Remarks on the history of Lagrangian vertical coordinates in atmospheric modeling

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Speaker’s first encounter with this topic:

“The time has come to get serious about building an isentropic-coordinate weather prediction model”

A non-verbatim quote from an NSF proposal by **Dutton, Johnson, Danielsen** (and possibly others), seen on Danielsen’s light table in 1965 or 1966 …. not funded, presumably.

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A QUASI-LAGRANGIAN SYSTEM OF HYDRODYNAMICAL EQUATIONS

By Victor P. Starr

The University of Chicago

(Manuscript Received, January 31, 1946)

ABSTRACT

In this paper a system of hydrodynamical equations is developed which utilizes as independent variables two space coordinates and a material coordinate together with time. The space coordinates retain the same significance as the corresponding space coordinates in the Eulerian system of equations, while the material coordinate has properties identical with one of the material coordinates in the Lagrangian system. The system here presented is therefore one of the possible hybrid combinations of the two classic systems. A Cartesian reference framework is used throughout, although other reference coordinates could be used. The fundamental principles are formulated, including various forms of the equation of continuity and the equations of motion.

The main purpose of the system here described is to render certain geophysical problems more directly tractable for analysis and solution.

A thorough treatise on layered equations, including derivations of Bjerknes' Circulation Theorem and Rossby's PV Theorem:

The potential vorticity theorem ….

If (79) is combined with the continuity equation (7), the result, namely

$$\frac{1}{f+\zeta}\frac{d}{dt}(f+\zeta) = \frac{1}{\rho\dfrac{\partial z}{\partial c}}\frac{d}{dt}\left(\rho\frac{\partial z}{\partial c}\right), \qquad (80)$$

is then capable of individual integration so that

$$\frac{f+\zeta}{\rho\dfrac{\partial z}{\partial c}} = \frac{f_0+\zeta_0}{\rho_0\dfrac{\partial z_0}{\partial c}}. \qquad (81)$$

Surprisingly, no reference to *potential vorticity* or to Rossby

One question which may need additional comment is why must the quantity c be a conservative property. The main difficulty which would arise if c were not conservative relates to the expansion of an individual time derivative. Equation (2) would then have an additional term on the right involving the individual rate of change of c with time. This would introduce a quantity which generally speaking would become quite troublesome in the subsequent development of the subject. However, if we limit outselves to topics

Today we no longer fear the *dc/dt* term (a sign of progress?)

A major milestone in our field ….

PROCEEDINGS OF THE INTERNATIONAL SYMPOSIUM ON NUMERICAL WEATHER PREDICTION IN TOKYO

NOVEMBER 7–13, 1960

Edited by

SIGEKATA SYŌNO, Chief

MASAHIKO AIHARA
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THE INTERNATIONAL UNION OF GEODESY AND GEOPHYSICS

Two articles of interest here …

Numerical Experiments with the Primitive Equations

By FREDERICK G. SHUMAN

Joint Numerical Weather Prediction Unit, Suitland, U.S.A.

On the Use of a Material Layer Model of the Atmosphere in Numerical Prediction

By ARNT ELIASSEN

University of Oslo, Norway

Shuman is haunted by noise problems in his multilevel sigma coordinate primitive equation model ….

The conclusion to be drawn from the results of Part II is that handling of the vertical advection terms in the atmospheric equations is crucial to stable computational behavior. I have tentatively gone one step further. The mere presence of these terms in the finite difference equations may very well be destructive. I doubt if they can be satisfactorily handled at all, at least within the framework of the objectives I have adopted—i. e., the design of a simple finite-difference system with no artificial controls.

Excerpts from discussion following Shuman's talk ….

Phillips: Can you comment on the seemingly greater instability you had in your test computation with meteorological fields as compared to the experience of Hinkelmann and his school? They have integrated after perhaps 48 hours: but your system seems to blow up in 12 hours.

A: They had an unusually large value of the coefficient of lateral viscosity; also a regular filtering procedure, none of which I have. There is no viscosity in these computations, no artificial filtering procedures.

On the Use of a Material Layer Model of the Atmosphere in Numerical Prediction

By ARNT ELIASSEN

University of Oslo, Norway

Concluding sentence:

The model presented has not yet been tested, and much work remains before we shall know its possible virtues and shortcomings.

Discussion following Eliassen's talk …

Shuman: I was lead to precisely the same formulation from considerations of computational stability. I have a stable system for computing the layer equations where you integrate superposed homogenous layers and these are quite stable and the equations are very similar, of course, to this set.

Charney: Have you considered what will happen when you take, say, several material layers? Over a long period of time these layers may get very close together. They will not intersect but they can become infinitesimally close.

A: Yes, of course, that will happen. I am sure that one will have to choose a new system of surfaces in the middle of the calculation, by interpolation, if this is going to be applied during more than, say, one or two days. So I don't think it is very suitable for predictions for long periods.

Mintz: Won't these, in the adiabatic case, belong to the surfaces of constant potential temperature? And how would you describe the initial state for potential temperature surfaces which intersect the ground?

A: They may not be surfaces of potential temperatures. For instance, you may choose one as 500 millibar surface in the beginning, at the initial time, and then, from that it will move as a material surface. You can choose them however you like.

6 years later: Shuman & Hovermale, 1968…

bounded by quasi-horizontal material surfaces. In such systems vertical variations appear as true differences across each surface bounding the various layers. "Layered" resolution thus leads directly to Starr's (1945) quasi-Lagrangian coordinate system. This was proposed by Shuman (1962), but as a solution to the wrong problem, the problem turning out to be a gross violation of conservation laws, as discussed in the foregoing. Valid experiments were run several years ago at NMC with Starr's coordinate system, experiments which rather clearly delineated its limitations. The basic limitation derives from the fact that a surface to be carried as a material surface during an integration will become multiple-valued in the vertical if initially drawn arbitrarily.

After limited experimentation with Starr's quasi-Lagrangian system, we turned to Phillips' (1957) σ-coordibounded by quasi-horizontal material surfaces. In such systems vertical variations appear as true differences across each surface bounding the various layers. "Layered" resolution thus leads directly to Starr's (1945) quasi-Lagrangian coordinate system. This was proposed by Shuman (1962), but as a solution to the wrong problem, the problem turning out to be a gross violation of conservation laws, as discussed in the foregoing. Valid experiments were run several years ago at NMC with Starr's coordinate system, experiments which rather clearly delineated its limitations. The basic limitation derives from the fact that a surface to be carried as a material surface during an integration will become multiple-valued in the vertical if initially drawn arbitrarily.

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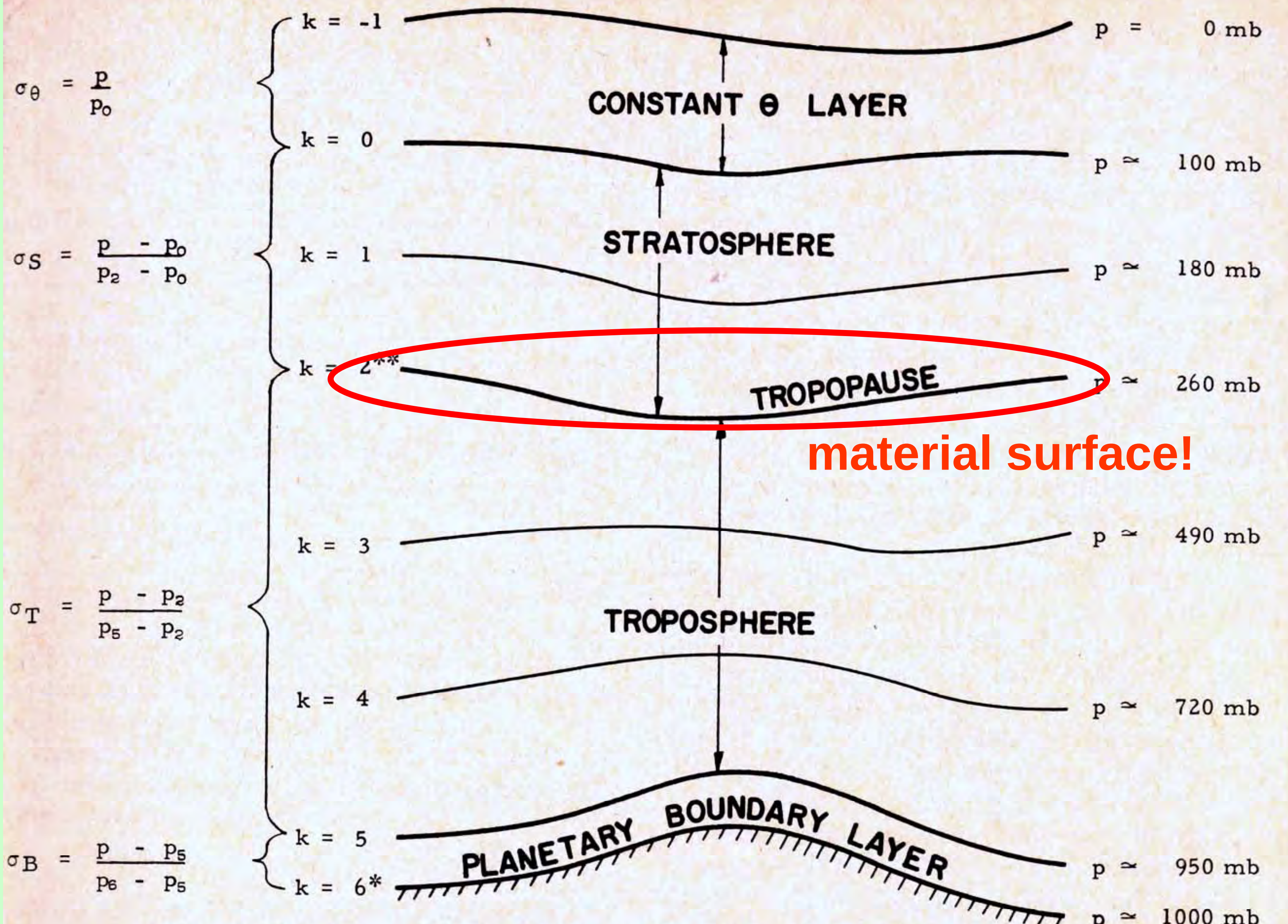

FIG. 2. Schematic diagram of the vertical structure of the six-layer model.

Back to …

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METEOROLOGISKE ANNALER BD. 5 NO. 2

A NUMERICAL INTEGRATION EXPERIMENT WITH A MODEL ATMOSPHERE BASED ON ISENTROPIC SURFACES

by

Arnt Eliassen and Elmer Raustein
Institute of Geophysics, University of Oslo

(Manuscript received Sept. 5 1967)

This work was performed under the sponsorship of U. S. Weather Bureau.

Abstract

A mathematical model of the atmosphere is presented, in which the information surfaces are sloping isentropic surfaces which may intersect the ground. The results of a numerical test integration of the primitive equations for this model are shown.

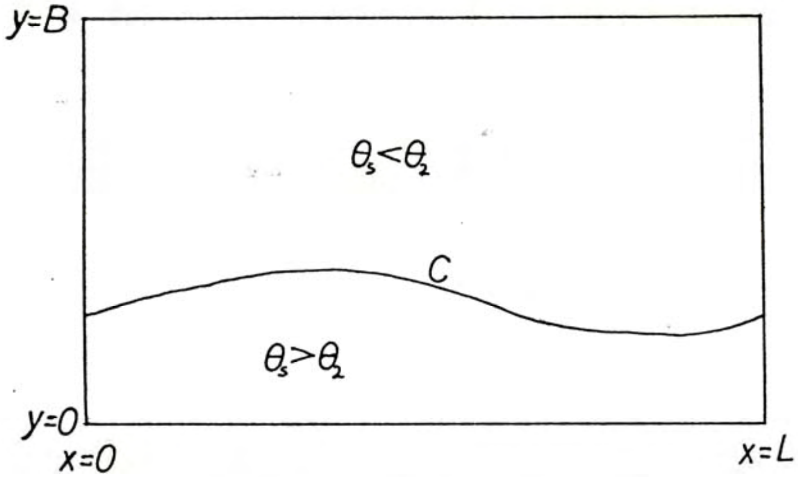
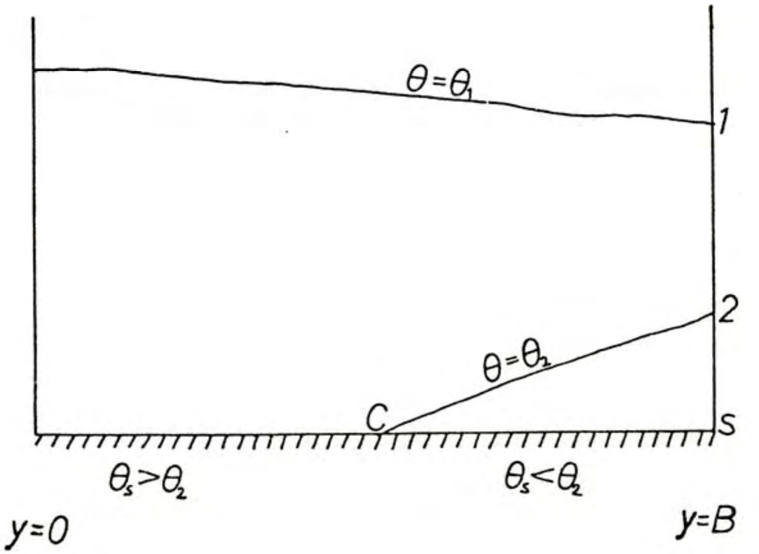
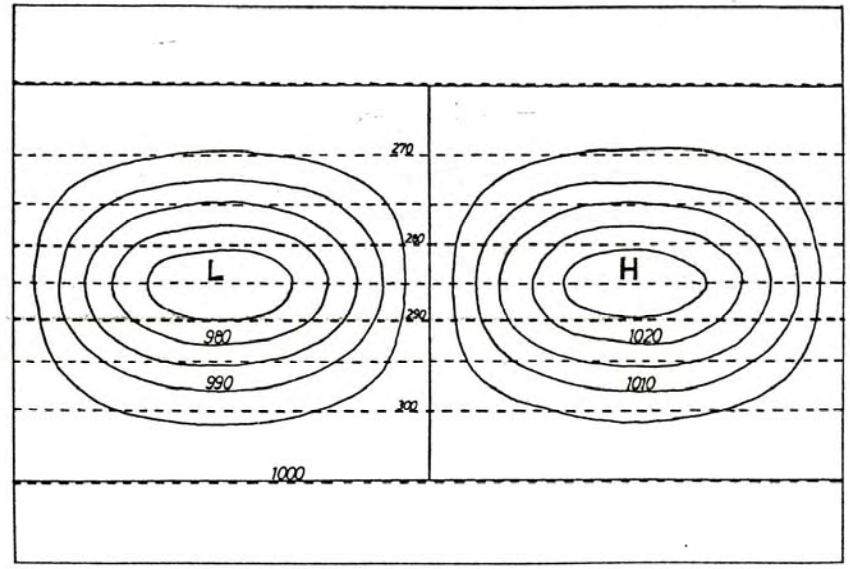


Figure 1. The region of integration.





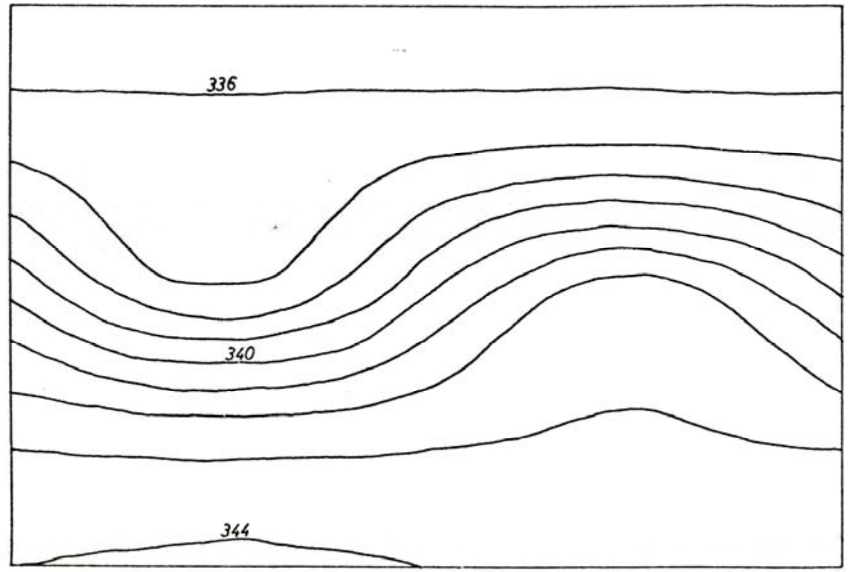


Fig. 3. Initial condition

Initial conditions, Eliassen-Raustein 2-layer model (1967)

72-hr simulation, Eliassen-Raustein 2-layer isentropic model (1967)

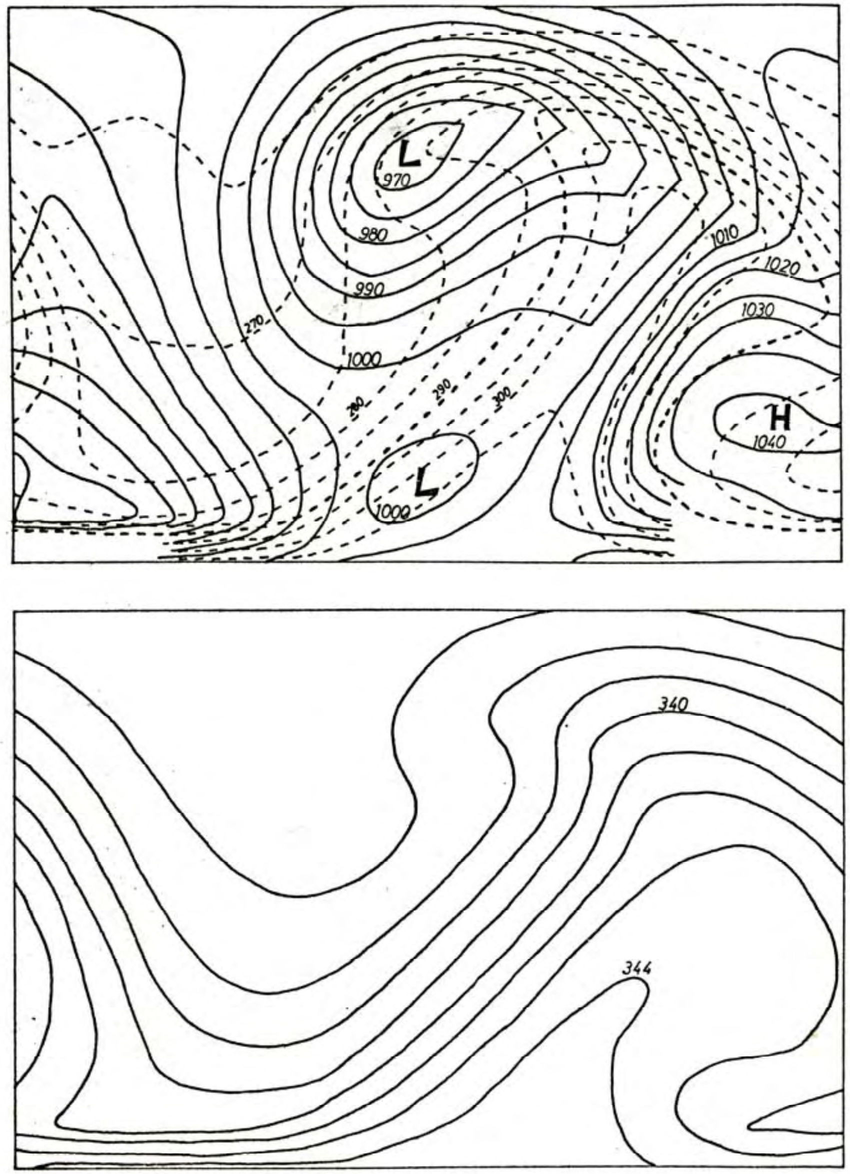


Fig. 11. After 72 hours; non-linear diffusion. Isolines as in Fig. 3.

Wrap-up: 50 years ago, the centers of action (as far as layer modeling is concerned) were Oslo and Washington, D.C. While the Norwegian work led to the first multilayer isentropic model, U.S. efforts culminated in an eminently successful "hybrid" model in which troposphere and stratosphere were represented by two material layers, each one subdivided into 3 sigma-like layers.

The End