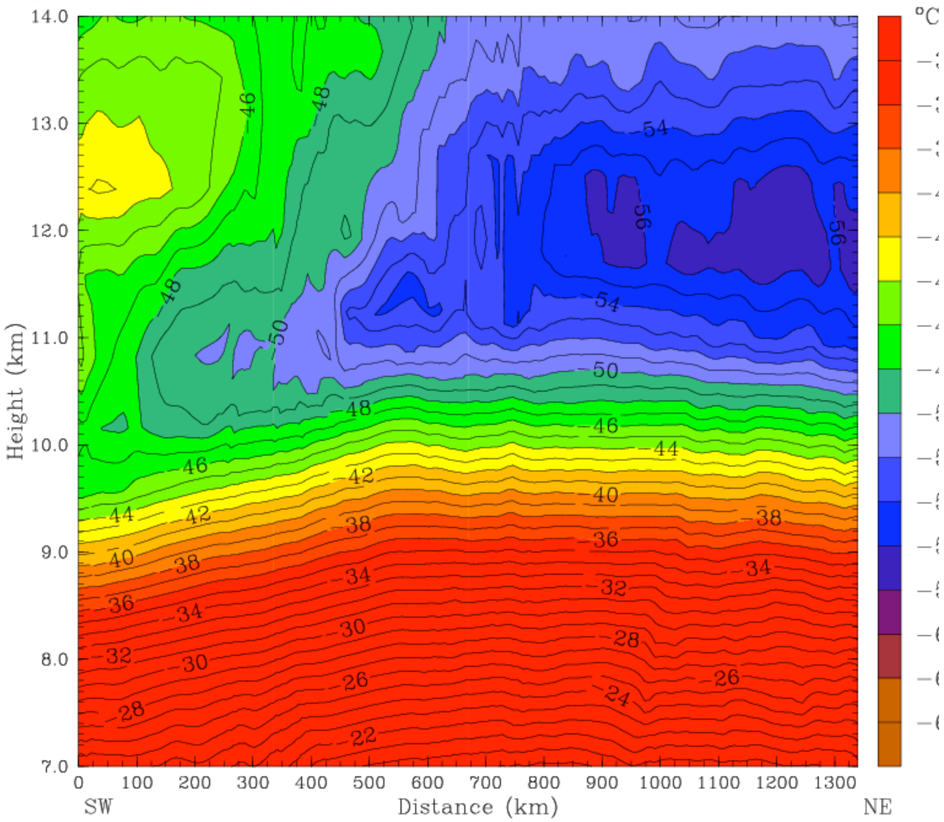




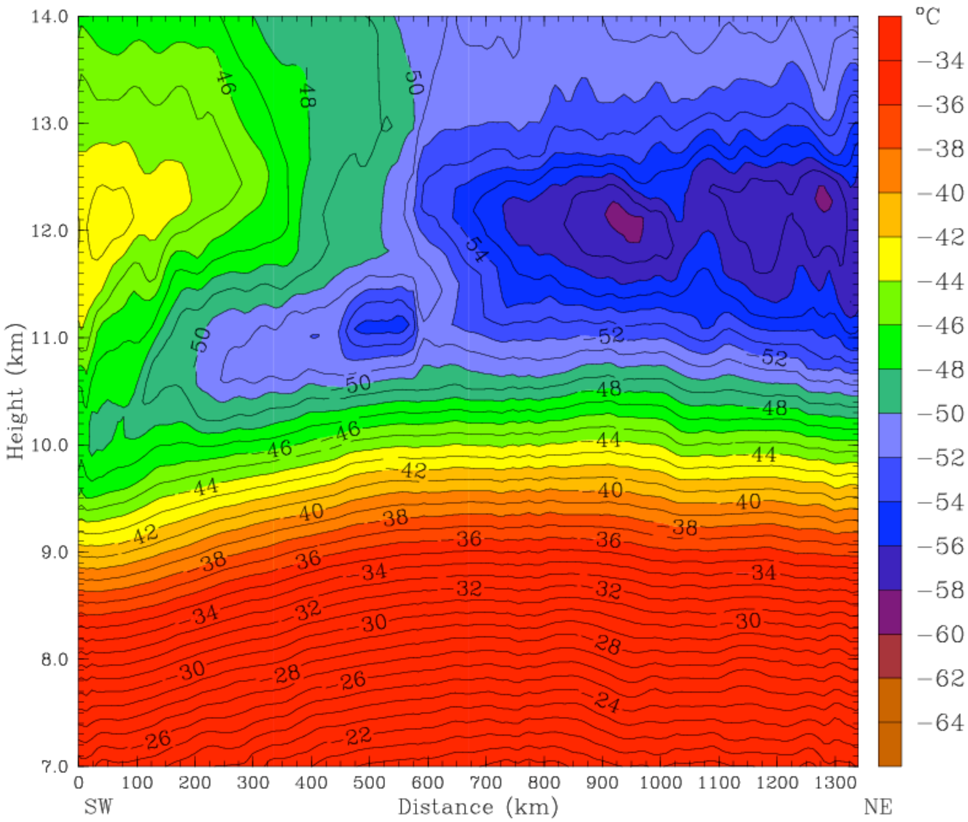
Corresponding structure of the coordinate surfaces

Temperature field

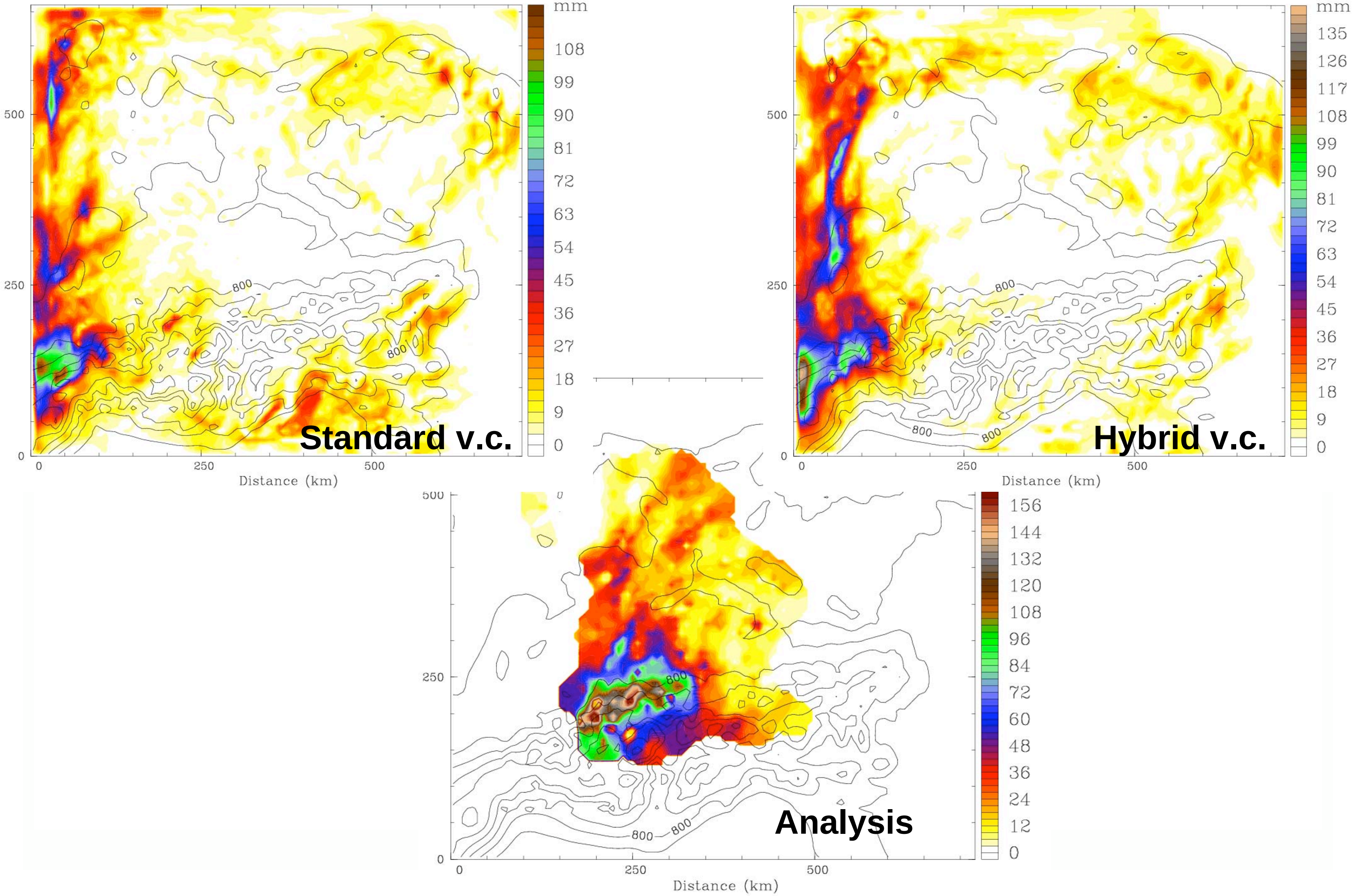
standard coordinate



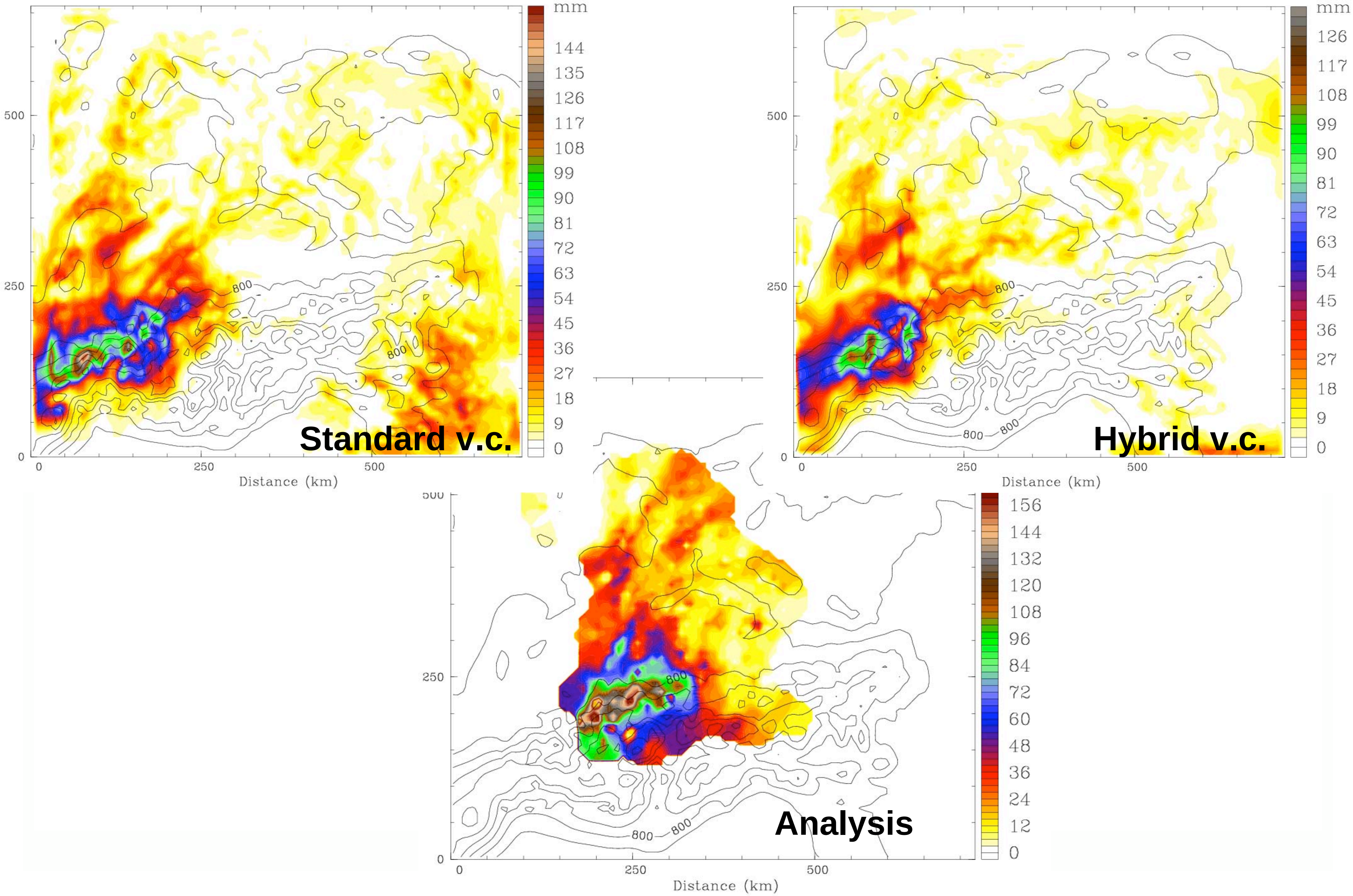
adaptive coordinate



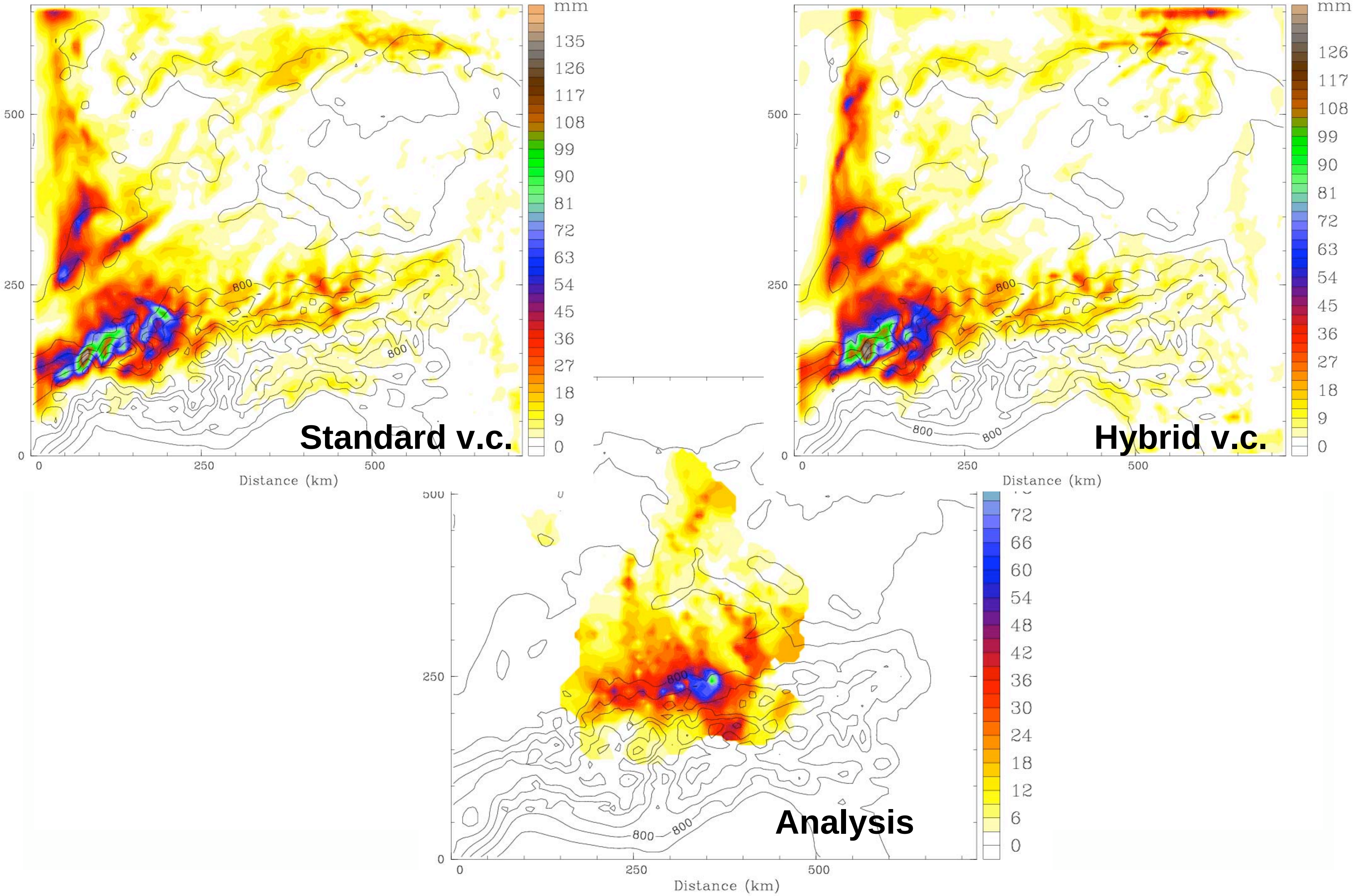
24h-acc. precipitation 22-23 August 05 (pressure-level initialization)



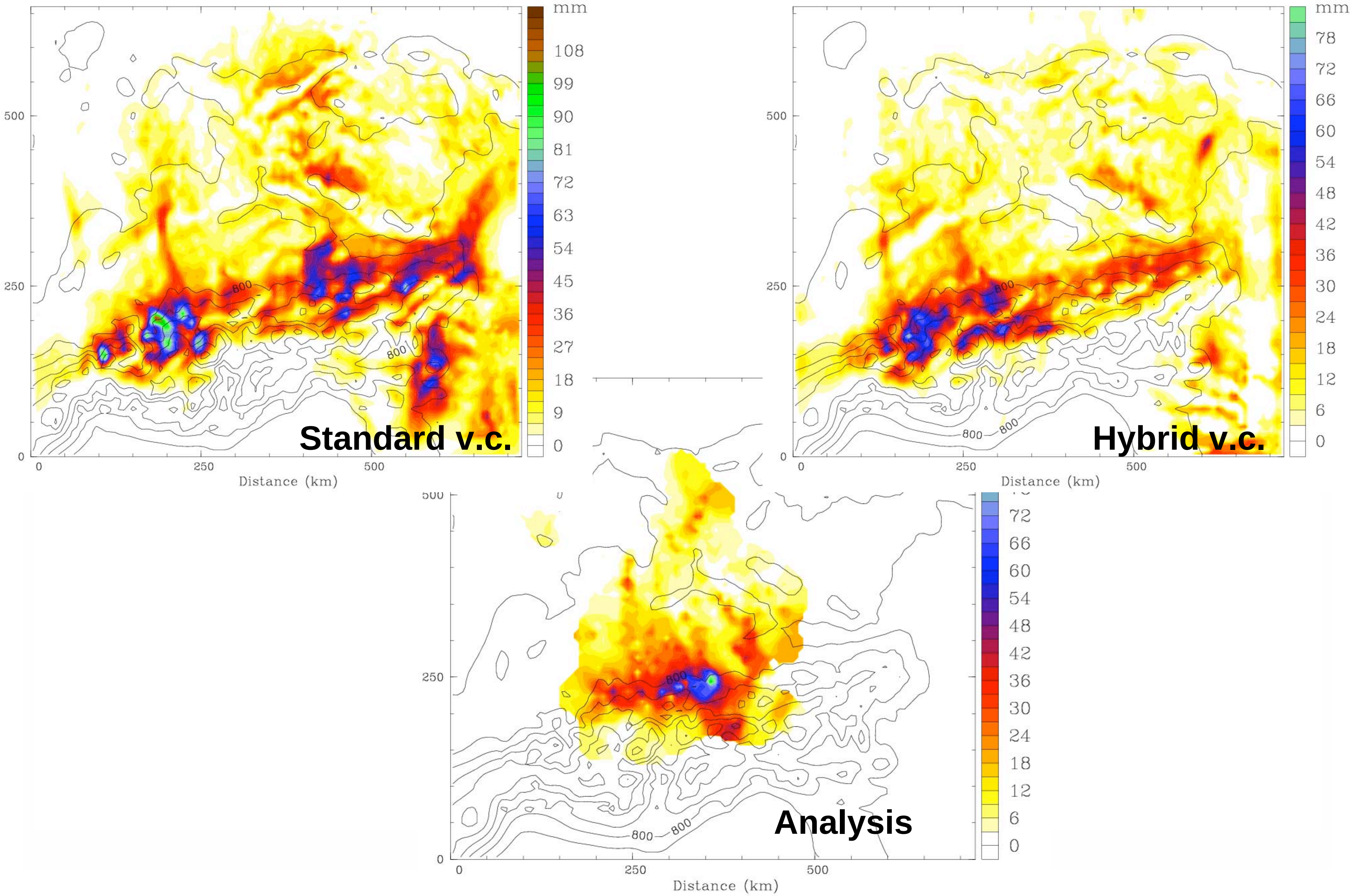
24h-acc. precipitation 22-23 August 05 (model-level initialization)



24h-acc. precipitation 23-24 August 05 (pressure-level initialization)



24h-acc. precipitation 23-24 August 05 (model-level initialization)



Summary for real-case tests

- The hybrid coordinate significantly improves the representation of the tropopause in the model, particularly in combination with initial and lateral boundary conditions from model-level data
- For precipitation, no significant signal could be extracted; the difference between using pressure-level and model-level analysis data appears to be larger
- (Not shown:) The sea-level pressure is systematically higher (by 1-4 hPa) with model-level data than with pressure-level data