The Development and Performance of NCEP GFS in sigma-theta Coordinates

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Approach

- Instead of developing a sigma-theta hybrid coordinates dynamics code
- A generic coordinates dynamic code is implemented into NCEP GFS, so it can have possible sigma, sigma-pressure, sigma-theta, and sigma-theta-pressure combination hybrid coordinates.
- Thus we can compare different coordinates under the same system.

Contents

- Introducing a generic coordinate system with enthalpy as thermodynamic equation
- Discritization of the generic coordinate system
- Performance of the new dynamics code, especially in sigma-theta hybrid coordinates
- Problems of sigma-theta coordinate and possible solution for next version

The thermodynamics equation used in operational GFS is

$$\frac{dT_v}{dt} - \frac{\kappa T_v}{p}\frac{dp}{dt} = F_{T_v}$$

where

$$T_v = \left\langle 1 + \left(R_v / R_d - 1 \right) q \right\rangle T$$

$$\kappa = \frac{R_d}{C_P} = \frac{R_d}{C_{Pd} + \left(C_{Pv} - C_{Pd} \right) q} = \frac{\kappa_d}{1 + \left(C_{Pv} / C_{Pd} - 1 \right) q}$$

with ideal-gas law of $p = \rho R_d T_v$

including only standard atmospheric dry air and vapor, but current GFS, we have humidity, ozone, and cloud water, and other tracers will be added in.

From the ideal-gas law for individual gas as

$$p_i = \rho_i R_i T$$

from $$p = \sum_{i=1}^{N} p_i$$

through $$p = \sum_{i=1}^{N} \rho_i R_i T = \rho \sum_{i=1}^{N} \frac{\rho_i}{\rho} R_i T = \rho R T$$

we have $$p = \rho R T$$

where $$\rho = \sum_{i=1}^{N} \rho_i$$ and let $$q_i = \frac{\rho_i}{\rho}$$

we get $$R = \sum_{i=0}^{Ntracers} q_i R_i = (1 - \sum_{i=1}^{Ntracers} q_i) R_d + \sum_{i=1}^{Ntracers} R_i q_i$$

The thermodynamic equation, derived from internal energy equation, should be written as

$$\rho \frac{dC_P T}{dt} - \frac{dp}{dt} = \rho Q$$

and the same as R

$$C_P = \sum_{i=0}^{Ntracers} C_{P\,i} q_i = (1 - \sum_{i=1}^{Ntracers} q_i) C_{P\,d} + \sum_{i=1}^{Ntracers} C_{P\,i} q_i$$

Our current tracers are
specific humidity, ozone and cloud water, thus Ntracers=3

Ri	287.05	461.50	191.87
Cpi	1004.6	1846.0	39370.

Instead of using tracer equations to derive

$$\rho\frac{dC_P T}{dt}-\frac{dp}{dt}=\rho Q$$

into

$$\left(\sum_{i=1}^{\mathbf{N}} C_{P_i} q_i\right)\frac{dT}{dt}+T\left(\sum_{i=1}^{\mathbf{N}} C_{P_i}\frac{dq_i}{dt}\right)-\frac{1}{\rho}\frac{dp}{dt}=Q$$

we let $h=C_P T$ as a prognostic variable, enthalpy.

the above thermodynamics equation can be re-written as

$$\frac{dh}{dt}-\frac{\kappa h}{p}\frac{dp}{dt}=Q \qquad \text{comparing} \qquad \frac{dT_v}{dt}-\frac{\kappa T_v}{p}\frac{dp}{dt}=F_{T_v}$$

From horizontal pressure gradient

We have

$$-\frac{1}{\rho}(\nabla p)_z = -\frac{RT}{p}(\nabla p)_z = -\frac{\kappa h}{p}(\nabla p)_z$$

from generalized coordinate transform, above can be written

$$-\frac{\kappa h}{p}(\nabla p)_z = -\frac{\kappa h}{p}\left[(\nabla p)_\zeta - \frac{\partial p}{\partial \Phi}(\nabla \Phi)_\zeta\right]$$

from hydrostatic $\frac{\partial p}{\partial z} = -\rho g(z)$ and $\frac{\partial \Phi}{\partial z} = g(z)$ or $\Phi = \int_o^z g(z)dz$

the pressure gradient force and hydrostatic can be written as

$$-\frac{\kappa h}{p}(\nabla p)_z = -\frac{\kappa h}{p}(\nabla p)_\zeta - (\nabla \Phi)_\zeta$$

$$\frac{\partial \Phi}{\partial \zeta} = -\frac{\kappa h}{p}\frac{\partial p}{\partial \zeta}$$

We can define potential enthalpy Θ as following

$$\Theta = \frac{h}{\pi}$$ where $$\pi = \left(\frac{p}{p_0}\right)^{\kappa}$$

then total derivative of potential enthalpy can be derived as

$$\frac{d\Theta}{dt} = \frac{1}{\pi}\left(\frac{dh}{dt} - h\frac{d\ln\pi}{dt}\right) = \frac{1}{\pi}\left(\frac{dh}{dt} - \frac{\kappa h}{p}\frac{dp}{dt} - h\ln\frac{p}{p_0}\frac{d\kappa}{dt}\right) = \frac{Q}{\pi} - \frac{h}{\pi}\ln\left(\frac{p}{p_0}\right)\frac{d\kappa}{dt}$$

if adiabatic, we have $Q = 0$ and

if no sink/source $\frac{dq_i}{dt} = 0$ we have $\frac{dR}{dt} = \frac{dC_P}{dt} = \frac{d\kappa}{dt} = 0$

thus $\frac{d\Theta}{dt} = 0$ conservation of potential enthalpy

Put enthalpy into generic coordinate system

$$\frac{\partial u^*}{\partial t} = -m^2 u^* \frac{\partial u^*}{a\partial\lambda} - m^2 v^* \frac{\partial u^*}{a\partial\varphi} - \dot{\zeta}\frac{\partial u^*}{\partial\zeta} - \frac{\kappa h}{p}\frac{\partial p}{a\partial\lambda} - \frac{\partial\Phi}{a\partial\lambda} + f_s v^*$$

$$\frac{\partial v^*}{\partial t} = -m^2 u^* \frac{\partial v^*}{a\partial\lambda} - m^2 v^* \frac{\partial v^*}{a\partial\varphi} - \dot{\zeta}\frac{\partial v^*}{\partial\zeta} - \frac{\kappa h}{p}\frac{\partial p}{a\partial\varphi} - \frac{\partial\Phi}{a\partial\varphi} - f_s u^* - m^2 \frac{s^{*2}}{a}\sin\phi$$

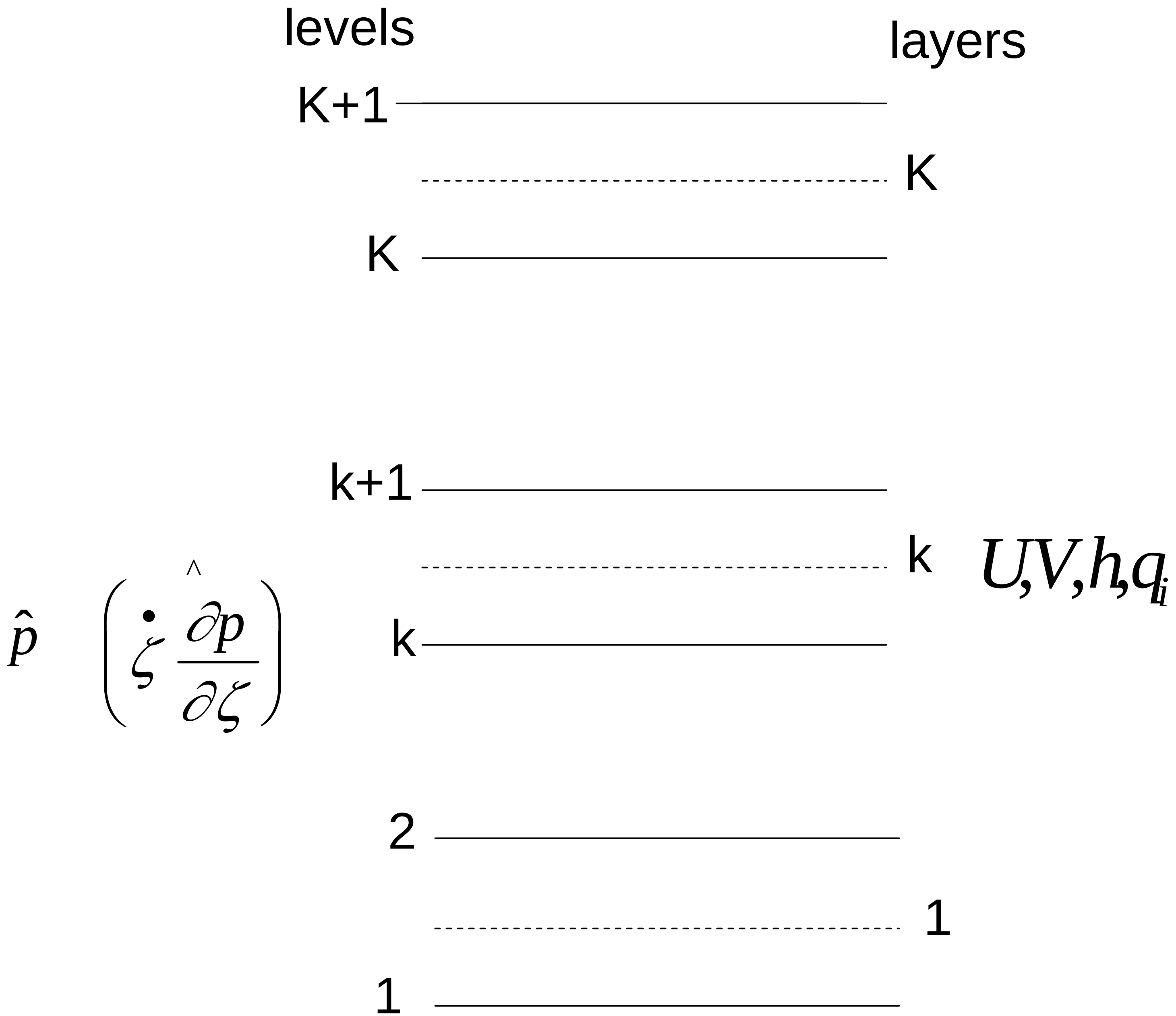
$$\frac{\partial h}{\partial t} = -m^2 u^* \frac{\partial h}{a\partial\lambda} - m^2 v^* \frac{\partial h}{a\partial\varphi} - \dot{\zeta}\frac{\partial h}{\partial\zeta} + \frac{\kappa h}{p}\frac{dp}{dt}$$

$$\frac{\partial(\partial p/\partial\zeta)}{\partial t} = -m^2\left(\frac{\partial u^*(\partial p/\partial\zeta)}{a\partial\lambda} + \frac{\partial v^*(\partial p/\partial\zeta)}{a\partial\varphi}\right) - \frac{\partial\dot{\zeta}(\partial p/\partial\zeta)}{\partial\zeta}$$

$$\frac{\partial q_i}{\partial t} = -m^2 u^* \frac{\partial q_i}{a\partial\lambda} - m^2 v^* \frac{\partial q_i}{a\partial\varphi} - \dot{\zeta}\frac{\partial q_i}{\partial\zeta}$$

Consider Multi-conserving

- The natural of the differential equations
- Conservation of momentum
- Conservation of total energy
- Conservation of mass
- Conservation of potential enthalpy
- (Juang 2005 NCEP Office Note #445)


levels
layers
K+1
K
K
k+1
k
U, V, h, q_i
k
2
1
1
$\hat{p}$
$\widehat{\left(\dot{\zeta}\frac{\partial p}{\partial \zeta}\right)}$

Conservation Constraint 1
Mass weighted vertically integration of PGF

$$\int_{\zeta_s}^{\zeta_T} \frac{\partial p}{\partial \zeta}\left(\nabla\Phi + \frac{\kappa h}{p}\nabla p\right)d\zeta = \int_{\zeta_s}^{\zeta_T}\left[\nabla\left(\frac{\partial p}{\partial \zeta}\Phi\right) - \Phi\nabla\frac{\partial p}{\partial \zeta} + \frac{\partial p}{\partial \zeta}\frac{\kappa h}{p}\nabla p\right]d\zeta$$

$$= \int_{\zeta_s}^{\zeta_T}\nabla\left(\frac{\partial p}{\partial \zeta}\Phi\right)d\zeta - \int_{\zeta_s}^{\zeta_T}\left(\Phi\nabla\frac{\partial p}{\partial \zeta} + \frac{\partial \Phi}{\partial \zeta}\nabla p\right)d\zeta$$

$$= \nabla\int_{\zeta_s}^{\zeta_T}\left(\frac{\partial p}{\partial \zeta}\Phi\right)d\zeta - \int_{\zeta_s}^{\zeta_T}\frac{\partial \Phi\nabla p}{\partial \zeta}d\zeta$$

$$= \nabla\int_{\zeta_s}^{\zeta_T}\left(\frac{\partial p}{\partial \zeta}\Phi\right)d\zeta - \Phi_T\nabla p_T + \Phi_s\nabla p_s$$

Since we need pressure at model layer and level

let $$p_k = f\left(\hat{p}_{k+1}, \hat{p}_k\right)$$

so $$\nabla p_k = \frac{\partial p_k}{\partial \hat{p}_{k+1}} \nabla \hat{p}_{k+1} + \frac{\partial p_k}{\partial \hat{p}_k} \nabla \hat{p}_k$$

then let $$\left(\frac{\partial p}{\partial \zeta}\right)_k = \frac{\hat{p}_{k+1} - \hat{p}_k}{\Delta \zeta_k}$$ and $$\nabla \hat{p}_{Top} = 0$$

the equation in the previous page can be written as

$$\sum_{k=1}^{K} [-\Phi_k \nabla (\hat{p}_{k+1} - \hat{p}_k) + (\hat{p}_{k+1} - \hat{p}_k) \left(\frac{\kappa h}{p}\right)_k \left(\frac{\partial p_k}{\partial \hat{p}_{k+1}} \nabla \hat{p}_{k+1} + \frac{\partial p_k}{\partial \hat{p}_k} \nabla \hat{p}_k\right)] = \Phi_s \nabla p_s$$

Expanding the above equation for all k, we will find there can be grouped based on $\nabla \hat{p}_k$

Regroup the previous equation, let each group=0, we have

$$-\Phi_k + (\hat{p}_{k+1} - \hat{p}_k)\left(\frac{\kappa h}{p}\right)_k \frac{\partial p_k}{\partial \hat{p}_{k+1}} + \Phi_{k+1} + (\hat{p}_{k+2} - \hat{p}_{k+1})\left(\frac{\kappa h}{p}\right)_{k+1} \frac{\partial p_{k+1}}{\partial \hat{p}_{k+1}} = 0$$

$$\Phi_1 + (\hat{p}_2 - p_s)\left(\frac{\kappa h}{p}\right)_1 \frac{\partial p_1}{\partial p_s} = \Phi_s$$

Thus hydrostatic between layers

$$\Phi_{k+1} - \Phi_k = -(\hat{p}_{k+2} - \hat{p}_{k+1})\left(\frac{\kappa h}{p}\right)_{k+1} \frac{\partial p_{k+1}}{\partial \hat{p}_{k+1}} - (\hat{p}_{k+1} - \hat{p}_k)\left(\frac{\kappa h}{p}\right)_k \frac{\partial p_k}{\partial \hat{p}_{k+1}}$$

And let hydrostatic between layer and level

$$\Phi_{k+1} - \hat{\Phi}_{k+1} = -(\hat{p}_{k+2} - \hat{p}_{k+1})\left(\frac{\kappa h}{p}\right)_{k+1} \frac{\partial p_{k+1}}{\partial \hat{p}_{k+1}}$$

$$\hat{\Phi}_{k+1} - \Phi_k = -(\hat{p}_{k+1} - \hat{p}_k)\left(\frac{\kappa h}{p}\right)_k \frac{\partial p_k}{\partial \hat{p}_{k+1}}$$

Conservation Constraint 2
Consistency in energy conversion term

From thermodynamic energy

$$\frac{\partial p}{\partial \zeta}\left[\frac{\partial h}{\partial t} = -m^2 u^* \frac{\partial h}{a\partial\lambda} - m^2 v^* \frac{\partial h}{a\partial\varphi} - \dot{\zeta}\frac{\partial h}{\partial \zeta} + \frac{\kappa h}{p}\frac{dp}{dt}\right]$$

$$h\left[\frac{\partial(\partial p/\partial\zeta)}{\partial t} = -m^2\left(\frac{\partial u^*(\partial p/\partial\zeta)}{a\partial\lambda} + \frac{\partial v^*(\partial p/\partial\zeta)}{a\partial\varphi}\right) - \frac{\partial\dot{\zeta}(\partial p/\partial\zeta)}{\partial\zeta}\right]$$

$$\frac{\partial(\frac{\partial p}{\partial\zeta})h}{\partial t} = -m^2\left(\frac{\partial u^*(\frac{\partial p}{\partial\zeta})h}{a\partial\lambda} + \frac{\partial v^*(\frac{\partial p}{\partial\zeta})h}{a\partial\varphi}\right) - \frac{\partial\dot{\zeta}(\frac{\partial p}{\partial\zeta})h}{\partial\zeta} + \boxed{\frac{\partial p}{\partial\zeta}\frac{\kappa h}{p}\frac{dp}{dt}}$$

From kinetic energy equation

$$\frac{\partial p}{\partial \zeta} u^* \left[\frac{\partial u^*}{\partial t} = -m^2 u^* \frac{\partial u^*}{a\partial \lambda} - m^2 v^* \frac{\partial u^*}{a\partial \varphi} - \dot{\zeta}\frac{\partial u^*}{\partial \zeta} - \frac{\kappa h}{p}\frac{\partial p}{a\partial \lambda} - \frac{\partial \Phi}{a\partial \lambda} + f_s v^* \right]$$

$$\frac{\partial p}{\partial \zeta} v^* \left[\frac{\partial v^*}{\partial t} = -m^2 u^* \frac{\partial v^*}{a\partial \lambda} - m^2 v^* \frac{\partial v^*}{a\partial \varphi} - \dot{\zeta}\frac{\partial v^*}{\partial \zeta} - \frac{\kappa h}{p}\frac{\partial p}{a\partial \varphi} - \frac{\partial \Phi}{a\partial \varphi} - f_s u^* - m^2 \frac{s^{*2}}{a} \sin\phi \right]$$

$$\frac{1}{2}(u^{*2} + v^{*2}) \left[\frac{\partial(\partial p/\partial \zeta)}{\partial t} = -m^2 \left(\frac{\partial u^*(\partial p/\partial \zeta)}{a\partial \lambda} + \frac{\partial v^*(\partial p/\partial \zeta)}{a\partial \varphi} \right) - \frac{\partial \dot{\zeta}(\partial p/\partial \zeta)}{\partial \zeta} \right]$$

$$K^* = (\partial p/\partial \zeta)\frac{1}{2}(u^{*2} + v^{*2}) = \frac{(\partial p/\partial \zeta)}{m^2}\frac{1}{2}(u^2 + v^2)$$

$$\frac{DK^*}{Dt} = \frac{\partial K^*}{\partial t} + m^2 \nabla \bullet V K^* + \frac{\partial \dot{\zeta} K^*}{\partial \zeta}$$

$$\frac{DK^*}{Dt} = -\frac{\partial p}{\partial \zeta}\vec{V} \bullet \left(\nabla\Phi + \frac{\kappa h}{p}\nabla p \right)$$

$$= -\nabla \bullet \left(\frac{\partial p}{\partial \zeta}\Phi\vec{V} \right) + \Phi\nabla \bullet \left(\frac{\partial p}{\partial \zeta}\vec{V} \right) - \frac{\partial p}{\partial \zeta}\frac{\kappa h}{p}\vec{V} \bullet \nabla p$$

$$= -\nabla \bullet \left(\frac{\partial p}{\partial \zeta}\Phi\vec{V} \right) - \Phi\frac{1}{m^2}\left(\frac{\partial(\partial p/\partial \zeta)}{\partial t} + \frac{\partial \dot{\zeta}(\partial p/\partial \zeta)}{\partial \zeta} \right) - \frac{\partial p}{\partial \zeta}\frac{\kappa h}{p}\vec{V} \bullet \nabla p$$

$$= -\nabla \bullet \left(\frac{\partial p}{\partial \zeta}\Phi\vec{V} \right) - \Phi\frac{1}{m^2}\left(\frac{\partial(\partial p/\partial \zeta)}{\partial t} + \frac{\partial \dot{\zeta}(\partial p/\partial \zeta)}{\partial \zeta} \right) - \frac{\partial p}{\partial \zeta}\frac{\kappa h}{p}\frac{1}{m^2}\left(\frac{dp}{dt} - \frac{\partial p}{\partial t} - \dot{\zeta}\frac{\partial p}{\partial \zeta} \right)$$

$$= -\nabla \bullet \left(\frac{\partial p}{\partial \zeta}\Phi\vec{V} \right) - \frac{1}{m^2}\Phi\left(\frac{\partial(\partial p/\partial t)}{\partial \zeta} + \frac{\partial \dot{\zeta}(\partial p/\partial \zeta)}{\partial \zeta} \right) - \frac{1}{m^2}\frac{\partial \Phi}{\partial \zeta}\left(\frac{\partial p}{\partial t} + \dot{\zeta}\frac{\partial p}{\partial \zeta} \right) - \frac{1}{m^2}\frac{\partial p}{\partial \zeta}\frac{\kappa h}{p}\frac{dp}{dt}$$

$$= -\nabla \bullet \left(\frac{\partial p}{\partial \zeta}\Phi\vec{V} \right) - \frac{1}{m^2}\frac{\partial}{\partial \zeta}\left[\Phi\left(\frac{\partial p}{\partial t} + \dot{\zeta}\frac{\partial p}{\partial \zeta} \right) \right] - \frac{1}{m^2}\frac{\partial p}{\partial \zeta}\frac{\kappa h}{p}\frac{dp}{dt}$$

From the known discretization of hydrostatic relation

$$\Phi_k - \hat{\Phi}_k = -(\hat{p}_{k+1} - \hat{p}_k)\left(\frac{\kappa h}{p}\right)_k \frac{\partial p_k}{\partial \hat{p}_k}$$

$$\hat{\Phi}_{k+1} - \Phi_k = -(\hat{p}_{k+1} - \hat{p}_k)\left(\frac{\kappa h}{p}\right)_k \frac{\partial p_k}{\partial \hat{p}_{k+1}}$$

$$\left(\frac{\partial p}{\partial \zeta}\right)_k = \frac{1}{\Delta \zeta}(\hat{p}_{k+1} - \hat{p}_k)$$

the equation in the previous page can be written, after some manipulations, as following

$$\left(\frac{dp}{dt}\right)_k = \frac{\partial p_k}{\partial \hat{p}_{k+1}} \frac{\partial \hat{p}_{k+1}}{\partial t} + \frac{\partial p_k}{\partial \hat{p}_k} \frac{\partial \hat{p}_k}{\partial t} + m^2 \vec{V}_k \bullet \nabla p_k + \frac{\partial p_k}{\partial \hat{p}_{k+1}} \left(\dot{\zeta} \frac{\partial p}{\partial \zeta}\right)^{\wedge}_{k+1} + \frac{\partial p_k}{\partial \hat{p}_k} \left(\dot{\zeta} \frac{\partial p}{\partial \zeta}\right)^{\wedge}_k$$

For simplicity, we can have

$$\frac{\partial p_k}{\partial \hat{p}_k} = \frac{\partial p_k}{\partial \hat{p}_{k+1}} = \frac{1}{2}$$

so

$$p_k = \frac{1}{2}\left(\hat{p}_k + \hat{p}_{k+1}\right)$$

in order to satisfy previous equation as

$$\left(\frac{dp}{dt}\right)_k = \frac{1}{2}\left\langle \frac{\partial \hat{p}_k}{\partial t} + \frac{\partial \hat{p}_{k+1}}{\partial t} \right\rangle + m^2 \vec{V}_k \bullet \nabla p_k + \frac{1}{2}\left\langle \left(\dot{\zeta}\frac{\partial \hat{p}}{\partial \zeta}\right)_k + \left(\dot{\zeta}\frac{\partial \hat{p}}{\partial \zeta}\right)_{k+1} \right\rangle$$

It is obtained from in kinetic energy equation with momentum conservation, if we apply it to thermodynamic equation for potential energy equation

$$\frac{dh_k}{dt} = \frac{(\kappa h)_k}{p_k}\left(\frac{dp}{dt}\right)_k$$

then the total energy will be conserved.

Discretizing continuity equation

with $$\left(\frac{\partial p}{\partial \zeta}\right)_k = \frac{(\hat{p}_{k+1} - \hat{p}_k)}{\Delta \zeta}$$

$$\frac{\partial(\hat{p}_{k+1} - \hat{p}_k)}{\partial t} = -m^2\left((\hat{p}_{k+1} - \hat{p}_k)\left(\frac{\partial u_k^*}{a\partial\lambda} + \frac{\partial v_k^*}{a\partial\varphi}\right) + u_k^*\frac{\partial(\hat{p}_{k+1} - \hat{p}_k)}{a\partial\lambda} + v_k^*\frac{\partial(\hat{p}_{k+1} - \hat{p}_k)}{a\partial\varphi}\right)$$
$$-\left\langle\left(\dot{\zeta}\frac{\partial p}{\partial \zeta}\right)^{\wedge}_{k+1} - \left(\dot{\zeta}\frac{\partial p}{\partial \zeta}\right)^{\wedge}_{k}\right\rangle$$

vertical sum from top with $$\left(\dot{\zeta}\frac{\partial p}{\partial \zeta}\right)^{\wedge}_{K+1} = \left(\dot{\zeta}\frac{\partial p}{\partial \zeta}\right)^{\wedge}_{1} = 0$$

we obtain pressure equation for all levels, including Ps

$$\frac{\partial \hat{p}_k}{\partial t} = -m^2\sum_{i=k}^{K}\left((\hat{p}_i - \hat{p}_{i+1})\left(\frac{\partial u_i^*}{a\partial\lambda} + \frac{\partial v_i^*}{a\partial\varphi}\right) + u_i^*\frac{\partial(\hat{p}_i - \hat{p}_{i+1})}{a\partial\lambda} + v_i^*\frac{\partial(\hat{p}_i - \hat{p}_{i+1})}{a\partial\varphi}\right) - \left(\dot{\zeta}\frac{\partial p}{\partial \zeta}\right)^{\wedge}_{k}$$

Substitute following

$$\frac{\partial \hat{p}_{k+1}}{\partial t} = -m^2 \sum_{i=k+1}^{K} \left((\hat{p}_i - \hat{p}_{i+1}) \left(\frac{\partial u_i^*}{a\partial\lambda} + \frac{\partial v_i^*}{a\partial\varphi} \right) + u_i^* \frac{\partial(\hat{p}_i - \hat{p}_{i+1})}{a\partial\lambda} + v_i^* \frac{\partial(\hat{p}_i - \hat{p}_{i+1})}{a\partial\varphi} \right) - \left(\dot{\zeta} \frac{\partial \hat{p}}{\partial\zeta} \right)_{k+1}$$

$$\frac{\partial \hat{p}_{k}}{\partial t} = -m^2 \sum_{i=k}^{K} \left((\hat{p}_i - \hat{p}_{i+1}) \left(\frac{\partial u_i^*}{a\partial\lambda} + \frac{\partial v_i^*}{a\partial\varphi} \right) + u_i^* \frac{\partial(\hat{p}_i - \hat{p}_{i+1})}{a\partial\lambda} + v_i^* \frac{\partial(\hat{p}_i - \hat{p}_{i+1})}{a\partial\varphi} \right) - \left(\dot{\zeta} \frac{\partial \hat{p}}{\partial\zeta} \right)_{k}$$

into

$$\frac{dh_k}{dt} = \frac{(\kappa h)_k}{\hat{p}_k + \hat{p}_{k+1}} \left[\left(\frac{\partial \hat{p}_k}{\partial t} + \frac{\partial \hat{p}_{k+1}}{\partial t} \right) + m^2 \vec{V}_k \bullet \nabla(\hat{p}_k + \hat{p}_{k+1}) + \left\langle \left(\dot{\zeta} \frac{\partial \hat{p}}{\partial\zeta} \right)_k + \left(\dot{\zeta} \frac{\partial \hat{p}}{\partial\zeta} \right)_{k+1} \right\rangle \right]$$

We got energy conversion without vertical flux

$$\frac{dh_k}{dt} = \frac{(\kappa h)_k m^2}{\hat{p}_k + \hat{p}_{k+1}} \Bigg[V_k^* \bullet \nabla(\hat{p}_k + \hat{p}_{k+1}) - \sum_{i=k}^{K} \Big((\hat{p}_i - \hat{p}_{i+1}) D_i^* + V_i^* \bullet \nabla(\hat{p}_i - \hat{p}_{i+1}) \Big) - \sum_{i=k+1}^{K} \Big((\hat{p}_i - \hat{p}_{i+1}) D_i^* + V_i^* \bullet \nabla(\hat{p}_i - \hat{p}_{i+1}) \Big) \Bigg]$$

Vertical advection for momentum, tracers, & potential h
Total integral of total derivative should be zero if no force

$$\frac{\partial A}{\partial t} = -u^* \frac{\partial A}{a\partial\lambda} - v^* \frac{\partial A}{a\partial\phi} - \dot{\zeta}\frac{\partial A}{\partial\zeta} + F$$

$$\frac{\partial\rho}{\partial t} = -\left(\frac{\partial\rho u^*}{a\partial\lambda} + \frac{\partial\rho v^*}{a\partial\phi}\right) - \frac{\partial\rho\dot{\zeta}}{\partial\zeta}$$

Combine them, we have

$$\frac{\partial\rho A}{\partial t} = -\frac{\partial\rho A u^*}{a\partial\lambda} - \frac{\partial\rho A v^*}{a\partial\phi} - \frac{\partial\rho A\dot{\zeta}}{\partial\zeta} + \rho F_A$$

Then total integrate without forcing, we should have

$$\oiiint_{ijk} \frac{\partial\rho A}{\partial t} = -\oiiint_{kji} \frac{\partial\rho A u^*}{a\partial\lambda} - \oiiint_{kij} \frac{\partial\rho A v^*}{a\partial\phi} - \oiiint_{ijk} \frac{\partial\rho A\dot{\zeta}}{\partial\zeta} = 0$$

If we combine equations without dealing vertical advection as

$$\frac{\partial \rho A}{\partial t} = -\frac{\partial \rho A u^*}{a \partial \lambda} - \frac{\partial \rho A v^*}{a \partial \phi} - \rho \dot{\zeta} \frac{\partial A}{\partial \zeta} - A \frac{\partial \rho \dot{\zeta}}{\partial \zeta} + \rho F$$

Compare with the previous equation

$$\frac{\partial \rho A}{\partial t} = -\frac{\partial \rho A u^*}{a \partial \lambda} - \frac{\partial \rho A v^*}{a \partial \phi} - \frac{\partial \rho A \dot{\zeta}}{\partial \zeta} + \rho F$$

We have

$$\rho \dot{\zeta} \frac{\partial A}{\partial \zeta} + A \frac{\partial \rho \dot{\zeta}}{\partial \zeta} = \frac{\partial A \rho \dot{\zeta}}{\partial \zeta}$$

$$\left(\dot{\zeta} \frac{\partial A}{\partial \zeta} \right)_k = \frac{1}{\rho} \left(\frac{\partial A \rho \dot{\zeta}}{\partial \zeta} - A \frac{\partial \rho \dot{\zeta}}{\partial \zeta} \right)_k$$

$$= \frac{(A_{k-1} - A_k)\left(\dot{\zeta} \frac{\partial p}{\partial \zeta} \right)_k + (A_k - A_{k+1})\left(\dot{\zeta} \frac{\partial p}{\partial \zeta} \right)_{k+1}}{2(p_k - p_{k+1})}$$

Expand the thermodynamic equation as

$$\pi_k\left(\frac{\partial\Theta}{\partial t}+m^2\vec{V}\bullet\nabla\Theta\right)_k+\Theta_k\left(\frac{\partial\pi}{\partial t}+m^2\vec{V}\bullet\nabla\pi\right)_k+\left(\dot{\zeta}\frac{\partial h}{\partial\zeta}\right)_k$$

$$=\frac{(\kappa h)_k}{p_k}\left[\frac{1}{2}\left\langle\frac{\partial\hat{p}_k}{\partial t}+\frac{\partial\hat{p}_{k+1}}{\partial t}\right\rangle+m^2\vec{V}_k\bullet\nabla p_k+\frac{1}{2}\left\langle\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_k+\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_{k+1}\right\rangle\right]$$

apply

$$\left(\frac{\partial\pi}{\partial t}+m^2\vec{V}\bullet\nabla\pi\right)_k=\left[\frac{\partial\pi_k}{\partial p_k}\frac{\partial p_k}{\partial t}+m^2\frac{\partial\pi_k}{\partial p_k}\vec{V}_k\bullet\nabla p_k\right]$$

$$\frac{\partial\pi_k}{\partial p_k}=\frac{\kappa\pi_k}{p_k}$$

$$p_k=\frac{1}{2}\left(\hat{p}_k+\hat{p}_{k+1}\right)$$

$$\left(\frac{\partial\Theta}{\partial t}+m^2\vec{V}\bullet\nabla\Theta\right)_k+\left(\dot{\zeta}\frac{\partial\Theta}{\partial\zeta}\right)_k=0$$

We have

$$-\pi_k\left(\dot{\zeta}\frac{\partial\Theta}{\partial\zeta}\right)_k+\left(\dot{\zeta}\frac{\partial h}{\partial\zeta}\right)_k=\frac{(\kappa h)_k}{\hat{p}_k+\hat{p}_{k+1}}\left[\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_k+\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_{k+1}\right]$$

substitute advection term by potential h conserving

$$\left(\dot{\zeta}\frac{\partial\Theta}{\partial\zeta}\right)_k=\frac{1}{2(\hat{p}_k-\hat{p}_{k+1})}\left[\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_k\left(\left(\frac{h}{\pi}\right)_{k-1}-\left(\frac{h}{\pi}\right)_k\right)+\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_{k+1}\left(\left(\frac{h}{\pi}\right)_k-\left(\frac{h}{\pi}\right)_{k+1}\right)\right]$$

after some arrangement, we obtain

$$\left(\dot{\zeta}\frac{\partial h}{\partial\zeta}\right)_k=\frac{1}{2(\hat{p}_k-\hat{p}_{k+1})}\left\{\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_k\left(\frac{\pi_k}{\pi_{k-1}}h_{k-1}-\left(1-2\kappa_k\frac{\hat{p}_k-\hat{p}_{k+1}}{\hat{p}_k+\hat{p}_{k+1}}\right)h_k\right)\right.$$
$$\left.+\left(\dot{\zeta}\frac{\partial\hat{p}}{\partial\zeta}\right)_{k+1}\left(\left(1+2\kappa_k\frac{\hat{p}_k-\hat{p}_{k+1}}{\hat{p}_k+\hat{p}_{k+1}}\right)h_k-\frac{\pi_k}{\pi_{k+1}}h_{k+1}\right)\right\}$$

Summary for finite difference

$$\frac{du_k^*}{dt}=-\frac{(\kappa h)_k}{\hat p_k+\hat p_{k+1}}\left[\frac{\partial\hat p_k+\hat p_{k+1}}{a\partial\lambda}+\frac{\partial\hat p_k-\hat p_{k+1}}{a\partial\lambda}-\frac{(\hat p_k-\hat p_{k+1})}{(\hat p_k+\hat p_{k+1})}\frac{\partial\hat p_k+\hat p_{k+1}}{a\partial\lambda}\right]-2\sum_{i=1}^{k-1}\frac{(\kappa h)_i}{\hat p_i+\hat p_{i+1}}\left[\frac{\partial\hat p_i-\hat p_{i+1}}{a\partial\lambda}-\frac{\hat p_i-\hat p_{i+1}}{\hat p_i+\hat p_{i+1}}\frac{\partial\hat p_i+\hat p_{i+1}}{a\partial\lambda}\right]$$

$$-\frac{\partial\Phi_s}{a\partial\lambda}-\frac{\hat p_k-\hat p_{k+1}}{\hat p_k+\hat p_{k+1}}\frac{\partial(\kappa h)_k}{a\partial\lambda}-2\sum_{i=1}^{k-1}\frac{\hat p_i-\hat p_{i+1}}{\hat p_i+\hat p_{i+1}}\frac{\partial(\kappa h)_i}{a\partial\lambda}+f_s v_k^*$$

$$\frac{dv_k^*}{dt}=-\frac{(\kappa h)_k}{\hat p_k+\hat p_{k+1}}\left[\frac{\partial\hat p_k-\hat p_{k+1}}{a\partial\varphi}+\frac{\partial\hat p_k-\hat p_{k+1}}{a\partial\varphi}-\frac{(\hat p_k-\hat p_{k+1})}{(\hat p_k+\hat p_{k+1})}\frac{\partial\hat p_k+\hat p_{k+1}}{a\partial\varphi}\right]-2\sum_{i=1}^{k-1}\frac{(\kappa h)_i}{\hat p_i+\hat p_{i+1}}\left[\frac{\partial\hat p_i-\hat p_{i+1}}{a\partial\varphi}-\frac{\hat p_i-\hat p_{i+1}}{\hat p_i+\hat p_{i+1}}\frac{\partial\hat p_i+\hat p_{i+1}}{a\partial\varphi}\right]$$

$$-\frac{\partial\Phi_s}{a\partial\varphi}-\frac{\hat p_k-\hat p_{k+1}}{\hat p_k+\hat p_{k+1}}\frac{\partial(\kappa h)_k}{a\partial\varphi}-2\sum_{i=1}^{k-1}\frac{\hat p_i-\hat p_{i+1}}{\hat p_i+\hat p_{i+1}}\frac{\partial(\kappa h)_i}{a\partial\varphi}-f_s u_k^*-m^2\frac{s_k^{*2}}{a}\sin\phi$$

$$\frac{dh_k}{dt}=\frac{(\kappa h)_k}{\hat p_k+\hat p_{k+1}}m^2\left[\vec V_k\bullet\nabla(\hat p_k+\hat p_{k+1})-\sum_{i=k}^{K}\left((\hat p_i-\hat p_{i+1})D_i^*+V_i^*\bullet\nabla(\hat p_i-\hat p_{i+1})\right)-\sum_{i=k+1}^{K}\left((\hat p_i-\hat p_{i+1})D_i^*+V_i^*\bullet\nabla(\hat p_i-\hat p_{i+1})\right)\right]$$

$$\frac{\partial\hat p_k}{\partial t}=-m^2\sum_{i=k}^{K}\left((\hat p_i-\hat p_{i+1})\left(\frac{\partial u_i^*}{a\partial\lambda}+\frac{\partial v_i^*}{a\partial\varphi}\right)+u_i^*\frac{\partial(\hat p_i-\hat p_{i+1})}{a\partial\lambda}+v_i^*\frac{\partial(\hat p_i-\hat p_{i+1})}{a\partial\varphi}\right)-\left(\dot\zeta\frac{\partial\hat p}{\partial\zeta}\right)_k$$

$$\frac{dq_{i_k}}{dt}=0$$

where $$\frac{d()_k}{dt}=\frac{\partial()_k}{\partial t}+m^2\left(V^*\bullet\nabla()\right)_k+\frac{1}{2}\left\langle\left(\dot\zeta\frac{\partial\hat p}{\partial\zeta}\right)_k\frac{()_{k-1}-()_k}{\hat p_k-\hat p_{k+1}}+\left(\dot\zeta\frac{\partial\hat p}{\partial\zeta}\right)_{k+1}\frac{()_k-()_{k+1}}{\hat p_k-\hat p_{k+1}}\right\rangle$$ for u,v,q

and $$\frac{dh_k}{dt}=\frac{\partial h_k}{\partial t}+m^2\left(V^*\bullet\nabla h\right)_k+\frac{1}{2}\left\langle\left(\dot\zeta\frac{\partial\hat p}{\partial\zeta}\right)_k\frac{\alpha_{k-1}h_{k-1}-\gamma_k h_k}{\hat p_k-\hat p_{k+1}}+\left(\dot\zeta\frac{\partial\hat p}{\partial\zeta}\right)_{k+1}\frac{\delta_k h_k-\beta_{k+1}h_{k+1}}{\hat p_k-\hat p_{k+1}}\right\rangle$$ for h

The generic vertical coordinate can be defined as

$$\zeta = F(p_{sfc},\ \ p,\ \ h)$$

The vertical flux can be obtained by

$$\left(\frac{\partial \hat{\zeta}}{\partial t}\right)_k = \left(\frac{\partial F}{\partial p_{sfc}}\right)_k \frac{\partial p_{sfc_k}}{\partial t} + \left(\frac{\partial F}{\partial p}\right)_k \frac{\partial \hat{p}_k}{\partial t} + \left(\frac{\partial F}{\partial h}\right)_k \frac{\partial \hat{h}_k}{\partial t} = 0$$

then, separating horizontal and vertical terms in equations

$$\frac{\partial p_{sfc}}{\partial t} = \left(\frac{\partial p_{sfc}}{\partial t}\right)_H$$

$$\frac{\partial \hat{p}_k}{\partial t} = \left(\frac{\partial p_k}{\partial t}\right)_H - \left(\dot{\zeta}\frac{\hat{\partial p}}{\partial \zeta}\right)_k$$

$$\frac{\partial \hat{h}_k}{\partial t} = \frac{1}{2}\left(\frac{\partial h_k}{\partial t} + \frac{\partial h_{k-1}}{\partial t}\right)_H + fun\left\langle \left(\dot{\zeta}\frac{\hat{\partial p}}{\partial \zeta}\right)_{k-1}, \left(\dot{\zeta}\frac{\hat{\partial p}}{\partial \zeta}\right)_k, \left(\dot{\zeta}\frac{\hat{\partial p}}{\partial \zeta}\right)_{k+1} \right\rangle$$

A specific hybrid coordinate can be defined as

$$\hat{p}_k = \hat{A}_k + \hat{B}_k p_s + \hat{C}_k \left(\frac{h_{k-1} + h_k}{h_{0k-1} + h_{0k}} \right)^{C_{pd}/R_d}$$

The vertical flux can be obtained by

$$\frac{\partial \hat{p}_k}{\partial t} = \hat{B}_k \frac{\partial p_s}{\partial t} + \frac{\hat{C}_k}{h_{k-1} + h_k} \frac{C_{p_d}}{R_d} \left(\frac{h_{k-1} + h_k}{h_{0k-1} + h_{0k}} \right)^{C_{pd}/R_d} \left(\frac{\partial h_{k-1}}{\partial t} + \frac{\partial h_k}{\partial t} \right) = \hat{B}_k \frac{\partial p_s}{\partial t} + \hat{C}_{Tk} \left(\frac{\partial h_{k-1}}{\partial t} + \frac{\partial h_k}{\partial t} \right)$$

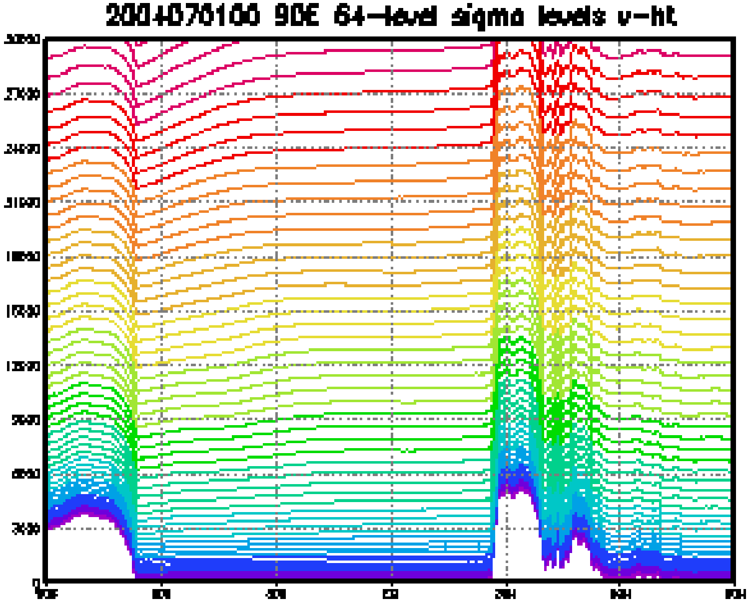
then, again, separating horizontal and vertical terms after some arrangement, we have

$$\hat{C}_{Tk} \frac{\left(\delta_k h_k - \beta_{k+1} h_{k+1}\right)}{\left(\hat{p}_k - \hat{p}_{k+1}\right)} \left(\dot{\zeta} \frac{\partial \hat{p}}{\partial \zeta} \right)_{k+1}$$

$$+ \left\{ \hat{C}_{Tk} \left\langle \left(\frac{\left(\delta_{k-1} h_{k-1} - \beta_k h_k\right)}{\left(\hat{p}_{k-1} - \hat{p}_k\right)} \right) + \left(\frac{\left(\alpha_{k-1} h_{k-1} - \gamma_k h_k\right)}{\left(\hat{p}_k - \hat{p}_{k+1}\right)} \right) \right\rangle - 1 \right\} \left(\dot{\zeta} \frac{\partial \hat{p}}{\partial \zeta} \right)_k$$

$$+ \hat{C}_{Tk} \frac{\left(\alpha_{k-2} h_{k-2} - \gamma_{k-1} h_{k-1}\right)}{\left(\hat{p}_{k-1} - \hat{p}_k\right)} \left(\dot{\zeta} \frac{\partial \hat{p}}{\partial \zeta} \right)_{k-1} = -\left(\frac{\partial \hat{p}}{\partial t} \right)_k^H + \hat{B}_k \frac{\partial p_s}{\partial t} + \hat{C}_{Tk} \left[\left(\frac{\partial h}{\partial t} \right)_{k-1}^H + \left(\frac{\partial h}{\partial t} \right)_k^H \right]$$

A=0
C=0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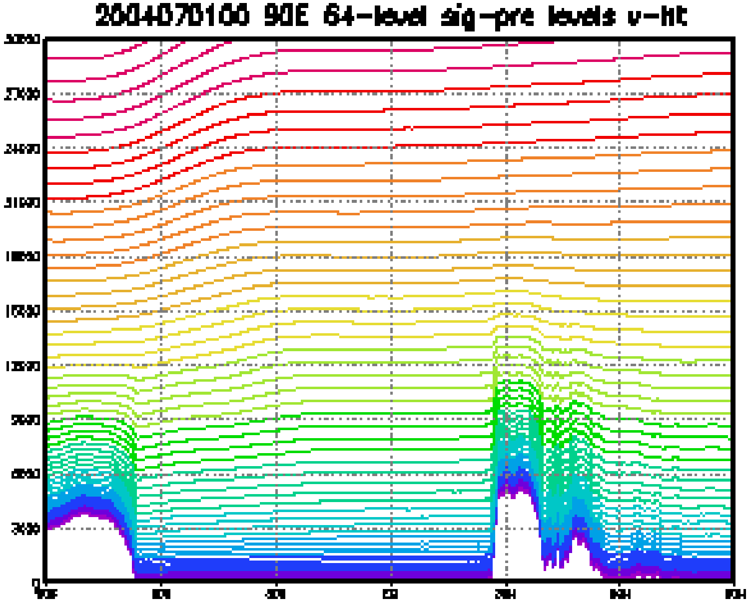
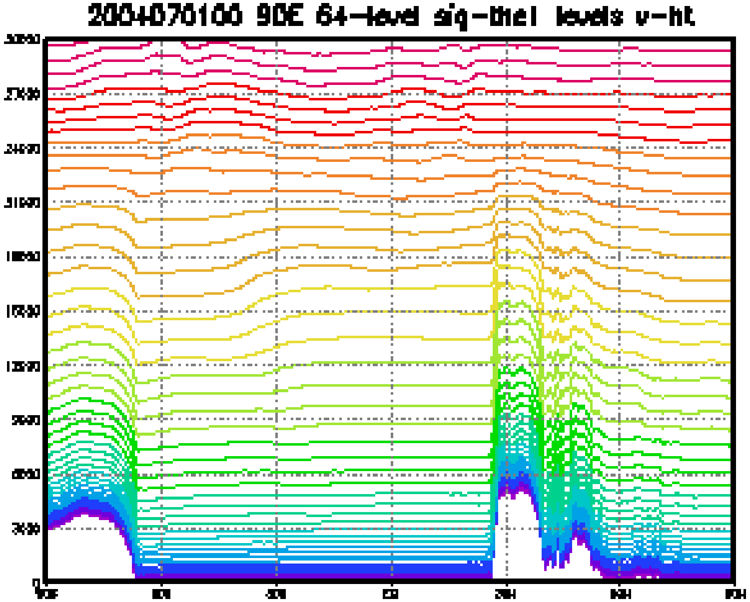
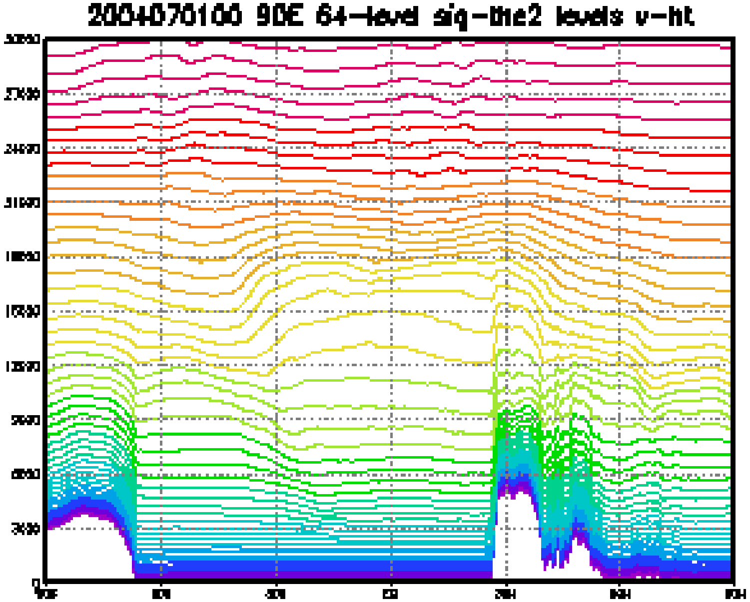

2004070100 90E 64-level sig-pre levels v-ht

C=0

B=0


2004070100 90E 64-level sig-the1 levels v-ht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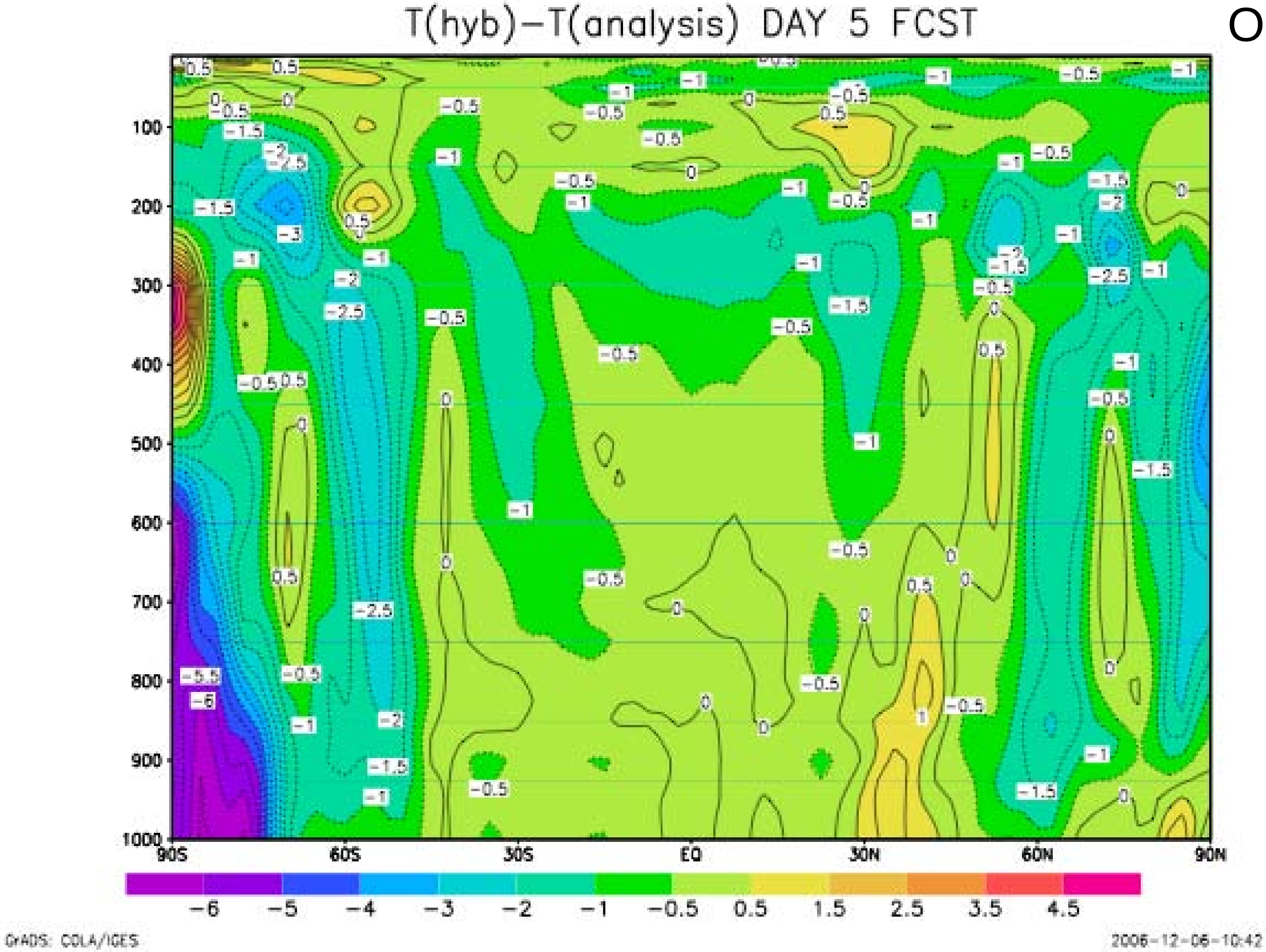
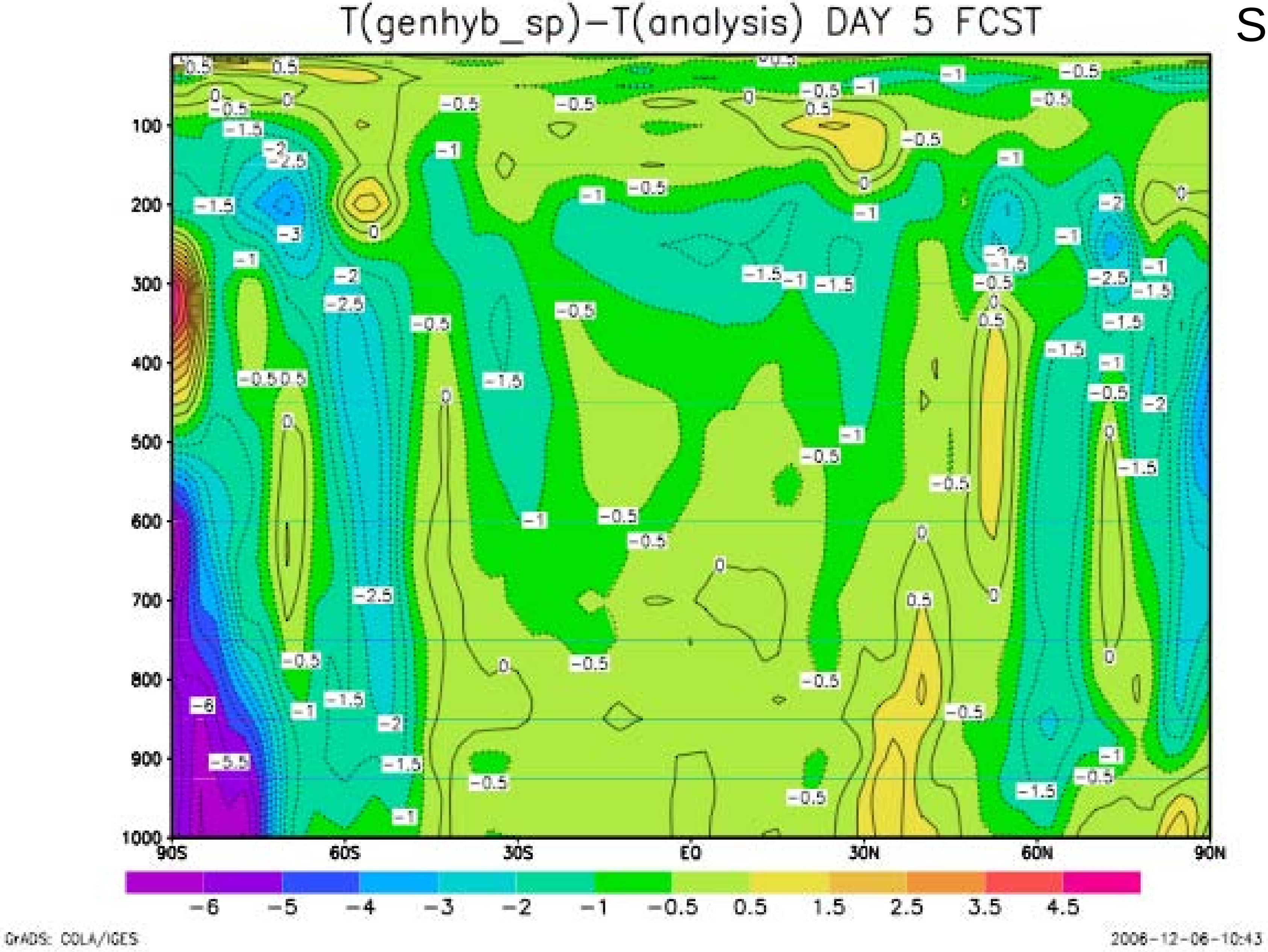
B=0

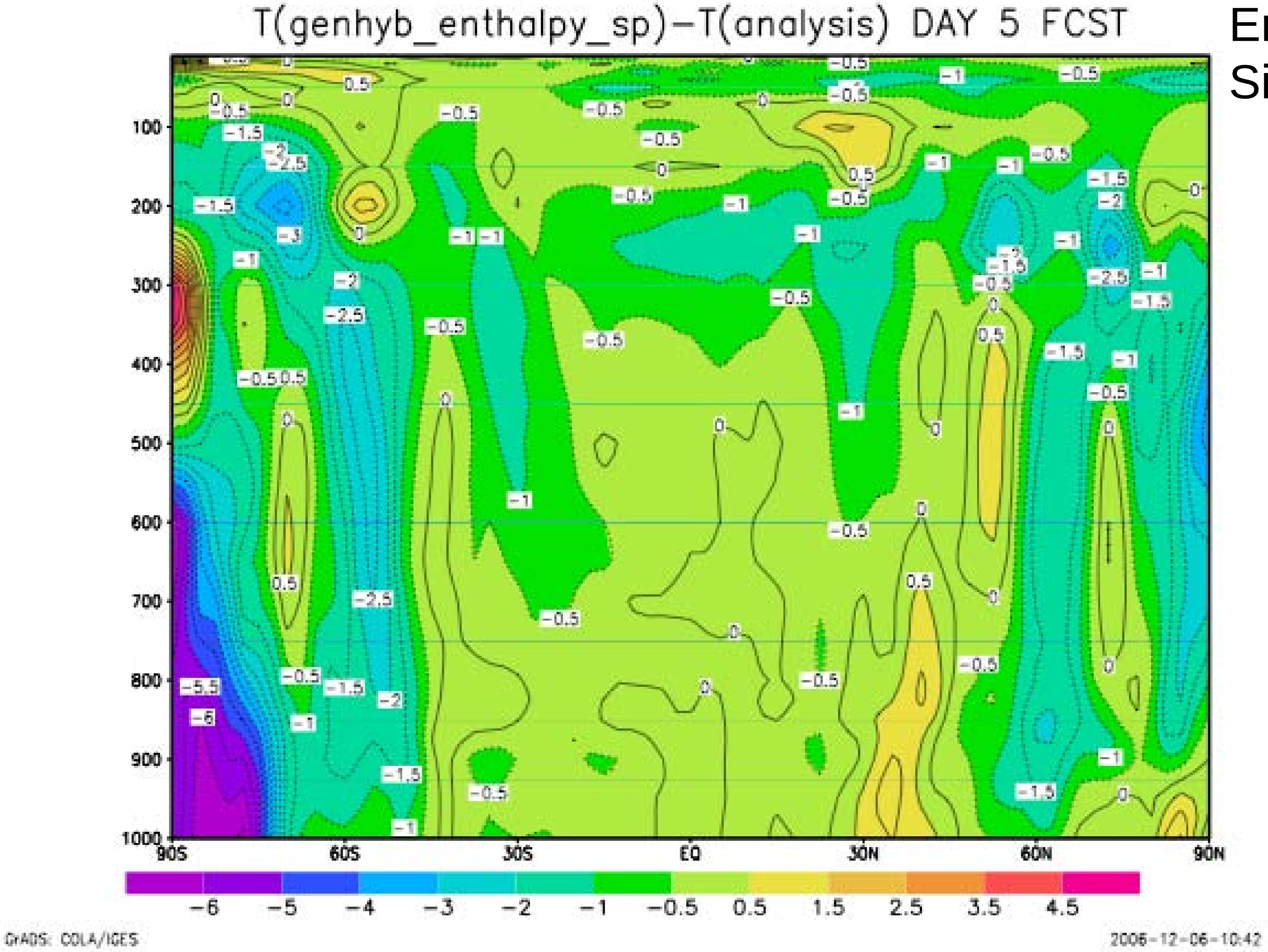
Opr GFS


T(hyb)−T(analysis) DAY 5 FCST
100
200
300
400
500
600
700
800
900
1000
90S
60S
30S
EQ
30N
60N
90N
−6
−5
−4
−3
−2
−1
−0.5
0.5
1.5
2.5
3.5
4.5
GrADS: COLA/IGES
2006−12−06−10:42

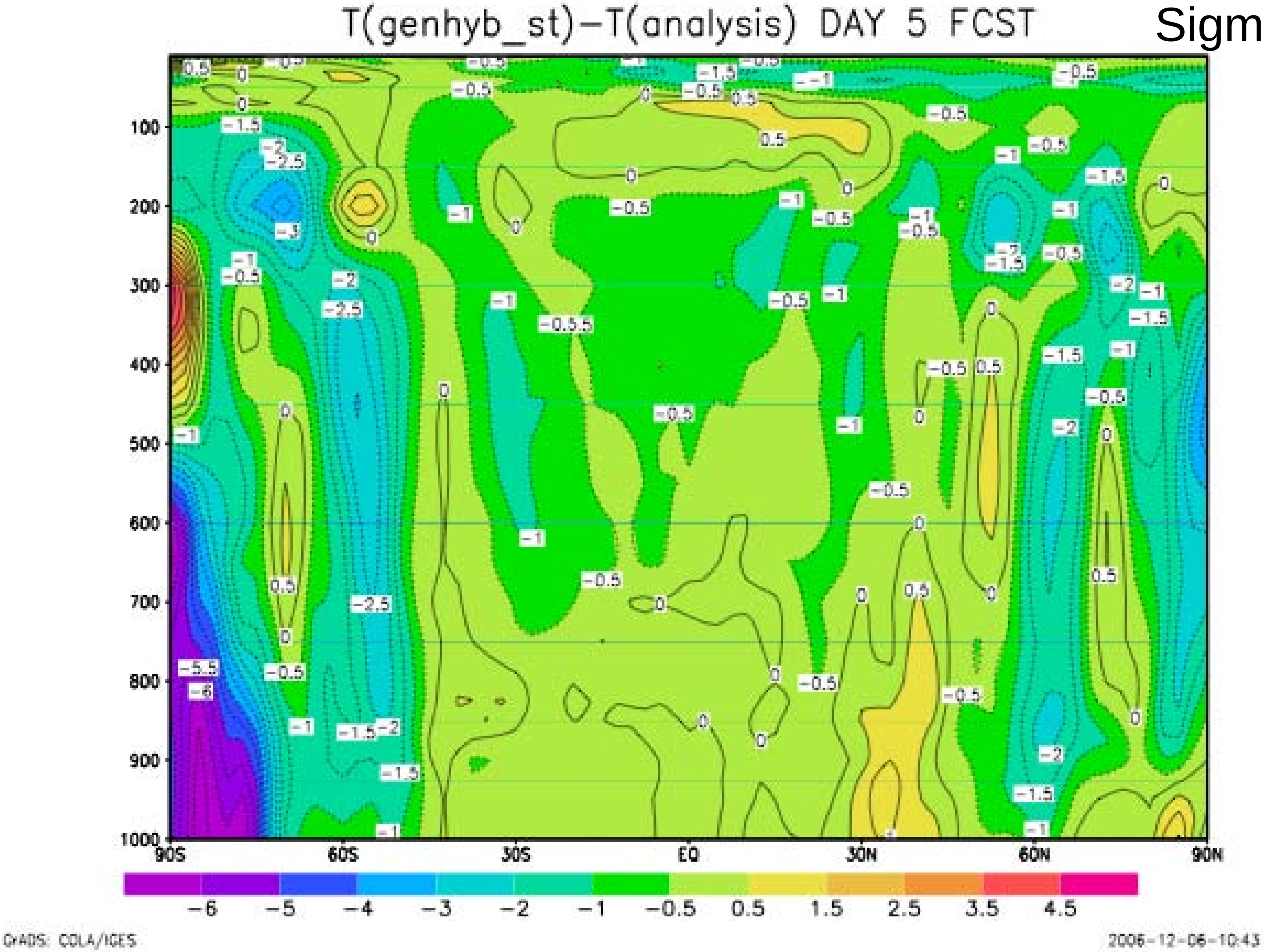
Sigma-P


T(genhyb_sp)−T(analysis) DAY 5 FCST
100
200
300
400
500
600
700
800
900
1000
90S
60S
30S
EQ
30N
60N
90N
−6
−5
−4
−3
−2
−1
−0.5
0.5
1.5
2.5
3.5
4.5
GrADS: COLA/IGES
2006−12−06−10:43

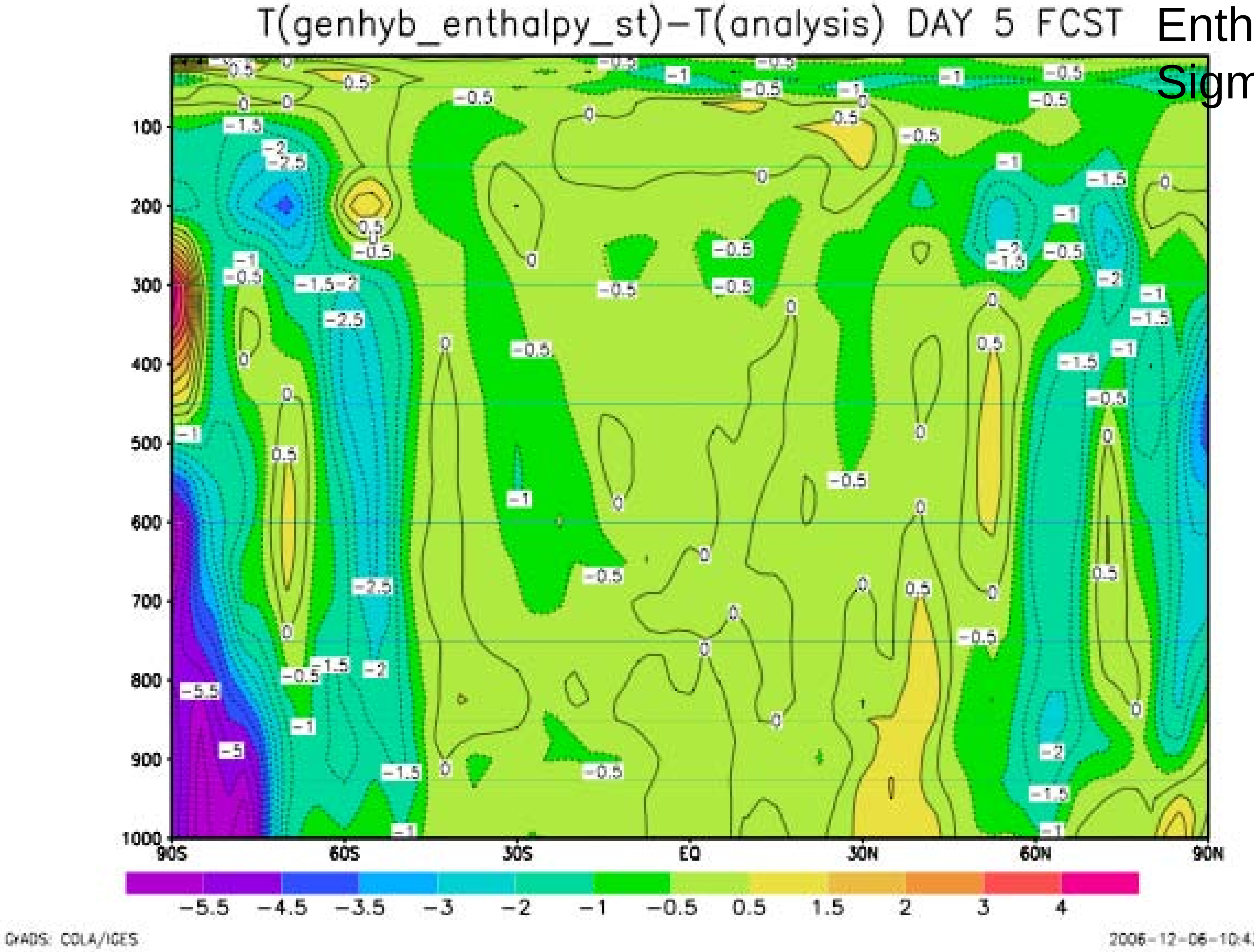
Enthalpy Sigma-P

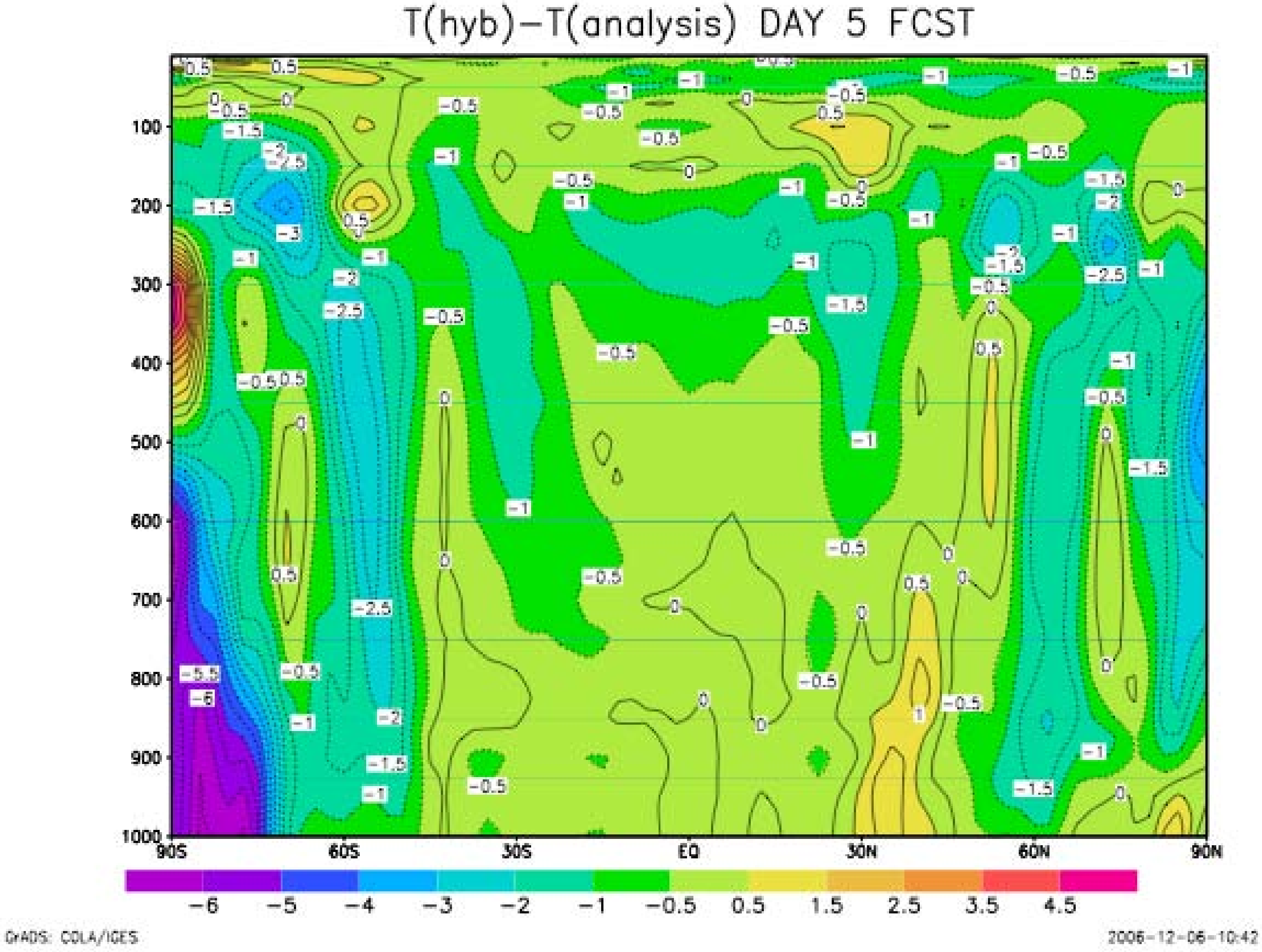


Sigma-theta

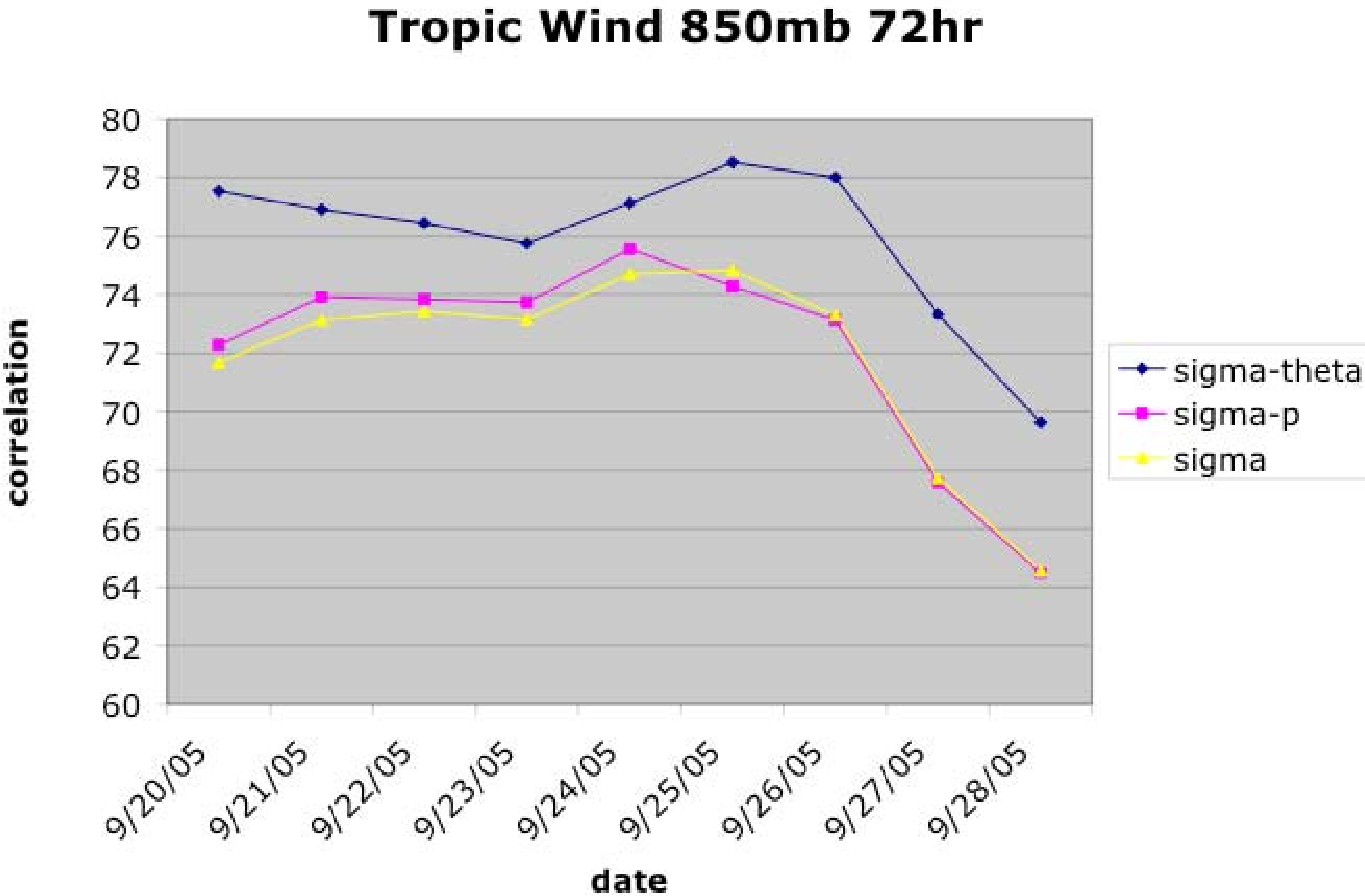

T(genhyb_st)−T(analysis) DAY 5 FCST
100
200
300
400
500
600
700
800
900
1000
90S
60S
30S
EQ
30N
60N
90N
−6
−5
−4
−3
−2
−1
−0.5
0.5
1.5
2.5
3.5
4.5
GrADS: COLA/IGES
2006−12−06−10:43

Enthalpy
Sigma-theta



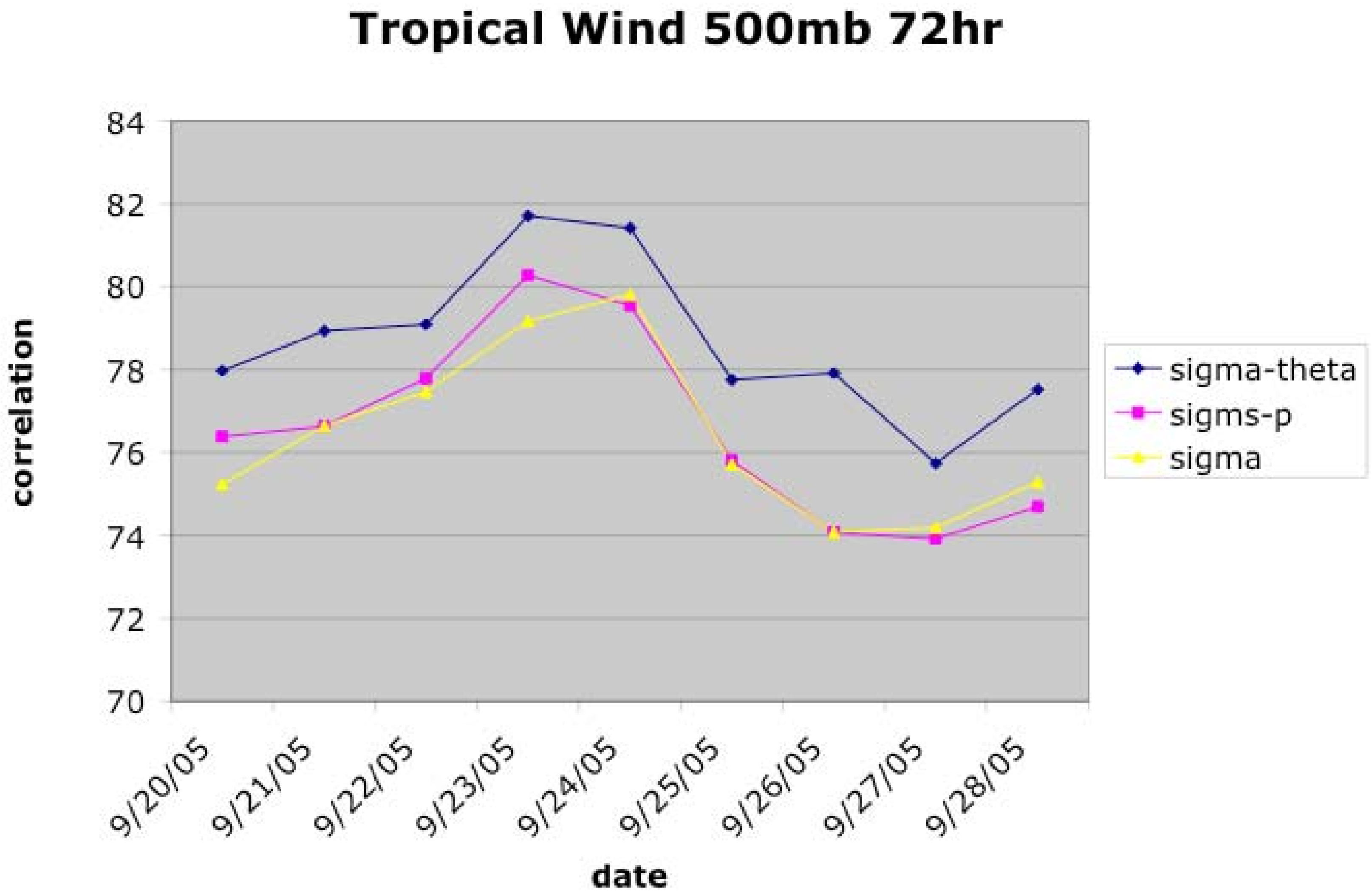

T(hyb)−T(analysis) DAY 5 FCST
100
200
300
400
500
600
700
800
900
1000
90S
60S
30S
EQ
30N
60N
90N
−6
−5
−4
−3
−2
−1
−0.5
0.5
1.5
2.5
3.5
4.5
GrADS: COLA/IGES
2006-12-06-10:42

T382L64



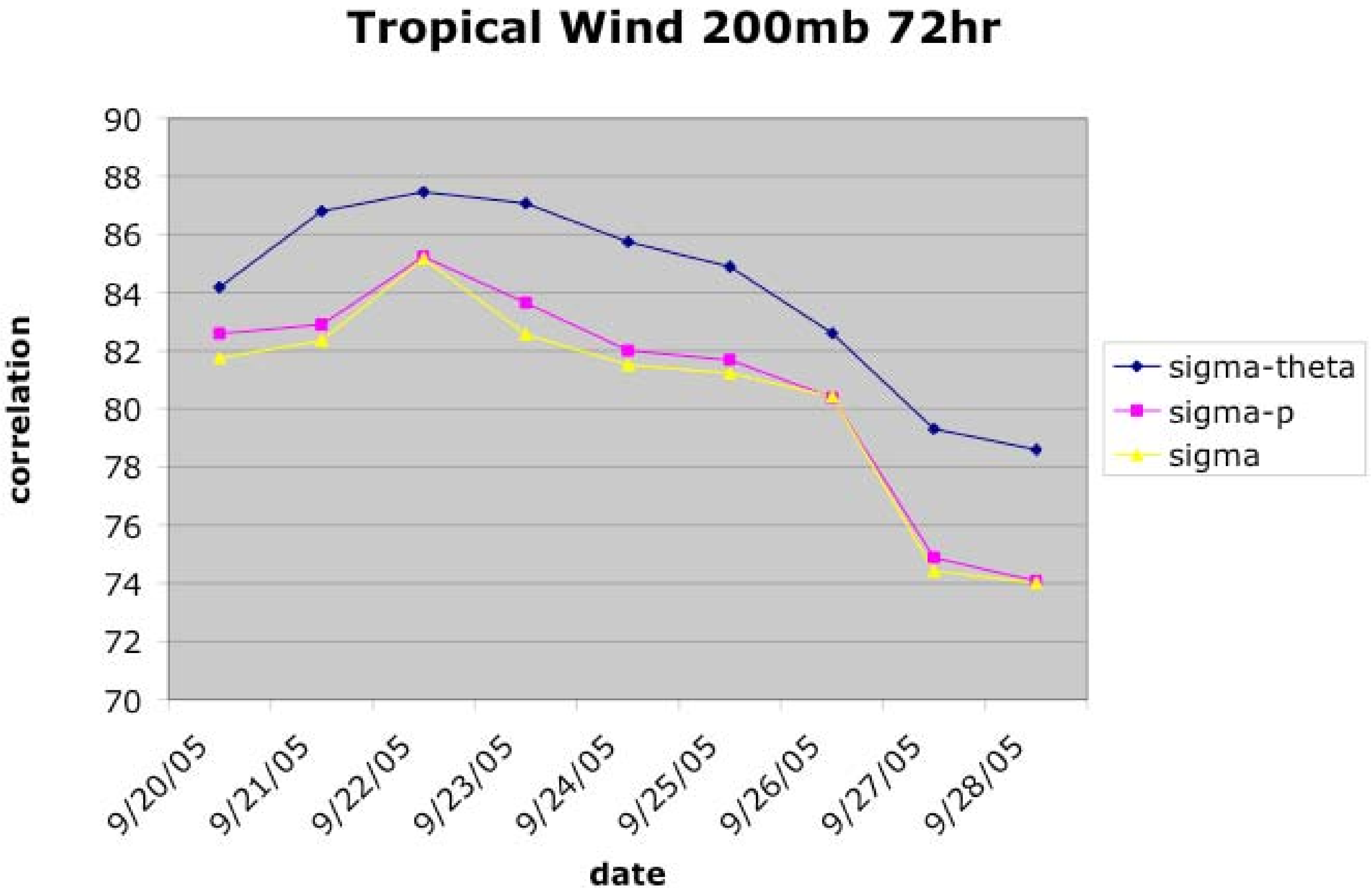
Tropic Wind 850mb 72hr
correlation
80
78
76
74
72
70
68
66
64
62
60
9/20/05
9/21/05
9/22/05
9/23/05
9/24/05
9/25/05
9/26/05
9/27/05
9/28/05
date
sigma-theta
sigma-p
sigma

T382L64



Tropical Wind 500mb 72hr
84
82
80
78
76
74
72
70
correlation
9/20/05
9/21/05
9/22/05
9/23/05
9/24/05
9/25/05
9/26/05
9/27/05
9/28/05
date
sigma-theta
sigms-p
sigma

T382L64



Tropical Wind 200mb 72hr
90
88
86
84
82
80
78
76
74
72
70
correlation
9/20/05
9/21/05
9/22/05
9/23/05
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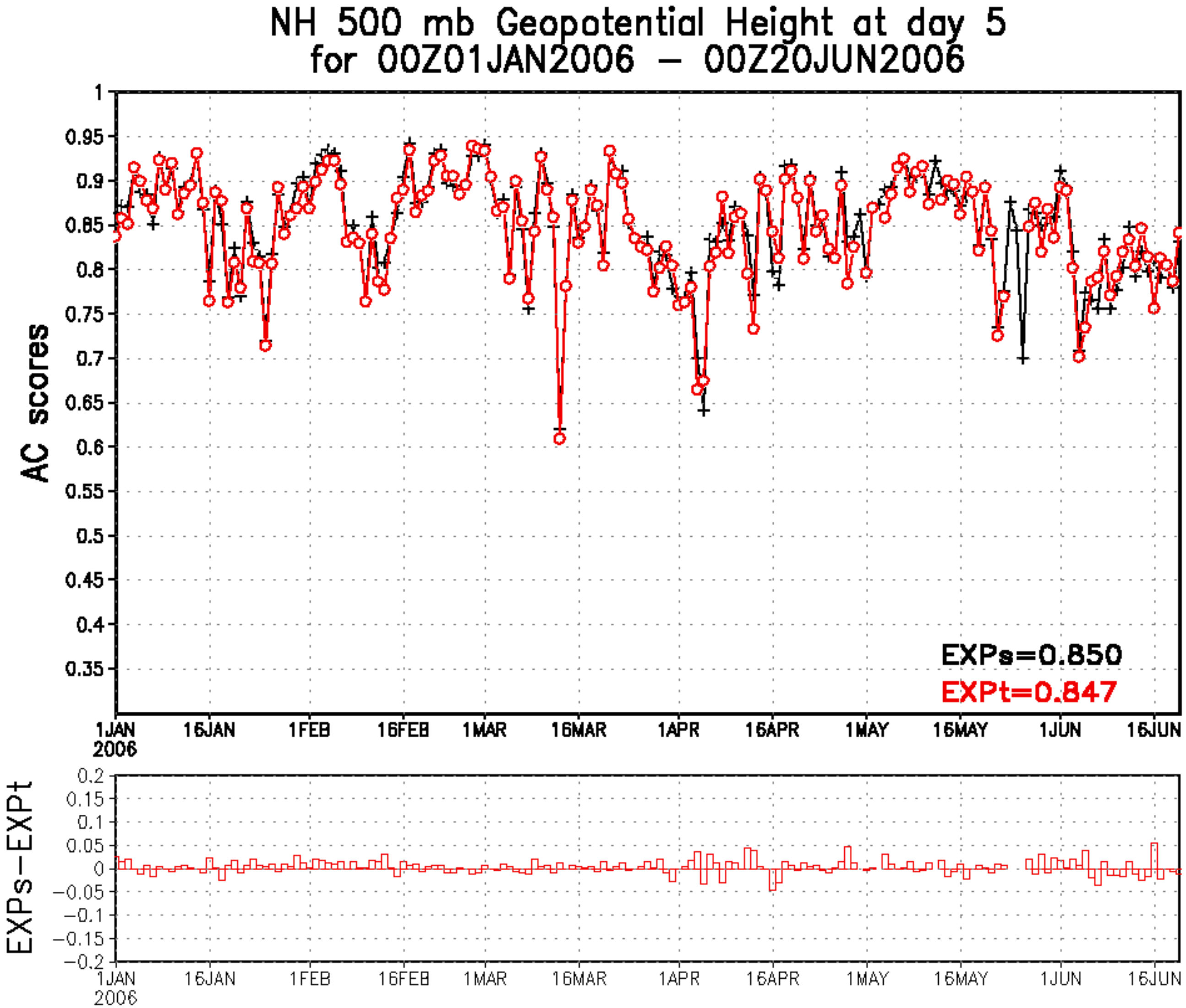
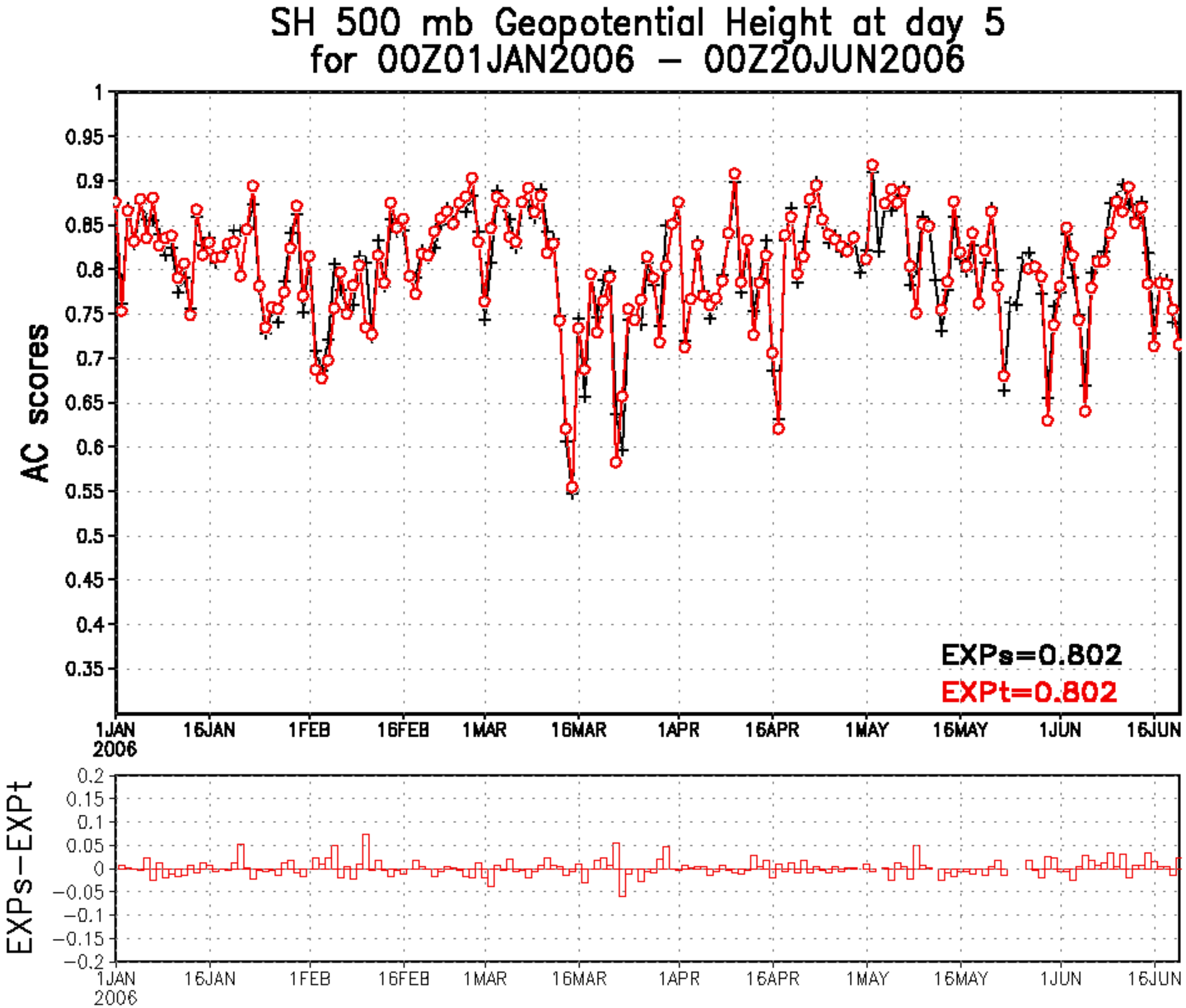
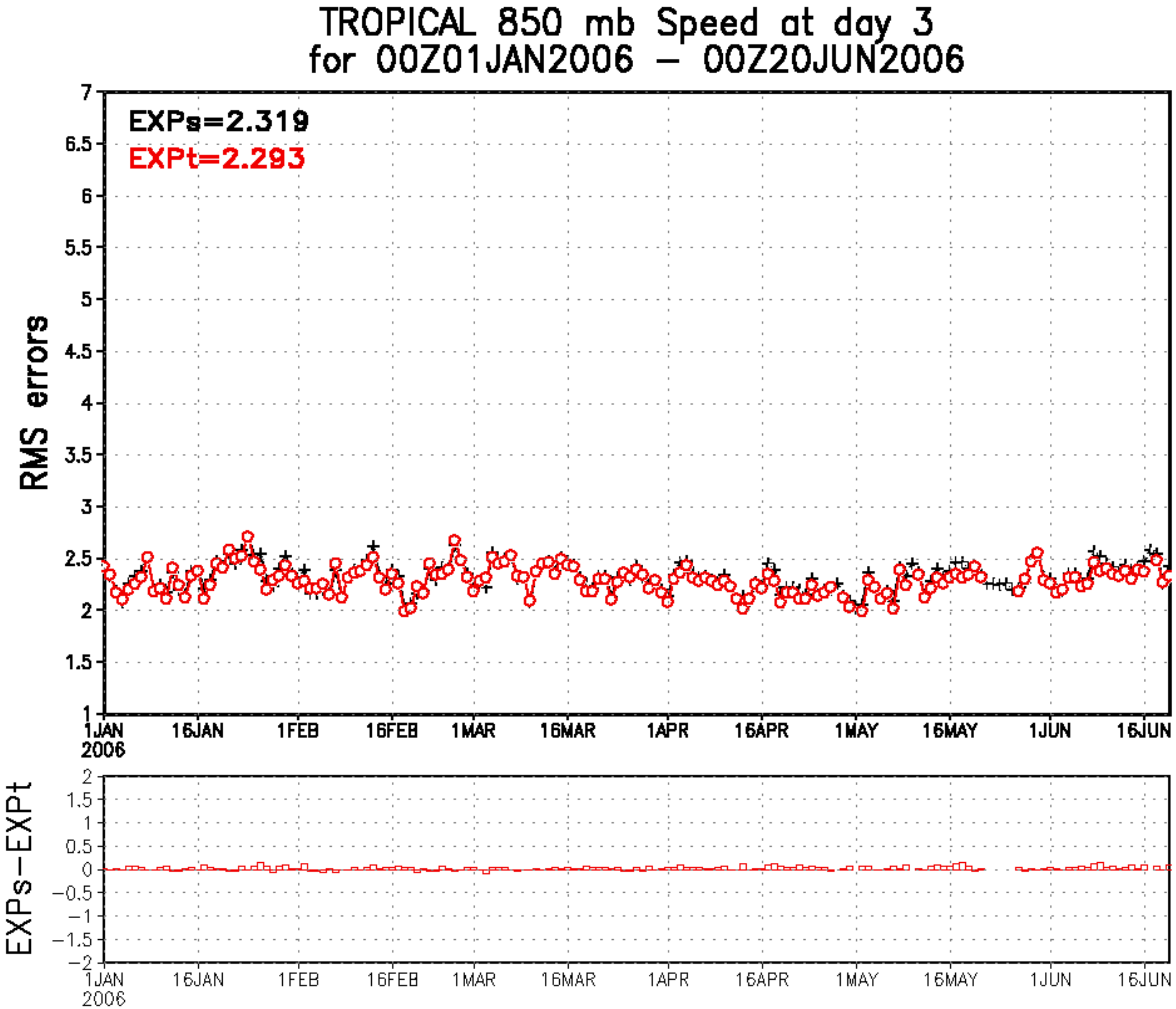
9/27/05
9/28/05
date
sigma-theta
sigma-p
sigma



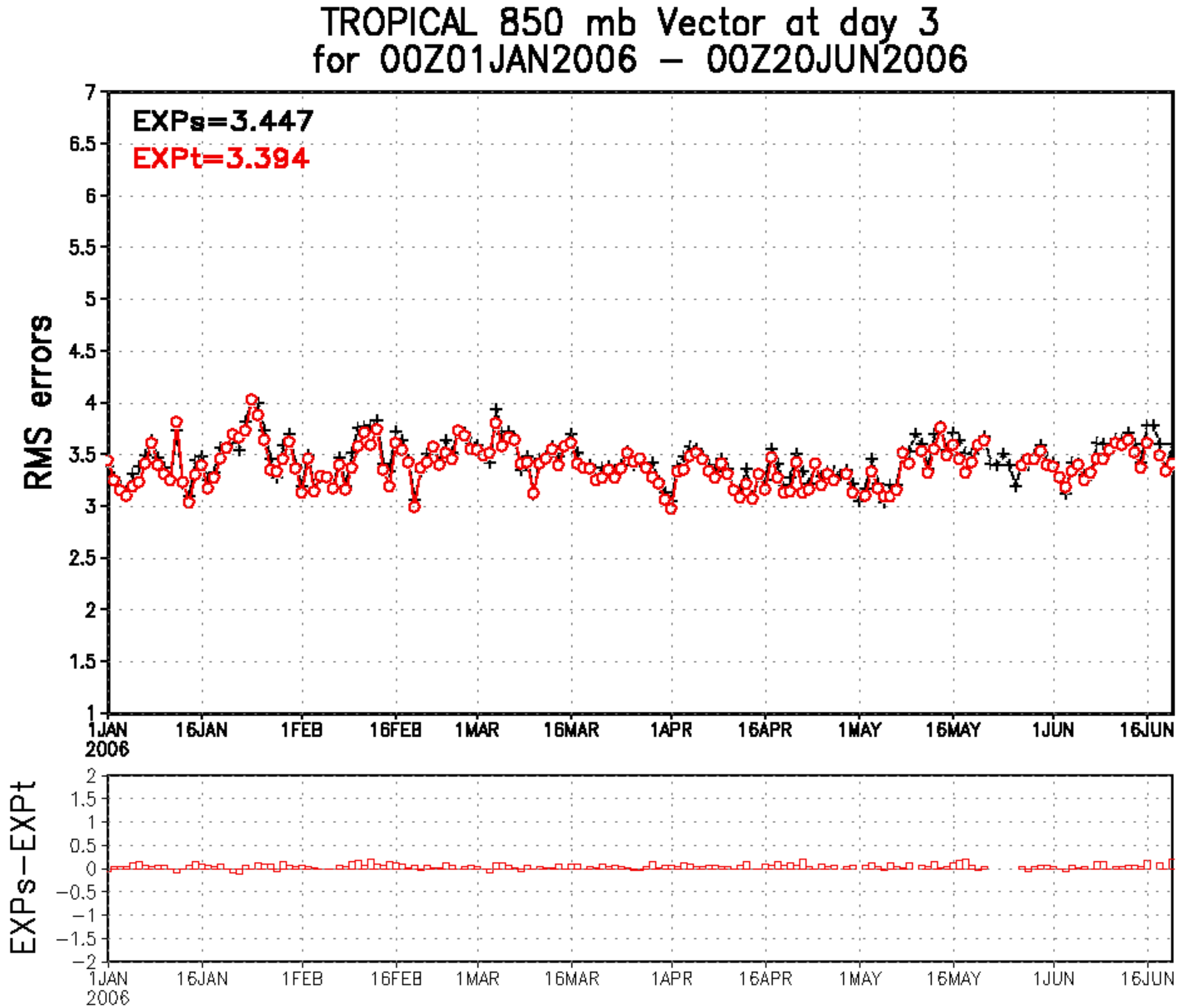
Black s: operational GFS   Red t: sigma-theta GFS



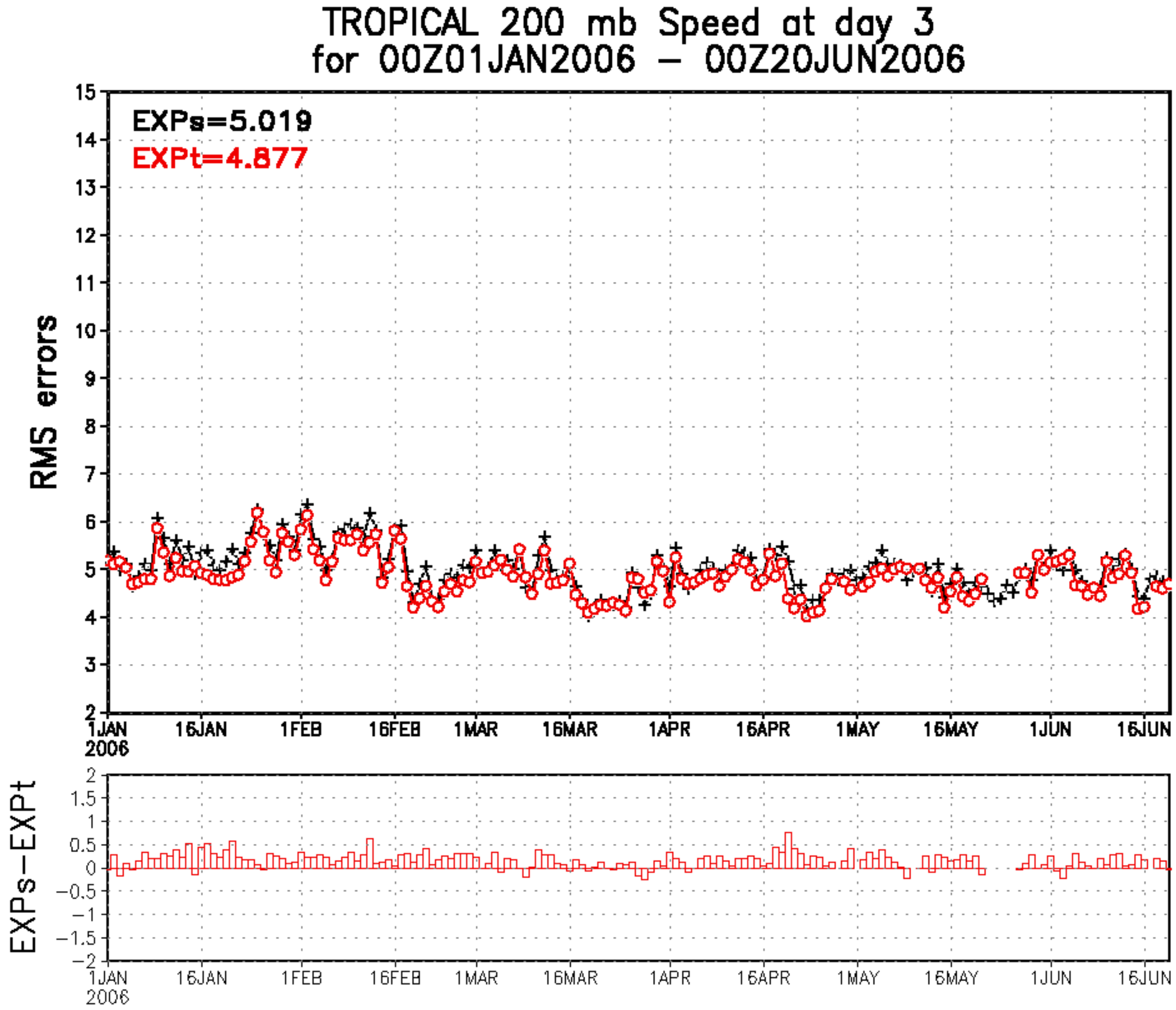
Black s: operational GFS   Red t: sigma-theta GFS



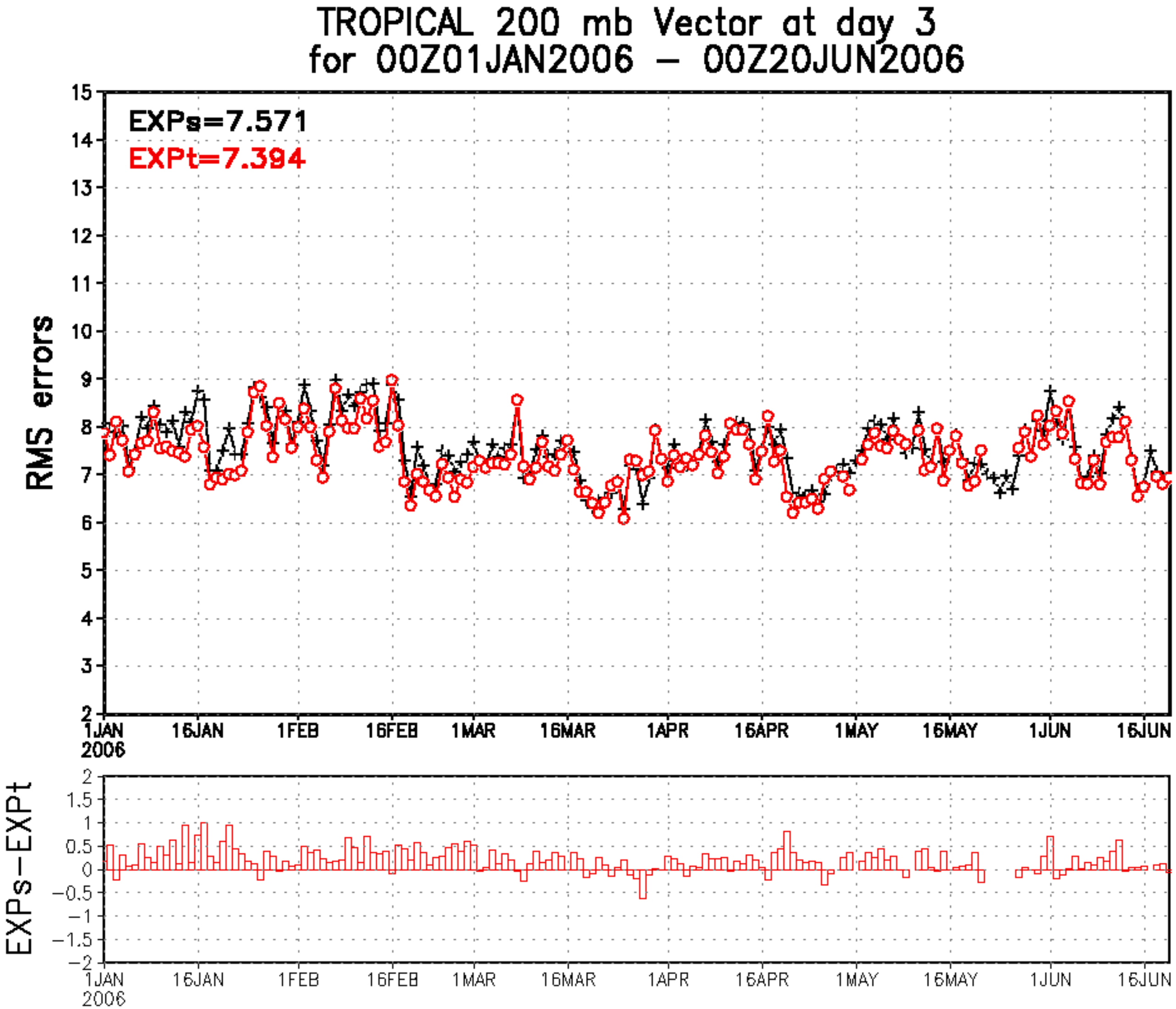
Black s: operational GFS Red t: sigma-theta GFS



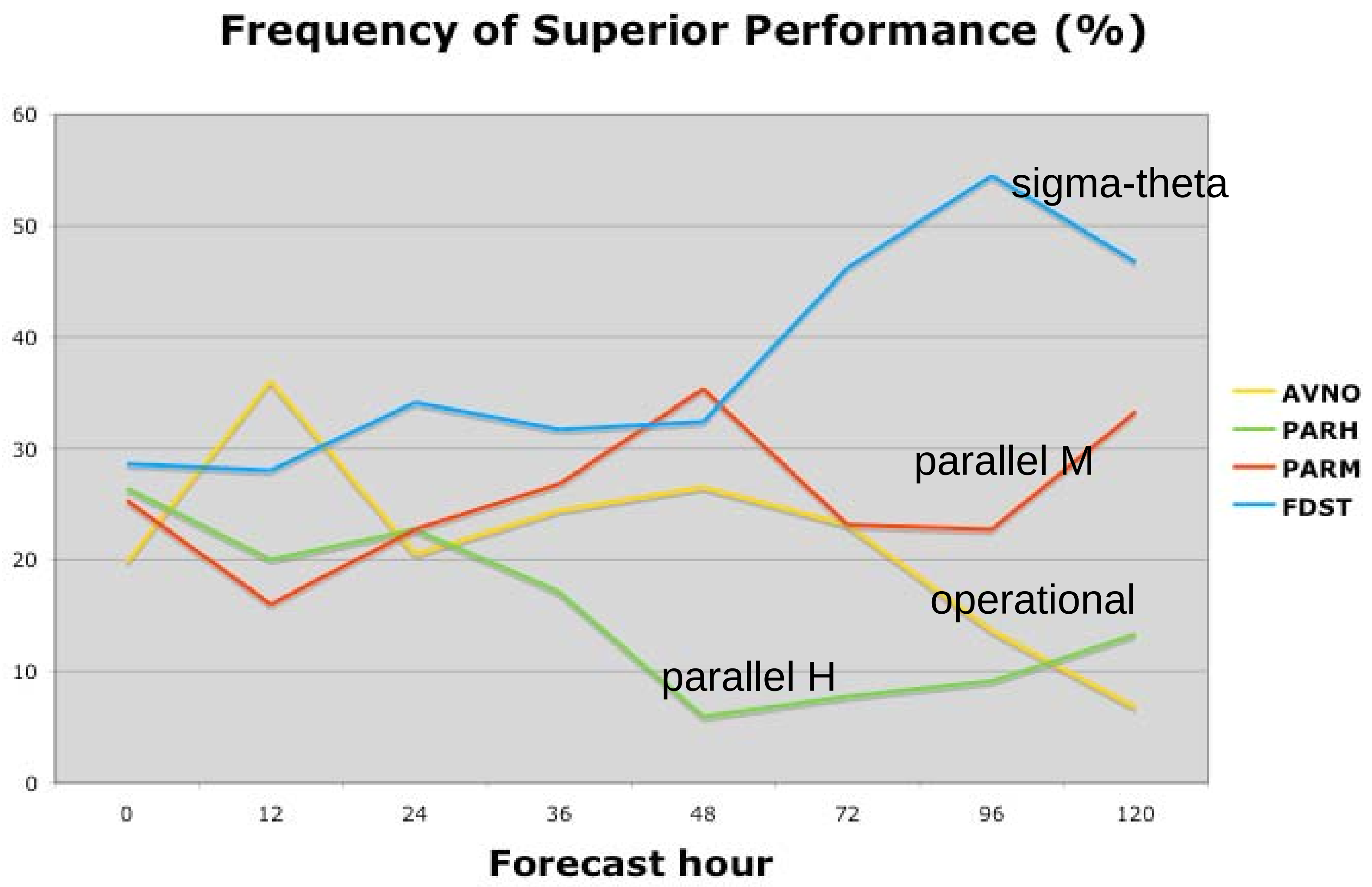
Black s: operational GFS   Red t: sigma-theta GFS



Black s: operational GFS Red t: sigma-theta GFS

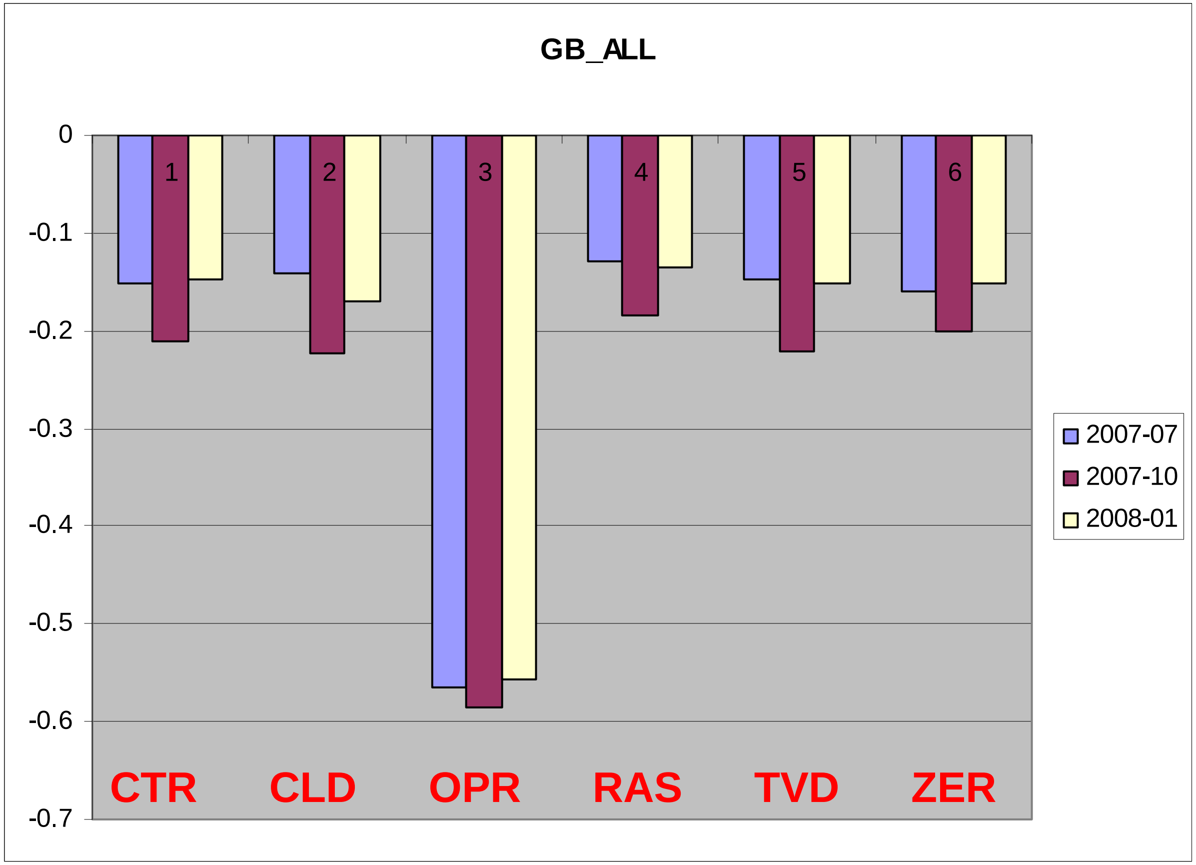


Black s: operational GFS Red t: sigma-theta GFS



2005 hurricane season

3 ICs versus 6 experiments for GB_ALL tracer



0.1- 0.2 %, except for OPR

Accomplish/Problem/Solution

- Enthalpy sigma-p version is ready for CFSRR and next GFS implementation
- But enthalpy sigma-theta coordinates run into negative mass through current advection scheme, in 2 to 3 days per month, then model stops.
- Thus the positive mass is required to have stable integration, it implies that we need positive defined advection, which will be introduced and called as nislfv scheme.

For mass conservation and positive advection, let's start from

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho \dot{\zeta}}{\partial \zeta} = 0$$

$$\rho = \Delta p$$

Consider 1-D and rewrite it in advection form, we have

$$\left(\frac{\partial \rho}{\partial t}\right)_{X-direction} + u\frac{\partial \rho}{\partial x} = -\rho\frac{\partial u}{\partial x}$$

$$\left(\frac{d\rho}{dt}\right)_{X-direction} = -\rho\frac{\partial u}{\partial x}$$

Advection form is for semi-Lagrangian,
but it is not conserved if divergence is treated as force at mid-point,
So divergence term should be treated with advection

Divergence term in Lagrangian sense is the change of the volume if mass is conserved, so we can write divergence form as

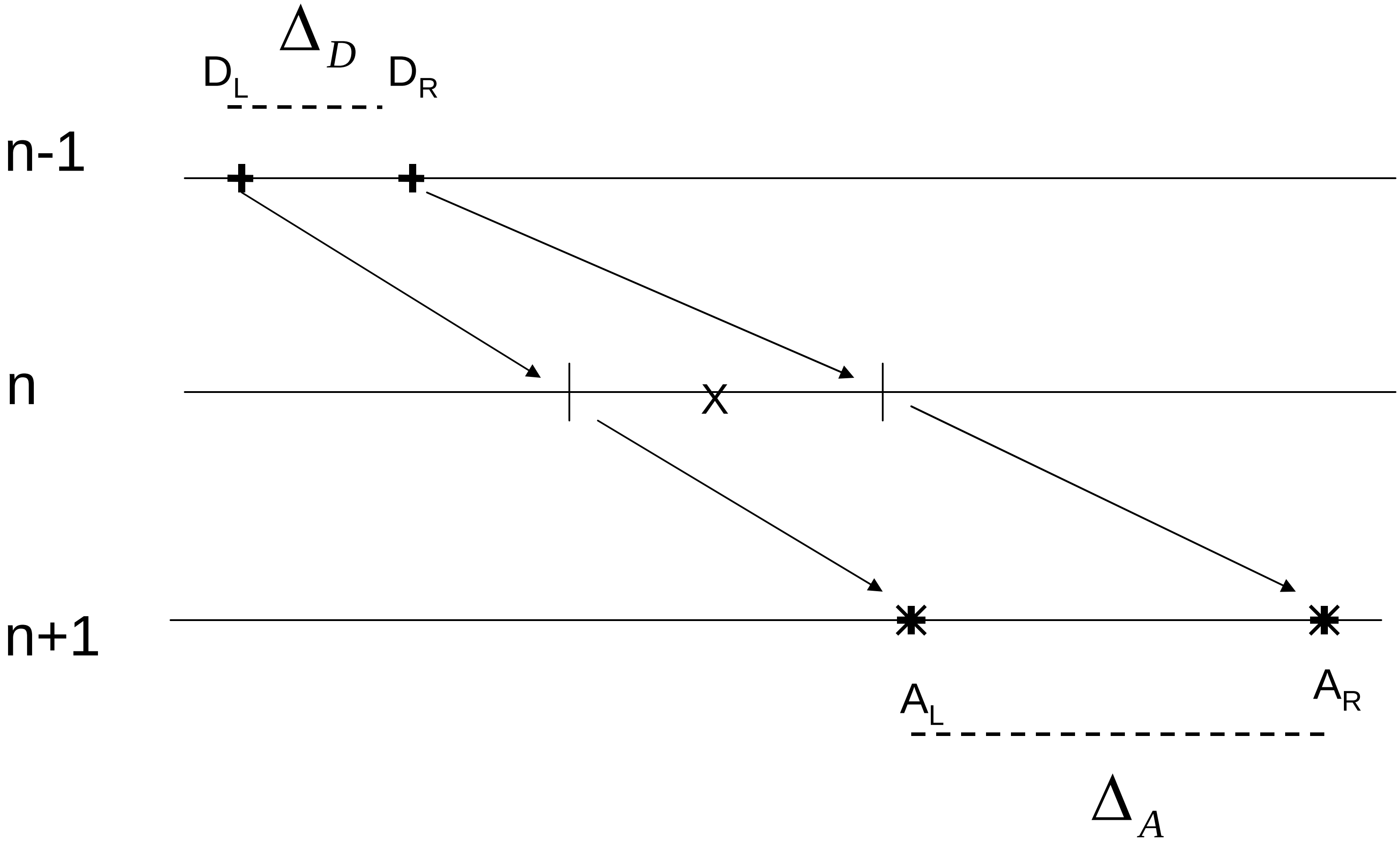
$$\left(\frac{\partial u}{\partial x}\right)_{Lagrangian\_sense} = \frac{1}{\Delta_x}\frac{d\Delta_x}{dt}$$

Put it into the previous continuity equation, we have

$$\left(\frac{d\rho\Delta_x}{dt}\right)_{X-direction} = 0$$

$$\left(\frac{\partial\rho\Delta_x}{\partial t}\right)_{X-direction} + u\frac{\partial\rho\Delta_x}{\partial x} = 0$$

which can be seen as $(\rho\Delta_x)_{departure} = (\rho\Delta_x)_{arrival}$



$$\rho_D^{n-1}\Delta_D = \rho_A^{n+1}\Delta_A$$

The given value can be presented piece-wisely by

$$\rho = S(x)$$

so the previous mass equality can be replaced as following

$$\int_{D_L}^{D_R} S_D^{n-1}(x)dx = \int_{A_L}^{A_R} S_A^{n+1}(x)dx$$

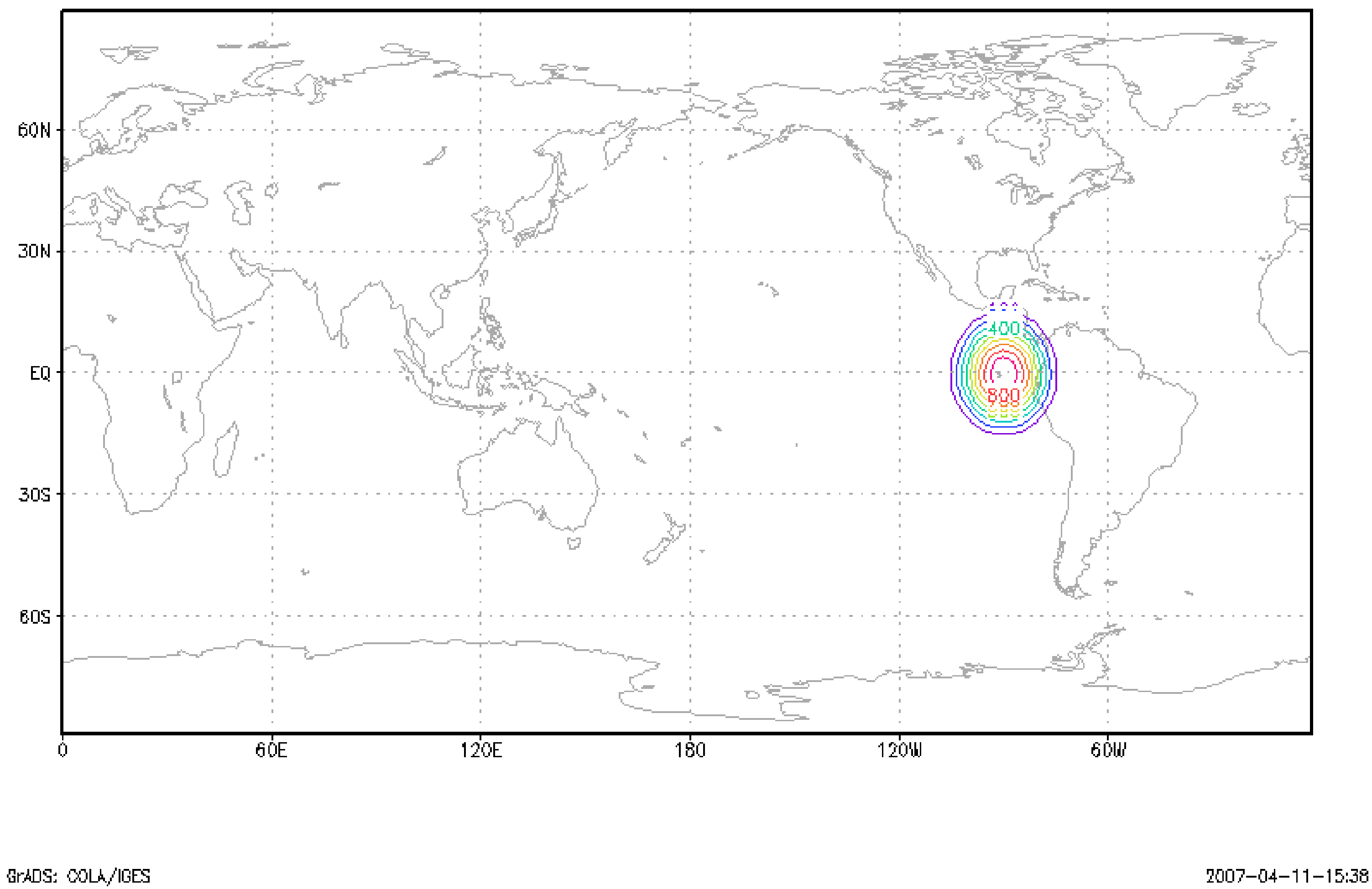
Also we want to make sure that total mass is conserved as

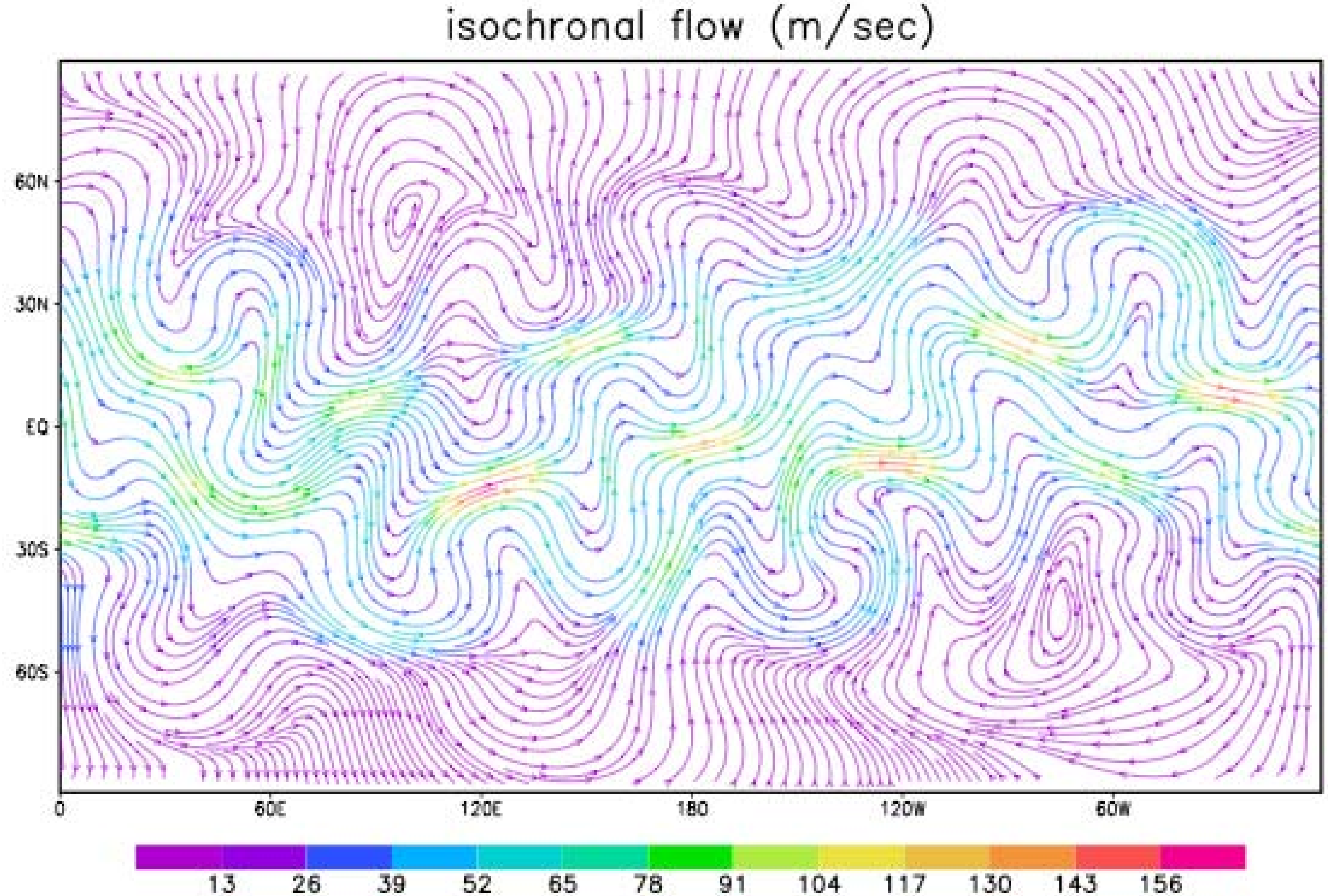
$$\oint S_R^{n-1}(x)dx = \oint S_D^{n-1}(x)dx = \oint S_A^{n+1}(x)dx = \oint S_R^{n+1}(x)dx$$

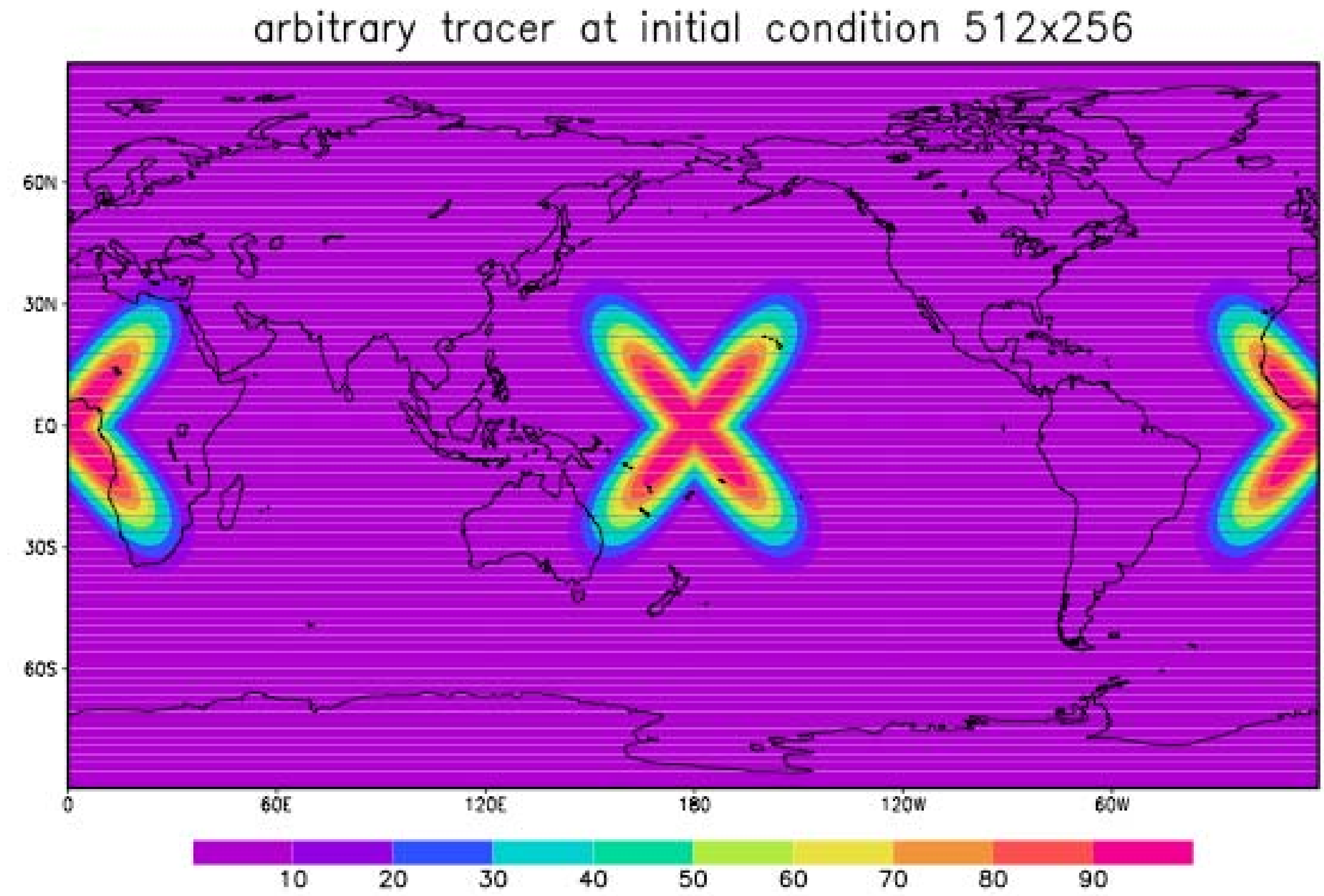
where subscript R is regular grid
D is departure grid
A is arrival grid for

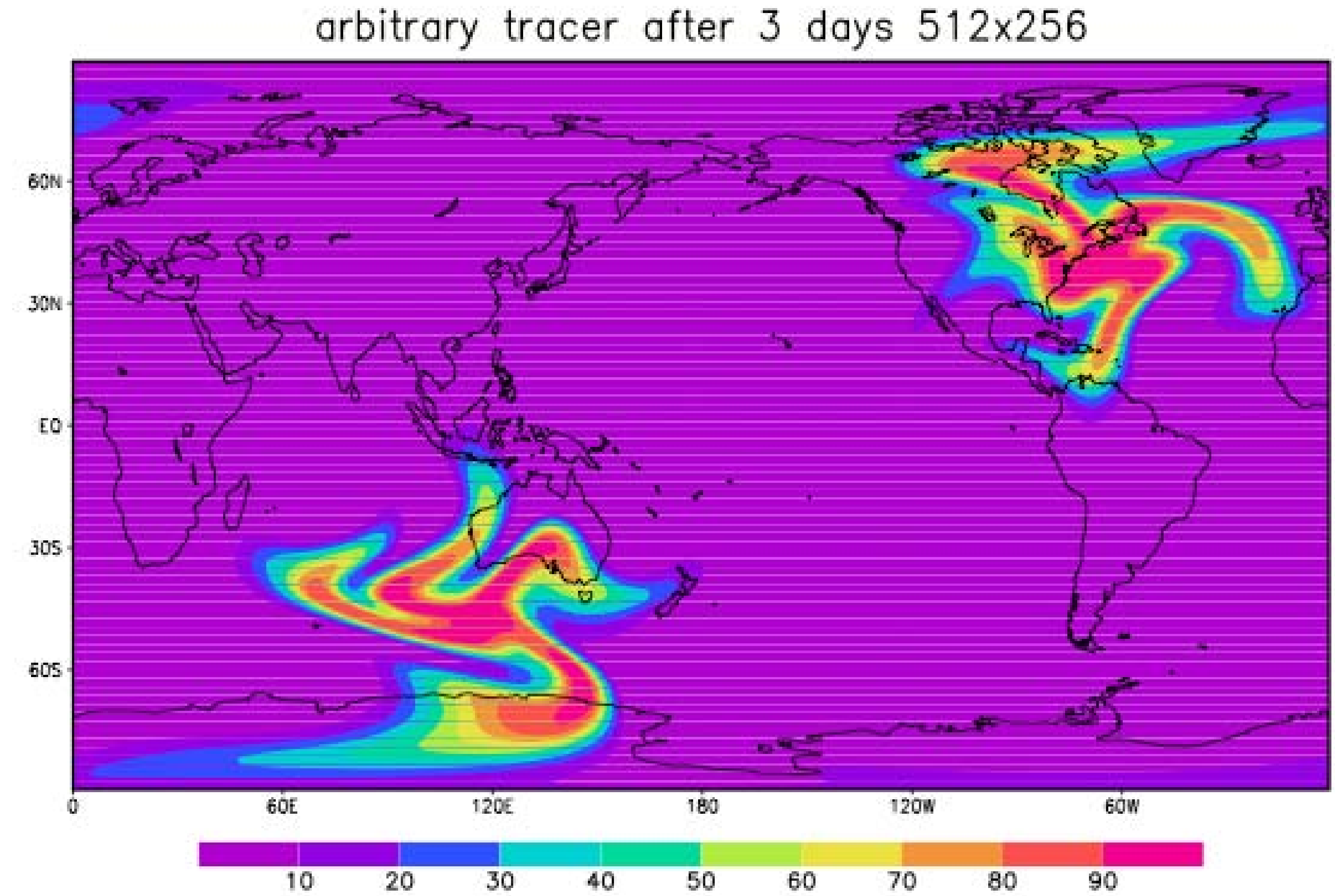
This implies that mass conservation should be used during interpolation from regular cell to departure cell and from arrival cell to regular cell. thus, we apply monotonic PPM for S(x).

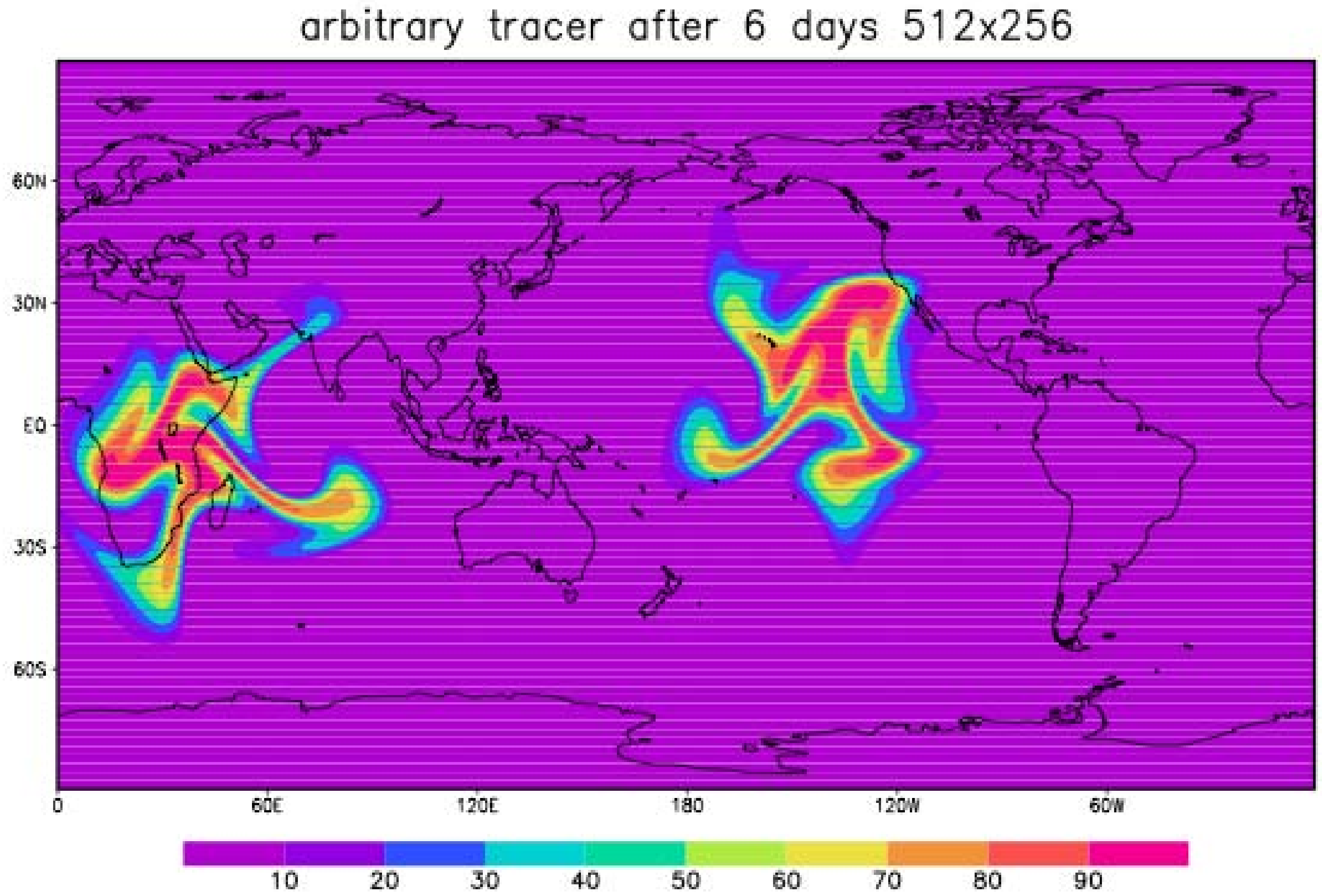
Gaussian 256 x 128 with time step of 1800 sec

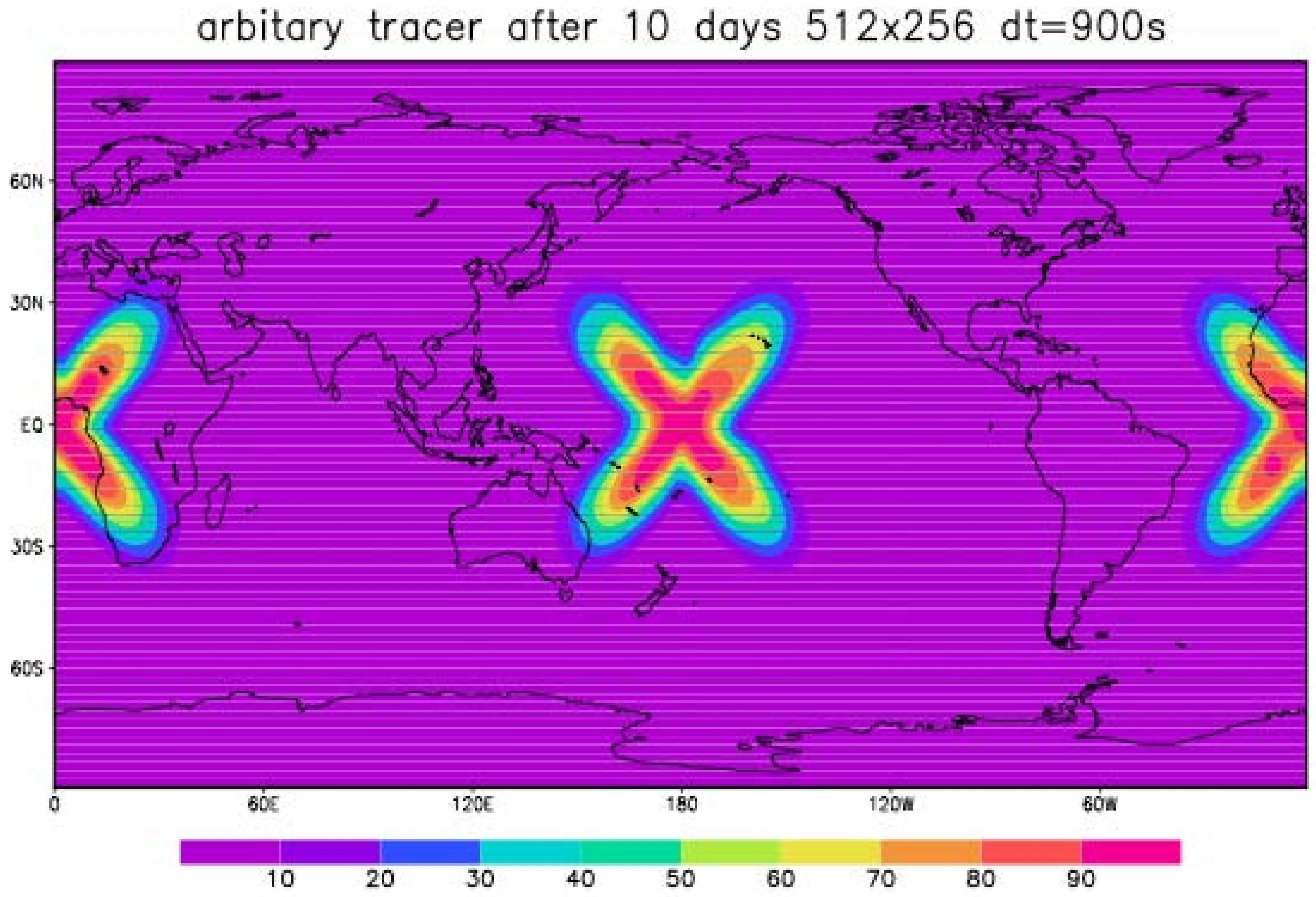



isochronal flow (m/sec)
60N
30N
EQ
30S
60S
0
60E
120E
180
120W
60W
13
26
39
52
65
78
91
104
117
130
143
156


arbitrary tracer at initial condition 512x256
60N
30N
EQ
30S
60S
0
60E
120E
180
120W
60W
10
20
30
40
50
60
70
80
90


arbitrary tracer after 3 days 512x256
60N
30N
EQ
30S
60S
0
60E
120E
180
120W
60W
10
20
30
40
50
60
70
80
90

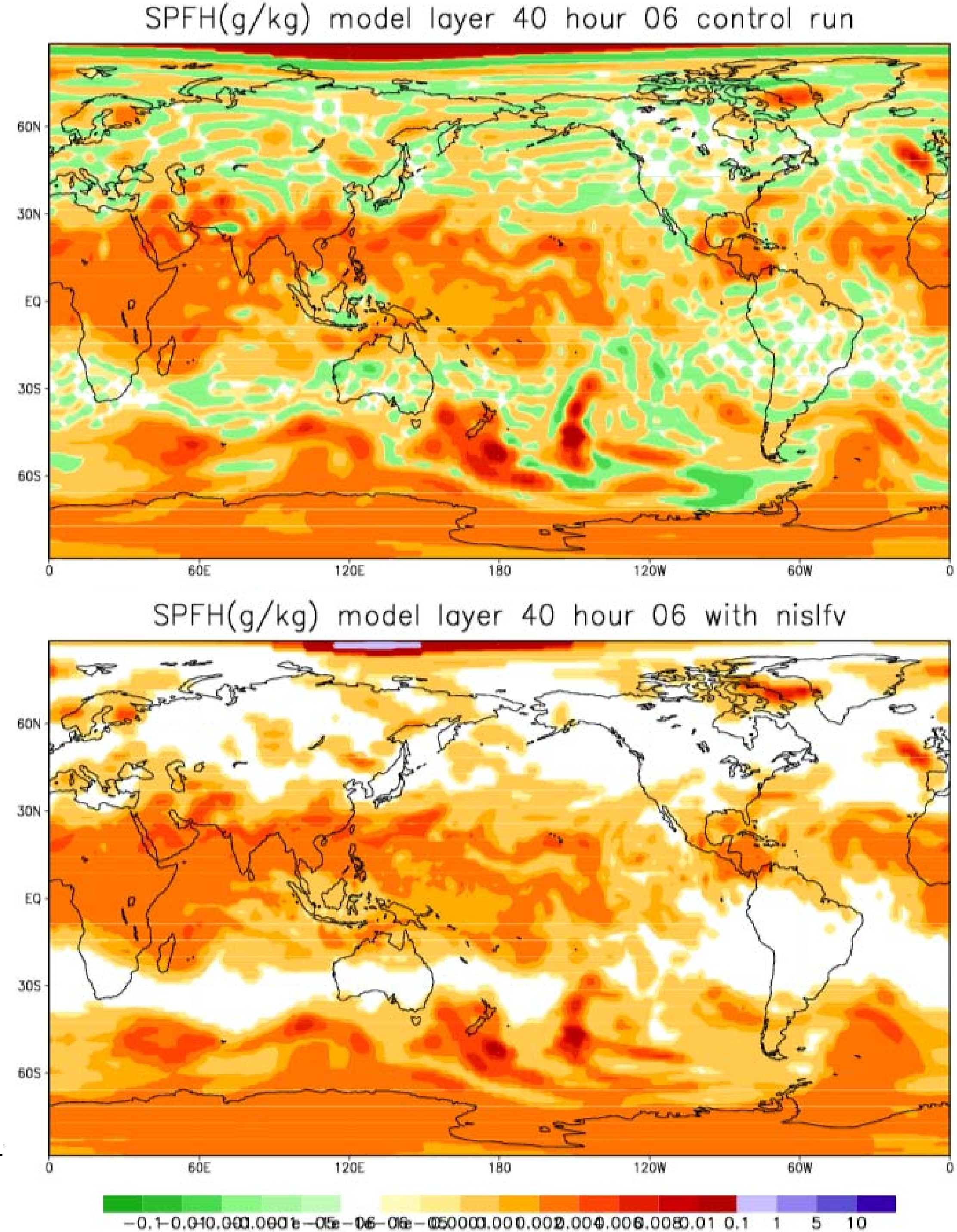

arbitrary tracer after 6 days 512x256
60N
30N
EQ
30S
60S
0
60E
120E
180
120W
60W
10
20
30
40
50
60
70
80
90


arbitary tracer after 10 days 512x256 dt=900s
60N
30N
EQ
30S
60S
0
60E
120E
180
120W
60W
10
20
30
40
50
60
70
80
90

control

06h fcst specific humidity
 at model layer 40

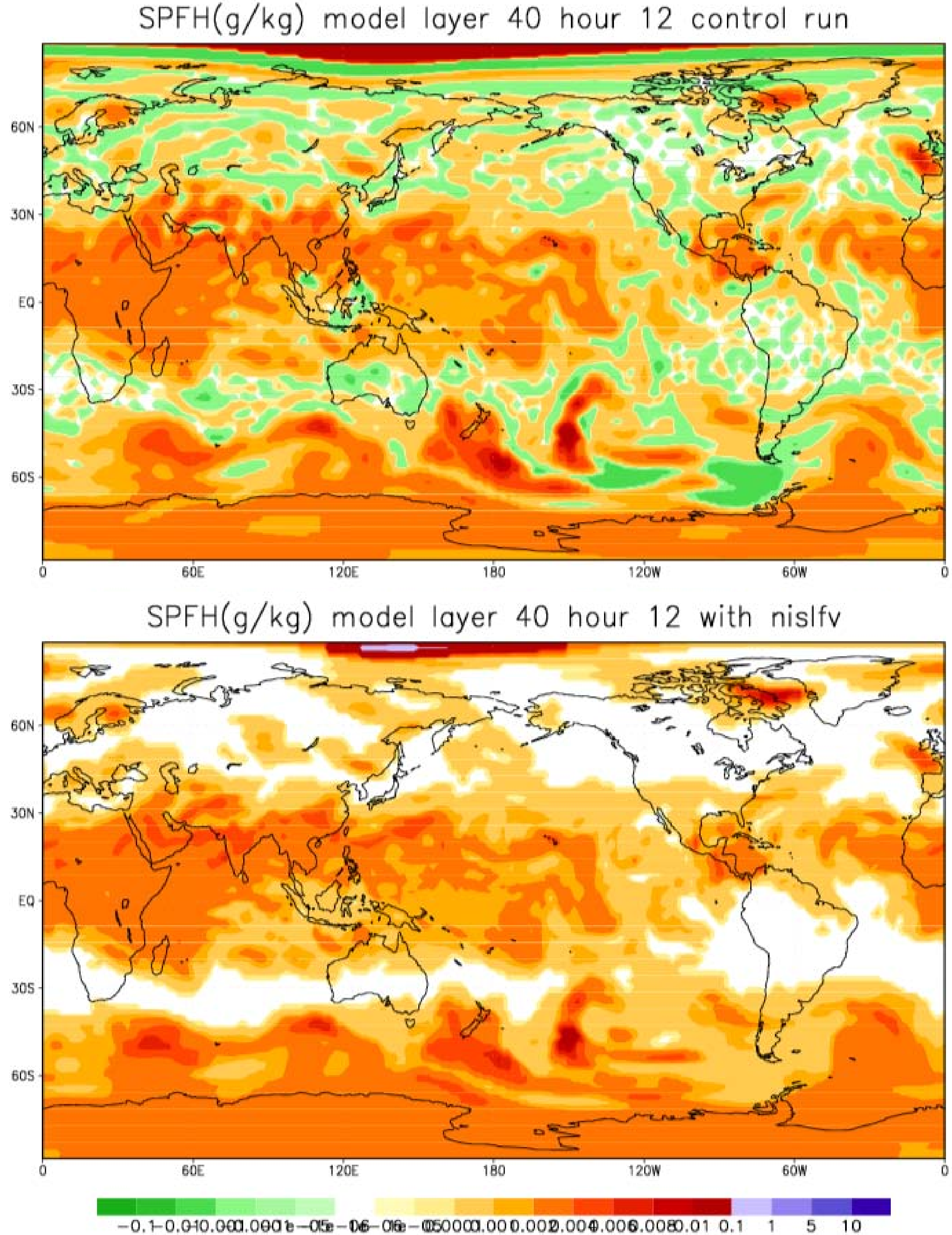
nislfv



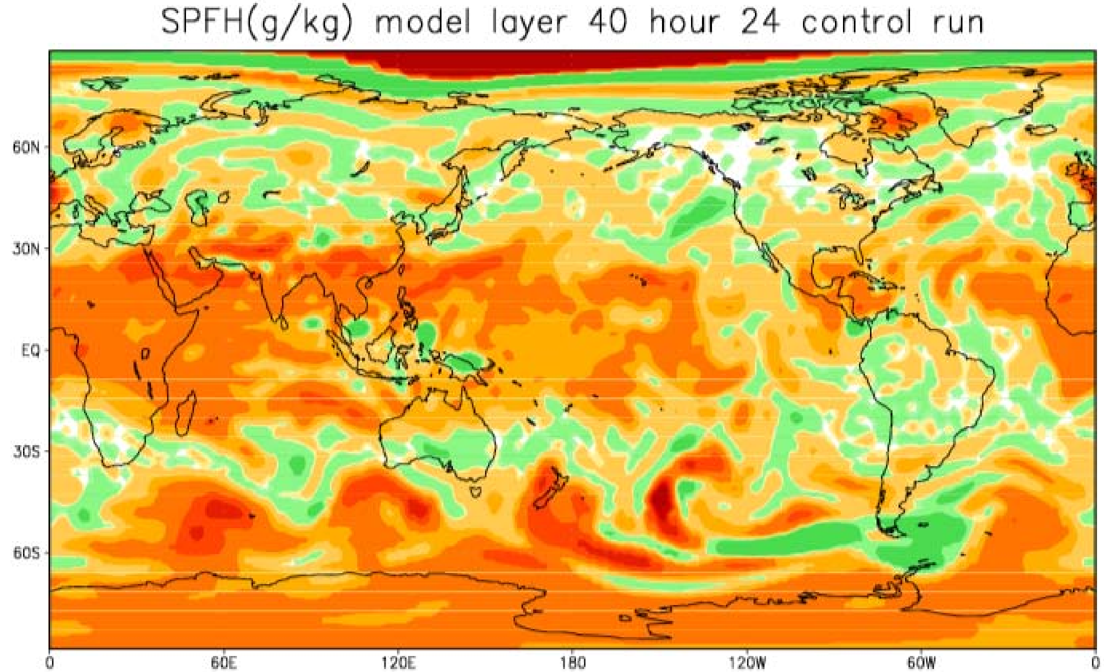
control

12h fcst specific humidity
at model layer 40

nislfv

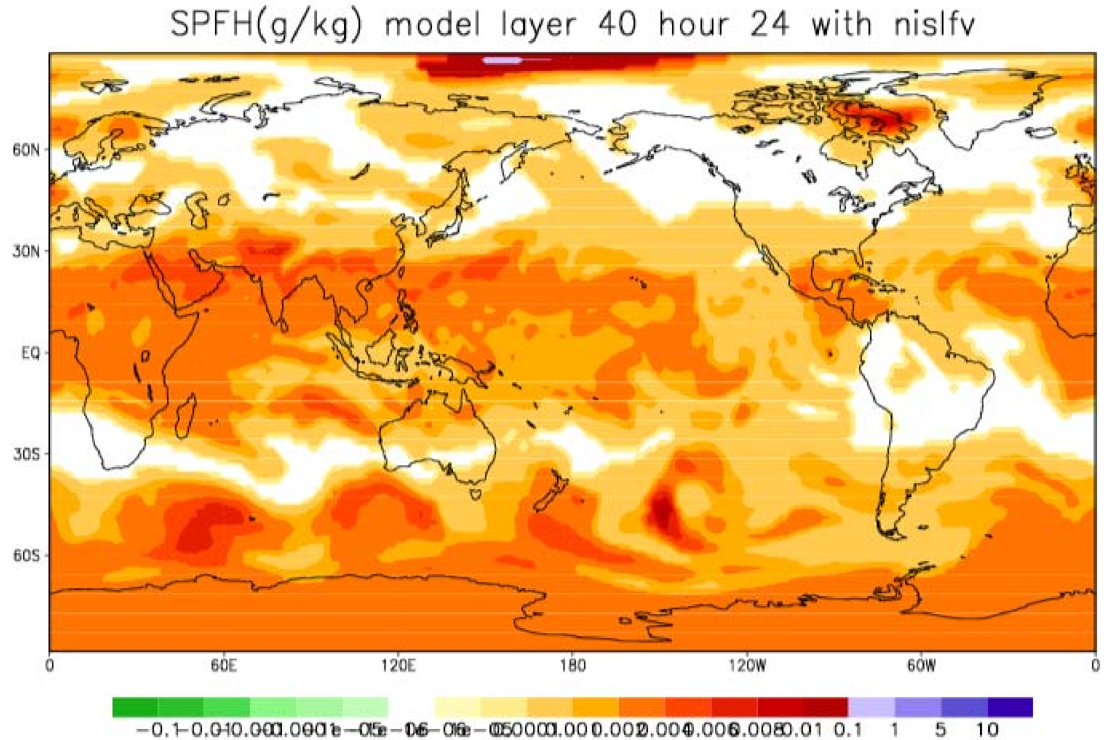


control



24h fcst specific humidity
at model layer 40

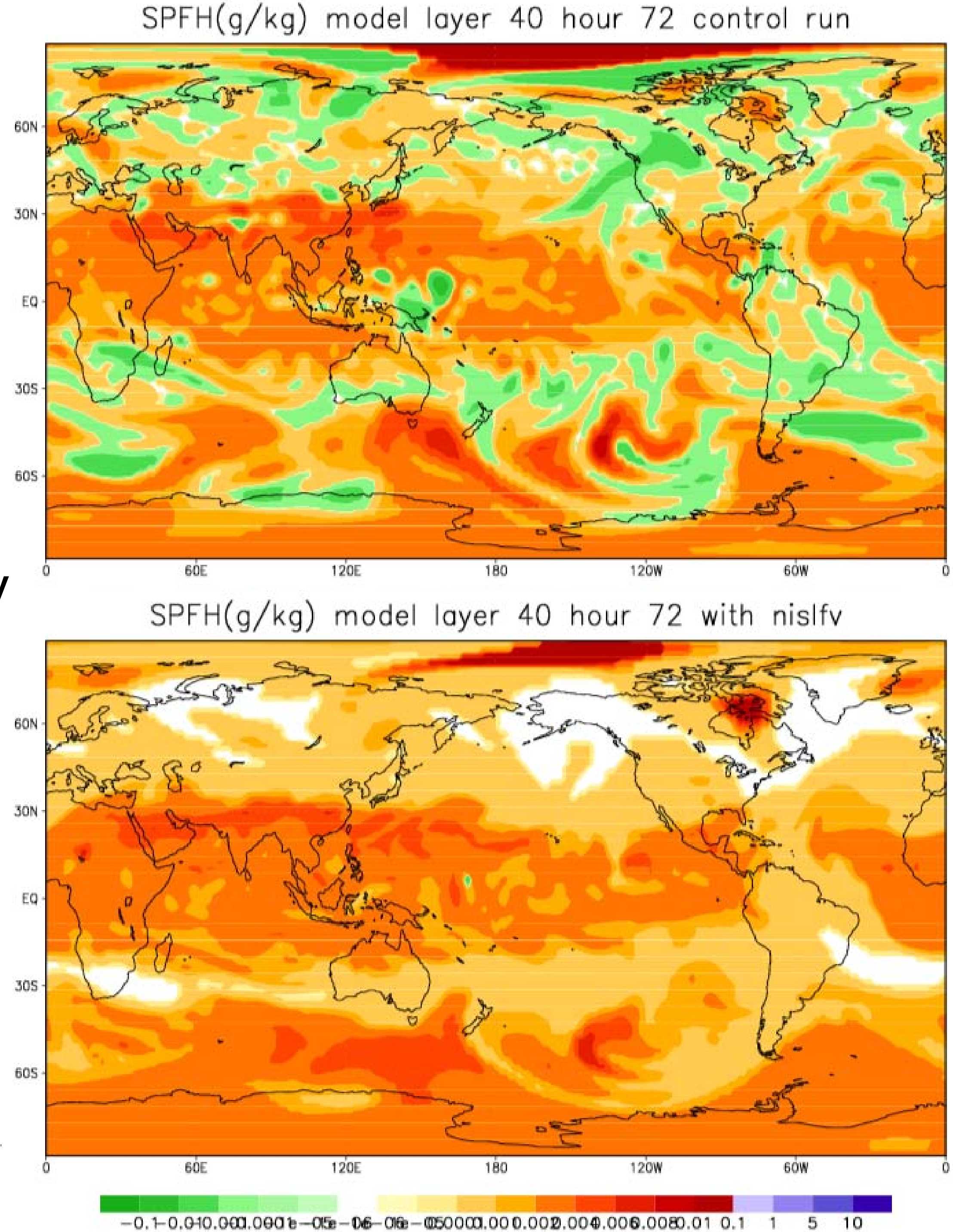
nislfv



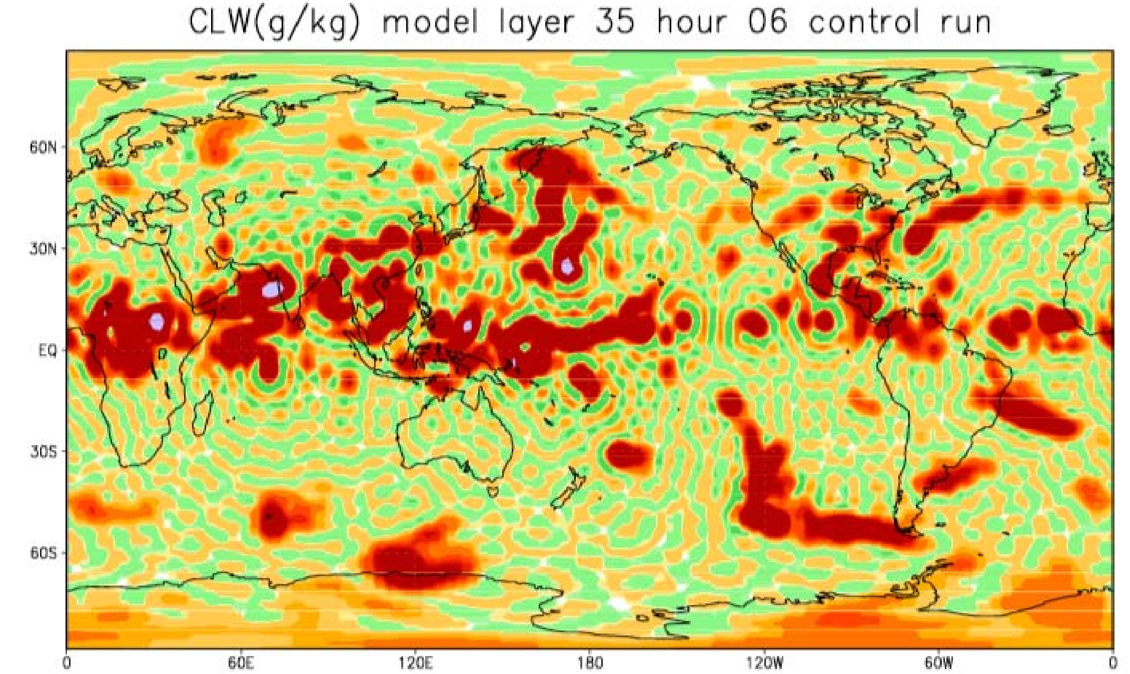
control

72h fcst specific humidity
at model layer 40

nislfv

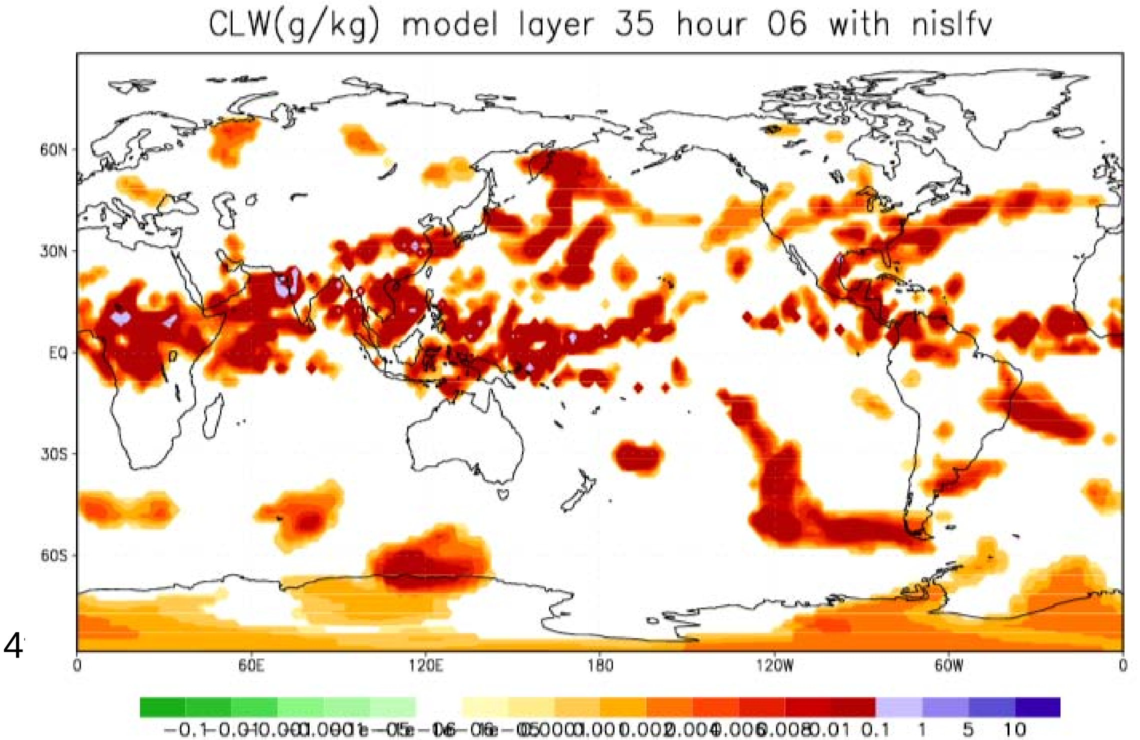


control



6hr fcst cloud water at model layer 35

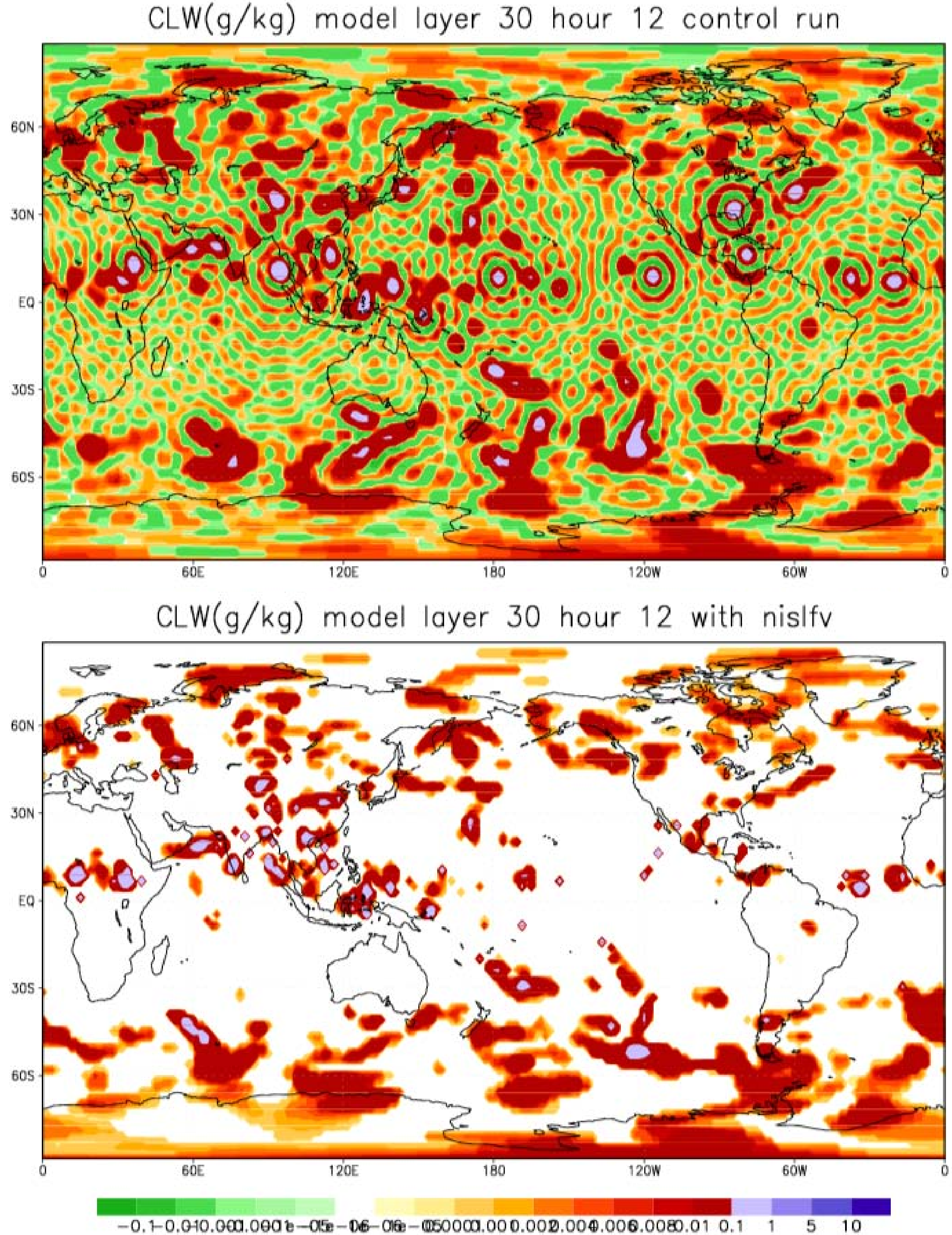
nislfv



control

12hr fcst cloud water
at model layer 30

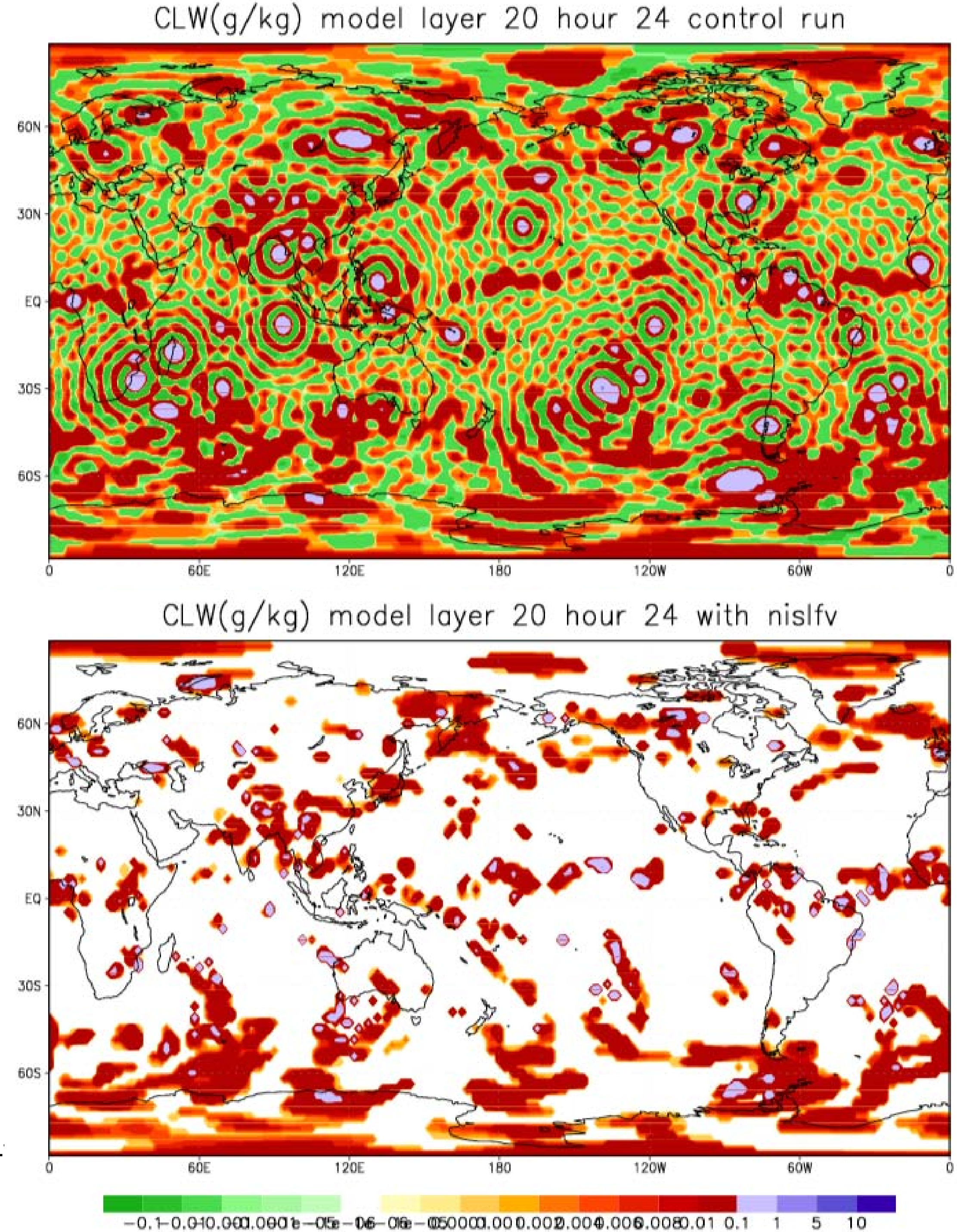
nislfv



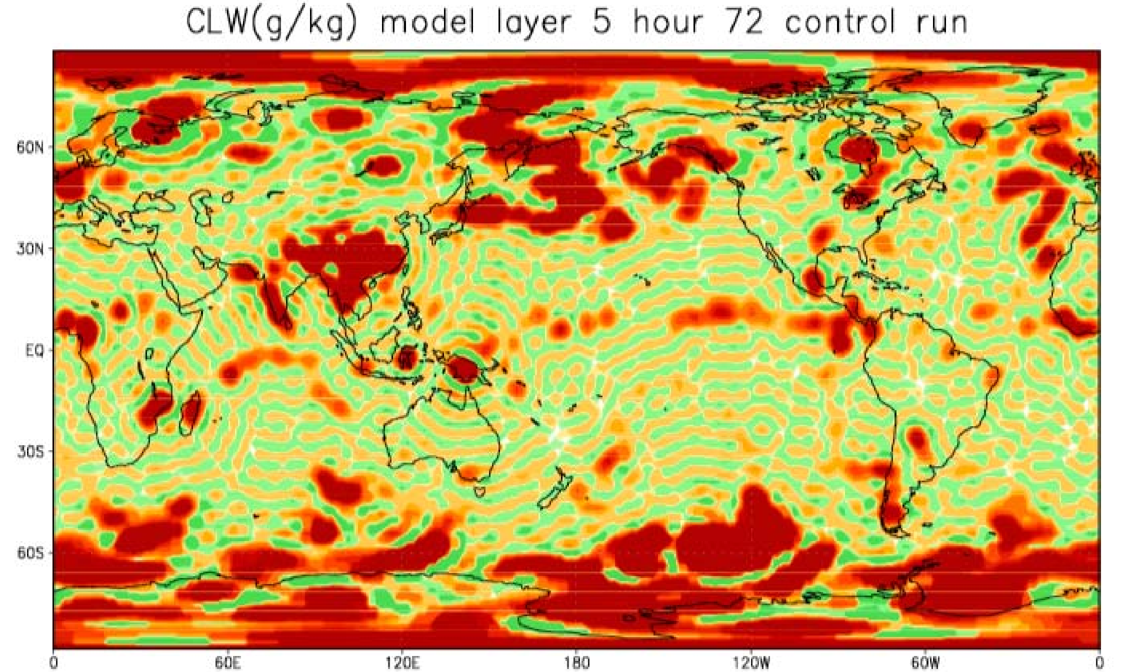
control

24hr fcst cloud water
at model layer 30

nislfv

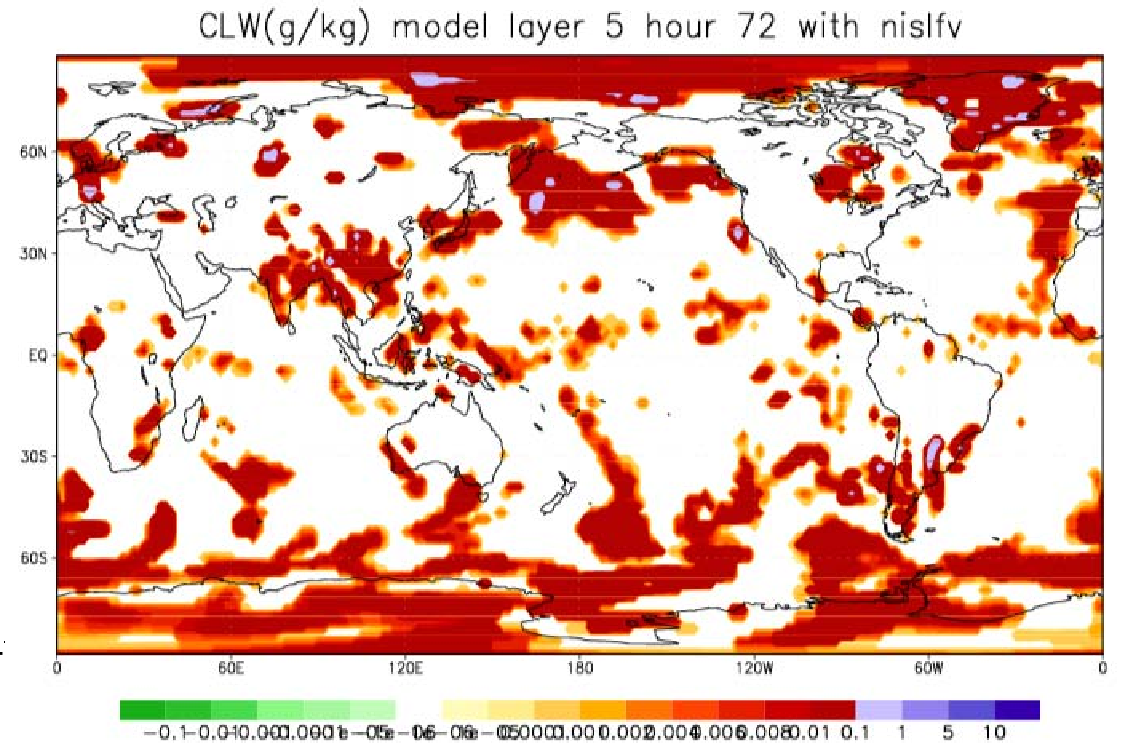


control



72hr fcst cloud water
at model layer 5

nislfv



Conclusion & Future Works

- Enthalpy generic vertical coordinate has been implemented into NCEP GFS for CFSRR and operational GFS in sigma-p version, with multi-conserving schemes.
- Mass conserving positive advection with semi-Lagrangain should be a stable scheme for large time step in sigma-theta hybrid coordinate.
- Final step to have sigma-theta requires
  – Completing semi-implicit semi-Lagragian mass conserving positive advection
  – Long period of parallel runs to obtain statistic scores