

**Reassessing methods to estimate population size
and sustainable mortality limits for the
Yellowstone Grizzly Bear
Workshop Document Supplement¹
19–21 June 2006**

This supplement is the result of a Workshop held at the AMK Ranch in Grand Teton National Park, 19–21 June 2006. The purpose of this workshop was to establish the scientific rationale and conduct additional analyses needed to adequately address concerns and issues raised by professional peer reviews and by the general public during the public comment period of the original document *Reassessing Methods to Estimate Population Size and Sustainable Mortality Limits for the Yellowstone Grizzly Bear* (Interagency Grizzly Bear Study Team [IGBST] 2005). We do not address all comments expressed during the public review period explicitly in this document because those have been addressed in a separate document titled *Responses to Public Comments on the Reassessing Methods Document* and are available online at <http://mountain-prairie.fws.gov/species/mammals/grizzly/yellowstone.htm>.

Items addressed here focus on 2 issues: (1) the wide variation about the original method proposed to index population size using annual estimates of females with cubs of the year as derived from the Chao2 estimator ($FCOY_{Chao2}$), and (2) the uncertainty about the estimate of independent females, independent males, and dependent young in the population.

Professional peer reviewers expressed concern about the wide swings in the index of population size using annual counts derived from estimates of FCOY and the use of a constant in the denominator when extrapolating $FCOY_{Chao2}$ to an index of independent females, independent males, and dependent young. In the original *Reassessing Methods* document, the group rejected using a running average over multiple years to address the variability about the annual population indices because of “possible unknown statistical biases” (IGBST 2005:25). Instead, we chose to smooth the mortality limit provided to managers “to dampen variability and provide managers with inter-annual stability in the threshold.” Consequently, we recommended that allowable mortality limits be based on a 3-year running average derived from the annual index of population size (IGBST 2005:7–8).

We anticipated that the normal process (biological) variation associated with grizzly bear reproduction in the Greater Yellowstone Ecosystem (GYE) would result in wide swings in counts of FCOY and the resultant $FCOY_{Chao2}$ estimate (see Schwartz et al. 2006a:20, Figure 6). Female bears tend to produce litters in the year following an autumn with highly abundant naturally occurring autumn foods. Hence, using a constant

¹ This document is the product of team work. Participants from the original workshops contributed to its production. Please cite as follows: Interagency Grizzly Bear Study Team. 2006. Reassessing methods to estimate population size and sustainable mortality limits for the Yellowstone grizzly bear: workshop document supplement. U.S. Geological Survey, Northern Rocky Mountain Science Center, Montana State University, Bozeman, Montana, USA.

in the denominator to extrapolate $FCOY_{Chao2}$ to index independent females, independent males, and dependent young failed to remove this process variation.

After considerable discussion, the group concluded that it was more appropriate to use $FCOY_{Chao2}$ as an initial estimate of FCOY. This was used along with all the data and information-theoretic model selection methods (Burnham and Anderson 2002) to select the best model for estimation of FCOY. We considered both linear and quadratic models and model averaging of the $FCOY_{Chao2}$. Model averaging has the effect of putting the numerator (model averaged estimates of number of FCOY) on the same temporal scale as the denominator (mean transition probability derived from 1983–2003) based on previous work (IGBST 2005:60–65) and thus addresses concerns about process variation causing wide swings in population estimates. The model averaging method and its application are presented in the following sections.

Estimation of number and trend for females with cubs of the year

The Chao2 estimator (Chao 1989, Keating et al. 2002, Cherry et al. 2007) is used annually to estimate the number of females with cubs of the year ($FCOY_{Chao2}$) for year i . For convenience, we will change notation and define \hat{N}_i to be the value of $FCOY_{Chao2}$ in year i . The trend in this segment of the population and its rate of change (λ) can also be estimated from these annual estimates. Although the Chao2 estimator accounts for sampling heterogeneity, annual estimates of FCOY can vary because of sampling error (sampling variance) associated with the annual estimates, and because of pulsed or synchronized reproductive output by a segment of the female population (process variance). Consequently, using each annual estimate independently each year can result in wide swings in the estimate of total population size, producing results that may be inconsistent with expected changes in true population size, which complicates management. This annual variability was criticized during professional peer review. Therefore, we investigated methods to smooth these potential swings.

Methods

Monitoring numbers and λ using females with cubs. We fit the natural logarithm of the number of females with cubs [$\log(\hat{N}_i)$] with a linear model of year (y_i):

$$\log(\hat{N}_i) = \beta_0 + \beta_1 y_i + \varepsilon_i$$

so that the population size at time zero is estimated as $\hat{N}_0 = \exp(\hat{\beta}_0)$. An additional benefit of this model is that it allows (under reasonable assumptions) estimation of the rate of population change (λ) as $\hat{\lambda} = \exp(\hat{\beta}_1)$, giving $\hat{N}_i = \hat{N}_0 \hat{\lambda}^{y_i}$. Confidence intervals on λ can be estimated as the exponential of the confidence bounds on β_1 , providing an asymmetric confidence bound. Standard errors and confidence intervals for $\log(\hat{N}_i)$ can be computed with the usual linear model methods, and confidence intervals for N_i can be estimated as the exponential of the confidence bounds on $\log(N_i)$.

Changes in the numbers of FCOY are representative of the rate of change of the entire population, but with additional process variation coming from the proportion of the female population that has cubs of the year (COY). Thus, random noise of \hat{N}_i is coming

from both sampling variation from the Chao2 estimator and the proportion of the population with COY. When we assume a reasonably stable age and sex structure for the total population, the model provides an estimate of λ , which represents the rate of change of the entire population and a modeled estimate of FCOY for the current year. Fitting a linear relationship makes the standard assumptions of least squares regression.

Quadratic regression can be used to detect a change in $\hat{\lambda}$ (i.e., the slope of the log-linear model) through time. We fit the model

$$\log(\hat{N}_i) = \beta_0 + \beta_1 y_i + \beta_2 y_i^2 + \varepsilon_i,$$

and the estimate of β_2 provides a metric for assessing whether λ has changed through time. We expect that the estimate of β_2 will become negative as the population reaches carrying capacity and λ approaches 1. Information-theoretic model selection methods (Burnham and Anderson 2002) can be used to select between the linear and quadratic models, and hence to detect changes in $\hat{\lambda}$ and \hat{N}_i as additional data are collected. We used model averaging with the linear and quadratic models of the predicted population sizes of females with cubs to estimate population sizes through time (i.e., \hat{N}_i), and thus smooth the variation of the Chao2 estimates. We used Akaike's information criterion weights corrected for small sample size (AIC_c ; Burnham and Anderson 2002) to weight the estimates from the linear and quadratic models to produce our best estimate of the current number of females with cubs and λ .

Power analysis of using \hat{N} to estimate λ . To assess the behavior of our proposed model selection procedure, we (i) added 2 hypothetical years of data for 2006 and 2007, assuming $\lambda = 0.9$ for both additional years, and (ii) added 4 hypothetical years of data, assuming $\lambda = 1.0$ for all additional years. In other words, we assumed that λ was equal to 0.9 for 2006 and 2007, or λ was 1.0 for 4 consecutive years.

Simply adding hypothetical years with altered λ , as above, would not constitute a power analysis of the proposed trend monitoring method, because future years' data will also contain process and sampling variation. To estimate the power of these data to detect a true reduction in λ (i.e., correctly choose the quadratic model), we estimated variance components of the Chao2 estimates from 1983–2005 and applied these in Monte Carlo projections for 10 additional years under assumed values of λ .

To separate sampling variance associated with each population estimate, ($\text{var}(\hat{N}_i)$) from process variance, we fit the linear model (above), assuming that the error term ε_i was the sum of the sampling variance and process variances (earlier analyses provided no evidence for significant serial correlation; unpublished data). For the Chao2 estimator, $\text{var}(\hat{N}_i)$ was estimated with bootstrap resampling of the data, and the variance of the resampling distribution was the estimate of $\text{var}(\hat{N}_i)$. Note that the variance of $\log(\hat{N}_i)$ is estimated, using the delta method, as $\text{var}(\log(\hat{N}_i)) = \text{var}(\hat{N}_i) / \hat{N}_i^2$.

To estimate the process standard deviation from the 1983–2006 Chao2 estimates, we used PROC NLMIXED in SAS. This procedure maximizes the likelihood of $\log(\hat{N}_i)$ for β_0, β_1 , and the process SD, with the likelihood specified as a normal distribution with mean predicted by $\log(\hat{N}_i) = \beta_0 + \beta_1 y_i$ and variance

$\text{var}(\log(\hat{N}_i)) + (\text{Process SD})^2$. This model thus explicitly includes the sampling variance of $\log(\hat{N}_i)$ plus the process variance that is estimated by the procedure. Process SD was estimated to be 0.176 with SE 0.0461 and 95% confidence interval 0.0808–0.271

To estimate the expected sampling variance of future Chao2 estimates (which assumes that future sampling effort will remain approximately the same as used to collect the 1983–2006 data), the mean of the sampling variances of the log population estimates for the 1983–2006 data was computed. The sampling variance of future Chao2 estimates was sampled from a normally distributed population with mean zero and standard deviation equal to the square root of mean sampling variance. From this procedure, the estimated sampling standard deviation was 0.34.

To evaluate sensitivity of the linear and quadratic models to changes in \hat{N} over 1 to 10-year intervals, we projected forward the 2006 population estimate of $N_{2006} = 52.356$ (obtained by model averaging the linear and quadratic model estimates from the 1983–2006 data), assuming alternative λ values of 0.95, 0.975, 1, 1.025, and 1.05, and using our estimates of process and sampling variation (above). Population size for each succeeding year was generated with the recursive relation $\log(N_{i+1}) = \log(N_i) + \log(\lambda) + \delta_i$, where the process variation was added as δ_i , a normally distributed random variable with mean zero and standard deviation of 0.176. The estimated population size (corresponding to the Chao2 estimates) was taken as $\log(N_{i+1}) + \varepsilon_{i+1}$, where the sampling variation ε_{i+1} was added as a normally distributed random variable with mean zero and standard deviation of 0.34. Each replicate was simulated independently (i.e., new data were added to the 1983–2006 data for each simulation).

One thousand replicates of each of the 50 scenarios (5 alternative $\lambda \times 10$ alternative time-frames) were generated, from which we estimated the mean AIC_c weight of the quadratic model, the proportion of iterations in which the quadratic term was selected (weight > 0.5), and the power of the t -test to reject the null hypothesis that the quadratic term was equal to zero. This realistically simulated the data and analyses managers would have available to them to make decisions about whether the true population had changed its trajectory.

Results

Monitoring numbers and λ using females with cubs. Data for 1983–2005 (Table 1) were used to estimate the rate of population change (Figure 1). The parameter estimates and AIC_c weights for the linear and quadratic models (Table 2) suggest that only the linear model was needed to model changes in the $FCOY_{\text{Chao2}}$ population during this period. The estimate of λ using the linear model was 1.0479 with 95% confidence interval of 1.031 to 1.065 and was quite close to the independent estimates of Harris et al. (2006:48) using data from radiocollared bears (mean estimates of 1.04 or 1.07 under slightly different assumptions). The estimated quadratic effect (-0.00071104 , SE = 0.00133) was not significant ($P = 0.6$), with 79% of the AIC_c weight associated with the linear model. Thus, the linear model was the best approximating model for 1983–2005, but we also provide the model averaged estimates (Figure 1).

Table 1. Observations of females with cubs of the year (FCOY) in the Greater Yellowstone Ecosystem, 1983–2005, where m is the number of unique individuals observed after n samples and f_j is the number of individuals observed 1 or 2 times. The annual and modeled estimates (1983–2005) of $FCOY_{Chao2}$ are also provided.

Year	n^a	m^a	Sighting frequency		Chao2 estimate	
			f_1	f_2	Annual	Modeled
1983	12	10	8	2	19.33	18.46238
1984	40	17	7	3	22.25	19.40793
1985	17	8	5	0	18.00	20.39578
1986	82	24	7	5	27.50	21.42746
1987	20	12	7	3	17.25	22.50457
1988	36	17	7	4	21.20	23.62873
1989	28	14	7	5	17.50	24.80158
1990	49	22	7	6	25.00	26.02483
1991	62	24	11	3	37.75	27.30021
1992	37	23	15	5	40.50	28.62948
1993	30	18	8	8	21.11	30.01446
1994	29	18	9	7	22.50	31.45699
1995	25	17	13	2	43.00	32.95893
1996	45	28	15	10	37.55	34.52222
1997	65	29	13	7	38.75	36.14879
1998	75	33	11	13	36.93	37.84063
1999	96	30	9	5	36.00	39.59974
2000	76	34	18	8	51.00	41.42819
2001	84	39	16	12	48.23	43.32803
2002	145	49	17	14	58.07	45.30139
2003	54	35	19	14	46.40	47.35039
2004	202	48	15	10	57.55	49.47720
2005	86	29	6	8	30.67	51.68401

^aValues differ from Keating et al. (2002) because we included females throughout the Greater Yellowstone Ecosystem. Only observations made without the benefit of radiotelemetry are included.

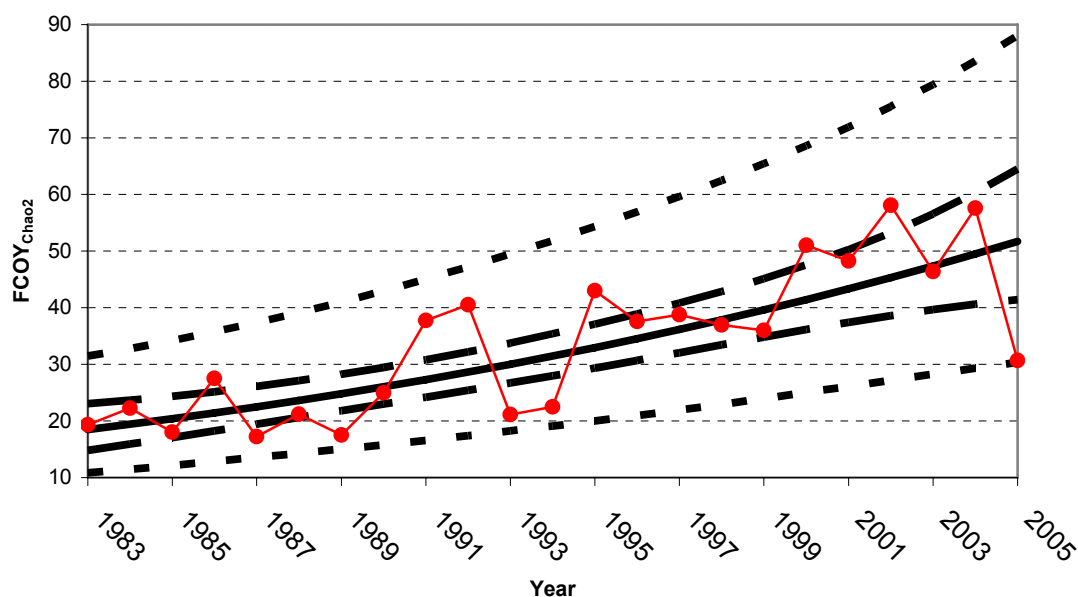


Figure 1. Model-averaged estimates of $FCOY_{Chao2}$ for 1983–2005, where the linear and quadratic models of $\log(FCOY_{Chao2})$ were fitted. The inner dashed lines represent a 95% confidence interval on the predicted population size, whereas the outer dashed lines represent a 95% confidence interval for individual population estimates. The red dotted line represents number of unique FCOY observed.

Table 2. Estimates and model selection results from fitting the $FCOY_{Chao2}$ population estimates from the Chao2 model, 1983–2005.

Model	Parameter	Estimate	Standard error	t	$Pr(>t)$
Linear					
	β_0	2.88051	0.10628	27.10	<0.0001
	β_1	0.04679	0.00775	6.04	<0.0001
	SSE ^a	1.27685			
	AIC _c	-59.2320			
	AIC _c weight	0.78870			
Quadratic					
	β_0	2.80941	0.17165	16.37	<0.0001
	β_1	0.06386	0.03295	1.94	0.0669
	β_2	0.00071104	0.00133	-0.53	0.5997
	SSE	1.25895			
	AIC _c	-56.5978			
	AIC _c weight	0.21130			

^aSum of squared errors.

Power analysis of using \hat{N} to estimate λ . When 2 years with $\lambda = 0.9$ were added to these data, the resulting quadratic model had an AIC_c weight of 0.67847 and an estimated quadratic effect of -0.0028 (SE = 0.0012) that differed from zero ($P = 0.03$). Thus, had the Chao2 counts declined by 10% each year, our model selection would have detected this fundamental change within 2 years. Two years would not have been sufficient to detect a change to stationary Chao2 counts (Table 3), but by the third year, model weights would have shifted to favor the quadratic model, suggesting that population growth had stopped.

Table 3. Behavior of linear and quadratic models of population growth assuming identical Chao2 estimates following 2005, showing AIC_c weights (w_i) for the linear and quadratic models and P values for the quadratic term in the quadratic model.

Years of Chao2 estimates identical to 2005 values	Linear model w_i	Quadratic model w_i	Quadratic term P
2	0.73241	0.26759	0.1902
3	0.46623	0.53377	0.0561
4	0.20702	0.79298	0.0168
5	0.07439	0.92561	0.0053

When our best estimates of process and sampling variation were added to hypothetical years 1 through 10, approximately 5 years were required of the population decreasing 5% yearly (i.e., $\lambda = 0.95$) before the preponderance of evidence (AIC_c weight > 0.5) favored the quadratic model (i.e., fundamental change in state from linear increase, Figure 2). Under the scenario in which population size stabilized after year 2006 (i.e., $\lambda = 1.0$), 7 or 8 years were required for the preponderance of evidence to favor the quadratic model (depending on the criterion used, Figure 3). Power to detect a yearly decline of 2.5% was intermediate between these 2 examples. Power was lower to detect changes in λ to 1.025 or 1.05 (unpublished data), but this was neither unexpected nor worrisome under the baseline linear estimate of λ of 1.0479.

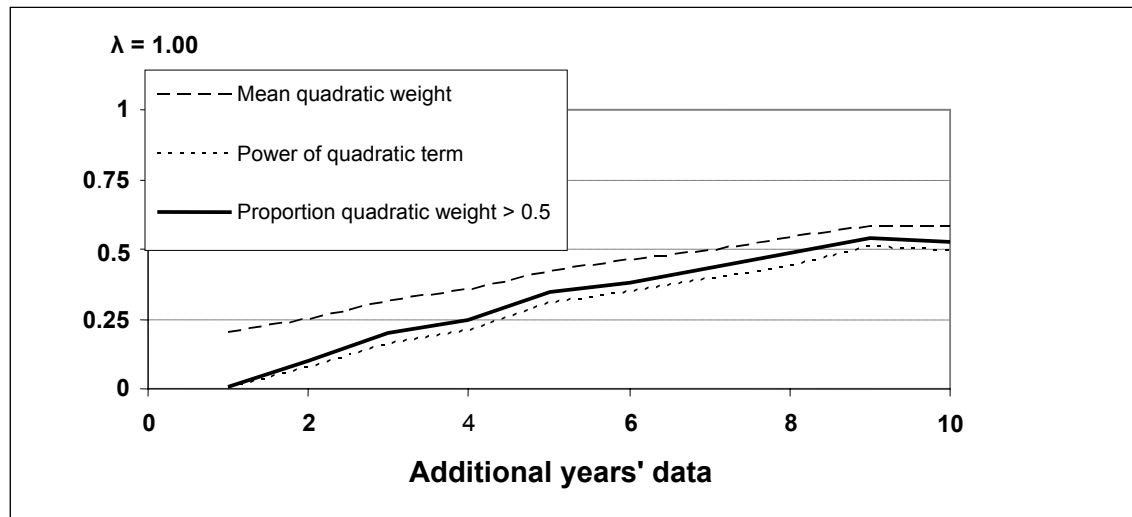


Figure 3. Mean AIC_c weight of the (negative) quadratic term, proportion of simulations in which the quadratic model had greater AIC_c weight than the linear model, and power of the quadratic term (i.e., probability of rejecting the linear model) when expected λ changed to 1.0 following the 1983–2006 series of estimates of females with cubs, for additional years 1 to 10 and using estimates of process and sampling variation from the data.

Discussion

FCOY are the critical segment of the population driving reproduction. Thus, we appropriately use all the data to estimate the number of FCOY each year and the rate of change of this segment as a measure of the rate of change of the entire population. Both reproductive effort and mortality of the entire population are driven by the performance of the FCOY segment.

According to the 1993 Recovery Plan (U.S. Fish and Wildlife Service 1993:20) “[a]ny attempt to use this parameter [FCOY] to indicate trends or precise population size would be an invalid use of these data.” However, subsequent to the drafting of the 1993 Recovery Plan, several researchers developed methods to address varying effort and heterogeneity in sightings of females with cubs of the year, the underpinnings for the above quote. When Knight et al. (1995) published the methods used to distinguish unique females from replicate sighting of the same female and presented a method to estimate trend, there were no methods available to correct for problems of observer effort and sighting heterogeneity. Subsequent to that publication, a number of researchers provided improved methods that address varying effort and heterogeneity of sighting probabilities and use the FCOY index to estimate trend (Eberhardt et al. 1999, Boyce et al. 2001, Keating et al. 2002). The method we recommended is an extension of that research.

Summary of workshop recommendations for grizzly bear monitoring

We propose using the linear and quadratic models as described above to estimate changes in λ over time and the predicted numbers of FCOY as the best estimate of the number of FCOY annually. The results will then be used to estimate the number of

independent females, independent males, and dependent young following procedures outlined in the original *Reassessing Methods Document* (Interagency Grizzly Bear Study Team 2005). We recommend this new weighted model method replace the older method proposed in the *Reassessing Methods Document* that used the annual estimate $FCOY_{Chao2}$.

The new method addresses normal process variation and associated swings in annual counts of FCOY and dampens fluctuations arising from sampling variation because it uses the entire string of data. Details on how the methods will be applied to calculate the index of independent females, independent males, and dependent young are below.

The estimated λ and associated confidence interval demonstrate an increase in the FCOY numbers, and hence the total population. The proposed set of models will also allow managers to detect a decline in λ , and thus recognize when the population is approaching carrying capacity or decreasing. We recommend this method of estimating λ be used as an independent measure of population trajectory that can be compared to estimates derived from data using radiocollared bears as recommended in the *Reassessing Methods Document* (IGBST 2005:42–44).

For future monitoring, we recommend continued monitoring of females with cubs, fitting both linear and quadratic models to the data set, and using AIC_c to evaluate the strength of these competing models. Weight favoring the quadratic term is evidence that population growth has slowed or reversed, but lack of such evidence is not necessarily proof that change has not occurred. Under the best of circumstances, this monitoring protocol leaves uncertainty about the system state during the most recent years. Gradually increasing evidence for the quadratic model over a few years (assuming a negative quadratic slope) should keep biologists and managers alert to a possible change in system state. We recommend continued monitoring of demographic rates from a sample of radiomarked females and their offspring. Although also characterized by variability and time-lags, such monitoring provides an independent measure of population vigor and is likely to be helpful in explaining hypothesized changes in numbers of females with cubs. We recommend that if the AIC_c weight favors the quadratic term (i.e., >0.5) in modeling the rate of change of females with cubs in any year, a full review of the population's demographics be undertaken to better understand its status.

Because we are refitting the model with new data each year, estimates from previous years will change slightly after each iteration. We recognize that this will occur, but do not recommend retrospectively adjusting previous population estimates and accompanying mortality limits. The purpose of the model is to get the best possible estimate of the current number of females with cubs of the year borrowing information from past estimates, recognizing that with each iteration some change is expected.

Occasionally, a dead bear is reported in a year(s) subsequent to the actual year of mortality. We recommend that the IGBST, to the best of their ability, attempt to estimate actual year of death and sex and age of the individual. These mortalities would then be added into the mortality tally for year of death, and mortality totals recomputed (including estimates of unknown and unreported deaths). If adding extra bear(s) retrospectively results in exceeding the threshold in that year, the excess (tallied mortality minus threshold) would be deducted from the current years threshold (i.e., the threshold would be reduced by the difference). For example if a dead bear reported in 2006 died in 2005, that bear (and the estimated unknown and unreported mortality) would be counted

in 2005 and the updated mortality total compared to the 2005 threshold. If the 2005 threshold is exceeded, the difference would be deducted from the current years' threshold.

Establishing confidence intervals around estimates of independent females, independent males, and dependent young

The second issue raised during public and professional peer review of the *Reassessing Methods Document* (Interagency Grizzly Bear Study Team 2005) was the need to display uncertainty around the estimates of independent females, independent males, dependent young, and total population size. Here we detail methods used and present confidence intervals around those estimates.

Methods

We estimated the uncertainty associated with an estimate $\hat{\theta}$ of a parameter θ using a formula derived from the delta method (Seber 1982:7). For estimates of the form

$$\hat{\theta} = \frac{\hat{\beta}_1 \hat{\beta}_2 \dots \hat{\beta}_k}{\hat{\beta}_{k+1} \hat{\beta}_{k+2} \dots \hat{\beta}_n}$$

the variance of $\hat{\theta}$ was approximated by

$$\hat{\text{var}}(\hat{\theta}) = \hat{\theta}^2 \sum_{i=1}^n \text{CV}(\hat{\beta}_i)^2$$

where $\hat{\text{var}}(\hat{\theta})$ is the estimated variance of the index $\hat{\theta}$ (independent females, independent males, cubs, or yearlings). For estimates of the form

$$\hat{\theta} = \hat{\beta}_1 + \hat{\beta}_2 \dots + \hat{\beta}_k$$

the variance of $\hat{\theta}$ was approximated by

$$\hat{\text{var}}(\hat{\theta}) = \sum_{i=1}^n \text{var}(\hat{\beta}_i)$$

where $\hat{\text{var}}(\hat{\theta})$ is the estimated variance of the index $\hat{\theta}$ (dependent young or population size). For both methods used to estimate variance, we assumed that covariances (correlations) of the various inputs were zero because we lacked the ability to determine their structure.

The coefficient of variation for the ratio of females 4 years and older in the population of females 2 years and older (4+ females:2+ females), and the ratio of males 2 years and older in the population of females 2 years and older (2+ males:2+ females) were derived using back-transformed logit normal distributions to model the survival parameters: cub survival, yearling survival, and adult (age 2+) survival. The variable m_x was modeled with a beta distribution so as to reproduce, as nearly as possible, the mean and 95% confidence limits about the mean, as reported in the monograph (Schwartz et al. 2006c). We used the PopTools extension on Excel to run Monte Carlo iterations from all distributions simultaneously, each time. We ran 10,000 iterations for each of the 2 possible mean independent female survival rates (0.922 and 0.950) and 2 possible mean

independent male survival rates (0.874 and 0.823) to generate the expected relationship between the number of 4+ and 2+ females (4+ females:2+ females) and 2+ males and 2+ females (2+ males:2+ females) when stable age distribution was achieved. We used PopTools to convert the life-table formats in the Leslie matrix formats and took age ratios from the eigenvector (i.e., stable age distribution) associated with each iteration. Variation about the ratio of adult females (age 4+) to independent females (age 2+) was derived from these simulations (Table 4). Variation about the ratio of independent males (age 2+) to independent females (age 2+) was derived from a second series of simulations (Table 5). These estimates did not include temporal variation in rates.

For estimating the number of 2+ females based on the estimated ratio of 4+ females:2+ females, and for the estimate of the proportion of 2+ males based on the ratio of 2+ males:2+ females, we used the mean and variance from the assumed dead (AD) estimate rather than the censored (C) estimate because the former included more uncertainty about estimates. Because of the random simulation process, values presented in Tables 4 and 5 differ slightly from the *Reassessing Methods Document* (0.773, 4+ females:2+ females, and 0.605, 2+ males:2+ females). We recommend using the new estimates.

Table 4. Mean, variance, and upper and lower 95% confidence limits around the ratio (4+ females:2+ females) when mean vital rates during 1983–2002 varied randomly. Line AD was when adult survival was estimated assuming all females with unresolved fates died at last contact, line C was when adult survival was estimated censoring unresolved females (as in Haroldson et al. 2006). This ratio provides a way to estimate the number of females older than yearling based on an estimate of the number of females ≥ 4 years old.

	Mean	Variance	Lower CL	Upper CL
AD	0.77699	0.00081	0.72459	0.83546
C	0.78446	0.00075	0.73504	0.84156

Table 5. Mean, variance, and upper and lower 95% confidence limits around the ratio (2+ males:2+ females) when mean vital rates during 1983–2002 varied randomly. Line AD was when adult survival was estimated assuming all adults with unresolved fates died at last contact, whereas line C was when adult survival was estimated censoring unresolved losses (as in Haroldson et al. 2006). This ratio provides a way to estimate the number of independent males older than yearling based on an estimate of the number of females ≥ 2 years old.

	Mean	Variance	Lower CL	Upper CL
AD	0.63513	0.002457	0.528489	0.720547
C	0.61093	0.001992	0.515741	0.691977

Estimates of variation for transition probabilities were presented in the *Reassessing Methods Document* (Interagency Grizzly Bear Study Team 2005:Appendix C, page 62, Table 6). Estimates of variation for litter size and cub survival can be found in Schwartz et al. (2006a:19) and Schwartz et al. (2006b:27), respectively.

Results

We used estimates of FCOY derived from model averaged estimates (Table 1). Data from counts of FCOY used to generate the annual Chao2 estimate are provided in Table 1.

Using this formula, we generated 95% confidence intervals around the estimate of independent females (Table 6), independent males (Table 7), dependent young (Table 8), and total population size (Table 9).

Table 6. Model average estimate of $FCOY_{Chao2}$, the derived estimate of independent females (age ≥ 2 year old), the estimated variance, and the 95% confidence interval about the estimate. Data are based on observations of females with cubs of the year in the Greater Yellowstone Ecosystem, 1983–2005.

Year	Model averaged	\hat{N}_i 2+ females	Estimated variance	95% confidence interval	
				Lower	Upper
1983	18.46	82	52.23	68	96
1984	19.41	86	57.63	72	101
1985	20.40	91	63.59	75	106
1986	21.43	95	70.14	79	112
1987	22.50	100	77.33	83	117
1988	23.63	105	85.23	87	123
1989	24.80	110	93.88	91	129
1990	26.02	116	103.35	96	136
1991	27.30	122	113.72	101	142
1992	28.63	127	125.05	106	149
1993	30.01	134	137.43	111	157
1994	31.46	140	150.95	116	164
1995	32.96	147	165.70	122	172
1996	34.52	154	181.79	127	180
1997	36.15	161	199.32	133	189
1998	37.84	169	218.41	140	197
1999	39.60	176	239.19	146	207
2000	41.43	184	261.79	153	216
2001	43.33	193	286.36	160	226
2002	45.30	202	313.05	167	236
2003	47.35	211	342.02	175	247
2004	49.48	220	373.46	182	258
2005	51.68	230	407.55	191	270

Table 7. Derived estimate of independent males (age ≥ 2 year old), the estimated variance, and the 95% confidence interval about the estimate. Data are based on observations of females with cubs of the year in the Greater Yellowstone Ecosystem, 1983–2005.

Year	\hat{N}_i 2+ males	Estimated variance	95% confidence interval	
			Lower	Upper
1983	52	37.70	40	64
1984	55	41.57	42	68
1985	58	45.88	44	71
1986	61	50.62	47	75
1987	64	55.82	49	78
1988	67	61.53	51	82

1989	70	67.78	54	86
1990	74	74.63	57	91
1991	77	82.12	59	95
1992	81	90.30	62	100
1993	85	99.25	65	104
1994	89	109.01	69	109
1995	93	119.67	72	115
1996	98	131.29	75	120
1997	102	143.95	79	126
1998	107	157.74	82	132
1999	112	172.74	86	138
2000	117	189.07	90	144
2001	123	206.81	94	151
2002	128	226.08	99	158
2003	134	247.00	103	165
2004	140	269.69	108	172
2005	146	294.30	113	180

Table 8. Derived estimate of dependent young (cubs and yearlings), the estimated variance, and the 95% confidence interval about the estimate. Data are based on observations of females with cubs of the year in the Greater Yellowstone Ecosystem, 1983–2005.

Year	\hat{N}_i			
	dependent young	Estimated variance	95% confidence interval	
			Lower	Upper
1983 ^a				
1984	64	12.59	57	71
1985	67	13.90	60	74
1986	70	15.33	63	78
1987	74	16.91	66	82
1988	78	18.64	69	86
1989	81	20.54	73	90
1990	85	22.63	76	95
1991	90	24.91	80	99
1992	94	27.40	84	104
1993	99	30.13	88	109
1994	103	33.12	92	115
1995	108	36.37	96	120
1996	113	39.92	101	126
1997	119	43.80	106	132
1998	124	48.02	111	138
1999	130	52.61	116	144
2000	136	57.61	121	151
2001	142	63.05	127	158
2002	149	68.97	133	165
2003	156	75.39	139	173
2004	163	82.37	145	181
2005	170	89.94	151	189

^aNumber of yearlings estimated from the previous years estimate of cubs. Data not available.

Table 9. Derived estimate of total population size, the estimated variance, and the 95% confidence interval about the estimate. Data are based on observations of females with cubs of the year in the Greater Yellowstone Ecosystem, 1983–2005.

Year	\hat{N}_i All bears	Estimated variance	95% confidence interval	
			Lower	Upper
1983				
1984	205	111.79	184	226
1984	215	123.37	194	237
1986	226	136.09	204	249
1987	238	150.07	214	262
1988	250	165.40	224	275
1989	262	182.20	236	289
1990	275	200.60	247	303
1991	288	220.74	259	318
1992	303	242.76	272	333
1993	317	266.81	285	349
1994	332	293.08	299	366
1995	348	321.74	313	383
1996	365	353.00	328	402
1997	382	387.06	343	421
1998	400	424.16	360	440
1999	419	464.54	376	461
2000	438	508.47	394	482
2001	458	556.22	412	504
2002	479	608.09	431	527
2003	501	664.41	450	551
2004	523	725.52	470	576
2005	546	791.79	491	602

Discussion

The confidence intervals we provide were derived with a Taylor series expansion (delta method) and may be only rough approximations. Because we lacked the ability to estimate the underlying covariance structure, intervals may be too narrow (or too broad). Uncertainty is a fact that we must deal with regarding data collected on the Yellowstone grizzly bear. However, as stated by Beissinger and Westphal (1998:836) “[u]ncertainty is inherent in decision-making but is not an excuse for not making decisions.” We agree. In the *Reassessing Methods Document*, we elected not to generate confidence intervals around our estimates of independent females, independent males, dependent young, and population size because we lacked valid statistical methods to do so. Here we provide approximate estimates of uncertainty because many commenters requested them. It is important to recognize that in the *Reassessing Methods Document* and this supplement, we recommend methods to estimate bear numbers and sustainable mortality limits. However, we also recommended using the point estimate and not intervals of uncertainty. We focused on point estimates because statistically they represent the best approximation of reality. Some will argue that not knowing the uncertainty about our estimates could mislead us when making recommendations or when managers are forced to make decisions. This is a valid point in general; however, we feel that the monitoring protocols established for the Yellowstone grizzly bear are multifaceted and when considered as a whole, provide us with a reasonable understanding of the current health and status of the population. Further, when faced with making decisions, the group made

recommendations that if wrong, err on the conservative side. In other words, if uncertainty leads us astray, we are more likely to underestimate bear numbers and sustainable mortality limits as opposed to overestimating them. We have made every attempt to build in conservative recommendations to cushion against uncertainty but in the real world, managers still must make decisions.

Summary of proposed methods

We recognize that the methods we originally proposed (IGBST 2005) and the newer methods proposed here might be difficult to assimilate. The Interagency Grizzly Bear Study Team will use the following procedures to establish and track sustainable mortality for grizzly bears in the Greater Yellowstone Ecosystem:

1. Raw observations of sightings of females with cubs of the year will be separated into observations of unique females and repeat observations of the same female using the methods of Knight et al. (1995).
2. The Chao2 estimator will be applied to sighting frequencies of unique females to estimate the number of females with cubs of the year in the population.
3. The number of unique females obtained from the Chao2 estimator each year will be added to the dataset and the model averaging process described above repeated.
4. The predicted number of females with cubs obtained from the model fit will be used as the best estimate of the total number of independent females in the population accompanied by cubs of the year for that year.
5. The purpose of the model is to get the best estimate of the current number of females with cubs of the year borrowing information from past estimates, recognizing that with each iteration some change is expected. We do not recommend retrospectively adjusting estimates from previous years.
6. The predicted number of females with cubs will be divided by the proportion of females ≥ 4 years old estimated to be accompanied by cubs of the year (transition probability = 0.289). The resulting value represents the best estimate of the total number of females in the population ≥ 4 years old.
7. The number of females ≥ 4 years old will be divided by the estimated proportion of females ≥ 4 years old in the population of females ≥ 2 years old (0.77699). The resulting value is the best estimate of the number of independent females (≥ 2 years old) in the population that year.
8. The sustainable mortality limit for independent females will be set at 9% of the population estimate of independent females.
9. Unknown and unreported mortality will be estimated based on the methods of Cherry et al. (2002) as described in the *Reassessing Methods Document*.
10. The number of independent males in the population will be based on the estimated ratio of independent males:independent females (0.63513) derived via stochastic modeling described above. The number of independent females in the population will be multiplied by 0.63513 and the resulting value represents the best estimate of the number of independent males that year.

11. The sustainable mortality limit for independent males will be set at 15% of the population estimate of independent males.
12. The number of cubs in the annual population estimate will be calculated directly from the model-predicted estimate of females with cubs of the year. The number of cubs will be estimated by multiplying the modeled estimate by the mean litter size (2.04) observed from 1983–2002.
13. The number of yearlings will be estimated by multiplying the estimated number of cubs from the previous year by the mean survival rate for cubs (0.638) observed from 1983–2001.
14. The sustainable mortality limit for dependent young (cubs and yearlings) will be set at 9% of the annual estimate of dependent young. Only human-caused deaths (reported known and probable) will be tallied against the threshold.
15. Unknown and unreported mortality will not be estimated for dependent young.
16. Allowable mortality limits will be established annually following methods detailed here. Because we are using modeled predictions, annual variability among years has been addressed. Consequently, we do not recommend basing annual limits on a 3-year running average as proposed in the *Reassessing Methods Document*. Rather, we recommend annual mortality limits based on the current year.
17. Estimates of uncertainty about the number of independent females, independent males, dependent young, and total population size will be derived following methods detailed in this report.
18. We recommend the demographic objective originally proposed in the *Reassessing Methods Document* (Interagency Grizzly Bear Study Team 2005:44–45) of 48 FCOY_{Chao2} remains the same; however, we recommend using the predicted number based on model averaging.
19. We recommend a biology and monitoring review should this predicted estimate decline below 48 for any 2 consecutive years.
20. We also recommend the management agencies attempt to limit female mortality if the model predicted estimate of Chao2 drops below 48 in any given year.
21. We recommend a biology and monitoring review if independent female mortality exceeds the 9% limit in any 2 consecutive years.
22. We recommend a biology and monitoring review if independent male mortality exceeds the 15% limit in any 3 consecutive years.
23. We recommend a biology and monitoring review if dependent young mortality exceeds the 9% limit in any 3 consecutive years.
24. We recommend that if the AIC_c weight favors the quadratic term (i.e., >0.5) in modeling the rate of change of females with cubs, a full review of the population's demographics be undertaken to better understand its status.
25. We recommend that dead bears reported in years subsequent to actual year of mortality be tallied against year of death and mortality total be recalculated. If mortality exceeds the threshold for that year, the difference (total mortality minus threshold) should be counted against the current years' threshold. If sex cannot be

- Interagency Grizzly Bear Study Team. 2005. Reassessing methods to estimate population size and sustainable mortality limits for the Yellowstone grizzly bear. Interagency Grizzly Bear Study Team, U.S. Geological Survey, Northern Rocky Mountain Science Center, Montana State University, Bozeman, Montana, USA.
- Keating, K. A., C. S. Schwartz, M. A. Haroldson, and D. Moody. 2002. Estimating numbers of females with cubs-of-the-year in the Yellowstone grizzly bear population. *Ursus* 13:161–174.
- Knight, R. R., B. M. Blanchard, and L. L. Eberhardt. 1995. Appraising status of the Yellowstone grizzly bear population by counting females with cubs-of-the-year. *Wildlife Society Bulletin* 23:245–248.
- Schwartz C. C., and M. A. Haroldson. 2001. Yellowstone grizzly bear investigations: annual report of the Interagency Grizzly Bear Study Team, 2000. U.S. Geological Survey, Bozeman, MT.
- , ———, and S. Cherry. 2006a. Reproductive performance of grizzly bears in the Greater Yellowstone Ecosystem, 1983–2001. Pages 18–24 *in* C. C. Schwartz, M. A. Haroldson, G. C. White, R. B. Harris, S. Cherry, K. A. Keating, D. Moody, and C. Servheen. Temporal, spatial and environmental influences on the demographics of grizzly bears in the Greater Yellowstone Ecosystem. *Wildlife Monographs* 161.
- , ———, and G. C. White. 2006b. Survival of cub and yearling grizzly bears in the Greater Yellowstone Ecosystem, 1983–2001. Pages 25–31 *in* C. C. Schwartz, M. A. Haroldson, G. C. White, R. B. Harris, S. Cherry, K. A. Keating, D. Moody, and C. Servheen. Temporal, spatial and environmental influences on the demographics of grizzly bears in the Greater Yellowstone Ecosystem. *Wildlife Monographs* 161.
- , ———, ———, R. B. Harris, S. Cherry, K. A. Keating, D. Moody, and C. Servheen. 2006c. Temporal, spatial, and environmental influences on the demographics of grizzly bears in the Greater Yellowstone Ecosystem. *Wildlife Monographs* 161.
- Seber, G. A. F. 1982. The estimation of animal abundance and related parameters. Macmillian Publishing Company, Incorporated, New York, New York, USA.
- U.S. Fish and Wildlife Service. 1993. Grizzly bear recovery plan. U.S. Fish and Wildlife Service, Missoula, Montana, USA.