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A Simplified Realization for the Gaussian Filter in Surface Metrology

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Abstract

A simplified realization for the Gaussian filter in surface metrology is presented in this paper. The sampling function $\sin u/u$ is used for simplifying the Gaussian function. According to the central limit theorem, when n approaches infinity, the function $(\sin u/u)^n$ approaches the form of a Gaussian function. So designed, the Gaussian filter is easily realized with high accuracy, high efficiency and without phase distortion. The relationship between the Gaussian filtered mean line and the mid-point locus (or moving average) mean line is also discussed.

Key Words: surface roughness, mean line, sampling function, Gaussian filter

1. Introduction

The Gaussian filter has been recommended by ISO 11562-1996 and ASME B46-1995 standards for determining the mean line in surface metrology [1-2]. Its weighting function is given by

$$h(t) = \frac{1}{\alpha\lambda_c} e^{-\pi(t/\alpha\lambda_c)^2} \quad , \tag{1}$$

where $\alpha = 0.4697$, t is the independent variable in the spatial domain, and λ_c is the cut-off wavelength of the filter (in the units of t). If we use $x(t)$ to stand for the primary unfiltered profile, $m(t)$ for the Gaussian filtered mean line, and $r(t)$ for the roughness profile, then

$$m(t) = x(t) * h(t) \tag{2}$$

and

$$r(t) = x(t) - m(t) , \tag{3}$$

where the * represents a convolution of two functions. Therefore, the key issue is how to calculate the mean line $m(t)$. Many researchers [3-7] have worked on this problem and faced the same difficulties that arise from the collective requirements for high accuracy, fast speed, no phase distortion, and simplicity in the computer algorithm. A number of methods have been developed, including the direct convolution integral method [3], FFT fast filtering method [4], fast filtering method based on the symmetry and recurrence of the weighting function of the Gaussian filter [5], and various kinds of approximation methods [2,6,7,8]. In this paper, we present a simple new method using the sampling function $\sin u/u$ for Gaussian filtering. This method not only is practical, but also indicates quantitatively how one approaches a Gaussian filtered mean line with successive mid-point locus (or moving average) mean lines [9-11]. For comparable accuracy, the method presented here is even faster than a similar approximation method presented previously [8] using $(1+u^2)^{-n}$ as the basis function.

2. Basic Theory

2.1 Sampling Function $\sin u/u$ and Gaussian Function e^{-u^2}

Consider the limiting form of the Gaussian function as follows

$$\lim_{u\to 0} e^{-u^2} = 1 \ . \tag{4}$$

This fact implies that 1 is a special approximation of the Gaussian function. In the spatial frequency domain, if the transmission characteristic of a filter $H(\omega)=\begin{cases}1 & |\omega|\le\omega_c\\ 0 & |\omega|>\omega_c\end{cases}$, then this filter is an ideal low pass one below $\omega=\omega_c$. We can refer to this ideal low pass filter as an approximation of the Gaussian filter. Here ω is the spatial angular frequency, equal to $2\pi/\lambda$, where λ is the spatial wavelength, and ω_c is equal to $2\pi/\lambda_c$. In the time domain, if $h(t)=\begin{cases}1 & |t|\le\ell/2\\ 0 & |t|>\ell/2\end{cases}$, we can consider $h(t)$ to be a low pass filter. It has equal weighting coefficients and is also an approximation of the Gaussian filter. Here ℓ represents a spatial length. The time domain filter with equal weighting coefficients is useful for designing an accurate Gaussian filter. Since the Fourier transform of a Gaussian function is also a Gaussian distribution, we can deduce that the Fourier transform of the special function $h(t)=\begin{cases}1 & |t|\le\ell/2\\ 0 & |t|>\ell/2\end{cases}$ also approximates the Gaussian function. In fact, the Fourier transform of this special function is a function with the shape of $\sin u/u$. That is, the sampling function $\sin u/u$ is a prototype for approximating the Gaussian function. The two formulas,

$$\frac{\sin u}{u} = 1 - \frac{u^2}{3!} + \frac{u^4}{5!} - \frac{u^6}{7!} + \frac{u^8}{9!} - \cdots \tag{5}$$

and

$$e^{-u^2} = 1 - u^2 + \frac{u^4}{2!} - \frac{u^6}{3!} + \frac{u^8}{4!} - \cdots \quad , \tag{6}$$

closely resemble each other. Further, according to the central limit theorem [12], the self-multiplication of the $\sin u/u$, i.e., $(\sin u/u)^n$, approaches the shape of the Gaussian distribution. The larger the *n*, the higher the approximation accuracy. In brief, that is

$$\lim_{n\to\infty}\left(\frac{\sin c_n u}{c_n u}\right)^n = e^{-u^2} \quad , \tag{7}$$

where c_n is a constant related to *n*.

2.2 Gaussian Filter for Surface Metrology

The Fourier transform of the Gaussian weighting function in Eq. (1) is a Gaussian transfer function $H(\omega)$

$$H(\omega) = e^{-\pi(\alpha\omega/\omega_c)^2} \quad . \tag{8}$$

We choose now to express this function in terms of the normalized spatial wavelength λ/λ_c, so that

$$H(\lambda_c/\lambda) = e^{-\pi(\alpha\lambda_c/\lambda)^2} \quad . \tag{9}$$

With Eqs. (7) and (9) in mind, we can construct a series of approximation filters, $H_n(\lambda_c/\lambda)$, of the Gaussian filtering characteristic. The form of these approximation filters is as follows

$$H_n(\lambda_c/\lambda) = \left(\frac{\sin(c_n\pi\lambda_c/\lambda)}{c_n\pi\lambda_c/\lambda}\right)^n \quad , \tag{10}$$

where c_n is a constant to be determined by the condition that when $\lambda = \lambda_c$, $H_n(\lambda_c/\lambda) = 50\%$. Some values of c_n are given in Table 1.

Table 1. Coefficient c_n and Filtering Order *n*

n	1	2	3	4	8	16	32
c_n	0.6034	0.4429	0.3661	0.3189	0.2275	0.1616	0.1145

The transmission characteristics of the Gaussian filter and these approximation filters are shown in Table 2 and Fig. 1a, while the approximation error is shown in Fig. 1b.

Table 2. Gaussian Filter and Some Approximation Filters, $H_n(\lambda_c/\lambda)$

λ/λ_c	$e^{-\pi(\alpha\lambda_c/\lambda)^2}$	H_1	H_2	H_3	H_4	H_8	H_{16}
0.1	0.0%	0.6%	0.5%	0.0%	0.0%	0.0%	0.0%
0.2	0.0%	-0.6%	0.8%	-0.1%	0.1%	0.0%	0.0%
0.3	0.0%	0.6%	4.6%	-0.5%	0.0%	0.0%	0.0%
1/3	0.2%	-9.9%	4.2%	-0.1%	0.0%	0.1%	0.1%
0.5	6.3%	-16.0%	1.6%	3.4%	4.2%	5.3%	5.8%
0.7	24.3%	15.5%	21.2%	22.4%	22.9%	23.7%	24.0%
1.0	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%
1.5	73.5%	75.4%	74.4%	74.1%	73.9%	73.7%	73.6%
2.0	84.1%	85.7%	84.9%	84.6%	84.5%	84.3%	84.2%
2.5	89.5%	90.7%	90.1%	89.9%	89.8%	89.6%	89.6%
3.0	92.6%	93.5%	93.0%	92.9%	92.8%	92.7%	92.6%

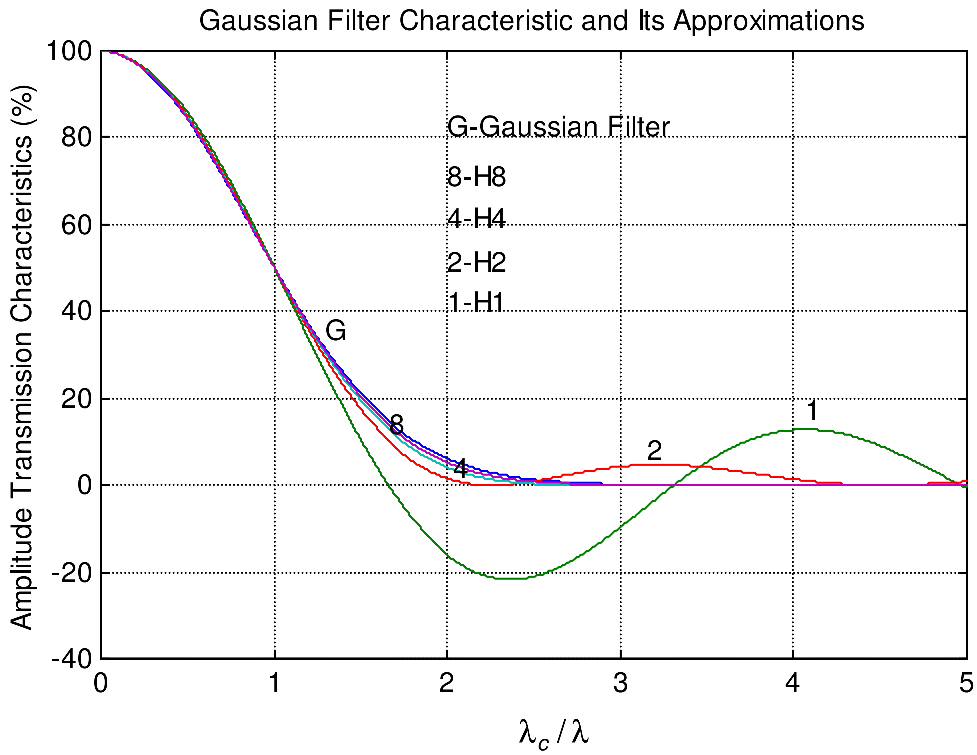

Fig. 1a Amplitude Transmission Characteristics of the Gaussian Filter and Its Approximation Filters

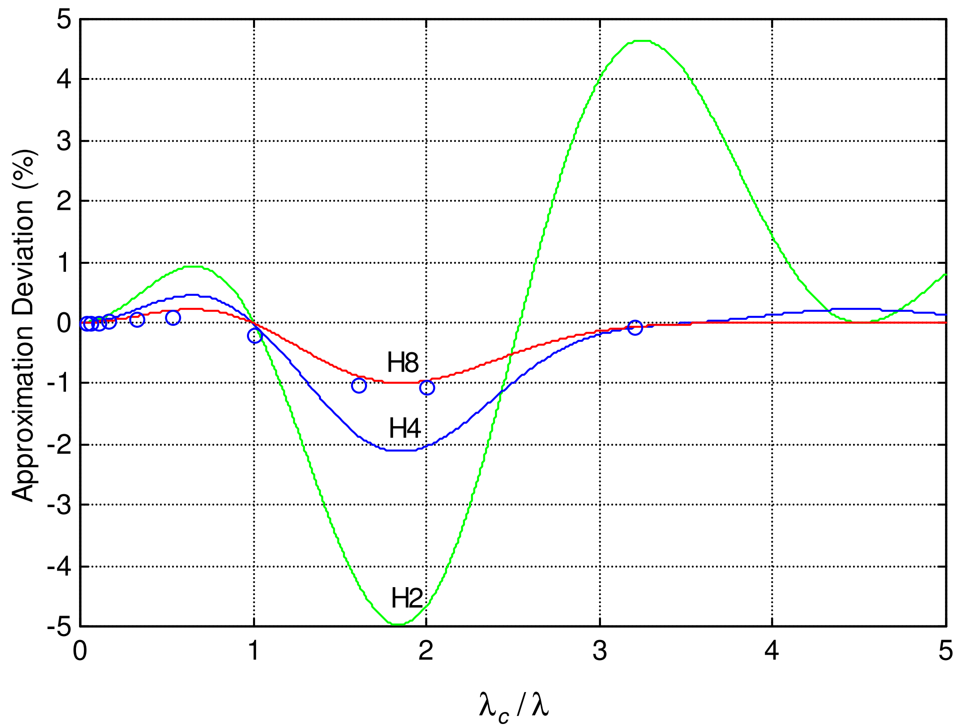

Fig. 1b The Transmission Characteristic Deviations of the Approximation Filters from the Gaussian Filter. The data points show, for selected spatial wavelengths, the results of calculations using an implementation of the H_8 filter described in Sec. 3.

In fact, when $n=1$, $H_1(\lambda_c/\lambda)$ represents the transmission characteristic of the mid-point locus (or moving average) mean line method [9-11]. In other words, the mid-point locus mean line filter is the first-order approximation to the Gaussian filter. The mid-point locus mean line is very simple conceptually and is easily realized in instruments.

When $n=2$, $H_2(\lambda_c/\lambda)$ is the second-order approximation to the Gaussian filter. It is equivalent to a triangular function in the spatial domain, an approximation discussed in the ASME B46 standard [2]. This is shown by Fig. 2, which is a re-plot of the data of Fig. 1b so that it can be compared to a similar graph in the ASME B46 standard. The results for H_2 here agree with those for the triangular function in the standard.

When n is odd, there are spatial wavelength regions where the function $H_n(\lambda_c/\lambda)$ is less than zero, thus producing filtered profile components 180° out of phase from the input profile components in those regions. Although this problem becomes less important with n increasing, we prefer even values of n to odd ones. In general, when $n\geq 2$, $H_n(\lambda_c/\lambda)$ can be called a high order approximation for the Gaussian filter.

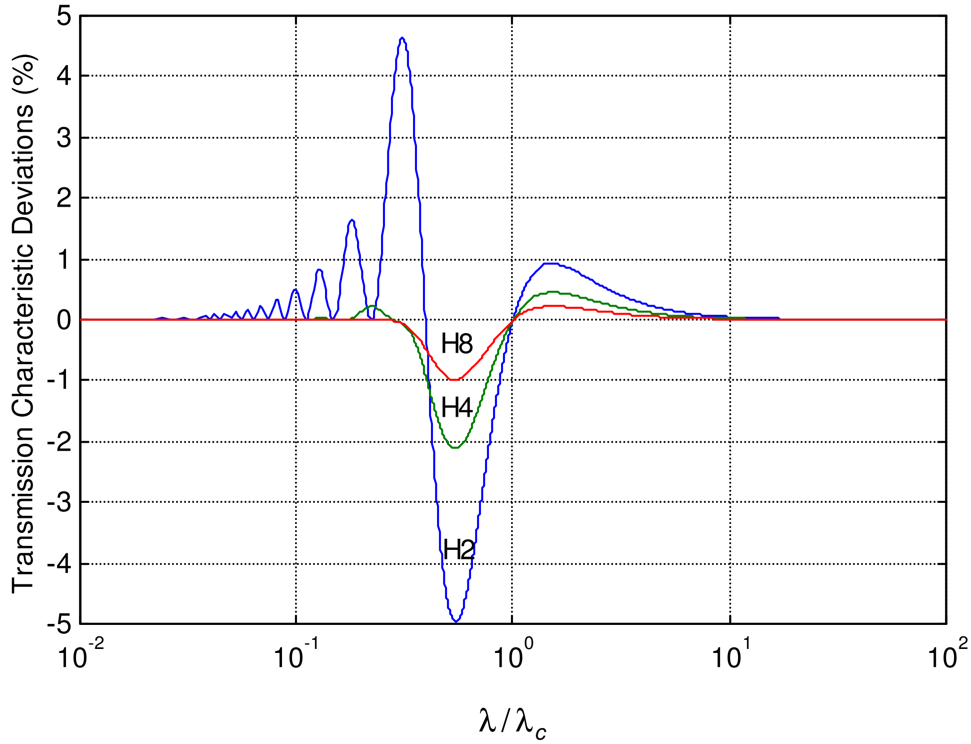

Fig. 2 The Transmission Characteristic Deviations of the Approximation Filters from the Gaussian Filter

2.3 Mid-Point Locus Mean Line Filter

The weighting function of the mid-point locus mean line filter is

$$h_1(t) = \begin{cases} 1 & |t| \le (c_1 \cdot \lambda_c)/2 \\ 0 & |t| > (c_1 \cdot \lambda_c)/2 \end{cases} . \quad (11)$$

Therefore, the mid-point locus mean line $m_1(t)$ can be computed by

$$m_1(t) = \frac{1}{c_1 \cdot \lambda_c} \int_{t-(c_1 \cdot \lambda_c)/2}^{t+(c_1 \cdot \lambda_c)/2} x(\tau) d\tau \ . \quad (12)$$

Obviously, the transmission characteristic equation of the mid-point locus mean line is $H_1(\lambda_c / \lambda)$, i.e.,

$$H_1(\lambda_c / \lambda) = \frac{\sin(c_1 \pi \lambda_c / \lambda)}{c_1 \pi \lambda_c / \lambda} \ . \quad (13)$$

Under the condition of digital measurements, if $x(i)$ represents the equally spaced, digitized surface profile, then the mid-point locus mean line $m_1(i)$ is given by

$$m_1(i) = \frac{1}{2k+1} \cdot \sum_{j=i-k}^{i+k} x(j) \quad , \tag{14}$$

where $2k+1$ is the number of sampled points within the length $c_1 \cdot \lambda_c$. Correspondingly, the transfer function $H_1(z)$ in z-transform space [13-14] is as follows

$$H_1(z) = \frac{1}{2k+1} \cdot \frac{z^{-k}(1-z^{2k+1})}{1-z} \quad . \tag{15}$$

From Eq. (15), the digital transmission characteristic of the mid-point locus mean line method is represented by

$$H_1(N/N_c) = \frac{1}{2k+1} \cdot \frac{\sin\left((2k+1)\pi/N\right)}{\sin(\pi/N)} \quad , \tag{16}$$

where N_c is the number of sampled points within a cut-off length λ_c; N is the number of sampled points within a wavelength λ, i.e., $N = (\lambda/\lambda_c) \cdot N_c$; k is an integer determined by

$$2k+1 = c_1 \cdot N_c \ ; \tag{17}$$

and the value of k is chosen to make $|\,2k+1-c_1 \cdot N_c\,|$ minimum.

2.4 High Order Approximation Filters

Analogously, the digital forms of these high order approximation filters must be

$$H_n(N/N_c) = \left(\frac{1}{2k+1} \cdot \frac{\sin\left((2k+1)\pi/N\right)}{\sin(\pi/N)}\right)^n \quad , \tag{18}$$

where k satisfies the condition of $\min|\,2k+1-c_n \cdot N_c\,|$.

By comparison with the mid-point locus mean line method, the z-transformation transfer functions, $H_n(z)$, of the high order approximation filters are represented by

$$H_n(z) = \left(\frac{1}{2k+1} \cdot \frac{z^{-k}(1-z^{2k+1})}{1-z} \right)^n \quad . \tag{19}$$

So we can use the same form of difference equation n times and finally obtain the approximation Gaussian filtered mean line. The computational processes are as follows. The unfiltered profile m_0 is given by

$$m_0(1) = x(1),\ m_0(2) = x(2),\ \cdots,\ m_0(M) = x(M) \ . \tag{20}$$

Then, for any intermediate or final filter stage p such that $p = 1, 2, 3, \cdots, n$, the filtered line m_p is given by

$$m_p(i) = \sum_{j=(p-1)k+1}^{(p+1)k+1} m_{p-1}(j) \qquad \text{for} \quad i = pk+1 ,$$

$$m_p(i) = m_p(i-1) + m_{p-1}(i+k) - m_{p-1}(i-(k+1)) , \quad \text{for} \tag{21}$$

$$i = pk+2, pk+3, \cdots, M-pk \quad .$$

A normalizing factor is then included at the end:

$$m(i) = m_n(i) \big/ (2k+1)^n ,$$

$$i = nk+1,\ nk+2,\ \cdots,\ M-nk \ , \tag{22}$$

where M is the number of sampled points within a traversing length and $m(i)$, $i = nk+1, \cdots, M-nk$ is the accurate Gaussian filtered mean line. In general, the sampling condition for Gaussian filtering must be satisfied with $M - 2nk \geq 5 \times N_c$. These $5N_c$ data points are used for surface assessment, and the points at both ends are omitted. A QBASIC program for the Gaussian filter for $n = 8$ is presented in the Appendix.

3. <u>Experiments</u>

Two approximation filters, $H_8(z)$ and $H_{16}(z)$, are selected as examples. Assuming a value of $N_c = 1600$, a typical setup condition for one of our instruments, the values of k are 181 and 129 for the two filters, respectively. The results of their amplitude transmission characteristics calculated from Eq. (18) are shown in Table 3.

From Table 3, it can be seen that the maximum error of the approximation is about 1% for $H_8(N/N_c)$, and about 0.4% for $H_{16}(N/N_c)$.

Table 3. Transmission Characteristics of the Approximation Filters

N/N_c	$e^{-\pi(\alpha N_c/N)^2}$	$H_8(N/N_c)$	$H_{16}(N/N_c)$
0.1	0.0%	0.0%	0.0%
0.2	0.0%	0.0%	0.0%
0.3	0.0%	0.0%	0.0%
1/3	0.2%	0.1%	0.1%
0.5	6.3%	5.5%	5.9%
0.7	24.3%	24.0%	24.1%
1.0	50.0%	50.4%	50.1%
1.5	73.5%	74.0%	73.7%
2.0	84.1%	84.4%	84.2%
2.5	89.5%	89.8%	89.6%
3.0	92.6%	92.8%	92.7%

A simulated surface profile with traversing length $7\lambda_c = 5.6$ mm is shown in Fig. 3a that consists of ten harmonic components with known frequencies ranging from $0.1/\lambda_c$ to $25/\lambda_c$. So we can compute its theoretical Gaussian filtered mean line according to the standard Gaussian filtering transmission characteristic. For this example, the sampling interval is 0.5 μm, and therefore the total number of the sampled data points, $7N_c = 11200$. The small differences between the theoretical Gaussian filtered mean line and $H_8(z)$ filtered mean line are shown in Figs. 3b and 3c, respectively. At the two ends of the profile, totaling 2×8×181=2896 data points, the $H_8(z)$ filtered mean line cannot be carried out. These are the "end effects" of the filter $H_8(z)$.

We have installed the H_8 approximation into a BASIC program for instrument control of a system for measuring sinusoidal roughness standard reference materials [15] and for analysis of the measured surface profiles. The QBASIC code for the filter is given in the Appendix. We have performed numerical tests of the filter transmission characteristic for $\lambda_c = 0.8$ mm as a function of spatial frequency of the sinusoidal input ranging from 25 mm to 0.08 mm. The results, some of which are shown in Fig. 1b, indicate a maximum error of 1.05% in the amplitude transmission characteristic for the selected spatial frequencies and excellent agreement with the theoretical error function for H_8. The system, including this new filter, has been used to calibrate the most recent batch of NIST SRM 2071 [16], sinusoidal roughness blocks with nominal Ra of 0.3 μm and nominal spatial wavelength of 100 μm.

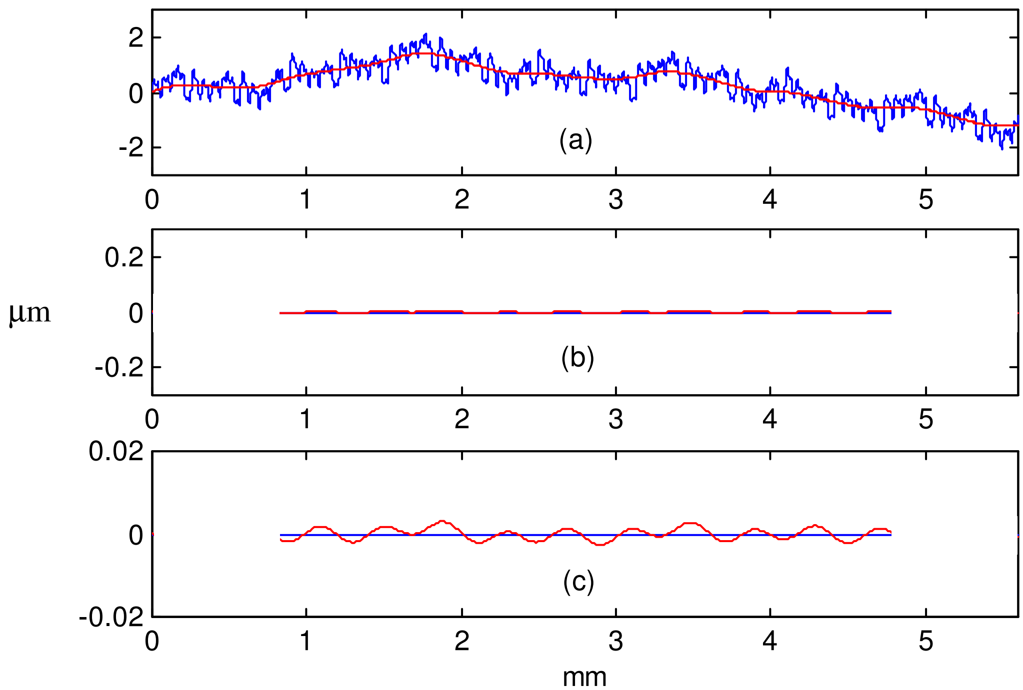

Fig. 3 (a) Surface Profile and Its Filtered Mean Lines;
(b) Difference Curve between the Theoretical Gaussian Filtered Mean Line and the Practical Approximation H_8;
(c) Difference Curve of (b) Amplified further

4. Summary

Based on the relationship between the sampling function $\sin u/u$ and the Gaussian function e^{-u^2}, a new simplified realization method for the Gaussian filter in surface metrology has been set up. The new method has a simple form, high accuracy, and fast computational speed and has no phase distortion. Even in the modestly efficient programming language QBASIC 4.5 using a 486/33MHz computer, less than 5 seconds are required using the H_8 level of approximation for the Gaussian filter to process a surface profile containing 11200 sampling data points.

The new method also shows quantitatively how successive mid-point locus mean line filters approximate the Gaussian filter. As shown by its amplitude transmission characteristic, the mid-point locus mean line filter is the lowest order approximation to the Gaussian filter. In other words, many cascades of the mid-point locus mean line filter can realize the Gaussian filter in surface metrology. The Gaussian filter approximation method discussed in the ISO 11562-1996 and ASME B46.1-1995 standards is composed of two cascades of the mid-point locus mean line filter.

One version of the new approach has been implemented in an existing calibration system for surface roughness measurement.

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Appendix: Example of a QBASIC Program for the Gaussian Approximation Filter $H_8(z)$

The Gaussian filtering algorithms above have been programmed in QBASIC for our purposes. It is also easy to write down the programs using C language. The QBASIC program for the filter $H_8(z)$ is as follows:

```
DIM X(11200), R(11200), M(11200)                : X(11200) measured profile
    K=181                                       : Filtering constant for H8(z)
    FOR I=1 TO 11200
        M(I)=X(I)
    NEXT I
    FOR N=1 TO 4                                 : Filtering
        J=K+1
        R(J)=0
        FOR I=J-K TO J+K
            R(J)=R(J)+M(I)
        NEXT I
        FOR J=(K+1)+1 TO 11200-(K+1)
            R(J)=R(J-1)+M(J+K)-M(J-K-1)
        NEXT J
        J=K+1
        M(J)=0
        FOR I=J-K TO J+K
            M(J)=M(J)+R(I)
        NEXT I
        FOR J=(K+1)+1 TO 11200-(K+1)
            M(J)=M(J-1)+R(J+K)-R(J-K-1)
        NEXT J
    NEXT N
    FOR I=1600 TO 11200-1600
        M(I)=M(I)/(2*K+1) /(2*K+1) /(2*K+1) /(2*K+1)
        M(I)=M(I)/(2*K+1) /(2*K+1) /(2*K+1) /(2*K+1)  : M(I) filtered mean line
        R(I)=X(I)-M(I)                               : R(I) roughness profile
    NEXT I
END
```