# Chapter 7

# Algebra and Functions

## Content Strand Description

The Algebra and Functions content strand extends from work with simple patterns at grade 4 to basic algebra concepts at grade 8 to more advanced questions at grade 12. The strand includes not only algebra, but also pre-calculus and some topics from discrete mathematics. On the NAEP 1996 mathematics assessment, students were expected to use algebraic notation and to solve mathematical problems set in real-world contexts. Across the grades, students also were expected to demonstrate an increasing understanding of the use of algebraic functions and geometry as representational tools.

At grade 4, the assessment involved informal demonstration of students' abilities to generalize from patterns and justify such generalizations, translate between mathematical representations, use simple equations, and construct basic graphs. At grade 8, the assessment included more algebraic notation, stressing the meaning of variables and an informal understanding of the use of symbolic representation in problem-solving contexts. Students also were expected to use basic concepts of functions as a way of describing relationships and to solve simple equations and inequalities. At grade 12, students were expected to be adept at appropriately choosing and applying algebraic representations in a variety of problem-solving situations, including using functions in representing and describing more complex relationships.

## Examples of Individual Questions and Student Performance

Several questions from the Algebra and Functions content strand of the NAEP 1996 mathematics assessment are shown in this chapter. Presentation of the questions is organized around four areas of emphasis within the Algebra and Functions content strand: 1) *patterns and functional relationships;* 2) *number lines and graphs;* 3) *equations and inequalities,* which includes algebraic representations and solving equations and inequalities; and 4) *advanced functions topics and trigonometry.* As was true for the other content strands, questions within all four areas tested students' conceptual understanding and procedural knowledge, as well as their abilities to reason, communicate, and make connections.

All sample questions from this content strand are mapped onto the NAEP composite mathematics scale as shown in Figure 7.1. Specific instructions on how to interpret this map are given at the end of Chapter 2. The map is included to provide an indication of the relative difficulty of each sample question and, thus, to indicate the type of material mastered within this content strand by students with varying degrees of mathematics proficiency. As mentioned in previous chapters, it is important to remember that the difficulty of a question is a function of the characteristics specific to the question (e.g., format, absence or presence of graphics, real-world application), as well as the specific mathematics content associated with the question, and students' opportunities to learn this content. Remember also that overall performance on the Algebra and Functions content strand is not determined solely by performance on the examples presented here. These examples illustrate only some of what students know and can do.

### Patterns and functional relationships

Most of the questions in this area that required students to solve problems related to patterns of numbers, letters, or figures were found at grade 4. In simpler patterns, all elements changed in the same way (e.g., adding 5, rotating figure one unit). In more complex patterns, elements changed in different ways. The most difficult questions covered relationships between patterns. Questions asked students to identify the next element(s) in the pattern, fill in missing elements, perform computations with missing elements, or explain patterns.

Two sample questions are presented for this area — an eighth-grade short constructed-response question and a twelfth-grade extended constructed-response question. Both questions assess students' problem-solving skills.

The question selected for the first example listed the number of diagonals that can be drawn from any vertex of various polygons and then asked how many diagonals could be drawn from any vertex of a 20-sided polygon. In order to answer the question correctly, students had to analyze the information presented and determine the pattern present, that is, that the number of diagonals is always three less than the number of sides of the polygon. Then they needed to apply that pattern to the 20-sided polygon to compute the correct answer of 17. Any other answer was considered "incorrect."

From any vertex of a 4-sided polygon, 1 diagonal can be drawn
From any vertex of a 5-sided polygon, 2 diagonals can be drawn
From any vertex of a 6-sided polygon, 3 diagonals can be drawn
From any vertex of a 7-sided polygon, 4 diagonals can be drawn

**10.** How many diagonals can be drawn from any vertex of a 20-sided polygon?

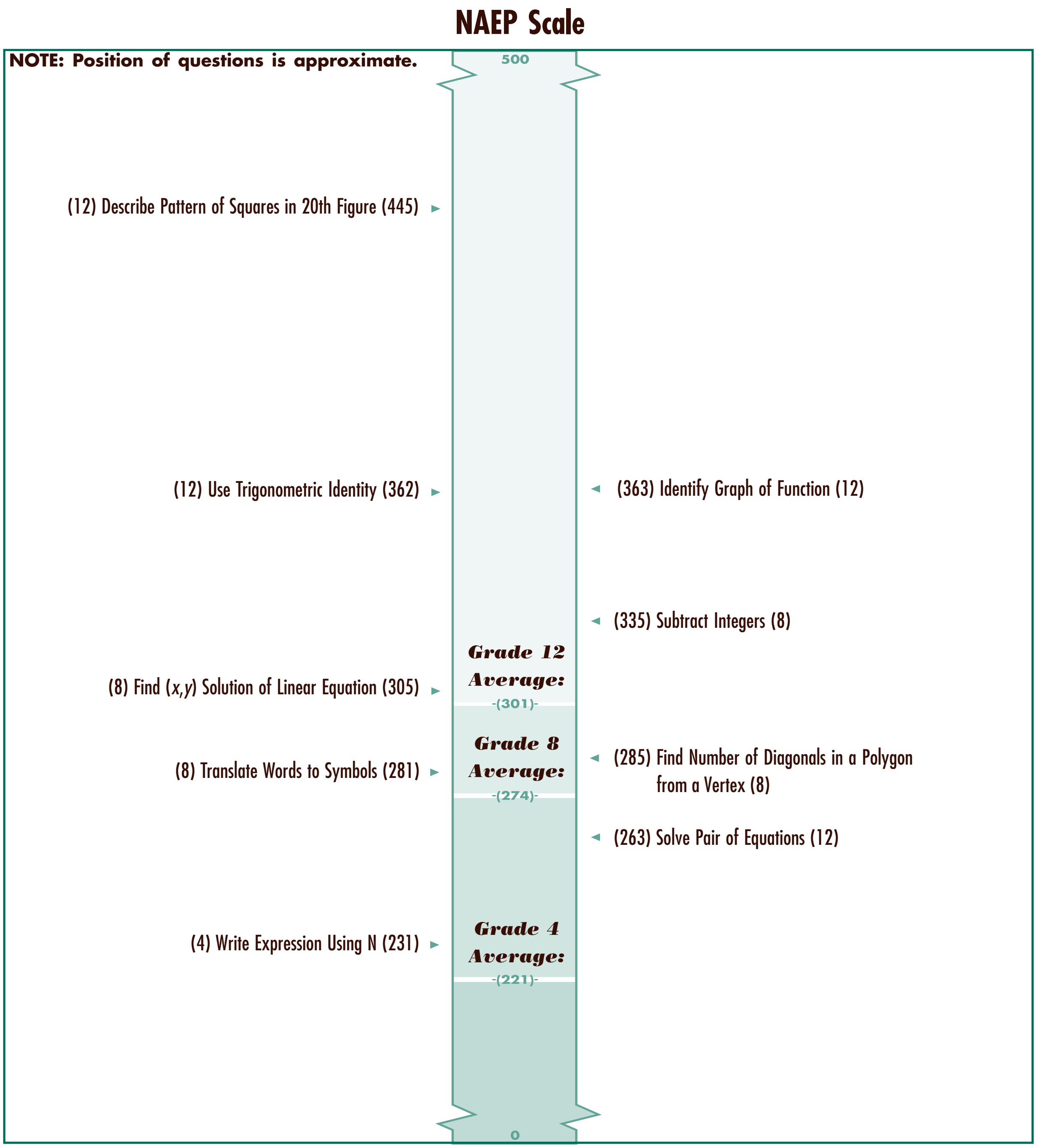
Answer: ____________________________

The correct answer is 17.

**Figure 7.1**

## Map of Selected Algebra and Functions Questions on the NAEP Composite Mathematics Scale (Item Map)

THE NATION'S REPORT CARD




NOTE: Each mathematics question was mapped onto the NAEP 0 to 500 mathematics scale. The position of the question on the scale represents the scale score obtained by students who had a 65 percent probability of successfully answering the question. (The probability was 74 percent for a 4-option multiple-choice question and 72 percent for a 5-option multiple-choice question.) Only selected questions are presented. The number 4, 8, or 12 in parentheses is the grade level at which the question was asked.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Student performance data are presented in Table 7.1, and the percentage of students within each achievement-level interval who successfully answered the question is presented in Table 7.2. Fifty-four percent of the students answered the question correctly. Female students outperformed males on this question. Students taking eighth-grade mathematics or pre-algebra performed similarly, while those taking algebra performed better than the other two groups. Sixty percent of students at the *Basic* level, 84 percent at the *Proficient* level, and more than 90 percent of students at the *Advanced* level gave the "correct" response. The question mapped at a composite scale score of 285.

**Table 7.1** ***Percentage Correct for "Find Number of Diagonals in a Polygon from a Vertex"***

THE NATION'S REPORT CARD naep

| Grade 8 | Percentage Correct |
|---|---|
| Overall | 54 |
| Males | 50 |
| Females | 59 |
| White | 61 |
| Black | 35 |
| Hispanic | 41 |
| Asian/Pacific Islander | – – |
| American Indian | *** |
| Mathematics Course Taking: | |
| Eighth-Grade Mathematics | 47 |
| Pre-Algebra | 51 |
| Algebra | 69 |

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

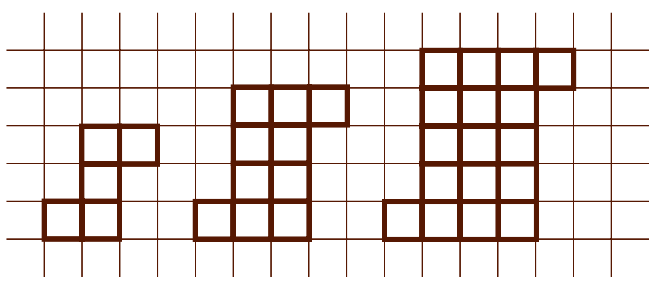
| Table 7.2 | Percentage Correct Within Achievement-Level Intervals for "Find Number of Diagonals in a Polygon from a Vertex" |
|---|---|

| | NAEP Grade 8 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 54 | 27 | 60 | 84 | 96! |

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The next example is an extended-constructed response question for twelfth-grade students. It included directions to the students to show all of their work and explain their reasoning. Students were told that they could use drawings, words, or numbers in their explanations and that the answer needed to be clear enough that another person could read it and understand their thinking. In the problem, students were shown the first three figures in a pattern of tiles that they were told contained a total of 50 figures. They were asked to describe the 20th figure in the pattern and to explain the reasoning they used to determine this solution. They then were asked to write a general description that could be used to define any of the 50 figures in the pattern.

**9.** The first 3 figures in a pattern of tiles are shown below. The pattern of tiles contains 50 figures.



Describe the 20th figure in this pattern, including the total number of tiles it contains and how they are arranged. Then explain the reasoning that you used to determine this information. Write a description that could be used to define any figure in the pattern.

In rating student responses, readers could rate a response as “extended,” “satisfactory,” “partial,” “minimal,” or “incorrect.” A response was considered “extended” if it included the following elements: 1) a correct count of 442 tiles for the 20th figure; 2) a verbal or graphical explanation of the reasoning the student used (e.g., a description of a figure with a row of 21 tiles across the top, a row of 21 across the bottom, and a 20 x 20 square between these rows with the top row extending one tile to the right of the square, and the bottom row extending one tile to the left); and 3) an accurate generalization, based on inductive reasoning. The following sample “extended” response contains all of these elements.

### *Sample “extended” response*

* Each figure increases 1 layer in height and one middle layer in width for every succession, relative to the first.

For example, for the $n^{th}$ section the figure will be n+1 units across at the base, n units wide, n+1 units across at the top, and n+2 units high. This is the pattern

The 20th figure will be 21 units across on the bottom length, 20 units wide in the middle, 22 units high; and 21 units wide at the top. The increase is linear.

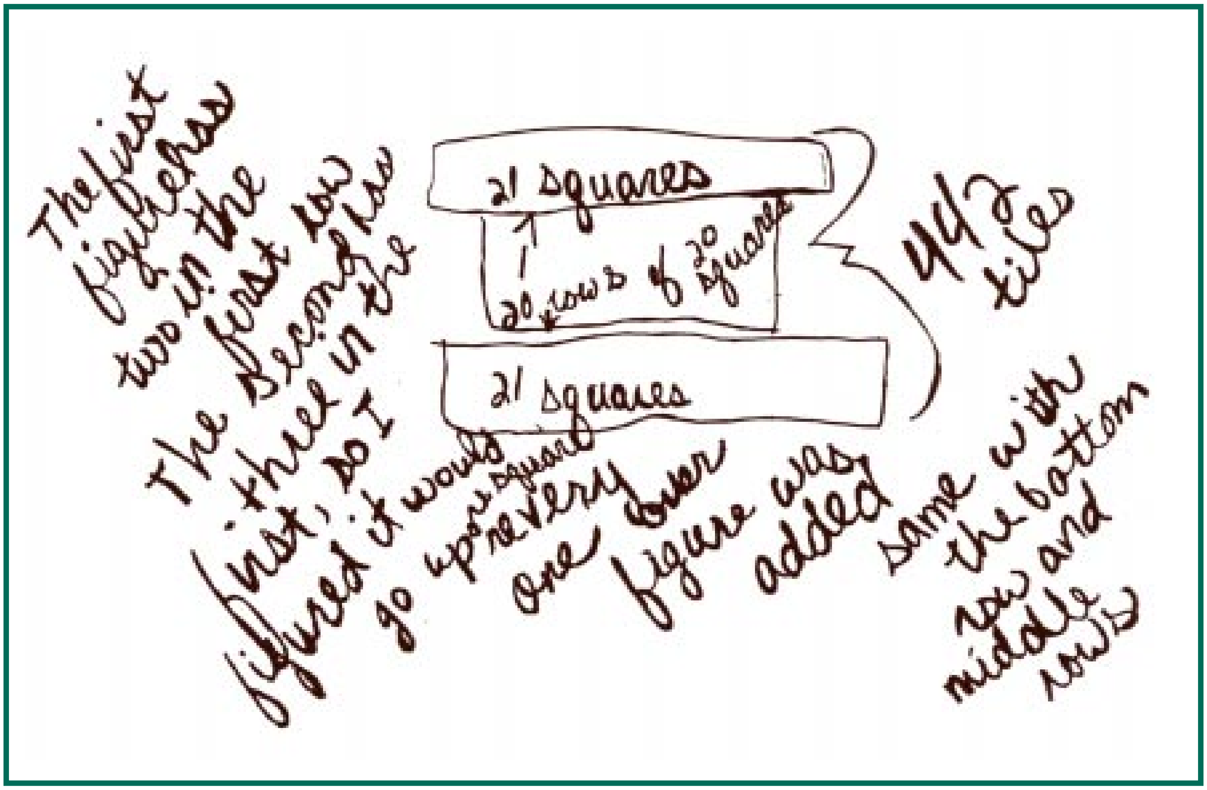
Total number of tiles it contains:

$21 + (20 \times 20) + 21 = 442$

The inner square is always (n×n) units in area ($n^2$)

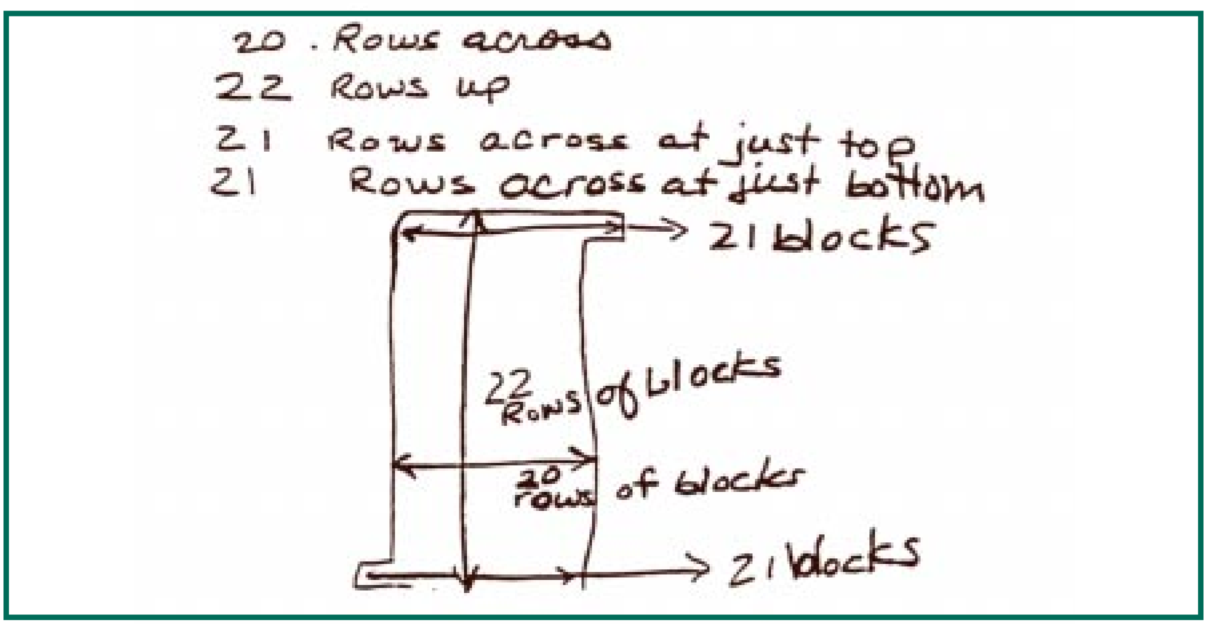
A response was considered “satisfactory” if the student described the 20th figure, gave the number of tiles, and provided some evidence of sound reasoning. However, “satisfactory” responses either included errors in computation or lacked clarity in the explanation. The following sample “satisfactory” response is an example of a response that contained most of the elements asked for in the question but that lacked a clear explanation or generalization.

### ***Sample "satisfactory" response***



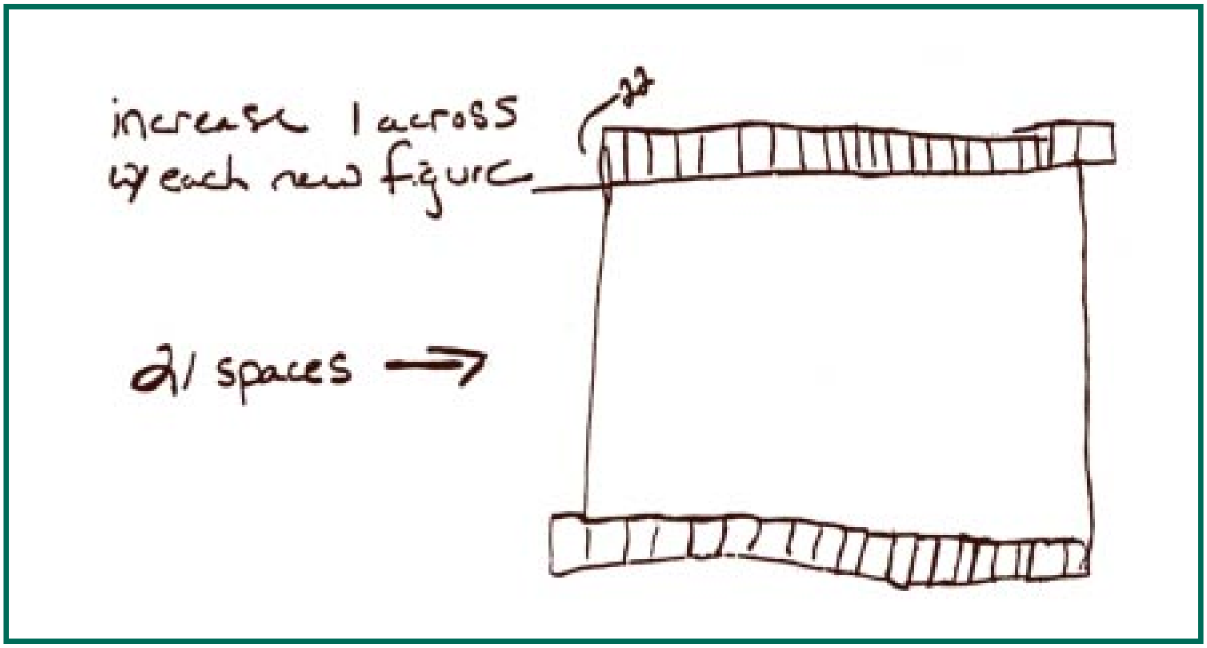

A response was considered "partial" if the student illustrated or described at least one additional figure in the pattern correctly or stated that there are 442 tiles in the 20th figure but did no more. In the following sample "partial" response, the student correctly diagrammed the 20th figure but did not tell how many tiles were in the figure, explain his or her reasoning, or provide a generalization.

### ***Sample "partial" response***



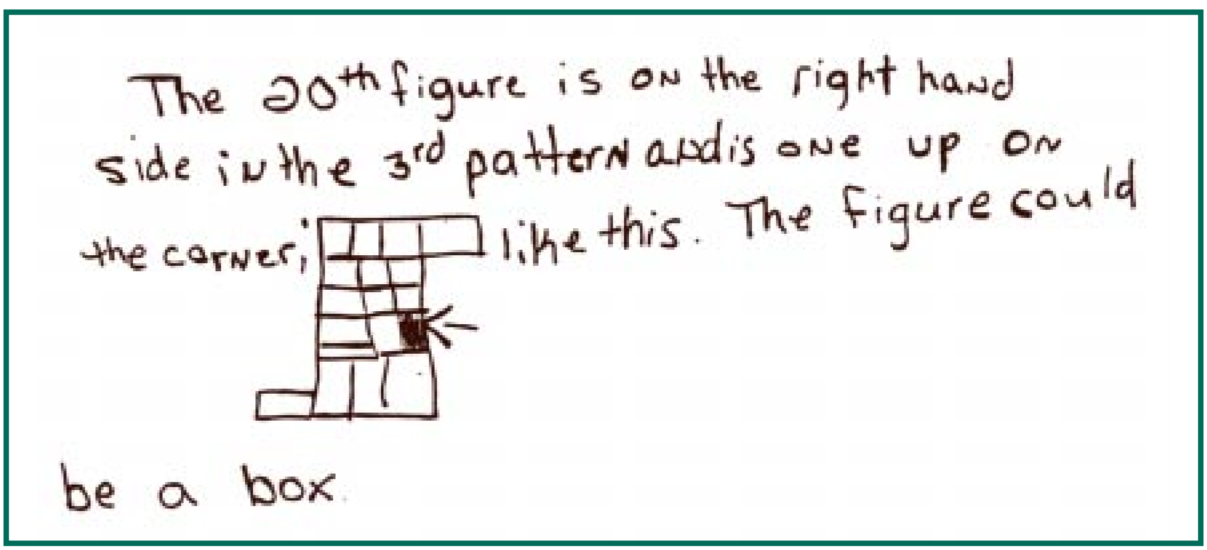
A response was considered "minimal" if the student attempted to draw or describe the given pattern or an additional figure in the pattern or made some attempt to go beyond what was shown in the question. The following "minimal" response is an example of a student who made an attempt to draw the 20th figure but did not correctly describe it or tell the number of tiles it contains. The student's reasoning was not clearly explained, and there was no description that could be generalized to any figure in the pattern.

***Sample "minimal" response***




A response was considered "incorrect" if the student showed no attempt to go beyond what was shown in the question. The following sample "incorrect" response is an example of a student who just repeated one of the figures shown in the original question and gave a verbal answer that appeared to show lack of comprehension of the question.

***Sample "incorrect" response***



Information on student performance on this question is presented in Table 7.3. This question was quite difficult for students, and when the question was anchored to the NAEP scale, the "extended" and "satisfactory" rating categories were collapsed. While 80 percent of twelfth-graders attempted to answer the question, only 4 percent provided a response that was rated as "satisfactory" or higher. Another 18 percent provided "partial" responses, and more than 50 percent provided responses rated as "minimal" or "incorrect."[1] Students whose highest mathematics course was calculus were more likely to provide a response considered to be at least partially correct than students with fewer courses in the algebra-through-calculus sequence, and students whose highest course was third-year algebra were more likely to provide a response considered to be at least partially correct than those whose highest course was first-year algebra.

**Table 7.3** ***Score Percentages for "Describe Pattern of Squares in 20th Figure"***

THE NATION'S REPORT CARD naep

| Grade 12 | Extended | Satisfactory | Partial | Minimal | Incorrect | Omit |
|---|---|---|---|---|---|---|
| Overall | 2 | 2 | 18 | 28 | 25 | 20 |
| Males | 2 | 2 | 19 | 26 | 27 | 19 |
| Females | 1 | 2 | 17 | 31 | 23 | 21 |
| White | 3 | 2 | 19 | 33 | 23 | 15 |
| Black | 0! | 0! | 9 | 17 | 35 | 36 |
| Hispanic | 0! | 1! | 18 | 19 | 26 | 33 |
| Asian/Pacific Islander | 2! | 3 | 30 | 20 | 20 | 22 |
| American Indian | *** | *** | *** | *** | *** | *** |
| Geometry Taken | 2 | 3 | 20 | 30 | 26 | 19 |
| Highest Algebra-Calculus Course Taken: | | | | | | |
| Pre-Algebra | *** | *** | *** | *** | *** | *** |
| First-Year Algebra | 2 | 1 | 13 | 24 | 26 | 28 |
| Second-Year Algebra | 2 | 2 | 18 | 30 | 24 | 21 |
| Third-Year Algebra/Pre-Calculus | 1! | 7 | 22 | 32 | 22 | 15 |
| Calculus | 7 | 3 | 38 | 28 | 12 | 8 |

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

[1] Student responses for this and all other constructed-response questions also could have been scored as "off task," which means that the student provided a response, but it was deemed not related in content to the question asked. There are many examples of these types of responses, but a simple one would be "I don't like this test." Responses of this sort could not be rated. In contrast, responses scored as "incorrect" were valid attempts to answer the question that were simply wrong.

The percentage of students within each achievement-level interval who provided a response that was rated at least satisfactory is shown in Table 7.4. Only 2 percent of students at the *Basic* level and 15 percent of students at the *Proficient* level submitted responses that were considered at least "satisfactory." The question mapped at 445.

**Table 7.4**

**Percentage at Least Satisfactory Within Achievement-Level Intervals for "Describe Pattern of Squares in 20th Figure"**

THE NATION'S REPORT CARD naep

| | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| **Overall** | **Below Basic** | **Basic** | **Proficient** | **Advanced** |
| 4 | 0! | 2 | 15 | *** |

*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

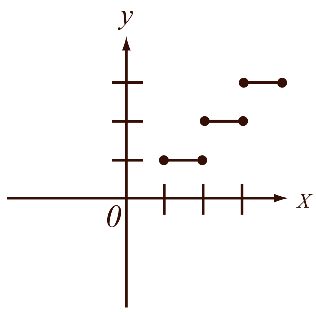
### Number lines and graphs

Questions in this area asked students to use number lines and rectangular coordinate systems. At grade 4, students primarily were asked to locate points on graphs, trace paths, and identify points on number lines. At grade 8, many questions asked students to identify coordinates, and at grade 12, questions asked students to represent equations, inequalities, and functions on number lines and graphs.

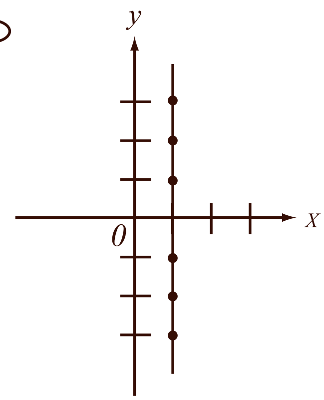
The following example is a twelfth-grade question that asked students to identify which of five figures shown could be the graph of a function. To answer the question correctly students needed to understand the definition of a function and be able to test each of the figures against that definition. The question mapped at a score of 363 on the NAEP composite mathematics scale.

**9.** Which of the following could be the graph of a function?

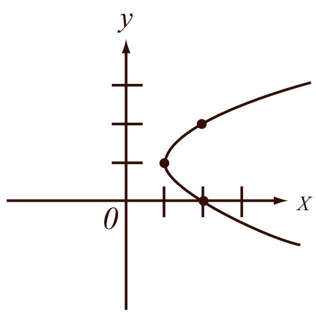
A



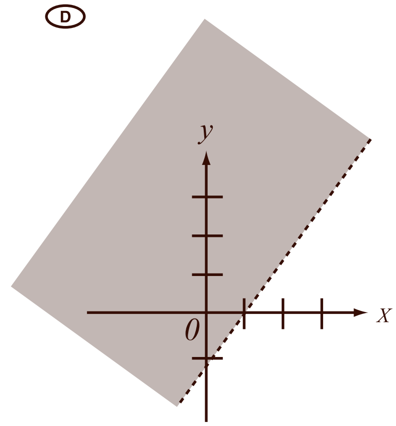

B



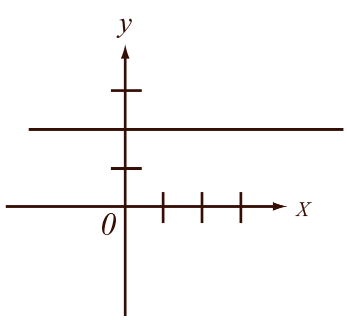

C




D




E




The correct option is E.

Performance data for this question are presented in Tables 7.5 and 7.6. The question was fairly difficult for students and mapped at a score of 363 on the composite scale. Twenty percent of the students selected the correct option, E, while another approximately 20 percent chose Option D, and 32 percent chose Option C. It is possible that students selected Option C because they did not know the definition of a function, or could not recognize Option E as representative of a function because $y$ does not vary with values of $x$, and simply selected the figure that appeared most complicated.

As might be expected, familiarity with functions appears to depend on a student's curriculum. Students who had taken more advanced mathematics courses were more likely to respond correctly to the question than students who had not taken these courses. Students who cited at least third-year algebra/pre-calculus as their highest mathematics course taken performed better than those who had taken fewer courses in the algebra-through-calculus sequence. That the question was difficult for students can be seen by the fact that only 17 percent of students at the *Basic* level and 56 percent of those at the *Proficient* level answered correctly.

**Table 7.5** ***Percentage Correct for "Identify Graph of Function"***

THE NATION'S REPORT CARD naep

| Grade 12 | Percentage Correct |
|---|---|
| Overall | 20 |
| Males | 21 |
| Females | 20 |
| White | 20 |
| Black | 16 |
| Hispanic | 22 |
| Asian/Pacific Islander | 43 |
| American Indian | *** |
| Geometry Taken | 22 |
| Highest Algebra-Calculus Course Taken: | |
| Pre-Algebra | 9 |
| First-Year Algebra | 10 |
| Second-Year Algebra | 17 |
| Third-Year Algebra/Pre-Calculus | 36 |
| Calculus | 55 |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

| Table 7.6 | Percentage Correct Within Achievement-Level Intervals for "Identify Graph of Function" |
|---|---|

| Overall | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| | Below Basic | Basic | Proficient | Advanced |
| 20 | 8 | 17 | 56 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Equations and inequalities

Questions assessing student knowledge of, and skill in using, equations and inequalities ranged from those asking for simple algebraic representations of situations and problems to those asking students to solve systems of equations and inequalities. At grade 4, students had to identify simple algebraic representations involving number sentences or pictures. They also were asked to identify missing numbers in simple equations and inequalities. Eighth-grade students were asked to solve more complex equations and inequalities, often involving two missing variables. At times they were asked to predict the resultant effect on the value of one variable when the value of another variable had been changed. Questions for students in grade 12 sometimes involved exponents and square roots as well as systems of equations and inequalities.

The next two sample questions assessed students' conceptual understanding of algebraic representations. The first question is a multiple-choice question for grade 4. It presented a short word problem about stamps that included a variable, *N*. Students were asked to identify the symbolic representation of the correct answer.

**3.** *N* stands for the number of stamps John had. He gave 12 stamps to his sister. Which expression tells how many stamps John has now?

Ⓐ $N + 12$

Ⓑ $N - 12$

Ⓒ $12 - N$

Ⓓ $12 \times N$

The correct option is B.

To answer the question correctly, students needed to understand that "giving stamps away" corresponds to subtracting them from the total, *N*, and is correctly represented by a minus sign. The question was fairly easy for fourth-grade students and mapped at a composite scale score of 231. Student performance is shown in Tables 7.7 and 7.8. Two-thirds of fourth-grade students selected the correct option; the remaining students were fairly evenly divided between Options A and C, with only around three percent selecting Option D. Female students performed better than males. Seventy-three percent of students at the *Basic* level and 90 percent of students at the *Proficient* level selected the correct response. Fewer than half of the students classified as below *Basic* were able to choose the correct response.

**Table 7.7** **Percentage Correct for "Write Expression Using N"**

THE NATION'S REPORT CARD naep

| Grade 4 | Percentage Correct |
|---:|:---:|
| Overall | 67 |
| Males | 64 |
| Females | 70 |
| White | 71 |
| Black | 56 |
| Hispanic | 58 |
| Asian/Pacific Islander | 70 |
| American Indian | *** |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table 7.8** **Percentage Correct Within Achievement-Level Intervals for "Write Expression Using N"**

THE NATION'S REPORT CARD naep

| | NAEP Grade 4 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 67 | 44 | 73 | 90 | *** |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The next example in this area is a grade 8 multiple-choice question similar to the previous one. It presented a word problem and then asked students to identify the symbolic representation of the solution. In this question, a plumber's hourly rate was given, plus travel charges. Students also were told to let *h* represent the number of hours worked and then were asked which expression could be used to calculate the plumber's total charge. In order to answer correctly, they needed to know what computations were required to solve the word problem and how those computations should be expressed in an equation. The question was fairly easy and mapped at 281 on the composite scale.

**9.** A plumber charges customers $48 for each hour worked plus an additional $9 for travel. If *h* represents the number of hours worked, which of the following expressions could be used to calculate the plumber's total charge in dollars?

(A) $48 + 9 + h$

(B) $48 \times 9 \times h$

(C) $48 + (9 \times h)$

(D) $(48 \times 9) + h$

(E) $(48 \times h) + 9$

The correct option is E.

Student performance data are presented in Table 7.9, and the percentage of students within each achievement-level interval who successfully answered the question is presented in Table 7.10. Fifty-eight percent of the students answered the question correctly. Incorrect responses were fairly evenly distributed across the other options. Students currently enrolled in algebra performed better than those in pre-algebra or eighth-grade mathematics, whereas students in the latter two courses performed similarly. Sixty-six percent of students at the *Basic* level and more than 90 percent of students at the *Proficient* level selected the correct response. Only one-fourth of the students below the *Basic* level were able to respond correctly.

| Table 7.9 | Percentage Correct for "Translate Words to Symbols" |
|---|---|

| Grade 8 | Percentage Correct |
|---|---|
| Overall | 58 |
| Males | 55 |
| Females | 60 |
| White | 64 |
| Black | 39 |
| Hispanic | 46 |
| Asian/Pacific Islander | – – |
| American Indian | *** |
| Mathematics Course Taking: | |
| Eighth-Grade Mathematics | 48 |
| Pre-Algebra | 54 |
| Algebra | 76 |

*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

| Table 7.10 | Percentage Correct Within Achievement-Level Intervals for "Translate Words to Symbols" |
|---|---|

| | NAEP Grade 8 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 58 | 24 | 66 | 94 | 99! |

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The following three sample questions assess students' knowledge of procedures for solving equations and inequalities. The first question is a grade 8 multiple-choice question that asked students to identify a solution to a linear equation with two unknowns, $x$ and $y$. To answer correctly, students could solve the equation by trial and error, working their way through the pairs of $x$ and $y$ values given in the response options and, in each case, determining whether the resultant expression equaled 6, as specified by the equation. Alternatively, a student could graph the equation and test which of the points specified by the $(x, y)$ coordinates in the response options fell onto the graphed line. The question mapped at 305 on the NAEP composite scale.

8. Which of the following ordered pairs $(x, y)$ is a solution to the equation $2x - 3y = 6$ ?

(A) (6, 3)

(B) (3, 0)

(C) (3, 2)

(D) (2, 3)

(E) (0, 3)

The correct option is B.

Student performance data are presented in Table 7.11. Over 40 percent of students answered the question correctly. The remaining students were distributed fairly evenly among Options A, C, and D, with less than five percent selecting Option E. Students currently taking pre-algebra or eighth-grade mathematics performed similarly, whereas those currently taking algebra performed better than students in the other two groups.

**Table 7.11** ***Percentage Correct for "Find (x, y) Solution of Linear Equation"***

THE NATION'S REPORT CARD naep

| Grade 8 | Percentage Correct |
|---|---|
| Overall | 42 |
| Males | 42 |
| Females | 40 |
| White | 46 |
| Black | 30 |
| Hispanic | 29 |
| Asian/Pacific Islander | – – |
| American Indian | *** |
| Mathematics Course Taking: | |
| Eighth-Grade Mathematics | 31 |
| Pre-Algebra | 36 |
| Algebra | 64 |

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The percentage of students within each achievement-level interval who correctly answered the question is presented in Table 7.12. More than 90 percent of students classified as *Advanced*, 75 percent of those classified as *Proficient*, and 44 percent of those classified as *Basic* selected the correct response.

**Table 7.12** **Percentage Correct Within Achievement-Level Intervals for "Find (x, y) Solution of Linear Equation"**

THE NATION'S REPORT CARD naep

| Overall | NAEP Grade 8 Composite Scale Range | | | |
|---|---|---|---|---|
| | Below Basic | Basic | Proficient | Advanced |
| 41 | 15 | 44 | 75 | 93! |

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

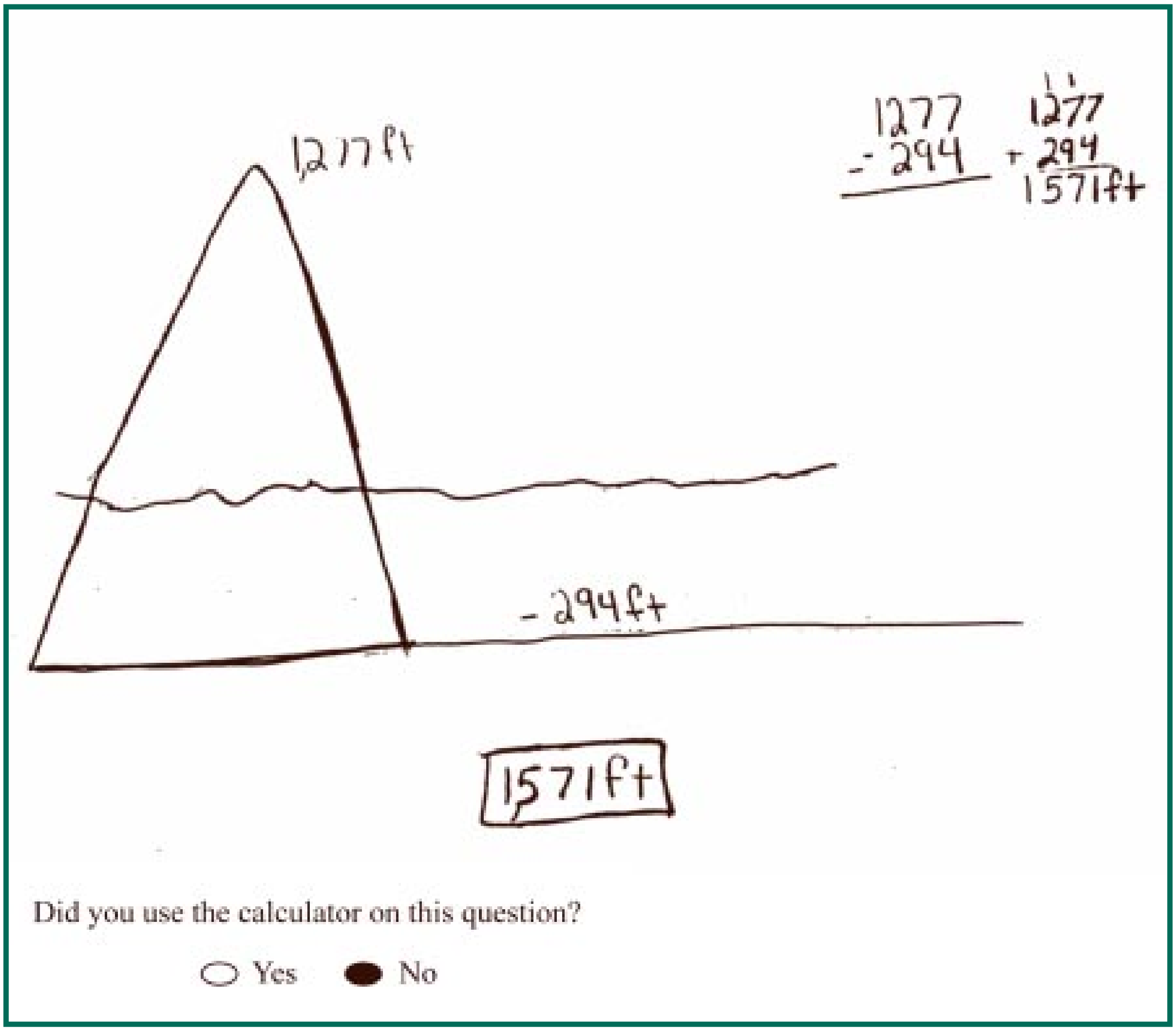
The next example in this area is a short constructed-response question for grade 8 students. Students were asked to find the difference between a high point above sea level and a low point below sea level and to show their work. In order to answer the question correctly, students had to recognize that the correct procedure would be to sum the two numbers, equivalent to subtracting a negative number, rather than to subtract one from the other.

**2.** The lowest point of the St. Lawrence River is 294 feet below sea level. The top of Mt. Jacques Cartier is 1,277 feet above sea level. How many feet higher is the top of Mt. Jacques Cartier than the lowest point of the St. Lawrence River? Show your work.

The correct response is 1,571 feet.

Readers could rate responses as "correct," "partial," or "incorrect." A response was considered "correct" if the answer given was 1,571 feet, even if the student's work was not shown. A response was considered "partial" if it showed the correct procedure, either $1{,}277 - (-294)$ or $1{,}277 + 294$, but did not have the correct answer. Anything else was considered "incorrect." Sample student responses follow. In the sample "partial" response, the student showed the correct procedure but made an arithmetic error. In the "incorrect" response, the student subtracted the two numbers even after drawing a figure that indicated some understanding of the relationship between the top of the mountain and the bottom of the river.

### *Sample "correct" response*



### *Sample "partial" response*

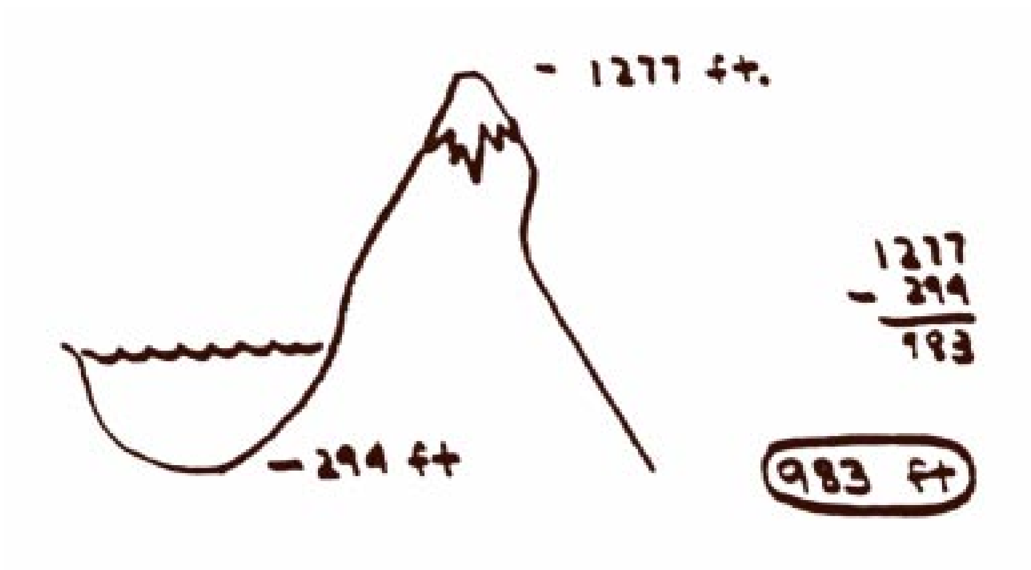
$$\begin{array}{r} 1277 \\ -\ {-294} \\ \hline \end{array}$$

$$\begin{array}{r} 1277 \\ +\ {+294} \\ \hline 1541 \end{array}$$

Did you use the calculator on this question?

● Yes ○ No

## ***Sample "incorrect" response***




Did you use the calculator on this question?

○ Yes ● No

Table 7.13 shows student performance on this question. While more than 95 percent of eighth-grade students attempted to answer the question, only 25 percent provided a response that was rated at least "partial." When the question was anchored to the NAEP scale, the "correct" and "partial" rating categories were collapsed. The question mapped at 335 on the composite scale. As may be expected, students currently taking algebra outperformed those taking eighth-grade mathematics or pre-algebra. Male students performed better than females.

## Table 7.13 Percentage Correct for "Subtract Integers"

THE NATION'S REPORT CARD naep

| Grade 8 | Correct | Partial | Incorrect | Omit |
|---|---|---|---|---|
| Overall | 22 | 3 | 70 | 4 |
| Males | 25 | 3 | 65 | 6 |
| Females | 19 | 2 | 76 | 2 |
| White | 26 | 3 | 68 | 2 |
| Black | 10 | 3 | 78 | 9 |
| Hispanic | 14 | 2 | 72 | 12 |
| Asian/Pacific Islander | – – | – – | – – | – – |
| American Indian | *** | *** | *** | *** |
| Mathematics Course Taking: | | | | |
| Eighth-Grade Mathematics | 18 | 2 | 75 | 5 |
| Pre-Algebra | 15 | 2 | 81 | 2 |
| Algebra | 38 | 5 | 54 | 2 |

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 7.14 presents the percentages of students whose responses were rated "correct" overall and within each of the achievement-level intervals. That this question was fairly difficult for students can be seen by the fact that only 18 percent of those classified as *Basic* and 46 percent of those classified as *Proficient* answered the question correctly.

## Table 7.14 Percentage Correct Within Achievement-Level Intervals for "Subtract Integers"

THE NATION'S REPORT CARD naep

| | NAEP Grade 8 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 22 | 9 | 18 | 46 | 81 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The following example is a multiple-choice question for grade 12 that required students to solve a pair of equations. The question showed two equations, each with two boxes for missing numbers, and asked what single number could be placed in all four boxes to make both equations true. In order to answer the question correctly, students had to realize that the only number that could be multiplied by both 4 and 3 and remain unchanged is 0. However, even if students did not realize this immediately and set about answering the question by trial and error (i.e., substituting the numbers presented in the options into the equations to solve for the answer), they would quickly obtain the correct answer, as it was presented in the first option. Presumably, this would be the first number tried by the students.

**6.** $4 \times \square = \square$ and $\square \times 3 = \square$

What number if placed in each box above would make both equations true?

Ⓐ 0
Ⓑ 1
Ⓒ 2
Ⓓ 3
Ⓔ 4

The correct option is A.

Student performance data are presented in Tables 7.15 and 7.16. The question was not difficult for students, as can be seen by the high percentage of students (88%) overall who answered correctly. Ninety-eight percent of students classified at the *Proficient* level, 96 percent of those classified at the *Basic* level, and 69 percent of those classified as below the *Basic* level answered correctly. Performance on this question appears less dependent on advanced curriculum than does performance on some of the more difficult questions; it appears that the concepts assessed in this question are taught in the lower level algebra courses. Thus, students in calculus, third-year algebra/pre-calculus, and second-year algebra performed similarly, whereas students in second-year algebra performed better than those in first-year algebra, and those in first-year algebra performed better than those in pre-algebra. Female students performed better than males on this question. The question mapped at 263 on the NAEP composite mathematics scale.

**Table 7.15**

## Percentage Correct for "Solve Pair of Equations"

THE NATION'S REPORT CARD naep

| Grade 12 | Percentage Correct |
|---|---|
| Overall | 88 |
| Males | 85 |
| Females | 90 |
| White | 91 |
| Black | 79 |
| Hispanic | 78 |
| Asian/Pacific Islander | 90 |
| American Indian | *** |
| Geometry Taken | 92 |
| Highest Algebra-Calculus Course Taken: | |
| Pre-Algebra | 57 |
| First-Year Algebra | 83 |
| Second-Year Algebra | 92 |
| Third-Year Algebra/Pre-Calculus | 96 |
| Calculus | 95 |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table 7.16**

## Percentage Correct Within Achievement-Level Intervals for "Solve Pair of Equations"

THE NATION'S REPORT CARD naep

| | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 88 | 69 | 96 | 98! | *** |

*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### *Advanced functions topics and trigonometry*

Questions testing advanced algebraic concepts asked students to describe functions and their properties, apply properties of functions, and apply functions to real-world situations. There also were questions that assessed students' familiarity with trigonometry. The following grade 12 example is a multiple-choice question that assessed students' knowledge of a trigonometric identity. To answer the question correctly, students had to know, or be able to derive, the identity that demonstrates that the value of the expression $cos^2 x + sin^2 x$ equals 1 for any real number $x$. The question mapped at 362 on the composite scale.

**8.** $\cos^2(3x) + \sin^2(3x) =$

- Ⓐ 0
- Ⓑ 1
- Ⓒ 3
- Ⓓ 6
- Ⓔ 9

The correct option is B.

Student performance data are presented in Table 7.17. Overall, 27 percent of the students who attempted the question answered correctly. Approximately 18 percent selected each of Options C and D, 15 percent selected Option E, and 9 percent chose Option A. Almost 13 percent of the students omitted the question. Students whose highest course was pre- or first-year algebra performed similarly. However, above that level, each additional course in the algebra-through-calculus sequence was associated with an increase in the proportion of students who could answer the question correctly.

**Table 7.17**

## Percentage Correct for "Use Trigonometric Identity"

| Grade 12 | Percentage Correct |
|---|---|
| Overall | 27 |
| Males | 26 |
| Females | 27 |
| White | 28 |
| Black | 17 |
| Hispanic | 25 |
| Asian/Pacific Islander | 56 |
| American Indian | *** |
| Geometry Taken | 30 |
| Highest Algebra-Calculus Course Taken: | |
| Pre-Algebra | 13 |
| First-Year Algebra | 15 |
| Second-Year Algebra | 26 |
| Third-Year Algebra/Pre-Calculus | 43 |
| Calculus | 64 |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

When performance is disaggregated by achievement level, Table 7.18 shows that 11 percent of students below the *Basic* level, 26 percent of students at the *Basic* level, and 60 percent of those at the *Proficient* level answered the question correctly. As might be expected, this question was difficult for students performing at the *Basic* level and below.

**Table 7.18**

## Percentage Correct Within Achievement-Level Intervals for "Use Trigonometric Identity"

| | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 27 | 11 | 26 | 60 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## *Summary*

Questions in this content strand assessed students' knowledge of and ability to solve problems in four areas: *patterns and functional relationships, number lines and graphs, equations and inequalities,* and *advanced functions topics and trigonometry*. The majority of students at all grade levels appeared to understand basic algebraic representations and simple equations, as well as how to find simple patterns. Students at grades 8 and 12 had difficulty with questions requiring knowledge of linear equations, algebraic functions, and trigonometric identities. Students in these grades also found that questions requiring them to identify and generate complex patterns and solve real-world problems were challenging. In general, for eighth- and twelfth-grade students, those with more advanced coursework performed better on this content strand.

# Chapter 8

# Course-Taking Patterns

When students do well in mathematics they are likely either to select or be placed in more advanced courses earlier in their school careers than students experiencing less success. This allows them to take a greater number of increasingly difficult courses as they progress through high school — courses that expose them to more advanced content, as well as provide them the opportunities to practice and apply more powerful mathematical techniques in problem settings. In contrast, lower performing students may select, or be assigned to, less demanding curricular offerings — placements that provide them with fewer challenging opportunities, offer slower progress toward more advanced coursework, or even increase the likelihood that they will terminate their study of mathematics earlier in their school careers than more successful students.

This chapter is about student course-taking patterns. It includes information on the types of mathematics courses in which eighth-grade students were enrolled at the time of the NAEP 1996 assessment and on the mathematics course-taking histories of twelfth-grade students participating in the assessment. It also presents course-taking information for different gender and racial/ethnic subgroups. One of the reasons for monitoring course taking by gender and racial/ethnic groups is that research indicates that males and White students are likely to study algebra before females and some minority students.[1] Perhaps more importantly, taking algebra early appears to be related to student outcomes of taking more mathematics overall as well as more advanced coursework in mathematics.[2]

## Eighth-Grade Course Taking

In 1996, less than one percent (0.2%) of eighth-grade students indicated that they were not taking a mathematics course. Table 8.1 presents self-reported information on mathematics course taking by eighth-grade students. The average mathematics scores of students with different course enrollments also are shown.

[1] Fennema, E., & Leder, G. C. (Eds.) (1990). *Mathematics and gender*. New York: Teachers College Press; Kifer, E. (1992). Opportunities, talents, and participation. In L. Burstein (Ed.), *The IEA student of mathematics III: Student growth and classroom processes.* (pp. 279–307). New York: Pergamon Press.

[2] Smith, J. B. (1996). Does an extra year make any difference? The impact of early access to algebra on long-term gains in mathematics attainment. *Educational Evaluation and Policy Analysis*, *18*(2), 141–153.

## Table 8.1

### Average Scale Score by Mathematics Course Enrollment and by Gender, Race/Ethnicity, and Whether School Offers Algebra for High School Credit or Placement, Grade 8

THE NATION'S REPORT CARD naep

| Grade 8 | Assessment Year | Algebra: Percentage of Students | Algebra: Average Scale Score | Pre-Algebra: Percentage of Students | Pre-Algebra: Average Scale Score | Eighth-Grade Mathematics: Percentage of Students | Eighth-Grade Mathematics: Average Scale Score | Other Mathematics: Percentage of Students | Other Mathematics: Average Scale Score |
|---|---|---|---|---|---|---|---|---|---|
| All Students | 1996 | 25*† | 295 | 27* | 270 | 43* | 262*† | 5 | 270* |
| | 1992 | 20 | 299 | 28* | 273 | 49* | 256 | 3 | 257 |
| | 1990 | 16 | 295 | 20 | 271 | 61 | 252 | 3 | 257 |
| Females | 1996 | 26*† | 294 | 27 | 271 | 42* | 261* | 5 | 278*† |
| | 1992 | 21 | 300 | 28 | 272 | 48 | 255 | 3 | 251 |
| | 1990 | 16 | 293 | 21 | 268 | 60 | 252 | 4 | *** |
| Males | 1996 | 25* | 297 | 27* | 269 | 43* | 264*† | 5 | 262 |
| | 1992 | 19 | 299 | 28* | 273 | 49* | 256 | 4 | 250 |
| | 1990 | 16 | 298 | 19 | 275 | 62 | 253 | 3 | *** |
| White | 1996 | 27* | 305 | 29 | 277 | 40* | 271* | 4 | 284† |
| | 1992 | 22 | 306 | 31 | 278 | 45 | 266 | 3 | 259 |
| | 1990 | 18 | 300 | 21 | 276 | 57 | 260 | 3 | 265 |
| Black | 1996 | 20* | 258 | 25 | 240 | 48* | 237 | 7 | 254 |
| | 1992 | 13 | 259 | 23 | 247 | 60 | 231 | 4 | *** |
| | 1990 | 9 | *** | 16 | 246 | 72 | 234 | 3 | *** |
| Hispanic | 1996 | 20*† | 262 | 22 | 260 | 52* | 249 | 6 | *** |
| | 1992 | 12 | 277 | 21 | 256 | 62 | 241 | 5 | *** |
| | 1990 | 7 | *** | 14 | 260 | 76 | 240 | 4 | *** |
| Asian/Pacific Islander | 1996 | – – | – – | – – | – – | – – | – – | – – | – – |
| | 1992 | 42 | 313 | 24 | *** | 32 | 265 | 2 | *** |
| | 1990 | 39 | *** | 22 | *** | 33 | *** | 6 | *** |
| American Indian | 1996 | 14 | *** | 18 | *** | 63 | *** | 6 | *** |
| | 1992 | 7 | *** | 30 | *** | 57 | 253 | 5 | *** |
| | 1990 | 6 | *** | 8 | *** | 84 | *** | 3 | *** |
| School Offers Algebra for High School Credit or Placement: | | | | | | | | | |
| Yes | 1996 | 28* | 298 | 28 | 271 | 39* | 262* | 5 | 276*† |
| | 1992 | 23 | 302 | 29 | 274 | 45* | 256 | 4 | 249 |
| | 1990 | 18 | 301 | 20 | 271 | 58 | 254 | 4 | 254 |
| No | 1996 | 16 | 279 | 24 | 268 | 56 | 266 | 5 | *** |
| | 1992 | 10 | 285 | 28 | 270 | 60 | 257 | 2 | *** |
| | 1990 | 7 | *** | 19 | 272 | 73 | 252 | 1 | *** |

NOTE: Row percentages may not total 100 due to rounding.
* Significantly different from 1990.
† Significantly different from 1992.
*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Among the eighth-grade students who provided usable answers about the mathematics course in which they were enrolled, 43 percent indicated that they were in a basic eighth-grade mathematics class; 27 percent were in a pre-algebra class; 25 percent were in an algebra class; and 5 percent were in some other mathematics course.[3] These "other mathematics" courses included applied mathematics (also referred to as technical preparation mathematics) and integrated or sequential mathematics. Perhaps not surprisingly, students in algebra classes outperformed students in pre-algebra, eighth-grade mathematics, and "other mathematics" courses on the NAEP 1996 assessment, and students in pre-algebra outperformed students in eighth-grade mathematics classes.

In 1996, the eighth-grade course-taking patterns and NAEP mathematics performance of *females and males* in the same mathematics courses were similar to each other. For example, 42 percent of female students and 43 percent of male students were enrolled in eighth-grade mathematics. Similarly, the percentages of female students enrolled in pre-algebra, algebra, and "other mathematics" classes were similar to the percentages of male students enrolled in the same type of class. Furthermore, within each mathematics course, female students and male students performed similarly on the NAEP 1996 mathematics assessment; for example, female students in algebra classes had an average scale score of 294, while the average scale score for male students in algebra classes was 297.

However, when one looks at performance differences across mathematics courses, the pattern differs by gender group. For female students, those taking algebra outperformed female students in pre-algebra classes, and female students in algebra, pre-algebra, and "other mathematics" courses outperformed those in eighth-grade mathematics. For male students, those taking algebra outperformed those in pre-algebra, eighth-grade mathematics, and "other mathematics" courses, while male students in any of the courses other than algebra performed similarly to each other.

An examination of the 1996 percentages of the different racial/ethnic groups enrolled in each type of mathematics course shows no significant differences, except in algebra, where the percentage of White students was higher than the percentage of American Indian students.[4] There were, however, some differences in the overall performance of different racial/ethnic groups in specific mathematics courses. In algebra, White students outperformed Black and Hispanic students. In pre-algebra, White and Hispanic students outperformed Black students, and White students also outperformed Hispanic students. In eighth-grade mathematics, White and Hispanic students outperformed Black students. In "other mathematics" courses, White students outperformed Black students.

Comparisons of percentages of students enrolled in different mathematics courses by *whether or not their school offered algebra for high school credit or placement* show that offering algebra for high school credit appears to make some difference. The percentage of eighth-grade

[3] About two percent of eighth-grade students taking the NAEP 1996 assessment either omitted this question or provided multiple responses. These were considered nonlegitimate answers, and, therefore, these students were excluded from analyses involving eighth-grade course-taking patterns.

[4] The reader is reminded that statements about significant differences are based on statistical tests that consider the magnitude of the difference among the percentages or averages and the standard errors of those statistics. Therefore, differences that appear to be large may turn out to be statistically nonsignificant. More details on statistical inferences using NAEP data are available in Appendix A.

students enrolled in algebra in schools that offered algebra for high school credit was higher than the percentage of students enrolled in algebra in schools that did not offer this option. Because course enrollments were self-reported, students in schools that did not offer algebra for high school credit, but who indicated that they were taking algebra, may have erroneously reported their enrollment status. However, in this report, we have assumed that the responses of these students were correct and either their schools offered a nontransferable algebra course or that they were taking algebra at an alternative site such as a local high school or community college.

As might be expected, the pattern of eighth-grade mathematics enrollment by whether or not the school offered algebra for high school credit was the converse of the algebra enrollment pattern. That is, the percentage of students enrolled in eighth-grade mathematics in schools that offered algebra for high school credit was lower than the percentage enrolled in eighth-grade mathematics in other schools. Comparisons of students' performance on the NAEP 1996 assessment show that algebra students from schools that offered algebra for high school credit performed better than algebra students from schools that did not.

Because some current mathematics reform efforts advocate that students take more difficult mathematics courses earlier in their school careers and specifically suggest that students be prepared to take algebra in eighth grade, we examined *enrollment patterns over time* to determine whether enrollment in eighth-grade algebra was increasing. Indeed, the data indicate that a higher percentage of eighth-grade students was enrolled in algebra in 1996 (25%) than had been enrolled in algebra in 1992 or in 1990 (20% and 16%, respectively). However, despite these increases in the percentages enrolled, students enrolled in algebra in 1996 performed similarly on the NAEP mathematics assessment to students enrolled in algebra in 1992 and 1990.

The percentage of eighth-grade students enrolled in pre-algebra in 1996 did not increase from 1992 but was higher than the percentage of students enrolled in pre-algebra in 1990. As with students in algebra, performance on the mathematics assessment for students in pre-algebra in 1996 was similar to the performance for pre-algebra students in 1992 and 1990. The percentage of eighth-grade students enrolled in eighth-grade mathematics in 1996 was similar to the percentage enrolled in 1992, but was lower than the percentage enrolled in 1990. It is possible that the curriculum of eighth-grade mathematics had also been changing over this period because, in 1996, students in eighth-grade mathematics performed better in mathematics than eighth-grade mathematics students did in 1992 and 1990.

The small percentage of students enrolled in "other mathematics" courses in 1996 was similar to the percentages enrolled in 1992 and 1990. Students in "other mathematics" courses in 1996 outperformed students in "other mathematics" courses in 1992.

Comparisons of gender groups over time show that a higher percentage of female students was enrolled in algebra in 1996 than was enrolled in algebra in 1992 and 1990. However, for male students, the percentage enrolled in algebra in 1996 was only significantly higher than the percentage enrolled in 1990.

Enrollment patterns in algebra over time differed among racial/ethnic subgroups. The percentages of White students and Black students enrolled in algebra were significantly higher in 1996 than they were in 1990, but not significantly higher than they were in 1992, while the percentage of Hispanic students enrolled in algebra in 1996 was higher than the percentage enrolled in 1992 or 1990.

In schools that offered algebra for high school credit, student enrollment in algebra increased from 1990 to 1996. A significant increase in enrollment in algebra was not observed for schools that did not offer algebra for high school credit.

## *Mathematics Course Taking in High School*

The NAEP background survey of twelfth-grade students collected considerable detail about students' current and past course-taking patterns. In 1996, three percent of the nation's twelfth-grade students attended schools that required 4 years of mathematics (taken in grades 9–12) for high school graduation, and 51 percent attended schools with a 3-year requirement. In schools that have less than a 4-year requirement, students generally take their mathematics classes earlier in their high school careers. This means that when these students graduate from high school, many have not been involved in the formal study of mathematics on a regular basis for a year or more. The chances, therefore, are likely that by the time they graduate and enter the work world or go on to higher education, many students probably will have forgotten much of what they learned or at least will be less facile with what they remember.

Table 8.2 shows that, in 1996, slightly less than two-thirds of the nation's twelfth-grade students (64%) were enrolled in a mathematics class. Being enrolled in mathematics, however, did not necessarily mean that these students had all had 4 years of high school mathematics or were enrolled in advanced courses. For some individuals, taking mathematics in their senior year might have been the result of having either failed previous classes or delayed taking a required class.[5] Nevertheless, taken as a group, students enrolled in mathematics in their twelfth-grade year outperformed students who were not enrolled in mathematics on the NAEP 1996 mathematics assessment. The average scale score of those who were taking mathematics was 311, while those who were not had an average scale score of 292.[6]

Similar percentages of female and male twelfth-grade students were enrolled in mathematics in 1996, 63 percent and 66 percent respectively. In terms of racial/ethnic groups, a higher percentage of Asian/Pacific Islander twelfth-grade students (77%) was enrolled in mathematics classes compared with White students (63%), Hispanic students (63%), and American Indian students (56%).

The percentage of students enrolled in a mathematics class their senior year was higher in 1996 than it was in 1990. Similarly, the percentage of female students enrolled in a mathematics class their senior year was higher in 1996 than in 1990.

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5 Here and throughout this report, the term "senior year" refers to students' twelfth-grade year.

6 The source of these data is the NAEP 1996 mathematics assessment. The data are available on the World Wide Web at: <http://nces.ed.gov/naep/>.

| Table 8.2 | Percentage of Students Currently Enrolled in a Mathematics Course by Gender and Race/Ethnicity, Grade 12 |
|---|---|

THE NATION'S REPORT CARD naep

| | Assessment Year | Percentage of Students |
|---|---|---|
| **Grade 12** | | |
| All Students | 1996 | 64* |
| | 1992 | 63 |
| | 1990 | 59 |
| Females | 1996 | 63* |
| | 1992 | 61* |
| | 1990 | 52 |
| Males | 1996 | 66 |
| | 1992 | 66 |
| | 1990 | 66 |
| White | 1996 | 63 |
| | 1992 | 62 |
| | 1990 | 58 |
| Black | 1996 | 70 |
| | 1992 | 64 |
| | 1990 | 62 |
| Hispanic | 1996 | 63* |
| | 1992 | 62 |
| | 1990 | 53 |
| Asian/Pacific Islander | 1996 | 77 |
| | 1992 | 85 |
| | 1990 | 76 |
| American Indian | 1996 | 56 |
| | 1992 | *** |
| | 1990 | *** |

* Significantly different from 1990.
*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

## First-Year Algebra

As shown by the data on mathematics course taking, in 1996 over a third of twelfth-grade students appear to have chosen to opt out of mathematics before their senior year of high school. Especially for these students, but for other students as well, to have taken any advanced mathematics courses, they would have had to take algebra as early as possible in their school careers. Information about when students initially took first-year algebra is an indicator of students' preparedness to enter a mathematics sequence that would lead to advanced courses. Data on eighth-grade course taking in 1996 shows that one-fourth of our nation's eighth-grade students were enrolled in algebra, which (in addition to geometry) is a prerequisite for higher level mathematics courses.[7] The responses of twelfth-grade students, who were asked to provide information on when they initially took first-year algebra, are presented in Table 8.3.

**Table 8.3** **Percentage of Students by Year They Initially Took a First-Year Algebra Course, Grade 12**

THE NATION'S REPORT CARD naep

| Grade 12 | Assessment Year | Before 9th Grade | 9th Grade | 10th Grade | 11th or 12th Grade | Not Taken |
|---|---|---|---|---|---|---|
| All Students | 1996 | 29† | 51 | 13 | 3 | 3† |
| | 1992 | 23 | 51 | 15 | 5 | 6 |
| Females | 1996 | 29† | 52 | 13 | 3 | 3† |
| | 1992 | 23 | 52 | 15 | 5 | 5 |
| Males | 1996 | 30† | 49 | 13 | 4 | 4† |
| | 1992 | 24 | 49 | 15 | 5 | 7 |
| White | 1996 | 30† | 52 | 12 | 3 | 3† |
| | 1992 | 24 | 52 | 14 | 4 | 6 |
| Black | 1996 | 27 | 48 | 17 | 5 | 4 |
| | 1992 | 18 | 48 | 19 | 9 | 7 |
| Hispanic | 1996 | 21 | 51 | 16 | 7 | 5 |
| | 1992 | 17 | 45 | 23 | 9 | 7 |
| Asian/Pacific Islander | 1996 | 50 | 37 | 8 | 2 | 2 |
| | 1992 | 40 | 44 | 10 | 4 | 2 |
| American Indian | 1996 | 13 | 52 | 27 | 8 | 1 |
| | 1992 | *** | *** | *** | *** | *** |

NOTE: Row percentages may not total 100 due to rounding.
† Significantly different from 1992.
*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

[7] This is not to imply that algebra is the most advanced mathematics course available to eighth-grade students. In fact, students may be in an integrated-mathematics course that substitutes for algebra, or some students may even be in courses more advanced than algebra. However, as shown in Table 8.1, only five percent of students were in mathematics courses other than algebra, pre-algebra, or eighth-grade mathematics.

In 1996, the majority of twelfth-grade students (51%) indicated that they initially took first-year algebra in the ninth grade and 29 percent took it before the ninth grade.[8] Regardless of whether twelfth-grade students were currently taking a mathematics class or not, about half of them indicated that they had initially taken algebra in the ninth grade.

The patterns for initially taking first-year algebra were similar for male and female students. The percentages by racial/ethnic groups, however, show some differences. For example, although the modal response of White, Black, Hispanic, and American Indian students indicated initially taking first-year algebra in the ninth grade, the modal response of Asian/Pacific Islander students indicated initially taking first-year algebra *before* the ninth grade.

Comparisons over time show that for all twelfth-grade students, for female and male students, and for White students, the percentages of students who initially took first-year algebra before the ninth grade were higher in 1996 than they were in 1992. Moreover, for all of those groups, the percentages of twelfth-grade students who had not taken a first-year algebra course at all were lower in 1996 than they were in 1992. These numbers appear to signify a positive trend in light of current mathematics reform efforts.

## Number and Types of Mathematics Courses Taken

As shown in Table 8.4, in 1996 almost half of the nation's twelfth-grade students indicated having taken seven or more *semesters of mathematics* during their high school career (i.e., grades 9 to 12). Seven or more semesters of mathematics translates into more than 3 years of mathematics courses. This appears encouraging, given that only three percent of twelfth-grade students were enrolled in schools that required more than 3 years of mathematics courses for high school graduation. That is, students appear to be taking more mathematics than schools require for graduation. However, the reader should keep in mind that some of these semesters of coursework may reflect repeats of courses for students who failed to reach levels of performance that would have allowed them to move forward.

It also is encouraging that, in 1996, the percentage of female students with seven or more semesters of mathematics was similar to the percentage of male students.

[8] Discrepancies between these data on grade 12 students in 1996 and data reported in Table 8.1 on grade 8 students in 1992 (the same population of students) may be explained by the fact that these data are based on students' self-reports. Memory limitations or confusion about the different levels of algebra may influence the accuracy of students' responses, especially when students are asked about their course-taking experiences retrospectively at grade 12.

## Table 8.4 Percentage of Students by Number of Semesters of Mathematics Taken (Grades 9 through 12) by Gender and Race/Ethnicity, Grade 12

THE NATION'S REPORT CARD naep

| Grade 12 | Assessment Year | 7 or More Semesters | 5–6 Semesters | 3–4 Semesters | 1–2 Semesters | No Semesters |
|---|---|---|---|---|---|---|
| All Students | 1996 | 48 | 22 | 26 | 4 | 1 |
| | 1992 | 48 | 23 | 25 | 3 | 1 |
| | 1990 | 45 | 23 | 27 | 6 | 0 |
| Females | 1996 | 47 | 23 | 26 | 4 | 0 |
| | 1992 | 46 | 25 | 25 | 3 | 0 |
| | 1990 | 40 | 27 | 28 | 5 | 0 |
| Males | 1996 | 49 | 20 | 26 | 5 | 1 |
| | 1992 | 50 | 21 | 25 | 3 | 1 |
| | 1990 | 50 | 18 | 26 | 6 | 1 |
| White | 1996 | 50 | 23 | 23 | 4 | 0 |
| | 1992 | 50 | 24 | 23 | 3 | 1 |
| | 1990 | 46 | 23 | 25 | 5 | 0 |
| Black | 1996 | 37 | 17 | 40 | 6 | 1 |
| | 1992 | 38 | 18 | 38 | 5 | 1 |
| | 1990 | 33 | 21 | 39 | 7 | 1 |
| Hispanic | 1996 | 44 | 23 | 28 | 5 | 1 |
| | 1992 | 38 | 28 | 28 | 5 | 1 |
| | 1990 | 38 | 22 | 35 | 5 | 0 |
| Asian/Pacific Islander | 1996 | 66 | 16 | 17 | 1 | 1 |
| | 1992 | 69 | 18 | 12 | 2 | 0 |
| | 1990 | 66 | 23 | 7 | 3 | 0 |
| American Indian | 1996 | 22 | 28 | 38 | 10 | 2 |
| | 1992 | *** | *** | *** | *** | *** |
| | 1990 | *** | *** | *** | *** | *** |

NOTE: Row percentages may not total 100 due to rounding.
*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Comparisons across racial/ethnic groups by semester categories show that, in 1996, the percentage of Asian/Pacific Islander students taking seven or more semesters of mathematics was higher than the percentages of students in the other racial/ethnic groups; and the percentage of White students was higher than the percentage of Black and American Indian students. For most of the racial/ethnic groups, the modal number of semesters was seven or more. However, for Black students and American Indian students, the modal number was 3–4 semesters.

Reform efforts to improve the mathematics achievement of the nation's students advocate taking more mathematics, as well as taking courses that are at more advanced levels of mathematics. In 1996, students were asked to indicate the level of exposure they had to different *types of mathematics courses* during their high school years. This information is presented in Table 8.5 along with student data from 1992 and 1990 where available.

In 1996, the relative percentages of students who indicated that they had taken one school year or more of each type of mathematics course were not unexpected. In terms of what are generally considered lower-level courses, 63 percent of twelfth-grade students indicated having taken a year or more of pre-algebra, and 53 percent indicated having taken a year or more of general mathematics. Among the higher level mathematics courses, the highest percentage of twelfth-grade students indicated having taken a year or more of first-year algebra (90%), followed by 80 percent who indicated having taken geometry. Two relatively new mathematics courses were added after the 1990 administration for students' consideration. In 1992, "unified, integrated, or sequential mathematics" was added to the list of mathematics courses, and, in 1996, nine percent of twelfth-grade students indicated having taken a year or more of that course. In 1996, students also were asked about "applied mathematics," also known as "technical preparation mathematics"; 15 percent of students indicated having taken a year or more of such work.

Given the belief that more students should be taking higher level mathematics, in general, the course-taking patterns of twelfth-grade students have improved over time. For example, between 1990 and 1996, there were increases in the percentages of students who had taken a full year of pre-algebra, first-year algebra, second-year algebra, pre-calculus (also known as third-year algebra), calculus, or probability or statistics. On the other hand, the percentage of students who reported having taken more than one year of general mathematics was also higher.

**Table 8.5**

**Percentage of Students by Mathematics Courses and Years of Study, Grade 12**

THE NATION'S REPORT CARD naep

| Grade 12 | Assessment Year | Years of Study: More Than One Year | One School Year | One-Half Year or Less | Not Studied |
|---|---|---|---|---|---|
| General Mathematics | 1996 | 33*† | 20 | 3 | 44 |
| | 1992 | 27 | 23 | 3 | 48 |
| | 1990 | 27 | 23 | 3 | 47 |
| Business or Consumer Mathematics | 1996 | 4 | 16† | 9 | 71† |
| | 1992 | 5 | 21* | 9 | 66 |
| | 1990 | 5 | 17 | 9 | 6 |
| Introduction to Algebra or Pre-Algebra | 1996 | 11*† | 52* | 7 | 30*† |
| | 1992 | 9 | 48 | 7 | 37 |
| | 1990 | 9 | 43 | 7 | 41 |
| First-Year Algebra | 1996 | 9 | 81* | 4 | 6*† |
| | 1992 | 8 | 79* | 4 | 9* |
| | 1990 | 8 | 73 | 4 | 14 |
| Geometry | 1996 | 5† | 75 | 7 | 13† |
| | 1992 | 4* | 72 | 5 | 19 |
| | 1990 | 5 | 66 | 5 | 25 |
| Second-Year Algebra | 1996 | 4 | 66*† | 7 | 23*† |
| | 1992 | 3 | 58 | 8 | 32* |
| | 1990 | 3 | 53 | 6 | 38 |
| Trigonometry | 1996 | 2 | 20 | 23 | 55* |
| | 1992 | 2 | 19 | 22 | 58 |
| | 1990 | 2 | 15 | 19 | 64 |
| Pre-Calculus, Third-Year Algebra | 1996 | 2 | 22* | 12 | 65*† |
| | 1992 | 1 | 18 | 10 | 70 |
| | 1990 | 2 | 14 | 10 | 75 |
| Calculus | 1996 | 1 | 11* | 4 | 84* |
| | 1992 | 1 | 9 | 3 | 87 |
| | 1990 | 1 | 7 | 3 | 88 |
| Probability or Statistics | 1996 | 2 | 6*† | 13* | 79* |
| | 1992 | 1 | 4 | 12 | 83 |
| | 1990 | 1 | 3 | 9 | 88 |
| Unified, Integrated, or Sequential Mathematics | 1996 | 4 | 5 | 4 | 87 |
| | 1992 | 2 | 4 | 5 | 89 |
| Applied Mathematics (Technical Preparation) | 1996 | 5 | 10 | 6 | 79 |

NOTE: Row percentages may not total 100 due to rounding.
* Significantly different from 1990.
† Significantly different from 1992.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

## *Algebra and Calculus Coursework in High School*

In Table 8.6, twelfth-grade students are categorized according to the highest level mathematics course they indicated having taken in an algebra-through-calculus sequence. The algebra-through-calculus sequence for this analysis was created in accordance with the typical sequential order of these courses in high schools in the United States.[9] The lowest level in this sequence is "Not having taken at least a pre-algebra or introduction-to-algebra course" and the highest level is calculus, with intermediate steps as follows: pre-algebra, first-year algebra, second-year algebra, and pre-calculus (also referred to as third-year algebra or analysis). Students were credited with having taken a particular course only if they indicated that they had taken one school year or more of that course.

In 1996, almost half of the twelfth-grade students indicated that second-year algebra was the highest course in an algebra-through-calculus sequence that they had taken for one school year or more. Second-year algebra was the modal response of both female students and male students; however, a higher percentage of female students than male students indicated second-year algebra as the highest algebra-through-calculus course they had taken. Comparisons between the percentages of female students and male students at each course level showed no other significant difference.

There were few significant differences in the course-taking patterns of different racial/ethnic groups. For example, the percentages of White, Black, Hispanic, Asian/Pacific Islander, and American Indian students who indicated that second-year algebra was the highest algebra-through-calculus course they had taken were similar to each other. The only differences found were for first-year algebra and calculus. The percentage of American Indian students indicating first-year algebra as their highest level course taken was higher than the percentage of any other racial/ethnic group, and the percentage of Hispanic students indicating this was higher than the percentage of Asian/Pacific Islander students. For calculus, the percentage of White students was higher than the percentages of Black and Hispanic students.

The trend toward students taking more advanced-level courses also is apparent when we focus on the highest level courses students have taken in the algebra-through-calculus sequence. For example, in 1996, four percent of twelfth-grade students indicated not having taken pre-algebra; this was lower than the nine percent in 1990. In addition, four percent of students indicated pre-algebra as their highest level algebra-through-calculus course, and this was lower than the six percent of students who so indicated in 1992. Even the 23 percent of students who indicated that first-year algebra was their highest level algebra-through-calculus course was lower than the 29 percent in 1992. At the other end of the spectrum, seven percent of students in 1996 indicated that calculus was the highest algebra-through-calculus course taken; this was higher than the five percent of students in 1992, or the three percent of students in 1990, who indicated that calculus was their highest course.

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[9] Chaney, B., Burgdorf, K., & Atash, N. (1997). Influencing achievement through high school graduation requirements. *Educational Evaluation and Policy Analysis*, *19*(3), 229–244.

## Table 8.6

### Percentage of Students by Highest Algebra-Through-Calculus Course Taken, Grade 12

THE NATION'S REPORT CARD naep

| Grade 12 | Assessment Year | Not Taken Pre-Algebra | Pre-Algebra | First-Year Algebra | Second-Year Algebra | Pre-Calculus or Third-Year Algebra | Calculus |
|---|---|---|---|---|---|---|---|
| All Students | 1996 | 4* | 4† | 23*† | 48* | 14 | 7*† |
| | 1992 | 6* | 6 | 29 | 44 | 11 | 5* |
| | 1990 | 9 | 8 | 28 | 43 | 9 | 3 |
| Females | 1996 | 3*† | 4*† | 21† | 51* | 14* | 6* |
| | 1992 | 5 | 6 | 28 | 45 | 11 | 5* |
| | 1990 | 8 | 8 | 28 | 45 | 9 | 3 |
| Males | 1996 | 5* | 5* | 24 | 45 | 13 | 8* |
| | 1992 | 6* | 6 | 29 | 42 | 11 | 5 |
| | 1990 | 10 | 7 | 27 | 41 | 10 | 4 |
| White | 1996 | 3* | 4* | 22 | 49 | 15 | 8 |
| | 1992 | 5 | 5 | 27 | 45 | 12 | 5 |
| | 1990 | 8 | 7 | 27 | 44 | 10 | 4 |
| Black | 1996 | 5* | 5* | 24*† | 52*† | 11 | 3 |
| | 1992 | 8 | 8 | 37 | 37 | 7 | 3 |
| | 1990 | 13 | 10 | 31 | 39 | 6 | 0 |
| Hispanic | 1996 | 8 | 6 | 27 | 46 | 10* | 4 |
| | 1992 | 7 | 9 | 34 | 40 | 6 | 4 |
| | 1990 | 17 | 10 | 31 | 37 | 3 | 1 |
| Asian/Pacific Islander | 1996 | 3 | 4 | 14 | 39 | 19 | 20 |
| | 1992 | 1 | 4 | 20 | 45 | 12 | 17 |
| | 1990 | 5 | 10 | 24 | 42 | 13 | 5 |
| American Indian | 1996 | 5 | 6 | 46 | 38 | 4 | 2 |
| | 1992 | *** | *** | *** | *** | *** | *** |
| | 1990 | *** | *** | *** | *** | *** | *** |

NOTE: Row percentages may not total 100 due to rounding.
* Significantly different from 1990.
† Significantly different from 1992.
*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

The patterns for both female students and male students over time are encouraging, especially for female students. In 1996, there were declines from 1990 and/or 1992 in the percentages of females who reported that their highest level course in the algebra-through-calculus sequence was none, pre-algebra, or first-year algebra. At the same time, there were increases in the percentages of female students who reported having taken pre-calculus or calculus. For male students, there were declines in the percentages who reported having stopped with no year-long courses in the algebra-through-calculus sequence; there was also an increase in the percentage who reported having taken calculus.

Differences over time for the racial/ethnic groups were as follows: among White students, there was a significant increase in the percentage of students taking calculus, whereas declines were observed in the percentages of students who did not take pre-algebra or who did not advance beyond pre-algebra. For Black students, there were increases in the percentages who reported having taken second-year algebra or calculus as their highest course; there were also declines in the percentages who reported stopping with no pre-algebra, only pre-algebra, or only first-year algebra. Among Hispanic students, there was an increase in the percentage who reported having taken pre-calculus.

## Geometry Coursework in High School

Although researchers have found that geometry is generally taken after first-year algebra and before second-year algebra, this sequence is not always the rule; therefore, we chose to examine course taking in geometry separate from algebra-calculus courses.[10] Furthermore, the role of geometry in the American educational system has changed over the years. Some educational researchers have cited geometry as the new "gatekeeper" course for access to higher education, because most colleges are now requiring the completion of a course in geometry prior to entrance.[11] Therefore, it seemed important to examine geometry course taking apart from course taking in an algebra-through-calculus sequence. The data in Table 8.7 indicate that in 1996, over 80 percent of twelfth-grade students had taken a year or more of geometry during their high school years. Similar percentages of female and male students in the twelfth grade reported having taken a school year or more of geometry. Over half of the students in all racial/ethnic groups indicated having taken geometry.

The 1996 percentage of twelfth-grade students who indicated having taken geometry was higher than the percentage in 1990. This pattern was similar for female and male students and for White, Black, and Hispanic students.

[10] Ibid; National Assessment of Educational Progress (NAEP) 1996 mathematics assessment.

[11] Pelavin, S., & Kane, M. (1990). *Changing the odds: Factors increasing access to college*. New York: College Board Publications; U.S. Department of Education. (1997). *Getting ready for college early: A handbook for parents of students in the middle and junior high school years.* Available on the Word Wide Web at: <http://www.ed.gov/pubs/GettingReadyCollegeEarly/>.

Table 8.7

## Percentage of Students by Whether They Have Taken a Geometry Course and by Gender and Race/Ethnicity, Grade 12

| Grade 12 | Assessment Year | Taken a Geometry Course: Yes | Taken a Geometry Course: No |
|---|---|---|---|
| All Students | 1996 | 80* | 20* |
| | 1992 | 76* | 24* |
| | 1990 | 71 | 29 |
| Females | 1996 | 82* | 18* |
| | 1992 | 77* | 23* |
| | 1990 | 71 | 29 |
| Males | 1996 | 78* | 22* |
| | 1992 | 75 | 25 |
| | 1990 | 70 | 30 |
| White | 1996 | 81* | 19* |
| | 1992 | 78 | 22 |
| | 1990 | 73 | 27 |
| Black | 1996 | 82* | 18* |
| | 1992 | 72 | 28 |
| | 1990 | 61 | 39 |
| Hispanic | 1996 | 77* | 23* |
| | 1992 | 67 | 33 |
| | 1990 | 58 | 42 |
| Asian/Pacific Islander | 1996 | 84 | 16 |
| | 1992 | 86 | 14 |
| | 1990 | 85 | 15 |
| American Indian | 1996 | 58 | 42 |
| | 1992 | *** | *** |
| | 1990 | *** | *** |

NOTE: Row percentages may not total 100 due to rounding.
* Significantly different from 1990.
*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

## *Summary*

This chapter examined the course-taking patterns of the nation's eighth- and twelfth-grade students in 1996 and over time. In 1996, over half of the eighth-grade students were enrolled in pre-algebra or algebra, while most of the remaining students were enrolled in eighth-grade mathematics. The percentages of female students enrolled in each of these three mathematics courses were similar to the percentages of male students enrolled in them. Enrollment percentages in each of these three types of mathematics classes were also similar for the different racial/ethnic groups except that the percentage of White students enrolled in algebra was higher than the percentage of American Indian students enrolled in that course. Trends over time appear to show that more eighth-grade students were taking more advanced mathematics courses. For example, the percentage of female students enrolled in algebra in 1996 was higher than the percentage enrolled in algebra in 1992 and 1990, and the percentage of male students, White students, and Black students enrolled in algebra in 1996 was higher than the percentage enrolled in 1990.

In 1996, approximately two-thirds of twelfth-grade students reported being enrolled in a mathematics class. Similar percentages of female and male students were taking mathematics, while the percentage of Asian/Pacific Islander students taking mathematics was higher than the percentages of White, Hispanic, and American Indian students. The percentage of twelfth-grade students enrolled in mathematics in 1996 was higher than in 1990; this was also true for the percentage of female students enrolled in mathematics.

In 1996, 29 percent of twelfth-grade students reported that they initially took first-year algebra before the ninth grade. This was true of both female and male students. In terms of racial/ethnic groups, half of the Asian/Pacific Islander students initially took first-year algebra before the ninth grade, which was higher than the percentage of White or Hispanic students taking first-year algebra this early. Information appears to show that over time, more students were taking first-year algebra and taking it earlier in their school careers.

In 1996, nearly half of all twelfth-grade students, both female and male students, reported taking seven or more semesters of mathematics. Large majorities of students reported having taken first-year algebra, geometry, and second-year algebra. In addition, there have been significant increases over time in the percentages of students taking courses at all levels of the algebra-through-calculus sequence, including the most advanced mathematics courses. Almost half of the twelfth-grade students indicated second-year algebra as the highest course taken in the algebra-through-calculus sequence. Twenty-one percent of students indicated taking a higher level course (such as pre-calculus or calculus) and 31 percent of students indicated taking a lower-level course (such as first-year algebra or pre-algebra) as their highest course in this sequence. With the exception of second-year algebra, where the percentage of female students was higher than the percentage of male students, there were no significant gender differences in the highest algebra-through-calculus course taken.

Comparisons over time also indicate a rise in the percentage of twelfth-grade students who have taken geometry. In 1996, four out of five twelfth-grade students indicated that they had taken a year or more of geometry. This was true of female and male students as well as students from the different racial/ethnic groups, with the exception of American Indian students.

# Chapter 9

# *Classroom Practices*

The NAEP 1996 mathematics assessment sought to embody many of the curricular emphases and objectives laid out in the curriculum and evaluation standards developed by the National Council of Teachers of Mathematics (NCTM).[1] Among the key features of the NAEP 1996 mathematics assessment were the following:

- *movement away from earlier assessments emphasizing only number properties and operations to also measure ability in number sense and estimations, as well as problem solving, communication, reasoning, and connections;*

- *inclusion of questions that require students to work through an extended problem and explain their reasoning through writing, giving examples, or drawing diagrams;*

- *increased use of calculators; and*

- *increased use of manipulatives such as geometric shapes to provide students with concrete representations to use in problem-solving situations.*[2]

The importance of these key features in current mathematics reform efforts, as well as the prominence given to them in the NAEP mathematics assessments since 1990, invites the question of the extent to which the mathematics instruction offered in our nation's classrooms reflects these same features. Background questions asked of students who participated in NAEP and of their teachers and principals were used to gather information about the instructional practices students were experiencing in their mathematics classrooms. For example, teachers were asked about the emphasis they placed on different mathematics content strands and on different mathematics skills. Teachers and students also were asked about the frequency with which students engaged in a variety of pedagogical and assessment practices in their mathematics classes, including questions about the use of calculators to do mathematics schoolwork.

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[1] National Assessment Governing Board (1996). *Mathematics framework for the 1996 National Assessment of Educational Progress.* Washington, DC: National Assessment Governing Board.

[2] White, S. (1994). *Overview of NAEP assessment frameworks*. Washington, DC: National Center for Education Statistics, p. 51.

Chapters 2 through 7 focused on student performance in mathematics overall, in the content strands, and on individual mathematics questions. This chapter focuses on information gathered by NAEP about the mathematics instruction our nation's students are experiencing in their classrooms. The information presented in this chapter about fourth- and eighth-grade students was provided either by their mathematics teachers or by the students themselves. Teachers of twelfth-grade students were not surveyed; therefore, information about twelfth-grade students was obtained solely through students' self-reports. In addition, because the questions focused on practices directly related to mathematics instruction, most of the information about twelfth-grade students was limited to those students who reported that they were presently enrolled in a mathematics class.

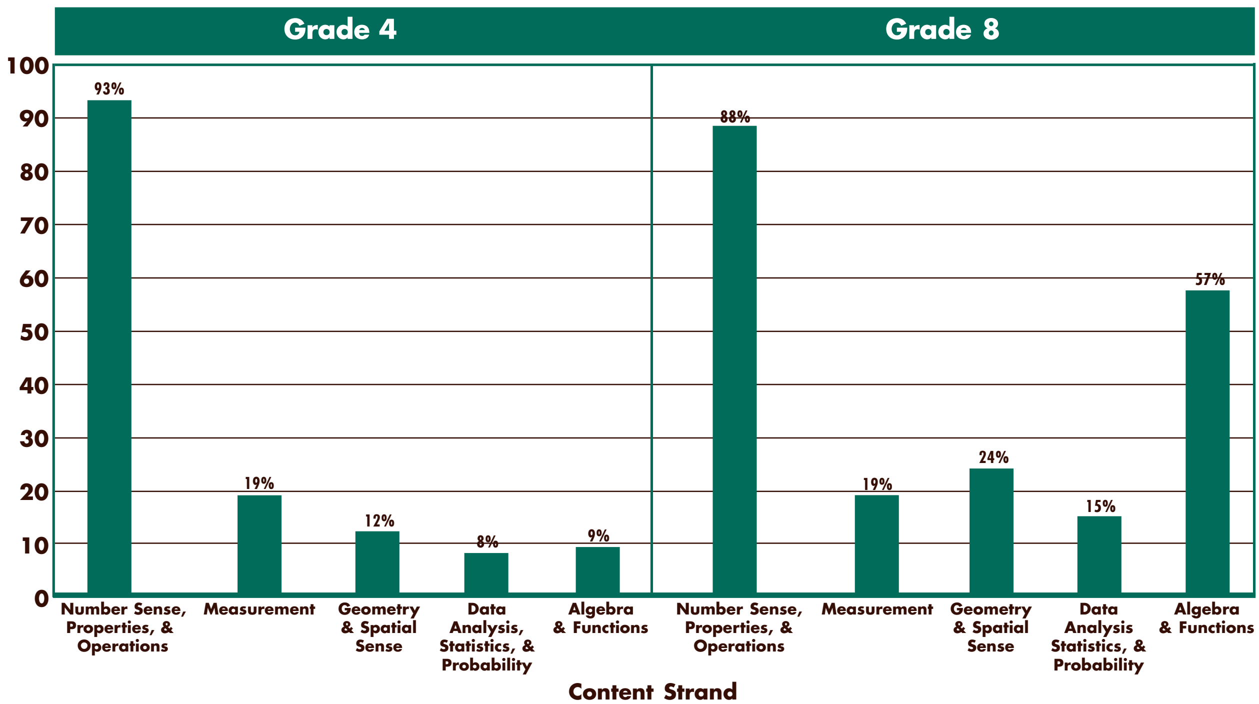
## *Emphasis on Content Strands*

In the 1996 mathematics assessment, teachers of mathematics were asked about the level of emphasis they placed in their mathematics curriculum on each of the five mathematics content strands that are part of the NAEP mathematics framework: Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; Data Analysis, Statistics, and Probability; and Algebra and Functions. Because the data are based on written self-reports using a three-category response scale ("a lot," "some," or "a little or no" emphasis), there is a certain inherent ambiguity in the findings in that one teacher's reading of "a lot" may be another teacher's "some," and so on. Nevertheless, patterns do emerge and provide an important picture of the state of mathematics instruction in our nation's classrooms. Figure 9.1 shows the percentages of fourth- and eighth-grade students whose teachers reported placing "a lot" of emphasis on each of the five content strands. More detailed information on teachers' responses for each of the different content strands is presented in Tables 9.1–9.5.

The data in Figure 9.1 show that at both grades 4 and 8, a large percentage of students had teachers who placed "a lot" of emphasis on Number Sense, Properties, and Operations. At the fourth-grade level, fewer than one in five students had teachers who placed "a lot" of emphasis on any one of the remaining four content strands. In contrast, at the eighth-grade level, in addition to the prominence of the Number Sense, Properties, and Operations strand, teachers of over half of the students reported placing "a lot" of emphasis on Algebra and Functions. "A lot" of emphasis on the other content strands, however, was still infrequent. In addition, as the data in Tables 9.1–9.5 show, with the exception of Algebra and Functions, the emphasis placed on the different content strands did not differ by type of eighth-grade mathematics course in which students were enrolled.

**Figure 9.1**

## Percentage of Students Whose Teachers Place "A Lot" of Emphasis on Specific Content Strands by Grade and Content Strand




SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### *Number Sense, Properties, and Operations*

As shown in Figure 9.1, and again in Table 9.1, "a lot" of emphasis on Number Sense, Properties, and Operations was very common in mathematics classes for both grade 4 and grade 8.

**Table 9.1**

**Percentage of Students by Teachers' Reports on Emphasis Placed on Number Sense, Properties, and Operations, Grades 4 and 8, 1996**

THE NATION'S REPORT CARD naep

| | A Lot | Some | Little or None |
|---|---|---|---|
| **Grade 4** | | | |
| All Students | 93 | 7 | 0 |
| **Grade 8** | | | |
| All Students | 88 | 10 | 2 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 87 | 11 | 1 |
| Pre-Algebra | 92 | 6 | 1 |
| Algebra | 87 | 10 | 3 |

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### *Measurement*

The data in Table 9.2 show that the modal response from teachers of both fourth- and eighth-grade mathematics was "some" emphasis on the Measurement content strand. Nearly two-thirds of fourth-grade students were being taught mathematics by teachers who reported placing "some" emphasis on this strand and over one-half of eighth-grade students were in classes with "some" emphasis on this strand.

**Table 9.2** ***Percentage of Students by Teachers' Reports on Emphasis Placed on Measurement, Grades 4 and 8, 1996***

THE NATION'S REPORT CARD naep

| | A Lot | Some | Little or None |
|---|---|---|---|
| **Grade 4** | | | |
| All Students | 19 | 64 | 17 |
| **Grade 8** | | | |
| All Students | 19 | 58 | 23 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 22 | 60 | 18 |
| Pre-Algebra | 19 | 57 | 24 |
| Algebra | 16 | 54 | 30 |

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Geometry and Spatial Sense

Data on the curricular emphasis given to Geometry and Spatial Sense are presented in Table 9.3. As with the Measurement content strand, mathematics teachers of the majority of students at grades 4 and 8 reported placing "some" emphasis on Geometry. Fifty-eight percent of fourth-grade students had mathematics teachers who indicated placing "some" emphasis on Geometry, and at the eighth-grade level, 54 percent of students had teachers who placed "some" emphasis on Geometry in their mathematics classes.

**Table 9.3** **Percentage of Students by Teachers' Reports on Emphasis Placed on Geometry and Spatial Sense, Grades 4 and 8, 1996**

THE NATION'S REPORT CARD naep

| | Level of Emphasis | | |
|---|---|---|---|
| | A Lot | Some | Little or None |
| **Grade 4** | | | |
| All Students | 12 | 58 | 30 |
| **Grade 8** | | | |
| All Students | 24 | 54 | 22 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 30 | 50 | 20 |
| Pre-Algebra | 16 | 59 | 25 |
| Algebra | 19 | 56 | 25 |

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### ***Data Analysis, Statistics, and Probability***

Although the Data Analysis, Statistics, and Probability content strand has received substantial attention in mathematics reform at all grade levels in recent years,[3] classroom emphasis placed on this strand appears to be less than the emphasis on Number Sense, Properties, and Operations; Measurement; or Geometry and Spatial Sense. However, the data, which appear in Table 9.4, indicate that there may be somewhat more emphasis at the eighth-grade level than at the fourth-grade level.

In 1996, only eight percent of fourth-grade students were taught mathematics by teachers who reported placing "a lot" of emphasis on Data Analysis, Statistics, and Probability and 41 percent of students had teachers who reported "some" emphasis. At the eighth-grade level, 15 percent of students had teachers of mathematics who reported placing "a lot" of emphasis on this content strand, and 47 percent had teachers who reported "some" emphasis.

**Table 9.4** ***Percentage of Students by Teachers' Reports on Emphasis Placed on Data Analysis, Statistics, and Probability, Grades 4 and 8, 1996***

THE NATION'S REPORT CARD naep

| | Level of Emphasis | | |
|---|---|---|---|
| | A Lot | Some | Little or None |
| **Grade 4** | | | |
| All Students | 8 | 41 | 50 |
| **Grade 8** | | | |
| All Students | 15 | 47 | 38 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 17 | 52 | 31 |
| Pre-Algebra | 12 | 45 | 43 |
| Algebra | 17 | 42 | 41 |

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

[3] National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards*. Reston, VA: Author.

## *Algebra and Functions*

The data on emphasis on Algebra and Functions are presented in Table 9.5. Teachers of eighth-grade students reported placing much more emphasis on this content strand than did teachers of fourth-grade students. In 1996, only nine percent of fourth-grade students had teachers who reported "a lot" of emphasis on Algebra and Functions, while the majority of eighth-grade students (57%) had teachers who indicated "a lot" of emphasis on this content area.

An examination across types of eighth-grade mathematics courses by level of emphasis shows some significant, and perhaps expected, differences. Eighty-five percent of algebra students had teachers who reported "a lot" of emphasis on Algebra and Functions; this percentage was higher than the percentage of pre-algebra students (58%) or the percentage of eighth-grade mathematics students (40%) whose teachers reported placing "a lot" of emphasis on this content strand. Thirteen percent of algebra students were in mathematics classes with "some" emphasis on Algebra and Functions; this percentage was lower than the percentage of pre-algebra students (36%) or the percentage of eighth-grade mathematics students (45%). Finally, 15 percent of students in eighth-grade mathematics had teachers who reported "little or no" emphasis on Algebra and Functions, which was higher than the five percent of pre-algebra students and the two percent of algebra students.

**Table 9.5** ***Percentage of Students by Teachers' Reports on Emphasis Placed on Algebra and Functions, Grades 4 and 8, 1996***

THE NATION'S REPORT CARD naep

| | Level of Emphasis | | |
|---|---|---|---|
| | A Lot | Some | Little or None |
| **Grade 4** | | | |
| All Students | 9 | 30 | 60 |
| **Grade 8** | | | |
| All Students | 57 | 34 | 9 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 40 | 45 | 15 |
| Pre-Algebra | 58 | 36 | 5 |
| Algebra | 85 | 13 | 2 |

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## *Emphasis on Mathematical Processes*

In addition to learning disciplinary content, students are expected to acquire mathematical skills and abilities that cut across content strands. Teachers of mathematics at grades 4 and 8 were asked questions about the extent to which they emphasized the following mathematical processes:

- learning mathematics facts and concepts;
- learning skills and procedures to solve routine problems;
- developing reasoning abilities to solve unique problems; and
- learning how to communicate ideas in mathematics.

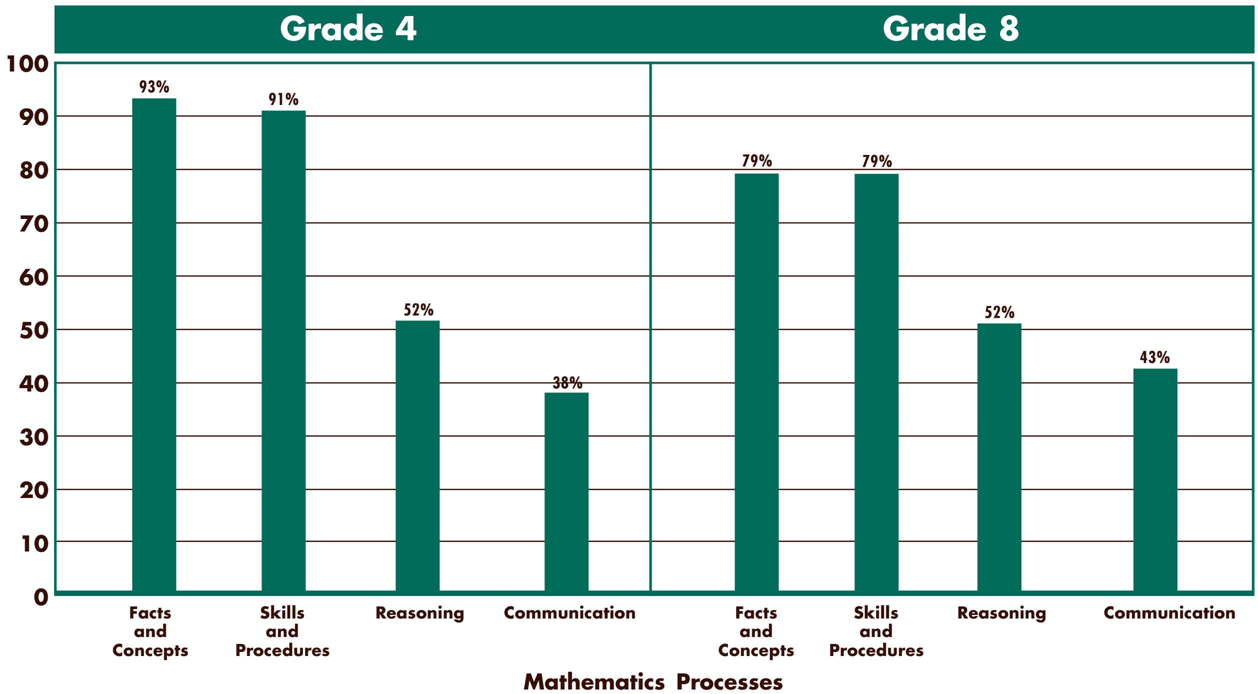
Together, these mathematical skills provide students with the ability to do mathematics successfully. They also reflect the mathematical abilities and the construct of mathematical power described in the NAEP mathematics framework. Figure 9.2 presents data on students whose teachers reported placing "a lot" of emphasis on the different mathematical processes. Tables 9.6 through 9.9 provide more detailed information on teachers' responses regarding the level of emphasis they place on these processes in their mathematics instruction.

The data in Figure 9.2 show that teachers of the majority of fourth- and eighth-grade students reported placing "a lot" of emphasis on learning facts and concepts, learning skills and procedures to solve routine problems, and developing reasoning ability to solve unique mathematics problems. Although fewer students had teachers who reported placing "a lot" of emphasis on communicating ideas in mathematics effectively, over one-third of fourth-grade students and 43 percent of eighth-grade students had such teachers.

As with mathematics content, with the exception of developing reasoning abilities to solve unique problems, the emphasis placed on the different mathematical processes was not found to be related to the mathematics class in which students were enrolled.

**Figure 9.2**

### Percentage of Students Whose Teachers Place "A Lot" of Emphasis on Specific Mathematics Processes by Grade and Mathematics Processes




SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### *Learning mathematics facts and concepts*

To do mathematics successfully, students must have knowledge of basic mathematics facts and a reasonable understanding of different mathematical concepts. The information provided by teachers on learning facts and concepts does not allow us to determine the relative focus on facts compared with concepts. Nevertheless, the data, which are presented in Table 9.6, provide a picture of the importance teachers of fourth- and eighth-grade students appear to place on learning mathematics facts and concepts. In 1996, 93 percent of fourth-grade students and 79 percent of eighth-grade students were taught mathematics by teachers who reported placing "a lot" of emphasis in their mathematics classes on learning mathematics facts and concepts.

**Table 9.6** ***Percentage of Students by Teachers' Reports on Emphasis Placed on Learning Mathematics Facts and Concepts, Grades 4 and 8, 1996***

THE NATION'S REPORT CARD naep

| | Level of Emphasis | | |
|---|---|---|---|
| | A Lot | Some | Little or None |
| **Grade 4** | | | |
| All Students | 93 | 7 | 0! |
| **Grade 8** | | | |
| All Students | 79 | 16 | 5 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 77 | 18 | 5 |
| Pre-Algebra | 82 | 15 | 3 |
| Algebra | 79 | 15 | 6 |

NOTE: Row percentages may not total 100 due to rounding.
! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### *Learning skills and procedures needed to solve routine problems*

Knowing facts and concepts is an essential beginning. To use this knowledge to solve problems, students must acquire procedural knowledge and problem-solving skills.[4] Information from teachers on the emphasis they place on learning skills and procedures to solve routine problems is presented in Table 9.7.

Teachers of both fourth- and eighth-grade students place similar emphasis on learning skills and procedures to solve routine problems as on learning mathematics facts and concepts. In 1996, 91 percent of fourth-grade students were taught mathematics by teachers who reported placing "a lot" of emphasis on learning these skills and procedures, whereas 79 percent of eighth-grade students had such teachers.

**Table 9.7** ***Percentage of Students by Teachers' Reports on Emphasis Placed on Learning Skills and Procedures Needed to Solve Routine Problems, Grades 4 and 8, 1996***

THE NATION'S REPORT CARD naep

| | Level of Emphasis | | |
|---|---|---|---|
| | A Lot | Some | Little or None |
| **Grade 4** | | | |
| All Students | 91 | 8 | 0! |
| **Grade 8** | | | |
| All Students | 79 | 18 | 3 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 80 | 19 | 1 |
| Pre-Algebra | 79 | 18 | 3 |
| Algebra | 78 | 16 | 6 |

NOTE: Row percentages may not total to 100 due to rounding.
! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

[4] Wakefield, A. P. (1997). Supporting math thinking. *Phi Delta Kappan*, *79*(3), 233–236.

### *Developing reasoning ability to solve unique problems*

Doing mathematics successfully means a lot of things. It means being able to follow procedures and solve computational problems, and it means having the ability to solve classes of problems that become relatively routine through repeated exposure. In addition, it means being able to use one's knowledge and reasoning ability to solve mathematical problems in contexts that have not been encountered previously. The NAEP 1996 data on developing reasoning ability suggest that the task of helping students develop these capabilities may be somewhat more difficult to incorporate into mathematics instruction than the tasks of teaching students facts and concepts or how to apply more routine skills and procedures. That is, as the data in Table 9.8 show, compared with the mathematics processes discussed above, fewer students have teachers who reported placing "a lot" of emphasis on developing reasoning abilities. This is true in both fourth- and eighth-grade mathematics classes. Nevertheless, the majority of both fourth- and eighth-grade students still had teachers who reported "a lot" of emphasis on developing students' reasoning ability to solve unique mathematics problems, while most of the remainder had teachers who reported "some" emphasis.

**Table 9.8** **Percentage of Students by Teachers' Reports on Emphasis Placed on Developing Reasoning Ability to Solve Unique Problems, Grades 4 and 8, 1996**

THE NATION'S REPORT CARD naep

| | Level of Emphasis | | |
|---|---|---|---|
| | A Lot | Some | Little or None |
| **Grade 4** | | | |
| All Students | 52 | 41 | 8 |
| **Grade 8** | | | |
| All Students | 52 | 40 | 8 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 42 | 47 | 12 |
| Pre-Algebra | 53 | 41 | 6 |
| Algebra | 68 | 29 | 3 |

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Overall, the reported emphasis at the two grades was nearly identical. That is, 52 percent of fourth-grade students and 52 percent of eighth-grade students were taught mathematics with “a lot” of emphasis on developing reasoning ability to solve unique problems. Forty-one percent of students at grade 4 and 40 percent students at grade 8, were taught by teachers who reported “some” emphasis.

However, there were some differences in the emphasis experienced by eighth-grade students in different mathematics courses. The percentage of algebra students (68%) in classes with “a lot” of emphasis on developing reasoning abilities was higher than the percentage of eighth-grade mathematics students (42%) in such classes. Reciprocally, the percentages of eighth-grade mathematics students in classes with “some” (47%) or “little or no” emphasis (12%) on developing reasoning skills were both higher than the percentages of algebra students in such classes (29% for “some” and 3% for “little or no” emphasis). That is, students perceived to be more advanced mathematically appear to get more exposure to higher level processes.

### *Learning how to communicate ideas in mathematics effectively*

Not only do students need to acquire knowledge and be able to reason and solve problems, but they also need to be able to communicate ideas in mathematics effectively. More and more, NAEP and other mathematics assessments are assessing students’ ability to explain how they solve problems. In addition, in more classrooms, students are being asked to discuss and, either verbally or in writing, to explain solutions to problems. Information about the emphasis teachers place on learning how to communicate ideas in mathematics effectively is presented in Table 9.9.

In 1996, similar percentages of fourth-grade students were taught mathematics by teachers who reported “some” or “a lot” of emphasis on communicating ideas in mathematics — 45 percent and 38 percent, respectively. At the eighth-grade level similar percentages of students also had teachers who reported “a lot” or “some” emphasis on communicating ideas in mathematics — 43 percent and 42 percent, respectively.

## Table 9.9 Percentage of Students by Teachers' Reports on Emphasis Placed on Learning How to Communicate Ideas in Mathematics Effectively, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

| | Level of Emphasis | | |
|---|---|---|---|
| | A Lot | Some | Little or None |
| **Grade 4** | | | |
| All Students | 38 | 45 | 18 |
| **Grade 8** | | | |
| All Students | 43 | 42 | 16 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 41 | 40 | 20 |
| Pre-Algebra | 39 | 47 | 14 |
| Algebra | 50 | 39 | 11 |

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Instructional Practices

Current mathematics reform efforts promote the use of a variety of instructional practices that can help students achieve academically.[5] This section includes teachers' reports on the frequency of use of selected classroom practices at the fourth- and eighth-grade levels and students' reports at the twelfth-grade level. Their responses provide a general picture of some instructional practices that students currently are experiencing in our nation's classrooms.

### Use of manipulatives

Since the mid-1960s, mathematics educators have been promoting the use of manipulative materials to facilitate mathematics learning.[6] Such materials include Cuisenaire™ rods, geometric shapes, geoboards, Base 10 place value blocks, and a host of measuring instruments. Starting with the NAEP 1990 mathematics assessment, students were provided with rulers and protractors for use in some tasks on the assessments. With the 1992 assessment, students also received some geometric shapes to use in responding to questions requiring the analysis of relationships between these simple shapes and more complex shapes that could be formed from the pieces. The 1996 assessment expanded the practice of including manipulative materials. In order for students to use these manipulatives most appropriately and effectively in the NAEP mathematics assessment, they must have had previous experience with them; one of the best ways to provide such exposure is through classroom instruction.

[5] National Council of Teachers of Mathematics (1991). *Professional standards for teaching mathematics.* Reston, VA: Author.

[6] Bohan, H. J., & Shawaker, P. B. *Using manipulatives effectively: A drive down rounding road*. Available on the World Wide Web at: <http://www.enc.org/classroom/lessons/docs04083/4083.htm>.

As part of the NAEP 1996 assessment, teachers of fourth- and eighth-grade students were asked two separate questions about the frequency with which they used specifically named manipulatives in their mathematics instruction. Their responses are presented in Tables 9.10 and 9.11. Twelfth-grade students also were asked about their use of specific manipulatives, chosen to be more appropriate to their grade level. Information from twelfth-grade students who were taking mathematics is presented in Table 9.12. The data appear to show that working with these types of manipulatives is more common at lower grade levels and for lower level mathematics courses taken by eighth-grade students.

**Table 9.10** ***Percentage of Students by Teachers' Reports on Frequency with Which Students Work with Objects Like Rulers, Grades 4 and 8, 1996***

THE NATION'S REPORT CARD naep

| | Frequency | | | |
|---|---|---|---|---|
| | Almost Every Day | Once or Twice a Week | Once or Twice a Month | Never or Hardly Ever |
| **Grade 4** | | | | |
| All Students | 8 | 36 | 51 | 5 |
| **Grade 8** | | | | |
| All Students | 7 | 18 | 53 | 21 |
| Students Enrolled in: | | | | |
| Eighth-Grade Mathematics | 7 | 23 | 56 | 14 |
| Pre-Algebra | 5 | 16 | 58 | 22 |
| Algebra | 9 | 13 | 45 | 33 |

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

In 1996, a substantial portion of fourth- and eighth-grade students were reported to work with objects like rulers in their mathematics classes at least "once or twice a month." Teachers of just over half of fourth-grade students reported using such objects "once or twice a month," while teachers of another third reported using such objects "once or twice a week." At the eighth-grade level, 53 percent of students worked with objects like rulers "once or twice a month," and 18 percent of students worked with them "once or twice a week."

Frequency of use of objects such as rulers differed slightly depending on the mathematics classes in which eighth-grade students were enrolled. Students in eighth-grade mathematics were more likely to use such objects than students in algebra; that is, only 14 percent of students in eighth-grade mathematics "never or hardly ever" used such objects, which was significantly lower than the 33 percent of algebra students who were in mathematics classes in which they "never or hardly ever" used such objects.

In addition to objects such as rulers, teachers were asked about the frequency of use of manipulatives and teaching aids such as counting blocks and geometric shapes. The information about these manipulatives is presented in Table 9.11. The use of these types of manipulatives appears less common than the use of objects such as rulers.[7] In 1996, although teachers of 47 percent of fourth-grade students reported using counting blocks and geometric shapes "once or twice a month," teachers of 26 percent of students reported "never or hardly ever" using such manipulatives.

At the eighth-grade level, the use of these manipulatives appears even less common. The majority of students (54%) had teachers who reported "never or hardly ever" having their students work with counting blocks or geometric shapes.

The frequency of use of counting blocks and geometric shapes differed slightly depending on the type of mathematics class eighth-grade students were taking. Students in eighth-grade mathematics classes were reported to use these types of manipulatives more frequently than those in algebra classes. That is, the percentage of students in algebra classes (66%) whose teachers reported "never or hardly ever" working with counting blocks or geometric shapes was higher than the percentage of students in eighth-grade mathematics classes (44%) whose teachers reported this.

**Table 9.11** ***Percentage of Students by Teachers' Reports on Frequency with Which Students Work with Counting Blocks and Geometric Shapes, Grades 4 and 8, 1996***

THE NATION'S REPORT CARD naep

| | Frequency | | | |
|---|---|---|---|---|
| | Almost Every Day | Once or Twice a Week | Once or Twice a Month | Never or Hardly Ever |
| **Grade 4** | | | | |
| All Students | 5 | 22 | 47 | 26 |
| **Grade 8** | | | | |
| All Students | 1 | 7 | 38 | 54 |
| Students Enrolled in: | | | | |
| Eighth-Grade Mathematics | 1 | 9 | 46 | 44 |
| Pre-Algebra | 1 | 6 | 36 | 58 |
| Algebra | 0! | 5 | 29 | 66 |

NOTE: Row percentages may not total 100 due to rounding.
! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

[7] It is possible that some of this difference could be due to the wording of the two questions. The question about rulers asked about working with "objects like rulers," while the other question only mentioned working with "counting blocks and geometric shapes" rather than, for example, "manipulatives such as counting blocks and geometric shapes."

Twelfth-grade students were asked a single question that combined the use of measuring instruments and geometric solids. Of twelfth-grade students who indicated they were currently taking a mathematics class, the majority (53%) reported "never or hardly ever" working with measuring instruments or geometric solids in their mathematics classes.

**Table 9.12** — ***Percentage of Students by Frequency with Which They Work with Measuring Instruments or Geometric Solids, Grade 12, 1996***

THE NATION'S REPORT CARD naep

| | Frequency | | | |
|---|---|---|---|---|
| | Almost Every Day | Once or Twice a Week | Once or Twice a Month | Never or Hardly Ever |
| **Grade 12** | | | | |
| Students Taking Mathematics | 7 | 14 | 26 | 53 |

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### *Working in small groups or with a partner*

One of the pedagogical strategies recommended to foster increased learning and understanding of mathematics is the use of small-group activities. By working in small groups or with a partner, students are expected to be more actively involved in the learning process, and this is believed to increase student learning.[8] Information about the frequency with which students were reported to work with other students to solve problems is presented in Table 9.13.

Teachers of a large majority of students in both grades 4 and 8 reported that their students worked at least once a week with other students to solve mathematics problems. However, although the percentages were relatively small, seven percent of fourth-grade students and eight percent of eighth-grade students "never or hardly ever" had this opportunity.

Twelfth-grade students enrolled in mathematics reported less frequency of working with other students to solve problems than did teachers of fourth- and eighth-grade students. In 1996, about one in five twelfth-grade students reported "never or hardly ever" working this way, while less than 10 percent of fourth- and eighth-grade students had teachers who reported "never or hardly ever" having their students work with a partner or in small groups.

[8] Lacampagne, C. B. (1993). *State of the art, transforming ideas for teaching and learning mathematics.* Washington, DC: U.S. Department of Education, Office of Educational Research and Improvement; Lotan, R. A., & Benton, J. H. (1990). Finding out about complex instruction: Teaching math and science in heterogeneous classrooms. In N. Davidson (Ed). *Cooperative learning in mathematics*. New York: Addison-Wesley.

## Table 9.13 Percentage of Students by Frequency with Which They Solve Problems in Small Groups or with a Partner, Grades 4, 8, and 12, 1996*

| | Frequency | | | |
|---|---|---|---|---|
| | Almost Every Day | Once or Twice a Week | Once or Twice a Month | Never or Hardly Ever |
| **Grade 4** | | | | |
| All Students | 25 | 50 | 18 | 7 |
| **Grade 8** | | | | |
| All Students | 27 | 40 | 26 | 8 |
| Students Enrolled in: | | | | |
| Eighth-Grade Mathematics | 24 | 44 | 25 | 6 |
| Pre-Algebra | 24 | 39 | 30 | 8 |
| Algebra | 34 | 33 | 25 | 9 |
| **Grade 12** | | | | |
| Students Taking Mathematics | 26 | 32 | 21 | 21 |

NOTE: Row percentages may not total 100 due to rounding.
* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

### Writing in mathematics and reports/projects

Writing across the curriculum is one of the current instructional strategies being advocated by many educators to increase student learning and communication skills.[9] Writing in mathematics not only helps students improve their language arts skills, but also places an expectation on them to be able to communicate mathematical thinking and understanding to others. Over the years, NAEP assessments have presented students with increasing numbers of questions that require them to write out responses and, often, to explain their answers in writing. Students have typically found these questions more challenging than multiple-choice questions.[10] However, it is reasonable to assume that if students are not being exposed to content or processes that are assessed by NAEP, they cannot be expected to answer those questions correctly. Therefore, whether students, in fact, are writing more in their mathematics classes is of interest to interpreters of NAEP assessment results, as well as to mathematics educators more generally. Information about the frequency with which students were reportedly asked to write a few sentences or to write larger reports in mathematics classes is presented in Tables 9.14 and 9.15.

[9] Miller, L. D. (1991). Writing to learn mathematics. *Mathematics Teacher*, *84*(7), 516–521.

[10] Dossey, J. A., Mullis, I. V. S., & Jones, C. O. (1993). *Can students do mathematical problem solving?* Washington, DC: National Center for Education Statistics; Hawkins, E., Stancavage, F., Mitchell, J., Goodman, M., & Lazer, S. (1998). *Learning about our world and our past: Using the tools and resources of geography and U.S. history–A report of the 1994 NAEP assessment.* Washington, DC: National Center for Education Statistics.

In 1996, the majority of fourth-grade students had teachers who indicated that students wrote a few sentences about how to solve a mathematics problem "once or twice a month" or less. However, the percentages of students who were asked to write about solving problems "almost every day" or "once or twice a week" in 1996 were higher than the percentages in 1992, and the percentage of students who "never or hardly ever" wrote about solving problems decreased from 1992 to 1996.

The frequency with which eighth-grade students wrote about solving mathematics problems in 1996 appeared to be similar to that of fourth-grade students. However, percentage changes from 1992 to 1996 for eighth-grade students, although in the same direction as the changes for fourth-grade students, were not statistically significant.

The majority (61%) of twelfth-grade students taking mathematics reported that they "never or hardly ever" wrote a few sentences about how to solve a mathematics problem.

**Table 9.14** **Percentage of Students by Frequency with Which They Write a Few Sentences about How to Solve a Mathematics Problem, Grades 4, 8, and 12***

THE NATION'S REPORT CARD naep

| | Assessment Year | Frequency | | | |
|---|---|---|---|---|---|
| | | Almost Every Day | Once or Twice a Week | Once or Twice a Month | Never or Hardly Ever |
| **Grade 4** | | | | | |
| All Students | 1996 | 9† | 26† | 36 | 29† |
| | 1992 | 2 | 17 | 36 | 45 |
| **Grade 8** | | | | | |
| All Students | 1996 | 5 | 25 | 37 | 33 |
| | 1992 | 3 | 18 | 37 | 41 |
| Students Enrolled in: | | | | | |
| Eighth-Grade Mathematics | 1996 | 4 | 27 | 35 | 34 |
| | 1992 | 2 | 16 | 38 | 44 |
| Pre-Algebra | 1996 | 4 | 27 | 37 | 33 |
| | 1992 | 2 | 18 | 42 | 37 |
| Algebra | 1996 | 5 | 20 | 39 | 36 |
| | 1992 | 7 | 24 | 31 | 38 |
| **Grade 12** | | | | | |
| Students Taking Mathematics | 1996 | 7 | 13 | 18 | 61 |

NOTE: Row percentages may not total 100 due to rounding.
† Significantly different from 1992.
* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

In 1996, fewer students were reported to be writing reports or doing mathematics projects than were reported to be writing a few sentences about how to solve a mathematics problem. As shown in Table 9.15, teachers of 66 percent of fourth-grade students and 64 percent of eighth-grade students reported "never or hardly ever" asking their students to write reports or do projects in their mathematics classes. Most of the remaining students had teachers who reported assigning reports or projects "once or twice a month." Responses indicating daily or weekly frequency were quite uncommon, but this may reflect the fact that such assignments, by their nature, have longer time spans associated with them and so would be less frequently assigned. At the twelfth-grade level, 71 percent of students taking mathematics reported that they "never or hardly ever" wrote reports or did mathematics projects.

**Table 9.15** ***Percentage of Students by Frequency with Which They Write Reports or Do Mathematics Projects, Grades 4, 8, and 12****

THE NATION'S REPORT CARD naep

| | Assessment Year | Frequency: Almost Every Day | Once or Twice a Week | Once or Twice a Month | Never or Hardly Ever |
|---|---|---|---|---|---|
| **Grade 4** | | | | | |
| All Students | 1996 | 1 | 4† | 29† | 66† |
| | 1992 | 0! | 1 | 17 | 82 |
| **Grade 8** | | | | | |
| All Students | 1996 | 0! | 3 | 33† | 64† |
| | 1992 | 0! | 1 | 21 | 78 |
| Students Enrolled in: | | | | | |
| Eighth-Grade Mathematics | 1996 | 0! | 3 | 35 | 63 |
| | 1992 | 0! | 0 | 23 | 76 |
| Pre-Algebra | 1996 | 0! | 5 | 34 | 61† |
| | 1992 | 0! | 1 | 20 | 79 |
| Algebra | 1996 | 0! | 2 | 30† | 68† |
| | 1992 | 0! | 1 | 16 | 83 |
| **Grade 12** | | | | | |
| Students Taking Mathematics | 1996 | 2 | 4 | 24 | 71 |

NOTE: Row percentages may not total 100 due to rounding.
† Significantly different from 1992.
* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.
! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

Changes in percentages over time appear to show increases in the frequency with which the practices of writing reports or doing mathematics projects are being implemented in mathematics classrooms. For example, the percentages of fourth-grade students in 1996 whose teachers reported having students write reports or do projects “once or twice a week” or “once or twice a month” were both higher than the percentages at those frequencies in 1992. In addition, the percentage of fourth-grade students who “never or hardly ever” wrote reports or did projects in 1996 was 66 percent, which was significantly lower than the 82 percent in 1992.

At the eighth-grade level, the percentage of students (33%) who were reported to be writing reports and doing projects “once or twice a month” in 1996 was higher than the percentage of students (21%) doing so in 1992. Additionally, in 1996, the percentage of students (64%) who “never or hardly ever” wrote reports or did projects was lower than the percentage of students (78%) in this category in 1992. When eighth-grade students were grouped by type of mathematics course, changes over time also were apparent. For students in pre-algebra and algebra, the 1996 percentages of students who “never or hardly ever” wrote reports or did projects were lower than the 1992 percentages. For algebra students, the 1996 percentage of students who wrote reports and did projects “once or twice a month” was higher than the 1992 percentage.

### *Communicating and connecting mathematics*

To reflect what is happening in mathematics classrooms across the nation, NAEP has attempted to develop an assessment that presents students with questions that represent real-life problems and require students to use their abilities to communicate mathematically. Information in Tables 9.16 and 9.17 shows that in 1996, substantial proportions of fourth-, eighth-, and twelfth-grade students were regularly involved in discussing solutions to mathematics problems with other students. Similarly large proportions of fourth- and eighth-grade students were working or discussing mathematics problems that reflected real-life situations. On average for the different grade levels, the frequency with which students were engaged in these practices had not changed significantly from 1992 to 1996, except for eighth-grade students in the less advanced mathematics classes, where there were indications of increased frequency.

As shown in Table 9.16, in 1996, over one-third of fourth-grade students and almost half of eighth-grade students were being taught mathematics by teachers who reported that their students had discussions with other students about mathematics solutions “almost every day.” Similarly, almost half of twelfth-grade students taking mathematics also reported that they discuss mathematics solutions with other students “almost every day.”

Between 1992 and 1996, for most of the response categories, the frequency with which fourth- and eighth-grade students were reported to discuss mathematics solutions with other students did not change significantly. However, for students in eighth-grade mathematics, the numbers suggest an upward trend, and the percentage in 1996 who “never or hardly ever” had such discussions was lower than the 1992 percentage. Also, for students in pre-algebra, the 1996 percentage of students who had such discussions “once or twice a month” was higher than the 1992 percentage.

**Table 9.16**

## Percentage of Students by Frequency with Which They Discuss Solutions to Mathematics Problems with Other Students, Grades 4, 8, and 12*

THE NATION'S REPORT CARD naep

| | Assessment Year | Frequency: Almost Every Day | Once or Twice a Week | Once or Twice a Month | Never or Hardly Ever |
|---|---|---|---|---|---|
| **Grade 4** | | | | | |
| All Students | 1996 | 35 | 37 | 22 | 6 |
| | 1992 | 33 | 39 | 22 | 6 |
| **Grade 8** | | | | | |
| All Students | 1996 | 49 | 3 | 37 | 2 |
| | 1992 | 43 | 1 | 32 | |
| Students Enrolled in: | | | | | |
| Eighth-Grade Mathematics | 1996 | 44 | 3 | 39 | 2† |
| | 1992 | 37 | 0 | 33 | 9 |
| Pre-Algebra | 1996 | 52 | 5 | 37 | 2 |
| | 1992 | 44 | 1 | 33 | 4 |
| Algebra | 1996 | 54 | 2 | 32 | 1 |
| | 1992 | 58 | 1 | 29 | 2 |
| **Grade 12** | | | | | |
| Students Taking Mathematics | 1996 | 48 | 28 | 11 | 14 |

NOTE: Row percentages may not total 100 due to rounding.
† Significantly different from 1992.
* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

It is important that students are able to apply the mathematics they learn in the classroom to solve real-life problems. And solving mathematics problems that reflect real-life situations in the classroom can facilitate mathematics learning and understanding.[11] In 1996, substantial proportions of students from grades 4 and 8 were working and discussing mathematics that reflected real-life situations at least "once or twice a week." Teachers of 29 percent of fourth-grade students reported that their students did this "almost every day," while teachers of 45 percent reported that their students did this "once or twice a week."

[11] National Council of Teachers of Mathematics. (1989). op. cit.; Usiskin, Z. (1993). Lessons from the Chicago mathematics project. *Educational Leadership*, *50*, 14–18.

The percentages were similar for eighth-grade students: teachers of 27 percent reported that students worked and discussed mathematics problems that reflected real-life situations "almost every day," and teachers of 47 percent reported working and discussing these types of problems "once or twice a week."

**Table 9.17**

**Percentage of Students by Teachers' Reports on Frequency with Which Students Work and Discuss Mathematics Problems That Reflect Real-Life Situations, Grades 4 and 8**

THE NATION'S REPORT CARD naep

| | Assessment Year | Frequency: Almost Every Day | Once or Twice a Week | Once or Twice a Month | Never or Hardly Ever |
|---|---|---|---|---|---|
| **Grade 4** | | | | | |
| All Students | 1996 | 29 | 45 | 23 | 4 |
| | 1992 | 26 | 48 | 23 | 4 |
| **Grade 8** | | | | | |
| All Students | 1996 | 27 | 47 | 22 | 4 |
| | 1992 | 19 | 51 | 24 | 6 |
| Students Enrolled in: | | | | | |
| Eighth-Grade Mathematics | 1996 | 26 | 48 | 23 | 4 |
| | 1992 | 19 | 51 | 25 | 5 |
| Pre-Algebra | 1996 | 28 | 48 | 21 | 3 |
| | 1992 | 19 | 52 | 24 | 5 |
| Algebra | 1996 | 28 | 45 | 22 | 5 |
| | 1992 | 20 | 53 | 19 | 8 |

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

## Calculator Use

The use of calculators as an instructional strategy is being highlighted because of the emphasis placed on appropriate use of calculators in mathematics education by NCTM curriculum and evaluation standards as well as other mathematics reform efforts. The increasing accessibility to a variety of calculators suggests an expectation that students have the ability to use them appropriately in the workplace and in everyday life. Although there is concern that wider use of calculators in mathematics instruction may interfere with students' mastery of basic skills in mathematics, there is research that shows that the proper use of calculators can enhance learning at all stages.[12] Furthermore, the NAEP 1996 mathematics framework recommends the inclusion of more mathematics questions that require the use of a calculator for successful completion of those questions.[13]

In the NAEP 1996 and 1992 assessments, teachers and students were asked about their use of calculators for schoolwork and on mathematics tests. Their responses are reported in this section. We also report findings in this section regarding the extent to which students used calculators appropriately in the 1996 assessment. The basis for the latter data is as follows: the assessment was subdivided into separately timed sections, or "blocks," and students were allowed to use calculators on some of these blocks. When students were allowed to use calculators, they also were asked to indicate if, in fact, they had used a calculator for each question. Each of the questions was in turn identified as to whether the use of a calculator to solve the question was warranted. That is, each question was characterized as: (a) calculator neither required nor useful, (b) calculator not required but some students might choose to use it; and (c) calculator required. By combining these two types of information, it is possible to examine data on the extent to which students used the calculators appropriately during the assessment.

### *Students' access to calculators*

Increasing student use of calculators in mathematics assessment is most appropriate when all students have access to calculators for instruction. In 1996, teachers of 80 percent of fourth-grade students and 80 percent of eighth-grade students reported that their students had access to school-owned calculators to do their school work, and 95 percent of twelfth-grade students taking mathematics reported having a calculator available to do mathematics schoolwork.[14] In 1996, teachers of fourth- and eighth-grade students also were asked about the frequency with which they used calculators in their mathematics classes. As the data in Table 9.18 show, teachers of eighth-grade students reported much greater frequency of calculator use than teachers of fourth-grade students.

Teachers of 68 percent of fourth-grade students reported that their students used calculators in class "once or twice a month" or less. In contrast, 76 percent of eighth-grade students had teachers who reported that they used calculators at least "once or twice a week." Comparisons of percentages of eighth-grade students by mathematics class show that the percentage of algebra students (68%) whose teachers reported use of calculators "almost every day" was higher than the percentage of eighth-grade mathematics students (48%) whose teachers reported similar usage.

In 1996, over three-fourths of twelfth-grade students taking mathematics indicated that they used calculators for class work in mathematics "almost every day," and 14 percent reported using them "once or twice a week."

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[12] Lacampagne, C. B. (1993). op. cit.

[13] National Assessment Governing Board. (1996). op.cit.

[14] The source of these data is the NAEP 1996 mathematics assessment.

Data over time appear to show increased frequency of use of calculators; this is true for all students at grades 4 and 8 as well as for eighth-grade students in each of the three different types of mathematics classes. For fourth-grade students, the 1996 percentages reported to be using calculators either "almost every day," "once or twice a week," or "once or twice a month" were all higher than the corresponding 1992 percentages. In addition, the 1996 percentage of fourth-grade students whose teachers reported "never or hardly ever" using calculators was lower than the 1992 percentage.

For eighth-grade students overall, the 1996 percentage who used calculators "almost every day" was higher than the 1992 percentage, and the 1996 percentage who used calculators "never or hardly ever" was lower than the 1992 percentage. This same pattern held true for students in eighth-grade mathematics, pre-algebra, and algebra.

**Table 9.18** ***Percentage of Students by Frequency with Which Students Use Calculators in Class, Grades 4, 8, and 12****

THE NATION'S REPORT CARD naep

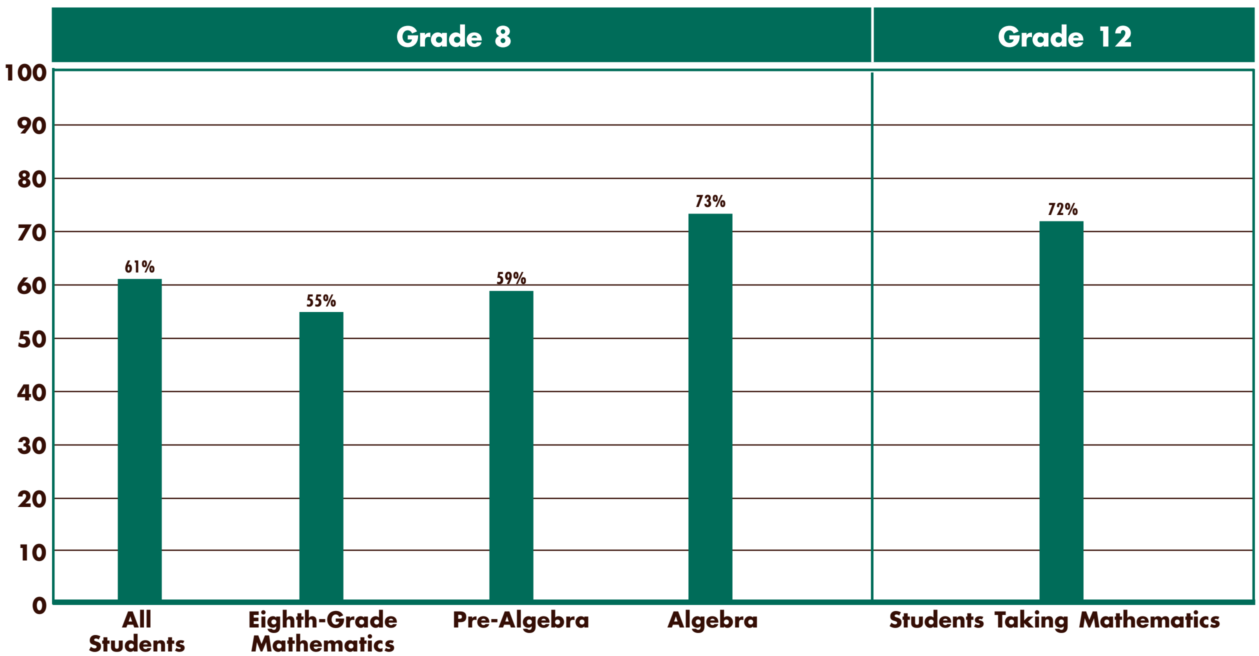
| | | Frequency | | | |
|---|---|---|---|---|---|
| | Assessment Year | Almost Every Day | Once or Twice a Week | Once or Twice a Month | Never or Hardly Ever |
| **Grade 4** | | | | | |
| All Students | 1996 | 5† | 28† | 42† | 26† |
| | 1992 | 1 | 15 | 32 | 51 |
| **Grade 8** | | | | | |
| All Students | 1996 | 55† | 21 | 14 | 9† |
| | 1992 | 34 | 22 | 21 | 24 |
| Students Enrolled in: | | | | | |
| Eighth-Grade Mathematics | 1996 | 48† | 24 | 16 | 12† |
| | 1992 | 27 | 25 | 24 | 24 |
| Pre-Algebra | 1996 | 57† | 20 | 16 | 7† |
| | 1992 | 36 | 21 | 18 | 25 |
| Algebra | 1996 | 68† | 16 | 9 | 7† |
| | 1992 | 49 | 18 | 13 | 19 |
| **Grade 12** | | | | | |
| Students Taking Mathematics | 1996 | 78 | 14 | 3 | 5 |

NOTE: Row percentages may not total 100 due to rounding.
† Significantly different from 1992.
* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

Students in grades 8 and 12 were asked whether they use *scientific or graphing calculators* for their mathematics schoolwork, and data on their responses are presented in Figures 9.3 and 9.4. As perhaps expected, higher percentages of both eighth-grade students and twelfth-grade students taking mathematics reported using scientific calculators than reported using graphing calculators. In addition, the percentage of twelfth-grade students taking mathematics who use scientific calculators was higher than the percentage of eighth-grade students overall who do so; this also was true for the use of graphing calculators. At grade 8, the percentages of algebra students who indicated using scientific and graphing calculators were higher than the percentages of pre-algebra or eighth-grade mathematics students who reported using them.

**Figure 9.3** ***Percentage of Students Who Report Using Scientific Calculators, Grades 8 and 12, 1996***

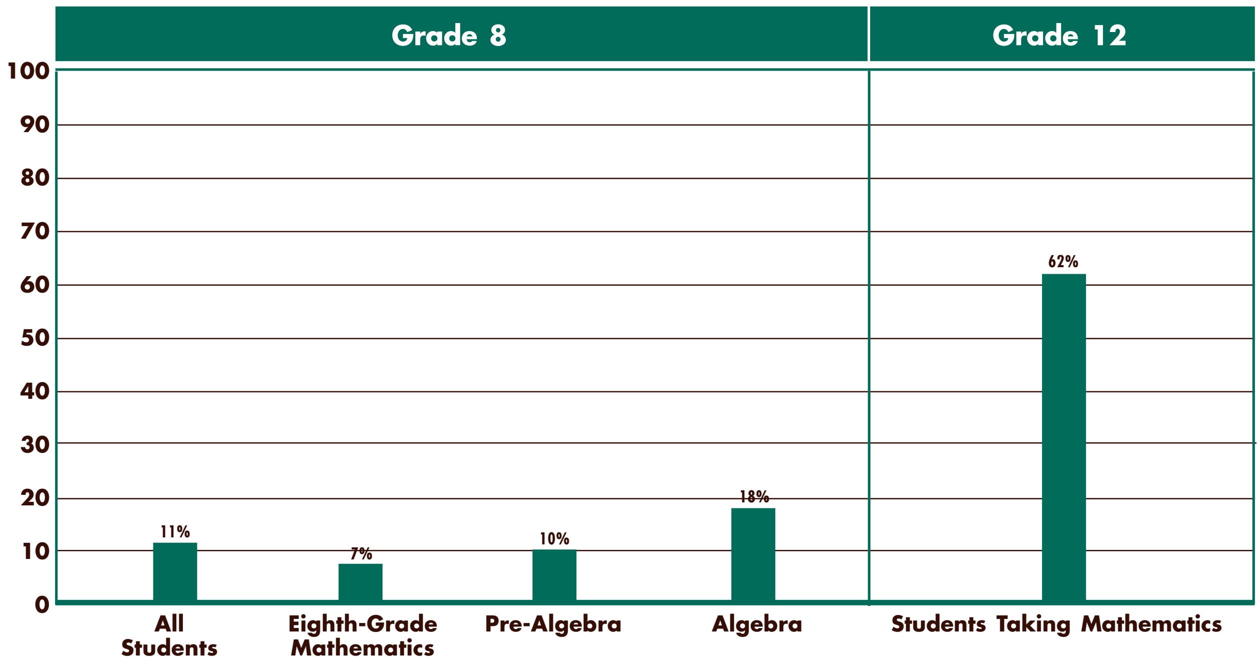
THE NATION'S REPORT CARD naep




SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

**Figure 9.4**

## Percentage of Students Who Report Using Graphing Calulators, Grades 8 and 12, 1996




SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

## ***Policies for using calculators in mathematics class***

Classroom policies regarding the use of calculators can help students learn to use them appropriately and effectively.[15] In NAEP assessments, teachers of mathematics were asked if they allowed unrestricted use of calculators in their classes and also whether they allowed calculators on mathematics tests. Information based on their responses is provided in Table 9.19.

In 1996, 13 percent of fourth-grade students had teachers who reported that they allowed unrestricted use of calculators, and 10 percent of fourth-grade students had teachers who reported that they allowed calculators to be used on mathematics tests. A higher percentage of eighth-grade (47%) than fourth-grade students was allowed unrestricted use of calculators in mathematics classes, and a higher percentage (67%) also was allowed to use calculators on mathematics tests. Higher percentages of students taking algebra than students taking eighth-grade mathematics or pre-algebra had teachers who reported allowing unrestricted use of calculators and allowing calculators to be used on mathematics tests.

Between 1992 and 1996, there appears to have been an increase in the percentage of students being allowed unrestricted classroom use of calculators and use of calculators on mathematics tests. At the fourth-grade level, there were increases in both practices. This also was true for eighth-grade students and students taking eighth-grade mathematics. The differences between 1992 and 1996 for students in pre-algebra and algebra classes were significant only for the percentages being permitted to use calculators on mathematics tests.

**Table 9.19** ***Percentage of Students by Teacher Reported Uses of Calculators, Grades 4 and 8***

THE NATION'S REPORT CARD naep

| | Assessment Year | Teachers Allow Unrestricted Use in Classroom | Teachers Allow Use on Mathematics Tests |
|---|---|---|---|
| **Grade 4** | | | |
| All Students | 1996 | 13† | 10† |
| | 1992 | 5 | 5 |
| **Grade 8** | | | |
| All Students | 1996 | 47† | 67† |
| | 1992 | 30 | 48 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 1996 | 42† | 62† |
| | 1992 | 23 | 43 |
| Pre-Algebra | 1996 | 42 | 66† |
| | 1992 | 28 | 45 |
| Algebra | 1996 | 62 | 79† |
| | 1992 | 50 | 65 |

† Significantly different from 1992.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

[15] National Council of Teachers of Mathematics (1991, February). *Calculators and the education of youth.* NCTM Position Statement. Reston, VA: Author.

As noted earlier, several blocks of questions in the NAEP 1996 assessment allowed students to use calculators. For these questions, students were asked to indicate if, in fact, they used a calculator in solving the problem or not. Students' responses were used in conjunction with information on whether or not the question was calculator-appropriate to categorize students into two groups: an "Appropriate calculator use" group and an "Other" group. Students in the "Appropriate calculator use" group used the calculator for at least 65 percent of the calculator-suitable questions and for no more than one of the calculator-unsuitable questions. Students in the "Other" group used the calculator for less than 65 percent of the calculator-suitable questions and/or for more than one of the calculator-unsuitable questions. Information on calculator use by different instructional practices is presented in Table 9.20. Student mathematics performance information also is presented in the table.

## Table 9.20

### *Percentage of Students by Calculator Use, Grades 4, 8, and 12, 1996*

THE NATION'S REPORT CARD naep

| | Calculator Use | | | |
|---|---|---|---|---|
| | Appropriate Calculator Use Group | | Other Group | |
| | Percentage of Students | Average Scale Score | Percentage of Students | Average Scale Score |
| **Grade 4** | | | | |
| All Students | 21 | 221 | 79 | 224 |
| Unrestricted Classroom Use | 19 | 217 | 81 | 226 |
| Restricted Classroom Use | 21 | 222 | 79 | 224 |
| Allowed Use on Classroom Tests | 16 | 224 | 84 | 225 |
| Not Allowed Use on Classroom Tests | 21 | 221 | 79 | 225 |
| **Grade 8** | | | | |
| All Students | 20 | 285 | 80 | 269 |
| Unrestricted Classroom Use | 24 | 293 | 76 | 277 |
| Restricted Classroom Use | 17 | 278 | 83 | 265 |
| Allowed Use on Classroom Tests | 22 | 292 | 78 | 276 |
| Not Allowed Use on Classroom Tests | 17 | 271 | 83 | 261 |
| **Grade 12** | | | | |
| All Students | 27 | 318 | 73 | 299 |
| Use in Classwork: | | | | |
| Almost Every Day | 32 | 321 | 68 | 304 |
| Once or Twice a Week | 22 | 317 | 78 | 296 |
| Once or Twice a Month | 13 | *** | 87 | 286 |
| Never or Hardly Ever | 16 | 293 | 84 | 285 |
| Use on Tests or Quizzes: | | | | |
| Almost Every Day | 34 | 323 | 66 | 309 |
| Once or Twice a Week | 26 | 318 | 74 | 297 |
| Once or Twice a Month | 28 | 322 | 72 | 300 |
| Never or Hardly Ever | 16 | 292 | 84 | 285 |

NOTE: Students in the "Appropriate Calculator Use" group used the calculator for at least 65 percent of the calculator-suitable questions and for no more than one of the calculator-unsuitable questions. Students in the "Other" group used the calculator for less than 65 percent of the calculator-suitable questions and/or used it for more than one of the calculator-unsuitable questions.
NOTE: Row percentages may not total 100 due to rounding.
*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

In the NAEP 1996 mathematics assessment, 21 percent of fourth-grade students, 20 percent of eighth-grade students, and 27 percent of twelfth-grade students used the calculator appropriately, as we have defined appropriate use. The average mathematics scale score for fourth-grade students who used the calculator appropriately was similar to the average scale score of students who did not. However, at the eighth- and twelfth-grade levels, students who appropriately used calculators outperformed students who did not.

At the fourth-grade level, the appropriateness of students' use of calculators on the assessment was not related to whether they were allowed unrestricted use of calculators in the classroom or whether they were allowed to use calculators on classroom tests. On the other hand, at the eighth-grade level, students in classrooms that allowed unrestricted use of calculators were more likely than others to use calculators on NAEP appropriately. Furthermore, students in classrooms that allowed unrestricted use outperformed students in classrooms that did not allow unrestricted use. The findings with regard to use of calculators on classroom tests were similar. That is, the percentage of students who used calculators appropriately on NAEP was higher in classrooms where calculators were used on mathematics tests. In addition, students from classrooms in which calculators were used on classroom tests performed better on the 1996 mathematics NAEP than did students from classrooms in which calculators were not used on tests.

At the twelfth-grade level, it appeared that the more often students used calculators for class work and on classroom tests, the more likely they were to be appropriate users of calculators on the 1996 mathematics assessment. For example, the percentage who applied calculators appropriately on NAEP was higher among those who used calculators for class work "almost every day" than among those who used calculators for class work less often. Additionally, appropriate usage was more frequent among students who reported using calculators "once or twice a week" than among those who reported using calculators "once or twice a month." Among students who were able to use the calculator appropriately on NAEP, twelfth-grade students who reported "never or hardly ever" using calculators in the classroom performed lower on the NAEP 1996 assessment than students in other frequency-of-use groups.

In terms of frequency of use on classroom tests, the percentage of twelfth-grade students who used calculators appropriately on NAEP was higher among those who reported "almost every day" use of calculators on tests than among those who reported using calculators on tests less frequently.

## Assessment Methods

The dialogue about assessment of students' academic achievement in mathematics continues to be an important one.[16] Most of the arguments focus on the inadequacies and inappropriateness of the format of assessment questions. For example, opponents of multiple-choice questions

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[16] Cain, R. W., & Kenney, P. A. (1992). A joint vision for classroom assessment. *Mathematics Teacher*, *85*(8), 612–615; Herman, J. L. (1997). *Large-scale assessment in support of school reform: Lessons in search of alternative measures.* Los Angeles: National Center for Research on Evaluation, Standards, and Student Testing; Glaser, R., & Silver, E. (1994). *Assessment, testing and instruction: Retrospective and prospect.* Los Angeles: National Center for Research on Evolution, Standards, and Student Testing; Romberg, T. A. (Ed.) (1995). *Reform in school mathematics and authentic assessment.* Albany, NY: State University of New York Press.

argue that these questions do not often provide students with the opportunity to show all that they know, and encourage movement to alternative methods of assessment such as performance-based assessments or project-based assessments.

In addition to arguments about the *validity* of current assessment formats, the education community has debated the *usefulness* of different forms of assessments for informing teachers and students about how to improve their teaching and learning. This section includes information from teachers' reports on the frequency with which they assess students and use different forms of assessment in mathematics.

In 1996, as shown in Table 9.21, the teachers of 64 percent of fourth-grade students reported that they gave mathematics tests "once or twice a month," and teachers of 32 percent of fourth-grade students reported that they gave mathematics tests "once or twice a week." At the eighth-grade level, the frequency of weekly tests increased somewhat, with 55 percent of students reportedly given tests "once or twice a month" and 45 percent reportedly given tests "once or twice a week."

Forty-one percent of twelfth-grade students who were taking mathematics reported that they took mathematics tests "once or twice a month," and 54 percent reported that they took mathematics tests "once or twice a week." Twelfth-grade students in mathematics reported taking mathematics tests with greater frequency than reported by teachers of eighth-grade students.

**Table 9.21** ***Percentage of Students by Frequency with Which Students Take Mathematics Tests, Grades 4, 8, and 12, 1996****

THE NATION'S REPORT CARD naep

| | Frequency | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Almost Every Day | | Once or Twice a Week | | Once or Twice a Month | | Never or Hardly Ever | |
| | Percentage of Students | Average Scale Score | Percentage of Students | Average Scale Score | Percentage of Students | Average Scale Score | Percentage of Students | Average Scale Score |
| **Grade 4** | | | | | | | | |
| All Students | 1 | 214 | 32 | 221 | 64 | 226 | 4 | 230 |
| **Grade 8** | | | | | | | | |
| All Students | 1 | *** | 45 | 273 | 55 | 275 | 0 | *** |
| Students Enrolled in: | | | | | | | | |
| Eighth-Grade Mathematics | 0 | *** | 43 | 261 | 57 | 266 | 0 | *** |
| Pre-Algebra | 0 | *** | 47 | 272 | 53 | 271 | 0 | *** |
| Algebra | 1 | *** | 46 | 295 | 53 | 300 | 0 | *** |
| **Grade 12** | | | | | | | | |
| Students Taking Mathematics | 4 | 297 | 54 | 308 | 41 | 318 | 2 | *** |

NOTE: Row percentages may not total 100 due to rounding.
*** Sample size is not sufficient to permit a reliable estimate.
* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Tables 9.22 through 9.25 provide information from fourth- and eighth-grade teachers on the types of assessments they used to assess students' progress. In 1996, teachers appeared to be responding to mathematics reform calls for less multiple-choice testing and more constructed-response testing. Teachers of nearly one-third of fourth-grade students reported that they "never or hardly ever" used multiple-choice tests to assess their students' progress in mathematics, although the modal response was to report using such tests "once or twice a month" (reported by teachers of 42 percent of grade 4 students). At the eighth-grade level, there was greater variability in the reported use of multiple-choice tests. Teachers of just over one-third of eighth-grade students indicated that they "never or hardly ever" used multiple-choice tests, 31 percent of students had teachers who indicated using multiple-choice tests "once or twice a year," and another 31 percent of students had teachers who indicated using such tests "once or twice a month."

**Table 9.22** ***Percentage of Students by Teachers' Reports on the Frequency with Which They Use Multiple-Choice Tests to Assess Their Students' Progress in Mathematics, Grades 4 and 8, 1996***

THE NATION'S REPORT CARD naep

| | Frequency | | | |
|---|---|---|---|---|
| | Once or Twice a Week | Once or Twice a Month | Once or Twice a Year | Never or Hardly Ever |
| **Grade 4** | | | | |
| All Students | 6 | 42 | 20 | 32 |
| **Grade 8** | | | | |
| All Students | 3 | 31 | 31 | 34 |
| Students Enrolled in: | | | | |
| Eighth-Grade Mathematics | 4 | 35 | 26 | 35 |
| Pre-Algebra | 3 | 30 | 36 | 32 |
| Algebra | 2 | 28 | 35 | 35 |

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

In 1996, as shown in Table 9.23, 26 percent of fourth-grade students were taught by teachers who indicated that they used short and long written responses to assess students' progress in mathematics "once or twice a week," and 36 percent of students had teachers who reported using written responses to assess progress "once or twice a month." The pattern of percentages was only slightly different for eighth-grade students.

**Table 9.23** **Percentage of Students by Teachers' Reports on the Frequency with Which They Use Short and Long Written Responses to Assess Their Students' Progress in Mathematics, Grades 4 and 8, 1996**

THE NATION'S REPORT CARD naep

| | Frequency | | | |
|---|---|---|---|---|
| | Once or Twice a Week | Once or Twice a Month | Once or Twice a Year | Never or Hardly Ever |
| **Grade 4** | | | | |
| All Students | 26 | 36 | 19 | 18 |
| **Grade 8** | | | | |
| All Students | 17 | 41 | 21 | 21 |
| Students Enrolled in: | | | | |
| Eighth-Grade Mathematics | 17 | 40 | 20 | 22 |
| Pre-Algebra | 20 | 44 | 18 | 19 |
| Algebra | 14 | 35 | 28 | 23 |

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

As the data in Table 9.24 show, the use of individual or group projects or presentations for assessment appears less common than the use of short or long written responses. In 1996, teachers of over half of fourth-grade students indicated using projects or presentations only "once or twice a year" or less. The percentage of eighth-grade students whose teachers reported very limited use was even higher: 66 percent of eighth-grade students had teachers who indicated using such methods to assess students' progress in mathematics only "once or twice a year" or less frequently.

**Table 9.24** ***Percentage of Students by Teachers' Reports on the Frequency with Which They Use Individual or Group Projects or Presentations to Assess Their Students' Progress in Mathematics, Grades 4 and 8, 1996***

THE NATION'S REPORT CARD naep

| | Frequency | | | |
|---|---|---|---|---|
| | Once or Twice a Week | Once or Twice a Month | Once or Twice a Year | Never or Hardly Ever |
| **Grade 4** | | | | |
| All Students | 16 | 30 | 31 | 24 |
| **Grade 8** | | | | |
| All Students | 7 | 27 | 43 | 23 |
| Students Enrolled in: | | | | |
| Eighth-Grade Mathematics | 7 | 29 | 44 | 19 |
| Pre-Algebra | 8 | 26 | 42 | 23 |
| Algebra | 6 | 25 | 38 | 30 |

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The data in Table 9.25 show that, in 1996, the use of portfolios appeared to be more frequent at the fourth-grade level than at the eighth-grade level. Forty-five percent of fourth-grade students had teachers who reported using portfolios for assessing students' progress in mathematics "once or twice a month" or more often, whereas 29 percent of eighth-grade students had teachers who used portfolios at least "once or twice a month."

**Table 9.25** ***Percentage of Students by Teachers' Reports on the Frequency with Which They Use Portfolio Collections of Each Student's Work to Assess Students' Progress in Mathematics, Grades 4 and 8, 1996***

THE NATION'S REPORT CARD naep

| | Frequency | | | |
|---|---|---|---|---|
| | Once or Twice a Week | Once or Twice a Month | Once or Twice a Year | Never or Hardly Ever |
| **Grade 4** | | | | |
| All Students | 15 | 30 | 17 | 39 |
| **Grade 8** | | | | |
| All Students | 10 | 19 | 21 | 50 |
| Students Enrolled in: | | | | |
| Eighth-Grade Mathematics | 10 | 17 | 23 | 50 |
| Pre-Algebra | 11 | 19 | 18 | 51 |
| Algebra | 10 | 20 | 21 | 50 |

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Summary

This chapter provided a picture of the instructional practices students in grades 4, 8, and 12 were experiencing in 1996 in their mathematics classrooms. In terms of *disciplinary content*, the majority of fourth- and eighth-grade students were receiving mathematics instruction with "a lot" of emphasis on Number Sense, Properties, and Operations and "some" emphasis on Measurement and Geometry and Spatial Sense. Higher percentages of eighth-grade students compared with fourth-grade students had mathematics instruction with somewhat more emphasis on Data Analysis, Statistics, and Probability, and Algebra and Functions. Except for Algebra and Functions, eighth-grade students in different types of mathematics classes were not experiencing differing levels of emphasis on the different content strands. For Algebra and Functions, a higher percentage of students in algebra classes had instruction with "a lot" of emphasis on this content strand compared with the percentage of pre-algebra and eighth-grade mathematics students receiving such emphasis.

With regard to *mathematical processes*, in 1996, high percentages of fourth- and eighth-grade students had teachers who reported placing "a lot" of emphasis on learning mathematics facts and concepts, and learning skills and procedures needed to solve routine problems. A slight majority of fourth- and eighth-grade students were in classes with "a lot" of emphasis on developing reasoning ability. At both grades 4 and 8, the percentage of students with "a lot" of emphasis on how to communicate ideas in mathematics effectively was similar to the percentage of students with "some" emphasis. Only for the process of developing reasoning ability was the percentage of algebra students whose instruction had "a lot" of emphasis higher than the percentage of eighth-grade mathematics students.

Data on specific *instructional practices* in 1996 show differences by grade level and a few by eighth-grade course taking. Additionally, there were a few changes over time. For example, working with objects like rulers and other manipulatives was more common at the lower grade levels and in less advanced mathematics courses taken by eighth-grade students. The majority of fourth- and eighth-grade students work at least once a week with other students to solve mathematics problems, while twelfth-grade students taking mathematics report working with other students to solve problems less frequently.

Writing a few sentences about how to solve a mathematics problem was relatively rare among fourth- and eighth-grade students; however, the percentages of fourth-grade students who were asked to write about solving problems "almost every day" or "once or twice a week" in 1996 was higher than the percentages in 1992. On average, fewer students were writing reports or doing mathematics projects than were writing a few sentences about how to solve a mathematics problem. However, changes over time appear to show increases in the frequency with which the practices of writing reports or doing mathematics projects are being implemented in mathematics classrooms.

In 1996, substantial proportions of students — over one-third of fourth-grade students, almost half of eighth-grade students, and almost half of twelfth-grade students taking mathematics — were discussing solutions to mathematics problems with other students "almost every day." Furthermore, substantial proportions of students from grades 4 and 8 were working on and discussing mathematics that reflected real-life situations at least "once or twice a week."

As Table 9.18 indicates, in 1996, the frequency with which *calculators were used* increased with increasing grades and with more advanced mathematics courses at the eighth-grade level. The data across time show increases in the frequency of use by fourth- and eighth-grade students, regardless of mathematics course. A majority of eighth-grade students and twelfth-grade students taking mathematics reported using scientific calculators to do schoolwork. Although a majority of twelfth-grade students taking mathematics also reported using graphing calculators, only 11 percent of eighth-grade students did. At the eighth-grade level, for both scientific and graphing calculators, the percentage of algebra students who indicated using them was higher than the percentage of pre-algebra or eighth-grade mathematics students.

As Table 9.19 shows, in 1996, smaller percentages of fourth- than eighth-grade students had teachers who reported allowing unrestricted use of calculators and use of calculators on mathematics tests. Higher percentages of students taking algebra than students taking eighth-grade mathematics or pre-algebra had teachers who reported allowing unrestricted use of calculators and use of calculators on mathematics tests. Between 1992 and 1996, there appears to have been an increase in both the percentage of students allowed unrestricted use of calculators and the percentage of students allowed use of calculators on mathematics tests.

In the NAEP 1996 assessment, the majority of fourth-grade, eighth-grade, and twelfth-grade students did not use calculators appropriately (see Table 9.20). Appropriate calculator use is defined as using a calculator on questions for which a calculator is either required or useful. Although the average mathematics scale score for fourth-grade students who used the calculator appropriately was similar to the average scale score of students who did not, at the eighth- and twelfth-grade levels, students who appropriately used calculators outperformed students who did not.

In 1996, the majority of students in grades 4 and 8 were *assessed* in mathematics classes "once or twice a month," while the majority of twelfth-grade students were assessed "once or twice a week." Teachers of grades 4 and 8 reported less testing with multiple-choice questions and more with constructed-response questions. The use of individual or group projects or presentations was less common than the use of written responses. Teachers' use of portfolios was more common with fourth- than with eighth-grade students.

# Chapter 10

# Student Attitudes Toward Mathematics

Having the necessary content knowledge and skills is essential to being successful in using mathematics. However, some support also exists for the notion that students' attitudes and beliefs about mathematics can influence their persistence and achievement in the subject.[1] Over the years, in NAEP assessments, students have been presented with statements pertaining to their attitudes toward mathematics. To each of these statements students were asked to indicate whether they agreed with, disagreed with, or were undecided about the statement. Students' responses to the following three statements are discussed in this chapter:

- "I like mathematics";
- "If I had a choice, I would not take any more mathematics"; and
- "Everyone can do well in mathematics if they try."

As shown in Table 10.1, in 1996, over half of fourth- and eighth-grade students agreed with the statement "I like mathematics." However, the percentage of fourth-grade students who agreed was significantly higher than the percentage of eighth-grade students who agreed. An examination of data by mathematics course showed that the percentage of algebra students who disagreed with the statement "I like mathematics" was significantly lower than the percentage of pre-algebra or eighth-grade mathematics students who disagreed.

Among twelfth-grade students, 50 percent indicated liking mathematics. This percentage was lower than the percentages of eighth-grade and fourth-grade students. As might be expected, the frequency of positive responses was greater among twelfth-grade students who were currently taking mathematics than among those who were not taking mathematics. Furthermore, positive responses increased in frequency among students who reported having taken more advanced mathematics coursework. For example, the percentage of twelfth-grade students who had taken geometry and agreed with the statement "I like mathematics" was 53 percent, which was higher than the 38 percent among students who had not taken geometry.

[1] Kohn, A. (1994). The truth about self-esteem. *Phi Delta Kappan*, *76*(4), 272–283.

Similarly, when responses are examined by highest level algebra-through-calculus course taken, one observes that the percentage of students agreeing that they like mathematics was higher among those who had progressed to calculus or pre-calculus than among those whose highest course was second-year algebra, first-year algebra, or pre-algebra. There also was a higher rate of agreement among those whose highest algebra-through-calculus course was second-year algebra than among those whose highest course was first-year algebra or pre-algebra.

**Table 10.1** ***Percentages of Students by Their Response to the Statement: "I Like Mathematics," Grades 4, 8, and 12, 1996***

THE NATION'S REPORT CARD naep

| | Agreement | | |
|---|---|---|---|
| | **Agree** | **Disagree** | **Undecided** |
| **Grade 4** | | | |
| All Students | 69 | 14 | 17 |
| **Grade 8** | | | |
| All Students | 56 | 23 | 21 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 55 | 24 | 21 |
| Pre-Algebra | 54 | 24 | 22 |
| Algebra | 60 | 20 | 20 |
| **Grade 12** | | | |
| All Students | 50 | 33 | 17 |
| Students Who Are: | | | |
| Enrolled in Mathematics | 57 | 26 | 16 |
| Not Enrolled in Mathematics | 37 | 45 | 18 |
| Students Who Have: | | | |
| Taken Geometry | 53 | 30 | 17 |
| Not Taken Geometry | 38 | 44 | 18 |
| Highest Algebra-Calculus Course Taken: | | | |
| Pre-Algebra | 39 | 42 | 19 |
| First-Year Algebra | 39 | 42 | 20 |
| Second-Year Algebra | 51 | 33 | 16 |
| Third-Year Algebra/Pre-Calculus | 62 | 22 | 16 |
| Calculus | 74 | 11 | 14 |

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

There were no changes in the percentages agreeing or disagreeing with the statement "I like mathematics" from 1990 or 1992 to 1996 for either fourth-grade or eighth-grade students. This also was true for eighth-grade students regardless of the mathematics class they were taking. At the twelfth-grade level, however, the 1996 percentage of students who agreed with the statement "I like mathematics" (50%) was less than the 1990 percentage (54%).[2]

A second question addressed to the students was whether they agreed or disagreed with the statement, "If I had a choice, I would not study any more mathematics." In 1996, 72 percent of fourth-grade students disagreed with the statement, implying that, given a choice, they would choose to continue their studies in mathematics. The data presented in Table 10.2, suggest that, as students progress through their school careers, more students become disenchanted with mathematics and, if given a choice, would choose not to take any more mathematics. For example, the percentage of eighth-grade students who agreed that they would choose not to study any more mathematics (16%) was higher than the percentage of fourth-grade students who agreed (12%) and lower than the percentage of twelfth-grade students who agreed (31%).

Not surprisingly, students who had taken more mathematics were more likely to express interest in taking even more mathematics classes. Among eighth-grade students, the percentage of algebra students (70%) who indicated that they would choose to take more mathematics was higher than the percentage of pre-algebra (63%) or eighth-grade mathematics (63%) students who so indicated. Twelfth-grade students who were taking mathematics were more likely to indicate that they would take more mathematics (56%) than were those who were not taking mathematics (33%). Students who had taken geometry also were more likely to indicate that they would take more mathematics (50%) than those who had not taken geometry (38%). Students whose highest algebra-through-calculus class was calculus or pre-calculus were more likely to indicate that they would take more mathematics (70% and 62%, respectively) than students whose highest course was pre-algebra, first-year algebra, or second-year algebra (38%, 39%, and 46%, respectively). Students whose highest course was second-year algebra were more likely to indicate that they would take more mathematics than were students whose highest course was first-year algebra.

The 1996 percentage of fourth-grade students who disagreed with the statement (i.e., who implied they would take more mathematics; 72%) was lower than the 1992 percentage (76%).[3] This is somewhat discouraging, given current reform efforts to increase the accessibility of the mathematics curriculum as well as the amount of mathematics children take. Of course, fourth-grade students are not usually given the choice of taking or not taking mathematics. Nevertheless, this attitudinal trend does not reflect well on efforts to increase mathematics course taking. Between 1992 and 1996, there were no significant differences in the percentages of all eighth-grade students indicating agreement or disagreement. Similarly, over this time period, the opinions of eighth-grade students in the different mathematics classes did not change.

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[2] Sources of trend data are the NAEP 1996, 1992, and 1990 mathematics assessments. These data are available on the World Wide Web at: <http://nces.ed.gov/NAEP>.

[3] Sources of trend data are the NAEP 1996 and 1992 mathematics assessments. Fourth- and eighth-grade students were not asked to respond to this statement in the NAEP 1990 mathematics assessment; twelfth-grade students were not asked to respond to this statement in the NAEP 1992 and 1990 mathematics assessments.

**Table 10.2**

**Percentages of Students by Their Response to the Statement: "If I Had a Choice, I Would Not Study Any More Mathematics," Grades 4, 8, and 12, 1996**

THE NATION'S REPORT CARD naep

| | Agreement | | |
|---|---|---|---|
| | **Agree** | **Disagree** | **Undecided** |
| **Grade 4** | | | |
| All Students | 12 | 72 | 16 |
| **Grade 8** | | | |
| All Students | 16 | 65 | 19 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 16 | 63 | 21 |
| Pre-Algebra | 18 | 63 | 19 |
| Algebra | 13 | 70 | 17 |
| **Grade 12** | | | |
| All Students | 31 | 47 | 22 |
| Students Who Are: | | | |
| Enrolled in Mathematics | 24 | 56 | 21 |
| Not Enrolled in Mathematics | 42 | 33 | 25 |
| Students Who Have: | | | |
| Taken Geometry | 29 | 50 | 22 |
| Not Taken Geometry | 38 | 38 | 24 |
| Highest Algebra-Calculus Course Taken: | | | |
| Pre-Algebra | 40 | 38 | 22 |
| First-Year Algebra | 37 | 39 | 24 |
| Second-Year Algebra | 31 | 46 | 23 |
| Third-Year Algebra/Pre-Calculus | 20 | 62 | 18 |
| Calculus | 13 | 70 | 17 |

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

A potential motivator for students to persist in mathematics and to continue to work at improving their mathematics achievement is the belief that everyone can do well in mathematics. In 1996, students were asked whether they agreed with the statement, "Everyone can do well in mathematics if they try."[4] The data in Table 10.3 show that the nation's children were much more likely to agree than to disagree with the statement; 89 percent of fourth-grade students, 73 percent of eighth-grade students, and 50 percent of twelfth-grade students agreed with the statement. However, as the data also make clear, the percentage agreeing declined with grade level. Furthermore, increasing percentages of older students were unsure about how they

[4] Students were not asked to respond to this statement in the NAEP 1992 or 1990 mathematics assessments.

felt about the statement: 21 percent of twelfth-grade students, 15 percent of eighth-grade students, and 8 percent of fourth-grade students indicated that they were undecided in their opinion.

Perhaps surprisingly, an examination by course taking at the eighth-grade level shows that a higher percentage of students in eighth-grade mathematics (77%) than in algebra (67%) agreed with the statement. There were no significant differences in percentages by course taking at the twelfth-grade level.

**Table 10.3** ***Percentage of Students by Their Response to the Statement: "Everyone Can Do Well in Mathematics If They Try," Grades 4, 8, and 12, 1996***

THE NATION'S REPORT CARD naep

| | Agree | Disagree | Undecided |
|---|---|---|---|
| **Grade 4** | | | |
| All Students | 89 | 3 | 8 |
| **Grade 8** | | | |
| All Students | 73 | 12 | 15 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 77 | 10 | 13 |
| Pre-Algebra | 72 | 11 | 17 |
| Algebra | 67 | 15 | 18 |
| **Grade 12** | | | |
| All Students | 50 | 29 | 21 |
| Students Who Are: | | | |
| Enrolled in Mathematics | 51 | 28 | 21 |
| Not Enrolled in Mathematics | 47 | 31 | 22 |
| Students Who Have: | | | |
| Taken Geometry | 49 | 30 | 22 |
| Not Taken Geometry | 53 | 28 | 19 |
| Highest Algebra-Calculus Course Taken: | | | |
| Pre-Algebra | 54 | 25 | 20 |
| First-Year Algebra | 51 | 29 | 20 |
| Second-Year Algebra | 49 | 29 | 22 |
| Third-Year Algebra/Pre-Calculus | 47 | 32 | 22 |
| Calculus | 46 | 30 | 24 |

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## *Summary*

This chapter included information on student attitudes and beliefs about mathematics. In particular, it reported on students' agreement with three specific statements: "I like mathematics"; "If I had a choice, I would not study any more mathematics"; and "Everyone can do well in mathematics if they try." In general, the majority of students at each grade level rendered a response that was favorable to mathematics. However, the percentage offering a favorable response declined with grade level. For example, 72 percent of fourth graders, but only 65 percent of eighth graders and 47 percent of twelfth graders disagreed with the statement "If I had a choice, I would not study any more mathematics." Liking mathematics, and a willingness to study more mathematics, were both positively associated with the students' mathematics course taking. That is, favorable responses were more frequent among eighth-grade students enrolled in algebra, twelfth-grade students enrolled in any mathematics class, and twelfth-grade students who had completed more advanced course work. These associations with course taking were not, however, apparent in students' opinions on the relationship between effort and mathematics achievement. In fact, eighth-grade students enrolled in algebra were *less* likely than those enrolled in eighth-grade mathematics to agree that "everyone can do well in mathematics if they try."

# Chapter 11

# *Summary*

This report has presented three types of information derived from the NAEP 1996 mathematics assessment: 1) information on what students know and can do in mathematics, 2) information on course-taking patterns and current classroom practices in this subject area, and 3) information on student attitudes about mathematics. The first portion of this information is derived from an analysis of student performance on the actual assessment exercises; the latter two portions draw upon the questionnaires completed by the students who participated in the assessment and their mathematics teachers.

The chapters on student work were organized around the five content strands assessed by NAEP: Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; Data Analysis, Statistics, and Probability; and Algebra and Functions. Within these chapters, the discussion also highlighted students' proficiency on a number of cognitive skills that cut across the different content areas. These include conceptual understanding, procedural knowledge, and problem solving, as well as the ability to reason in mathematical situations, to communicate perception and conclusions drawn from a mathematical context, and to connect the mathematical nature of a situation with related mathematical knowledge and information gained from other disciplines or through observation.

## *Student Work*

### *Trend comparisons*

In 1990, NAEP gathered baseline achievement data for fourth-, eighth-, and twelfth-grade students, using a newly developed mathematics framework. Two subsequent assessments, based on the same framework and administered in 1992 and 1996, offered the opportunity to track trends in achievement. The results have been promising, indicating statistically significant improvements in overall mathematics performance at all three grade levels and in each of the five content strands. The gains were largest between 1990 and 1992, but additional gains also were evident between 1992 and 1996 on the overall composite scale and for some of the content strands. Specifically, student performance in Geometry and Spatial Sense and in Algebra and Functions improved at all grade levels; performance in Number Sense, Properties, and Operations and in Data Analysis, Statistics, and Probability improved at fourth grade; and student performance in Measurement and in Data Analysis, Statistics, and Probability improved at twelfth grade. When the achievement trends were disaggregated by race and gender, the direction of change still was generally positive for most comparisons. However, trend comparisons for some of the smaller or more diverse groups did not achieve statistical significance; as a result, one cannot say with certainty that these gains did not simply reflect chance variation due to sampling.

### *Subgroup comparisons*

Gender. In 1996, gender differences in performance favoring males were observed for overall proficiency and three content strands at grade 4 (Number Sense, Properties, and Operations; Measurement; and Algebra and Functions) and for two content strands at grade 12 (Measurement, and Geometry and Spatial Sense).

Race/Ethnicity. In 1996, White and Asian/Pacific Islander students at grades 4 and 12 and White students at grade 8 performed better than other racial/ethnic groups overall and in each of the content strands of mathematics.[1] Hispanic students performed better than Black students in Geometry and Spatial Sense at grade 4; in Measurement and in Geometry and Spatial Sense at grade 8; and in Measurement and in Data Analysis, Statistics, and Probability at grade 12. American Indian students performed better than Black and Hispanic students in all strands at grade 4 and outperformed Black students in all content strands and Hispanic students in all strands but Geometry and Spatial Sense at grade 8. At grade 12, Asian/Pacific Islander students performed better than White students in Algebra and Functions.

Course Taking. In general, taking more mathematics courses and more advanced mathematics courses were associated with improved mathematics performance in all content strands. Eighth-grade students enrolled in algebra performed better in all content strands than eighth-grade students enrolled in pre-algebra or eighth-grade mathematics, and eighth-grade students enrolled in pre-algebra performed better than students enrolled in eighth-grade mathematics in all but one of the content strands (Geometry and Spatial Sense).

Twelfth-grade results show a similar story. Students at any given point in the algebra-through-calculus sequence performed better than students whose mathematics exposure had stopped at the next lowest course in the sequence with one exception: students whose highest course had been pre-algebra did not perform significantly better than students who had taken neither pre-algebra nor algebra. Similarly, students who had taken geometry performed better in all content strands than those who had not taken geometry.

In addition, taking more mathematics courses in high school was related to higher mathematics performance, with one exception: students who took 3–4 semesters of mathematics did not perform significantly better in Measurement than students who took only 1–2 semesters.

[1] Results for eighth-grade Asian/Pacific Islander students are not included in the body of this report. See Appendix A for details.

## *Content strands*

Number Sense, Properties, and Operations. Students scoring in the *Basic* achievement level or above appeared to grasp many of the fundamental concepts and properties of and relationships between numbers, and displayed the skills required for manipulating numbers and completing computations. Questions assessing proportional thinking, requiring multistep solutions, or involving new concepts tended to be more difficult. Additionally, questions requiring students to solve problems and communicate their reasoning proved challenging, and often it was the communication aspect that provided the most challenge.

Measurement. Many of the measurement questions were difficult for students, particularly those requiring unit conversions, calculations of volume and circumference, and estimation.

Eighth-grade algebra students tended to perform better than other eighth-grade students, whereas eighth-grade students in pre-algebra or eighth-grade mathematics tended to perform similarly. At the twelfth-grade level, students whose highest course was second-year algebra tended to outperform those who had only reached first-year algebra, and students who reported calculus as their highest mathematics course tended to perform better than those who had taken less advanced mathematics courses. [2]

Geometry and Spatial Sense. Most of the questions in this content strand required a drawn or written response, and many were difficult for students. Questions in this content strand also relied upon students' visual-spatial skills. In several of the sample questions, a significant difference was found between the performance of male and female students. Here also, eighth-grade algebra students tended to outperform other eighth-grade students, whereas eighth-grade students in pre-algebra and those in eighth-grade mathematics performed similarly. In addition, on some of the questions, twelfth-grade students who had taken at least second-year algebra outperformed those who had not and, similarly, students who had taken at least third-year algebra or pre-calculus outperformed those who had not.

Data Analysis, Statistics, and Probability. In this content strand, students seemed to perform better on questions that asked them to make straightforward interpretations of graphs, charts, and tables as opposed to those requiring them to perform calculations with displayed data. Students had difficulty explaining why one method of reporting or displaying data was better than another, even though they may have recognized which was the better method. Questions asking students to determine chance or probability also were difficult.

Algebra and Functions. The majority of students at all grade levels appeared to understand basic algebraic representations and simple equations, as well as how to find simple patterns. The more proficient students at grades 8 and 12 were able to demonstrate knowledge of linear equations, algebraic functions, and trigonometric identities, but even those students found that questions requiring them to identify and generalize complex patterns and solve real-world problems were challenging. In general, for eighth- and twelfth-grade students, those with more advanced coursework performed better in this content strand.

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[2] Performance in Measurement and in Geometry and Spatial Sense was not analyzed with respect to whether students had taken a course in geometry because of the variability in mathematics course sequencing, the small percentage of students for whom the impact of geometry can be isolated, and the difficulty associated with identifying the effect of a particular curriculum on the performance of students in advanced mathematics. See discussion in Chapter 2.

## Classroom Teaching

### Course-taking patterns

In 1996, the modal group, but not the majority, of eighth-grade students, regardless of whether they were male or female, were enrolled in eighth-grade mathematics, and most of the remaining students were enrolled in pre-algebra or algebra. Trends over time show increases in the percentage of eighth-grade students taking more advanced mathematics courses.

These positive trends also were evident at the twelfth-grade level. For example, the 1996 percentage of twelfth-grade students enrolled in mathematics was significantly higher than the 1990 percentage. In addition, over time more students appear to be initially taking first-year algebra earlier in their school careers. Examination of the highest course taken by twelfth-grade students in an algebra-through-calculus sequence showed that in 1996, almost half of the twelfth-grade students indicated second-year algebra as their highest course taken. In the remaining half, fewer students indicated a course higher than second-year algebra as their highest course taken than indicated a lower level course as their highest course taken.

### Classroom practices

In 1996, teachers of fourth- and eighth-grade students were asked about the emphasis they placed on different mathematics content and processes in their mathematics instruction. The majority of fourth- and eighth-grade students were receiving mathematics instruction with more emphasis on Number Sense, Properties, and Operations; Measurement; and Geometry and Spatial Sense than on Data Analysis, Statistics, and Probability; and Algebra and Functions. Perhaps as expected, more emphasis was placed on Data Analysis, Statistics, and Probability and on Algebra and Functions at the eighth-grade level than at the fourth-grade level. In all of the eighth-grade mathematics classes, students experienced similar levels of emphasis on the mathematics content strands, except for Algebra and Functions, which was more heavily emphasized in the algebra classes. Mathematics instruction at grades 4 and 8 placed more emphasis on learning mathematics facts and concepts and on learning skills and procedures needed to solve routine problems than on developing reasoning ability or on learning how to communicate ideas in mathematics effectively.

Teachers of fourth- and eighth-grade students, as well as twelfth-grade students, were asked about a variety of instructional practices that were being implemented in their mathematics classes. In 1996, results showed differences in the frequencies of implementation of some practices at different grade levels. For example, working with objects like rulers and other manipulatives was more common at the fourth-grade level and in less advanced mathematics courses taken by eighth-grade students. Similarly, the majority of fourth- and eighth-grade students worked at least once a week with other students to solve mathematics problems, while this type of structured interaction was less frequent among twelfth-grade students.

Reports on these practices over time show some significant changes. For example, while the practice of writing a few sentences about how to solve a mathematics problem was relatively rare among fourth-grade students, there have been increases in frequency over time. On average, few students at grades 4 and 8 were writing reports or doing mathematics projects, but changes over time show increases in the frequency of implementation of this practice also.

In 1996, the frequency with which calculators were used increased with increasing grade level and with mathematics content at the eighth-grade level. Furthermore, the use of calculators has increased over time. The majority of eighth- and twelfth-grade students taking mathematics reported using scientific calculators to do schoolwork. At the eighth-grade level, the use of scientific and graphing calculators was more common in the higher level mathematics courses than in the lower level courses. A majority of the twelfth-grade students taking mathematics reported using graphing calculators, although only about one in ten eighth-grade students did. In addition, the unrestricted use of calculators and the use of calculators on mathematics tests were more common among eighth-grade than fourth-grade students and among eighth-grade students in higher level mathematics courses than among those in lower level courses.

Finally, students in grade 12 reported being tested more frequently in mathematics than teachers reported that fourth- and eighth-grade students were tested. Teachers of grades 4 and 8 reported less testing with multiple-choice questions than with constructed-response questions and less use of individual or group projects than of written responses. Teachers' use of portfolios was more common with fourth- than with eighth-grade students.

## Student Attitudes Toward Mathematics

The NAEP 1996 mathematics assessment probed student attitudes and beliefs about mathematics. In particular, it examined students' agreement with three specific statements: "I like mathematics"; "If I had a choice, I would not study any more mathematics"; and "Everyone can do well in mathematics if they try." In general, the majority of students at each grade level rendered a response that was favorable to mathematics. However, the percentage offering a favorable response declined with grade level.

Liking mathematics and being willing to study more mathematics were both positively associated with students' mathematics course taking. That is, favorable responses were more frequent among eighth-grade students enrolled in algebra, twelfth-grade students enrolled in any mathematics class, and twelfth-grade students who had completed more advanced coursework. These associations with course taking were not, however, apparent in students' opinions on the relationship between effort and mathematics achievement. In fact, eighth-grade students enrolled in algebra were *less* likely than those enrolled in eighth-grade mathematics to agree that "everyone can do well in mathematics if they try."

## Conclusions

Performance of U.S. students in mathematics continues to improve. Since 1990, improved performance overall at all three grade levels and in each of the five content strands has been observed. When the achievement trends observed in 1996 were disaggregated by race and gender, improvement in performance continued to be observed for most groups. In addition, taking more, and more advanced, coursework in mathematics was associated with improved performance in all content strands.

Examination of student work revealed that certain types of questions were harder for some students than others. In particular, questions involving new concepts or requiring multistep solutions, written (or drawn) explanations of students' reasoning, problem solving, estimation, or the use of spatial skills were difficult for students. Straightforward questions that required simple (decontextualized) calculations were easier.

While examination of 1996 course-taking patterns revealed that more students appear to be taking more, and more advanced, mathematics courses than before, a look at classroom practices indicated that students still need more exposure to communicating effectively about mathematics. In particular, students need more practice writing about how to solve mathematical problems and discussing how to solve problems reflecting real-life situations. Activities of this sort invite students to engage more fully with the content of mathematics, can serve to increase students' ability to think analytically, and are necessary for improving performance on more difficult cognitive questions.

# Appendix A

# *Procedures*

## *The NAEP 1996 Mathematics Assessment*

The 1996 assessment utilized the first update of the NAEP mathematics assessment framework since the release of the National Council of Teachers of Mathematics (NCTM*) Curriculum and Evaluation Standards for School Mathematics*.[1] This update sought to incorporate new knowledge about the teaching and learning of mathematics while also ensuring comparability of results across the 1990, 1992, and 1996 assessments.

## *The Assessment Design*

Each student participating in the assessment received a booklet containing three 15-minute segments, or blocks, of cognitive questions. NAEP uses an adaptation of matrix sampling called balanced incomplete block (BIB) spiraling — a design that enables broad coverage of mathematics content while minimizing the burden for any one student. The balanced incomplete block part of the design assigns blocks of questions to booklets; each pair of blocks appears together in at least one booklet, and each pair of booklets shares at least one block of questions. The spiraling part of the method cycles the booklets for administration, so that typically only a few students in any assessment session receive the same booklet.

Of the 17 blocks in the national sample at grade 4 and 19 blocks in the national sample at grades 8 and 12, three were carried forward from the 1990 assessment, and five were carried forward from the 1992 assessment, to allow for the measurement of trends across time. The remaining blocks of questions at each grade level contained new questions that were developed for the 1996 assessment as specified by the updated framework.

Each cognitive block of math questions consisted of multiple-choice and constructed-response questions. In addition, five to seven of the blocks at each grade allowed for the use of calculators. For several blocks, students were given manipulatives (including geometric shapes, three-dimensional models, and spinners). For two of the blocks, students were given rulers at grade 4 and rulers and protractors at grades 8 and 12.

[1] National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

Each student booklet also included three sets of student background questions. The first set included general background questions such as questions about the student's race or ethnicity, mother's and father's level of education, number and type of reading materials in the home, amount of time spent on homework, and student's academic expectations. The second set was directed specifically at the student's mathematics background and included questions about mathematics instructional activities, mathematics courses taken, use of specialized resources such as calculators in mathematics classes, and views on the utility and value of mathematics. These first two sets of background questions preceded the cognitive blocks in the assessment. The third set of questions followed the cognitive question blocks and contained five questions about students' motivation to do well on the assessment, their perception of the difficulty of the assessment, and their familiarity with the types of cognitive questions included. Students were given 5 minutes to complete each set of background questions, with the exception of fourth graders, who were given more time on the initial set of general background questions to allow those questions to be read aloud to them.

In addition to the student assessment booklets, two other instruments relevant to this report provided data relating to the assessment — a mathematics teacher questionnaire and a school characteristics and policy questionnaire.

The teacher questionnaires were administered to the mathematics teachers of each of the fourth- and eighth-grade students participating in the assessment. Because twelfth-grade students were not necessarily enrolled in mathematics, no questionnaires were administered to twelfth-grade mathematics teachers. The teacher questionnaire consisted of three sections and took approximately 20 minutes to complete. The first section focused on the teacher's general background and experience; the second section focused on the teacher's background related to mathematics; and the third section focused on classroom information about mathematics instruction. Because the sampling for the teacher questionnaire was based on participating students, the responses to the mathematics teacher questionnaire do not necessarily represent all fourth- or eighth-grade mathematics teachers in the nation or in a state. Rather, they represent teachers of the representative sample of students assessed. It is important to note that in this report, as in all NAEP reports, the student is always the unit of analysis, even when information from the teacher or school questionnaire is being reported. Using the student as the unit of analysis makes it possible to describe the educational context experienced by representative samples of students. Although this approach may provide a different perspective from that obtained by simply collecting information from teachers or schools, it is consistent with NAEP's goals of providing information about the educational context and performance of students.

The school characteristics and policy questionnaires were given to the principals or other administrators in each participating school and took about 20 minutes to complete. The questions asked about the principal's background and experience, school policies, programs, facilities, and the demographic composition and background of the students and teachers in that school.

## National Samples

The national results presented in this report are based on nationally representative probability samples of fourth-, eighth-, and twelfth-grade students. The samples were selected by Westat using a complex multistage sampling design that involved sampling students from selected schools within selected geographic areas across the country. For a more detailed description of the sampling procedures, see the *NAEP 1996 Mathematics Report Card for the Nation and the States.*[2]

## Students with Disabilities (SD) and Limited English Proficient (LEP) Students

It is NAEP's intent to assess all selected students. However, some students with disabilities or limited English proficiency are not capable of taking the assessment, or not capable of taking it under standard conditions. NAEP provides written guidelines in an effort to standardize local school decisions about which students will participate in the assessment and under what conditions.

The 1996 assessment marked a transition in NAEP guidelines for the inclusion of students with disabilities or limited English proficiency. New guidelines were developed in an effort to 1) increase inclusion rates, 2) be applied more consistently across states and jurisdictions, and 3) ensure that inclusion decisions would be related to the subject matter instruction given to the student rather than less relevant considerations. Under the new guidelines, students with disabilities should participate unless:

- the student's Individualized Education Plan (IEP) team (or equivalent) determined that the student cannot participate in assessments such as NAEP; *or*
- the student's cognitive functioning is so severely impaired that he or she cannot participate; *or*
- the student's IEP requires an accommodation or adaptation that NAEP and the school do not provide, and the student cannot demonstrate his or her knowledge without that accommodation.

The guidelines indicate that students with limited English proficiency should participate unless:

- the student has received language arts instruction primarily in English for less than three school years including the current year; *and*
- the student cannot demonstrate his or her knowledge of the subject being assessed in English even with an accommodation permitted by NAEP.

In all cases, schools are encouraged to include the student in instances of doubt.

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[2] Reese, C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). *NAEP 1996 mathematics report card for the nation and the states*. Washington, DC: National Center for Education Statistics.

In order to determine the impact of the change in criteria on the measurement of trends, the 1996 national mathematics sample was subdivided into three parts: S1, S2, and S3. Schools in S1 received the old inclusion guidelines, and schools in S2 and S3 received the new guidelines. In addition, schools in S3 were instructed to offer a series of specified accommodations to students who normally receive such accommodations for testing.

Initial analyses of the 1996 results demonstrated that the change in written inclusion guidelines did not adversely impact the cross-sectional or trend estimation of achievement. Therefore the S1 and S2 samples were combined for reporting. Data from students in S3, however, were held aside for further analysis of the impact of accommodations on the measurement of trend.[3]

## Data Collection and Scoring

As with all NAEP assessments, data collection was conducted by trained field staff. For the national assessment, this was accomplished by Westat staff. Materials collected as part of the 1996 assessment were shipped to National Computer Systems, where trained staff evaluated the responses to the constructed-response questions using scoring rubrics or guides prepared by the Educational Testing Service (ETS).

Each constructed-response question had a unique scoring rubric that defined the criteria used to evaluate students' responses. The extended constructed-response questions were evaluated with four- or five-level rubrics (e.g., no evidence of understanding, evidence of minimal understanding, evidence of partial understanding, and evidence of satisfactory or extended understanding), while the short constructed-response questions first appearing in the 1996 assessment were rated according to three-level rubrics that permitted partial credit (e.g., evidence of little or no understanding, evidence of partial understanding, and evidence of full understanding). Other short constructed-response questions that appeared in previous assessments were scored as either correct or incorrect. For more information, see *The NAEP 1996 Technical Report*.[4]

Student responses for constructed responses also could have been scored as "off task," which meant that the students provided a response that was deemed unrelated in content to the question asked. A simple example of this type of response is, "I don't like this test." Responses of this sort could not be rated. By contrast, responses scored as incorrect were valid attempts to answer the question that were simply wrong.

Scoring of the NAEP 1996 assessment included rescoring to monitor interrater reliability and trend reliability. In other words, scoring reliability was calculated within year (1996) and across years (1990, 1992, and 1996). The overall within-year percentages of agreement for the 1996 national reliability samples were 96 percent at grade 4, 96 percent at grade 8, and 96 percent at grade 12. For information on trend reliability, see the *NAEP 1996 Mathematics Report Card for the Nation and the States*.[5]

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[3] For further details, see Reese, C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). op. cit.; and Mazzeo, J., Carlson, J., Voekl, K., & Lutkus, A. (forthcoming). *Increasing the participation of students with disabilities and limited English proficient students in the National Assessment of Educational Progress: A special report on 1996 research activities.* Washington, DC: U.S. Department of Education.

[4] Allen, N. L., Carlson, J. E., & Zelenak, C. A. (1999). *The NAEP 1996 technical report*. Washington, DC: National Center for Education Statistics.

[5] Reese, C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). op. cit.

## *Data Analysis and IRT Scaling*

Subsequent to the professional scoring, all information was transcribed to the NAEP database at ETS. Each processing activity was conducted with rigorous quality control. After the assessment information had been compiled in the database, the data were weighted according to the population structure. The weighting for the national and state samples reflected the probability of selection for each student as a result of the sampling design, adjusted for nonresponse. Through stratification, the weighting assured that the representation of certain subpopulations corresponded to figures from the U.S. Census and the Current Population Survey.[6]

Analyses then were conducted to determine the percentages of students who gave various responses to each cognitive and background question. Item response theory (IRT) was used to estimate average scale score proficiency for the nation, various subgroups of interest within the nation, and for the states. IRT models the probability of answering a question correctly as a mathematical function of proficiency or skill. The main purpose of IRT analysis is to provide a common scale on which performance can be compared across groups, such as those defined by grades and subgroups (e.g., gender or race/ethnicity). Because of the BIB spiraling design used by NAEP, students do not receive enough cognitive questions about a specific content area to provide reliable information about individual performance. Traditional test scores for individual students, even those based on IRT, would lead to misleading estimates of population characteristics, such as subgroup means and percentages of students at or above a certain proficiency level. Instead, NAEP constructs sets of plausible values designed to represent the distribution of proficiency in the population. A plausible value for an individual is not a scale score for that individual but may be regarded as a representative value from the distribution of potential scale scores for all students in the population with similar characteristics and identical patterns of item (question) responses. Statistics describing performance on the NAEP proficiency scale are based on these plausible values. They estimate values that would have been obtained had individual proficiencies been observed — that is, had each student responded to a sufficient number of cognitive questions so that proficiency could be precisely estimated.[7]

A score scale ranging from 0 to 500 was created to report performance for each content strand (Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; Data Analysis, Statistics, and Probability; Algebra and Functions). The scales summarize examinee performance across all three question types used in the assessment (multiple-choice, short constructed-response, and extended constructed-response). Each content area scale was based on the distribution of student performance across all three grades assessed in the 1996 national assessment (grades 4, 8, and 12) and had a mean of 250 and a standard deviation of 50. A composite score was created as an overall measure of students' mathematics proficiency. The composite scale was a weighted average of the five content-strand scales, where the weight for each content strand was proportional to the relative importance assigned to the content strands in the specifications developed by the Mathematics Objectives Panel.

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[6] For additional information about the use of weighting procedures in NAEP, see Johnson, E. G. (December 1989). *Journal of Education Statistics*, *14*(4), pp. 303–334.

[7] For theoretical justification of the procedures employed, see Mislevy, R. J. (1988). Randomization-based inferences about latent variables from complex samples. *Psychometrika*, *56*(2), pp. 177–196.

The NAEP proficiency scales make it possible to examine relationships between students' performance and a variety of background factors measured by NAEP. The fact that a relationship exists between achievement and another variable, however, does not reveal the underlying cause of the relationship, which may be influenced by a number of other variables. Similarly, the assessments do not capture the influence of unmeasured variables. The results are most useful when they are considered in combination with other knowledge about the student population and the educational system, such as trends in instruction, changes in the school-age population, and societal demands and expectations.

Most of the data analyses were conducted by ETS. However, some of the results presented in this report are based on additional analyses conducted by the American Institutes for Research using data sets provided by ETS.

More detailed information about data analysis and item response theory is presented in *The NAEP 1996 Technical Report*.[8]

## Reporting Groups

In this report, some of the results are provided for subgroups of students with shared characteristics: gender, race/ethnicity, course-taking patterns. Based on criteria described later in this appendix, results are reported for subpopulations only when sufficient numbers of students and adequate school representation are present. The minimum requirement is at least 62 students in a particular subgroup from at least five primary sampling units (PSUs).[9] Regardless of whether the subgroup was reported separately, the data for all students were included in computing overall results. Definitions of the subpopulations referred to in this report are presented below.

### Gender

Results are reported separately for males and females.

### Race/Ethnicity

The race/ethnicity variable is derived from two questions asked of students and school records, and it is used for race/ethnicity subgroup comparisons. Two questions from the set of general student background questions were used to determine race/ethnicity:

If you are Hispanic, what is your background?

- I am not Hispanic
- Mexican, Mexican American, or Chicano
- Puerto Rican
- Cuban
- Other Spanish or Hispanic background

[8] Allen, N. L., Carlson, J. E., & Zelenak, C. A. (1999). op. cit.

[9] For the national assessment, a PSU is a geographic region (a county, a group of counties, or metropolitan statistical areas).

Students who responded to this question by selecting "Mexican, Mexican American, or Chicano," "Puerto Rican," "Cuban," or "Other Spanish or Hispanic background" were considered Hispanic. Students who selected "I am not Hispanic," did not respond to the question, or provided information that was illegible or could not be classified were further classified based on their responses to the following question:

Which best describes you?

- White (not Hispanic)
- Black (not Hispanic)
- Hispanic ("Hispanic" means someone who is from a Mexican, Mexican American, Chicano, Puerto Rican, Cuban, or other Spanish or Hispanic background.)
- Asian or Pacific Islander ("Asian or Pacific Islander" means someone who is from a Chinese, Japanese, Korean, Filipino, Vietnamese, or other Asian or Pacific Islander background.)
- American Indian or Alaskan Native ("American Indian or Alaskan Native" means someone who is from one of the American Indian tribes or one of the original people of Alaska.)
- Other (specify) ________________________________

Students' race/ethnicity was then assigned on the basis of their responses. For students who selected "Other" and provided illegible information or information that could not be classified or who did not respond at all, race/ethnicity was assigned as determined by school records.

Race/ethnicity could not be determined for students who did not respond to either of the demographic questions and whose schools did not provide information about race/ethnicity.

Details of how race/ethnicity classifications were derived is presented so that readers can determine how useful the results are for their particular purposes. Also, some students indicated that they were from a Hispanic background (e.g., Puerto Rican or Cuban) and that a racial/ethnic category other than Hispanic best described them. These students were classified as Hispanic based on the rules described above. Furthermore, the information from the schools did not always correspond to how students described themselves. Therefore, the racial/ethnic results presented in this report attempt to provide a clear picture based on several sources of information.

As noted in Chapter 2, scale score and achievement level results for eighth-grade Asian/Pacific Islander students are not included in the main body of this report. The decision not to publish these results is discussed in detail at the end of this appendix.

### *Eighth-grade course taking*

Eighth-grade students responded to a question about what mathematics course they were taking. Students were provided with seven response options that included the following:

- I am not taking mathematics this year
- Eighth-grade mathematics
- Pre-algebra
- Algebra
- Integrated or sequential mathematics
- Applied mathematics (technical preparation)
- Other mathematics class

The course-taking grouping variable used in this report is based on the subset of students who responded that they were taking eighth-grade mathematics, pre-algebra, or algebra. Students who marked some other response are not included in the subpopulation analysis.

### *Twelfth-grade highest algebra-calculus course taken*

At the twelfth-grade level, the course-taking subpopulations are based on the highest level mathematics course students reported having taken in an algebra-through-calculus sequence. The grouping of students was based on students' reports on the amount of time they took the following mathematics courses:

- Introduction to algebra or pre-algebra
- First-year algebra
- Second-year algebra
- Pre-calculus, third-year algebra, elementary functions, or analysis
- Calculus

Students' responses were edited for consistency with the standard course-taking sequence. That is, the student was not credited as having taken a certain course unless his or her responses also indicated completion of the course prerequisites.

The twelfth-grade grouping variable has six categories:

1. Not Taken Pre-Algebra: These are students who had less than a year of introduction to algebra or pre-algebra.
2. Pre-Algebra: These are students who had a year or more of introduction to algebra or pre-algebra, but not first-year algebra.
3. First-Year Algebra: These are students who had a year or more of first-year algebra, but not second-year algebra.
4. Second-Year Algebra: These are students who had a year or more of second-year algebra, but not pre-calculus.

5. Pre-Calculus: These are students who had a year or more of pre-calculus, but not calculus.

6. Calculus: These are students who had a year or more of calculus.

## Guidelines for Analysis and Reporting

This report describes students', teachers', and principals' responses to background questions as well as mathematics performance for fourth-, eighth-, and twelfth-grade students. The report also compares the performance results for various groups of students within these populations (e.g., subgroups formed of those who responded to a specific background question in a particular way or by individual course-taking groups as described above). However, it does not include an analysis of the relationships among combinations of these subpopulations or background questions.

### Estimating variability

The statistics presented in this report are estimates of group and subgroup performance based on samples of students, and they therefore differ from statistics that could be calculated if every student in the nation answered every question. The degree of uncertainty associated with these sample-based estimates should, therefore, be taken into account. Two components of uncertainty are accounted for in the variability statistics based on student ability: 1) the uncertainty due to sampling only a relatively small number of students, and 2) the uncertainty due to sampling only a relatively small number of cognitive questions per student. The first component alone accounts for the variability associated with the estimated percentages of students who had certain background characteristics or who answered a certain cognitive question correctly.

Because NAEP uses complex sampling procedures, conventional formulas for estimating sampling variability that assume simple random sampling are inappropriate. NAEP uses a jackknife replication procedure to estimate standard errors. The jackknife standard error provides a reasonable measure of uncertainty for any student information that can be observed without error. However, because each student typically responds to only a few questions within any content strand, the scale score for any single student would be imprecise. In this case, plausible values technology can be used to describe the performance of groups or subgroups of students, but the underlying imprecision involved in this step adds another component of variability to statistics based on NAEP scale scores.[10]

Typically, when the standard error is based on a small number of students or when the group of students is enrolled in a small number of schools, the amount of uncertainty associated with the standard error may be quite large. Throughout this report, estimates of standard errors subject to a large degree of uncertainty are designated by a "!" symbol. In such cases, the standard errors — and any confidence intervals or significance tests involving these standard errors — should be interpreted cautiously. Additional details concerning procedures for identifying such standard errors are discussed in *The NAEP 1996 Technical Report*.[11]

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[10] For more details, see Johnson, E. G. & Rust, K. F. (1992). Population inferences and variance estimation for NAEP data. *Journal of Educational Statistics*, *17*(2), pp. 175–190.

[11] Allen, N. L., Carlson, J. E., & Zelenak, C. A. (1999). op. cit.

The reader is reminded that, like findings from all surveys, NAEP results are subject to other kinds of error, including the effects of imperfect adjustments for student and school nonresponse and unknown effects associated with the particular instrumentation and data collection methods. Nonsampling errors can be attributed to a number of sources: inability to obtain complete information about all selected schools in the sample (some students or schools refused to participate, or students participated but answered only certain questions); ambiguous definitions; differences in interpreting questions; inability or unwillingness to give correct information; mistakes in recording, coding, or scoring data; and other errors in collecting, processing, sampling, and estimating missing data. The extent of nonsampling error is difficult to estimate, and because of their nature, the impact of such errors cannot be reflected in the data-based estimates of uncertainty provided in NAEP reports.

### *Drawing inferences from the results*

As noted, the percentages of students and average scale scores used in reporting NAEP results are based on samples rather than on the entire population of fourth-, eighth-, or twelfth-graders in the nation or a jurisdiction. Consequently, the numbers reported are estimates and are subject to a measure of uncertainty, reflected in the standard error of the estimate. When the percentages or average scale scores of certain groups are compared, the standard error should be taken into account, and observed similarities or differences should not be relied on solely. Therefore, the comparisons discussed in this report are based on statistical tests that consider the standard errors of those statistics as well as the magnitude of the difference among the averages or percentages.

The results from the sample, taking into account the uncertainty associated with all samples, are used to make inferences about the population. Using confidence intervals based on the standard errors provides a way to make inferences about the population averages and percentages in a manner that reflects the uncertainty associated with the sample estimates. An estimated sample average scale score ± 2 standard errors approximates a 95 percent confidence interval for the corresponding population quantity. This statement means that one can conclude with approximately a 5 percent level of significance that the average performance of the entire population of interest (e.g., all fourth-grade students in public schools in a jurisdiction) is within ± 2 standard errors of the sample average.

As an example, suppose that the average mathematics scale score of the students in a particular group was 256, with a standard error of 1.2. A 95 percent confidence interval for the population quantity would be as follows:

Average ± 2 standard errors
$256 \pm 2 \times 1.2$
$256 \pm 2.4$
253.6, 258.4

Thus, one can conclude with a 5 percent level of confidence that the average scale score for the entire population of students in that group is between 253.6 and 258.4.

Similar confidence intervals can be constructed for percentages, if the percentages are not extremely large or extremely small. For extreme percentages, confidence intervals constructed in the above manner may not be appropriate, and accurate confidence intervals can be constructed only by using procedures that are quite complicated.

Extreme percentages, defined by both the magnitude of the percentage and the size of the sample from which it was derived, should be interpreted with caution. *The NAEP 1996 Technical Report* contains a more complete discussion of extreme percentages.[12]

### *Analyzing group differences in averages and percentages*

Statistical tests are used to determine whether the evidence, based on the data from the groups in the sample, is strong enough to conclude that the averages or percentages are actually different for those groups in the population. If the evidence is strong (i.e., the difference is statistically significant), the report describes the group averages or percentages as being different (e.g., one group performed higher than or lower than another group), regardless of whether the sample averages or percentages appear to be approximately the same. If the evidence is not sufficiently strong (i.e., the difference is not statistically significant), the averages or percentages are described as being not significantly different, regardless of whether the sample averages or percentages appear to be approximately the same or widely discrepant.

The reader is cautioned to rely on the results of the statistical tests rather than on the apparent magnitude of the difference between sample averages or percentages when determining whether the sample differences are likely to represent actual differences among the groups in the population.

To determine whether a real difference exists between the average scale scores (or percentages of a certain attribute) for two groups in the population, one needs to obtain an estimate of the degree of uncertainty associated with the difference between the averages (or percentages) of these groups for the sample. This estimate of the degree of uncertainty, called the standard error of the difference between the groups, is obtained by taking the square of each group's standard error, summing the squared standard errors, and taking the square root of that sum.

$$\text{Standard Error of the Difference} = SE_{A\text{-}B} = \sqrt{(SE_A^{\,2} + SE_B^{\,2})}$$

Similar to how the standard error for an individual group average or percentage is used, the standard error of the difference can be used to help determine whether differences among groups in the population are real. The difference between the averages or percentages of the two groups ± 2 standard errors of the difference represents an approximate 95 percent confidence interval. If the resulting interval includes zero, there is insufficient evidence to claim that a real difference between the groups is statistically significant (different) at the five percent level. In this report, differences among groups that involve poorly defined variability estimates or extreme percentages are not discussed.

[12] Allen, N. L., Carlson, J. E., & Zelenak, C. A. (1999). op. cit.

As an example, to determine whether the average mathematics scale score of Group A is higher than that of Group B, suppose that the sample estimates of the average scale score and standard errors were as follows:

| Group | Average Scale Score | Standard Error |
|---|---|---|
| A | 218 | 0.9 |
| B | 216 | 1.1 |

The difference between the estimates of the average scale scores of Groups A and B is two points (218–216). The standard error of this difference is:

$$\sqrt{(0.9^2 + 1.1^2)} = 1.4$$

Thus, an approximate 95 percent confidence interval for this difference is:

$$\begin{aligned} &\text{Difference} \pm 2 \text{ standard errors of the difference} \\ &2 \pm 2 \times 1.4 \\ &2 \pm 2.8 \\ &-0.8, 4.8 \end{aligned}$$

The value zero is within the confidence interval; therefore, there is insufficient evidence to claim that Group A outperformed Group B.

The procedures described in this section and the certainty ascribed to intervals (e.g., a 95 percent confidence interval) are based on statistical theory that assumes that only one confidence interval or test of statistical significance is being performed. However, in this report, many different groups are being compared (i.e., multiple sets of confidence intervals are being analyzed). In sets of confidence intervals, statistical theory indicates that the certainty associated with the entire set of intervals is less than that attributable to each individual comparison from the set. To hold the significance level for the set of comparisons at a particular level (e.g., 0.05), adjustments (called multiple comparison procedures) must be made to the methods described in the previous section. One such procedure, the Bonferroni method, was used in the analyses described in this report to determine confidence intervals for the differences among groups when sets of comparisons were considered.[13] Thus, the confidence intervals for the sets of comparisons in the text are more conservative than those described on the previous pages.

Most of the multiple comparisons in this report pertain to relatively small sets or families of comparisons. For example, for discussions concerning comparisons of parents' level of education, six comparisons were conducted — all pairs of the four parental education levels. In these situations, Bonferroni procedures were appropriate. A detailed description of the Bonferroni procedure appears in *The NAEP 1996 Technical Report*.[14]

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[13] Miller, R. G. (1996). *Simultaneous statistical inference*. New York: Wiley.

[14] Allen, N. L., Carlson, J. E., & Zelenak, C. A. (1999). op. cit.

## Revisions to the NAEP 1990 and 1992 Mathematics Findings

After the NAEP 1994 assessment was conducted, a technical problem was discovered in the procedures used to develop the NAEP mathematics scale used to report the 1992 mathematics assessment. This error affected the mathematics scale scores reported in 1992. The technical error has been corrected, and the revised national and state scale score results for 1992 are presented in the NAEP 1996 mathematics reports. The technical problem is described in greater detail in *The NAEP 1996 Technical Report*.[15] A brief summary of the problem is presented in the *NAEP 1996 Mathematics Report Card for the Nation and the States*.[16]

## Discussion of the Grade 8 Asian/Pacific Islander Sample

As noted earlier, scale score and achievement level results for eighth grade Asian/Pacific Islander students are not included in the main body of this report. The decision to exclude these results was made following a thorough investigation by the current NAEP grantees (Westat and ETS)[17, 18] into the quality and credibility of these results, as well as an independent review by a committee of statisticians from the National Institute of Statistical Sciences (NISS).[19] Collateral results from the grade 8 state assessment program in mathematics suggested that the 1996 national results may substantially underestimate actual achievement of the Asian/Pacific Islander group. Because of its potential to misinform, NCES decided to omit the national grade 8 Asian/Pacific Islander results from the body of the report. The results are, however, included in this appendix along with a description of the findings that led to this decision.

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[15] Ibid.

[16] Reese, C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). op. cit.

[17] Carlson, J., & Williams, P. (1996, October 29) ETS/NAEP Technical Memorandum on 1996 Mathematics Grade 8 results for Asian/Pacific Island Subpopulation.

[18] Rust, K. (1996, November 1) Westat Memorandum to Gary Phillips on 1996 Mathematics Grade 8 Results for Asian and Pacific Islander Students.

[19] Letter from Jerome Sacks to Gary Phillips, dated November 21, 1996.

Concerns about the accuracy of the grade 8 Asian/Pacific Islander results were initially noted during routine quality control of the NAEP 1996 mathematics assessment results. Despite statistically significant gains from 1992 to 1996 in average scale scores for the nation as a whole at all three grade levels, a large apparent decline in average scores was observed for the grade 8 Asian/Pacific Islander subgroup. Table A.1 contains average mathematics scale score estimates, and their standard errors, for the Asian/Pacific Islander subgroup for the 1990, 1992, and 1996 assessment years. From 1992 to 1996, the estimated decline in average scores for this subgroup was approximately 14 scale score points (about .4 within-grade standard deviation units) on the NAEP 500-point scale. Despite the large magnitude of this apparent decline, it is not statistically significant at the .05 level, after controlling for multiple comparisons.

**Table A.1** ***Average Mathematics Scale Scores for the Grade 8 Asian/Pacific Islander Subgroup***

THE NATION'S REPORT CARD naep

| | 1990 | | 1992 | | 1996 | |
|---|---|---|---|---|---|---|
| | Percentage | Average Scale Score | Percentage | Average Scale Score | Percentage | Average Scale Score |
| All Students | 100 | 263 (1.3) | 100 | 268 (0.9)* | 100 | 272 (1.1)*† |
| Students Who Indicated Their Race/Ethnicity as... | | | | | | |
| Asian/Pacific Islander | 2 (0.5)! | 279 (4.8)! | 3 (0.2) | 288 (5.4) | 3 (0.2) | 274 (3.9) |

The standard errors of the estimated percentages and average scale scores appear in parentheses.
* Indicates a significant difference from 1990.
† Indicates a significant difference from 1992.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistic does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

The data from the NAEP state assessment program in mathematics provided an independent data source to aid in evaluating the accuracy of the national grade 8 NAEP results for Asian/Pacific Islander students as well as for other subgroups. Forty states and the District of Columbia participated in the state assessment. Results based on the combined data from these jurisdictions are quite stable in that they are based on a sample of approximately 4,000 schools and over 100,000 students. Because of the voluntary nature of the state assessment program, these aggregated state results are not nationally representative. They can, however, be compared to restricted national results, calculated using public-school data from only those states participating in the state assessment, to obtain valuable insight into the quality of the national estimates for the grade 8 race/ethnicity subgroups.

Table A.2 contains restricted national results. Results are presented separately for four of the race/ethnicity subgroups: White, Black, Hispanic, and Asian/Pacific Islander. Aggregated state results are also presented for these same four subgroups. For three of the four subgroups, the difference between the restricted national estimates and aggregated state estimates are quite small. However, for the Asian/Pacific Islander subgroup, the difference between the two estimates, though again within reasonable bounds of sampling variability, is of considerably greater magnitude and the restricted national estimates are substantially lower than those obtained from the aggregated state data. These results suggest that the national grade 8 Asian/Pacific Islander results may substantially underestimate the performance of this subgroup. NCES was concerned that publishing the national results in the absence of the kind of discussion included in this appendix was potentially misinforming. Hence, NCES made the decision to omit the results from the body of the report and to include them in this appendix.

**Table A.2** ***Average Mathematics Scale Scores by Race/Ethnicity for Restricted National and Aggregated State Samples***

THE NATION'S REPORT CARD

| | Restricted National Sample | Aggregated State Sample | Difference |
|---|---|---|---|
| **Grade 8** | | | |
| Students Who indicated Their Race/Ethnicity as… | | | |
| White | 280.7 | 280.0 | 0.7 |
| Black | 242.8 | 242.3 | 0.5 |
| Hispanic | 250.4 | 250.3 | 0.1 |
| Asian/Pacific Islander | 272.0 | 281.7 | −9.7 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

It is important to note that all NAEP results are estimates and are subject to some degree of sampling variability. If different samples of schools or students had been obtained, results for some subgroups would be higher than reported here and some would be lower. In most subgroups, particularly large subgroups or subgroups for which special sampling procedures are employed, estimates of performance are likely to remain similar from one sample to another. However, the national population of Asian/Pacific Islander students is small (about 3 percent of the national population), heterogeneous with respect to academic achievement, and highly clustered in certain locations and schools — factors that are associated with large sampling variability in survey results and reflected in the large standard errors associated with performance estimates for this subgroup. Furthermore, the sampling plan for the national assessment does not include explicit stratification procedures designed to mitigate these factors. It was the judgment of all three organizations (ETS, Westat, and NISS) that investigated these results that the occurrence of this large, but statistically nonsignificant, change in the grade 8 Asian/Pacific Islander results was a consequence of these three factors: (1) the heterogeneous nature of the Asian/Pacific Islander population, (2) the current NAEP sampling design, and (3) the sample sizes that were assessed.

NCES, working with its current NAEP contractors and other advisory groups, will continue to investigate cost-effective ways of improving the accuracy and stability of NAEP results beginning with the 1998 assessment. NCES will also continue to seek improvements as part of an ongoing redesign of NAEP for the year 2000 and beyond.

# Appendix B

# *Standard Errors*

The comparisons presented in this report are based on statistical tests that consider the magnitude of the difference between group averages or percentages and the standard errors of those statistics. The following appendix contains the standard errors for the averages and percentages discussed in Chapters 2 through 10. For ease of reference, the format and headings of each table in this appendix match the corresponding chapter table, although the numbers that appear are actually standard errors.

## Figure B2.2

### *Standard Errors for Average Proficiency in Mathematics Content Strands, Grades 4, 8, and 12*

THE NATION'S REPORT CARD

| | 1996 | 1992 | 1990 |
|---|---|---|---|
| **Grade 4** | | | |
| Overall Proficiency | 0.9 | 0.7 | 0.9 |
| Number Sense, Properties, & Operations | 1.0 | 0.8 | 1.1 |
| Measurement | 1.1 | 0.8 | 1.0 |
| Geometry & Spatial Sense | 0.8 | 0.6 | 0.9 |
| Data Analysis, Statistics, & Probability | 1.1 | 0.9 | – |
| Algebra & Functions | 1.0 | 0.9 | 0.9 |
| **Grade 8** | | | |
| Overall Proficiency | 1.1 | 0.9 | 1.3 |
| Number Sense, Properties, & Operations | 1.0 | 0.8 | 1.3 |
| Measurement | 1.4 | 1.2 | 1.6 |
| Geometry & Spatial Sense | 1.1 | 0.9 | 1.3 |
| Data Analysis, Statistics, & Probability | 1.5 | 1.0 | 1.6 |
| Algebra & Functions | 1.1 | 1.0 | 1.2 |
| **Grade 12** | | | |
| Overall Proficiency | 1.0 | 0.9 | 1.1 |
| Number Sense, Properties, & Operations | 1.2 | 0.9 | 1.1 |
| Measurement | 1.1 | 0.9 | 1.3 |
| Geometry & Spatial Sense | 1.1 | 1.0 | 1.3 |
| Data Analysis, Statistics, & Probability | 1.0 | 1.0 | 1.2 |
| Algebra & Functions | 1.2 | 1.0 | 1.2 |

– 1990 data are not available.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

**Table B2.1**

## Standard Errors for Average Proficency in Mathematics Content Strands by Gender, Grades 4, 8, and 12

THE NATION'S REPORT CARD naep

| | 1996 | | | 1992 | | | 1990 | | |
|---|---|---|---|---|---|---|---|---|---|
| | All Students | Male | Female | All Students | Male | Female | All Students | Male | Female |
| **Grade 4** | | | | | | | | | |
| Overall Proficiency | 0.9 | 1.1 | 1.0 | 0.7 | 0.8 | 1.0 | 0.9 | 1.2 | 1.1 |
| Number Sense, Properties, & Operations | 1.0 | 1.2 | 1.1 | 0.8 | 0.9 | 1.1 | 1.1 | 1.4 | 1.3 |
| Measurement | 1.1 | 1.2 | 1.2 | 0.8 | 1.0 | 1.0 | 1.0 | 1.3 | 1.3 |
| Geometry & Spatial Sense | 0.8 | 1.1 | 1.0 | 0.6 | 0.8 | 0.9 | 0.9 | 1.2 | 1.2 |
| Data Analysis, Statistics, & Probability | 1.1 | 1.4 | 1.3 | 0.9 | 0.9 | 1.2 | – | – | – |
| Algebra & Functions | 1.0 | 1.1 | 1.2 | 0.9 | 1.1 | 1.5 | 0.9 | 1.3 | 1.1 |
| **Grade 8** | | | | | | | | | |
| Overall Proficiency | 1.1 | 1.4 | 1.1 | 0.9 | 1.1 | 1.0 | 1.3 | 1.6 | 1.3 |
| Number Sense, Properties, & Operations | 1.0 | 1.4 | 1.1 | 0.8 | 1.0 | 1.0 | 1.3 | 1.6 | 1.3 |
| Measurement | 1.4 | 1.7 | 1.6 | 1.2 | 1.4 | 1.5 | 1.6 | 2.0 | 1.5 |
| Geometry & Spatial Sense | 1.1 | 1.3 | 1.3 | 0.9 | 1.1 | 1.0 | 1.3 | 1.6 | 1.3 |
| Data Analysis, Statistics, & Probability | 1.5 | 1.8 | 1.5 | 1.0 | 1.3 | 1.2 | 1.6 | 1.9 | 1.6 |
| Algebra & Functions | 1.1 | 1.5 | 1.1 | 1.0 | 1.2 | 1.2 | 1.2 | 1.6 | 1.3 |
| **Grade 12** | | | | | | | | | |
| Overall Proficiency | 1.0 | 1.1 | 1.1 | 0.9 | 1.1 | 1.0 | 1.1 | 1.4 | 1.3 |
| Number Sense, Properties, & Operations | 1.2 | 1.3 | 1.3 | 0.9 | 1.0 | 1.0 | 1.1 | 1.3 | 1.2 |
| Measurement | 1.1 | 1.3 | 1.3 | 0.9 | 1.2 | 1.1 | 1.3 | 1.5 | 1.5 |
| Geometry & Spatial Sense | 1.1 | 1.2 | 1.4 | 1.0 | 1.2 | 1.2 | 1.3 | 1.5 | 1.6 |
| Data Analysis, Statistics, & Probability | 1.0 | 1.2 | 1.1 | 1.0 | 1.1 | 1.1 | 1.2 | 1.4 | 1.5 |
| Algebra & Functions | 1.2 | 1.3 | 1.2 | 1.0 | 1.2 | 1.1 | 1.2 | 1.4 | 1.3 |

– 1990 data are not available.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure B2.3

## Standard Errors for Average Mathematics Proficiency, Composite Scale by Race/Ethnicity, Grades 4, 8, and 12

THE NATION'S REPORT CARD naep

| | 1996 Average Scale Score | 1992 Average Scale Score | 1990 Average Scale Score |
|---|---|---|---|
| **Grade 4** | | | |
| All Students | 0.9 | 0.7 | 0.9 |
| White | 0.9 | 0.9 | 1.1 |
| Black | 2.3 | 1.3 | 1.8 |
| Hispanic | 2.1 | 1.4 | 2.0 |
| Asian/Pacific Islander | 4.1 | 2.3 | 3.5 |
| American Indian | 2.3 | 3.1 | 3.9 |
| **Grade 8** | | | |
| All Students | 1.1 | 0.9 | 1.3 |
| White | 1.2 | 1.0 | 1.4 |
| Black | 2.0 | 1.3 | 2.7 |
| Hispanic | 2.0 | 1.2 | 2.8 |
| Asian/Pacific Islander | – – | 5.4 | 4.8! |
| American Indian | 3.0! | 2.8 | 9.4! |
| **Grade 12** | | | |
| All Students | 1.0 | 0.9 | 1.1 |
| White | 1.0 | 0.9 | 1.2 |
| Black | 2.2 | 1.7 | 1.9 |
| Hispanic | 1.8 | 1.7 | 2.8 |
| Asian/Pacific Islander | 4.8 | 3.5 | 5.2 |
| American Indian | 8.9! | *** | *** |

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure B2.4

## Standard Errors for Average Proficiency in Number Sense, Properties, and Operations by Race/Ethnicity, Grades 4, 8, and 12

THE NATION'S REPORT CARD naep

| | 1996 Average Scale Score | 1992 Average Scale Score | 1990 Average Scale Score |
|---|---|---|---|
| **Grade 4** | | | |
| All Students | 1.0 | 0.8 | 1.1 |
| White | 1.0 | 0.9 | 1.3 |
| Black | 2.7 | 1.3 | 1.9 |
| Hispanic | 2.2 | 1.8 | 2.2 |
| Asian/Pacific Islander | 4.8 | 2.5 | 3.6 |
| American Indian | 2.6 | 3.3 | 4.0 |
| **Grade 8** | | | |
| All Students | 1.0 | 0.8 | 1.3 |
| White | 1.2 | 0.9 | 1.3 |
| Black | 2.3 | 1.3 | 2.8 |
| Hispanic | 1.9 | 1.5 | 2.7 |
| Asian/Pacific Islander | – – | 5.2 | 4.5! |
| American Indian | 3.9! | 2.7 | 10.1! |
| **Grade 12** | | | |
| All Students | 1.2 | 0.9 | 1.1 |
| White | 1.2 | 0.9 | 1.2 |
| Black | 2.4 | 1.5 | 1.8 |
| Hispanic | 1.7 | 1.8 | 2.9 |
| Asian/Pacific Islander | 5.1 | 3.8 | 4.8 |
| American Indian | 10.6! | *** | *** |

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

## Figure B2.5

### *Standard Errors for Average Proficiency in Measurement by Race/Ethnicity, Grades 4, 8, and 12*

THE NATION'S REPORT CARD naep

| | 1996 Average Scale Score | 1992 Average Scale Score | 1990 Average Scale Score |
|---|---|---|---|
| **Grade 4** | | | |
| All Students | 1.1 | 0.8 | 1.0 |
| White | 1.2 | 1.0 | 1.3 |
| Black | 2.5 | 1.7 | 2.3 |
| Hispanic | 2.5 | 1.6 | 2.3 |
| Asian/Pacific Islander | 4.6 | 3.4 | 4.8 |
| American Indian | 2.8 | 3.5 | 4.8 |
| **Grade 8** | | | |
| All Students | 1.4 | 1.2 | 1.6 |
| White | 1.5 | 1.3 | 1.7 |
| Black | 2.6 | 1.9 | 3.2 |
| Hispanic | 2.8 | 1.7 | 3.3 |
| Asian/Pacific Islander | – – | 7.1 | 6.4! |
| American Indian | 4.5! | 4.1 | 10.2! |
| **Grade 12** | | | |
| All Students | 1.1 | 0.9 | 1.3 |
| White | 1.1 | 1.0 | 1.4 |
| Black | 2.2 | 1.8 | 2.2 |
| Hispanic | 2.1 | 1.8 | 3.0 |
| Asian/Pacific Islander | 6.6 | 4.0 | 6.1 |
| American Indian | 11.9! | *** | *** |

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure B2.6

## Standard Errors for Average Proficiency in Geometry and Spatial Sense by Race/Ethnicity, Grades 4, 8, and 12

THE NATION'S REPORT CARD naep

| | 1996 Average Scale Score | 1992 Average Scale Score | 1990 Average Scale Score |
|---|---|---|---|
| **Grade 4** | | | |
| All Students | 0.8 | 0.6 | 0.9 |
| White | 0.9 | 0.8 | 1.1 |
| Black | 1.5 | 1.4 | 1.6 |
| Hispanic | 2.3 | 1.3 | 1.9 |
| Asian/Pacific Islander | 4.2 | 2.5 | 4.6 |
| American Indian | 2.8 | 3.4 | 4.0 |
| **Grade 8** | | | |
| All Students | 1.1 | 0.9 | 1.3 |
| White | 1.3 | 1.1 | 1.4 |
| Black | 2.5 | 1.7 | 3.1 |
| Hispanic | 2.4 | 1.2 | 2.5 |
| Asian/Pacific Islander | – – | 5.1 | 5.0! |
| American Indian | 3.5! | 3.3 | 8.5! |
| **Grade 12** | | | |
| All Students | 1.1 | 1.0 | 1.3 |
| White | 1.2 | 1.1 | 1.5 |
| Black | 2.4 | 1.8 | 2.1 |
| Hispanic | 2.6 | 2.4 | 2.9 |
| Asian/Pacific Islander | 4.3 | 3.5 | 5.8 |
| American Indian | 7.9! | *** | *** |

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

## Figure B2.7

### *Standard Errors for Average Proficiency in Data Analysis, Statistics, and Probability by Race/Ethnicity, Grades 4, 8, and 12*

THE NATION'S REPORT CARD naep

| | 1996 Average Scale Score | 1992 Average Scale Score | 1990 Average Scale Score |
|---|---|---|---|
| **Grade 4** | | | |
| All Students | 1.1 | 0.9 | – |
| White | 1.1 | 1.1 | – |
| Black | 3.5 | 1.6 | – |
| Hispanic | 2.4 | 1.4 | – |
| Asian/Pacific Islander | 4.7 | 3.0 | – |
| American Indian | 2.5 | 3.2 | – |
| **Grade 8** | | | |
| All Students | 1.5 | 1.0 | 1.6 |
| White | 1.8 | 1.1 | 1.6 |
| Black | 2.2 | 1.7 | 3.2 |
| Hispanic | 2.5 | 1.5 | 3.3 |
| Asian/Pacific Islander | – – | 6.3 | 5.4! |
| American Indian | 4.5! | 2.9 | 11.5! |
| **Grade 12** | | | |
| All Students | 1.0 | 1.0 | 1.2 |
| White | 0.9 | 1.0 | 1.3 |
| Black | 2.4 | 1.9 | 2.3 |
| Hispanic | 2.0 | 2.2 | 3.7 |
| Asian/Pacific Islander | 5.7 | 4.4 | 5.5 |
| American Indian | 8.3! | *** | *** |

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

– 1990 data are not available.

! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

| Figure B2.8 | Standard Errors for Average Proficiency in Algebra and Functions by Race/Ethnicity, Grades 4, 8, and 12 |
|---|---|

THE NATION'S REPORT CARD naep

| | 1996 Average Scale Score | 1992 Average Scale Score | 1990 Average Scale Score |
|---|---|---|---|
| **Grade 4** | | | |
| All Students | 1.0 | 0.9 | 0.9 |
| White | 1.0 | 1.0 | 1.1 |
| Black | 2.5 | 1.6 | 1.8 |
| Hispanic | 2.4 | 1.7 | 2.2 |
| Asian/Pacific Islander | 3.8 | 3.1 | 3.4 |
| American Indian | 2.3 | 3.4 | 3.8 |
| **Grade 8** | | | |
| All Students | 1.1 | 1.0 | 1.2 |
| White | 1.2 | 1.2 | 1.4 |
| Black | 2.0 | 2.0 | 2.6 |
| Hispanic | 2.0 | 1.4 | 2.9 |
| Asian/Pacific Islander | – – | 5.3 | 5.1! |
| American Indian | 3.2! | 2.9 | 8.3! |
| **Grade 12** | | | |
| All Students | 1.2 | 1.0 | 1.2 |
| White | 1.2 | 1.0 | 1.3 |
| Black | 2.8 | 2.1 | 2.0 |
| Hispanic | 1.9 | 1.7 | 2.8 |
| Asian/Pacific Islander | 5.0 | 3.4 | 5.1 |
| American Indian | 8.0! | *** | *** |

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

## Figure B2.9

## Standard Errors for Average Proficiency in Mathematics Content Areas by Course Taking, Grade 8

THE NATION'S REPORT CARD naep

| | Assessment Year 1996 | | |
|---|---|---|---|
| | Eighth-Grade Mathematics | Pre-Algebra | Algebra |
| **Content Area** | Average Proficiency | Average Proficiency | Average Proficiency |
| Number Sense, Properties, & Operations | 1.4 | 1.4 | 1.6 |
| Measurement | 2.0 | 2.6 | 2.3 |
| Geometry & Spatial Sense | 1.5 | 1.5 | 1.7 |
| Data Analysis, Statistics, & Probability | 1.7 | 2.0 | 2.6 |
| Algebra & Functions | 1.4 | 1.4 | 1.6 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Figure B2.10

## Standard Errors for Average Proficiency in Mathematics Content Areas by Algebra and Calculus Courses Taken, Grade 12

THE NATION'S REPORT CARD naep

| | Course Taken For Assessment Year 1996 | | | | | |
|---|---|---|---|---|---|---|
| **Content Area** | Have Not Studied Algebra or Pre-Algebra | Only Taken Pre-Algebra | Only Taken Algebra I | Taken Algebra II But Not Beyond | Taken Algebra III or Pre-Calculus But Not Calculus | Calculus |
| Number Sense, Properties, & Operations | 2.7 | 2.2 | 1.6 | 1.2 | 1.6 | 2.3 |
| Measurement | 4.3 | 3.4 | 1.8 | 1.1 | 1.4 | 3.1 |
| Geometry & Spatial Sense | 4.2 | 2.7 | 1.9 | 1.1 | 1.5 | 2.0 |
| Data Analysis, Statistics, & Probability | 3.5 | 3.1 | 1.6 | 0.9 | 1.4 | 2.7 |
| Algebra & Functions | 3.1 | 2.4 | 1.8 | 1.1 | 1.6 | 2.1 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Figure B2.11

### Standard Errors for Average Proficiency in Mathematics Content Areas by Geometry Course Taken, Grade 12

THE NATION'S REPORT CARD naep

| | Assessment Year 1996 | |
|---|---|---|
| | Have Not Taken Geometry | Have Taken Geometry |
| Content Area | Average Proficiency | Average Proficiency |
| Number Sense, Properties, & Operations | 1.6 | 1.1 |
| Measurement | 2.2 | 0.9 |
| Geometry & Spatial Sense | 1.9 | 0.9 |
| Data Analysis, Statistics, & Probability | 2.1 | 0.8 |
| Algebra & Functions | 2.0 | 1.0 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Figure B2.12

### Standard Errors for Average Proficiency in Mathematics Content Areas by Probability or Statistics Course Taken, Grade 12

THE NATION'S REPORT CARD naep

| | Assessment Year 1996 | |
|---|---|---|
| | Have Not Taken Probability & Statistics | Have Taken Probability & Statistics |
| Content Area | Average Proficiency | Average Proficiency |
| Number Sense, Properties, & Operations | 1.2 | 2.7 |
| Measurement | 1.1 | 3.2 |
| Geometry & Spatial Sense | 1.1 | 2.7 |
| Data Analysis, Statistics, & Probability | 0.9 | 2.8 |
| Algebra & Functions | 1.2 | 2.7 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Figure B2.13**

## Standard Errors for Average Proficiency in Mathematics Content Areas by Number of Semesters of Mathematics Courses Taken in Grades 9 through 12, Grade 12

THE NATION'S REPORT CARD naep

| Content Area | Average Proficiency by Number of Semesters, Assessment Year 1996<br>1–2 Semesters | 3–4 Semesters | 5–6 Semesters | 7 or More Semesters |
|---|---|---|---|---|
| Number Sense, Properties, & Operations | 2.3 | 1.1 | 1.9 | 1.2 |
| Measurement | 5.5 | 1.7 | 1.8 | 1.0 |
| Geometry & Spatial Sense | 3.2 | 1.1 | 1.5 | 0.9 |
| Data Analysis, Statistics, & Probability | 3.1 | 1.3 | 1.4 | 1.1 |
| Algebra & Functions | 2.3 | 1.2 | 1.4 | 1.3 |

NOTE: Sample size for 0 semesters is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B3.1 Standard Errors for Score Percentages for "Evaluate Expression for Odd/Even"

THE NATION'S REPORT CARD naep

| | Correct | Incorrect | | Omit |
|---|---|---|---|---|
| | 3 Correct Entries | 1 or 2 Correct Entries | No Correct Entries | |
| **Grade 12** | | | | |
| Overall | 1.6 | 1.6 | 0.5 | 0.8 |
| Males | 2.1 | 2.1 | 1.0 | 1.4 |
| Females | 2.0 | 2.1 | 0.5 | 0.9 |
| White | 1.7 | 1.9 | 0.4 | 0.8 |
| Black | 4.2 | 3.5 | 2.6 | 2.1 |
| Hispanic | 3.3 | 3.5 | 2.1 | 3.3 |
| Asian/Pacific Islander | 7.2 | 6.8 | 1.4 | 2.9 |
| American Indian | *** | *** | *** | *** |
| Geometry Taken | 1.7 | 1.7 | 0.5 | 0.8 |
| Highest Algebra-Calculus Course Taken: | | | | |
| Pre-Algebra | 6.6 | 5.5 | 2.2 | 3.4 |
| First-Year Algebra | 3.1 | 3.5 | 1.5 | 1.5 |
| Second-Year Algebra | 2.1 | 2.2 | 0.4 | 0.7 |
| Third-Year Algebra/Pre-Calculus | 3.7 | 3.8 | 0.7 | 2.4 |
| Calculus | 7.1 | 5.5 | 0.8 | 3.9 |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B3.2 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Evaluate Expression for Odd/Even"

THE NATION'S REPORT CARD naep

| | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.6 | 2.3 | 2.1 | 4.8 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B3.3 Standard Errors for Percentage Correct for "Multiply Two Negative Integers"

THE NATION'S REPORT CARD naep

| Grade 8 | Percentage Correct |
|---|---|
| Overall | 2.1 |
| Males | 2.6 |
| Females | 2.8 |
| White | 3.0 |
| Black | 3.2 |
| Hispanic | 3.4 |
| Asian/Pacific Islander | – – |
| American Indian | *** |
| Mathematics Course Taking: | |
| Eighth-Grade Mathematics | 2.4 |
| Pre-Algebra | 4.0 |
| Algebra | 2.6 |

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B3.4 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Multiply Two Negative Integers"

THE NATION'S REPORT CARD naep

| | NAEP Grade 8 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 2.1 | 2.4 | 3.6 | 3.6 | 3.8 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

THE NATION'S REPORT CARD naep

## Table B3.5 Standard Errors for Percentage Correct for "Use Subtraction in a Problem"

| Grade 4 | Percentage Correct |
|---|---|
| Overall | 1.4 |
| Males | 1.9 |
| Females | 2.2 |
| White | 1.6 |
| Black | 4.6 |
| Hispanic | 3.7 |
| Asian/Pacific Islander | *** |
| American Indian | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

THE NATION'S REPORT CARD naep

## Table B3.6 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Subtraction in a Problem"

| | NAEP Grade 4 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.4 | 2.5 | 2.4 | 1.6 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B3.7**

## Standard Errors for Percentage Correct for "Choose a Number Sentence"

| Grade 4 | Percentage Correct |
|---|---|
| Overall | 1.5 |
| Males | 2.2 |
| Females | 2.0 |
| White | 1.9 |
| Black | 4.0 |
| Hispanic | 3.2 |
| Asian/Pacific Islander | 6.8 |
| American Indian | *** |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B3.8**

## Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Choose a Number Sentence"

| | NAEP Grade 4 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.5 | 2.4 | 2.4 | 3.8 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B3.9

### *Standard Errors for Score Percentages for "Reason to Maximize Difference"*

| Grade 8 | Extended | Satisfactory | Partial | Minimal | Incorrect | Omit |
|---|---|---|---|---|---|---|
| Overall | 0.3 | 1.0 | 1.0 | 1.4 | 1.3 | 0.9 |
| Males | 0.4 | 1.2 | 1.2 | 1.9 | 1.5 | 1.1 |
| Females | 0.5 | 1.5 | 1.8 | 2.1 | 2.1 | 1.0 |
| White | 0.4 | 1.4 | 1.2 | 1.8 | 1.7 | 1.2 |
| Black | - - - | 1.3 | 2.0 | 2.7 | 3.0 | 1.6 |
| Hispanic | - - - | 1.2 | 2.8 | 3.9 | 4.0 | 1.4 |
| Asian/Pacific Islander | – – | – – | – – | – – | – – | – – |
| American Indian | *** | *** | *** | *** | *** | *** |
| Mathematics Course Taking: | | | | | | |
| Eighth-Grade Mathematics | 0.5 | 1.5 | 1.7 | 2.2 | 2.4 | 1.9 |
| Pre-Algebra | - - - | 1.7 | 2.0 | 2.9 | 3.2 | 1.0 |
| Algebra | 0.8 | 2.5 | 2.0 | 2.0 | 2.3 | 0.9 |

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B3.10

### *Standard Errors for Percentage at Least Satisfactory Within Achievement-Level Intervals for "Reason to Maximize Difference"*

| | NAEP Grade 8 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.0 | 1.0 | 1.8 | 3.2 | 8.1 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

THE NATION'S REPORT CARD naep

## Table B3.11 Standard Errors for Score Percentages for "Solve a Multistep Problem"

| Grade 4 | Correct | Partial | Incorrect | Omit |
|---|---|---|---|---|
| Overall | 1.4 | 1.2 | 1.6 | 0.9 |
| Males | 2.1 | 1.9 | 2.3 | 1.3 |
| Females | 1.6 | 1.6 | 1.9 | 1.1 |
| White | 1.8 | 1.8 | 2.0 | 1.1 |
| Black | 1.7 | 2.1 | 4.4 | 3.0 |
| Hispanic | 2.0 | 2.9 | 3.6 | 2.0 |
| Asian/Pacific Islander | *** | *** | *** | *** |
| American Indian | *** | *** | *** | *** |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

THE NATION'S REPORT CARD naep

## Table B3.12 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Solve a Multistep Problem"

| | NAEP Grade 4 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.4 | 0.6 | 1.6 | 4.0 | *** |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B3.13 Standard Errors for Percentage Correct for "Relate a Fraction to 1"

| Grade 4 | Percentage Correct |
|---|---|
| Overall | 1.7 |
| Males | 2.0 |
| Females | 2.3 |
| White | 2.3 |
| Black | 4.0 |
| Hispanic | 3.8 |
| Asian/Pacific Islander | 6.7 |
| American Indian | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B3.14 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Relate a Fraction to 1"

| | NAEP Grade 4 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.7 | 2.9 | 2.6 | 3.4 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B3.15 — Standard Errors for Percentage Correct for "Find Amount of Restaurant Tip"

THE NATION'S REPORT CARD naep

| Grade 8 | Percentage Correct |
|---|---|
| Overall | 1.9 |
| Males | 2.3 |
| Females | 2.4 |
| White | 2.4 |
| Black | 4.3 |
| Hispanic | 3.2 |
| Asian/Pacific Islander | – – |
| American Indian | *** |
| Mathematics Course Taking: | |
| Eighth-Grade Mathematics | 2.5 |
| Pre-Algebra | 3.5 |
| Algebra | 2.2 |

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B3.16 — Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Find Amount of Restaurant Tip"

THE NATION'S REPORT CARD naep

| | NAEP Grade 8 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.9 | 2.4 | 2.9 | 3.6 | 9.4 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B3.17 — Standard Errors for Score Percentages for "Use Percent Increase"

THE NATION'S REPORT CARD naep

| | Correct | Partial | Incorrect | Omit |
|---|---|---|---|---|
| **Grade 8** | | | | |
| Overall | 0.2 | 1.3 | 1.5 | 0.9 |
| Males | 0.3 | 1.4 | 1.9 | 1.4 |
| Females | 0.4 | 2.0 | 2.0 | 1.1 |
| White | 0.3 | 1.9 | 1.8 | 1.0 |
| Black | - - - | 3.2 | 3.8 | 2.2 |
| Hispanic | - - - | 3.0 | 4.0 | 3.5 |
| Asian/Pacific Islander | – – | – – | – – | – – |
| American Indian | *** | *** | *** | *** |
| Mathematics Course Taking: | | | | |
| Eighth-Grade Mathematics | - - - | 1.8 | 2.1 | 1.6 |
| Pre-Algebra | - - - | 2.5 | 2.9 | 1.7 |
| Algebra | 0.8 | 3.2 | 3.1 | 1.8 |
| **Grade 12** | | | | |
| Overall | 0.5 | 1.7 | 1.6 | 0.9 |
| Males | 0.8 | 2.1 | 2.0 | 1.4 |
| Females | 0.5 | 1.9 | 2.1 | 1.1 |
| White | 0.7 | 1.8 | 2.0 | 1.4 |
| Black | - - - | 3.6 | 4.7 | 4.1 |
| Hispanic | 0.8 | 3.3 | 4.2 | 3.0 |
| Asian/Pacific Islander | 2.7 | 6.5 | 6.2 | 6.7 |
| American Indian | *** | *** | *** | *** |
| Geometry Taken | 0.5 | 1.8 | 1.5 | 1.0 |
| Highest Algebra-Calculus Course Taken: | | | | |
| Pre-Algebra | *** | *** | *** | *** |
| First-Year Algebra | 0.7 | 2.2 | 3.2 | 2.3 |
| Second-Year Algebra | 0.5 | 1.5 | 1.6 | 1.1 |
| Third-Year Algebra/Pre-Calculus | 2.4 | 6.2 | 4.3 | 1.4 |
| Calculus | 3.4 | 6.7 | 8.0 | 2.8 |

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B3.18

### Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Percent Increase"

THE NATION'S REPORT CARD naep

| | Overall | NAEP Grades 8 and 12 Composite Scale Ranges | | | |
|---|---|---|---|---|---|
| | | Below Basic | Basic | Proficient | Advanced |
| Grade 8 | 0.2 | - - - | - - - | - - - | - - - |
| Grade 12 | 0.5 | - - - | 0.5 | 3.5 | *** |

*** Sample size is insufficient to permit a reliable estimate.
- - - Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B3.19

### Standard Errors for Percentage Correct for "Solve a Rate Versus Time Problem"

THE NATION'S REPORT CARD naep

| Grade 12 | Percentage Correct |
|---|---|
| Overall | 1.4 |
| Males | 2.2 |
| Females | 2.0 |
| White | 1.5 |
| Black | 3.3 |
| Hispanic | 3.3 |
| Asian/Pacific Islander | 6.0 |
| American Indian | *** |
| Geometry Taken | 1.5 |
| Highest Algebra-Calculus Course Taken: | |
| Pre-Algebra | 6.8 |
| First-Year Algebra | 3.0 |
| Second-Year Algebra | 2.3 |
| Third-Year Algebra/Pre-Calculus | 3.6 |
| Calculus | 5.4 |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.20

## Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Solve a Rate Versus Time Problem"

THE NATION'S REPORT CARD naep

| | NAEP Grade12 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.4 | 2.5 | 2.1 | 4.8 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B4.1**

## *Standard Errors for Percentage Correct for "Recognize Best Unit of Measurement"*

THE NATION'S REPORT CARD naep

| | Percentage Correct |
|---|---|
| **Grade 8** | |
| Overall | 1.5 |
| Males | 2.1 |
| Females | 2.1 |
| White | 1.9 |
| Black | 3.5 |
| Hispanic | 4.0 |
| Asian/Pacific Islander | – – |
| American Indian | *** |
| Mathematics Course Taking: | |
| Eighth-Grade Mathematics | 2.0 |
| Pre-Algebra | 2.5 |
| Algebra | 2.7 |
| **Grade 12** | |
| Overall | 1.0 |
| Males | 1.5 |
| Females | 1.2 |
| White | 1.1 |
| Black | 3.7 |
| Hispanic | 3.6 |
| Asian/Pacific Islander | 4.1 |
| American Indian | *** |
| Geometry Taken | 1.2 |
| Highest Algebra-Calculus Course Taken: | |
| Pre-Algebra | *** |
| First-Year Algebra | 2.3 |
| Second-Year Algebra | 1.3 |
| Third-Year Algebra/Pre-Calculus | 2.9 |
| Calculus | 1.3 |

*** Sample size is insufficient to permit a reliable estimate.

–– Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B4.2 — Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Recognize Best Unit of Measurement"

THE NATION'S REPORT CARD naep

| | Overall | NAEP Grades 8 and 12 Composite Scale Ranges | | | |
|---|---|---|---|---|---|
| | | Below Basic | Basic | Proficient | Advanced |
| Grade 8 | 1.5 | 3.0 | 2.2 | 1.4 | - - - |
| Grade 12 | 1.0 | 2.8 | 1.0 | - - - | *** |

*** Sample size is insufficient to permit a reliable estimate.

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B4.3 — Standard Errors for Percentage Correct for "Use Conversion Units of Length"

THE NATION'S REPORT CARD naep

| | Percentage Correct |
|---|---|
| **Grade 8** | |
| Overall | 1.6 |
| Males | 2.6 |
| Females | 2.1 |
| White | 2.3 |
| Black | 2.6 |
| Hispanic | 2.9 |
| Asian/Pacific Islander | – – |
| American Indian | *** |
| Mathematics Course Taking: | |
| Eighth-Grade Mathematics | 2.3 |
| Pre-Algebra | 2.7 |
| Algebra | 2.8 |

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B4.4 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Conversion Units of Length"

THE NATION'S REPORT CARD naep

| | NAEP Grade 8 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.6 | 1.5 | 2.9 | 3.6 | 8.0 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B4.5 Standard Errors for Score Percentages for "Use Protractor to Draw a 235° Arc on a Circle"

THE NATION'S REPORT CARD naep

| | Correct | | Incorrect | | | Omit |
|---|---|---|---|---|---|---|
| | ( ± 2° ) | ( ± 3–5° ) | No "A" Endpoint | Arc Not Indicated | Other | |
| **Grade 12** | | | | | | |
| Overall | 1.1 | 0.8 | 0.2 | 0.5 | 1.4 | 1.0 |
| Males | 1.6 | 1.6 | - - - | 0.7 | 2.2 | 1.3 |
| Females | 1.5 | 1.1 | 0.3 | 0.7 | 1.6 | 1.4 |
| White | 1.4 | 1.1 | 0.2 | 0.6 | 1.8 | 1.0 |
| Black | 1.6 | 1.7 | - - - | 1.6 | 3.4 | 3.2 |
| Hispanic | 2.2 | 1.9 | - - - | 1.6 | 4.4 | 5.2 |
| Asian/Pacific Islander | 7.7 | 4.3 | - - - | 1.5 | 7.0 | 2.5 |
| American Indian | *** | *** | *** | *** | *** | *** |
| Geometry Taken | 1.3 | 1.0 | 0.2 | 0.6 | 1.6 | 1.0 |
| Highest Algebra-Calculus Course Taken: | | | | | | |
| Pre-Algebra | 5.6 | - - - | 0.0 | 3.5 | 6.5 | 2.7 |
| First-Year Algebra | 1.9 | 2.4 | 0.0 | 1.5 | 3.2 | 2.0 |
| Second-Year Algebra | 1.7 | 1.1 | 0.3 | 0.6 | 1.8 | 1.4 |
| Third-Year Algebra/Pre-Calculus | 2.5 | 2.8 | - - - | 2.0 | 3.7 | 1.5 |
| Calculus | 4.8 | 3.5 | 0.0 | 0.7 | 5.2 | 1.2 |

*** Sample size is insufficient to permit a reliable estimate.
- - - Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B4.6

### *Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Protractor to Draw a 235° Arc on a Circle"*

THE NATION'S REPORT CARD naep

| | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| **Overall** | **Below Basic** | **Basic** | **Proficient** | **Advanced** |
| 1.6 | 1.8 | 2.3 | 4.5 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B4.7

### *Standard Errors for Percentage Correct for "Relate Perimeter to Side Length"*

THE NATION'S REPORT CARD naep

| | Percentage Correct |
|---|---|
| **Grade 4** | |
| Overall | 1.4 |
| Males | 1.6 |
| Females | 2.0 |
| White | 1.8 |
| Black | 2.9 |
| Hispanic | 2.8 |
| Asian/Pacific Islander | 7.1 |
| American Indian | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B4.8

### *Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Relate Perimeter to Side Length"*

THE NATION'S REPORT CARD naep

| | NAEP Grade 4 Composite Scale Range | | | |
|---|---|---|---|---|
| **Overall** | **Below Basic** | **Basic** | **Proficient** | **Advanced** |
| 1.4 | 2.0 | 2.5 | 4.2 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

| Table B4.9 | *Standard Errors for Score Percentages for "Find Volume of a Cylinder"* | | | THE NATION'S REPORT CARD naep |
|---|---|---|---|---|

| | Correct | Partial | Incorrect | Omit |
|---|---|---|---|---|
| **Grade 8** | | | | |
| Overall | 1.1 | 1.2 | 1.4 | 1.0 |
| Males | 1.4 | 1.5 | 2.2 | 1.4 |
| Females | 1.7 | 1.9 | 2.1 | 1.5 |
| White | 1.4 | 1.7 | 2.0 | 1.3 |
| Black | 1.2 | 2.1 | 2.9 | 2.9 |
| Hispanic | 1.8 | 1.8 | 3.9 | 3.8 |
| Asian/Pacific Islander | –– | –– | –– | –– |
| American Indian | *** | *** | *** | *** |
| Mathematics Course Taking: | | | | |
| Eighth-Grade Mathematics | 1.4 | 1.9 | 2.3 | 1.9 |
| Pre-Algebra | 1.7 | 2.2 | 2.7 | 1.6 |
| Algebra | 2.2 | 2.7 | 2.6 | 1.4 |
| **Grade 12** | | | | |
| Overall | 1.5 | 1.2 | 1.6 | 0.9 |
| Males | 2.1 | 1.4 | 2.5 | 1.4 |
| Females | 2.1 | 2.0 | 2.3 | 1.0 |
| White | 1.9 | 1.6 | 2.2 | 1.0 |
| Black | 3.6 | 3.4 | 3.8 | 2.9 |
| Hispanic | 4.1 | 3.4 | 4.0 | 4.2 |
| Asian/Pacific Islander | 6.3 | 3.4 | 5.5 | 1.8 |
| American Indian | *** | *** | *** | *** |
| Geometry Taken | 1.6 | 1.3 | 1.7 | 0.7 |
| Highest Algebra-Calculus Course Taken: | | | | |
| Pre-Algebra | *** | *** | *** | *** |
| First-Year Algebra | 2.0 | 2.3 | 2.8 | 2.1 |
| Second-Year Algebra | 1.8 | 1.6 | 1.8 | 0.9 |
| Third-Year Algebra/Pre-Calculus | 3.7 | 3.7 | 4.0 | 0.9 |
| Calculus | 6.5 | 6.6 | 3.5 | - - - |

*** Sample size is insufficient to permit a reliable estimate.

–– Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B4.10**

## Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Find Volume of a Cylinder"

THE NATION'S REPORT CARD naep

| | Overall | NAEP Grades 8 and 12 Composite Scale Ranges | | | |
|---|---|---|---|---|---|
| | | Below Basic | Basic | Proficient | Advanced |
| Grade 8 | 1.1 | - - - | 1.6 | 3.6 | 10.1 |
| Grade 12 | 1.5 | 1.6 | 2.1 | 4.6 | *** |

*** Sample size is insufficient to permit a reliable estimate.
- - - Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B4.11**

## Standard Errors for Score Percentages for "Use a Ruler to Find the Circumference of a Circle"

THE NATION'S REPORT CARD naep

| | Correct | | Incorrect | | Omit |
|---|---|---|---|---|---|
| | 15.7 cm | 15.0–16.4 cm Not Including 15.7 cm | Any Response in Inches | Other | |
| **Grade 12** | | | | | |
| Overall | 1.6 | 0.6 | 0.2 | 1.6 | 0.9 |
| Males | 1.9 | 1.0 | 0.3 | 2.1 | 1.4 |
| Females | 1.9 | 0.6 | 0.3 | 1.9 | 1.2 |
| White | 2.1 | 0.8 | 0.2 | 2.4 | 1.0 |
| Black | 2.6 | 0.8 | 0.5 | 2.7 | 3.0 |
| Hispanic | 2.6 | 1.3 | - - - | 4.0 | 3.4 |
| Asian/Pacific Islander | 6.2 | 3.6 | - - - | 5.2 | 2.7 |
| American Indian | *** | *** | *** | *** | *** |
| Geometry Taken | 1.7 | 0.7 | 0.2 | 1.8 | 0.7 |
| Highest Algebra-Calculus Course Taken: | | | | | |
| Pre-Algebra | 4.6 | 0.9 | - - - | 7.7 | 5.3 |
| First-Year Algebra | 2.1 | 1.8 | 0.4 | 3.0 | 2.0 |
| Second-Year Algebra | 1.7 | 0.6 | 0.3 | 1.8 | 1.3 |
| Third-Year Algebra/Pre-Calculus | 4.6 | 1.9 | - - - | 4.5 | 1.7 |
| Calculus | 5.2 | 2.5 | - - - | 5.8 | 0.0 |

*** Sample size is insufficient to permit a reliable estimate.
- - - Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B4.12**

## *Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use a Ruler to Find the Circumference of a Circle"*

THE NATION'S REPORT CARD naep

| | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.8 | 1.6 | 1.9 | 4.6 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B4.13**

## *Standard Errors for Score Percentages for "Describe Measurement Task"*

THE NATION'S REPORT CARD naep

| | Correct | Partial | Incorrect | Omit |
|---|---|---|---|---|
| **Grade 4** | | | | |
| Overall | 0.7 | 1.5 | 1.6 | 1.0 |
| Males | 0.9 | 1.9 | 1.9 | 1.6 |
| Females | 0.9 | 2.3 | 2.4 | 1.2 |
| White | 1.0 | 1.9 | 2.1 | 1.0 |
| Black | 0.8 | 2.8 | 2.7 | 2.2 |
| Hispanic | 0.7 | 2.8 | 3.5 | 2.7 |
| Asian/Pacific Islander | *** | *** | *** | *** |
| American Indian | *** | *** | *** | *** |

*** Sample size is insufficient to provide a reliable estimate
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B4.14**

## Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Describe Measurement Task"

THE NATION'S REPORT CARD naep

| Overall | NAEP Grade 4 Composite Scale Range | | | |
|---|---|---|---|---|
| | Below Basic | Basic | Proficient | Advanced |
| 0.7 | - - - | 1.1 | 2.4 | *** |

*** Sample size is insufficient to permit a reliable estimate.
- - - Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B4.15**

## Standard Errors for Score Percentages for "Compare Areas of Two Shapes," Grade 4

THE NATION'S REPORT CARD naep

| | Correct | Incorrect | | Omit |
|---|---|---|---|---|
| | | Bob–No Adequate Explanation | Not Bob | |
| **Grade 4** | | | | |
| Overall | 0.7 | 1.3 | 1.3 | 0.1 |
| Males | 1.0 | 1.8 | 1.7 | 0.2 |
| Females | 0.8 | 1.6 | 1.8 | - - - |
| White | 0.9 | 1.6 | 1.6 | 0.1 |
| Black | - - - | 3.6 | 3.6 | - - - |
| Hispanic | - - - | 2.6 | 2.6 | - - - |
| Asian/Pacific Islander | 3.8 | 4.2 | 5.0 | - - - |
| American Indian | *** | *** | *** | *** |

*** Sample size is insufficient to permit a reliable estimate.
- - - Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B4.16**

## Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Compare Areas of Two Shapes," Grade 4

THE NATION'S REPORT CARD naep

| Overall | NAEP Grade 4 Composite Scale Range | | | |
|---|---|---|---|---|
| | Below Basic | Basic | Proficient | Advanced |
| 0.7 | - - - | 1.1 | 2.9 | *** |

*** Sample size is insufficient to permit a reliable estimate.
- - - Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

THE NATION'S REPORT CARD naep

**Table B4.17** ***Standard Errors for Score Percentages for "Compare Areas of Two Shapes," Grades 8 and 12***

| | Correct | Incorrect | | Omit |
|---|---|---|---|---|
| | | Bob–No Adequate Explanation | Not Bob | |
| **Grade 8** | | | | |
| Overall | 1.4 | 0.9 | 1.4 | 0.5 |
| Males | 2.2 | 1.5 | 2.1 | 0.8 |
| Females | 1.6 | 1.5 | 1.7 | 0.5 |
| White | 1.9 | 1.1 | 1.7 | 0.4 |
| Black | 1.7 | 2.8 | 2.9 | 1.2 |
| Hispanic | 2.6 | 3.5 | 4.3 | 2.0 |
| Asian/Pacific Islander | – – | – – | – – | – – |
| American Indian | *** | *** | *** | *** |
| Mathematics Course Taking: | | | | |
| Eighth-Grade Mathematics | 2.2 | 1.6 | 2.2 | 0.4 |
| Pre-Algebra | 2.0 | 1.7 | 1.9 | 0.7 |
| Algebra | 2.9 | 1.9 | 2.8 | 0.7 |
| **Grade 12** | | | | |
| Overall | 1.3 | 1.1 | 1.3 | 0.5 |
| Males | 2.0 | 2.0 | 2.3 | 0.8 |
| Females | 1.5 | 1.0 | 1.7 | 0.6 |
| White | 1.5 | 1.4 | 1.7 | 0.5 |
| Black | 1.8 | 2.2 | 2.5 | 1.9 |
| Hispanic | 5.2 | 2.7 | 5.1 | 2.5 |
| Asian/Pacific Islander | 8.2 | 4.1 | 7.2 | 2.2 |
| American Indian | *** | *** | *** | *** |
| Geometry Taken | 1.3 | 1.2 | 1.3 | 0.6 |
| Highest Algebra-Calculus Course Taken: | | | | |
| Pre-Algebra | 5.1 | 3.2 | 6.0 | 2.3 |
| First-Year Algebra | 2.9 | 2.5 | 2.6 | 1.0 |
| Second-Year Algebra | 1.9 | 1.1 | 1.7 | 0.6 |
| Third-Year Algebra/Pre-Calculus | 3.4 | 2.2 | 3.3 | 1.6 |
| Calculus | 5.4 | 3.4 | 5.6 | 1.6 |

*** Sample size is insufficient to permit a reliable estimate.

–– Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B4.18 — Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Compare Areas of Two Shapes," Grades 8 and 12

THE NATION'S REPORT CARD naep

| | Overall | NAEP Grades 8 and 12 Composite Scale Ranges | | | |
|---|---|---|---|---|---|
| | | Below Basic | Basic | Proficient | Advanced |
| Grade 8 | 1.4 | 1.7 | 2.8 | 3.2 | *** |
| Grade 12 | 1.3 | 2.1 | 1.8 | 3.9 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B4.19 — Standard Errors for Score Percentages for "Find Perimeter (Quadrilateral)"

THE NATION'S REPORT CARD naep

| | Correct | Incorrect | | | Omit |
|---|---|---|---|---|---|
| | Between 6 and 7 | Between 7 and 8 | Between 5 and 6 | Other | |
| **Grade 8** | | | | | |
| Overall | 1.3 | 0.8 | 0.6 | 1.6 | 1.2 |
| Males | 1.7 | 1.0 | 0.9 | 2.0 | 1.6 |
| Females | 1.7 | 1.1 | 0.9 | 2.4 | 1.8 |
| White | 1.7 | 1.1 | 0.7 | 2.2 | 1.5 |
| Black | 1.9 | 0.9 | 0.6 | 3.3 | 3.3 |
| Hispanic | 2.7 | 0.5 | 1.8 | 3.7 | 2.8 |
| Asian/Pacific Islander | –– | –– | –– | –– | –– |
| American Indian | *** | *** | *** | *** | *** |
| Mathematics Course Taking: | | | | | |
| Eighth-Grade Mathematics | 2.2 | 1.0 | 0.8 | 2.4 | 2.2 |
| Pre-Algebra | 2.3 | 1.2 | 1.1 | 3.4 | 2.1 |
| Algebra | 2.8 | 1.5 | 1.3 | 2.7 | 1.8 |

*** Sample size is insufficient to permit a reliable estimate.
–– Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.20

## *Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Find Perimeter (Quadrilateral)"*

THE NATION'S REPORT CARD naep

| Overall | NAEP Grade 8 Composite Scale Range | | | |
|---|---|---|---|---|
| | Below Basic | Basic | Proficient | Advanced |
| 1.3 | 1.4 | 2.9 | 4.0 | 5.8 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B5.1

### *Standard Errors for Percentage Correct for "Compare Two Geometric Shapes"*

THE NATION'S REPORT CARD naep

| Grade 4 | Extended | Satisfactory | Partial | Minimal | Incorrect | Omit |
|---|---|---|---|---|---|---|
| Overall | 0.1 | 1.1 | 1.6 | 1.2 | 1.4 | 0.6 |
| Males | 0.2 | 1.3 | 1.8 | 1.8 | 2.0 | 1.1 |
| Females | 0.1 | 1.3 | 2.1 | 1.8 | 1.8 | 0.7 |
| White | 0.2 | 1.2 | 2.1 | 1.4 | 1.6 | 0.6 |
| Black | - - - | 1.9 | 3.1 | 3.1 | 3.5 | 2.3 |
| Hispanic | - - - | 2.0 | 3.5 | 3.7 | 3.9 | 2.1 |
| Asian/Pacific Islander | - - - | 4.4 | 3.7 | 4.9 | 5.9 | 2.3 |
| American Indian | *** | *** | *** | *** | *** | *** |

*** Sample size is insufficient to permit a reliable estimate.

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B5.2

### *Standard Errors for Percentage Satisfactory Within Achievement-Level Intervals for "Compare Two Geometric Shapes"*

THE NATION'S REPORT CARD naep

| Overall | NAEP Grade 4 Composite Scale Range | | | |
|---|---|---|---|---|
| | Below Basic | Basic | Proficient | Advanced |
| 1.1 | 1.1 | 1.6 | 2.6 | *** |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B5.3

### Standard Errors for Percentage Correct for "Use Similar Triangles"

THE NATION'S REPORT CARD naep

| Grade 12 | Percentage Correct |
|---|---|
| Overall | 1.4 |
| Males | 2.1 |
| Females | 1.6 |
| White | 1.6 |
| Black | 4.6 |
| Hispanic | 3.0 |
| Asian/Pacific Islander | 6.6 |
| American Indian | *** |
| Geometry Taken | 1.4 |
| Highest Algebra-Calculus Course Taken: | |
| Pre-Algebra | *** |
| First-Year Algebra | 3.4 |
| Second-Year Algebra | 2.0 |
| Third-Year Algebra/Pre-Calculus | 3.6 |
| Calculus | 5.2 |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B5.4

### Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Similar Triangles"

THE NATION'S REPORT CARD naep

| | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.4 | 2.4 | 1.8 | 4.2 | *** |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B5.5 Standard Errors for Score Percentages for "Draw a Parallelogram with Perpendicular Diagonals"

THE NATION'S REPORT CARD naep

| | Correct | | Incorrect | | Omit |
|---|---|---|---|---|---|
| | Rhombus that is Not a Square | Square | Quadrilateral with Incorrect Diagonals | Other | |
| **Grade 12** | | | | | |
| Overall | 0.8 | 0.9 | 1.5 | 1.4 | 1.1 |
| Males | 1.0 | 1.3 | 1.7 | 2.0 | 1.5 |
| Females | 1.4 | 1.1 | 2.1 | 1.4 | 1.6 |
| White | 1.0 | 1.2 | 1.6 | 1.4 | 1.2 |
| Black | 0.9 | 0.9 | 3.5 | 3.5 | 3.5 |
| Hispanic | 1.3 | 1.7 | 4.6 | 4.8 | 3.7 |
| Asian/Pacific Islander | 4.9 | 4.8 | 8.8 | 4.0 | 4.1 |
| American Indian | *** | *** | *** | *** | *** |
| Geometry Taken | 1.0 | 1.0 | 1.6 | 1.4 | 1.0 |
| Highest Algebra-Calculus Course Taken: | | | | | |
| Pre-Algebra | --- | --- | 6.8 | 6.0 | 4.9 |
| First-Year Algebra | 1.2 | 1.6 | 2.5 | 2.3 | 2.1 |
| Second-Year Algebra | 1.2 | 1.4 | 2.2 | 1.8 | 1.5 |
| Third-Year Algebra/Pre-Calculus | 2.3 | 3.1 | 3.6 | 2.0 | 1.5 |
| Calculus | 5.6 | 5.0 | 4.3 | 4.1 | 2.3 |

*** Sample size is insufficient to permit a reliable estimate.
--- Standard error estimate cannot be accurately determined
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B5.6 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Draw a Parallelogram with Perpendicular Diagonals"

THE NATION'S REPORT CARD naep

| | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.3 | 0.7 | 1.6 | 3.7 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B5.7 Standard Errors for Score Percentages for "Use Protractor to Draw Perpendicular Line and Measure Angle"

| | Correct | Incorrect | | | Omit |
|---|---|---|---|---|---|
| | | Line, Correct Angle | Angle, Correct Line | Other | |
| **Grade 12** | | | | | |
| Overall | 1.4 | 0.5 | 0.9 | 1.5 | 0.7 |
| Males | 1.6 | 0.8 | 1.4 | 2.1 | 1.1 |
| Females | 1.9 | 0.7 | 1.3 | 2.0 | 1.0 |
| White | 1.8 | 0.7 | 1.2 | 2.0 | 0.9 |
| Black | 1.8 | 0.9 | 2.8 | 3.1 | 2.1 |
| Hispanic | 2.9 | 1.4 | 2.3 | 4.1 | 2.1 |
| Asian/Pacific Islander | 7.0 | 1.5 | 4.5 | 8.0 | 0.8 |
| American Indian | *** | *** | *** | *** | *** |
| Geometry Taken | 1.6 | 0.6 | 1.2 | 1.6 | 0.5 |
| Highest Algebra-Calculus Course Taken: | | | | | |
| Pre-Algebra | *** | *** | *** | *** | *** |
| First-Year Algebra | 1.7 | 1.0 | 2.0 | 2.7 | 1.1 |
| Second-Year Algebra | 2.0 | 0.7 | 1.5 | 2.5 | 0.7 |
| Third-Year Algebra/Pre-Calculus | 4.5 | 1.5 | 3.1 | 2.8 | 2.3 |
| Calculus | 7.0 | 2.8 | 4.3 | 4.2 | --- |

*** Sample size is insufficient to permit a reliable estimate.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B5.8 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Protractor to Draw Perpendicular Line and Measure Angle"

| Overall | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| | Below Basic | Basic | Proficient | Advanced |
| 1.4 | --- | 1.6 | 4.0 | *** |

*** Sample size is insufficient to permit a reliable estimate.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B5.9**

## *Standard Errors for Percentage Correct for "Assemble Pieces to Form a Square"*

THE NATION'S REPORT CARD naep

| | Percentage Correct |
|---|---|
| **Grade 4** | |
| Overall | 1.3 |
| Males | 1.6 |
| Females | 1.7 |
| White | 1.3 |
| Black | 3.8 |
| Hispanic | 4.2 |
| Asian/Pacific Islander | 4.8 |
| American Indian | *** |
| **Grade 8** | |
| Overall | 0.8 |
| Males | 1.3 |
| Females | 1.1 |
| White | 0.7 |
| Black | 3.0 |
| Hispanic | 3.1 |
| Asian/Pacific Islander | – – |
| American Indian | *** |
| Mathematics Course Taking: | |
| Eighth-Grade Mathematics | 1.3 |
| Pre-Algebra | 1.6 |
| Algebra | 2.4 |

***Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

THE NATION'S REPORT CARD naep

## Table B5.10 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Assemble Pieces to Form a Square"

| | Overall | NAEP Grades 4 and 8 Composite Scale Ranges | | | |
|---|---|---|---|---|---|
| | | Below Basic | Basic | Proficient | Advanced |
| Grade 4 | 1.3 | 3.1 | 1.5 | - - - | *** |
| Grade 8 | 0.8 | 1.8 | 1.1 | 0.7 | *** |

*** Sample size is insufficient to permit a reliable estimate.

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B5.11**

## Standard Errors for Score Percentages for "Assemble Pieces to Form Shape"

THE NATION'S REPORT CARD naep

| | Correct: Rhombus | Correct: Not a Rhombus | Incorrect | Omit |
|---|---|---|---|---|
| **Grade 4** | | | | |
| Overall | 0.9 | 0.2 | 1.2 | 0.7 |
| Males | 1.4 | 0.4 | 1.7 | 1.0 |
| Females | 1.3 | 0.3 | 1.7 | 0.8 |
| White | 1.1 | 0.3 | 1.4 | 0.7 |
| Black | 1.7 | - - - | 3.0 | 2.2 |
| Hispanic | 2.2 | - - - | 3.1 | 2.9 |
| Asian/Pacific Islander | 4.9 | - - - | 6.0 | - - - |
| American Indian | *** | *** | *** | *** |
| **Grade 8** | | | | |
| Overall | 1.2 | 0.7 | 1.3 | 0.4 |
| Males | 1.9 | 0.9 | 2.0 | 0.5 |
| Females | 1.7 | 0.8 | 1.8 | 0.6 |
| White | 1.7 | 0.8 | 1.8 | 0.4 |
| Black | 2.7 | 1.4 | 2.9 | 1.0 |
| Hispanic | 3.2 | 2.6 | 3.7 | 1.8 |
| Asian/Pacific Islander | – – | – – | – – | – – |
| American Indian | *** | *** | *** | *** |
| Mathematics Course Taking: | | | | |
| Eighth-Grade Mathematics | 2.2 | 0.9 | 2.0 | 0.5 |
| Pre-Algebra | 2.4 | 0.9 | 2.2 | 0.6 |
| Algebra | 2.6 | 1.8 | 2.5 | 0.8 |
| **Grade 12** | | | | |
| Overall | 1.2 | 0.6 | 1.1 | 0.4 |
| Males | 1.9 | 0.8 | 2.1 | 0.7 |
| Females | 1.8 | 0.8 | 1.6 | 0.6 |
| White | 1.4 | 0.6 | 1.5 | 0.4 |
| Black | 3.1 | 1.3 | 3.0 | 1.5 |
| Hispanic | 3.9 | 3.4 | 3.5 | 1.7 |
| Asian/Pacific Islander | 5.7 | 3.0 | 4.7 | – – |
| American Indian | *** | *** | *** | *** |
| Geometry Taken | 1.5 | 0.6 | 1.5 | 0.3 |
| Highest Algebra-Calculus Course Taken: | | | | |
| Pre-Algebra | 6.0 | 6.8 | 5.4 | 2.9 |
| First-Year Algebra | 3.0 | 1.1 | 2.6 | 0.8 |
| Second-Year Algebra | 1.9 | 0.9 | 1.9 | 0.5 |
| Third-Year Algebra/Pre-Calculus | 4.2 | 1.5 | 3.8 | 0.8 |
| Calculus | 5.2 | 2.8 | 5.0 | - - - |

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B5.12**

## *Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Assemble Pieces to Form Shape"*

THE NATION'S REPORT CARD naep

| | | NAEP Grades 4, 8, and 12 Composite Scale Ranges | | | |
|---|---|---|---|---|---|
| | **Overall** | **Below Basic** | **Basic** | **Proficient** | **Advanced** |
| Grade 4 | 0.9 | 1.0 | 1.8 | 4.6 | *** |
| Grade 8 | 1.4 | 2.3 | 2.8 | 4.2 | *** |
| Grade 12 | 1.1 | 2.7 | 1.8 | 3.6 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics SAssessment.

**Table B5.13**

## *Standard Errors for Percentage Correct for "Reason About Betweenness"*

THE NATION'S REPORT CARD naep

| Grade 8 | Percentage Correct |
|---|---|
| Overall | 1.4 |
| Males | 2.0 |
| Females | 1.5 |
| White | 1.8 |
| Black | 2.5 |
| Hispanic | 2.7 |
| Asian/Pacific Islander | – – |
| American Indian | *** |
| Mathematics Course Taking: | |
| Eighth-Grade Mathematics | 2.0 |
| Pre-Algebra | 2.3 |
| Algebra | 2.8 |

*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B5.14 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Reason About Betweenness"

THE NATION'S REPORT CARD naep

| | NAEP Grade 8 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.4 | 1.9 | 2.6 | 4.6 | 8.7 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B5.15 Standard Errors for Score Percentages for "Describe Geometric Process for Finding Center of Disk"

THE NATION'S REPORT CARD naep

| | Extended | Satisfactory | Partial | Minimal | Incorrect | Omit |
|---|---|---|---|---|---|---|
| **Grade 12** | | | | | | |
| Overall | 0.2 | 1.2 | 1.2 | 1.7 | 1.2 | 1.3 |
| Males | 0.4 | 1.6 | 2.3 | 2.6 | 1.7 | 2.0 |
| Females | - - - | 1.5 | 1.0 | 1.9 | 1.7 | 1.9 |
| White | 0.3 | 1.4 | 1.7 | 2.2 | 1.4 | 1.5 |
| Black | - - - | 1.7 | 1.6 | 2.8 | 4.1 | 4.5 |
| Hispanic | - - - | 2.5 | 2.7 | 3.8 | 3.2 | 3.5 |
| Asian/Pacific Islander | 1.7 | 7.4 | 6.5 | 5.0 | 5.4 | 4.3 |
| American Indian | *** | *** | *** | *** | *** | *** |
| Geometry Taken | 0.2 | 1.4 | 1.4 | 1.7 | 1.5 | 1.5 |
| Highest Algebra-Calculus Course Taken: | | | | | | |
| Pre-Algebra | 0.0 | 2.4 | - - - | 5.7 | 6.1 | 5.4 |
| First-Year Algebra | - - - | 1.9 | 1.7 | 3.7 | 2.9 | 3.5 |
| Second-Year Algebra | 0.3 | 1.9 | 1.1 | 1.7 | 1.8 | 1.9 |
| Third-Year Algebra/Pre-Calculus | 1.0 | 2.7 | 2.0 | 3.3 | 2.8 | 3.1 |
| Calculus | - - - | 4.4 | 3.3 | 6.4 | 7.3 | 4.9 |

*** Sample size is insufficient to permit a reliable estimate.

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B5.16 Standard Errors for Percentage Satisfactory Within Achievement-Level Intervals for "Describe Geometric Process for Finding Center of Disk"

THE NATION'S REPORT CARD naep

| | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.2 | 1.0 | 1.6 | 4.3 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B6.1 Standard Errors for Percentage Correct for "Read a Bar Graph"

THE NATION'S REPORT CARD naep

| | Percentage Correct |
|---|---|
| **Grade 4** | |
| Overall | 1.4 |
| Males | 2.1 |
| Females | 1.8 |
| White | 1.7 |
| Black | 3.4 |
| Hispanic | 3.2 |
| Asian/Pacific Islander | *** |
| American Indian | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B6.2 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Read a Bar Graph"

THE NATION'S REPORT CARD naep

| | NAEP Grade 4 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.4 | 2.0 | 2.4 | 3.0 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B6.3 — Standard Errors for Score Percentages for "Use Data from a Chart"

| | Correct | Incorrect | | | Omit |
|---|---|---|---|---|---|
| | Shape N–Correct Explanation | Shape N–No, or Incorrect, Explanation | Shape Q | Other | |
| **Grade 4** | | | | | |
| Overall | 1.4 | 1.3 | 1.7 | 1.1 | 0.5 |
| Males | 1.8 | 1.6 | 2.0 | 1.7 | 0.8 |
| Females | 2.0 | 1.6 | 2.5 | 1.5 | 0.8 |
| White | 1.8 | 1.4 | 2.0 | 1.1 | 0.6 |
| Black | 2.7 | 2.5 | 3.9 | 3.3 | 0.8 |
| Hispanic | 2.6 | 3.5 | 4.0 | 5.0 | 2.2 |
| Asian/Pacific Islander | 5.7 | 4.1 | 5.8 | 4.6 | - - - |
| American Indian | *** | *** | *** | *** | *** |
| **Grade 8** | | | | | |
| Overall | 2.0 | 1.5 | 1.2 | 1.1 | 0.2 |
| Males | 2.9 | 2.1 | 1.7 | 1.6 | 0.3 |
| Females | 2.2 | 1.6 | 1.2 | 1.2 | - - - |
| White | 2.2 | 1.8 | 1.3 | 1.0 | - - - |
| Black | 3.8 | 3.4 | 3.0 | 3.1 | 1.2 |
| Hispanic | 4.0 | 1.9 | 3.4 | 2.6 | - - - |
| Asian/Pacific Islander | –– | –– | –– | –– | –– |
| American Indian | *** | *** | *** | *** | *** |
| Mathematics Course Taking: | | | | | |
| Eighth-Grade Mathematics | 2.5 | 2.2 | 1.9 | 1.3 | 0.0 |
| Pre-Algebra | 4.2 | 2.7 | 2.6 | 1.8 | 0.0 |
| Algebra | 3.3 | 1.9 | 1.6 | 2.9 | 0.0 |
| **Grade 12** | | | | | |
| Overall | 1.5 | 1.3 | 0.7 | 0.8 | 0.3 |
| Males | 2.1 | 2.1 | 1.1 | 1.0 | 0.5 |
| Females | 1.7 | 1.4 | 1.0 | 0.9 | 0.3 |
| White | 1.7 | 1.4 | 0.8 | 0.7 | 0.3 |
| Black | 3.7 | 2.6 | 3.0 | 1.8 | 1.2 |
| Hispanic | 4.3 | 3.9 | 2.1 | 2.7 | 1.5 |
| Asian/Pacific Islander | 5.6 | 2.9 | 3.6 | 4.6 | 1.2 |
| American Indian | *** | *** | *** | *** | *** |
| Geometry Taken | 1.4 | 1.3 | 0.9 | 0.8 | 0.3 |
| Highest Algebra-Calculus Course Taken: | | | | | |
| Pre-Algebra | 6.0 | 4.9 | 3.1 | 3.1 | 1.8 |
| First-Year Algebra | 3.0 | 3.3 | 2.0 | 1.6 | 0.9 |
| Second-Year Algebra | 1.7 | 1.5 | 1.1 | 1.0 | 0.3 |
| Third-Year Algebra/Pre-Calculus | 3.8 | 3.7 | 1.6 | 1.6 | 0.3 |
| Calculus | 4.9 | 4.0 | 3.4 | 2.4 | - - - |

*** Sample size is insufficient to permit a reliable estimate.

–– Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B6.4

### Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Data from a Chart"

THE NATION'S REPORT CARD naep

| | Overall | NAEP Grades 4, 8, and 12 Composite Scale Ranges | | | |
|---|---|---|---|---|---|
| | | Below Basic | Basic | Proficient | Advanced |
| Grade 4 | 1.4 | 1.4 | 2.7 | 4.3 | *** |
| Grade 8 | 2.0 | 3.0 | 2.8 | 3.6 | *** |
| Grade 12 | 1.5 | 2.4 | 1.8 | 5.0 | *** |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B6.5

### Standard Errors for Score Percentages for "Recognize Misleading Graph"

THE NATION'S REPORT CARD naep

| | Correct | Partial | | Incorrect | Omit |
|---|---|---|---|---|---|
| | Graph B–Complete Explanation | Graph B–Incomplete but Partially Correct Explanation | Graph B–No or Incorrect Explanation | | |
| **Grade 8** | | | | | |
| Overall | 0.4 | 1.3 | 1.5 | 1.5 | 1.6 |
| Males | 0.5 | 2.2 | 2.2 | 1.9 | 1.9 |
| Females | 0.7 | 1.8 | 2.2 | 1.7 | 2.0 |
| White | 0.5 | 1.8 | 1.8 | 1.9 | 1.7 |
| Black | - - - | 1.7 | 4.5 | 4.2 | 4.5 |
| Hispanic | - - - | 2.0 | 4.8 | 4.1 | 5.4 |
| Asian/Pacific Islander | – – | – – | – – | – – | – – |
| American Indian | *** | *** | *** | *** | *** |
| Mathematics Course Taking: | | | | | |
| Eighth-Grade Mathematics | 0.4 | 2.2 | 2.5 | 2.8 | 3.0 |
| Pre-Algebra | 1.1 | 1.7 | 2.9 | 1.9 | 2.8 |
| Algebra | 0.8 | 2.1 | 2.3 | 1.4 | 2.5 |

*** Sample size is insufficient to permit a reliable estimate.

–– Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B6.6

### Standard Errors for Percentage at Least Partial Within Achievement-Level Intervals for "Recognize Misleading Graph"

THE NATION'S REPORT CARD naep

| | NAEP Grade 8 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.5 | 1.9 | 2.8 | 3.6 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B6.7

### Standard Errors for Score Percentages for "Use Data in Table to Compute Average Hourly Wage and Determine When Wage Rate Changes"

THE NATION'S REPORT CARD naep

| | Correct | Partial | Incorrect | Omit |
|---|---|---|---|---|
| **Grade 12** | | | | |
| Overall | 1.0 | 1.7 | 1.6 | 0.6 |
| Males | 1.3 | 2.2 | 2.1 | 0.7 |
| Females | 1.6 | 2.3 | 2.0 | 0.8 |
| White | 1.3 | 2.1 | 2.1 | 0.4 |
| Black | 0.9 | 5.2 | 5.2 | 2.0 |
| Hispanic | 1.8 | 3.5 | 5.0 | 3.5 |
| Asian/Pacific Islander | 4.5 | 5.4 | 7.5 | 1.7 |
| American Indian | *** | *** | *** | *** |
| Geometry Taken | 1.3 | 1.9 | 1.8 | 0.6 |
| Highest Algebra-Calculus Course Taken: | | | | |
| Pre-Algebra | *** | *** | *** | *** |
| First-Year Algebra | 1.9 | 3.0 | 3.1 | 1.3 |
| Second-Year Algebra | 1.6 | 1.9 | 2.0 | 0.5 |
| Third-Year Algebra/Pre-Calculus | 2.7 | 3.8 | 3.6 | 0.8 |
| Calculus | 5.4 | 4.1 | 3.1 | 1.8 |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B6.8

### Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Data in Table to Compute Average Hourly Wage and Determine When Wage Rate Changes"

THE NATION'S REPORT CARD naep

| Overall | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| | Below Basic | Basic | Proficient | Advanced |
| 1.0 | 1.1 | 1.6 | 4.1 | *** |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B6.9

### Standard Errors for Score Percentages for "Reason About Sample Space"

THE NATION'S REPORT CARD naep

| | Number Correct | | | | | |
|---|---|---|---|---|---|---|
| | 4 | 3 | 2 | 1 | None | Omit |
| **Grade 8** | | | | | | |
| Overall | 1.1 | 0.9 | 0.4 | 0.5 | 0.3 | 0.1 |
| Males | 1.6 | 1.2 | 0.7 | 0.9 | 0.5 | 0.3 |
| Females | 1.5 | 1.3 | 0.5 | 0.6 | 0.3 | 0.1 |
| White | 1.1 | 1.0 | 0.5 | 0.5 | 0.2 | - - - |
| Black | 2.8 | 2.4 | 1.3 | 2.2 | 1.3 | - - - |
| Hispanic | 4.1 | 2.5 | 0.7 | 3.0 | 1.6 | - - - |
| Asian/Pacific Islander | – – | – – | – – | – – | – – | – – |
| American Indian | *** | *** | *** | *** | *** | *** |
| Mathematics Course Taking: | | | | | | |
| Eighth-Grade Mathematics | 1.9 | 1.3 | 0.9 | 1.0 | 0.4 | 0.3 |
| Pre-Algebra | 2.4 | 2.1 | 0.7 | 1.1 | 0.4 | 0.0 |
| Algebra | 1.9 | 1.4 | 0.4 | 1.3 | 0.6 | 0.0 |

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B6.10 — Standard Errors for Percentage with at Least Three Correct Within Achievement-Level Intervals for "Reason About Sample Space"

THE NATION'S REPORT CARD naep

| Overall | NAEP Grade 8 Composite Scale Range | | | |
|---|---|---|---|---|
| | Below Basic | Basic | Proficient | Advanced |
| 0.8 | 2.0 | 1.0 | - - - | - - - |

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B6.11 — Standard Errors for Percentage Correct for "Identify Representative Sample"

THE NATION'S REPORT CARD naep

| Grade 8 | Percentage Correct |
|---|---|
| Overall | 1.6 |
| Males | 1.8 |
| Females | 2.4 |
| White | 1.8 |
| Black | 3.7 |
| Hispanic | 3.9 |
| Asian/Pacific Islander | – – |
| American Indian | * * * |
| Mathematics Course Taking: | |
| Eighth-Grade Mathematics | 2.5 |
| Pre-Algebra | 2.6 |
| Algebra | - - - |

*** Sample size is insufficient to permit a reliable estimate.

–– Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B6.12

### *Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Identify Representative Sample"*

THE NATION'S REPORT CARD

| Overall | NAEP Grade 8 Composite Scale Range | | | |
|---|---|---|---|---|
| | Below Basic | Basic | Proficient | Advanced |
| 1.6 | 2.7 | 2.7 | 2.6 | - - - |

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B6.13

## Standard Errors for Score Percentages for "Compare Mean and Median"

THE NATION'S REPORT CARD naep

| | Extended | Satisfactory | Partial | Minimal | Incorrect | Omit |
|---|---|---|---|---|---|---|
| | Better Measure Both Theaters; Complete Explanation | Better Measure Both Theaters; Complete Explanation for 1 Theater | Better Measure and Complete Explanation 1 Theater; or Better Measure Both Theaters with No or Incomplete Explanation | Better Measure 1 Theater; No or Incomplete Explanation | | |
| **Grade 12** | | | | | | |
| Overall | 0.3 | 0.5 | 1.0 | 1.4 | 1.4 | 1.5 |
| Males | 0.5 | 0.7 | 1.7 | 1.9 | 2.2 | 2.5 |
| Females | - - - | 0.5 | 1.1 | 2.0 | 1.9 | 1.6 |
| White | 0.4 | 0.7 | 1.4 | 1.7 | 1.3 | 1.7 |
| Black | - - - | - - - | 1.3 | 4.0 | 3.9 | 4.4 |
| Hispanic | - - - | - - - | 2.2 | 3.3 | 7.1 | 7.4 |
| Asian/Pacific Islander | - - - | 2.1 | 3.0 | 6.0 | 5.3 | 6.2 |
| American Indian | *** | *** | *** | *** | *** | *** |
| Geometry Taken | 0.4 | 0.6 | 1.1 | 1.6 | 1.6 | 1.9 |
| Highest Algebra-Calculus Course Taken: | | | | | | |
| Pre-Algebra | *** | *** | *** | *** | *** | *** |
| First-Year Algebra | - - - | 0.8 | 1.8 | 2.8 | 3.4 | 3.1 |
| Second-Year Algebra | 0.2 | 0.5 | 1.3 | 2.0 | 2.1 | 2.2 |
| Third-Year Algebra/Pre-Calculus | - - - | 2.0 | 3.8 | 4.6 | 3.4 | 4.6 |
| Calculus | 2.7 | 2.1 | 3.7 | 4.1 | 4.4 | 3.5 |

*** Sample size is insufficient to permit a reliable estimate.

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B6.14

### *Standard Errors for Percentage at Least Satisfactory Within Achievement-Level Intervals for "Compare Mean and Median"*

THE NATION'S REPORT CARD naep

| Overall | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| | Below Basic | Basic | Proficient | Advanced |
| 0.6 | - - - | 0.9 | 3.9 | *** |

*** Sample size is insufficient to permit a reliable estimate.
- - - Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B6.15

### *Standard Errors for Percentage Correct for "Determine a Probability"*

THE NATION'S REPORT CARD naep

| | Percentage Correct |
|---|---|
| **Grade 4** | |
| Overall | 1.5 |
| Males | 2.2 |
| Females | 1.6 |
| White | 1.8 |
| Black | 3.0 |
| Hispanic | 4.0 |
| Asian/Pacific Islander | 3.7 |
| American Indian | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B6.16

### Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Determine a Probability"

| | NAEP Grade 4 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.5 | 2.3 | 2.3 | 3.6 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B6.17

### Standard Errors for Score Percentages for "Compare Probabilities"

| | Correct | Partial | Incorrect | | Omit |
|---|---|---|---|---|---|
| | Correct Answer to "Yes/No" Question; Correct Explanation | Correct Answer to "Yes/No" Question; Partial Explanation | Correct Answer to "Yes/No" Question; Incorrect Explanation | Incorrect Answer to "Yes/No" Question | |
| **Grade 12** | | | | | |
| Overall | 0.8 | 1.5 | 1.7 | 1.7 | 0.7 |
| Males | 1.3 | 2.3 | 2.4 | 2.1 | 0.9 |
| Females | 0.9 | 1.9 | 2.1 | 2.1 | 1.0 |
| White | 0.9 | 1.9 | 2.2 | 2.1 | 0.9 |
| Black | 1.9 | 2.9 | 3.0 | 3.4 | 2.0 |
| Hispanic | 2.9 | 2.7 | 5.9 | 6.9 | 3.0 |
| Asian/Pacific Islander | 3.6 | 5.7 | 6.1 | 7.5 | - - - |
| American Indian | *** | *** | *** | *** | *** |
| Geometry Taken | 1.0 | 1.7 | 1.9 | 1.8 | 0.7 |
| Highest Algebra-Calculus Course Taken: | | | | | |
| Pre-Algebra | *** | *** | *** | *** | *** |
| First-Year Algebra | 1.9 | 2.5 | 2.5 | 3.9 | 1.1 |
| Second-Year Algebra | 1.3 | 1.7 | 2.4 | 2.6 | 0.7 |
| Third-Year Algebra/Pre-Calculus | 2.2 | 4.8 | 3.3 | 4.4 | 3.1 |
| Calculus | 3.6 | 3.7 | 4.0 | 4.1 | 4.2 |

*** Sample size is insufficient to permit a reliable estimate.
- - - Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B6.18 — Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Compare Probabilities"

THE NATION'S REPORT CARD

| | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| **Overall** | **Below Basic** | **Basic** | **Proficient** | **Advanced** |
| 1.6 | 1.7 | 2.6 | 4.9 | *** |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B7.1 — Standard Errors for Percentage Correct for "Find Number of Diagonals in a Polygon from a Vertex"

THE NATION'S REPORT CARD

| Grade 8 | Percentage Correct |
|---|---|
| Overall | 1.6 |
| Males | 2.0 |
| Females | 2.1 |
| White | 1.7 |
| Black | 2.9 |
| Hispanic | 4.0 |
| Asian/Pacific Islander | –– |
| American Indian | *** |
| Mathematics Course Taking: | |
| Eighth-Grade Mathematics | 2.9 |
| Pre-Algebra | 3.3 |
| Algebra | 2.5 |

*** Sample size is insufficient to permit a reliable estimate.
–– Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

| Table B7.2 | Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Find Number of Diagonals in a Polygon from a Vertex" |
|---|---|

| Overall | NAEP Grade 8 Composite Scale Range | | | |
|---|---|---|---|---|
| | Below Basic | Basic | Proficient | Advanced |
| 1.6 | 2.2 | 2.1 | 2.6 | - - - |

- - - Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

| Table B7.3 | Standard Errors for Score Percentages for "Describe Pattern of Squares in 20th Figure" |
|---|---|

| Grade 12 | Extended | Satisfactory | Partial | Minimal | Incorrect | Omit |
|---|---|---|---|---|---|---|
| Overall | 0.4 | 0.5 | 1.2 | 1.4 | 1.4 | 1.3 |
| Males | 0.7 | 0.6 | 1.9 | 1.6 | 2.0 | 1.5 |
| Females | 0.5 | 0.9 | 1.8 | 2.2 | 1.9 | 1.9 |
| White | 0.5 | 0.7 | 1.5 | 1.7 | 1.6 | 1.2 |
| Black | - - - | - - - | 2.4 | 2.6 | 3.4 | 4.5 |
| Hispanic | - - - | - - - | 5.1 | 3.9 | 4.8 | 5.0 |
| Asian/Pacific Islander | - - - | 1.6 | 6.6 | 4.7 | 4.3 | 4.9 |
| American Indian | *** | *** | *** | *** | *** | *** |
| Geometry Taken | 0.4 | 0.6 | 1.3 | 1.7 | 1.6 | 1.3 |
| Highest Algebra-Calculus Course Taken: | | | | | | |
| Pre-Algebra | *** | *** | *** | *** | *** | *** |
| First-Year Algebra | 0.8 | 0.4 | 2.4 | 2.7 | 3.1 | 2.8 |
| Second-Year Algebra | 0.6 | 0.5 | 1.8 | 2.3 | 1.9 | 1.7 |
| Third-Year Algebra/Pre-Calculus | - - - | 2.8 | 3.4 | 3.7 | 3.6 | 2.4 |
| Calculus | 3.0 | 1.9 | 5.0 | 5.1 | 3.6 | 2.5 |

*** Sample size is insufficient to permit a reliable estimate.
- - - Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B7.4

### *Standard Errors for Percentage at Least Satisfactory Within Achievement-Level Intervals for "Describe Pattern of Squares in 20th Figure"*

THE NATION'S REPORT CARD naep

| Overall | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| | Below Basic | Basic | Proficient | Advanced |
| 0.8 | - - - | 0.7 | 3.6 | *** |

*** Sample size is insufficient to permit a reliable estimate.
- - - Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP)1996 Mathematics Assessment.

## Table B7.5

### *Standard Errors for Percentage Correct for "Identify Graph of Function"*

THE NATION'S REPORT CARD naep

| Grade 12 | Percentage Correct |
|---|---|
| Overall | 1.4 |
| Males | 1.8 |
| Females | 1.9 |
| White | 1.8 |
| Black | 2.8 |
| Hispanic | 4.0 |
| Asian/Pacific Islander | 6.4 |
| American Indian | *** |
| Geometry Taken | 1.6 |
| Highest Algebra-Calculus Course Taken: | |
| Pre-Algebra | 2.7 |
| First-Year Algebra | 1.9 |
| Second-Year Algebra | 2.0 |
| Third-Year Algebra/Pre-Calculus | 3.9 |
| Calculus | 7.9 |

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B7.6 — Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Identify Graph of Function"

| | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| Overall | Below Basic | Basic | Proficient | Advanced |
| 1.4 | 1.9 | 2.0 | 6.0 | *** |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B7.7 — Standard Errors for Percentage Correct for "Write Expression Using N"

| Grade 4 | Percentage Correct |
|---|---|
| Overall | 1.2 |
| Males | 1.7 |
| Females | 1.9 |
| White | 1.8 |
| Black | 3.0 |
| Hispanic | 4.6 |
| Asian/Pacific Islander | 4.4 |
| American Indian | *** |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B7.8**

## Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Write Expression Using N"

THE NATION'S REPORT CARD naep

| Overall | NAEP Grade 4 Composite Scale Range | | | |
|---|---|---|---|---|
| | Below Basic | Basic | Proficient | Advanced |
| 1.2 | 2.4 | 2.2 | 2.1 | *** |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B7.9**

## Standard Errors for Percentage Correct for "Translate Words to Symbols"

THE NATION'S REPORT CARD naep

| Grade 8 | Percentage Correct |
|---|---|
| Overall | 1.6 |
| Males | 2.1 |
| Females | 2.2 |
| White | 2.0 |
| Black | 3.1 |
| Hispanic | 3.5 |
| Asian/Pacific Islander | – – |
| American Indian | *** |
| Mathematics Course Taking: | |
| Eighth-Grade Mathematics | 3.1 |
| Pre-Algebra | 3.4 |
| Algebra | 2.1 |

*** Sample size is insufficient to permit a reliable estimate.

–– Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B7.10 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Translate Words to Symbols"

THE NATION'S REPORT CARD naep

| Overall | NAEP Grade 8 Composite Scale Range: Below Basic | Basic | Proficient | Advanced |
|---|---|---|---|---|
| 1.6 | 2.4 | 2.5 | 1.3 | - - - |

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B7.11 Standard Errors for Percentage Correct for "Find (x, y) Solution of Linear Equation"

THE NATION'S REPORT CARD naep

| Grade 8 | Percentage Correct |
|---|---|
| Overall | 1.7 |
| Males | 2.3 |
| Females | 2.0 |
| White | 2.1 |
| Black | 3.7 |
| Hispanic | 3.2 |
| Asian/Pacific Islander | – – |
| American Indian | *** |
| Mathematics Course Taking: | |
| Eighth-Grade Mathematics | 2.6 |
| Pre-Algebra | 2.8 |
| Algebra | 2.5 |

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996

## Table B7.12 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Find (x, y) Solution of Linear Equation"

THE NATION'S REPORT CARD naep

| Overall | NAEP Grade 8 Composite Scale Range | | | |
|---|---|---|---|---|
| | Below Basic | Basic | Proficient | Advanced |
| 1.7 | 1.9 | 3.2 | 3.6 | - - - |

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B7.13 Standard Errors for Percentage Correct for "Subtract Integers"

THE NATION'S REPORT CARD naep

| Grade 8 | Correct | Partial | Incorrect | Omit |
|---|---|---|---|---|
| Overall | 1.4 | 0.6 | 1.5 | 0.6 |
| Males | 2.0 | 1.0 | 2.2 | 0.9 |
| Females | 1.4 | 0.6 | 1.4 | 0.6 |
| White | 1.9 | 0.8 | 2.1 | 0.4 |
| Black | 2.1 | 1.8 | 4.0 | 2.3 |
| Hispanic | 2.9 | 0.7 | 3.5 | 3.2 |
| Asian/Pacific Islander | –– | –– | –– | –– |
| American Indian | *** | *** | *** | *** |
| Mathematics Course Taking: | | | | |
| Eighth-Grade Mathematics | 1.9 | 0.8 | 2.0 | 1.1 |
| Pre-Algebra | 2.2 | 0.6 | 2.4 | 0.6 |
| Algebra | 3.0 | 1.4 | 2.7 | 0.9 |

*** Sample size is insufficient to permit a reliable estimate.

–– Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B7.14

### *Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Subtract Integers"*

THE NATION'S REPORT CARD naep

| | NAEP Grade 8 Composite Scale Range | | | |
|---|---|---|---|---|
| **Overall** | **Below Basic** | **Basic** | **Proficient** | **Advanced** |
| 1.4 | 1.4 | 2.3 | 3.4 | 7.2 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B7.15

### *Standard Errors for Percentage Correct for "Solve Pair of Equations"*

THE NATION'S REPORT CARD naep

| Grade 12 | Percentage Correct |
|---|---|
| Overall | 1.0 |
| Males | 1.9 |
| Females | 1.1 |
| White | 1.1 |
| Black | 3.0 |
| Hispanic | 4.5 |
| Asian/Pacific Islander | 3.6 |
| American Indian | *** |
| Geometry Taken | 0.9 |
| Highest Algebra-Calculus Course Taken: | |
| Pre-Algebra | 6.1 |
| First-Year Algebra | 2.1 |
| Second-Year Algebra | 1.1 |
| Third-Year Algebra/Pre-Calculus | 1.6 |
| Calculus | 3.3 |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B7.16

### *Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Solve Pair of Equations"*

| Overall | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| | Below Basic | Basic | Proficient | Advanced |
| 1.0 | 2.8 | 1.3 | - - - | *** |

*** Sample size is insufficient to permit a reliable estimate.

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B7.17

### *Standard Errors for Percentage Correct for "Use Trigonometric Identity"*

| Grade 12 | Percentage Correct |
|---|---|
| Overall | 1.6 |
| Males | 1.8 |
| Females | 2.4 |
| White | 2.0 |
| Black | 3.0 |
| Hispanic | 3.1 |
| Asian/Pacific Islander | 6.7 |
| American Indian | *** |
| Geometry Taken | 1.8 |
| Highest Algebra-Calculus Course Taken: | |
| Pre-Algebra | 5.1 |
| First-Year Algebra | 2.4 |
| Second-Year Algebra | 2.6 |
| Third-Year Algebra/Pre-Calculus | 4.1 |
| Calculus | 6.3 |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B7.18 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Trigonometric Identity"

| Overall | NAEP Grade 12 Composite Scale Range | | | |
|---|---|---|---|---|
| | Below Basic | Basic | Proficient | Advanced |
| 1.6 | 1.8 | 2.4 | 5.1 | *** |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B8.1

# Standard Errors for Average Scale Score by Mathematics Course Enrollment and by Gender, Race/Ethnicity, and Whether School Offers Algebra for High School Credit or Placement, Grade 8

THE NATION'S REPORT CARD naep

| | | Mathematics Course | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Algebra | | Pre-Algebra | | Eighth-Grade Mathematics | | Other Mathematics | |
| Grade 8 | Assessment Year | Percentage of Students | Average Scale Score | Percentage of Students | Average Scale Score | Percentage of Students | Average Scale Score | Percentage of Students | Average Scale Score |
| All Students | 1996 | 1.5 | 1.7 | 1.8 | 1.5 | 2.2 | 1.3 | 0.6 | 4.7 |
| | 1992 | 1.0 | 1.8 | 2.2 | 1.5 | 2.6 | 1.3 | 0.4 | 4.1 |
| | 1990 | 1.1 | 2.6 | 1.8 | 2.3 | 2.0 | 1.4 | 0.4 | 5.3 |
| Females | 1996 | 1.6 | 1.8 | 1.9 | 1.9 | 2.2 | 1.4 | 0.7 | 5.4 |
| | 1992 | 1.3 | 2.1 | 2.2 | 1.8 | 2.7 | 1.5 | 0.5 | 5.4 |
| | 1990 | 1.6 | 2.8 | 2.1 | 2.8 | 2.4 | 1.5 | 0.7 | *** |
| Males | 1996 | 1.8 | 2.0 | 1.9 | 1.8 | 2.5 | 1.6 | 0.6 | 5.6 |
| | 1992 | 1.0 | 2.1 | 2.4 | 1.7 | 2.6 | 1.4 | 0.4 | 5.6 |
| | 1990 | 1.2 | 3.0 | 1.7 | 2.8 | 2.1 | 1.6 | 0.5 | *** |
| White | 1996 | 2.1 | 1.4 | 2.3 | 1.7 | 2.9 | 1.4 | 0.7 | 4.5 |
| | 1992 | 1.3 | 1.6 | 2.5 | 1.2 | 3.1 | 1.3 | 0.4 | 5.4 |
| | 1990 | 1.5 | 2.5 | 2.2 | 2.1 | 2.3 | 1.6 | 0.6 | 6.9 |
| Black | 1996 | 2.0 | 2.9 | 2.4 | 2.2 | 2.2 | 1.9 | 1.0 | 8.2 |
| | 1992 | 1.7 | 4.8 | 3.7 | 3.1 | 3.9 | 1.4 | 1.2 | *** |
| | 1990 | 2.1 | *** | 2.9 | 6.0 | 4.6 | 3.2 | 0.8 | *** |
| Hispanic | 1996 | 2.3 | 4.9 | 2.1 | 2.7 | 3.0 | 2.5 | 1.1 | *** |
| | 1992 | 1.2 | 4.3 | 2.5 | 2.6 | 2.8 | 1.5 | 0.8 | *** |
| | 1990 | 1.5 | *** | 3.5 | 4.9 | 4.1 | 2.7 | 0.9 | *** |
| Asian/Pacific Islander | 1996 | – – | – – | – – | – – | – – | – – | – – | – – |
| | 1992 | 5.1 | 5.0 | 3.4 | *** | 5.1 | 4.6 | 0.8 | *** |
| | 1990 | 6.6 | *** | 6.1 | *** | 6.3 | *** | 2.3 | *** |
| American Indian | 1996 | 2.9 | *** | 4.8 | *** | 7.8 | *** | 3.2 | *** |
| | 1992 | 2.7 | *** | 6.1 | *** | 6.2 | 3.8 | 1.1 | *** |
| | 1990 | 2.7 | *** | 6.8 | *** | 5.8 | *** | 1.9 | *** |
| School Offers Algebra for High School Credit or Placement: Yes | 1996 | 2.2 | 1.9 | 2.2 | 2.1 | 2.6 | 1.5 | 0.7 | 5.3 |
| | 1992 | 1.3 | 1.7 | 2.5 | 1.5 | 2.7 | 1.8 | 0.5 | 4.9 |
| | 1990 | 1.5 | 2.7 | 2.3 | 3.2 | 2.5 | 1.7 | 0.7 | 6.6 |
| No | 1996 | 2.8 | 5.6 | 4.4 | 4.6 | 6.4 | 4.2 | 1.2 | *** |
| | 1992 | 1.6 | 5.8 | 3.8 | 2.2 | 4.5 | 2.8 | 0.3 | *** |
| | 1990 | 2.1 | *** | 3.3 | 5.3 | 4.0 | 4.0 | 0.5 | *** |

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

## Table B8.2 Standard Errors for Percentage of Students Currently Enrolled in a Mathematics Course by Gender and Race/Ethnicity, Grade 12

| | Assessment Year | Percentage of Students |
|---|---|---|
| **Grade 12** | | |
| All Students | 1996 | 1.2 |
| | 1992 | 1.2 |
| | 1990 | 2.0 |
| Females | 1996 | 1.4 |
| | 1992 | 1.3 |
| | 1990 | 2.2 |
| Males | 1996 | 1.6 |
| | 1992 | 1.4 |
| | 1990 | 2.4 |
| White | 1996 | 1.5 |
| | 1992 | 1.4 |
| | 1990 | 2.5 |
| Black | 1996 | 2.0 |
| | 1992 | 2.4 |
| | 1990 | 3.4 |
| Hispanic | 1996 | 2.6 |
| | 1992 | 2.1 |
| | 1990 | 2.3 |
| Asian/Pacific Islander | 1996 | 2.9 |
| | 1992 | 2.8 |
| | 1990 | 4.9 |
| American Indian | 1996 | 6.0 |
| | 1992 | *** |
| | 1990 | *** |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

## Table B8.3 Standard Errors for Percentage of Students by Year They Initially Took a First-Year Algebra Course, Grade 12

THE NATION'S REPORT CARD

| | Assessment Year | Before 9th Grade | 9th Grade | 10th Grade | 11th or 12th Grade | Not Taken |
|---|---|---|---|---|---|---|
| **Grade 12** | | | | | | |
| All Students | 1996 | 1.2 | 1.1 | 0.9 | 0.3 | 0.4 |
| | 1992 | 1.0 | 1.4 | 0.8 | 0.5 | 0.5 |
| Females | 1996 | 1.3 | 1.4 | 1.1 | 0.3 | 0.5 |
| | 1992 | 1.1 | 1.6 | 1.0 | 0.6 | 0.5 |
| Males | 1996 | 1.5 | 1.4 | 1.0 | 0.5 | 0.5 |
| | 1992 | 1.1 | 1.5 | 1.0 | 0.6 | 0.6 |
| White | 1996 | 1.3 | 1.4 | 1.1 | 0.3 | 0.4 |
| | 1992 | 1.1 | 1.5 | 0.8 | 0.5 | 0.5 |
| Black | 1996 | 2.9 | 3.2 | 1.8 | 1.4 | 0.8 |
| | 1992 | 1.6 | 3.4 | 2.4 | 1.2 | 1.3 |
| Hispanic | 1996 | 2.0 | 3.3 | 1.8 | 1.4 | 1.6 |
| | 1992 | 2.0 | 3.0 | 2.4 | 1.4 | 2.3 |
| Asian/Pacific Islander | 1996 | 4.1 | 3.1 | 2.0 | 0.4 | 0.7 |
| | 1992 | 4.9 | 3.8 | 2.6 | 1.4 | 0.8 |
| American Indian | 1996 | 6.5 | 3.7 | 8.2 | 2.8 | - - - |
| | 1992 | *** | *** | *** | *** | *** |

*** Sample size is insufficient to permit a reliable estimate.

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

Table B8.4

# Standard Errors for Percentage of Students by Number of Semesters of Mathematics Taken (Grades 9 through 12) by Gender and Race/Ethnicity, Grade 12

THE NATION'S REPORT CARD naep

| Grade 12 | Assessment Year | 7 or More Semesters | 5–6 Semesters | 3–4 Semesters | 1–2 Semesters | No Semesters |
|---|---|---|---|---|---|---|
| All Students | 1996 | 1.3 | 0.9 | 0.8 | 0.5 | 0.1 |
| | 1992 | 1.2 | 0.8 | 1.0 | 0.4 | 0.1 |
| | 1990 | 1.8 | 1.2 | 1.1 | 0.9 | 0.1 |
| Females | 1996 | 1.3 | 0.9 | 0.9 | 0.5 | 0.1 |
| | 1992 | 1.5 | 1.1 | 1.1 | 0.5 | 0.1 |
| | 1990 | 2.1 | 1.5 | 1.3 | 1.2 | 0.1 |
| Males | 1996 | 1.7 | 1.1 | 1.1 | 0.6 | 0.2 |
| | 1992 | 1.3 | 1.0 | 1.2 | 0.4 | 0.2 |
| | 1990 | 2.1 | 1.5 | 1.5 | 0.9 | 0.2 |
| White | 1996 | 1.5 | 1.0 | 0.9 | 0.5 | 0.1 |
| | 1992 | 1.3 | 1.0 | 1.1 | 0.3 | 0.1 |
| | 1990 | 2.2 | 1.4 | 1.0 | 1.0 | 0.1 |
| Black | 1996 | 2.5 | 2.0 | 2.9 | 1.4 | 0.3 |
| | 1992 | 2.7 | 1.5 | 2.3 | 1.4 | 0.4 |
| | 1990 | 4.2 | 2.6 | 4.1 | 1.9 | 0.4 |
| Hispanic | 1996 | 2.6 | 1.7 | 2.2 | 1.2 | 0.3 |
| | 1992 | 2.9 | 2.2 | 2.6 | 1.5 | 0.5 |
| | 1990 | 3.8 | 3.5 | 4.3 | 1.9 | --- |
| Asian/Pacific Islander | 1996 | 4.7 | 3.1 | 2.8 | 0.4 | --- |
| | 1992 | 4.4 | 2.7 | 3.0 | 0.8 | --- |
| | 1990 | 8.8 | 7.2 | 2.9 | --- | --- |
| American Indian | 1996 | 9.7 | 3.0 | 6.3 | 4.4 | 1.4 |
| | 1992 | *** | *** | *** | *** | *** |
| | 1990 | *** | *** | *** | *** | *** |

*** Sample size is insufficient to permit a reliable estimate.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Table B8.5

## Standard Errors for Percentage of Students by Mathematics Courses and Years of Study, Grade 12

| | Assessment Year | Years of Study: More Than One Year | One School Year | One-Half Year or Less | Not Studied |
|---|---|---|---|---|---|
| **Grade 12** | | | | | |
| General Mathematics | 1996 | 1.3 | 0.9 | 0.3 | 1.3 |
| | 1992 | 1.0 | 0.8 | 0.3 | 1.1 |
| | 1990 | 1.8 | 1.2 | 0.3 | 2.3 |
| Business or Consumer Mathematics | 1996 | 0.4 | 0.7 | 0.4 | 0.9 |
| | 1992 | 0.4 | 1.0 | 0.7 | 1.0 |
| | 1990 | 0.5 | 1.0 | 1.0 | 1.3 |
| Introduction to Algebra or Pre-Algebra | 1996 | 0.5 | 1.1 | 0.6 | 1.1 |
| | 1992 | 0.5 | 1.0 | 0.4 | 1.2 |
| | 1990 | 0.6 | 1.7 | 0.7 | 1.8 |
| First-Year Algebra | 1996 | 0.4 | 0.8 | 0.5 | 0.5 |
| | 1992 | 0.4 | 0.9 | 0.3 | 0.7 |
| | 1990 | 0.7 | 1.1 | 0.5 | 1.0 |
| Geometry | 1996 | 0.4 | 1.5 | 0.6 | 1.1 |
| | 1992 | 0.4 | 1.3 | 0.4 | 1.2 |
| | 1990 | 0.5 | 1.7 | 0.7 | 1.5 |
| Second-Year Algebra | 1996 | 0.3 | 1.4 | 0.6 | 1.2 |
| | 1992 | 0.2 | 1.5 | 0.7 | 1.4 |
| | 1990 | 0.4 | 2.1 | 0.7 | 1.9 |
| Trigonometry | 1996 | 0.3 | 1.5 | 1.2 | 1.6 |
| | 1992 | 0.2 | 1.2 | 1.1 | 1.5 |
| | 1990 | 0.2 | 1.5 | 1.4 | 1.7 |
| Pre-Calculus, Third-Year Algebra | 1996 | 0.2 | 1.4 | 0.8 | 1.4 |
| | 1992 | 0.2 | 1.1 | 0.8 | 1.2 |
| | 1990 | 0.2 | 1.5 | 0.8 | 1.7 |
| Calculus | 1996 | 0.2 | 0.8 | 0.3 | 0.9 |
| | 1992 | 0.2 | 0.7 | 0.3 | 0.8 |
| | 1990 | 0.2 | 0.7 | 0.5 | 0.7 |
| Probability or Statistics | 1996 | 0.4 | 0.7 | 0.7 | 1.4 |
| | 1992 | 0.2 | 0.3 | 0.6 | 0.7 |
| | 1990 | 0.2 | 0.4 | 1.0 | 1.2 |
| Unified, Integrated, or Sequential Mathematics | 1996 | 1.1 | 0.5 | 0.4 | 1.7 |
| | 1992 | 0.4 | 0.4 | 0.4 | 0.9 |
| Applied Mathematics (Technical Preparation) | 1996 | 0.4 | 0.7 | 0.5 | 0.9 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Table B8.6

## Standard Errors for Percentage of Students by Highest Algebra-through-Calculus Course Taken, Grade 12

THE NATION'S REPORT CARD naep

| | Assessment Year | Not Taken Pre-Algebra | Pre-Algebra | First-Year Algebra | Second-Year Algebra | Pre-Calculus or Third-Year Algebra | Calculus |
|---|---|---|---|---|---|---|---|
| **Grade 12** | | | | | | | |
| All Students | 1996 | 0.5 | 0.3 | 1.0 | 1.3 | 0.9 | 0.5 |
| | 1992 | 0.5 | 0.5 | 1.3 | 1.7 | 0.8 | 0.6 |
| | 1990 | 0.8 | 0.7 | 1.6 | 1.6 | 1.1 | 0.5 |
| Females | 1996 | 0.4 | 0.3 | 1.2 | 1.5 | 1.2 | 0.6 |
| | 1992 | 0.6 | 0.5 | 1.5 | 1.9 | 1.0 | 0.6 |
| | 1990 | 1.0 | 1.0 | 1.9 | 1.7 | 1.2 | 0.5 |
| Males | 1996 | 0.7 | 0.4 | 1.2 | 1.5 | 1.1 | 0.8 |
| | 1992 | 0.7 | 0.6 | 1.3 | 1.8 | 0.9 | 0.6 |
| | 1990 | 1.2 | 0.7 | 1.7 | 1.9 | 1.2 | 0.6 |
| White | 1996 | 0.5 | 0.4 | 1.1 | 1.6 | 1.1 | 0.6 |
| | 1992 | 0.6 | 0.5 | 1.5 | 2.0 | 0.9 | 0.6 |
| | 1990 | 0.9 | 0.8 | 1.7 | 2.0 | 1.4 | 0.5 |
| Black | 1996 | 0.9 | 0.7 | 2.2 | 2.6 | 1.3 | 0.9 |
| | 1992 | 1.5 | 1.2 | 1.8 | 2.8 | 1.2 | 0.6 |
| | 1990 | 1.8 | 1.5 | 2.6 | 2.5 | 1.6 | 0.4 |
| Hispanic | 1996 | 1.7 | 1.0 | 1.8 | 3.3 | 1.6 | 1.0 |
| | 1992 | 2.1 | 1.2 | 2.6 | 4.0 | 1.0 | 0.8 |
| | 1990 | 3.3 | 2.2 | 3.2 | 3.4 | 0.9 | 0.7 |
| Asian/Pacific Islander | 1996 | 1.2 | 2.0 | 3.0 | 3.6 | 2.8 | 5.4 |
| | 1992 | 0.5 | 1.6 | 3.3 | 4.7 | 3.2 | 4.0 |
| | 1990 | 1.9 | 6.1 | 4.8 | 4.3 | 3.9 | 2.9 |
| American Indian | 1996 | 2.1 | 1.9 | 5.1 | 4.3 | 4.1 | 1.8 |
| | 1992 | *** | *** | *** | *** | *** | *** |
| | 1990 | *** | *** | *** | *** | *** | *** |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

**Table B8.7**

# Standard Errors for Percentage of Students by Whether They Have Taken a Geometry Course and by Gender and Race/Ethnicity, Grade 12

THE NATION'S REPORT CARD naep

| Grade 12 | Assessment Year | Taken a Geometry Course: Yes | Taken a Geometry Course: No |
|---|---|---|---|
| All Students | 1996 | 1.4 | 1.4 |
| | 1992 | 1.3 | 1.3 |
| | 1990 | 1.7 | 1.7 |
| Females | 1996 | 1.5 | 1.5 |
| | 1992 | 1.3 | 1.3 |
| | 1990 | 1.9 | 1.9 |
| Males | 1996 | 1.5 | 1.5 |
| | 1992 | 1.6 | 1.6 |
| | 1990 | 1.9 | 1.9 |
| White | 1996 | 1.6 | 1.6 |
| | 1992 | 1.3 | 1.3 |
| | 1990 | 1.8 | 1.8 |
| Black | 1996 | 2.4 | 2.4 |
| | 1992 | 3.5 | 3.5 |
| | 1990 | 3.0 | 3.0 |
| Hispanic | 1996 | 2.7 | 2.7 |
| | 1992 | 5.6 | 5.6 |
| | 1990 | 3.8 | 3.8 |
| Asian/Pacific Islander | 1996 | 3.9 | 3.9 |
| | 1992 | 2.7 | 2.7 |
| | 1990 | 4.7 | 4.7 |
| American Indian | 1996 | 15.4 | 15.4 |
| | 1992 | *** | *** |
| | 1990 | *** | *** |

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

**Figure B9.1** **Standard Errors for Percentage of Students Whose Teachers Place "A Lot" of Emphasis on Specific Content Strands by Grade and Content Strand**

| | |
|---|---|
| **Grade 4** | |
| Number Sense, Properties, & Operations | 1.1 |
| Measurement | 2.2 |
| Geometry & Spatial Sense | 1.7 |
| Data Analysis, Statistics, & Probability | 1.4 |
| Algebra & Functions | 1.9 |
| **Grade 8** | |
| Number Sense, Properties, & Operations | 1.9 |
| Measurement | 2.8 |
| Geometry & Spatial Sense | 2.4 |
| Data Analysis, Statistics, & Probability | 2.0 |
| Algebra & Functions | 3.3 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B9.1** **Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Number Sense, Properties, and Operations, Grades 4 and 8, 1996**

| | Level of Emphasis | | |
|---|---|---|---|
| | A Lot | Some | Little or None |
| **Grade 4** | | | |
| All Students | 1.1 | 1.2 | 0.1 |
| **Grade 8** | | | |
| All Students | 1.9 | 1.8 | 0.6 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 3.1 | 3.1 | 0.7 |
| Pre-Algebra | 1.8 | 1.7 | 0.8 |
| Algebra | 3.1 | 2.8 | 1.0 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B9.2

### Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Measurement, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

| | Level of Emphasis | | |
|---|---|---|---|
| | A Lot | Some | Little or None |
| **Grade 4** | | | |
| All Students | 2.2 | 2.3 | 1.8 |
| **Grade 8** | | | |
| All Students | 2.8 | 3.2 | 2.6 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 4.1 | 4.9 | 3.6 |
| Pre-Algebra | 3.6 | 4.7 | 4.3 |
| Algebra | 3.6 | 4.7 | 4.0 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B9.3

### Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Geometry and Spatial Sense, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

| | Level of Emphasis | | |
|---|---|---|---|
| | A Lot | Some | Little or None |
| **Grade 4** | | | |
| All Students | 1.7 | 2.2 | 2.4 |
| **Grade 8** | | | |
| All Students | 2.4 | 2.7 | 2.7 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 4.3 | 4.1 | 4.4 |
| Pre-Algebra | 2.9 | 3.8 | 3.7 |
| Algebra | 3.0 | 3.9 | 3.1 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B9.4

### Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Data Analysis, Statistics, and Probability, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

| | Level of Emphasis | | |
|---|---|---|---|
| | A Lot | Some | Little or None |
| **Grade 4** | | | |
| All Students | 1.4 | 2.6 | 2.5 |
| **Grade 8** | | | |
| All Students | 2.0 | 3.1 | 3.3 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 3.5 | 4.4 | 3.9 |
| Pre-Algebra | 2.5 | 4.6 | 4.7 |
| Algebra | 3.3 | 5.4 | 4.7 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B9.5

### Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Algebra and Functions, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

| | Level of Emphasis | | |
|---|---|---|---|
| | A Lot | Some | Little or None |
| **Grade 4** | | | |
| All Students | 1.9 | 2.7 | 3.0 |
| **Grade 8** | | | |
| All Students | 3.3 | 2.9 | 1.1 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 5.2 | 4.9 | 2.5 |
| Pre-Algebra | 4.7 | 4.5 | 1.3 |
| Algebra | 3.0 | 2.9 | 0.6 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Figure B9.2**

## Standard Errors for Percentage of Students Whose Teachers Place "A Lot" of Emphasis on Specific Mathematics Processes by Grade and Mathematics Processes

THE NATION'S REPORT CARD naep

| | |
|---|---|
| **Grade 4** | |
| Facts and Concepts | 1.0 |
| Skills and Procedures | 1.2 |
| Reasoning | 2.4 |
| Communication | 2.4 |
| **Grade 8** | |
| Facts and Concepts | 2.7 |
| Skills and Procedures | 2.5 |
| Reasoning | 3.0 |
| Communication | 3.0 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

**Table B9.6**

## Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Learning Mathematics Facts and Concepts, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

| | Level of Emphasis | | |
|---|---|---|---|
| | A Lot | Some | Little or None |
| **Grade 4** | | | |
| All Students | 1.0 | 1.0 | - - - |
| **Grade 8** | | | |
| All Students | 2.7 | 2.2 | 1.4 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 4.2 | 3.7 | 2.7 |
| Pre-Algebra | 3.2 | 3.0 | 0.9 |
| Algebra | 3.7 | 2.9 | 1.9 |

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B9.7

### Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Learning Skills and Procedures Needed to Solve Routine Problems, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

| | Level of Emphasis | | |
|---|---|---|---|
| | A Lot | Some | Little or None |
| **Grade 4** | | | |
| All Students | 1.2 | 1.2 | - - - |
| **Grade 8** | | | |
| All Students | 2.5 | 2.4 | 0.9 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 3.6 | 3.7 | 0.3 |
| Pre-Algebra | 3.3 | 3.0 | 1.2 |
| Algebra | 3.3 | 2.7 | 2.1 |

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B9.8

### Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Developing Reasoning Ability to Solve Unique Problems, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

| | Level of Emphasis | | |
|---|---|---|---|
| | A Lot | Some | Little or None |
| **Grade 4** | | | |
| All Students | 2.4 | 2.4 | 1.1 |
| **Grade 8** | | | |
| All Students | 3.0 | 3.1 | 1.5 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 4.0 | 4.8 | 2.9 |
| Pre-Algebra | 4.6 | 4.4 | 1.1 |
| Algebra | 3.7 | 3.7 | 0.8 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B9.9

### Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Learning How to Communicate Ideas in Mathematics Effectively, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

| | Level of Emphasis | | |
|---|---|---|---|
| | A Lot | Some | Little or None |
| **Grade 4** | | | |
| All Students | 2.4 | 2.5 | 1.8 |
| **Grade 8** | | | |
| All Students | 3.0 | 3.0 | 1.9 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 4.0 | 4.4 | 3.4 |
| Pre-Algebra | 4.3 | 4.0 | 1.9 |
| Algebra | 3.8 | 3.8 | 2.4 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B9.10

### Standard Errors for Percentage of Students by Teachers' Reports on Frequency with Which Students Work with Objects Like Rulers, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

| | Frequency | | | |
|---|---|---|---|---|
| | Almost Every Day | Once or Twice a Week | Once or Twice a Month | Never or Hardly Ever |
| **Grade 4** | | | | |
| All Students | 1.0 | 2.3 | 2.6 | 0.9 |
| **Grade 8** | | | | |
| All Students | 2.0 | 2.4 | 2.7 | 2.2 |
| Students Enrolled in: | | | | |
| Eighth-Grade Mathematics | 2.0 | 3.4 | 3.7 | 2.6 |
| Pre-Algebra | 2.4 | 2.8 | 4.7 | 3.3 |
| Algebra | 2.9 | 2.5 | 4.7 | 3.6 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B9.11 Standard Errors for Percentage of Students by Teachers' Reports on Frequency with Which Students Work with Counting Blocks and Geometric Shapes, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

| | Frequency | | | |
|---|---|---|---|---|
| | Almost Every Day | Once or Twice a Week | Once or Twice a Month | Never or Hardly Ever |
| **Grade 4** | | | | |
| All Students | 0.9 | 1.8 | 2.8 | 2.4 |
| **Grade 8** | | | | |
| All Students | 0.5 | 1.8 | 3.3 | 3.4 |
| Students Enrolled in: | | | | |
| Eighth-Grade Mathematics | 0.7 | 3.0 | 5.0 | 5.0 |
| Pre-Algebra | 0.8 | 2.5 | 4.1 | 4.3 |
| Algebra | - - - | 1.5 | 4.2 | 4.2 |

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B9.12 Standard Errors for Percentage of Students by Frequency with Which They Work with Measuring Instruments or Geometric Solids, Grade 12, 1996

THE NATION'S REPORT CARD naep

| | Frequency | | | |
|---|---|---|---|---|
| **Grade 12** | Almost Every Day | Once or Twice a Week | Once or Twice a Month | Never or Hardly Ever |
| Students Taking Mathematics | 0.5 | 0.7 | 1.0 | 1.4 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B9.13

### Standard Errors for Percentage of Students by Frequency with Which They Solve Problems in Small Groups or with a Partner, Grades 4, 8, and 12, 1996*

THE NATION'S REPORT CARD naep

| | Frequency | | | |
|---|---|---|---|---|
| | Almost Every Day | Once or Twice a Week | Once or Twice a Month | Never or Hardly Ever |
| **Grade 4** | | | | |
| All Students | 2.0 | 1.9 | 1.4 | 1.5 |
| **Grade 8** | | | | |
| All Students | 2.9 | 3.2 | 3.3 | 1.4 |
| Students Enrolled in: | | | | |
| Eighth-Grade Mathematics | 4.0 | 5.2 | 4.5 | 2.0 |
| Pre-Algebra | 3.8 | 4.8 | 4.3 | 1.7 |
| Algebra | 4.1 | 3.3 | 4.2 | 2.7 |
| **Grade 12** | | | | |
| Students Taking Mathematics | 1.4 | 1.2 | 0.8 | 1.0 |

* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

## Table B9.14

### Standard Errors for Percentage of Students by Frequency with Which They Write a Few Sentences about How to Solve a Mathematics Problem, Grades 4, 8, and 12*

THE NATION'S REPORT CARD naep

| | Assessment Year | Frequency | | | |
|---|---|---|---|---|---|
| | | Almost Every Day | Once or Twice a Week | Once or Twice a Month | Never or Hardly Ever |
| **Grade 4** | | | | | |
| All Students | 1996 | 1.4 | 2.1 | 2.6 | 2.4 |
| | 1992 | 0.6 | 1.9 | 1.9 | 2.3 |
| **Grade 8** | | | | | |
| All Students | 1996 | 1.1 | 2.8 | 2.8 | 3.3 |
| | 1992 | 0.9 | 2.0 | 2.5 | 2.5 |
| Students Enrolled in: | | | | | |
| Eighth-Grade Mathematics | 1996 | 1.3 | 4.2 | 3.8 | 4.2 |
| | 1992 | 0.6 | 2.5 | 3.1 | 3.3 |
| Pre-Algebra | 1996 | 1.5 | 4.3 | 3.9 | 4.0 |
| | 1992 | 0.9 | 3.1 | 3.8 | 3.1 |
| Algebra | 1996 | 1.7 | 3.5 | 5.4 | 5.2 |
| | 1992 | 3.0 | 3.0 | 3.5 | 3.6 |
| **Grade 12** | | | | | |
| Students Taking Mathematics | 1996 | 0.5 | 0.6 | 0.8 | 1.0 |

* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

**Table B9.15**

## Standard Errors for Percentage of Students by Frequency with Which They Write Reports or Do Mathematics Projects, Grades 4, 8, and 12*

THE NATION'S REPORT CARD naep

| | Assessment Year | Frequency: Almost Every Day | Frequency: Once or Twice a Week | Frequency: Once or Twice a Month | Frequency: Never or Hardly Ever |
|---|---|---|---|---|---|
| **Grade 4** | | | | | |
| All Students | 1996 | 0.5 | 0.8 | 2.3 | 2.4 |
| | 1992 | --- | 0.4 | 2.0 | 2.1 |
| **Grade 8** | | | | | |
| All Students | 1996 | --- | 1.1 | 3.2 | 3.3 |
| | 1992 | --- | 0.3 | 1.9 | 2.0 |
| Students Enrolled in: | | | | | |
| Eighth-Grade Mathematics | 1996 | --- | 1.0 | 4.6 | 4.8 |
| | 1992 | --- | 0.2 | 2.6 | 2.7 |
| Pre-Algebra | 1996 | --- | 2.3 | 4.7 | 4.9 |
| | 1992 | --- | 0.7 | 2.9 | 3.1 |
| Algebra | 1996 | --- | 1.2 | 4.0 | 4.1 |
| | 1992 | --- | 0.7 | 2.3 | 2.5 |
| **Grade 12** | | | | | |
| Students Taking Mathematics | 1996 | 0.2 | 0.3 | 1.2 | 1.3 |

* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

**Table B9.16**

## Standard Errors for Percentage of Students by Frequency with Which They Discuss Solutions to Mathematics Problems with Other Students, Grades 4, 8, and 12*

THE NATION'S REPORT CARD naep

| | Assessment Year | Frequency: Almost Every Day | Once or Twice a Week | Once or Twice a Month | Never or Hardly Ever |
|---|---|---|---|---|---|
| **Grade 4** | | | | | |
| All Students | 1996 | 2.1 | 2.1 | 1.7 | 1.3 |
| | 1992 | 2.3 | 1.9 | 2.0 | 0.8 |
| **Grade 8** | | | | | |
| All Students | 1996 | 3.2 | 3.1 | 1.9 | 0.7 |
| | 1992 | 2.3 | 2.1 | 1.9 | 1.1 |
| Students Enrolled in: | | | | | |
| Eighth-Grade Mathematics | 1996 | 4.8 | 4.4 | 3.1 | 1.1 |
| | 1992 | 3.1 | 3.2 | 2.4 | 1.6 |
| Pre-Algebra | 1996 | 4.0 | 3.7 | 1.5 | 0.9 |
| | 1992 | 3.6 | 2.4 | 3.3 | 1.5 |
| Algebra | 1996 | 4.6 | 4.3 | 2.9 | 0.4 |
| | 1992 | 3.7 | 3.4 | 2.1 | 0.6 |
| **Grade 12** | | | | | |
| Students Taking Mathematics | 1996 | 1.1 | 0.9 | 0.5 | 0.7 |

* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

**Table B9.17**

## Standard Errors for Percentage of Students by Teachers' Reports on Frequency with Which Students Work and Discuss Mathematics Problems That Reflect Real-Life Situations, Grades 4 and 8

| | Assessment Year | Frequency: Almost Every Day | Once or Twice a Week | Once or Twice a Month | Never or Hardly Ever |
|---|---|---|---|---|---|
| **Grade 4** | | | | | |
| All Students | 1996 | 2.1 | 2.1 | 1.9 | 0.9 |
| | 1992 | 2.1 | 2.4 | 1.7 | 1.1 |
| **Grade 8** | | | | | |
| All Students | 1996 | 2.6 | 2.9 | 2.7 | 1.1 |
| | 1992 | 1.6 | 2.2 | 2.1 | 1.0 |
| Students Enrolled in: | | | | | |
| Eighth-Grade Mathematics | 1996 | 3.3 | 3.7 | 3.8 | 1.6 |
| | 1992 | 2.1 | 2.7 | 2.9 | 1.1 |
| Pre-Algebra | 1996 | 4.2 | 4.7 | 3.6 | 1.4 |
| | 1992 | 2.9 | 2.6 | 3.1 | 1.7 |
| Algebra | 1996 | 4.0 | 5.0 | 3.5 | 1.8 |
| | 1992 | 3.6 | 4.2 | 2.8 | 1.9 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

## Table B9.18 Standard Errors for Percentage of Students by Frequency with Which Students Use Calculators in Class, Grades 4, 8, and 12*

THE NATION'S REPORT CARD naep

| | Assessment Year | Frequency: Almost Every Day | Once or Twice a Week | Once or Twice a Month | Never or Hardly Ever |
|---|---|---|---|---|---|
| **Grade 4** | | | | | |
| All Students | 1996 | 0.9 | 2.2 | 2.4 | 2.4 |
| | 1992 | 0.4 | 1.9 | 2.0 | 2.5 |
| **Grade 8** | | | | | |
| All Students | 1996 | 2.7 | 2.5 | 2.1 | 1.5 |
| | 1992 | 2.7 | 2.1 | 2.0 | 2.4 |
| Students Enrolled in: | | | | | |
| Eighth-Grade Mathematics | 1996 | 3.9 | 3.7 | 2.8 | 2.5 |
| | 1992 | 3.3 | 2.7 | 3.4 | 2.9 |
| Pre-Algebra | 1996 | 4.0 | 3.3 | 3.4 | 1.8 |
| | 1992 | 4.6 | 3.2 | 2.4 | 3.9 |
| Algebra | 1996 | 4.2 | 3.4 | 1.6 | 1.6 |
| | 1992 | 3.9 | 3.1 | 2.4 | 2.7 |
| **Grade 12** | | | | | |
| Students Taking Mathematics | 1996 | 1.1 | 0.9 | 0.3 | 0.5 |

* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

## Figure B9.3 Standard Errors for Percentage of Students Who Report Using Scientific Calculators, Grades 8 and 12, 1996

THE NATION'S REPORT CARD naep

| | |
|---|---|
| **Grade 8** | |
| All Students | 2.1 |
| Eighth-Grade Mathematics | 2.2 |
| Pre-Algebra | 2.9 |
| Algebra | 3.0 |
| **Grade 12** | |
| Students Taking Mathematics | 1.3 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Figure B9.4 Standard Errors for Percentage of Students Who Report Using Graphing Calulators, Grades 8 and 12, 1996

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| | |
|---|---|
| **Grade 8** | |
| All Students | 1.1 |
| Eighth-Grade Mathematics | 0.8 |
| Pre-Algebra | 2.3 |
| Algebra | 2.3 |
| **Grade 12** | |
| Students Taking Mathematics | 2.0 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B9.19 Standard Errors for Percentage of Students by Teacher Reported Uses of Calculators, Grades 4 and 8

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| | Assessment Year | Teachers Allow Unrestricted Use in Classroom | Teachers Allow Use on Mathematics Tests |
|---|---|---|---|
| **Grade 4** | | | |
| All Students | 1996 | 1.8 | 1.7 |
| | 1992 | 1.1 | 1.1 |
| **Grade 8** | | | |
| All Students | 1996 | 2.9 | 2.6 |
| | 1992 | 2.3 | 3.0 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 1996 | 4.0 | 3.7 |
| | 1992 | 2.7 | 3.6 |
| Pre-Algebra | 1996 | 4.8 | 3.9 |
| | 1992 | 4.2 | 4.1 |
| Algebra | 1996 | 5.0 | 3.1 |
| | 1992 | 4.0 | 3.6 |

SOURCE: National Center for Education Statistic, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

## Table B9.20 Standard Errrors for Percentage of Students by Calculator Use, Grades 4, 8, and 12, 1996

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| | Calculator Use | | | |
|---|---|---|---|---|
| | Appropriate Calculator Use Group | | Other Group | |
| | Percentage of Students | Average Scale Score | Percentage of Students | Average Scale Score |
| **Grade 4** | | | | |
| All Students | 0.8 | 1.5 | 0.8 | 1.1 |
| Unrestricted Classroom Use | 2.4 | 5.0 | 2.4 | 2.7 |
| Restricted Classroom Use | 0.9 | 1.6 | 0.9 | 1.4 |
| Allowed Use on Classroom Tests | 2.8 | 5.2 | 2.8 | 3.0 |
| Not Allowed Use on Classroom Tests | 0.9 | 1.6 | 0.9 | 1.2 |
| **Grade 8** | | | | |
| All Students | 0.9 | 1.8 | 0.9 | 1.0 |
| Unrestricted Classroom Use | 1.6 | 3.0 | 1.6 | 1.8 |
| Restricted Classroom Use | 0.9 | 2.2 | 0.9 | 1.8 |
| Allowed Use on Classroom Tests | 1.3 | 2.5 | 1.3 | 1.5 |
| Not Allowed Use on Classroom Tests | 1.2 | 2.9 | 1.2 | 2.1 |
| **Grade 12** | | | | |
| All Students | 0.7 | 1.2 | 0.7 | 1.0 |
| Use in Classwork: | | | | |
| Almost Every Day | 0.7 | 1.2 | 0.7 | 1.1 |
| Once or Twice a Week | 1.6 | 3.4 | 1.6 | 1.7 |
| Once or Twice a Month | 2.4 | *** | 2.4 | 3.0 |
| Never or Hardly Ever | 1.5 | 2.5 | 1.5 | 1.7 |
| Use on Tests or Quizzes: | | | | |
| Almost Every Day | 0.8 | 1.4 | 0.8 | 1.3 |
| Once or Twice a Week | 1.4 | 1.7 | 1.4 | 1.4 |
| Once or Twice a Month | 1.7 | 2.5 | 1.7 | 1.9 |
| Never or Hardly Ever | 1.0 | 2.8 | 1.0 | 1.4 |

NOTE: Students in the "Appropriate Calculator Use" group used the calculator for at least 65 percent of the calculator-suitable questions and used the calculator for no more than one of the calculator-unsuitable questions. Students in the "Other" group used the calculator for less than 65 percent of the calculator-suitable questions and/or used it for more than one of the calculator-unsuitable questions.

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B9.21 Standard Errors for Percentage of Students by Frequency with Which Students Take Mathematics Tests, Grades 4, 8, and 12, 1996*

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| | Frequency | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Almost Every Day | | Once or Twice a Week | | Once or Twice a Month | | Never or Hardly Ever | |
| | Percentage of Students | Average Scale Score | Percentage of Students | Average Scale Score | Percentage of Students | Average Scale Score | Percentage of Students | Average Scale Score |
| **Grade 4** | | | | | | | | |
| All Students | 0.4 | 4.7 | 2.3 | 1.7 | 2.5 | 1.0 | 1.1 | 4.9 |
| **Grade 8** | | | | | | | | |
| All Students | 0.3 | *** | 3.1 | 1.7 | 3.1 | 1.7 | 0.1 | *** |
| Students Enrolled in: | | | | | | | | |
| Eighth-Grade Mathematics | 0.2 | *** | 3.9 | 2.7 | 3.8 | 2.0 | 0.0 | *** |
| Pre-Algebra | 0.3 | *** | 5.1 | 1.8 | 5.1 | 2.4 | 0.2 | *** |
| Algebra | 0.8 | *** | 5.8 | 2.7 | 5.9 | 2.2 | 0.1 | *** |
| **Grade 12** | | | | | | | | |
| Students Taking Mathematics | 0.4 | 3.6 | 1.3 | 1.2 | 1.2 | 1.5 | 0.2 | *** |

* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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**Table B9.22** *Standard Errors for Percentage of Students by Teachers' Reports on the Frequency with Which They Use Multiple-Choice Tests to Assess Their Students' Progress in Mathematics, Grades 4 and 8, 1996*

| | Frequency | | | |
|---|---|---|---|---|
| | Once or Twice a Week | Once or Twice a Month | Once or Twice a Year | Never or Hardly Ever |
| **Grade 4** | | | | |
| All Students | 1.0 | 2.7 | 2.1 | 2.2 |
| **Grade 8** | | | | |
| All Students | 0.9 | 3.1 | 3.1 | 2.8 |
| Students Enrolled in: | | | | |
| Eighth-Grade Mathematics | 1.1 | 4.1 | 4.2 | 4.5 |
| Pre-Algebra | 1.0 | 4.2 | 4.5 | 4.3 |
| Algebra | 0.8 | 4.1 | 4.0 | 3.9 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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**Table B9.23** *Standard Errors for Percentage of Students by Teachers' Reports on the Frequency with Which They Use Short and Long Written Responses to Assess Their Students' Progress in Mathematics, Grades 4 and 8, 1996*

| | Frequency | | | |
|---|---|---|---|---|
| | Once or Twice a Week | Once or Twice a Month | Once or Twice a Year | Never or Hardly Ever |
| **Grade 4** | | | | |
| All Students | 2.6 | 2.4 | 2.0 | 1.9 |
| **Grade 8** | | | | |
| All Students | 2.8 | 3.7 | 2.5 | 2.3 |
| Students Enrolled in: | | | | |
| Eighth-Grade Mathematics | 3.6 | 4.9 | 3.2 | 3.6 |
| Pre-Algebra | 4.3 | 5.2 | 2.9 | 3.0 |
| Algebra | 2.8 | 4.1 | 4.0 | 3.2 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B9.24

### Standard Errors for Percentage of Students by Teachers' Reports on the Frequency with Which They Use Individual or Group Projects or Presentations to Assess Their Students' Progress in Mathematics, Grades 4 and 8, 1996

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| | Frequency | | | |
|---|---|---|---|---|
| | Once or Twice a Week | Once or Twice a Month | Once or Twice a Year | Never or Hardly Ever |
| **Grade 4** | | | | |
| All Students | 2.2 | 1.7 | 2.4 | 2.4 |
| **Grade 8** | | | | |
| All Students | 1.5 | 3.2 | 3.5 | 3.0 |
| Students Enrolled in: | | | | |
| Eighth-Grade Mathematics | 1.6 | 4.5 | 4.9 | 3.8 |
| Pre-Algebra | 3.1 | 4.4 | 4.2 | 3.5 |
| Algebra | 2.0 | 4.5 | 4.5 | 3.6 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B9.25

### Standard Errors for Percentage of Students by Teachers' Reports on the Frequency with Which They Use Portfolio Collections of Each Student's Work to Assess Students' Progress in Mathematics, Grades 4 and 8, 1996

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| | Frequency | | | |
|---|---|---|---|---|
| | Once or Twice a Week | Once or Twice a Month | Once or Twice a Year | Never or Hardly Ever |
| **Grade 4** | | | | |
| All Students | 1.8 | 2.4 | 1.9 | 2.2 |
| **Grade 8** | | | | |
| All Students | 2.1 | 2.5 | 2.5 | 3.7 |
| Students Enrolled in: | | | | |
| Eighth-Grade Mathematics | 3.2 | 2.5 | 4.1 | 5.1 |
| Pre-Algebra | 2.7 | 3.4 | 2.4 | 4.8 |
| Algebra | 2.9 | 4.2 | 2.8 | 5.2 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B10.1 Standard Errors for Percentage of Students by Their Response to the Statement: "I Like Mathematics," Grades 4, 8, and 12, 1996

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| | Agreement | | |
|---|---|---|---|
| | Agree | Disagree | Undecided |
| **Grade 4** | | | |
| All Students | 0.9 | 0.8 | 0.6 |
| **Grade 8** | | | |
| All Students | 1.1 | 0.7 | 0.8 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 1.6 | 1.2 | 1.1 |
| Pre-Algebra | 1.6 | 1.2 | 1.1 |
| Algebra | 1.5 | 1.0 | 1.4 |
| **Grade 12** | | | |
| All Students | 0.8 | 0.8 | 0.6 |
| Students Who Are: | | | |
| Enrolled in Mathematics | 1.1 | 0.8 | 0.6 |
| Not Enrolled in Mathematics | 1.1 | 1.4 | 0.9 |
| Students Who Have: | | | |
| Taken Geometry | 0.9 | 0.8 | 0.5 |
| Not Taken Geometry | 1.9 | 1.9 | 1.4 |
| Highest Algebra-Calculus Course Taken: | | | |
| Pre-Algebra | 3.4 | 3.5 | 2.5 |
| First-Year Algebra | 2.1 | 2.2 | 1.4 |
| Second-Year Algebra | 1.2 | 1.1 | 0.8 |
| Third-Year Algebra/Pre-Calculus | 2.4 | 1.8 | 1.5 |
| Calculus | 3.6 | 1.6 | 2.6 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B10.2

# Standard Errors for Percentage of Students by Their Response to the Statement: "If I Had a Choice, I Would Not Study Any More Mathematics," Grades 4, 8, and 12, 1996

| | Agreement | | |
|---|---|---|---|
| | Agree | Disagree | Undecided |
| **Grade 4** | | | |
| All Students | 0.6 | 0.9 | 0.7 |
| **Grade 8** | | | |
| All Students | 0.6 | 0.9 | 0.6 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 1.0 | 1.5 | 1.0 |
| Pre-Algebra | 1.1 | 1.3 | 1.3 |
| Algebra | 1.1 | 1.3 | 1.0 |
| **Grade 12** | | | |
| All Students | 0.8 | 0.9 | 0.6 |
| Students Who Are: | | | |
| Enrolled in Mathematics | 0.7 | 1.0 | 0.6 |
| Not Enrolled in Mathematics | 1.6 | 1.2 | 1.1 |
| Students Who Have: | | | |
| Taken Geometry | 0.9 | 1.1 | 0.7 |
| Not Taken Geometry | 1.8 | 2.0 | 1.7 |
| Highest Algebra-Calculus Course Taken: | | | |
| Pre-Algebra | 3.8 | 2.6 | 3.1 |
| First-Year Algebra | 1.7 | 1.8 | 1.2 |
| Second-Year Algebra | 1.2 | 1.3 | 0.9 |
| Third-Year Algebra/Pre-Calculus | 1.4 | 2.1 | 1.3 |
| Calculus | 1.8 | 3.8 | 2.6 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

## Table B10.3 Standard Errors for Percentage of Students by Their Response to the Statement: "Everyone Can Do Well in Mathematics If They Try," Grades 4, 8, and 12, 1996

| | Agreement | | |
|---|---|---|---|
| | Agree | Disagree | Undecided |
| **Grade 4** | | | |
| All Students | 0.5 | 0.3 | 0.5 |
| **Grade 8** | | | |
| All Students | 0.8 | 0.6 | 0.5 |
| Students Enrolled in: | | | |
| Eighth-Grade Mathematics | 1.2 | 1.0 | 0.8 |
| Pre-Algebra | 1.4 | 0.8 | 1.0 |
| Algebra | 1.5 | 1.2 | 0.9 |
| **Grade 12** | | | |
| All Students | 0.8 | 0.7 | 0.6 |
| Students Who Are: | | | |
| Enrolled in Mathematics | 1.0 | 0.8 | 0.8 |
| Not Enrolled in Mathematics | 1.2 | 1.4 | 1.1 |
| Students Who Have: | | | |
| Taken Geometry | 1.0 | 0.8 | 0.7 |
| Not Taken Geometry | 1.6 | 1.6 | 1.0 |
| Highest Algebra-Calculus Course Taken: | | | |
| Pre-Algebra | 3.5 | 3.3 | 2.7 |
| First-Year Algebra | 1.5 | 1.3 | 1.3 |
| Second-Year Algebra | 1.1 | 1.0 | 1.0 |
| Third-Year Algebra/Pre-Calculus | 2.3 | 1.6 | 1.8 |
| Calculus | 2.7 | 2.7 | 1.9 |

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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