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# AN APPLICATION OF PARAMETER ESTIMATION THEORY IN LOW FREQUENCY ACCELEROMETER CALIBRATION

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Low frequency accelerometers, velocity transducers, and seismometers are used extensively to investigate vibrations on mechanical structures. The measurement of low frequency (1-100 Hz), low amplitude (<10 mV) signals from these transducers has been a problem for conventional data acquisition systems. This paper describes a system for low-frequency transducer measurements which first digitizes the voltage signal utilizing a commercial high speed digitizer. Software routines developed at NBS for a desktop computer then estimate the rms amplitude, dc offset, and any distortion components in the transducer signal.

Using a software based system with a high-speed sampling voltmeter provided great flexibility in developing a system for this particular application. Compared with previous systems for low-frequency vibration measurements at the National Bureau of Standards, the present approach is also more accurate.

## INTRODUCTION

This paper describes a useful extension to a system for calibrating accelerometers in the frequency range 1 Hz to 200 Hz. The system is documented in a recent publication of the ISA [1]. The present paper is a supplement to the ISA paper in that it uses the same experimental setup, except for the data acquisition system and the signal processing software. The emphasis of this paper is on the new data acquisition hardware and software. The system uses a high-speed dc digital voltmeter for measurements of low-frequency ac transducer signals. Because of its high sampling rate and IEEE interface capability, this voltmeter is ideal for a computer-controlled low-frequency voltage measurement system.

## DESCRIPTION OF THE CALIBRATION SYSTEM

The calibration system uses a fringe-counting interferometer for displacement measurement. The interferometer shown in fig 1 consists of the laser light source, two beam splitters mounted directly on the head of the laser and a 1/2 inch retro-reflector mounted on the shaker table. The interferometer measures the displacement of the shaker table with reference to the beam-splitters mounted on the laser. The light emerging from the interferometer produces interference fringes on the photo-detector. The laser has a wavelength, $\lambda$, of 632.8 nm and the detector is a silicon photo-diode with a signal amplifier (gain of 20) having a bandwidth of approximately 1 kHz to 3 MHz. The counter measures the number of fringes corresponding to the shaker displacement amplitude [2]. The acceleration is given by eq (1), where f is the vibration frequency, n is the number of fringes/cycle , d is the displacement amplitude in meters, and g is the standard acceleration of free fall, 9.80665 m/s$^2$.

$$a = (2\pi f)^2 d/g = \lambda n \pi^2 f^2/2g \qquad (1)$$

The sensitivity of the accelerometer is calculated by:

$$S = E/a \qquad (2)$$

where E is the amplitude of the voltage output from the accelerometer. The present paper describes the procedure used to measure E.

## LOW-SIGNAL-LEVEL MEASUREMENTS

One difficulty in obtaining accurate readings of accelerometer signals encountered in low-frequency measurements has been the low voltages produced. A typical accelerometer sensitivity is nominally 10 mV/g. For a 2-Hz signal and a displace-

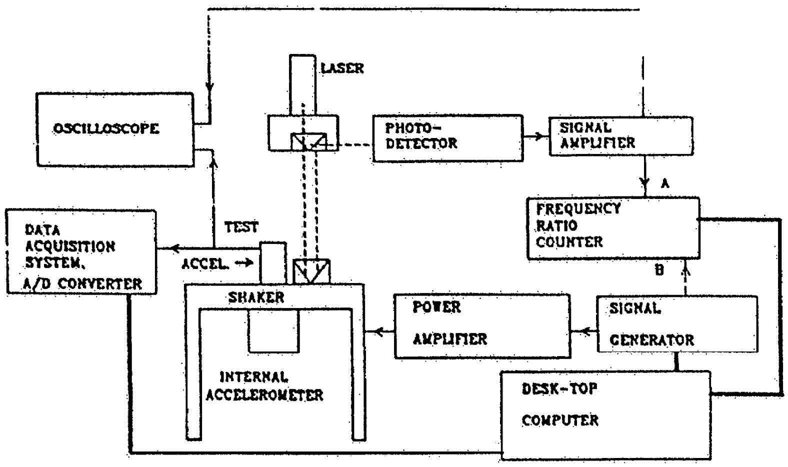


FIGURE 1. FRINGE-COUNTING INTERFEROMETER CALIBRATION SYSTEM

ment of one inch, a signal of approximately 2 mV is produced. This low voltage is difficult to measure for most commercial ac digital voltmeters. But it is often desirable to make measurements at much lower voltages since most low-frequency accelerometers can respond to micro-meter displacements. Table 1 gives voltages corresponding to several displacements, at 2 Hz for an accelerometer of 10 mV/g nominal sensitivity.

In the following section outlining the parameter estimation procedure, it is shown how accurate measurements can be obtained at the displacement levels given in the above table, even for accelerometers with low sensitivity.

TABLE 1

Displacement and Corresponding Acceleration and Voltage at 2 Hz for a 10 mV/g Accelerometer

| Displacement double amplitude mm | inch (approx.) | Acceleration amplitude g | Voltage amplitude mV |
|---|---|---|---|
| 25 | 1 | 0.2 | 2.0 |
| 10 | 0.39 | 0.08 | 0.8 |
| 5 | 0.20 | 0.04 | 0.4 |
| 2 | 0.08 | 0.016 | 0.16 |

## PARAMETER ESTIMATION ALGORITHM

The algorithm developed for processing the noisy low-frequency signals in the present application is a special case of a more general parameter estimation problem. Only an outline of the special-case theory will be given here; the ramifications of the bigger problem will be discussed in a later paper. It is planned also at that time to compare the results obtained by several approaches, including synchronous detection, Fourier analysis, and maximum likelihood detection.

For this application the task is to estimate the amplitude $\sqrt{(A_1^2+B_1^2)}$ of a sinusoidal signal from data $x(t)$ which may contain noise $\eta(t)$ and a dc offset $A$.

$$x(t) = A + A_1 \cos\omega t + B_1 \sin\omega t + \eta(t) \quad (3)$$

The angular frequency $\omega$ is presumed to be known and the noise is assumed to have a Gaussian distribution with mean $\eta(t) = 0$ and a constant variance

$$\overline{\eta^2(t)} = \sigma^2 . \quad (4)$$

Following [3], we define

$$\varphi_1(k\Delta t) = \cos\omega k\Delta t - \frac{\sin[(N+1/2)\,\omega\Delta t]}{(2N+1)\,\sin(\omega\Delta t/2)} \quad (5)$$

and $$\varphi_2(k\Delta t) = \sin\omega k\Delta t \quad (6)$$

where $\Delta t$ is the sampling period, and $k$ is an integer.

For $2N+1$ samples taken uniformly over a time interval $[-L,L]$,

$$x_k = x(k\Delta t) = A_0 + A_1\,\varphi_1(k\Delta t) + B_1\,\varphi_2(k\Delta t) + \eta(k\Delta t) \quad (7)$$

where $k = -N, \ldots, -2,-1,0,1,2, \ldots N$,

$$\Delta t = L/N \quad (8)$$

and

$$A_0 = A + A_1 \frac{\sin[(N+1/2)\,\omega\Delta t]}{(2N+1)\,\sin(\omega\Delta t/2)} . \quad (9)$$

Applying the method of maximum likelihood [4] to the sampled function $x_k$, the following formulas for the estimates $A_0^*, A_1^*$, and $B_1^*$ can be derived:

$$A_0^* = \frac{1}{2N+1} \sum_{k=-N}^{N} x_k \quad (10)$$

$$A_1^* = \frac{\sum_{k=-N}^{N} x_k\,\varphi_1(k)}{\sum_{k=-N}^{N} \varphi_1^2(k)} \quad (11)$$

$$B_1^* = \frac{\sum_{k=-N}^{N} x_k\,\varphi_2(k)}{\sum_{k=-N}^{N} \varphi_2^2(k)} \quad (12)$$

where $\varphi_1(k)$ and $\varphi_2(k)$ are calculated from (5) and (6). The estimated dc offset is calculated from eq (9), using the estimated values of $A_0$ and $A_1$:

$$A^* = A_0^* - A_1^* \frac{\sin[(N+1/2)\,\omega_1 L/N]}{(2N+1)\,\sin(\omega_1 L/2N)} . \quad (13)$$

Using the estimates given by eqs. (11), (12), and (13) the defining eq (7), may be rewritten as

$$x(kL/N) = A^* + E^*\cos(\omega t + \arctan(-B^*_1/A^*_1)) + \eta(kL/N) \quad (14).$$

where the amplitude of the ac voltage is

$$E^* = \sqrt{[(A_1^*)^2 + (B_1^*)^2]} \quad (15)$$

Using eq (14), the signal waveform can be reconstructed from the estimated parameters and the residue can be be tested for randomness.

The noise can be calculated from eq(4) and eq(7):

$$\sigma^* = \Big(\frac{1}{2N+1} \sum_{k=-N}^{N} [\,x_k - A_0^* - A_1^*\varphi_1(k) - B_1^*\varphi_2(k)\,]^2\Big)^{1/2} \quad (16)$$

Without any additional measurements we can compute confidence limits for the estimated parameter values. In the present application, a number of useful simplifications are possible because of the following conditions: (1) the number of samples is very large, $(2N+1) \gg 1$, and (2) the length of the record covers two cycles of the sinewave, that is, $L=2\pi/\omega$. The most important consequences of these conditions are that the t-distribution is closely approximated by the Gaussian distribution; the variances of $A_1^*$ and $B_1^*$ are approximately equal, and all results are identical with those obtained on the basis of a least-squares fit [5].

Thus, the confidence interval for $A^*$ becomes

$$[A^* \pm t_\alpha \sigma^*/\sqrt{(2N)}]. \tag{17}$$

where $t_\alpha$ is the value of

$$t = \frac{A^* - A}{\sigma^*/\sqrt{(2N)}},$$

corresponding to the confidence coefficient $(1-\alpha)$.

Similarly, according to reference [6] the approximate confidence interval for the function $E^*$ can be computed as follows:

$$E^* \pm t_\alpha \left[ \left(\frac{\partial\sqrt{(A_1^2+B_1^2)}}{\partial A_1}\right)^2 s^2(A_1) + \left(\frac{\partial\sqrt{(A_1^2+B_1^2)}}{\partial A_2}\right)^2 s^2(B_1^*)\right]^{1/2} \tag{18}$$

where $s^2(A_1^*)$ and $s^2(B_1^*)$ are the variances of $A_1^*$ and $B_1^*$ respectively. Since for integral number of cycles and N large, $s(A_1^*) \approx s(B_1^*) \approx \sigma^*/\sqrt{(2N)}$ [3], this expression reduces to

$$E^* \pm t_\alpha \sigma^*/\sqrt{(2N)} . \tag{19}$$

Remembering that $A_1$ and $B_1$ represent peak values and $\sigma^*$, a rms value, a "relative" confidence interval can be stated in the following simple, but meaningful, form:

$$1 \pm \frac{t_\alpha}{\sqrt{(N)}} / \frac{Signal_{RMS}}{Noise_{RMS}} . \tag{20}$$

Figure 2 shows a family of confidence limits computed from this expression.

## APPLYING THE ALGORITHM TO ACCELEROMETER CALIBRATIONS

This measurement technique was applied to accelerometer calibration at low frequencies (1-200 Hz) by the use of a high-speed voltmeter which can sample as fast as 100,000 voltage samples/second and ranges automatically down to 40 mV full scale with a resolution of 10 $\mu$V. The output from the accelerometer under test is connected to the sampling voltmeter. This voltmeter under computer control can be programmed to select the sampling rate and number of samples. The captured data samples are transferred to a desk-top computer for analysis. The voltage of the test accelerometer is calculated by eq (15) given above. Also the dc-offset voltage and the noise are calculated by equations (13) and (16) respectively.

The ability of the parameter estimation algorithm to extract small sinusoidal signals from noisy transducer signals provides a means of measuring much lower acceleration levels than previous measurement systems.

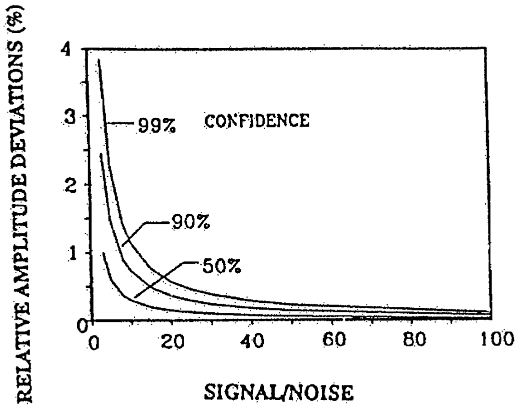


FIGURE 2. CURVES FOR CALCULATING CONFIDENCE LIMITS

## ESTIMATING ERRORS

In order to check the functional relationships shown in figure 2, a computer program was written in Pascal to simulate typical data and to calculate the expected errors. The program generated a sinusoidal waveform with random noise, dc-offset, and phase shift according to eq (14). The algorithm was then used to calculate the sinusoidal voltage component, the dc-offset, and the signal to noise ratio.

The example shown in figure 3 is a computer generated 1 volt, 2-Hz sinusoidal signal with added random noise. The voltage corresponding to the sinuscidal 2 Hz component was computed from eq. 15 and the noise from eq. 16. A sample run consisted of 1001 digitized data points (two cycles) for a 2-Hz, 1-volt signal, using

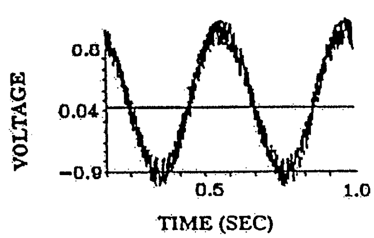


FIGURE 3. COMPUTER GENERATED SIGNAL WITH NOISE AND OFFSET

a phase of 25 deg and a dc-offset of 10 percent. Five hundred such sample runs were taken and their estimated voltages were compared to the original 1 Volt. The number of voltages outside 5%,4%, 3%, 2%, 1%, 0.5%, 0.35, 0.2%, and 0.1% was recorded in each case. By introducing differing amounts of random noise into the simulated signal, a set of estimated voltage-amplitude deviations was computed for a wide range of S/N ratios. Figure 4 shows a family of errors-versus-confidence curves for 3 values of S/N ratios computed from eq (20). The results of the simulated tests just described are also shown in figure 4. For example, for a S/N = 3, the number of sample runs whose estimated voltage lies outside a 3% band was 20 out of 500 runs. This is plotted as a 3% relative amplitude deviation at a 96% confidence level.

Table 2 shows data for an accelerometer with a nominal sensitivity of 500 mV/g, calibrated at 10 Hz over a wide acceleration range. The sensitivities shown are the average of 25 runs ( of 1001 samples each).

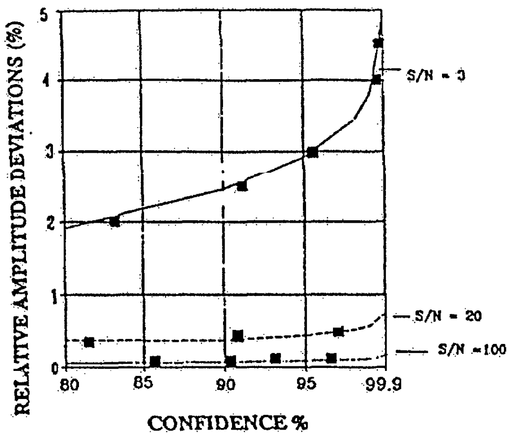


FIGURE 4. CONFIDENCE LIMITS FOR SOME TYPICAL S/N RATIOS

TABLE 2

Calibration Data for a 500 mV/g Accelerometer at 10 Hz

| Peak Acceleration (g) | Measured Sensitivity ( V/g) | Fringes/ cycle (approx.) | Peak(mV) Voltage (approx) | Sig/ Noise (approx) | Voltage Error (%) Expected |
|---|---|---|---|---|---|
| 0.96 | 0.5254 | 30,000 | 504 | 190 | <0.1 |
| 0.48 | 0.5255 | 15,000 | 252 | 480 | <0.1 |
| 0.10 | 0.5256 | 3280 | 52 | 335 | <0.1 |
| 0.06 | 0.5258 | 1820 | 30 | 300 | <0.1 |
| 0.004 | 0.5243 | 120 | 2 | 20 | 0.3 |
| 0.0006 | 0.5103 | 18 | 0.3 | 3 | 1.5 |

The expected voltage error in table 2 was estimated from figure 2. Not only the error in estimating the voltage signal, but also the low fringe count contribute to the uncertainty of the sensitivity measurement in the lowest acceleration shown in table 2. The frequency counter is set to count over approximately a 10-s interval to minimize the errors due to a small fringe count. The lowest acceleration shown in table 2 (0.0006 g) would not be considered a valid calibration and is shown only to illustrate the errors involved in measurements at this extreme.

In order to test the advantages of averaging, twenty five simulated sample runs (of 1001 digitized data points) were taken and the average voltage and S/N ratio were computed for the 25 runs. Ten such sets of 25 runs each were computed and the errors shown in figure 5 were calculated from the maximum spread in the voltages of the ten runs. By introducing

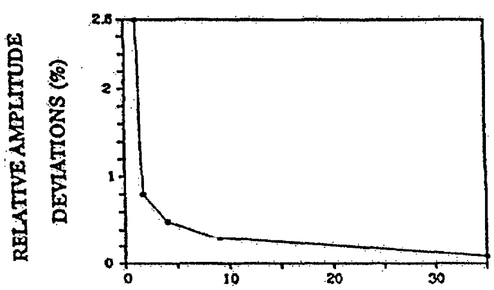


FIGURE 5. EXPECTED DEVIATIONS USING SIGNAL AVERAGING

Table 3
Effect of Averaging on Expected Error

| Accelerometer Sensitivity | Acceleration g/1000 | Voltage mV | S/N | Expected Voltage Error No Averaging | Expected Voltage Error With Averaging |
|---|---|---|---|---|---|
| 500 mV/g | 4 | 2 | 20 | 0.3 | 0.2 |
| | 0.6 | 0.3 | 3 | 1.6 | 0.6 |
| 10 mV/g | 30 | 0.3 | 5 | 1.5 | 0.5 |
| | 80 | 0.8 | 6 | 1.2 | 0.4 |

differing amounts of random noise a set of errors was calculated for a wide range of S/N ratios. Examples were also computed using different phases and offsets. Varying the phase and offset did not change the general results shown in figure 5. Other examples using smaller voltage amplitudes down to less than 0.5 mV were computed to test the calculations for round-off errors. The results for smaller voltages are similar to the data in figure 5.

Table 3 shows data for the the accelerometer of Table 2 and also an accelerometer of 10 mV/g using averaging of 25 sets of data.

## SUMMARY

A low-frequency calibration system for accelerometers has been developed which uses a fringe-counting interferometer and a high-speed digital acquisition system for accurate low-frequency, low-voltage measurements. The low-voltage measurements are possible due to the use of a parameter estimation algorithm which calculates the sinusoidal voltage output of a transducer in the presence of noise and dc-offset voltage. Confidence limits have been calculated for measurements in the presence of constant offsets and random Gaussian noise. Some examples of the type of measurements possible using the parameter estimation are the following (for an accelerometer of 500 mV/g sensitivity). Accelerations of 0.004 g were measured with a 90% confidence limit, with a voltage error estimate of 0.3% (S/N=20). By signal averaging of 25 sets of data, the error estimate can be reduced to 0.2%. Similarly, for a 10 mV/g accelerometer, an acceleration of 0.08 g was measured at a 90% confidence limit with 1.2% estimated error (S/N=6), which reduces to 0.4% after signal averaging.

## ACKNOWLEDGEMENTS

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