Chapter 4

Measurement

Content Strand Description

The Measurement content strand focuses on an understanding of the process of measurement and on the use of measurement to describe and compare mathematical and real-world objects. Students were asked to identify attributes of measurement, select appropriate units and tools, apply measurement concepts, and communicate measurement-related ideas.

At the fourth-grade level, the focus was on measurement of time, money, temperature, length, perimeter, area, weight/mass, and angles. At the eighth- and twelfth-grade levels, the measurement problems were more complex and involved volume and surface areas in addition to the aforementioned topics. Questions also involved reasoning with proportions, such as is required in scale drawing and map reading.

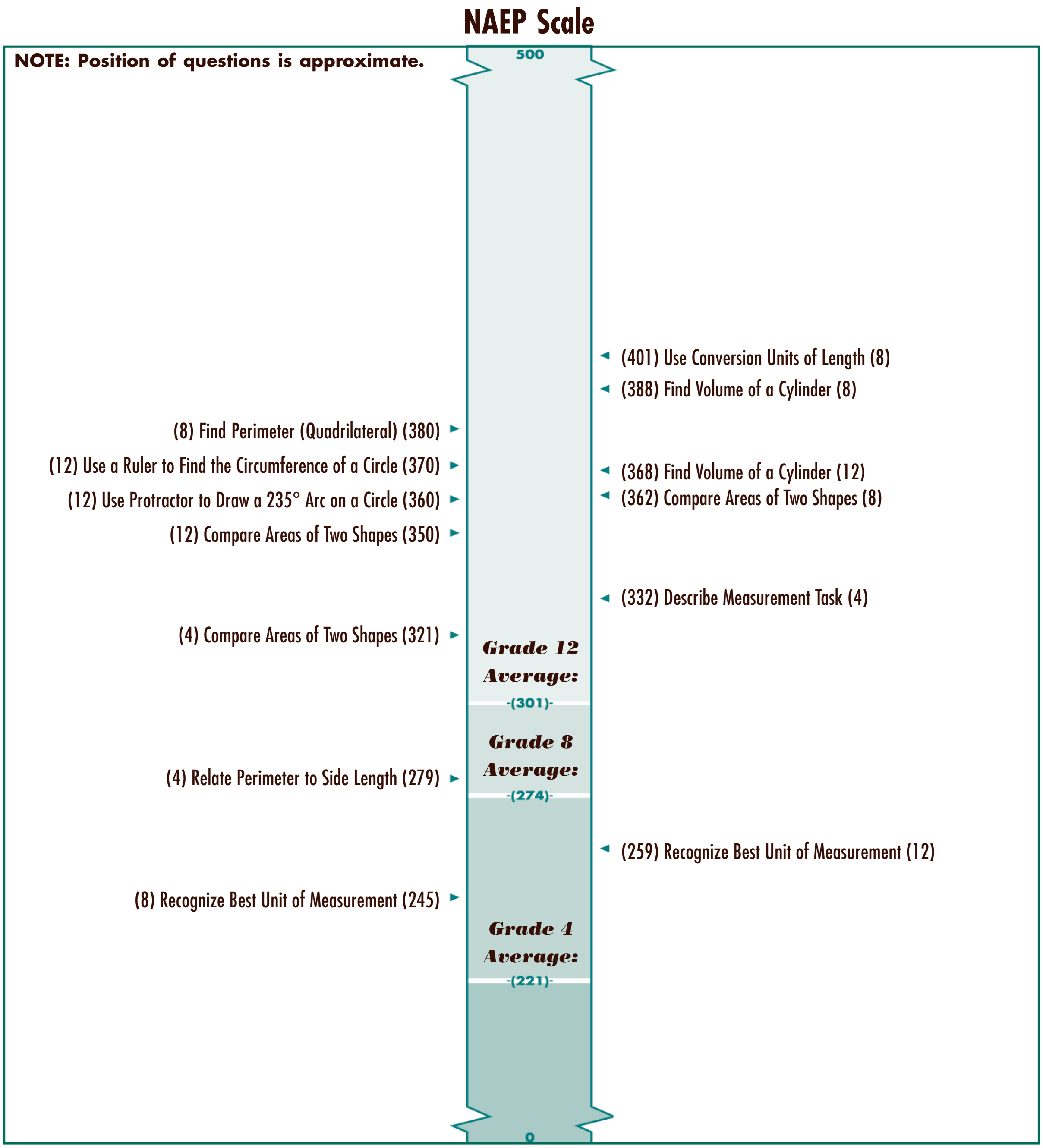
Examples of Individual Questions and Student Performance

Several assessment questions from the Measurement content strand of the NAEP 1996 assessment follow. For ease of discussion, presentation of the questions is organized around four areas of emphasis within the Measurement content strand: 1) *units of measurement*; 2) *measurement instruments*; 3) *perimeter, area, and volume*; and 4) *estimation of measurements*. Questions within all four areas tested students' conceptual understanding and procedural knowledge, as well as their abilities to reason, communicate, and make connections.

The sample questions from this content strand are mapped onto the NAEP composite mathematics scale as shown in Figure 4.1. Specific instructions on how to interpret this map are detailed at the end of Chapter 2. The map is included to provide an indication of the relative difficulty of each sample question and, thus, to suggest the type of material mastered within this content strand by students with varying degrees of mathematics proficiency. It should be remembered, however, that the difficulty of a question is influenced by many factors, including characteristics specific to the question (e.g., format, absence or presence of graphics, real-world application) as well as the particular mathematics content associated with the question and student opportunities to learn this content. It also must be remembered that overall performance on the Measurement content strand is not determined solely by performance on the few examples presented here. These examples illustrate only some of what students know and can do.

THE NATION'S REPORT CARD

Figure 4.1 ***Map of Selected Measurement Questions on the NAEP Composite Mathematics Scale (Item Map)***



NOTE: Each mathematics question was mapped onto the NAEP 0 to 500 mathematics scale. The position of the question on the scale represents the scale score obtained by students who had a 65 percent probability of successfully answering the question. (The probability was 74 percent for a 4-option multiple-choice question and 72 percent for a 5-option multiple-choice question.) Only selected questions are presented. The number 4, 8, or 12 in parentheses is the grade level at which the question was asked.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Asssessment.

The performance of students on the questions in the Measurement content strand is examined with respect to gender, race/ethnicity, and, for grades 8 and 12, the types of mathematics courses taken. However, as described in Chapter 2, the impact of taking geometry on student performance is not discussed for several reasons. First, there is only a small pool of students on which the specific influence of geometry could be isolated, given that most students who have taken geometry also have taken at least two years of algebra. Moreover, because more able students are likely to progress further in the mathematics course sequence, it is difficult to separate the impact of a particular curriculum from the impact of a student's overall strength in mathematics. Although comments on the impact of geometry course taking on performance on the questions in this content strand might be expected, these confounding effects make it difficult to isolate the specific impact of geometry. The data, however, are presented in the tables.

Units of measurement

These questions primarily assessed students' conceptual understanding and procedural knowledge of measurement units. Students had to understand what various units of measurement represent and the relationships between units. Questions typically required students to choose the best unit for a particular problem, and questions for older students required finer distinctions. Units that were assessed included standard and metric units of length, distance, volume, speed, and temperature, as well as units of time. Some questions required students to convert from one unit of measurement to another within the same system of measurement (e.g., feet to yards, quarts to pints) or to make and read scale drawings.

The following sample question was administered to both eighth- and twelfth-grade students. It is a multiple-choice question that tested students' conceptual understanding of appropriate measurement units. The question asked for the best unit for measuring plant growth during a 2-week period. To answer the question correctly, students had to realize that the daily growth of a plant would be small, and they needed to be familiar with different units of length in order to recognize which of those listed was small enough to make such a measurement. The question was not difficult for students and mapped at 245 for grade 8 and at 259 for grade 12 on the NAEP composite mathematics scale.

3. Of the following, which is the best unit to use when measuring the growth of a plant every other day during a 2-week period?

(A) Centimeter

(B) Meter

(C) Kilometer

(D) Foot

(E) Yard

Did you use the calculator on this question?

◯ Yes ◯ No

The correct option is A.

Student performance is reported in Tables 4.1 and 4.2. That the question was fairly easy can be seen by the fact that almost 80 percent of eighth-grade students and almost 90 percent of twelfth-grade students who answered the question selected the correct option. At the eighth-grade level, 54 percent of students classified as below the *Basic* level, 90 percent of students classified as *Basic*, and 97 percent of those classified as *Proficient* answered correctly. As might be expected, the percentage of twelfth-grade students at each achievement level who answered correctly was even higher: 72 percent of those classified as below *Basic*, 93 percent of those classified as *Basic*, and 98 percent of those classified as *Proficient*.

Eighth-grade students taking algebra were more likely to select the correct answer than those taking eighth-grade mathematics. The performance of students who were in pre-algebra, however, was not significantly different from the performance of students in either eighth-grade mathematics or algebra.

At the twelfth-grade level, students whose highest course was calculus in the algebra-through-calculus sequence performed better than those whose highest course was first- or second-year algebra.

Table 4.1

Percentage Correct for "Recognize Best Unit of Measurement"

THE NATION'S REPORT CARD naep

	Percentage Correct
Grade 8	
Overall	78
Males	78
Females	78
White	84
Black	63
Hispanic	66
Asian/Pacific Islander	– –
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	74
Pre-Algebra	80
Algebra	87
Grade 12	
Overall	87
Males	87
Females	87
White	90
Black	77
Hispanic	83
Asian/Pacific Islander	92
American Indian	***
Geometry Taken	90
Highest Algebra-Calculus Course Taken:	
Pre-Algebra	***
First-Year Algebra	85
Second-Year Algebra	89
Third-Year Algebra/Pre-Calculus	93
Calculus	96

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 4.2 ***Percentage Correct Within Achievement-Level Intervals for "Recognize Best Unit of Measurement"***

	Overall	NAEP Grades 8 and 12 Composite Scale Ranges			
		Below Basic	Basic	Proficient	Advanced
Grade 8	78	54	90	97	100!
Grade 12	87	72	93	98!	***

*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The next example in the *units of measurement* area also is a multiple-choice question for eighth-grade students that presented a real-world situation to assess students' problem-solving skills. The question gave a car odometer reading in miles, told the student there would be a detour some number of feet ahead, and then asked what the odometer would read when the car reached the detour. The conversion factor between feet and miles was given. To calculate the answer, students had to correctly convert the distance to the detour, given in feet, to a decimal fraction of a mile and then add that decimal fraction to the original odometer reading. Alternatively, students could have scanned the response options and selected the one response that corresponds to an increase of less than one mile.

7. A car odometer registered 41,256.9 miles when a highway sign warned of a detour 1,200 feet ahead. What will the odometer read when the car reaches the detour? (5,280 feet = 1 mile)

Ⓐ 42,456.9

Ⓑ 41,279.9

Ⓒ 41,261.3

Ⓓ 41,259.2

Ⓔ 41,257.1

Did you use the calculator on this question?

○ Yes ○ No

The correct option is E.

Performance data are shown in Tables 4.3 and 4.4. Twenty-six percent of the students selected the correct option. Another 37 percent selected Option A, the option corresponding to simply adding the number of feet to the odometer reading without first converting the distance to miles. Students taking pre-algebra or eighth-grade mathematics performed similarly, whereas students currently taking algebra performed better than the other two groups. Males performed better than females. Twenty-five percent of students at the *Basic* level, 50 percent at the *Proficient* level, and 70 percent of students at the *Advanced* level answered correctly. Only 11 percent of students classified as below the *Basic* level answered the question correctly. The question mapped at 401 on the NAEP mathematics composite scale.

Table 4.3 ***Percentage Correct for "Use Conversion Units of Length"***

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	26
Males	30
Females	21
White	30
Black	16
Hispanic	15
Asian/Pacific Islander	– –
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	22
Pre-Algebra	21
Algebra	39

*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 4.4 ***Percentage Correct Within Achievement-Level Intervals for "Use Conversion Units of Length"***

THE NATION'S REPORT CARD naep

	NAEP Grade 8 Composite Scale Range			
Overall	**Below Basic**	**Basic**	**Proficient**	**Advanced**
26	11	25	50	70

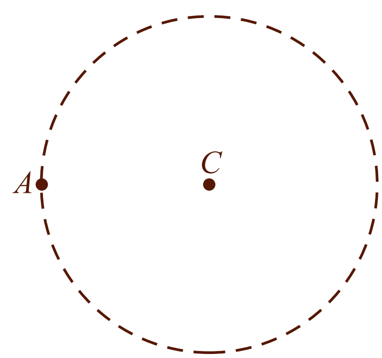
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Measurement instruments

Questions in this area assessed students' understanding of and ability to use measurement tools and instruments. Students had to identify appropriate instruments for certain situations. They also had to read representations of measurement instruments such as rulers, thermometers, gauges, and dials. Some questions in this area required students actually to use tools such as rulers, protractors, or compasses to measure and construct shapes. Questions for younger students involved more common instruments and required less accurate measurements than did questions for older students.

The following example is a short constructed-response question for grade 12 that required students to use a measurement instrument, provided with the assessment, to solve a problem. The question presented a dashed circle with center, *C*, and a point, *A*, marked on the circumference and asked students to use a protractor to draw and label an arc, *AB*, with a 235º angle. Because protractors only provide measurements up to 180 degrees, students needed to understand how to work with the difference between 235º and 180º to draw the obtuse angle required for the solution.

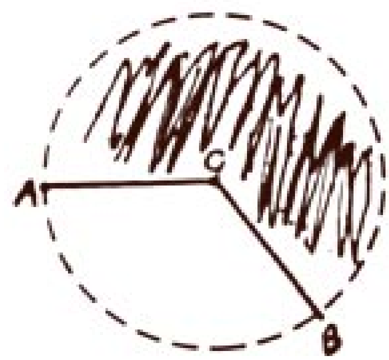
7. On the circle with center *C* shown below, use the protractor to locate and label a point *B* that creates an arc *AB* with measure 235˚. Darken this arc.



Responses were rated "correct" if they portrayed an obtuse angle *ACB* measuring within ±5º of 235º. More accurate responses, which were accurate within ±2º, were tabulated separately, as shown in Table 4.5.

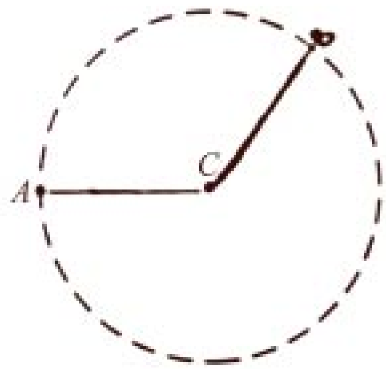
Responses were rated "incorrect" if point *B* was placed correctly on the circle (i.e., within ±5º of the correct location) but the arc was not clearly indicated, or if a sector or arc of 235º (±5º) was shown that did not have an endpoint at point *A*. Responses also were rated "incorrect" if they were incorrect for any other reason (e.g., the angle was incorrect). Three sample responses follow: one rated "correct" and two rated "incorrect." The "correct" response shows an angle *ACB* of 235º with a shaded area of the circle corresponding to the arc *AB*.

Sample "correct" response

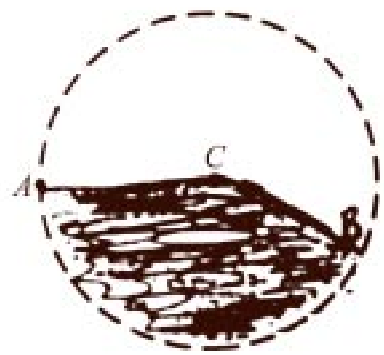


The first sample “incorrect” response has the angle ACB of 235º ±5º correctly drawn, but does not indicate the arc AB, whereas the second sample “incorrect” response has neither the angle nor the arc correctly indicated.

Sample “incorrect” response 1



Sample “incorrect” response 2



Information on student responses is presented in Table 4.5. One-fourth of the students provided responses rated "correct," 62 percent provided responses rated "incorrect," and approximately 12 percent did not respond to the question.[1] Students whose highest mathematics course was calculus were more likely to provide a response considered to be "correct" than those whose highest course was pre-calculus, and students who had taken pre-calculus were more likely to provide a response rated "correct" than those whose highest course was second-year algebra. Males were more likely than females to provide a response rated "correct."

Table 4.5

Score Percentages for "Use Protractor to Draw a 235° Arc on a Circle"

THE NATION'S REPORT CARD naep

	Correct		Incorrect			Omit
	(± 2°)	(± 3–5°)	No "A" Endpoint	Arc Not Indicated	Other	
Grade 12						
Overall	15	10	0	4	58	12
Males	18	12	0!	5	55	10
Females	12	9	0	3	61	14
White	18	12	0	4	57	9
Black	5	5	0!	2	70	18
Hispanic	8	5	0!	4	57	27
Asian/Pacific Islander	24	23	0!	2	46	6
American Indian	***	***	***	***	***	***
Geometry Taken	17	12	0	4	57	10
Highest Algebra-Calculus Course Taken:						
Pre-Algebra	12	4!	0	6	67	6
First-Year Algebra	10	9	0	5	64	10
Second-Year Algebra	14	10	1	3	58	14
Third-Year Algebra/Pre-Calculus	19	16	0!	5	54	6
Calculus	39	17	0	1	41	2

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

[1] Student responses for this and all other constructed-response questions also could have been scored as "off task," which means that the student provided a response, but it was deemed not related in content to the question asked. There are many examples of these types of responses, but a simple one would be "I don't like this test." Responses of this sort could not be rated. In contrast, responses scored as "incorrect" were valid attempts to answer the question that were simply wrong.

The percentage of students within each achievement level who provided a response that was considered "correct" is shown in Table 4.6. Twenty-six percent of students at the *Basic* level and 59 percent of those at the *Proficient* level provided responses rated "correct." The question was very difficult for students classified as below the *Basic* level; only five percent provided a response rated "correct." The question mapped at 360 on the composite mathematics scale.

Table 4.6

Percentage Correct Within Achievement-Level Intervals for "Use Protractor to Draw a 235° Arc on a Circle"

THE NATION'S REPORT CARD naep

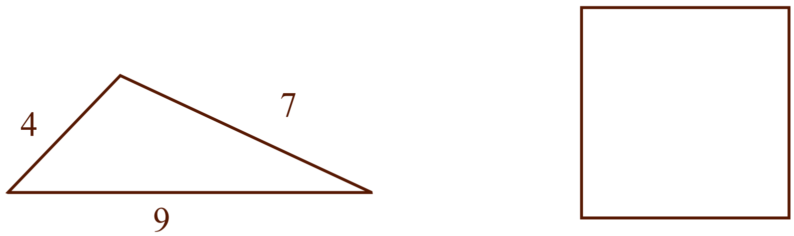
Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
26	5	26	59	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Perimeter, area, and volume

This area included questions measuring student procedural knowledge and problem-solving abilities applied to the concepts of perimeter, area, and volume. Questions at grade 4 asked students to calculate perimeters or areas of simple figures. Questions at grades 8 and 12 involved more complex figures, including three-dimensional figures. Some also required calculations of volume or circumference as well as an understanding of the relationship between perimeter, area, and volume. A number of the questions provided built-in aids, such as grids, to help students in their calculations. Assistance of this sort was especially common at the lower grade levels. As was true throughout the assessment, real-life problem situations were employed for many of the questions.

The first example in this area is a multiple-choice question for grade 4. The question presented two figures: a triangle with the lengths of the sides shown and a square. Students were asked what the length of each side of the square would be if the square and the triangle had the same perimeter. In order to answer correctly, students had to know that the perimeter is the distance around a figure. They needed to correctly sum the numbers shown on the triangle and then divide that sum by four to obtain the length of one side of the square.



8. If both the square and the triangle above have the same perimeter, what is the length of each side of the square?

Ⓐ 4

Ⓑ 5

Ⓒ 6

Ⓓ 7

The correct option is B.

Student performance data are presented in Table 4.7. Overall, 26 percent of the students selected the "correct" option, B, while 36 percent selected Option A, and 25 percent chose Option D. Only 10 percent selected Option C. Both Option A and Option D contain a number that is equal to one of the numbers on the sides of the triangle. Therefore, students who were unable to solve the problem may have been attracted to these options.

Table 4.7 ***Percentage Correct for "Relate Perimeter to Side Length"***

THE NATION'S REPORT CARD

	Percentage Correct
Grade 4	
Overall	26
Males	29
Females	23
White	30
Black	19
Hispanic	15
Asian/Pacific Islander	27
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The percentage of students within each achievement-level interval who successfully answered the question is presented in Table 4.8. Eleven percent of students classified as below *Basic*, 20 percent of those classified as *Basic,* and 58 percent of those classified as *Proficient* selected the correct response. The question mapped at 279 on the NAEP mathematics composite scale.

Table 4.8 **Percentage Correct Within Achievement-Level Intervals for "Relate Perimeter to Side Length"**

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 4 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
26	11	20	58	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The following example is a short constructed-response question administered to both grades 8 and 12. The question is a word problem that gave the dimensions of a cylindrical cereal box and asked for the volume of the box to the nearest cubic inch. The formula for the volume of a cylinder also was presented. To answer correctly, students had to know how to substitute the specified values for height and radius into the formula, solve the equation, and then round the answer correctly. It was not necessary to know the value of pi because the calculators provided for the assessment had pi keys on them.

6. A cereal company packs its oatmeal into cylindrical containers. The height of each container is 10 inches and the radius of the bottom is 3 inches. What is the volume of the box to the nearest cubic inch? (The formula for the volume of a cylinder is $V = \pi r^2 h$.)

Answer: ______________________ cubic inches

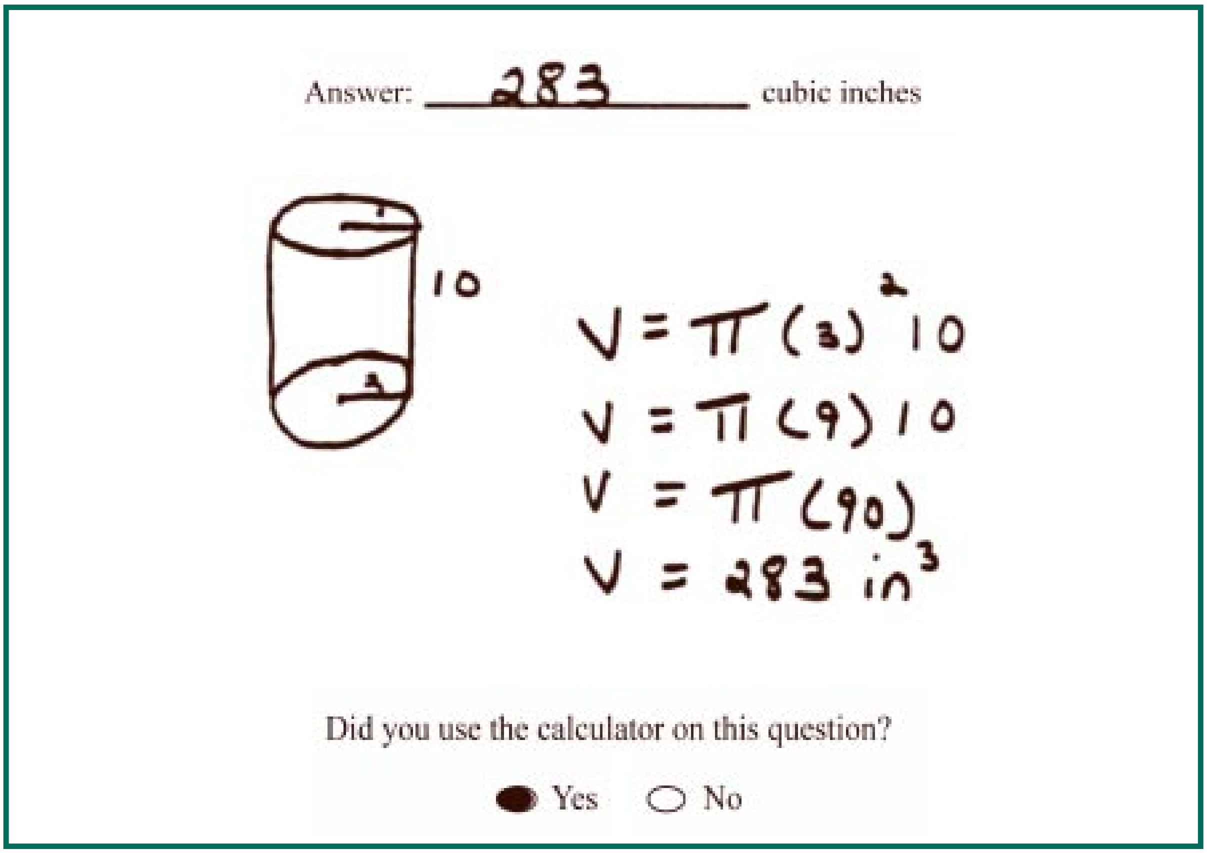
Did you use the calculator on this question?

◯ Yes ◯ No

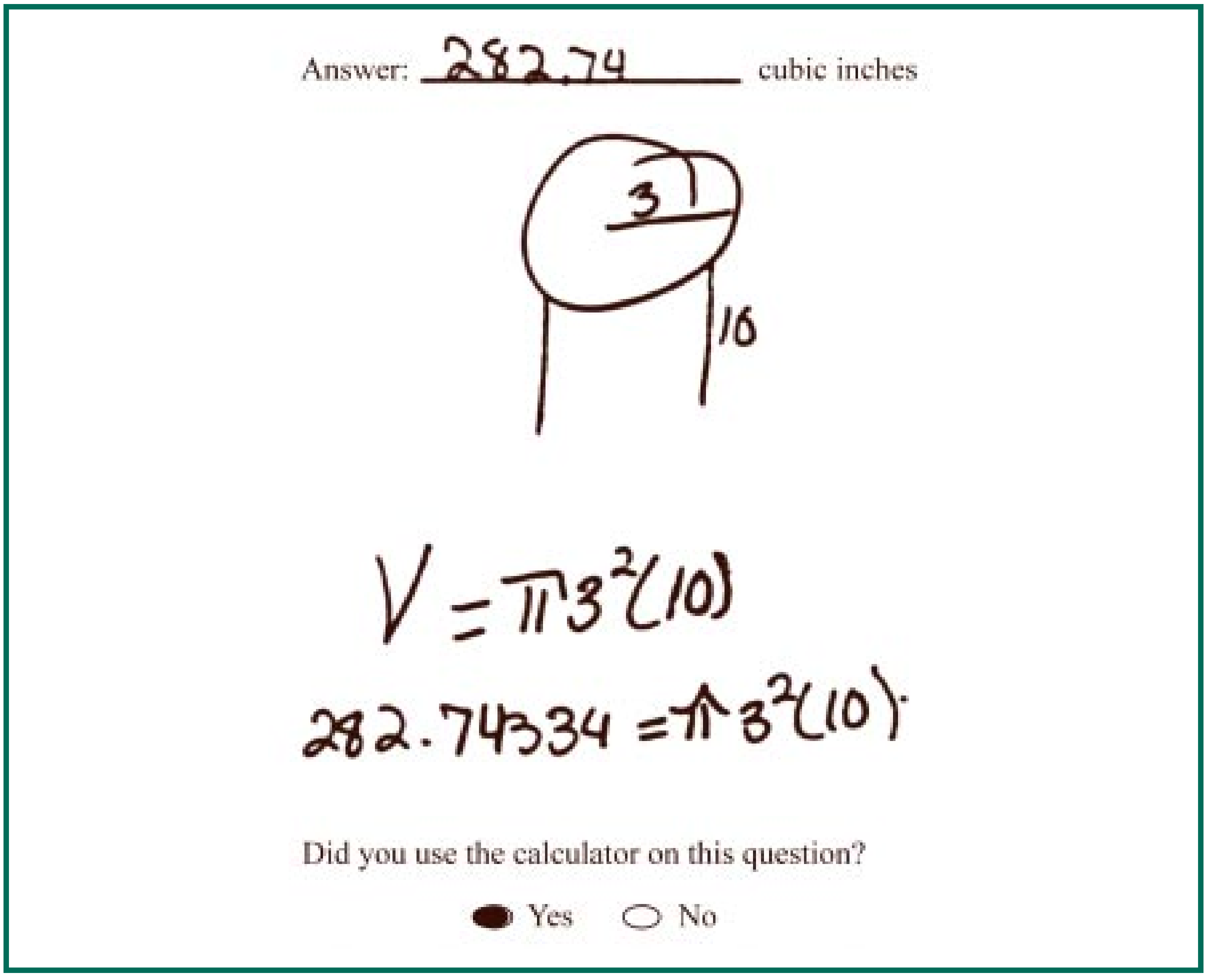
The correct answer is 283 cubic inches.

Responses were rated "correct," "partial," or "incorrect." A response was considered "correct" if the correct numerical answer of "283" was given with correct accompanying work or with no work shown. A response was considered "partial" if it showed any of the following: 1) correct substitutions into the formula but incorrect rounding; 2) 282.74334, suggesting multiplication by the pi key on the calculator, but with no work shown; 3) 282.6, suggesting multiplication by 3.14 on the calculator, but with no work shown; or 4) 282.8571, suggesting multiplication by 22/7 on the calculator, but again with no work shown. All other responses were considered "incorrect." Sample responses follow. The sample "correct" response contains the correct answer with accompanying work. The "partial" response shows correct substitution into the formula but no rounding, and the "incorrect" response shows no work and provides an incorrect answer. The question mapped at 388 for grade 8 and at 368 for grade 12.

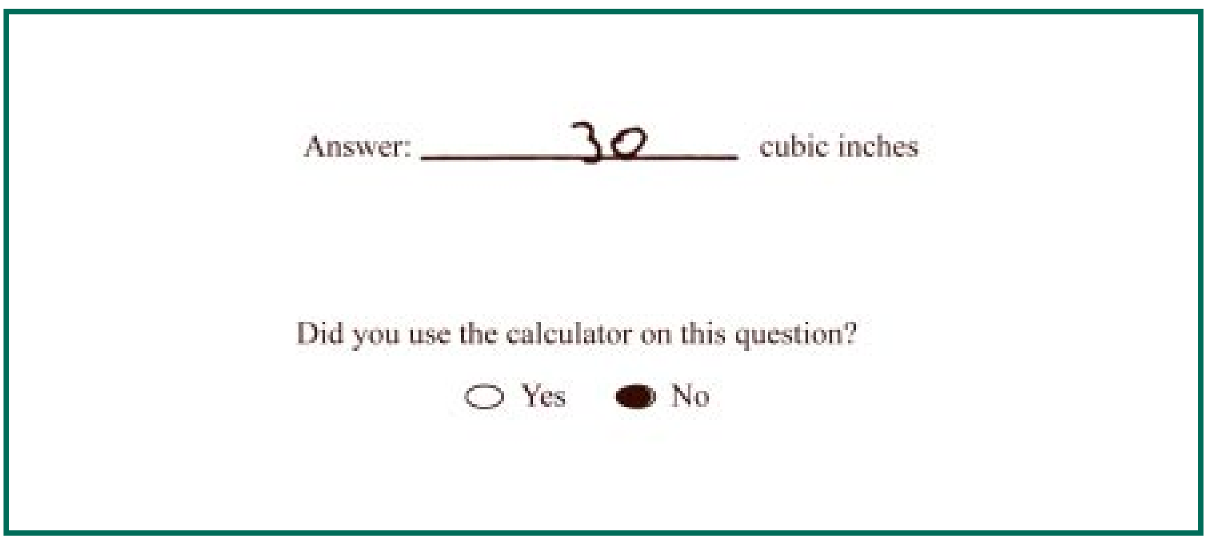
Sample "correct" response



Sample "partial" response



Sample "incorrect" response



Student performance is reported in Table 4.9. The question was fairly difficult for eighth-grade students, as can be seen by the fact that less than one-third of the students submitted responses considered to be at least partially correct. However, eighth-grade students enrolled in algebra performed better than their peers: 27 percent received full credit, and an additional 25 percent received partial credit.

As may be expected, twelfth-grade students had less trouble with the question; 55 percent of students submitted a response that was considered to be at least partially correct. Students who had taken second-year algebra as their highest mathematics course in the algebra-through-calculus sequence were more likely than those who had taken only first-year algebra to submit a response considered to be at least partially correct, and students whose highest course was calculus were more likely than those with less mathematics to submit a response rated "correct."

At both grade levels, female students were more likely than males to submit a response considered to be at least partially correct.

Table 4.9 ***Score Percentages for "Find Volume of a Cylinder"***

THE NATION'S REPORT CARD naep

	Correct	Partial	Incorrect	Omit
Grade 8				
Overall	13	17	57	12
Males	11	14	59	13
Females	14	19	54	11
White	16	19	56	8
Black	4	12	63	20
Hispanic	6	7	56	28
Asian/Pacific Islander	– –	– –	– –	– –
American Indian	***	***	***	***
Mathematics Course Taking:				
Eighth-Grade Mathematics	8	13	63	15
Pre-Algebra	8	15	63	10
Algebra	27	25	42	6
Grade 12				
Overall	29	26	36	8
Males	30	20	38	10
Females	28	31	33	7
White	32	27	34	6
Black	18	25	40	14
Hispanic	19	25	34	19
Asian/Pacific Islander	42	14	38	5
American Indian	***	***	***	***
Geometry Taken	32	28	34	6
Highest Algebra-Calculus Course Taken:				
Pre-Algebra	***	***	***	***
First-Year Algebra	16	22	46	13
Second-Year Algebra	31	28	33	7
Third-Year Algebra/Pre-Calculus	37	30	31	1
Calculus	56	26	18	0!

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The percentage of students within each achievement-level interval who successfully answered the question is presented in Table 4.10. At the eighth-grade level, 9 percent of students classified as *Basic*, 34 percent of those classified as *Proficient,* and 62 percent of those classified as *Advanced* provided a response rated "correct," whereas at grade 12, 6 percent of students below the *Basic* level, 32 percent at the *Basic* level, and 60 percent at the *Proficient* level submitted a "correct" response.

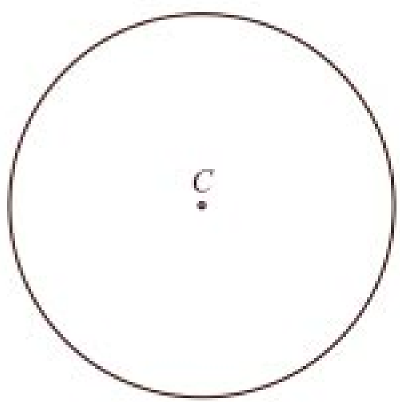
Table 4.10 — ***Percentage Correct Within Achievement-Level Intervals for "Find Volume of a Cylinder"***

THE NATION'S REPORT CARD

	Overall	NAEP Grades 8 and 12 Composite Scale Ranges			
		Below Basic	**Basic**	**Proficient**	**Advanced**
Grade 8	13	0	9	34	62
Grade 12	29	6	32	60	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The next example in the area of *perimeter, area, and volume* is a short constructed-response question for grade 12. In this question, students were shown a picture of a circle with the center, *C*, marked and were instructed to use a centimeter ruler to find the circumference of the circle. No calculator was available. They were told the value of pi but were not given the formula for circumference. The answer blank specified an answer in centimeters.



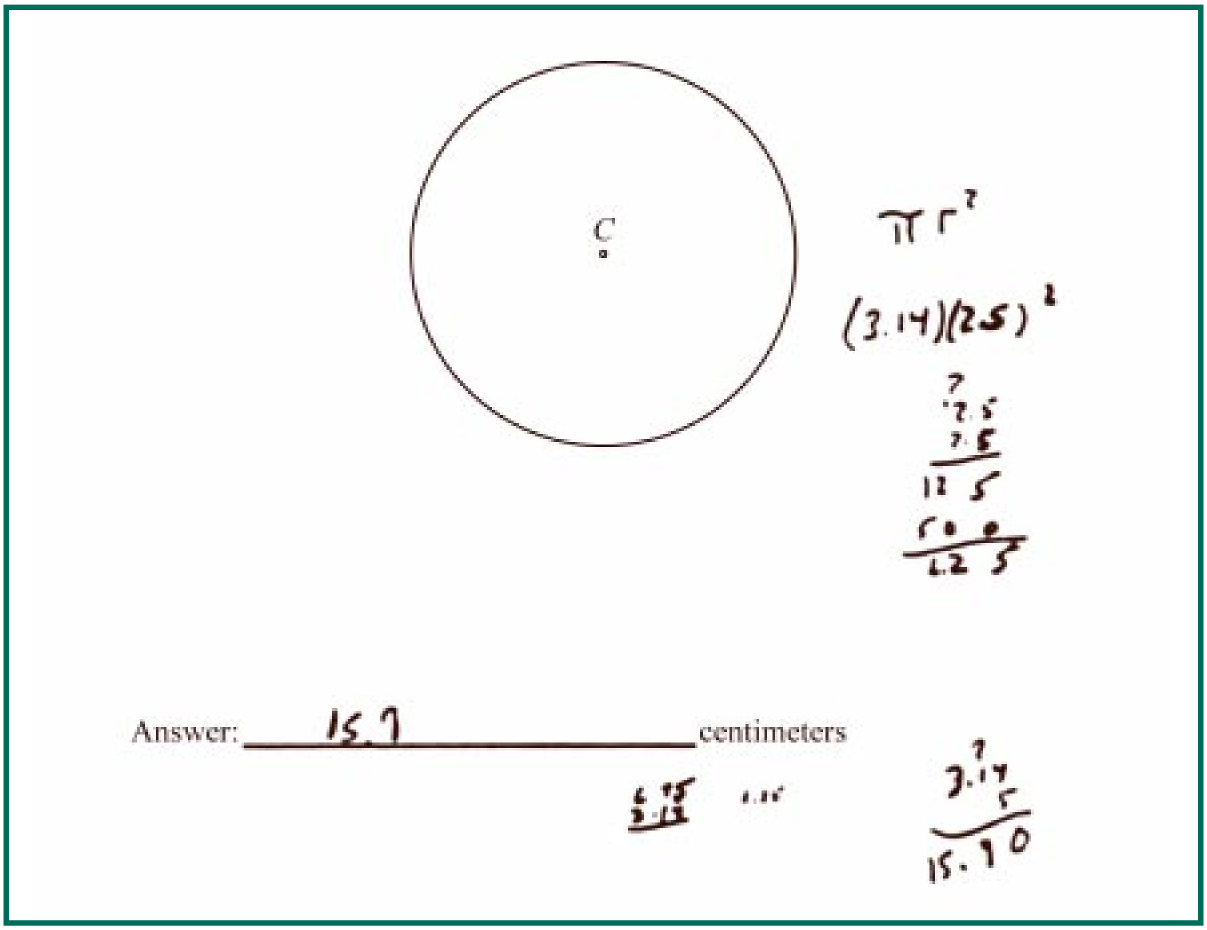
4. Using the centimeter ruler provided, find the circumference of the circle with center *C* above. (Use $\pi = 3.14$.)

Answer: ______________________________ centimeters

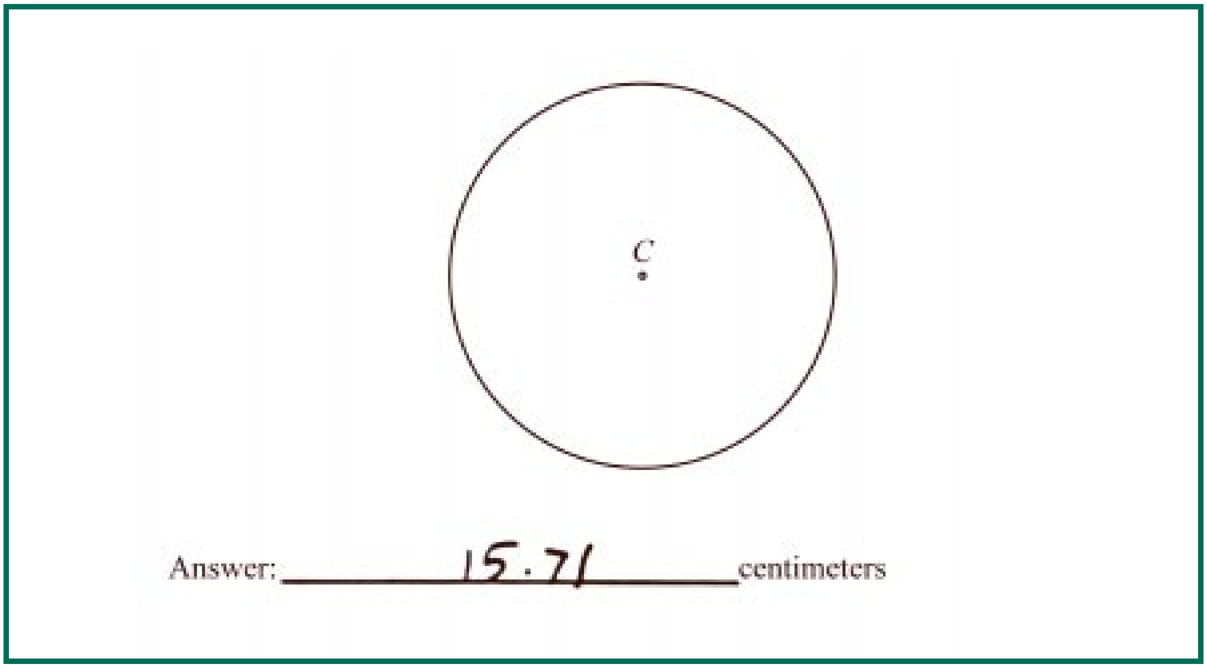
The correct answer is 15.70 centimeters.

In order to answer the question correctly, students had to know how the circumference of a circle is computed, make the correct measurement, and perform the multiplication correctly. Responses of either “15.7” or “15.70” centimeters were considered “correct,” as were other answers between 15.0 and 16.4 centimeters. Any answer in centimeters outside this range, as well as any response in inches, was considered “incorrect.” Two sample “correct” responses and one sample “incorrect” response are shown to illustrate these rating categories. In the first sample “correct” response, the student has given the correct answer of “15.7,” while in the second sample, the student has given an answer within the acceptable range but not exactly 15.7.

Sample “correct” response 1

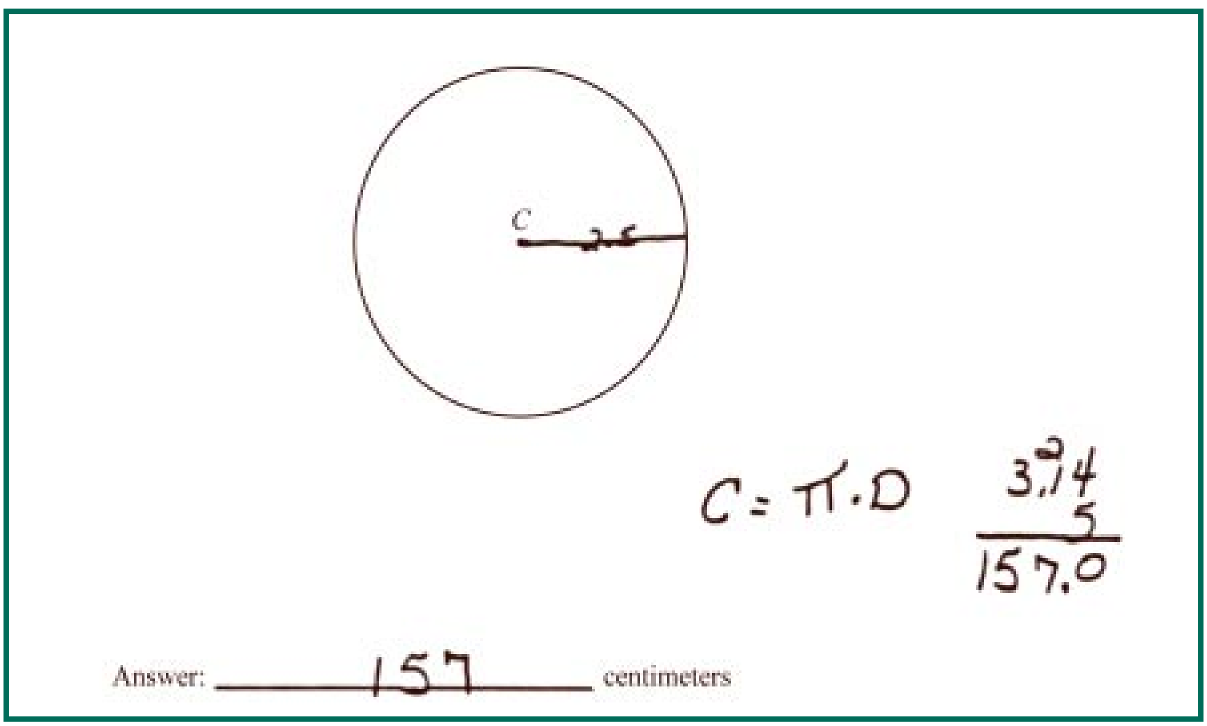


Sample "correct" response 2



In the following sample "incorrect" response, the student undertook the correct calculation of pi times the diameter of the circle, but made a decimal error.

Sample "incorrect" response



Information on student performance is presented in Table 4.11, and the percentage of students at each achievement level who provided a “correct” response is shown in Table 4.12. Overall, 29 percent of students provided a response rated “correct.” Of those students, only a small percent (3%) did not get the exact answer of 15.7 cm. Only one percent had their responses rated “incorrect” because they were given in inches. Students who had taken at least pre-calculus were more likely than those in the less advanced mathematics classes to submit a response rated “correct.” Ten percent of students classified as below the *Basic* level, 30 percent at the *Basic* level, and 62 percent at the *Proficient* level answered the question correctly. The question mapped at 370 on the composite scale.

Table 4.11 ***Score Percentages for “Use a Ruler to Find the Circumference of a Circle”***

THE NATION'S REPORT CARD

	Correct		Incorrect		Omit
	15.7 cm	15.0–16.4 cm Not Including 15.7 cm	Any Response in Inches	Other	
Grade 12					
Overall	26	3	1	57	13
Males	25	4	1	58	12
Females	27	3	1	56	14
White	29	3	1	58	8
Black	16	2	1	60	21
Hispanic	16	2	0!	52	30
Asian/Pacific Islander	42	6	1!	43	9
American Indian	***	***	***	***	***
Geometry Taken	29	4	1	57	9
Highest Algebra-Calculus Course Taken:					
Pre-Algebra	13	1	2	68	1
First-Year Algebra	18	3	0!	60	16
Second-Year Algebra	24	2	1	61	11
Third-Year Algebra/Pre-Calculus	44	7	0!	43	5
Calculus	52	6	1!	41	0

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 4.12 — *Percentage Correct Within Achievement-Level Intervals for "Use a Ruler to Find the Circumference of a Circle"*

Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
29	10	30	62	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Estimation of measurements

Questions in this area assessed students' abilities to estimate absolute and relative measurements, including size, weight, time, and distance. Questions for older students frequently required more accurate estimates or finer comparisons.

The first example in this area is a short constructed-response question for grade 4 that assessed students' abilities to reason, make connections, and communicate in mathematics. The question presented a measurement task that "Brett" needed to do without using a measuring instrument and asked the students to write directions to tell "Brett" how to accomplish the task. In order to answer the question correctly, students had to understand what "four equal pieces of string" means (i.e., draw on their knowledge of fractions) and visualize a method for obtaining these pieces. Then they had to communicate their idea in writing.

8. Brett needs to cut a piece of string into four equal pieces without using a ruler or other measuring instrument.

Write directions to tell Brett how to do this.

____

____

____

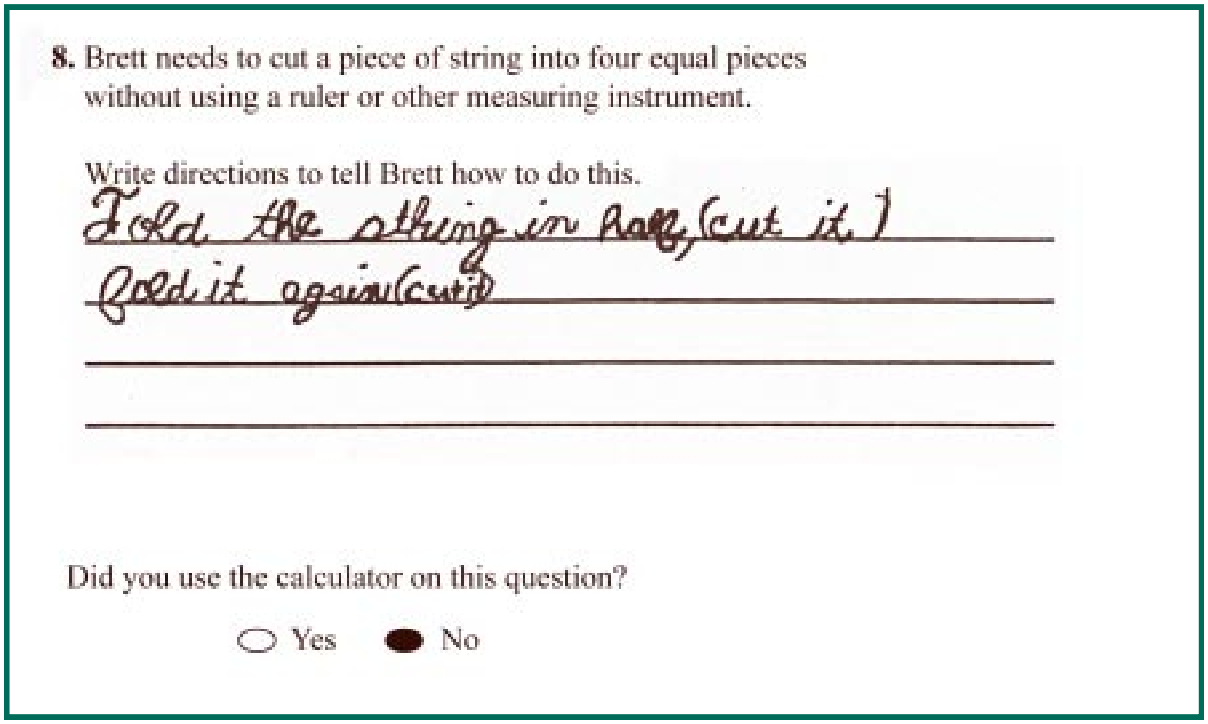
____

Did you use the calculator on this question?

○ Yes ○ No

A response was considered “correct” if it contained directions to fold the string in half and cut it and then to fold each of the resulting pieces in half and cut them. A response was considered “partial” if it mentioned folding the string in half once (e.g., “fold the string and cut”) or mentioned cutting in the middle and doing that to the pieces. Partial credit also was given if the student only addressed the question of how to get three more equal pieces once the first piece was made. All other responses were considered “incorrect,” including those that simply said to fold the string. Sample responses follow.

Sample “correct” response



8. Brett needs to cut a piece of string into four equal pieces without using a ruler or other measuring instrument.

Write directions to tell Brett how to do this.

Fold the string in half, (cut it)
fold it again (cut it)

Did you use the calculator on this question?

○ Yes ● No

Sample "partial" response

8. Brett needs to cut a piece of string into four equal pieces without using a ruler or other measuring instrument.

Write directions to tell Brett how to do this.

Brett should fold the string up into four and cut the four pieces

Did you use the calculator on this question?

○ Yes ● No

Sample "incorrect" response

8. Brett needs to cut a piece of string into four equal pieces without using a ruler or other measuring instrument.

Write directions to tell Brett how to do this.

cut a little bit off 3 times

Did you use the calculator on this question?

○ Yes ● No

Data on student performance are presented in Tables 4.13 and 4.14. This question was difficult for fourth-grade students and mapped at 332 on the composite scale. Overall, only six percent of students provided responses rated "correct." However, another 34 percent provided responses considered at least partially correct and rated "partial." Four percent of students at the *Basic* level and 14 percent at the *Proficient* level provided a response rated "correct."

Table 4.13 — ***Score Percentages for "Describe Measurement Task"***

THE NATION'S REPORT CARD naep

Grade 4	Correct	Partial	Incorrect	Omit
Overall	6	34	50	9
Males	5	32	52	10
Females	6	36	48	9
White	7	40	44	8
Black	1	15	66	15
Hispanic	1	24	61	13
Asian/Pacific Islander	***	***	***	***
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to provide a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

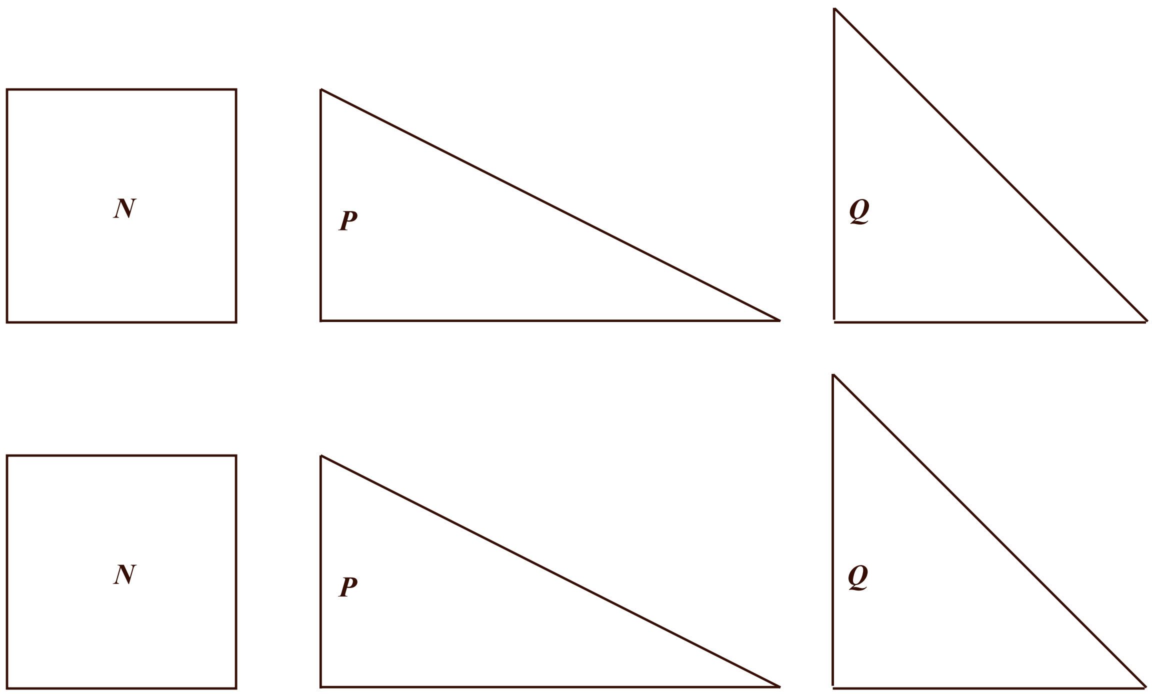
Table 4.14 — ***Percentage Correct Within Achievement-Level Intervals for "Describe Measurement Task"***

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 4 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
6	0!	4	14	***

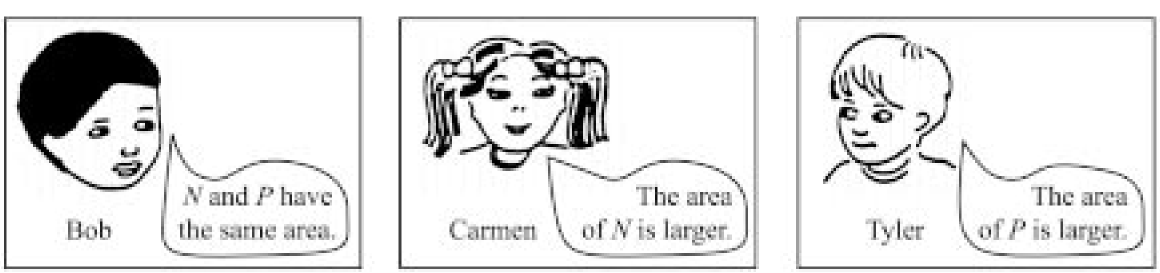
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The following example is another fourth-grade short constructed-response question. For the block of questions in which this question appeared, students were provided with two cardboard cutouts of each of three different shapes: squares labeled *N*, and two different types of right triangle labeled *P* and *Q*, respectively.



Students were shown cartoons of three different children, "Bob," "Carmen," and "Tyler," who were making statements comparing the areas of *N* and *P*, and they were asked who made the correct statement. They were instructed to use pictures and words to explain their answers.

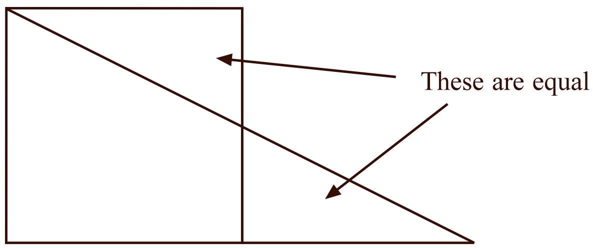
6. Bob, Carmen, and Tyler were comparing the areas of *N* and *P*.



Who was correct?

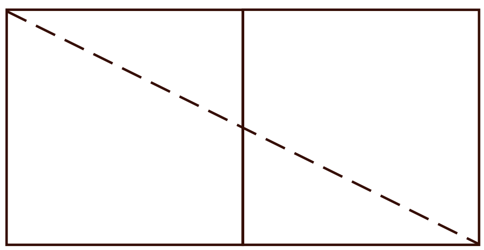
Use pictures and words to explain why.

A response was considered "correct" if an adequate explanation was presented with or without naming "Bob" as being correct. Adequate explanations included the following:



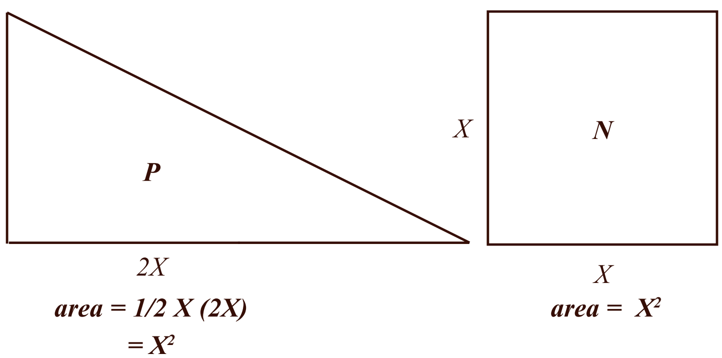
Part of *P* overlaps *N*, and part sticks out. The sticking-out part is equal to the left-out part of *N*.

OR



Two *P*s match two *N*s; therefore, they have the same area. (Therefore, one *N* has the same area as one *P*.)

OR

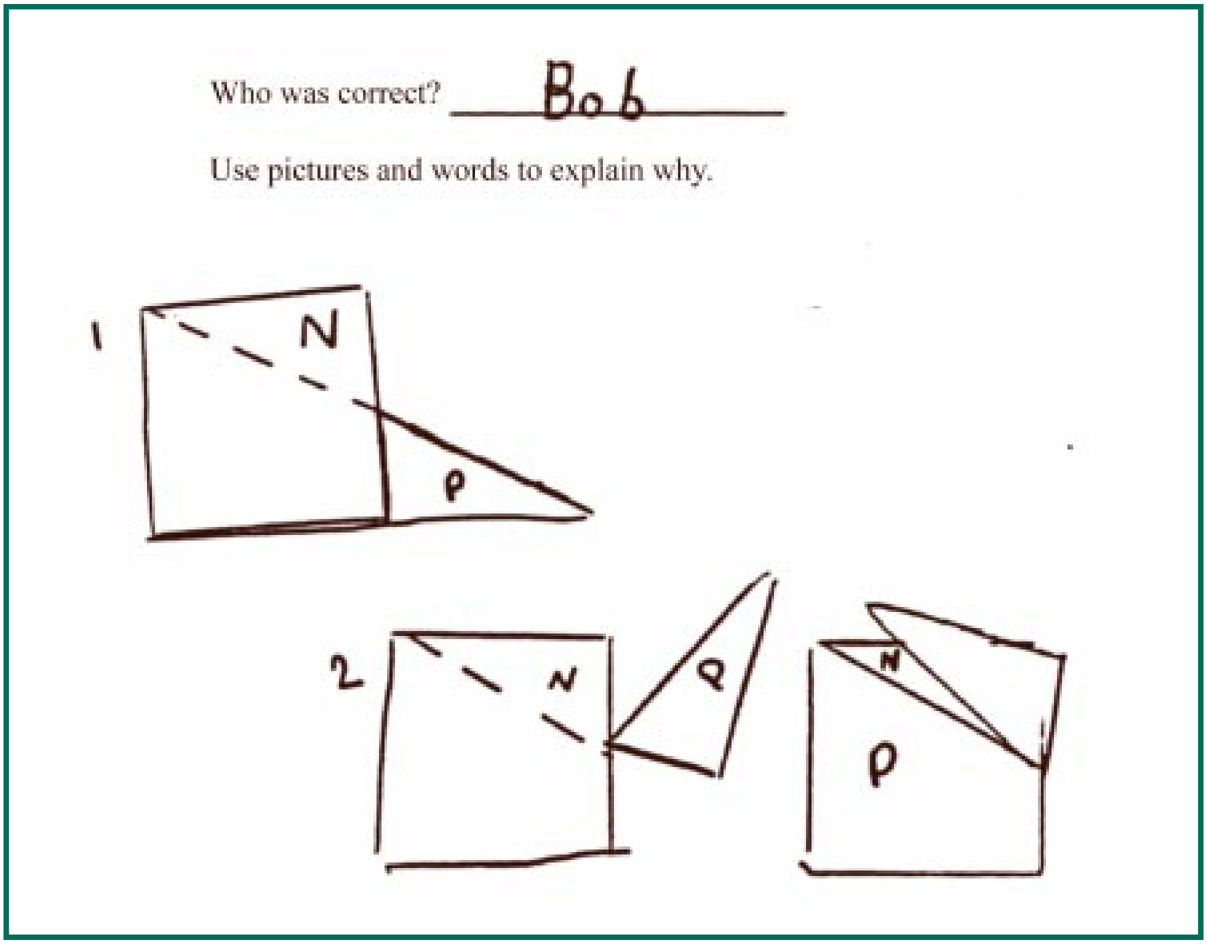


Areas are equal because the height of *P* is the same as the height of *N*, and the base of *P* is twice the base of *N*.

Responses were considered "incorrect" for two main reasons: 1) the student said "Bob" was correct but gave an inadequate or no explanation, or 2) the students named "Carmen" or "Tyler" as being correct or omitted a name and gave no satisfactory explanation. Sample student responses follow.

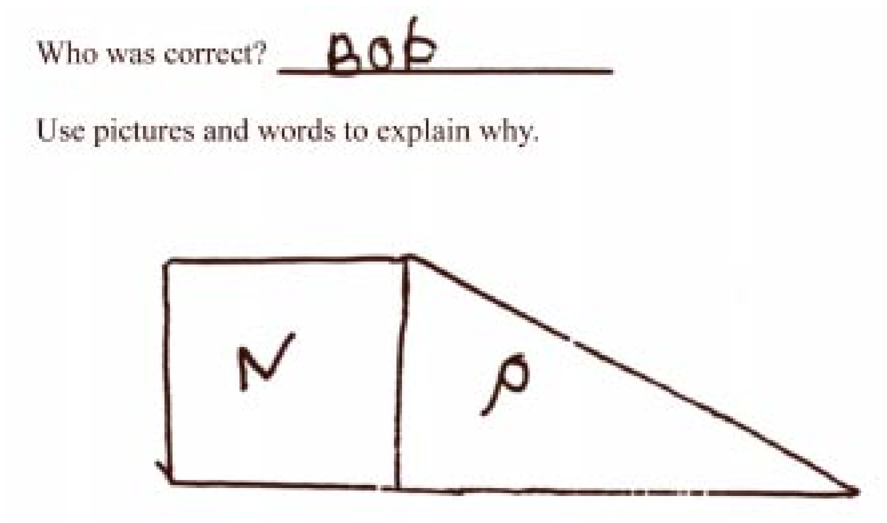
The first sample response was rated “correct” because the student conveyed a clear understanding of how the part of shape *P* that “sticks out” can be repositioned to form shape *N*. The drawings show that shapes *N* and *P* have the same area.

Sample “correct” response

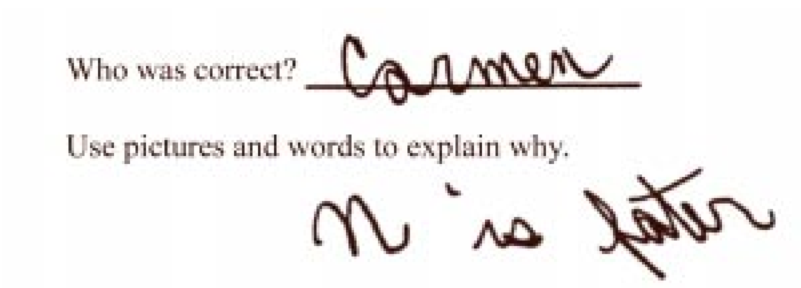


The next two sample responses were rated “incorrect.” In the first sample “incorrect” response, the student named “Bob” as being correct, but gave an inadequate explanation. In the second sample “incorrect” response, the student said that “Carmen” was correct.

Sample “incorrect” response 1



Sample “incorrect” response 2



Data on student performance are presented in Tables 4.15 and 4.16. This question also was difficult for fourth-grade students; as the table shows, only six percent of students provided responses rated "correct." Another 21 percent correctly answered "Bob" but could not provide an adequate explanation for their answer. The question mapped at 321 on the composite scale. Two percent of students at the *Basic* level and 14 percent at the *Proficient* level provided a response rated "correct."

Table 4.15 — ***Score Percentages for "Compare Areas of Two Shapes," Grade 4***

THE NATION'S REPORT CARD naep

	Correct	Incorrect		Omit
		Bob–No Adequate Explanation	Not Bob	
Grade 4				
Overall	6	21	74	0
Males	6	22	71	0
Females	5	19	76	0!
White	7	21	71	0
Black	0!	18	81	0!
Hispanic	0!	18	81	0!
Asian/Pacific Islander	6	21	73	0!
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 4.16 — ***Percentage Correct Within Achievement-Level Intervals for "Compare Areas of Two Shapes," Grade 4***

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 4 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
6	1!	2	14	***

*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The next example is essentially the same question as the one just shown for grade 4, but the question was presented as a word problem at grades 8 and 12 rather than in cartoon form. (The different presentation for grade 4 was used to reduce the amount of reading required for younger students.)

5. Bob, Carmen, and Tyler were comparing the areas of *N* and *P*. Bob said that *N* and *P* have the same area. Carmen said that the area of *N* is larger. Tyler said that the area of *P* is larger.

Who was correct? __________________

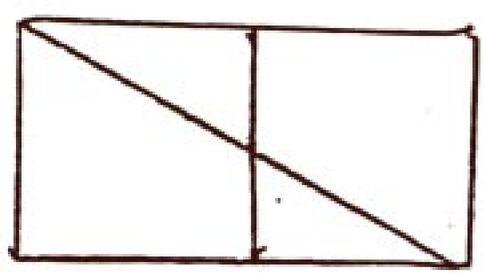
Use words or pictures (or both) to explain why.

The question was rated according to the criteria described above for grade 4, and sample responses follow. Similar to the fourth-grade sample "incorrect" responses, the first "incorrect" response shows "Bob" named as correct but has an inadequate explanation, and the second "incorrect" response says "Tyler" is correct.

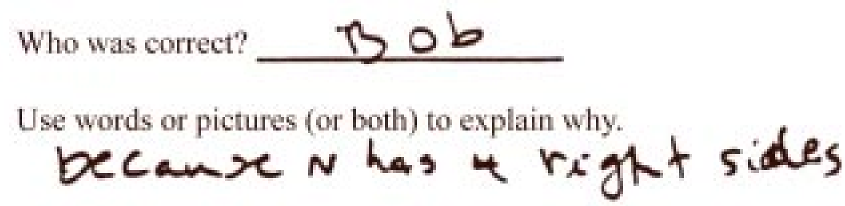
Sample "correct" response

Who was correct? Bob

Use words or pictures (or both) to explain why.



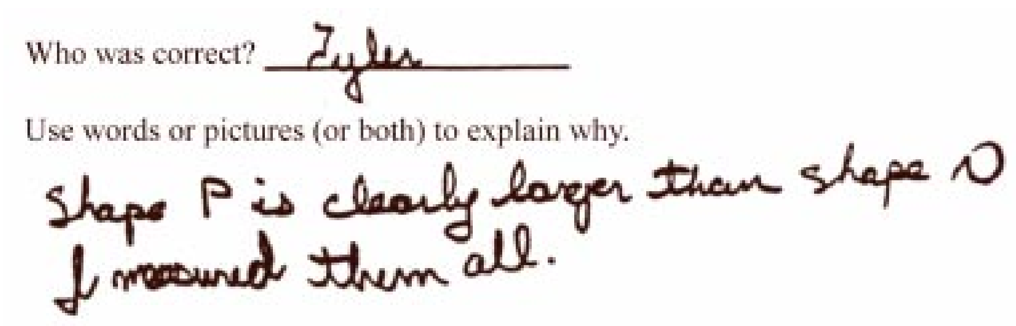
Sample "incorrect" response 1


Who was correct? Bob

Use words or pictures (or both) to explain why.

because N has 4 right sides

Sample "incorrect" response 2


Who was correct? Tyler

Use words or pictures (or both) to explain why.

Shape P is clearly larger than shape N I measured them all.

Information on student performance is presented in Table 4.17. Ninety-eight percent of eighth-grade students attempted the question, and 27 percent submitted a response that was rated "correct." Students currently taking eighth-grade mathematics or pre-algebra performed similarly, whereas students taking algebra performed better than students in the other two groups. At grade 8, the question mapped at 362 on the NAEP composite scale.

At grade 12, 95 percent of the students attempted the question, and 35 percent submitted a response that was rated "correct." Students who had calculus as their highest mathematics course performed better than those who had less mathematics. In addition, those whose highest course in the algebra-through-calculus sequence was second-year algebra outperformed those who had taken only first-year algebra. The question mapped at 350 on the composite scale.

Table 4.17

Score Percentages for "Compare Areas of Two Shapes," Grades 8 and 12

THE NATION'S REPORT CARD naep

	Correct	Incorrect		Omit
		Bob–No Adequate Explanation	Not Bob	
Grade 8				
Overall	27	16	54	2
Males	28	16	54	3
Females	26	16	56	2
White	32	16	50	2
Black	8	14	75	3
Hispanic	18	19	59	5
Asian/Pacific Islander	– –	– –	– –	– –
American Indian	***	***	***	***
Mathematics Course Taking:				
Eighth-Grade Mathematics	25	19	55	1
Pre-Algebra	23	14	60	2
Algebra	38	13	47	1
Grade 12				
Overall	35	14	46	5
Males	35	17	43	4
Females	35	12	48	5
White	40	15	42	3
Black	12	16	64	8
Hispanic	25	11	54	9
Asian/Pacific Islander	54	12	32	3
American Indian	***	***	***	***
Geometry Taken	38	15	43	4
Highest Algebra-Calculus Course Taken:				
Pre-Algebra	18	8	68	4
First-Year Algebra	25	18	51	5
Second-Year Algebra	39	14	43	3
Third-Year Algebra/Pre-Calculus	44	15	36	4
Calculus	62	9	26	3

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The percentage of students within each achievement level who provided a "correct" response is shown in Table 4.18. Perhaps not surprisingly, performance was better at twelfth grade, with more than 70 percent of students at the *Proficient* level submitting a response considered "correct."

Table 4.18 **Percentage Correct Within Achievement-Level Intervals for "Compare Areas of Two Shapes," Grades 8 and 12**

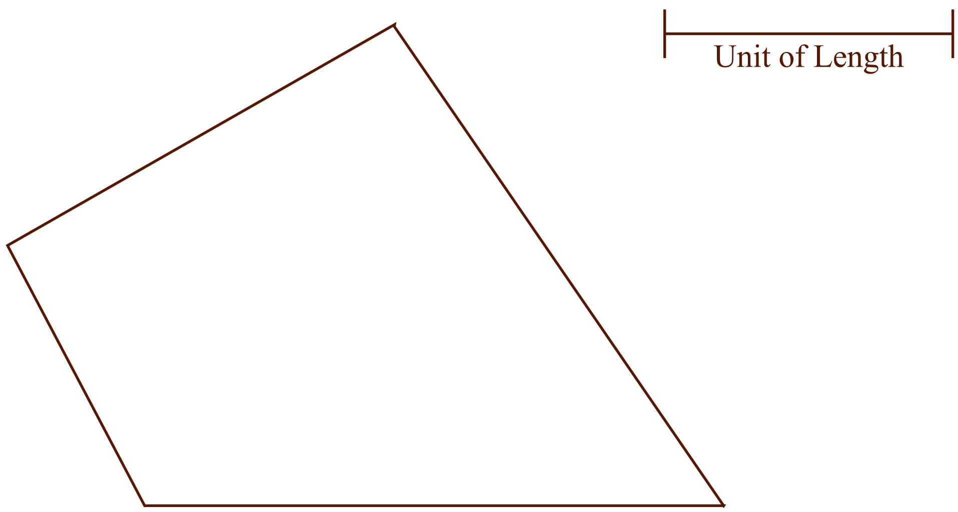
THE NATION'S REPORT CARD naep

		NAEP Grades 8 and 12 Composite Scale Ranges			
	Overall	Below Basic	Basic	Proficient	Advanced
Grade 8	27	8	26	58	***
Grade 12	35	10	39	72	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The last example in the area of *estimation of measurements* is a short constructed-response question for grade 8. Students were presented with a four-sided figure (quadrilateral) and a line labeled "unit of length" that did not give the dimensions of the unit. They were instructed to use the unit of length to estimate the perimeter of the figure and then to specify two consecutive whole-number units between which the length of the perimeter would lie. Since students did not have rulers for this question, they had to figure out other ways to estimate the number of "units" that were needed to go around the figure. They could do this by simple visual comparison or perhaps by adjusting the distance between the thumb and index finger to equal the unit of length given in the question and then applying that measure to the figure.

11. Use the unit of length below to estimate the perimeter of the figure shown. Between which two consecutive whole-number units does the perimeter lie?



Answer: Between ______________ and ______________

The correct answer is “between 6 and 7”. Students needed to have both 6 and 7 in their answer for it to be rated “correct.” Student responses of “between 5 and 6” and “between 7 and 8” were rated “incorrect,” as were any other incorrect responses. Information on student performance is shown in Tables 4.19 and 4.20. This question had a high omit rate of 21 percent. Another 21 percent submitted a response that was considered “correct.” Almost equal percentages of students (about 5%) submitted responses of “between 7 and 8” or “between 5 and 6,” possibly representing a miscounting of units. Students currently taking eighth-grade mathematics or pre-algebra performed similarly, whereas students taking algebra performed better than students in the other two groups. The question mapped at 380 on the NAEP composite mathematics scale.

When performance is disaggregated by achievement level, Table 4.20 shows that 19 percent of students at the *Basic* level, 44 percent of students at the *Proficient* level, and 69 percent of students at the *Advanced* level answered the question correctly. Only five percent of students below the *Basic* level were able to answer correctly.

Table 4.19

Score Percentages for "Find Perimeter (Quadrilateral)"

THE NATION'S REPORT CARD naep

	Correct	Incorrect			Omit
	Between 6 and 7	Between 7 and 8	Between 5 and 6	Other	
Grade 8					
Overall	21	6	5	47	21
Males	22	6	5	47	20
Females	20	5	5	48	22
White	26	7	5	45	17
Black	10	2	2	53	33
Hispanic	12	1	5	55	26
Asian/Pacific Islander	– –	– –	– –	– –	– –
American Indian	***	***	***	***	***
Mathematics Course Taking:					
Eighth-Grade Mathematics	16	4	5	50	23
Pre-Algebra	16	6	4	54	18
Algebra	34	8	7	37	12

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 4.20

Percentage Correct Within Achievement-Level Intervals for "Find Perimeter (Quadrilateral)"

THE NATION'S REPORT CARD naep

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
21	5	19	44	69

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Summary

This content strand assessed students' conceptual understanding and procedural knowledge of measurement units, their ability to use measurement tools and instruments, and their problem-solving abilities applied to the concepts of perimeter, area, and volume. In addition, several questions assessed students' abilities to estimate absolute and relative measurements. Many of the questions shown here as examples were difficult for students, particularly those requiring unit conversions, calculations of volume and circumference, and estimation of measurements.

Eighth-grade algebra students tended to perform better on the questions than other eighth-grade students, whereas eighth-grade students in pre-algebra or regular mathematics tended to perform similarly. At the twelfth-grade level, those students whose highest course in the algebra-through-calculus sequence was second-year algebra tended to outperform those who had only reached first-year algebra, whereas there were not always significant differences in performance between students who had taken pre-calculus/third-year algebra and those who had stopped with second-year algebra. In addition, students who reported calculus as their highest mathematics course tended to perform better than those who had only taken the less advanced mathematics courses.

Chapter 5

Geometry and Spatial Sense

Content Strand Description

At the foundation of successful performance in the Geometry and Spatial Sense content strand is a conceptual understanding of geometric figures and their properties. However, the questions classified under this content strand extended well beyond low-level identification of geometric shapes. Some of the questions required students to visualize and draw geometric figures after transforming them or combining them with other figures, and many required them to apply their understanding of geometry to reason through and solve problems. A large number of the questions from this content strand were constructed-response questions, including questions requiring drawn responses.

At the fourth-grade level, students were asked to demonstrate an understanding of the properties of shapes and to visualize shapes and figures under simple combinations and transformations. Fourth-grade students also were asked to use their mathematical communication skills to translate verbal descriptions into drawn figures. At the eighth-grade level, some questions measured concepts related to properties of angles and polygons. These included symmetry, congruence and similarity, and the Pythagorean theorem. Students also had to apply reasoning skills to make and validate conjectures about combinations and transformations of shapes. At the twelfth-grade level, students were expected to demonstrate a knowledge of more sophisticated geometric concepts and formulas and more sophisticated reasoning processes than at earlier grade levels. Questions sometimes involved proportional reasoning or coordinate geometry.

Examples of Individual Questions and Student Performance

Several questions from the NAEP 1996 mathematics assessment follow. Presentation of the questions is organized around four areas of emphasis within the Geometry and Spatial Sense content strand. The area of *basic geometric concepts and properties* includes questions that assessed a student's conceptual understanding of geometry. The area of *geometric procedures* includes questions that assessed a student's procedural knowledge of geometric constructions and computations. The area of *geometric transformations and spatial sense* includes questions that assessed students' abilities to visualize shapes and figures as well as transformations and combinations of shapes and figures. Finally, the area of *geometric models and problems* includes questions that measured students' abilities to represent problem situations with geometric models and to apply an understanding of the properties of different figures to solve problems. Questions within all four areas also required students to reason, communicate, and make connections.

All sample questions from this content strand are mapped onto the NAEP composite mathematics scale as shown in Figure 5.1. Specific instructions on how to interpret this map are given at the end of Chapter 2. The map is included to provide an indication of the relative difficulty of each example question and, thus, to indicate the type of material mastered within this content strand by students with varying degrees of mathematics proficiency. Keep in mind, however, that the difficulty of a question is influenced by many factors, including characteristics specific to the question (e.g., format, absence or presence of graphics, real-world application) as well as the particular mathematics content associated with the question and student opportunities to learn this content. Also, remember that overall performance on the Geometry and Spatial Sense content strand is not determined solely by performance on the examples presented here. These examples illustrate only some of what students know and can do.

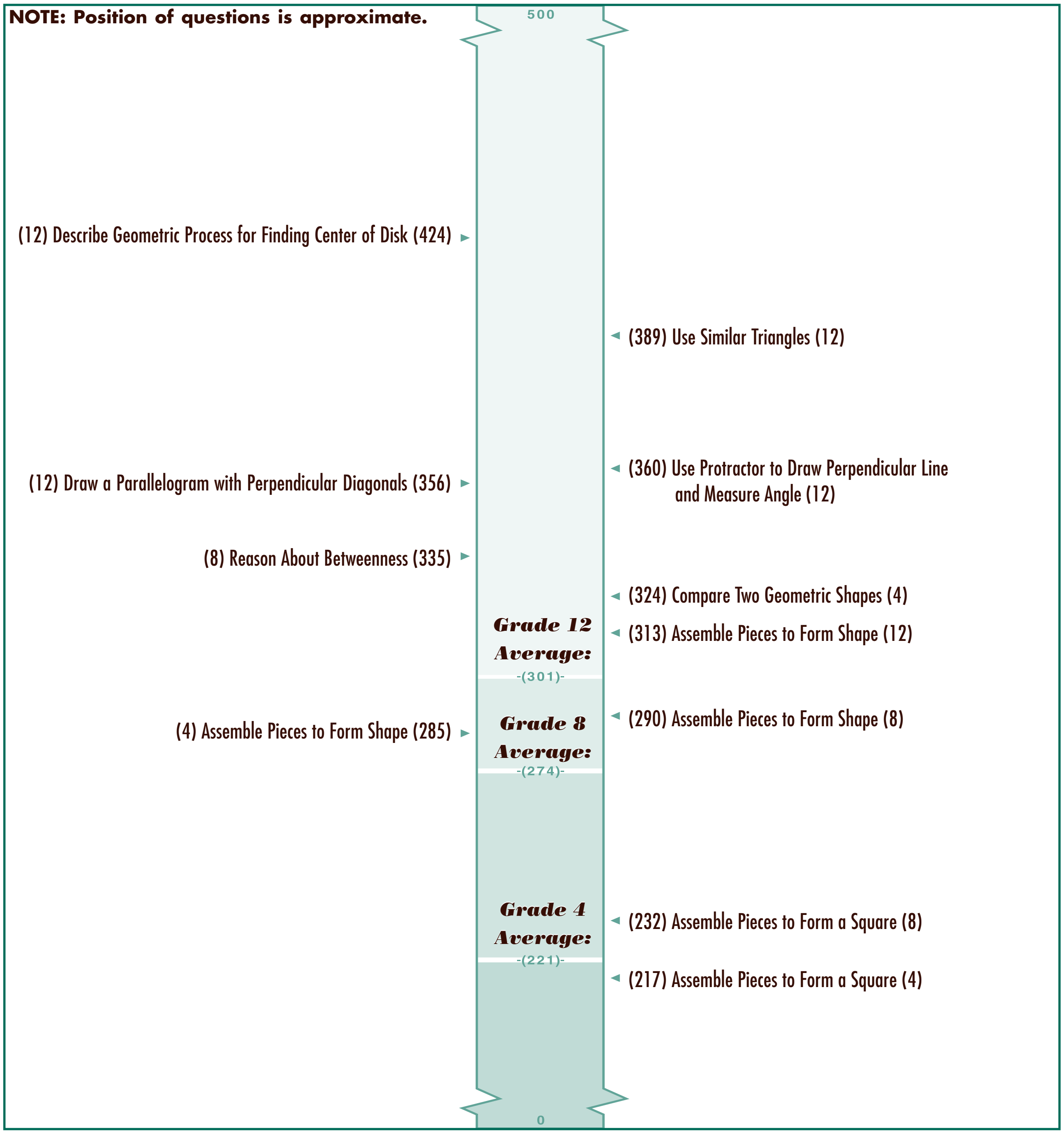
The performance of students on the questions in the Geometry and Spatial Sense content strand is examined with respect to gender, race/ethnicity, and, for grades 8 and 12, the types of mathematics courses taken. However, as described in Chapter 2, the impact of taking geometry on student performance is not discussed for several reasons. First, there is only a small pool of students on which the specific influence of geometry could be isolated, given that most students who have taken geometry also have taken at least 2 years of algebra. Moreover, because more able students are likely to progress further in the mathematics course sequence, it is difficult to separate the impact of a particular curriculum from the impact of a student's overall strength in mathematics. Although comments on the impact of geometry course taking on performance on the questions in this content strand might be expected, these confounding effects make it difficult to isolate the specific impact of geometry. The data, however, are presented in the tables.

Figure 5.1

Map of Selected Geometry and Spatial Sense Questions on the NAEP Composite Mathematics Scale (Item Map)

THE NATION'S REPORT CARD naep

NAEP Scale



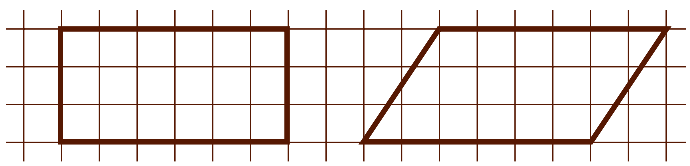
NOTE: Each mathematics question was mapped onto the NAEP 0 to 500 mathematics scale. The position of the question on the scale represents the scale score obtained by students who had a 65 percent probability of successfully answering the question. (The probability was 74 percent for a 4-option multiple-choice question and 72 percent for a 5-option multiple-choice question.) Only selected questions are presented. The number 4, 8, or 12 in parentheses is the grade level at which the question was asked.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Asssessment.

Basic geometric concepts and properties

Questions in this area required students to demonstrate a conceptual understanding of geometric figures, including an understanding of the properties of various figures and the definition of geometric terms. Some questions asked students to demonstrate their understanding by classifying or comparing various figures.

The example for this area is a fourth-grade extended constructed-response question. Students were presented with two four-sided figures (a rectangle and a parallelogram) and were asked to list the ways in which the figures are alike and the ways in which they are different. Students were instructed to list as many ways as they could.

Think carefully about the following question. Write a complete answer. You may use drawings, words, and numbers to explain your answer. Be sure to show all of your work.



10. In what ways are the figures above alike? List as many ways as you can.

In what ways are the figures above different? List as many ways as you can.

Some correct responses for how the figures are alike were the following:

- They both have four sides (or four corners or four angles).
- They both have parallel sides.
- They both have two sets of sides that are the same length.
- They have the same area.
- They have the same length (base).
- They have the same height.
- They have the same number of little squares.

A response of "they both have lines that are straight" was not accepted as correct. An answer that they both have four sides and four angles was considered to be only one reason.

Some correct responses for how the two figures are different were as follows:

- One has four equal angles, and the other does not.
- One has right angles or perpendicular lines, and the other does not.
- One is "slantier" than the other (or takes up full squares).
- They have different perimeters.

A response of "they are not both the same shape" was considered to be a rephrasing of the given information that the figures are different and was not accepted as correct. Furthermore, students did not need to make the comparisons in their responses; that is, they merely could have stated, for example, "one has four equal angles."

Student responses were rated as being either "extended," "satisfactory," "partial," "minimal," or "incorrect." However, when the question was anchored to the NAEP scale, the "extended" and "satisfactory" rating categories were collapsed. The rating guide for this question is presented below:

- "Extended": The student gave at least two valid reasons why the figures are alike and at least two valid reasons why they are different.
- "Satisfactory": The student gave two valid reasons why the figures are alike and at least one valid reason why they are different, *or* gave one valid reason why they are alike and two valid reasons why they are different.
- "Partial": The student gave one valid reason why the figures are alike and one valid reason why they are different, *or* gave two valid reasons why they are alike and no valid reasons why they are different, *or* gave two valid reasons why they are different and no valid reasons why they are alike.
- "Minimal": The student gave a nonspecific response (e.g., "the one on the right is skinnier") *or* gave only one correct reason why they are alike or why they are different.
- "Incorrect": Any response not fitting into the categories above.

Virtually no fourth-grade responses met the criteria for an "extended" response, that is, contained two valid reasons for why the figures are alike and two valid reasons for why they are different. However, the following is a sample of a "satisfactory" response.

Sample "satisfactory" response

10. In what ways are the figures above alike? List as many ways as you can.

they both are shapes
they both have four sides
they both have four courners
they both have the same amounts of smaller squares if you add peices together on one

In what ways are the figures above different? List as many ways as you can.

they aren't both the same shape
they don't both have full squares on both their courners

The student listed four reasons why the shapes are alike. However, one reason (they both are shapes) was not accepted as a valid reason, and the answers about both having four sides and four corners are considered to be the same reason. The student also listed two reasons why the shapes are different but was not given credit for, "they aren't both the same shape." Thus, the student was credited with providing two valid reasons why the shapes are alike and one valid reason why they are different. This met the criterion for a "satisfactory" response.

In the sample "partial" response below, the student listed three reasons why the shapes are alike and one reason why they are different, but among the reasons why they are alike, only one (they both have 18 squares) was considered correct. The response, therefore, was rated as "partial," because the student provided only one correct reason why the shapes are alike and one correct reason why they are different.

Sample "partial" response

10. In what ways are the figures above alike? List as many ways as you can.

- They both have 18 squares, they are both rectangles, they are both the same size.

In what ways are the figures above different? List as many ways as you can.

- one is slanted, one is not

The next sample response was rated as a "minimal" response. The student listed three reasons why the figures are alike and three reasons why they are different, but only one of these reasons (one is slanted/one is straight) was considered valid.

Sample "minimal" response

10. In what ways are the figures above alike? List as many ways as you can.

① They can both be square
② They can both be slanted
③ They can both turn many ways

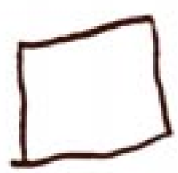
In what ways are the figures above different? List as many ways as you can.

① In the picture they are difrent.
② One is slanted.
③ One is furn and strate.

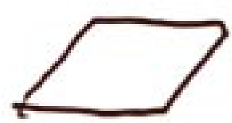
The next response, in which the student failed to list any information, was rated "incorrect."

Sample "incorrect" response

10. In what ways are the figures above alike? List as many ways as you can.



In what ways are the figures above different? List as many ways as you can.



Response rates for this question are reported in Tables 5.1 and 5.2. As stated earlier, virtually no students provided responses that were rated "extended." Only 11 percent of the students provided responses that were rated "satisfactory," and the remaining students' responses were divided fairly evenly among the "partial," "minimal," and "incorrect" categories.[1] Females were more likely than males to provide "satisfactory" or "partial" responses.

Table 5.1

Percentage Correct for "Compare Two Geometric Shapes"

THE NATION'S REPORT CARD naep

Grade 4	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
Overall	0	11	29	31	23	5
Males	0	9	25	32	25	7
Females	0	13	33	30	21	3
White	0	13	32	30	20	4
Black	0!	5	21	33	28	11
Hispanic	0!	6	22	29	34	8
Asian/Pacific Islander	0!	8	21	33	33	4
American Indian	***	***	***	***	***	***

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

About 20 percent of the students whose overall mathematics performance put them at the *Proficient* achievement level provided responses that were considered to be at least "satisfactory." As would be expected, "satisfactory" responses were even less frequent at the lower achievement levels. The question mapped at a score of 324 on the NAEP composite mathematics scale.

[1] Student responses for this and all other constructed-response questions also could have been scored as "off task," which means that the student provided a response, but it was deemed not related in content to the question asked. There are many examples of these types of responses, but a simple one would be "I don't like this test." Responses of this sort could not be rated. In contrast, responses scored as "incorrect" were valid attempts to answer the question that were simply wrong.

Table 5.2 **Percentage Satisfactory Within Achievement-Level Intervals for "Compare Two Geometric Shapes"**

THE NATION'S REPORT CARD naep

	NAEP Grade 4 Composite Scale Range			
Overall	**Below Basic**	**Basic**	**Proficient**	**Advanced**
11	4	12	19	***

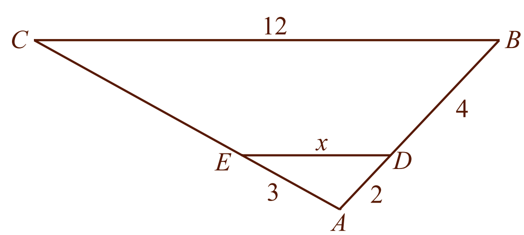
*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Geometric procedures

This area included questions that assessed students' procedural knowledge in geometry, including their ability to use the Pythagorean theorem or the properties of ratio and proportion; to draw shapes; and to use such tools as straightedges, compasses, and protractors.

Three examples of twelfth-grade questions are provided for this area. One example is a multiple-choice question, and two are short constructed-response questions that required drawn responses.

The first question presented students with two similar triangles, one within the other. Measurements were provided for two sides of the small triangle and two sides of the large triangle, and students were asked to determine the length of the third side of the small triangle.



1. If triangles *ADE* and *ABC* shown in the figure above are similar, what is the value of x?

- (A) 4
- (B) 5
- (C) 6
- (D) 8
- (E) 10

Did you use a calculator on this question?

◯ Yes ◯ No

The correct option is A.

To respond correctly to this question, students needed to know the properties of similar triangles — specifically, how to find the length of the sides of one triangle given the length of the corresponding sides of a similar triangle. In this question, the sides of the smaller triangle were one-third the length of the corresponding sides of the larger triangle. Once students recognized this, they could compute the length of small triangle side x as $^1/_3$ of large triangle side 12.

Performance data for this question are shown in Tables 5.3 and 5.4. Thirty-seven percent of twelfth-grade students provided a correct response. Half of the students whose highest mathematics course was third-year algebra or pre-calculus and more than 60 percent of the students who had taken calculus responded correctly to the question. Both of these groups of students performed better than students who had taken less math.

Forty-seven percent of the students chose "6" (Option C) as the correct response. These students may have calculated the length of *AB* instead of x, may have thought the smaller triangle was $^1/_2$ the length of the larger triangle, or may have used, in their calculations, the 4 to 2 relationship of *DB* to *AD* instead of the 6 to 2 relationship of *AB* to *AD*.

Table 5.3 ***Percentage Correct for "Use Similar Triangles"***

THE NATION'S REPORT CARD naep

Grade 12	Percentage Correct
Overall	37
Males	38
Females	36
White	40
Black	30
Hispanic	25
Asian/Pacific Islander	44
American Indian	***
Geometry Taken	38
Highest Algebra-Calculus Course Taken:	
Pre-Algebra	***
First-Year Algebra	34
Second-Year Algebra	32
Third-Year Algebra/Pre-Calculus	51
Calculus	62

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The question mapped at 389 on the NAEP composite mathematics scale. Over half of the students classified as *Proficient* responded correctly to the question compared with approximately one-third of those in the *Basic* category and approximately one-quarter of those whose overall performance was below *Basic*.

Table 5.4 **Percentage Correct Within Achievement-Level Intervals for "Use Similar Triangles"**

THE NATION'S REPORT CARD naep

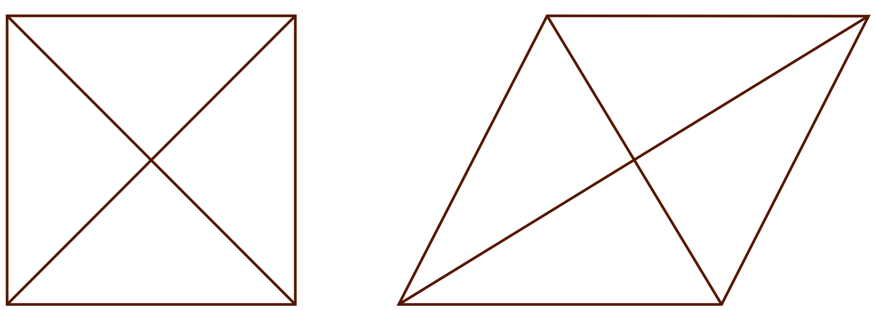
Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
37	26	37	56	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

In the next sample question for this area, students were asked to draw a parallelogram with perpendicular diagonals. To respond correctly, students needed to know the definitions of parallelogram, perpendicular, and diagonal. They also needed to be able to transfer this knowledge to a drawing.

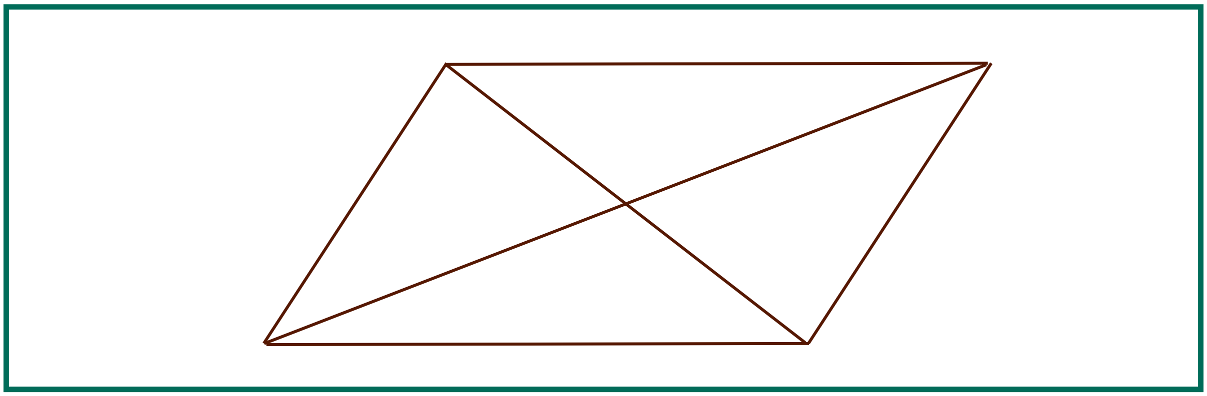
2. In the space below, use your ruler to draw a parallelogram that has perpendicular diagonals. Show the diagonals in your sketch.

"Correct" responses included drawings of a square or another rhombus with diagonals shown:



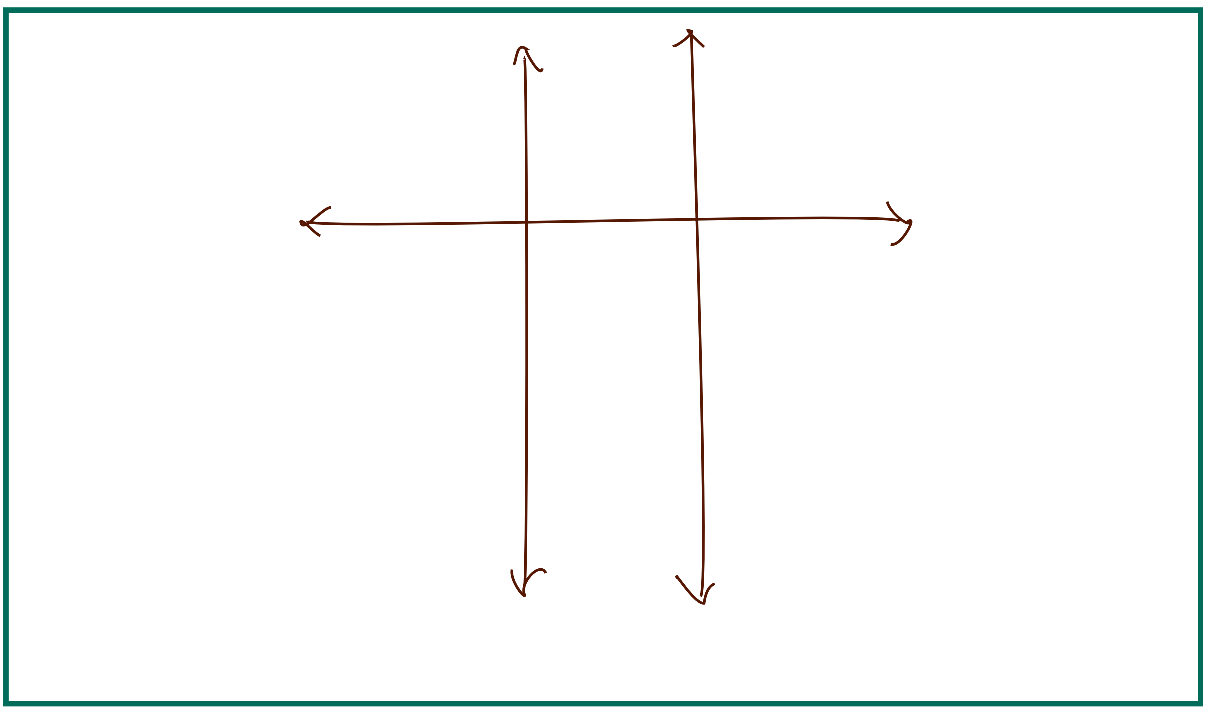
All other responses were considered "incorrect." A common "incorrect" response was a quadrilateral, with or without diagonals, that appeared to be a parallelogram other than those already described. One sample response of an incorrect quadrilateral follows:

Sample "incorrect" response 1



Another sample of an incorrect drawing is the following:

Sample "incorrect" response 2



Student performance data are presented in Tables 5.5 and 5.6. Nineteen percent of the students responded correctly to the question, including seven percent who drew a correct rhombus that was not a square. Forty-four percent of the students drew an incorrect quadrilateral, and 22 percent of the students submitted other types of "incorrect" responses. Students who had taken calculus performed better than all other groups of students (56% responded correctly), and students whose highest course in the algebra-calculus sequence was third-year algebra or pre-calculus performed better than those with less mathematics.

Table 5.5 — ***Score Percentages for "Draw a Parallelogram with Perpendicular Diagonals"***

THE NATION'S REPORT CARD naep

	Correct		Incorrect		Omit
	Rhombus that is Not a Square	Square	Quadrilateral with Incorrect Diagonals	Other	
Grade 12					
Overall	7	12	44	22	15
Males	7	13	42	23	15
Females	7	12	46	20	15
White	8	15	47	19	10
Black	2	4	38	31	26
Hispanic	3	3	37	30	27
Asian/Pacific Islander	16	20	35	14	15
American Indian	***	***	***	***	***
Geometry Taken	8	14	48	68	10
Highest Algebra-Calculus Course Taken:					
Pre-Algebra	0!	4!	40	32	20
First-Year Algebra	4	6	44	26	18
Second-Year Algebra	6	12	48	21	12
Third-Year Algebra/Pre-Calculus	12	21	49	12	6
Calculus	27	29	25	15	5

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Fifteen percent of the students did not attempt the question. The tendency to omit the question was higher for students whose highest course was either pre-algebra or first-year algebra than it was for students who had taken more advanced courses.

Table 5.6 shows student performance disaggregated by NAEP achievement levels. Over half of the students classified as *Proficient* responded correctly to the question, whereas only 17 percent of those classified as *Basic* and 1 percent of those classified as below *Basic* responded correctly. The question mapped at a score of 356 on the NAEP composite mathematics scale.

Table 5.6 **Percentage Correct Within Achievement-Level Intervals for "Draw a Parallelogram with Perpendicular Diagonals"**

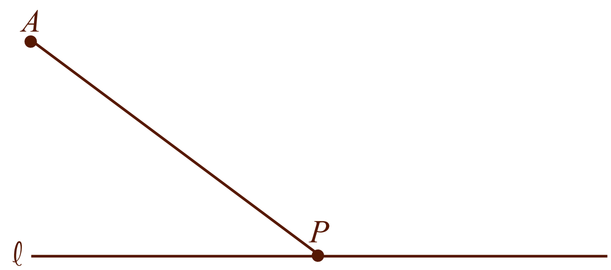
THE NATION'S REPORT CARD naep

	NAEP Grade 12 Composite Scale Range			
Overall	**Below Basic**	**Basic**	**Proficient**	**Advanced**
19	1	17	57	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The final sample question for this area follows. The question presented a line l containing a point P and an angle formed by line segment AP and line l. Students first were asked to draw a line (m) through point P that was perpendicular to segment AP. They then were asked to measure, with a protractor, the smaller angle formed by lines l and m. Correct responses required students to understand that perpendicular means 90 degrees and to know how to use the protractor to draw a 90-degree angle and measure another angle. Students also needed to recognize which angle to measure.

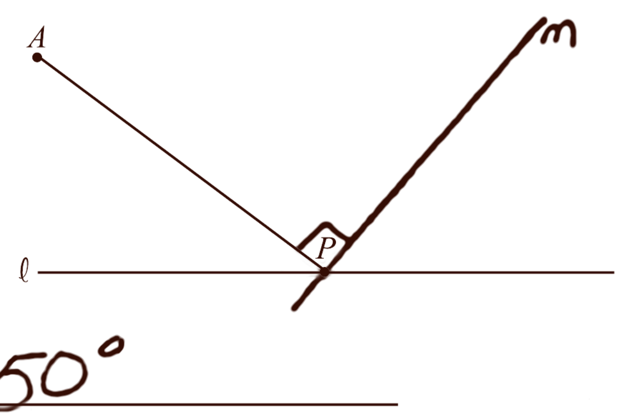
8. In the figure below, use the protractor to draw a line *m* through point *P* perpendicular to segment *AP*. In the answer space provided, give the measure of the smaller angle formed by lines ℓ and *m*.



Answer: ____________________

The correct response for the size of the angle was 50 degrees, but, to allow for measurement error, answers between 46 degrees and 54 degrees inclusive were accepted as "correct." All other answers were rated as "incorrect." The following is an example of a "correct" response.

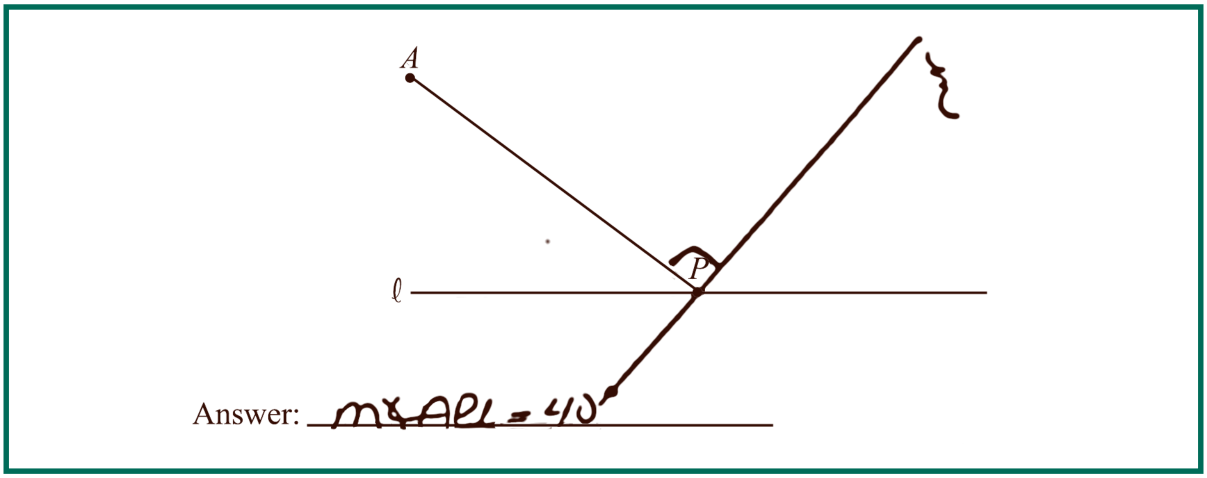
Sample "correct" response



Answer: ____________________

Some “incorrect” responses gave correct angle measurements but had incorrectly drawn lines. The following sample of an “incorrect” response shows an instance in which the line was drawn correctly, but the angle measurement was incorrect.

Sample “incorrect” response 1



In the following sample of an “incorrect” response, the student incorrectly drew a line that was perpendicular to line *l*.

Sample “incorrect” response 2

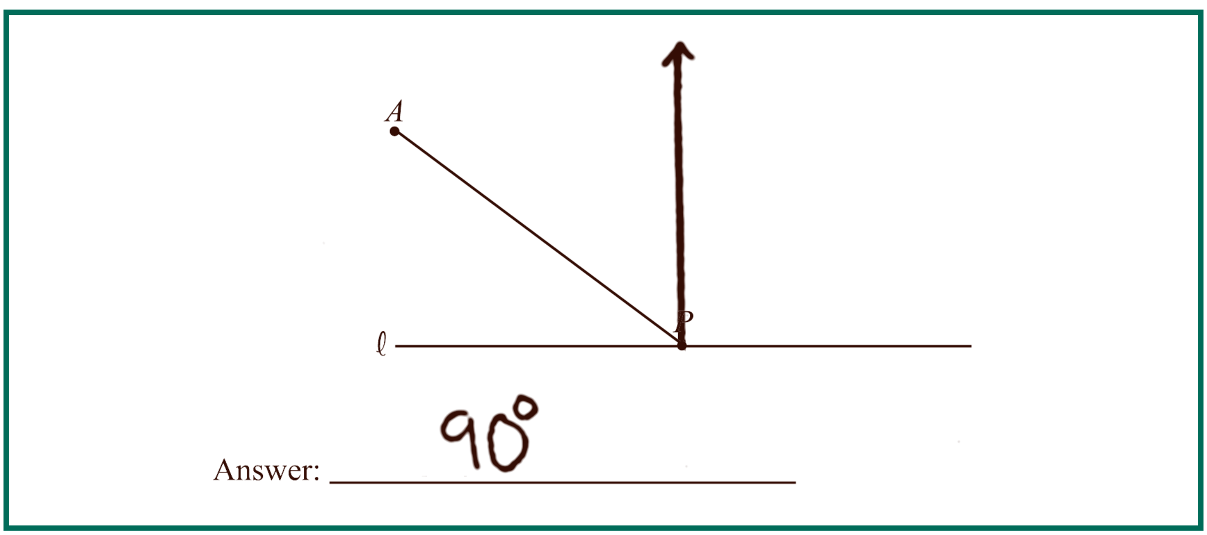


Table 5.7 shows student performance overall and by various subgroups. Twenty-three percent of the students drew line *m* correctly and provided a correct answer for the size of the angle; 4 percent drew incorrect lines but had correct angle measurements; 17 percent drew correct lines but had incorrect angle measurements; and half of the students had other "incorrect" responses. A higher percentage of males than females provided "correct" responses to the question, and the probability of a "correct" response was related to the student's mathematics preparation.

Table 5.7 **Score Percentages for "Use Protractor to Draw Perpendicular Line and Measure Angle"**

THE NATION'S REPORT CARD naep

	Correct	Incorrect			Omit
		Line, Correct Angle	Angle, Correct Line	Other	
Grade 12					
Overall	23	4	17	50	7
Males	26	4	17	46	7
Females	19	4	17	54	7
White	27	4	17	45	7
Black	6	3	15	67	9
Hispanic	11	3	13	67	6
Asian/Pacific Islander	35	3	25	36	1
American Indian	***	***	***	***	***
Geometry Taken	26	4	18	48	4
Highest Algebra-Calculus Course Taken:					
Pre-Algebra	***	***	***	***	***
First-Year Algebra	14	3	16	63	3
Second-Year Algebra	21	4	17	52	4
Third-Year Algebra/Pre-Calculus	34	5	21	32	8
Calculus	49	5	18	25	1!

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.

*** Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Performance by achievement levels, shown in Table 5.8, was similar to that of the previous question. The question mapped at a score of 360.

Table 5.8 **Percentage Correct Within Achievement-Level Intervals for "Use Protractor to Draw Perpendicular Line and Measure Angle"**

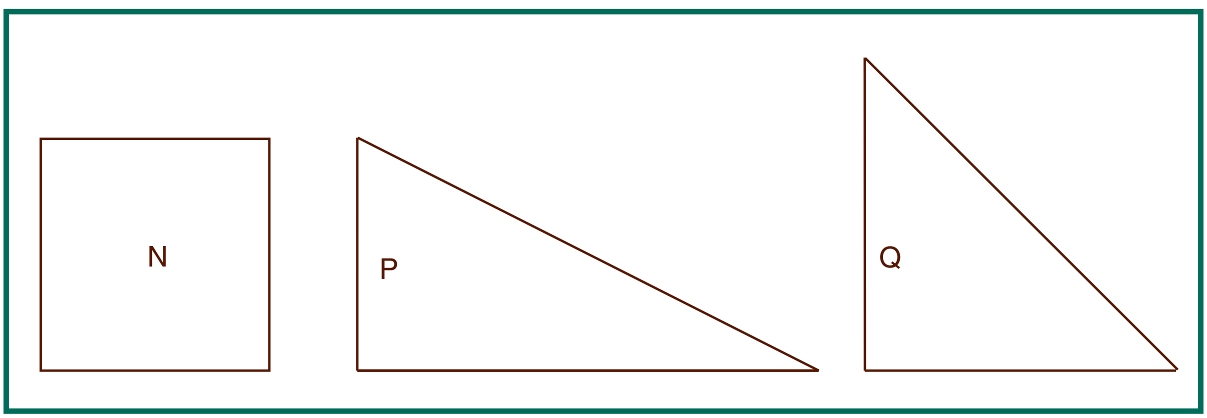
Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
23	1!	21	57	***

*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Geometric transformations and spatial sense

The area of geometric transformations and spatial sense includes questions that tapped students' visual-spatial skills. In many of the questions, students were presented with a figure and asked what the figure would look like if it were flipped, rotated, folded, unfolded, pulled apart, combined with another figure, or transformed in some other manner.

Three sample questions are presented for this area. The first two sample questions were included within a block of questions for which students were provided with two cardboard cutouts of each of the following shapes:

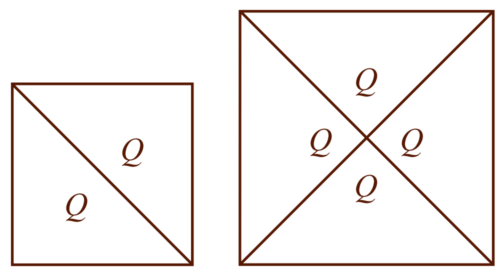


For the first question, students at grades 4 and 8 were asked to assemble the 2 pieces labeled *Q* (triangles) to make a square.

3. You will need the 2 pieces labeled *Q*. Please find those 2 pieces now.

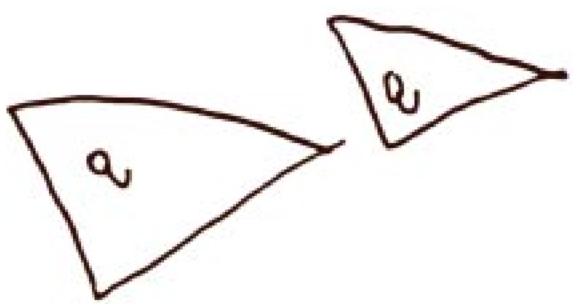
Use the 2 pieces labeled *Q* to make a square. Trace the square and draw the line to show where the 2 pieces meet.

Responses that were rated as "correct" could include either two or four replicates of *Q*, as long as the final shape was a square.



Diagonals had to be shown, although a slight space between the shapes was acceptable. Freehand drawings also were rated as "correct." Squares without diagonals drawn were rated "incorrect," as were other types of drawings not resembling those above. A sample of an "incorrect" response follows:

Sample "incorrect" response



Student performance data are presented in Tables 5.9 and 5.10. As might be expected, the question was fairly easy for students at both grade levels. Seventy-three percent of fourth-grade students and 89 percent of eighth-grade students assembled and drew the pieces correctly. The question mapped at a score of 217 for grade 4 and 232 for grade 8. At least three-quarters of the students classified in each of the achievement levels at each grade responded correctly, except for fourth-grade students performing below the level of *Basic*.

Table 5.9 ***Percentage Correct for "Assemble Pieces to Form a Square"***

THE NATION'S REPORT CARD naep

	Percentage Correct
Grade 4	
Overall	73
Males	75
Females	71
White	81
Black	46
Hispanic	55
Asian/Pacific Islander	81
American Indian	***
Grade 8	
Overall	89
Males	89
Females	89
White	93
Black	75
Hispanic	84
Asian/Pacific Islander	– –
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	89
Pre-Algebra	91
Algebra	91

***Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 5.10 Percentage Correct Within Achievement-Level Intervals for "Assemble Pieces to Form a Square"

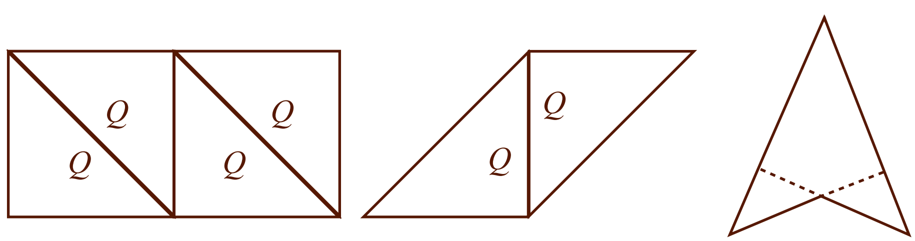
	Overall	NAEP Grades 4 and 8 Composite Scale Ranges: Below Basic	Basic	Proficient	Advanced
Grade 4	73	38	85	98!	***
Grade 8	89	77	95	99	***

*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The next question, which was presented to students at all three grade levels, was more difficult. For this question, students had to assemble the same two pieces used in the previous question to make a four-sided shape that was not a square. Students again had to trace the figure and draw a line to show where the pieces met.

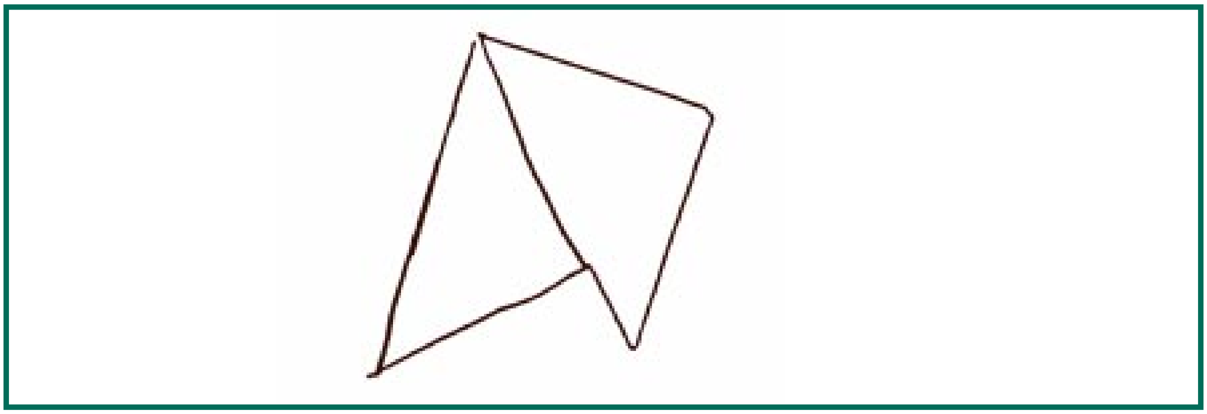
3. Use the 2 pieces labeled *Q* to make a 4-sided shape that is not a square. Trace the shape and draw the line to show where the 2 pieces meet.

The three types of drawings that were accepted as "correct" follow. As in the previous question, two or four replicates of *Q* could be used, but they had to be placed such that the resultant shape was rectangular or rhomboid, but not square. A four-sided figure composed by overlapping two *Q* shapes also was accepted as "correct." In the latter case, it was not necessary to show what would anyway have been an ambiguous concept, namely the line where the two pieces meet.



A sample "incorrect" response, in which the figure has more than four sides, is shown below.

Sample "incorrect" response



Tables 5.11 and 5.12 present performance results for this question. Only 16 percent of the students at grade 4 were able to provide a "correct" response to the question, while close to half of the eighth-grade students and over half of the twelfth-grade students provided a response rated "correct." Males were more likely than females to respond correctly. Furthermore, eighth-grade students in algebra performed better than those in pre-algebra or eighth-grade mathematics.

Table 5.11 Score Percentages for "Assemble Pieces to Form Shape"

	Correct		Incorrect	Omit
	Rhombus	Not a Rhombus		
Grade 4				
Overall	15	1	80	5
Males	18	1	75	6
Females	12	1	84	4
White	19	1	77	3
Black	3	0!	88	9
Hispanic	9	1!	80	11
Asian/Pacific Islander	17	2!	80	1!
American Indian	***	***	***	***
Grade 8				
Overall	45	3	49	2
Males	48	3	46	2
Females	42	3	52	3
White	52	3	43	2
Black	17	4	74	5
Hispanic	38	5	54	4
Asian/Pacific Islander	– –	– –	– –	– –
American Indian	***	***	***	***
Mathematics Course Taking:				
Eighth-Grade Mathematics	41	3	54	2
Pre-Algebra	42	2	54	2
Algebra	59	4	36	1
Grade 12				
Overall	53	5	39	3
Males	55	6	36	3
Females	51	4	42	3
White	59	5	34	2
Black	28	4	63	5
Hispanic	39	8	49	4
Asian/Pacific Islander	68	8	23	1!
American Indian	***	***	***	***
Geometry Taken	56	5	37	2
Highest Algebra-Calculus Course Taken:				
Pre-Algebra	35	11	46	6
First-Year Algebra	51	4	42	2
Second-Year Algebra	54	5	39	2
Third-Year Algebra/Pre-Calculus	60	5	32	2
Calculus	60	5	34	1!

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Among eighth- and twelfth-grade students, over half of those classified as *Basic* or *Proficient* responded correctly to the question. At grade 4, "correct" responses were provided by 15 percent of those classified as *Basic* and one-third of those classified as *Proficient*. The question mapped at 285 for grade 4, 290 for grade 8, and 313 for grade 12.

Table 5.12 ***Percentage Correct Within Achievement-Level Intervals for "Assemble Pieces to Form Shape"***

THE NATION'S REPORT CARD naep

	Overall	NAEP Grades 4, 8, and 12 Composite Scale Ranges			
		Below Basic	**Basic**	**Proficient**	**Advanced**
Grade 4	16	3	15	33	***
Grade 8	49	25	54	76	***
Grade 12	58	32	66	81	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Asssessment.

The final example for this area was a short constructed-response question for eighth-grade students. Students were given three facts about the spatial relationships among points *A*, *B*, and *C* and asked whether these facts supported the conclusion that *C* always had to fall between *A* and *B*. They also were asked to draw a diagram to explain their answer.

12. Jaime knows the following facts about points *A*, *B*, and *C*.

- Points *A*, *B*, and *C* are on the same line, but might not be in that order.
- Point *C* is twice as far from point *A* as it is from point *B*.

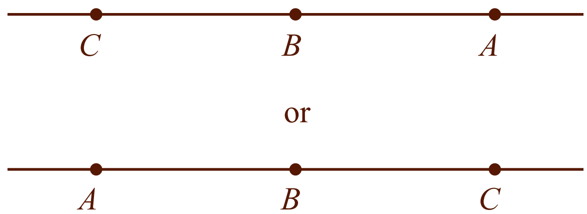
Jaime concluded that point *C* is always between points *A* and *B*.

Is Jaime's conclusion correct?

◯ Yes ◯ No

In the space provided, use a diagram to explain your answer.

To be considered "correct," a response had to disagree with the conclusion and include one of the following diagrams in which *B* is shown to fall halfway between points *A* and *C*. All other responses were considered "incorrect."



The following is an example of an "incorrect" response.

Sample "incorrect" response

Is Jaime's conclusion correct?

○ Yes ● No

In the space provided, use a diagram to explain your answer.

Student performance data are presented in Tables 5.13 and 5.14. The question was answered correctly by approximately one-quarter of the students. More females than males responded correctly. The percentage of eighth-grade students enrolled in algebra who answered the question correctly (34%) was greater than the percentage of those enrolled in pre-algebra or regular mathematics who answered correctly. Slightly under half of the students classified as *Advanced* or *Proficient* responded correctly. However, only 24 percent of the students classified at the *Basic* achievement level and 6 percent of those classified as below *Basic* responded correctly. The question mapped at a score of 335.

Table 5.13 ***Percentage Correct for "Reason About Betweenness"***

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	23
Males	21
Females	26
White	27
Black	10
Hispanic	16
Asian/Pacific Islander	– –
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	18
Pre-Algebra	19
Algebra	34

*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 5.14 ***Percentage Correct Within Achievement-Level Intervals for "Reason About Betweenness"***

THE NATION'S REPORT CARD naep

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
23	6	24	44	48

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Geometric models and problems

Questions falling into the final area of geometric models and problems required students to apply their geometric skills and understanding in order to represent situations with geometric models or solve practical problems. Many questions requiring extended responses fell into this area. The example shown is a twelfth-grade extended constructed-response question. Students were asked to describe a procedure for locating the point that is the center of a circular paper disk. They were provided with an actual disk and told they could manipulate it in any way. Students were asked to use "geometric definitions, properties, or principles" to justify their procedure.

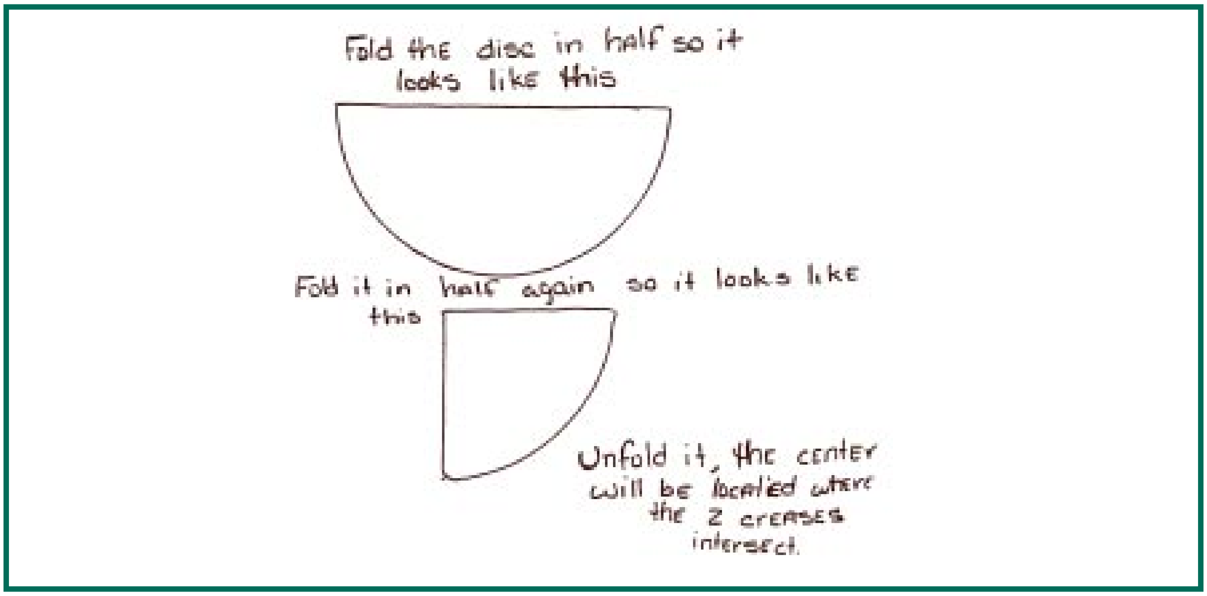
> This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show <u>all</u> your work.
>
> **10.** Describe a procedure for loacting the point that is the center of a circular paper disk. Use geometric definitions, properties, or principals to explain why your procedure is correct. Use the disk provided to help you formulate your procedure. You may write on it or fold it in any way that you find helpful, but it will not be collected.

Responses were rated "extended," "satisfactory," "partial," "minimal," or "incorrect." However, when the question was anchored to the NAEP scale, the "extended" and "satisfactory" rating categories were collapsed. A description of the ratings and sample responses for each rating category follow.

An "extended" response was one that described locating the center of the circle by folding or by compass and straightedge construction and that clearly and completely explained what geometric properties of circles justified the method chosen (e.g., two diameters intersect in the center of the circle, the intersection of two perpendicular bisectors of two nonparallel chords is the center of the circle). So few students received a rating of "extended" that no samples are available to present.

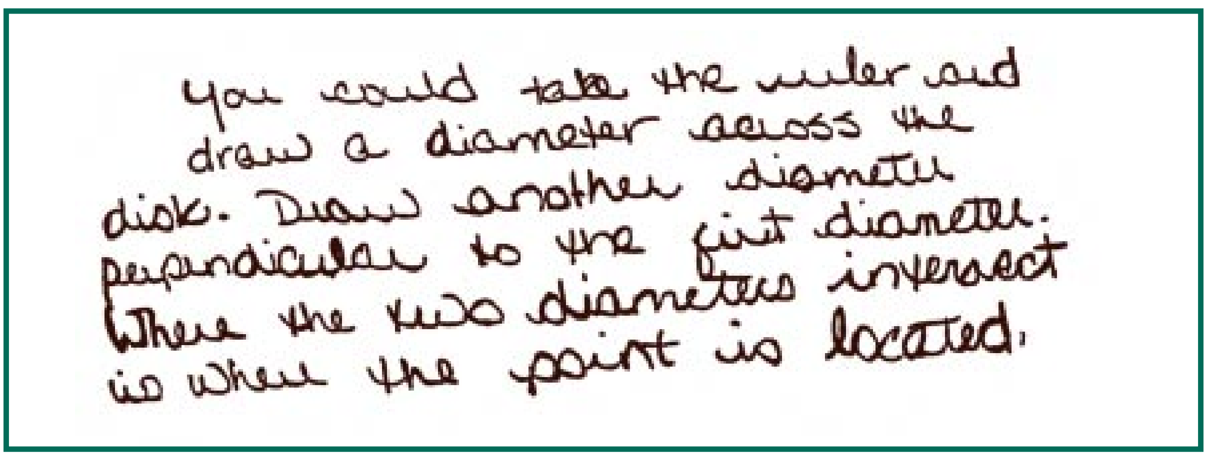
A "satisfactory" response was one in which the student described a method of locating the center point by folding or by compass-and-straightedge construction but did not use appropriate geometry terminology in the explanation. For example, the following sample response was rated as "satisfactory." In it the student described a method of folding the circle in half and then in half again, but did not use any geometric terminology.

Sample "satisfactory" response



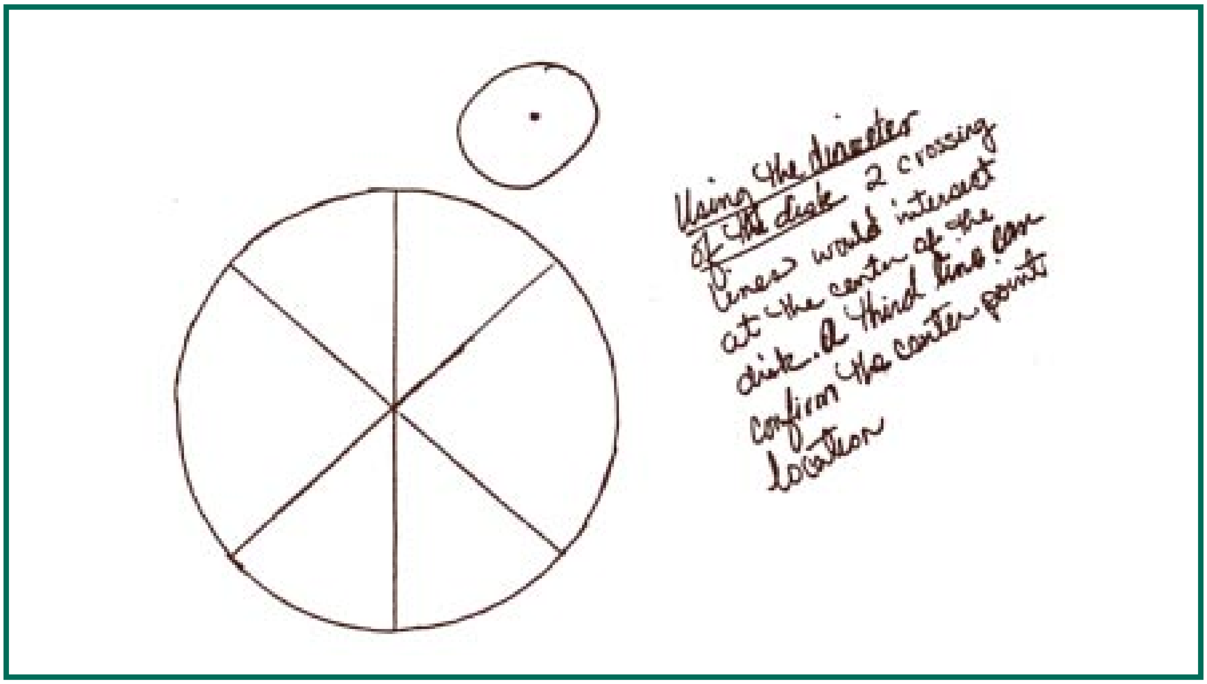
A "partial" response was one that located the center by folding or that described a compass and straightedge construction of the perpendicular bisectors of two nonparallel chords, but the explanation was incomplete. Responses that correctly explained a drawing of two diameters *or* the perpendicular bisectors of two nonparallel chords also were considered as "partial." The following response was rated "partial" because the student did not describe how to determine the diameter of the circle; thus, the response was incomplete.

Sample "partial" response


You could take the ruler and draw a diameter across the disk. Draw another diameter perpendicular to the first diameter. Where the two diameters intersect is where the point is located.

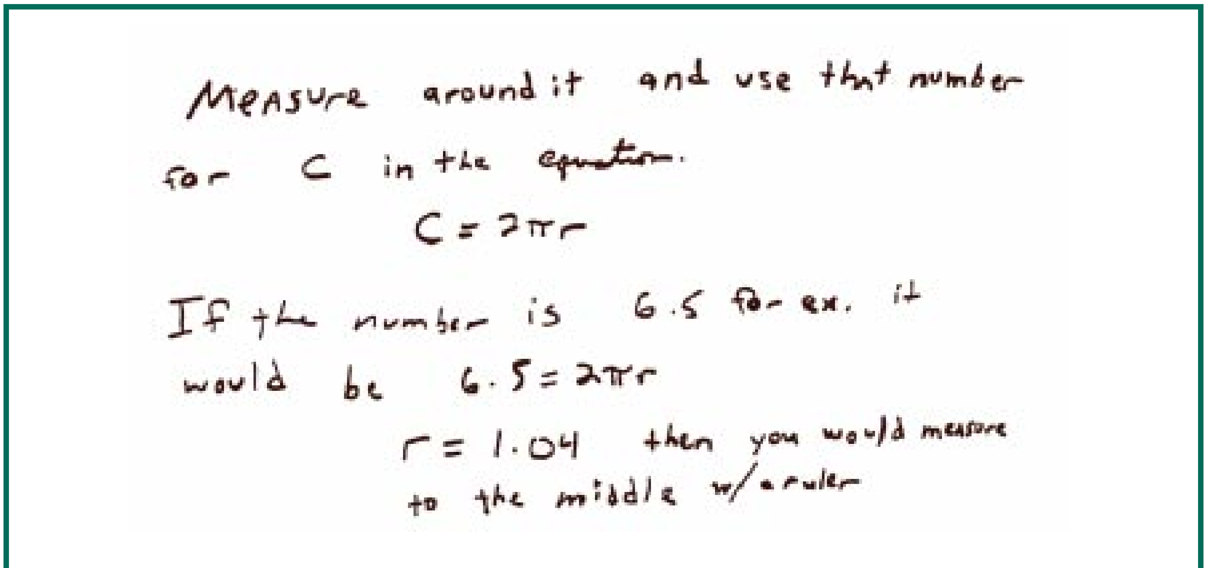
"Minimal" responses showed a line that appeared to include the center of the circle (e.g., a diameter or the perpendicular bisector of a chord), but the explanation was inaccurate, as in the following example.

Sample "minimal" response



All other responses were considered to be "incorrect."

Sample "incorrect" response



Tables 5.15 and 5.16 present student performance data on this question. Only 1 percent of the students provided a response that was considered to be "extended," and 13 percent provided "satisfactory" answers. Approximately one-quarter each provided answers that were rated as either "minimal" or "incorrect," and nearly as many did not respond at all. Students who had taken calculus were less likely than others to omit the question, and students who had taken at least third-year algebra, pre-calculus, or calculus were more likely to receive a rating of at least "satisfactory" than those who had not.

Table 5.15 ***Score Percentages for "Describe Geometric Process for Finding Center of Disk"***

THE NATION'S REPORT CARD naep

	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
Grade 12						
Overall	1	13	9	25	28	23
Males	1	13	10	25	28	22
Females	0!	13	7	26	29	24
White	1	15	9	28	27	19
Black	0!	4	5	13	36	42
Hispanic	0!	9	7	26	28	28
Asian/Pacific Islander	3	17	17	19	30	14
American Indian	***	***	***	***	***	***
Geometry Taken	1	14	9	26	29	22
Highest Algebra-Calculus Course Taken:						
Pre-Algebra	0	4	8!	25	33	25
First-Year Algebra	0!	8	11	26	27	25
Second-Year Algebra	1	13	8	26	30	21
Third-Year Algebra/Pre-Calculus	2	20	5	25	25	23
Calculus	0!	26	10	25	24	16

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The question mapped at a score of 424. One-third of the students at the *Proficient* achievement level and lower percentages at the levels below *Proficient* were able to respond correctly.

Table 5.16 **Percentage Satisfactory Within Achievement-Level Intervals for "Describe Geometric Process for Finding Center of Disk"**

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
13	4	14	31	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Summary

Many of the example questions for this content strand were difficult for students. The easier questions required students to construct or describe simple shapes, and more difficult questions required the application of knowledge about geometric properties to solve complex problems. Most of the questions required students to draw or explain a response. Questions in this content strand also relied upon students' visual-spatial skills. For many of the sample questions a significant difference between the performance of male and female students existed. Eighth-grade algebra students tended to perform better than other eighth-grade students, whereas eighth-grade students in pre-algebra or regular mathematics performed similarly. Also, an increase in performance was sometimes noted between twelfth-grade students who had taken at least second-year algebra and those who had not. Additionally, a further increase in performance was noted at times for students who had taken at least third-year algebra or pre-calculus.