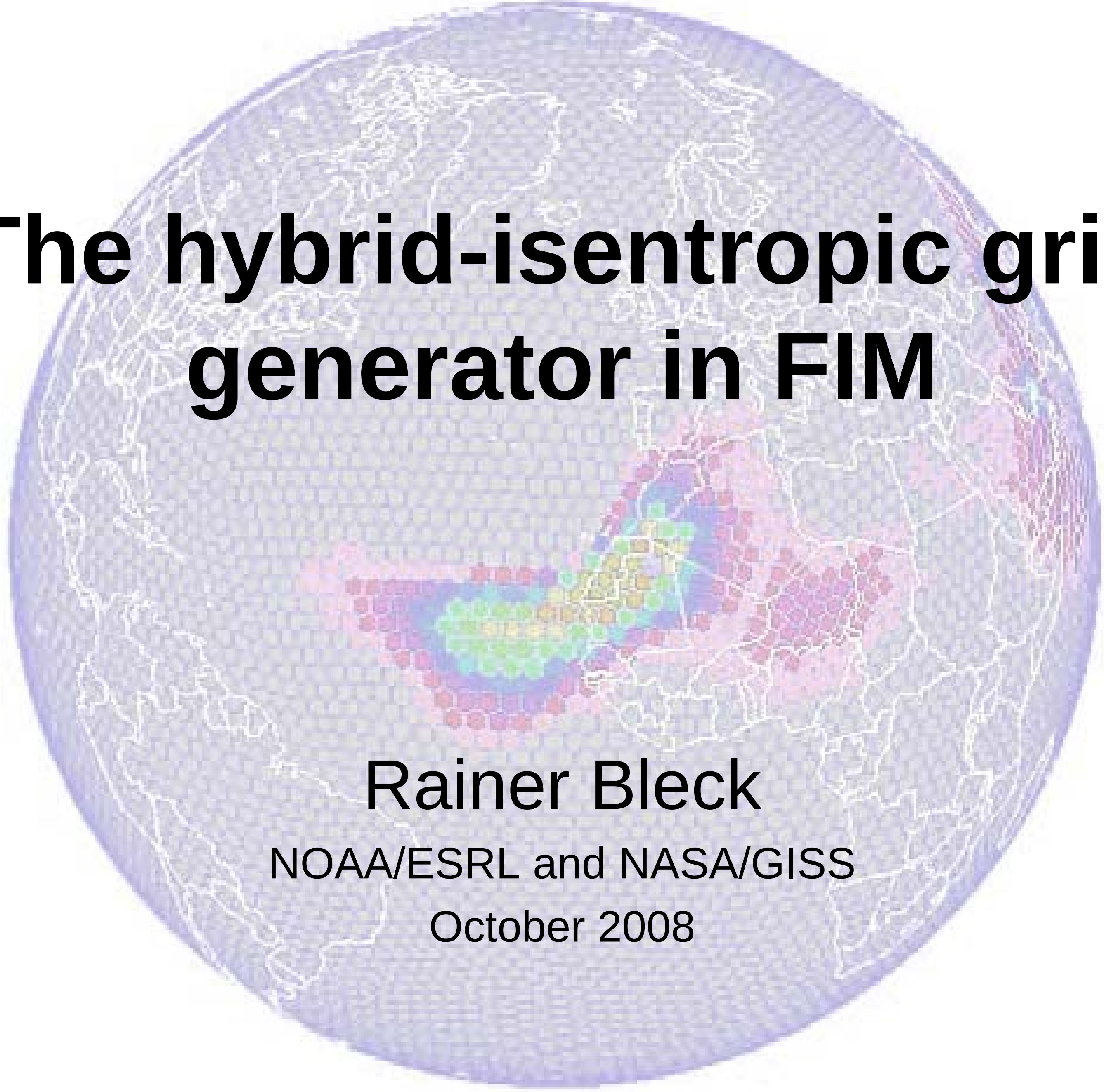


The hybrid-isentropic grid generator in FIM

Rainer Bleck

NOAA/ESRL and NASA/GISS

October 2008

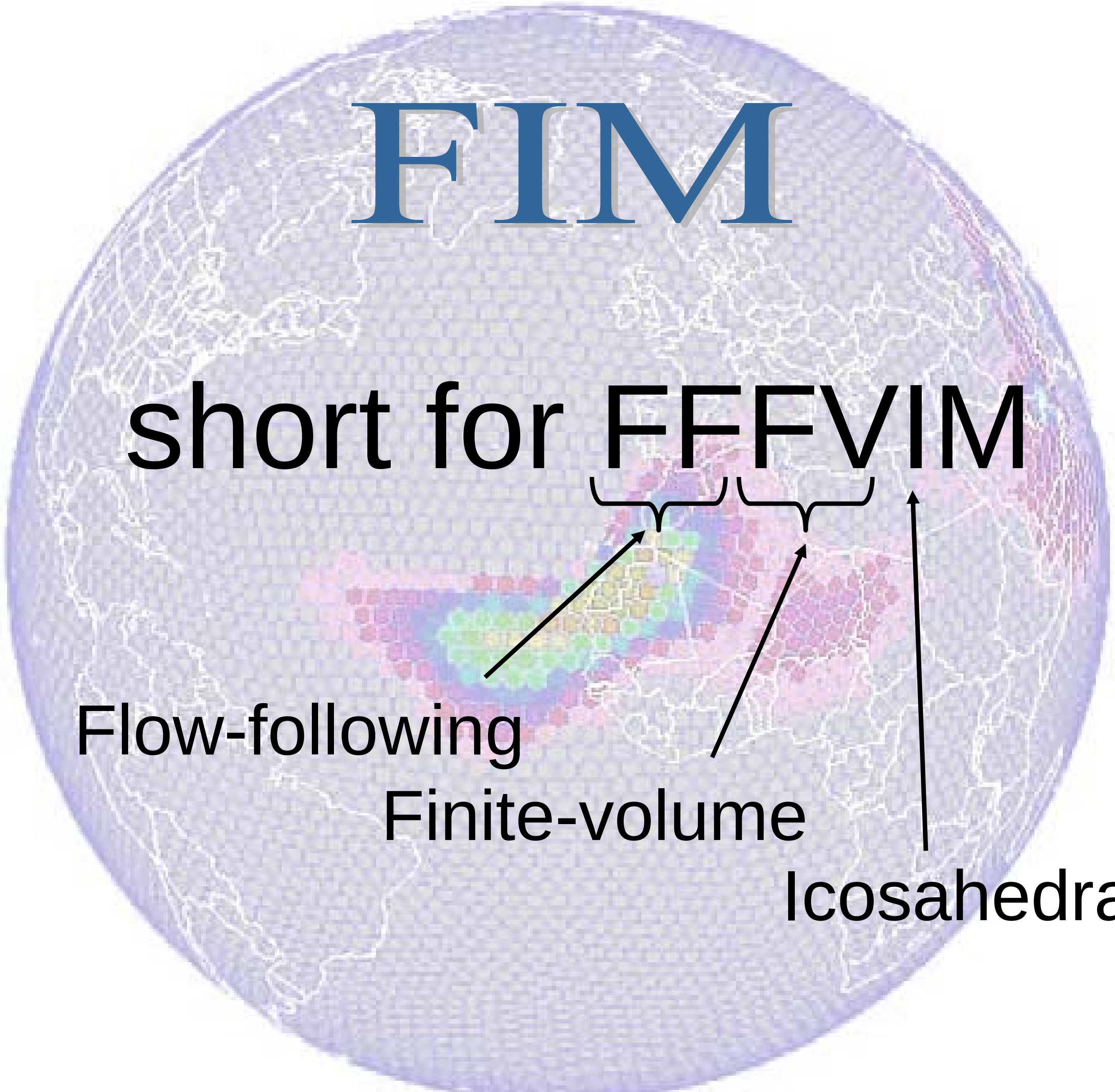
FIM

short for FFFVIM

Flow-following

Finite-volume

Icosahedral



Vertical grid considerations

- Geophysical fluids are "shallow" … but still rich in vertical structure. Inadvertent vertical mixing must be avoided.
- Strong flows often occur near boundaries (top, bottom, side). Grid should provide good resolution there and make it easy to apply boundary conditions. (σ coordinate )
- Grid points that follow vertical motion ("Lagrangian" grid) can prevent numerical dispersion during wave-induced vertical transport. (θ coordinate )
- Sloping coordinate surfaces can make it difficult to compute the horizontal pressure gradient. (*z, p,* or η coordinate )
- Fluids tend to form discontinuities (fronts). High resolution near fronts would be desirable. (θ coordinate )

Lagrangian vertical coordinate: Pros and Cons

("Lagrangian" = isentropic in atmospheric applications)

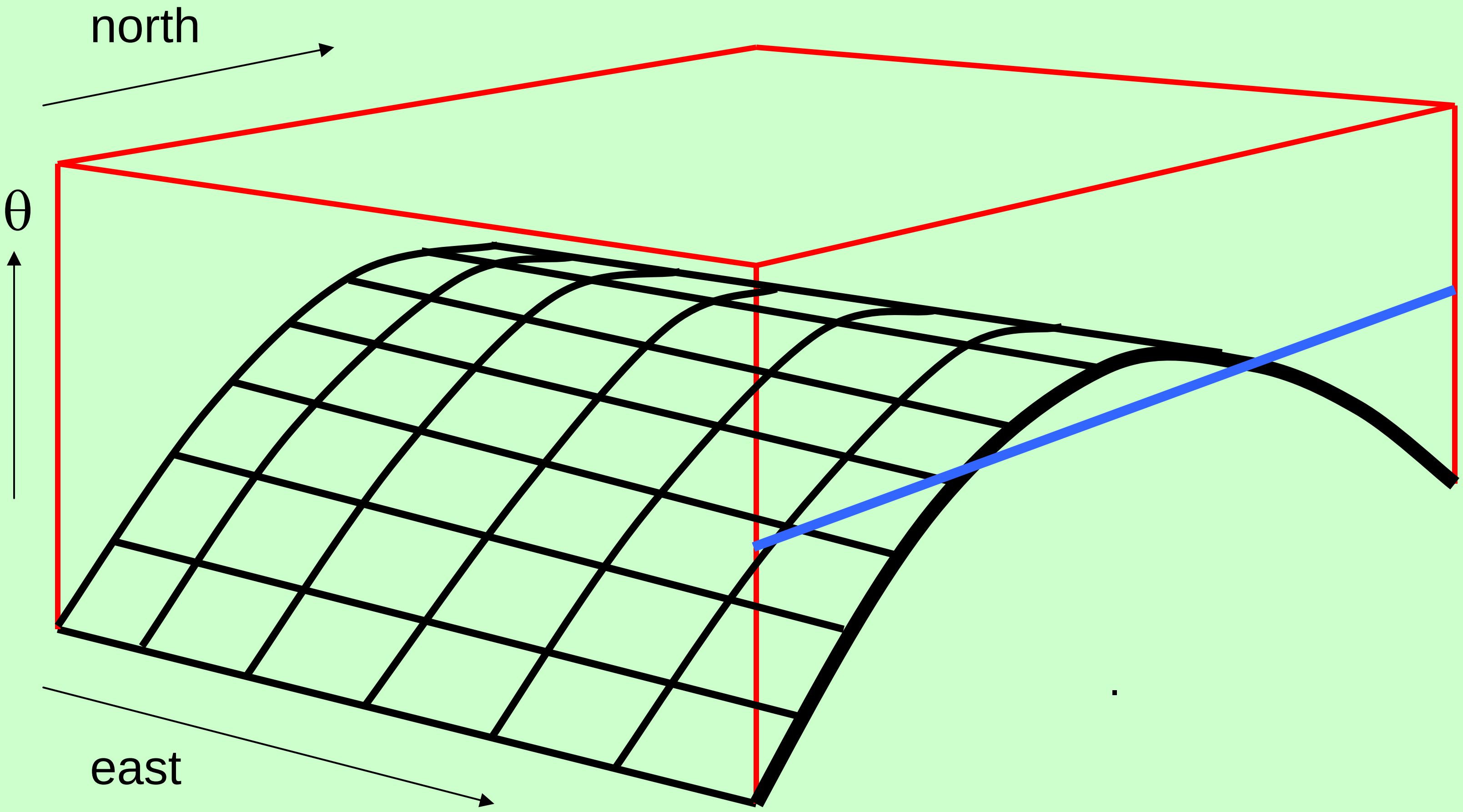
Major Pros:

- Subgridscale horizontal eddy **mixing** has no false diabatic component
- Numerical dispersion errors associated with vertical **transport** are minimized
- Optimal finite-difference representation of frontal zones & frontogenesis

Major Cons:

- Coordinate-ground intersections are inevitable (atmosphere doesn't fit snugly into x, y, θ grid box)
- Poor vertical resolution in weakly stratified regions
- Elaborate transport operators needed to achieve conservation

The x, y, θ grid box



Major Cons:

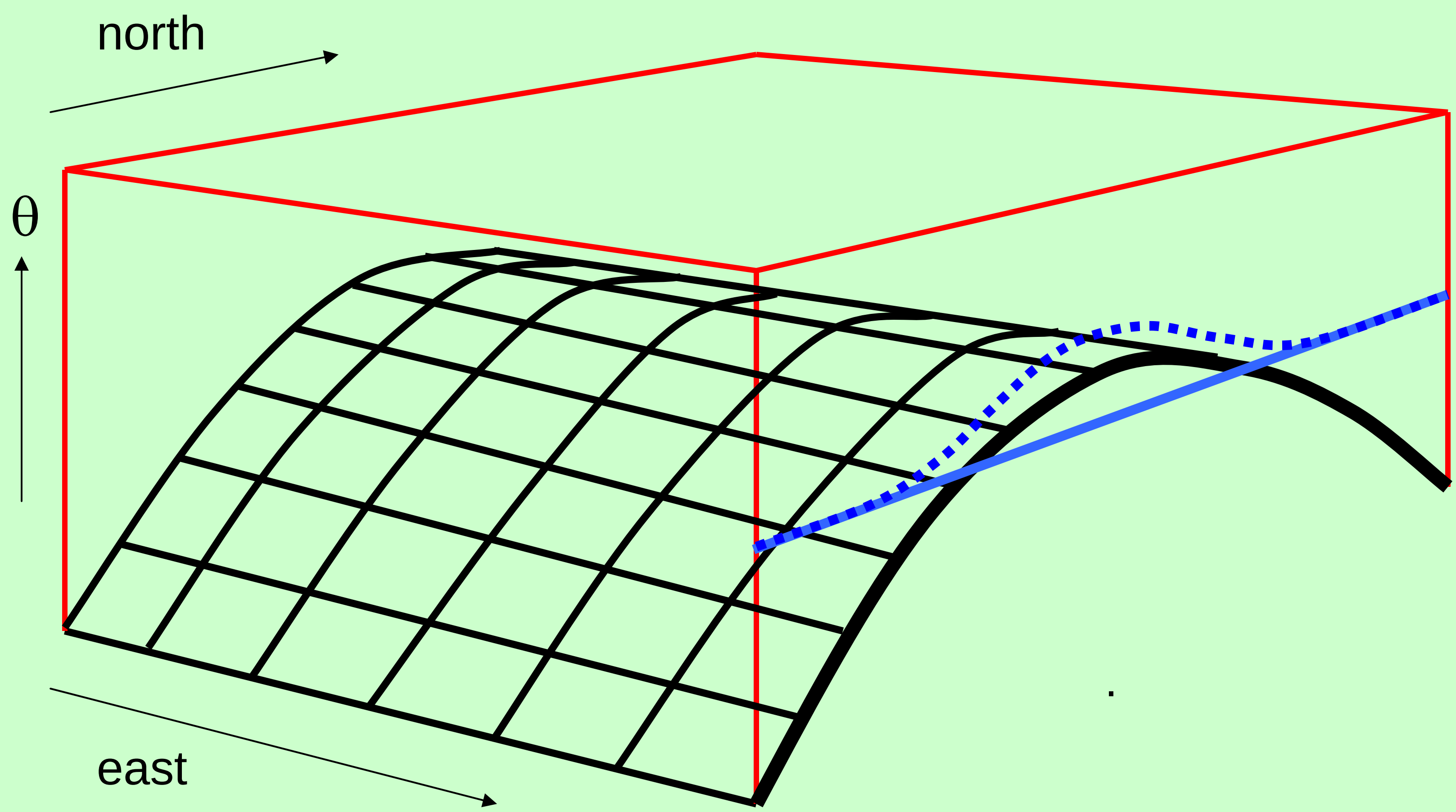
- Coordinate-ground intersections are inevitable (atmosphere doesn't fit snugly into x,y,θ grid box)
- Poor vertical resolution in weakly stratified regions

Fixes:

- Reassign grid points from underground portion of x,y,θ grid box to above-ground "*s*" surfaces
- Low stratification => large portion of x,y,θ grid box is underground => no shortage of grid points available for re-deployment as "*s*" points

=> A "hybrid" grid appears to have distinct advantages – both from a grid-economy and a vertical resolution perspective

The x, y, θ grid box



Grid degeneracy is main reason for introducing **hybrid** vertical coordinate

"Hybrid" means different things to different people:

- linear combination of 2 or more conventional coordinates (examples: p+sigma, p+theta, p+theta+sigma)
- ALE (Arbitrary Lagrangian-Eulerian) coordinate

ALE maximizes size of isentropic subdomain

ALE: "Arbitrary Lagrangian-Eulerian" coordinate

- Original concept (Hirt et al., 1974): maintain Lagrangian character of coordinate but "re-grid" intermittently to keep grid points from fusing.
- In FIM, we apply ALE in the vertical only and re-grid for 2 reasons:

  (1) to maintain minimum layer thickness;

  (2) to nudge an entropy-related thermo-dynamic variable toward a prescribed layer-specific "target" value by importing fluid from above or below.
- Process (2) renders the grid **quasi-isentropic**

The FIM grid generator

Design Principles:

- Choice of θ or pot. energy conservation
- Monotonicity-preserving (no new θ extrema during re-gridding)
- Layer too cold[1] => entrain warmer[1] air from above
- Layer too warm[1] => entrain colder[1] air from below
- Maintain finite layer thickness near surface but allow massless layers aloft
- Minimize diurnal vertical migration of coordinate layers by keeping non-isentropic layers near bottom of air column.

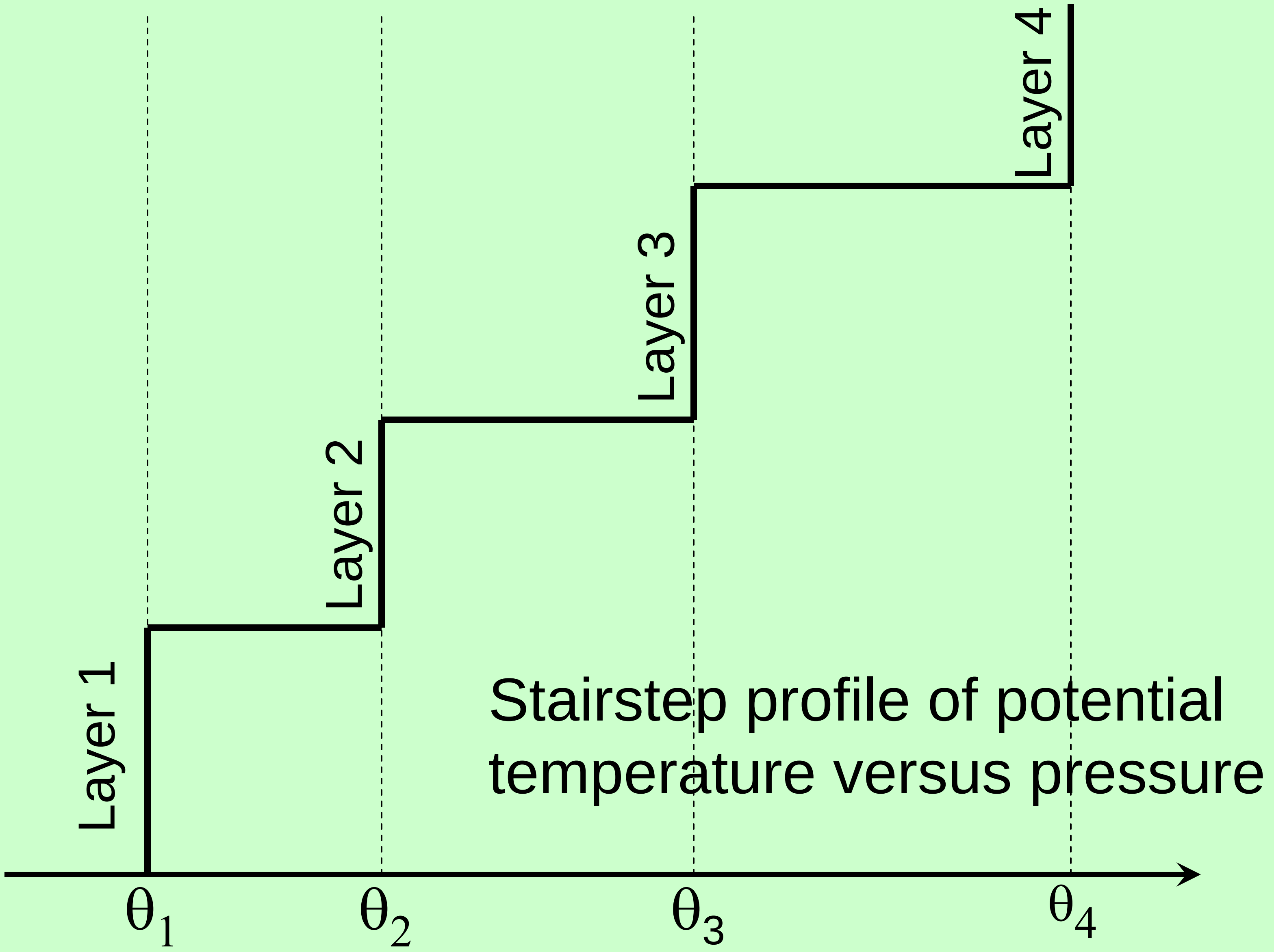
[1]in terms of potential temperature

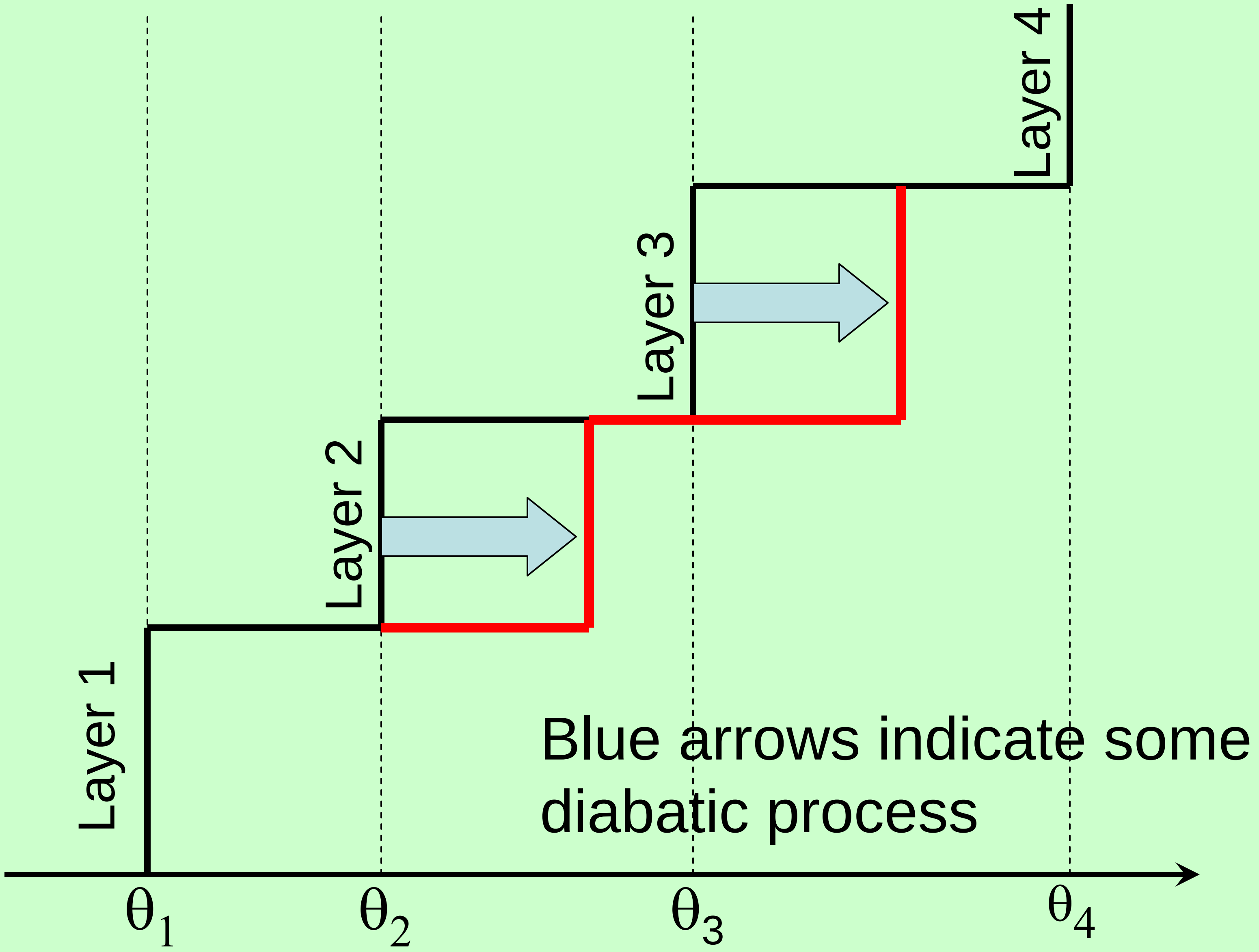
ALE: "Arbitrary Lagrangian-Eulerian" coordinate

- Original concept (Hirt et al., 1974): maintain Lagrangian character of coordinate but "re-grid" intermittently to keep grid points from fusing.
- In FIM, we apply ALE in the vertical only and re-grid for 2 reasons:

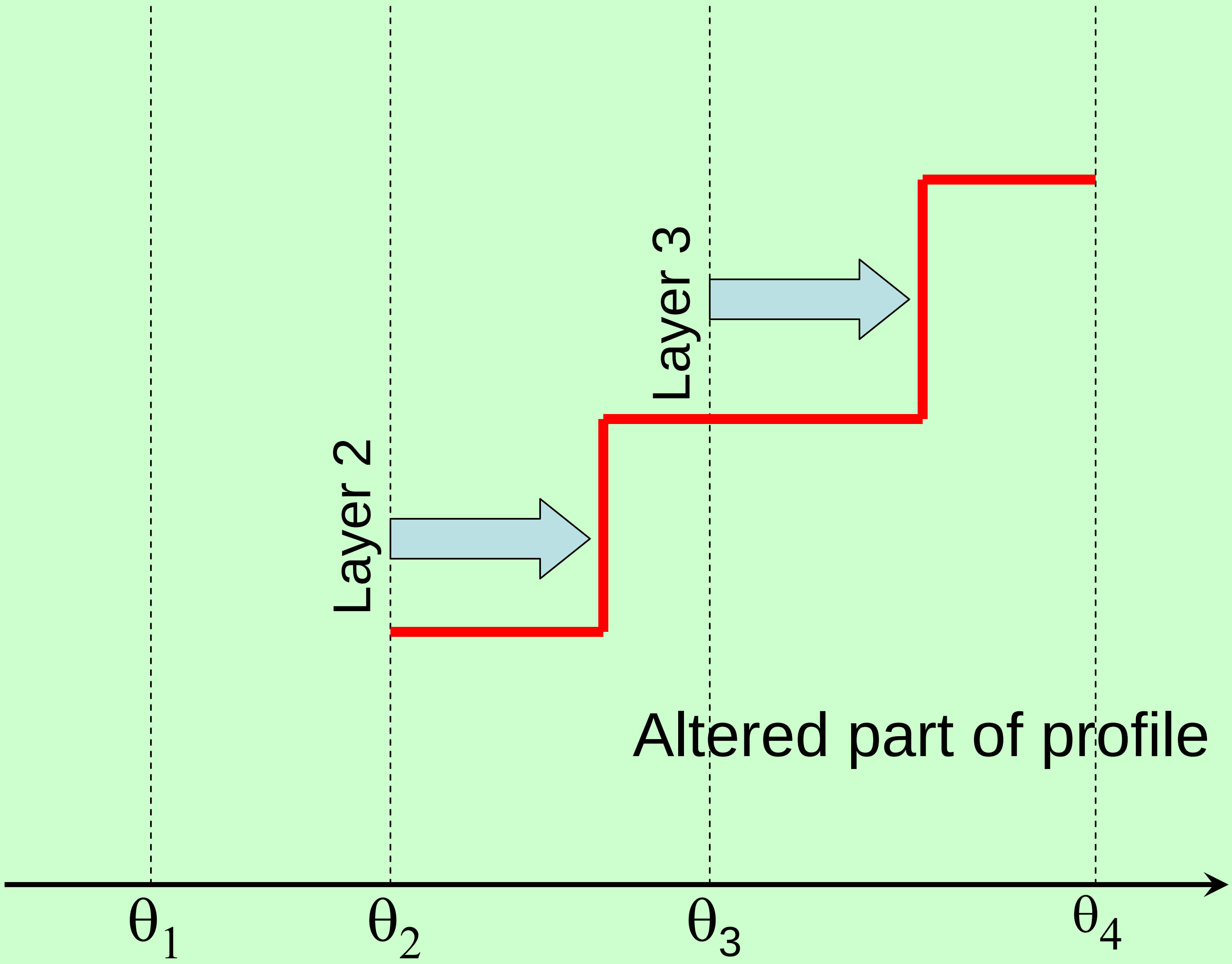
  (1) to maintain minimum layer thickness;

  (2) to nudge an entropy-related thermo-dynamic variable toward a prescribed layer-specific "target" value by importing fluid from above or below.
- Process (2) renders the grid **quasi-isentropic**

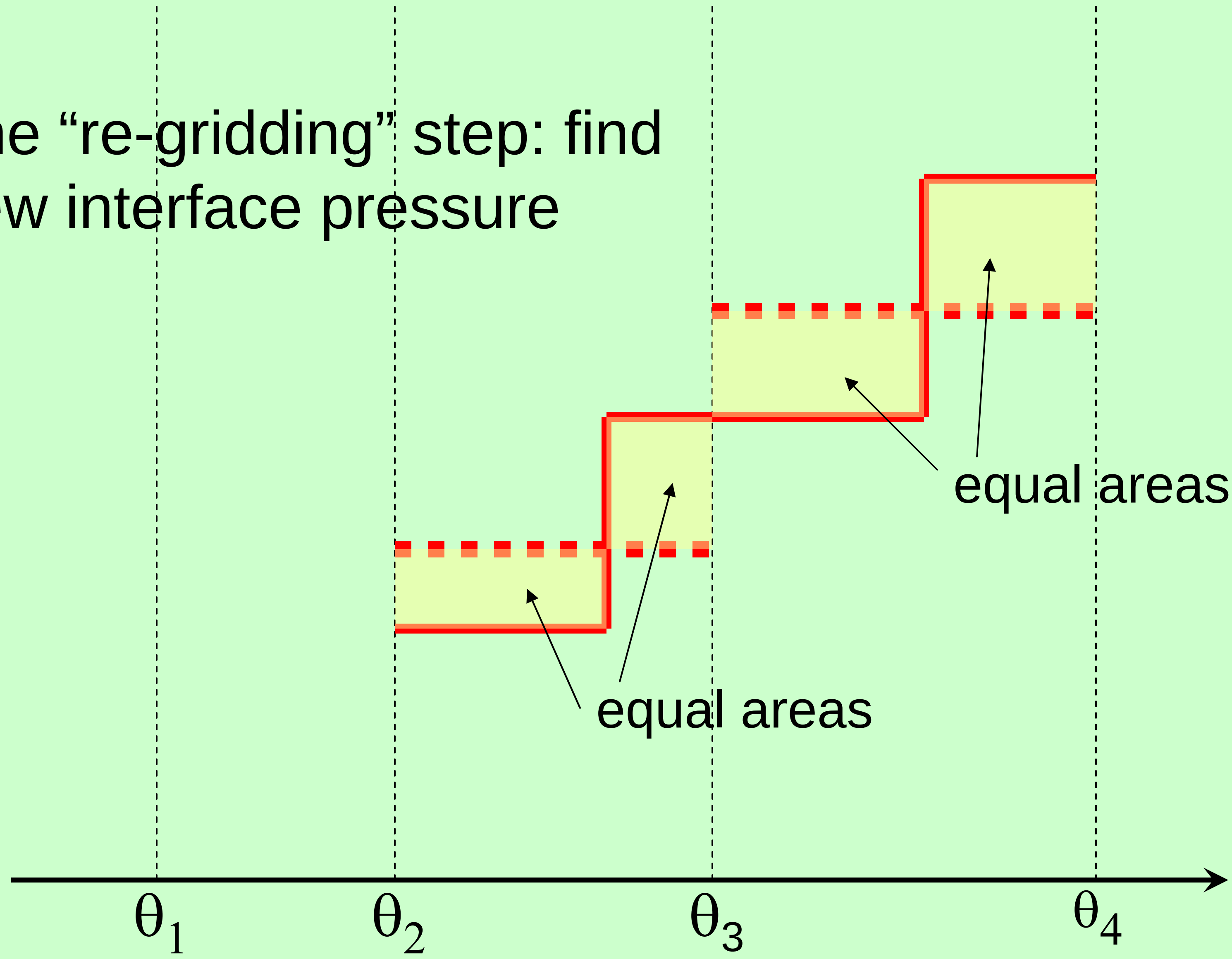

Layer 1
Layer 2
Layer 3
Layer 4
Stairstep profile of potential
temperature versus pressure
θ1
θ2
θ3
θ4

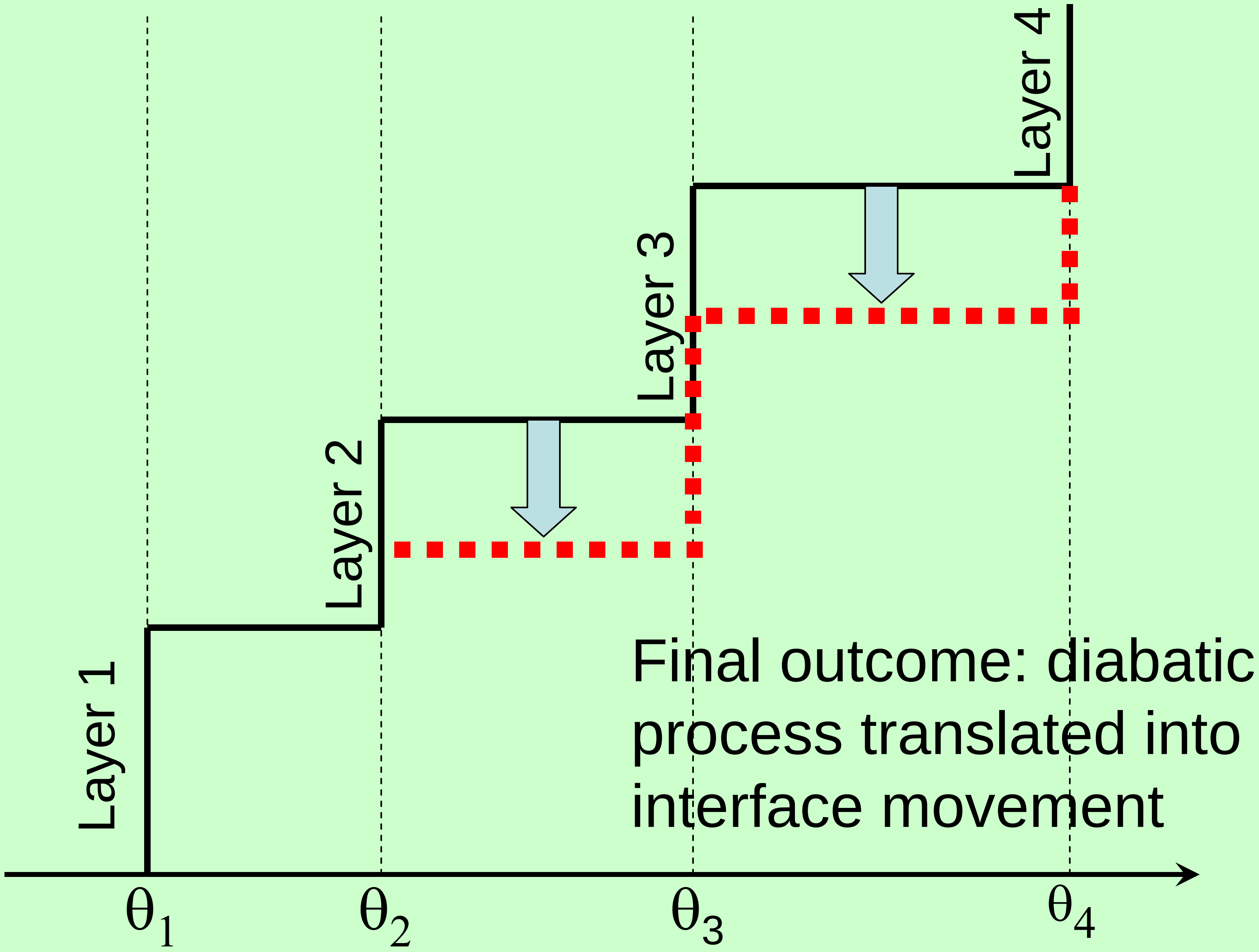


Layer 1
Layer 2
Layer 3
Layer 4
Blue arrows indicate some diabatic process
θ_1
θ_2
θ_3
θ_4


Layer 2
Layer 3
Altered part of profile
θ_1
θ_2
θ_3
θ_4

The “re-gridding” step: find new interface pressure




Layer 1
Layer 2
Layer 3
Layer 4
Final outcome: diabatic process translated into interface movement
θ1
θ2
θ3
θ4

The FIM grid generator (cont'd - 1)

- Determine how much air from the neighboring layer ("source layer") would be needed to restore target pot. temperature.
- The amount needed, Δp_{need}, may exceed the amount available, Δp_{avail}, in source layer.
- The amount ultimately transferred is $\min(\Delta p_{need}, \Delta p_{avail} - \Delta p_{min})$.
- The minimum thickness Δp_{min} is prescribed.

The FIM grid generator (cont'd - 2)

- The condition $\Delta p_{need} > \Delta p_{avail}$ typically occurs under the following conditions:
  - receiving layer is much warmer that target
  - restoration to target pot. temperature requires more mass from source layer than is available.
- The likelihood for this to happen is greatest at low latitudes immediately above the surface

  => low-latitude near-surface layers are more likely to end up with constant thickness than layers elsewhere.

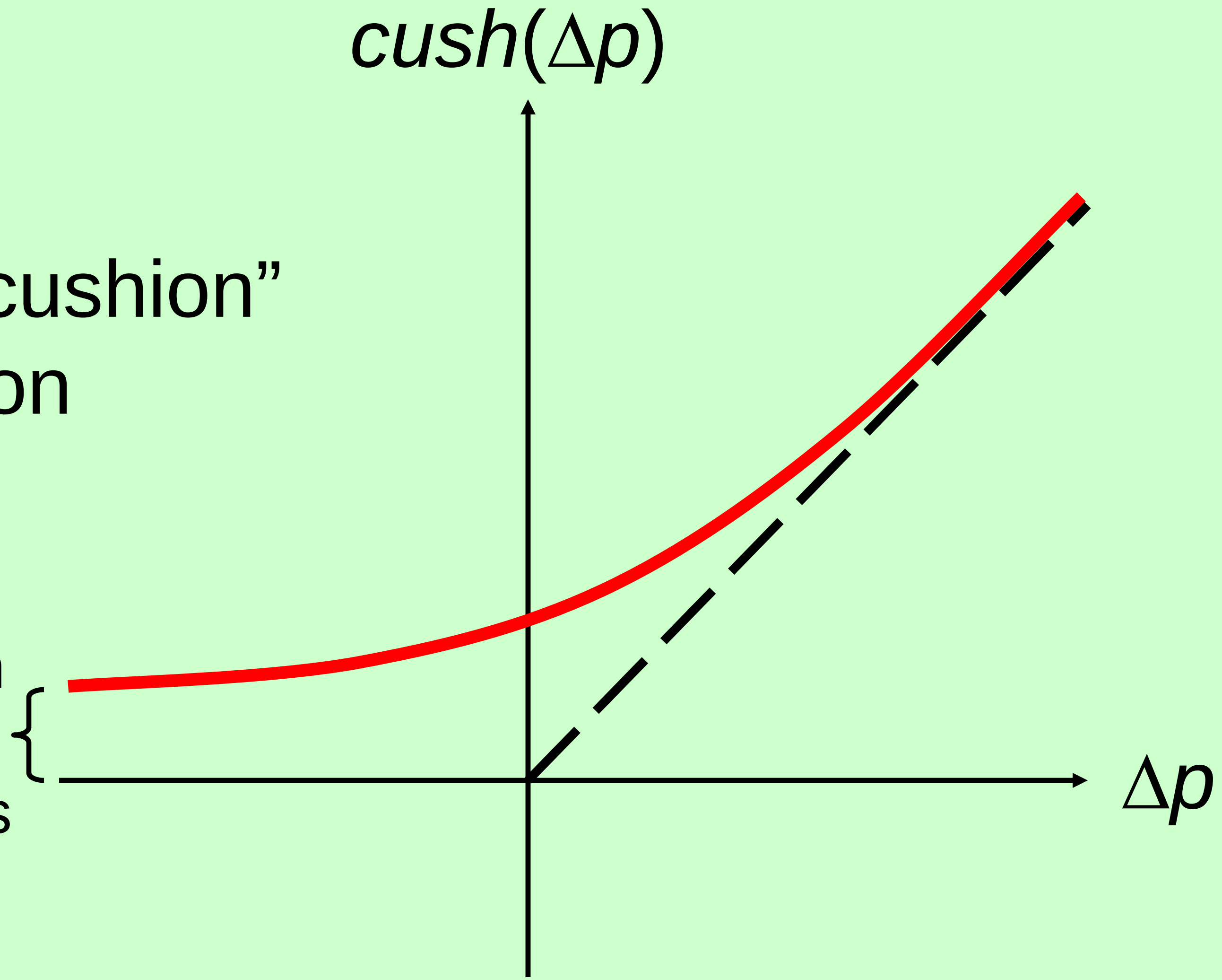
The FIM grid generator (cont'd - 3)

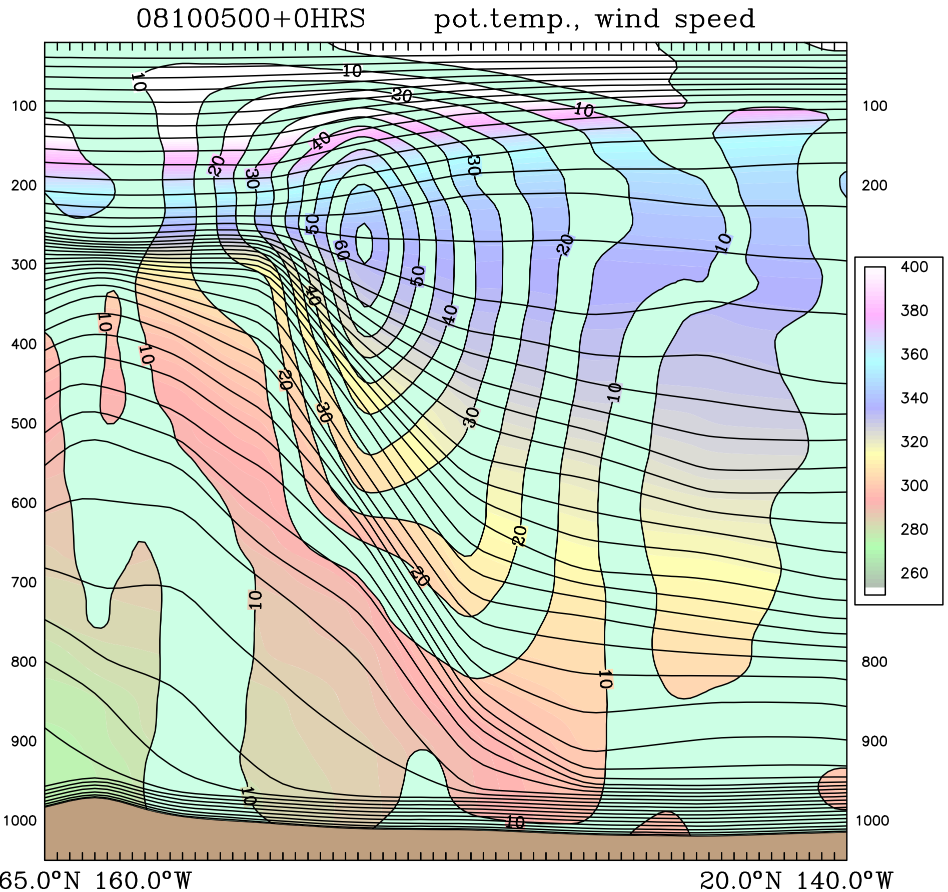
- Major challenge: achieve smooth **lateral** transition between prescribed-thickness and isentropic segments of a coordinate layer.
- Goal: avoid sideways-looking algorithms, i.e., accomplish transition through clever **vertical** re-gridding alone.
- Solution (at least a step in the right direction): employ a "cushion" function. Details of the algorithm are as follows ….

The FIM grid generator (cont'd - 4)

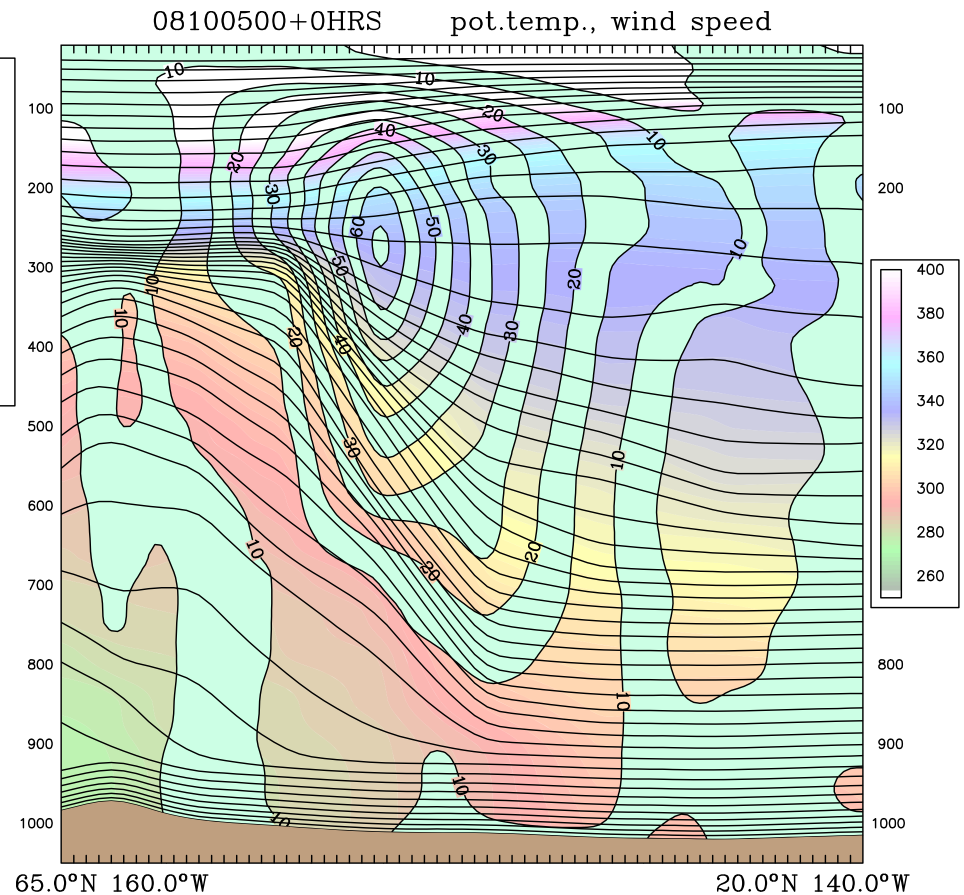
- The **cushion function,** which sets the final thickness of the source layer,
  - leaves large positive Δp values unchanged: $cush(\Delta p) = \Delta p \quad (\Delta p_{need} << \Delta p_{avail})$
  - returns a (small) constant value if Δp is large negative: $cush(\Delta p) = \text{const.} \quad (\Delta p_{need} >> \Delta p_{avail})$
  - links the two cases above by a smoothly varying function for intermediate values of Δp.

The “cushion” function





source: /tg2/projects/fim/bleck/trunk/FIMrun/fim5 50 4 1779796/



source: /tg2/projects/fim/bleck/trunk/FIMrun/fim5 50 4 1779881/

Continuity equation in generalized ("s") coordinates

$$\begin{pmatrix}\text{vertical}\\ \text{motion}\\ \textit{of}\\ \textit{s}\ \text{surface}\end{pmatrix} + \begin{pmatrix}\text{vertical}\\ \text{motion}\\ \textit{through}\\ \textit{s}\ \text{surface}\end{pmatrix} = \begin{pmatrix}\text{vertically}\\ \text{integrated}\\ \text{horizontal}\\ \text{mass flux}\\ \text{divergence}\end{pmatrix}$$

(set by FIM's "grid generator")

(generalized vertical velocity => advection terms in prognostic eqns.)

$$\begin{cases} u_t - \eta v + \left(\dot{s}\dfrac{\partial p}{\partial s}\right)\dfrac{\partial u}{\partial p} = -\dfrac{\partial\left(M + V^2/2\right)}{\partial x} + \Pi\dfrac{\partial \theta}{\partial x} \\ v_t + \eta u + \left(\dot{s}\dfrac{\partial p}{\partial s}\right)\dfrac{\partial v}{\partial p} = -\dfrac{\partial\left(M + V^2/2\right)}{\partial y} + \Pi\dfrac{\partial \theta}{\partial y} \\ \left(\dfrac{\partial p}{\partial s}\right)_t + \nabla_s \cdot \left(\vec{V}_h \dfrac{\partial p}{\partial s}\right) + \dfrac{\partial}{\partial s}\left(\dot{s}\dfrac{\partial p}{\partial s}\right) = 0 \\ \left(\theta\dfrac{\partial p}{\partial s}\right)_t + \nabla_s \cdot \left[\left(\vec{V}_h \dfrac{\partial p}{\partial s}\right)\theta\right] + \dfrac{\partial}{\partial s}\left[\left(\dot{s}\dfrac{\partial p}{\partial s}\right)\theta\right] = \dfrac{\partial p}{\partial s}\dfrac{\dot{H}}{C_p T} \\ \dfrac{\partial M}{\partial \theta} = \Pi \qquad \text{where} \quad \Pi = c_p (p/p_0)^{R/c_p}, \quad M = gz + \Pi\theta \end{cases}$$

$$\left(q\frac{\partial p}{\partial s}\right)_t + \nabla_s \cdot \left[\left(\vec{V}_h \frac{\partial p}{\partial s}\right)q\right] + \frac{\partial}{\partial s}\left[\left(\dot{s}\frac{\partial p}{\partial s}\right)q\right] = Source$$

Note: no explicit mixing terms

Closing remarks

- After some startup problems, the ALE-based grid generator meets design criteria and is working well.
- Vertical advection terms in FIM can be evaluated in several ways. We presently use the conservative, monotonicity-preserving, unconditionally stable piece-wise linear method (PLM).
- In light of FIM's future use as an AGCM, all transport & mixing algorithms are conservative (a step up from RUC).