The
Nation's
Report Card

Mathematics 2007
Performance of Public School Students in Puerto Rico

FOCUS ON THE CONTENT AREAS

National Assessment of Educational Progress at Grades 4 and 8

Contents

Photo by Norma Curet, Communications Office, Puerto Rico Department of Education

What is The Nation's Report Card™?

The Nation's Report Card™ informs the public about the academic achievement of elementary and secondary students in the United States. Report cards present the findings of the National Assessment of Educational Progress (NAEP), a continuing and nationally representative measure of achievement in various subjects over time.

For over three decades, NAEP assessments have been conducted periodically in reading, mathematics, science, writing, U.S. history, civics, geography, and other subjects. By collecting and reporting information on student performance at the national, state, and local levels, NAEP is an integral part of our nation's evaluation of the condition and progress of education. Only information related to academic achievement and relevant variables is collected. The privacy of individual students and their families is protected, and the identities of participating schools are not released.

NAEP is a congressionally authorized project of the National Center for Education Statistics (NCES) within the Institute of Education Sciences of the U.S. Department of Education. The Commissioner of Education Statistics is responsible for carrying out the NAEP project. The National Assessment Governing Board oversees and sets policy for NAEP.

Executive Summary

In 2007, public school students in Puerto Rico at grades 4 and 8 participated in a Spanish-language version of the National Assessment of Educational Progress (NAEP) in mathematics. A representative sample of approximately 2,800 students from 100 public schools was assessed at each grade.

This report contains performance results on NAEP mathematics questions for public school students in Puerto Rico and the nation. Results are presented as the average scores for the correct answers (see box below)–expressed as decimals ranging from 0.00 to 1.00–for all the questions in the NAEP mathematics assessment and for questions in each of the five mathematics content areas (as shown in figures A and B).

At grade 4

- The average of the question scores for students in Puerto Rico was lower than the score for students in the nation overall and within each content area.

- There was no statistically significant difference in performance between male and female students in Puerto Rico overall and in each content area.

At grade 8

- The overall average of the question scores for students in Puerto Rico was lower than the score for students in the nation. Results were similar for each content area.

- While there was no significant difference between the performance of male and female students in Puerto Rico overall, male students had a higher score than female students in the measurement content area, and female students had a higher score than their male peers in the data analysis and probability content area.

Figure A. Average of the question scores in NAEP mathematics at grade 4, by content area: 2007
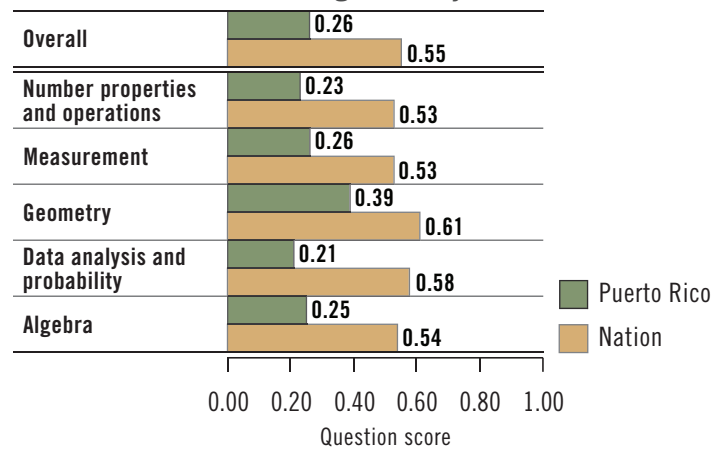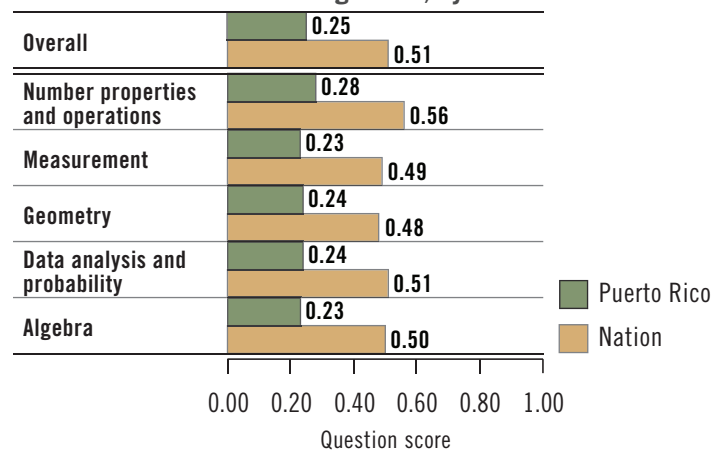

Figure B. Average of the question scores in NAEP mathematics at grade 8, by content area: 2007

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Interpreting Results for Puerto Rico

Question scores are calculated as the percentages of correct responses for multiple-choice questions and for constructed-response questions that are scored either correct or incorrect. For constructed-response questions that allow for partial credit, the question score is the sum of the percentage of students receiving full credit and a fraction of the percentage receiving partial credit. Individual question scores are then averaged together to report an average question score for the entire mathematics assessment or for each of the five content areas.

Because of technical concerns regarding the placement of the 2007 results for Puerto Rico on the NAEP mathematics scale, performance results could not be reported as average scale scores for Puerto Rico in this report, and students' performance in 2007 could not be compared to performance in previous assessments.

When comparing the results for students in Puerto Rico to students in the nation, it is important to consider some of the differences in demographics. For example, between 76 and 78 percent of fourth- and eighth-graders in Puerto Rico attended public schools compared to 91 percent in the nation. All of the public school students in Puerto Rico were eligible for the National School Lunch Program compared to between 41 and 46 percent of fourth- and eighth-graders in the nation.

The Mathematics Assessment

The Framework

The content of all NAEP assessments is determined by subject-area frameworks developed by the National Assessment Governing Board in a comprehensive process involving a broad spectrum of stakeholders, including teachers, curriculum specialists, subject-matter specialists, school administrators, parents, and members of the general public. Frameworks in NAEP are developed to survey student understanding over a broad range of content. The current frameworks are available at http://www.nagb.org/.

The NAEP mathematics framework serves as the blueprint for the assessment, describing the specific mathematical skills that should be assessed at grades 4 and 8. Two dimensions of mathematics, *content areas* and *mathematical complexity*, are used to guide the assessment. Each question is designed to measure one of five content areas. However, certain aspects of mathematics, such as computation, occur in all content areas.

The levels of complexity of a mathematics question are defined in the shaded box on the following page. This differs from the difficulty of a question, which is defined by the percentage of correct student responses; a lower percentage of students answering correctly indicates a more difficult question. For example, a question asking grade 8 students to interpret a number given in scientific notation has a low level of complexity but may be difficult, that is, few students answer it correctly. A question with a high level of complexity might ask students to explain or justify their solutions to a problem.

Mathematics Content Areas

Number properties and operations measures students' understanding of ways to represent, calculate, and estimate with numbers.

Measurement measures students' knowledge of measurement attributes, such as capacity and temperature, and geometric attributes, such as length, area, and volume.

Geometry measures students' knowledge and understanding of shapes in a plane and in space.

Data analysis and probability measures students' understanding of data representation, characteristics of data sets, experiments and samples, and probability.

Algebra measures students' understanding of patterns, using variables, algebraic representation, and functions.

Levels of Mathematical Complexity

Low complexity questions typically specify what a student is to do, which is often to carry out a routine mathematical procedure.

Moderate complexity questions involve more flexibility of thinking and often require a response with multiple steps.

High complexity questions make heavier demands and often require abstract reasoning or analysis in a novel situation.

Assessment Design

The NAEP mathematics framework specifies the percentage of questions to be assessed in each content area. The distribution of questions across content areas for grades 4 and 8 is shown in table 1. Topics in the Puerto Rico mathematics standards, *Estándares de Excelencia Programa de Matemáticas* (Departamento de Educación 2000) are organized around the same five content areas as the *Mathematics Framework for the 2007 National Assessment of Educational Progress* (National Assessment Governing Board 2006).

The entire NAEP mathematics assessment consists of 10 sections of mathematics questions at each grade. Each section includes 14 to 20 questions covering all five content areas. Because the assessment covers a breadth of content and includes more questions than any one student could reasonably answer, each student takes just a portion of the assessment, answering two sections of mathematics questions.

Students were asked to respond to multiple-choice questions, as well as constructed-response questions that required them to produce their own answers. Scorers evaluated student responses written in Spanish. Some questions at each grade level incorporated the use of calculators, rulers (grade 4), ruler/protractors (grade 8), or manipulatives such as geometric shapes or spinners that were provided

for students. On approximately one-third of the assessment, a four-function calculator was provided for students at grade 4, and a scientific calculator was provided for students at grade 8.

The 2007 NAEP mathematics assessments at grades 4 and 8 were translated into Spanish for administration in Puerto Rico (see the Technical Notes for more information). The content was the same as that for all students in the nation. Students in Puerto Rico were given a total of 70 minutes to complete the two sections of mathematics questions (20 minutes more than the time allotted for students assessed nationally).

Table 1. Target percentage distribution of NAEP mathematics questions, by grade and content area: 2007

Content area	Grade 4	Grade 8
Number properties and operations	40%	20%
Measurement	20%	15%
Geometry	15%	20%
Data analysis and probability	10%	15%
Algebra	15%	30%

SOURCE: U.S. Department of Education, National Assessment Governing Board, Mathematics Framework for the 2007 National Assessment of Educational Progress, 2006.

Reporting Student Results

Approximately 76 percent of fourth-graders and 78 percent of eighth-graders in Puerto Rico were enrolled in public schools in 2007 (table 2). Representative samples of public schools and their students at grades 4 and 8 in Puerto Rico participated in the NAEP mathematics assessment during the same time period in which students were assessed nationally. Approximately 2,800 students from 100 public schools at each grade were assessed in Puerto Rico.

Table 2. Percentage of students in Puerto Rico and the nation in NAEP mathematics, by grade and selected characteristics: 2007

| | Grade 4 | | Grade 8 | |
Characteristic	Puerto Rico	Nation	Puerto Rico	Nation
Private school	24	9	22	9
Public school	76	91	78	91
Eligible for free/ reduced-price school lunch	100	46	100	41
Identified as students with disabilities	20	14	14	13

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment. Percentages of Puerto Rico students in private and public schools provided by the Puerto Rico General Council of Education and the Puerto Rico Department of Education.

Performance Results

Student results are presented in this report as the average of the question scores for all the questions in the assessment and for questions in each content area. In the following pages, sample questions are shown as examples of what students in Puerto Rico know and are able to do within each of the five mathematics content areas. The percentages of students in each response category are presented in a table for each question. For a multiple-choice question, the response categories are the answer choices for the question. For a constructed-response question, the response categories are the ratings defined in the scoring guide for the question. The row with the percentages for the correct or most complete response is highlighted in the tables. Results for other released sample questions from the 2007 mathematics assessment are available at http://nces.ed.gov/nationsreportcard/itmrls/.

Interpreting Results for Puerto Rico

Because of technical concerns regarding the placement of 2007 results for Puerto Rico on the NAEP mathematics scale, students' performance results could not be reported as average scale scores for Puerto Rico in this report, and students' performance in 2007 could not be compared to performance in previous assessments. NCES is continuing to investigate ways to make meaningful comparisons between the performance of students in Puerto Rico and students in the nation (see the Technical Notes for more information). For comparison purposes, the average of the question scores and percentages of responses for sample questions are shown for public school students in the nation (excluding Puerto Rico).

NAEP results are based on samples of student responses, and there is a margin of error associated with every result. Any differences that are mentioned in the text as "higher" or "lower" are statistically significant at the .05 level. Statistical significance is not marked in the figures and tables in the body of this report, but in almost all cases, the average of the question scores and percentages of correct or most complete responses for Puerto Rico were lower than those for the nation.

Overall Results

At both grades 4 and 8, the overall average of the question scores on the NAEP mathematics assessment was lower for students in Puerto Rico than for public school students nationally (figures 1 and 2). In Puerto Rico, the averages of the question scores were 0.26 for grade 4 and 0.25 for grade 8, compared to national averages of 0.55 and 0.51, respectively.

The pattern of results in Puerto Rico by gender differed slightly from that in the nation. While the national average of the question scores for male students was higher than for female students at both grades 4 and 8, there was no significant difference in the average of the question scores for male and female students in Puerto Rico at either grade.

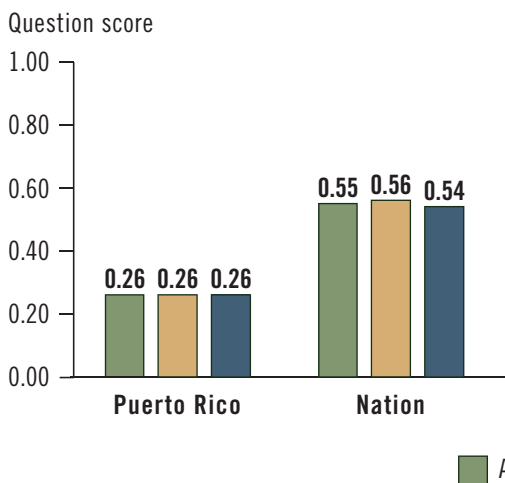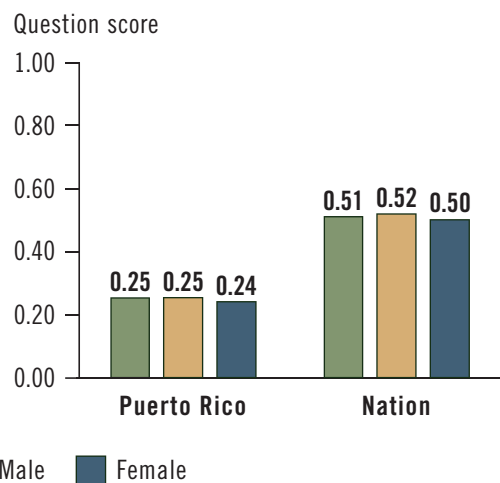**Figure 1. Average of the question scores in NAEP mathematics at grade 4, by gender: 2007**

Question score

Figure 2. Average of the question scores in NAEP mathematics at grade 8, by gender: 2007

Question score

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

The Question Score

For a multiple-choice or a constructed-response question that is scored either "Correct" or "Incorrect," the question score is the percentage of correct responses expressed as a decimal. For a constructed-response question in which students could earn partial credit if they did not have a completely correct response, the question score is computed by adding the percent of students receiving full credit to a fraction of the percent of students receiving partial credit. The average of the question scores for a set of assessment questions ranges from 0.00 to 1.00. See the Technical Notes for more details about the question score, including an example of how it is calculated. Question scores for released questions from the 2007 NAEP mathematics assessment are also presented in appendix tables A-3 and A-4.

Number Properties and Operations

Numbers are our main tools for describing the world quantitatively, so the ability to use number properties and operations is an important expectation of the 2007 NAEP mathematics framework. This content area focuses on students' understanding of ways to represent, calculate, and estimate numbers. In addition to basic computation skills, NAEP assesses students' ability to order and compare numbers and to solve problems in real-world settings using arithmetic operations.

At grade 4, students are expected to have a solid grasp of whole numbers and a beginning understanding of fractions. At grade 8, students are expected to be able to work with rational numbers (both fractions and decimals), ratios and proportional reasoning, scientific notation, and naturally occurring irrational numbers such as square roots and pi ( ).

Subtopics of the number properties and operations content area covered in the assessment are

• **Number sense**
• **Estimation**
• **Number operations**
• **Ratios and proportional reasoning**
• **Properties of number and operations**

Student Results

In Puerto Rico, the average of the question scores in the number properties and operations content area was 0.23 for fourth-graders (figure 3). The score for eighth-graders was 0.28 (figure 4). These scores were lower on average for students in Puerto Rico than for public school students in the nation.

At both grades 4 and 8, the average of the question scores in the number properties and operations content area did not differ significantly between male and female students in Puerto Rico. For students in both grades nationally, the scores in this content area for male students were higher than those for female students.

The next few pages contain sample questions from the number properties and operations content area in the 2007 NAEP mathematics assessment. These sample questions do not represent the entire range of content assessed in this area and are for illustrative purposes only. For each sample question, the percentages of students for each response category are reported for both Puerto Rico and the nation. The row corresponding to the correct or most complete response is highlighted.

Figure 3. **Average of the question scores in NAEP mathematics for number properties and operations at grade 4, by gender: 2007**
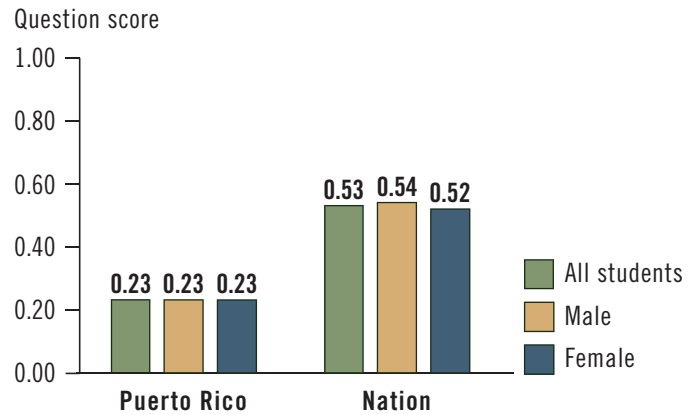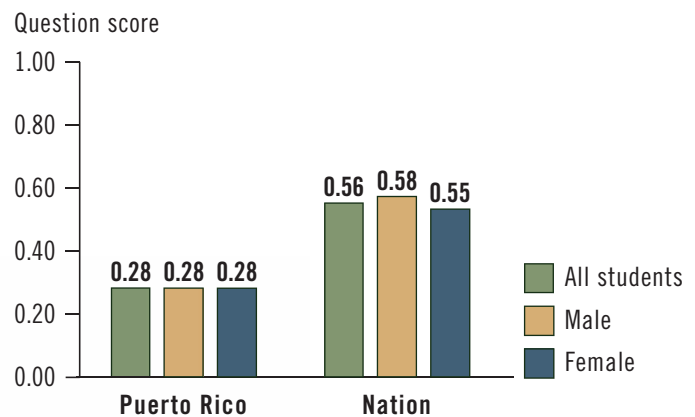

Figure 4. **Average of the question scores in NAEP mathematics for number properties and operations at grade 8, by gender: 2007**

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample question 1 addresses the number operations subtopic. This subtopic includes questions about computation, the effects of operations on numbers, relationships between operations, and application problems involving numbers and operations.

Sample question 2 addresses the properties of number and operations subtopic. This subtopic includes questions about odd and even numbers, factors of whole numbers, basic properties of operations, and explaining mathematical relationships.

Grade 4

Sample Multiple-Choice Question

Sample question 1 is a computation problem in a real-world setting. To solve the problem of finding how many more people went to the zoo requires subtraction with regrouping to find the difference between the number of people at the zoo on Saturday and Sunday: 983 – 789 = 194. This question was included in a section that allowed the use of a calculator. The framework objective measured in this question is about solving application problems involving numbers and operations.

In Puerto Rico, 17 percent of grade 4 students answered this question correctly (choice A). Some misconceptions and errors represented by the incorrect answer choices for this question are given below:

- Subtracting incorrectly (choice B and choice C)

- Adding the two numbers given in the problem instead of subtracting (choice D)

Sample Question 1

On Saturday 789 people went to the zoo. On Sunday 983 people went to the zoo. How many more people went to the zoo on Sunday than on Saturday?

- ⬭ 194
- Ⓑ 204
- Ⓒ 206
- Ⓓ 1,772

Percentage of students in each response category at grade 4: 2007

	Puerto Rico	Nation
Choice A	17	73
Choice B	3	2
Choice C	4	5
Choice D	74	19
Omitted	2	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Constructed-Response Question

Sample question 2 is a conceptual question about even and odd numbers. To answer the question, students must recognize that while an even number of objects can be paired, an odd number of objects will have one remaining when paired. This question is about pairing the students in three classes, and Mr. West's class, with 25 students, will have one child without a partner when the students line up with partners. The framework objective for this question asks students to identify odd and even numbers.

Student responses for this question were rated using the following three-level scoring guide:

Correct — A response that answered "Mr. West's Class" (or 25) with an explanation or picture indicating that there are an odd number of students in Mr. West's class

Partial — A response that answered "Mr. West's Class" (or 25) without an acceptable explanation for this choice

OR

A response with an explanation or a picture indicating an odd number of students without answering "Mr. West's Class" (or 25)

Incorrect — All incorrect responses

In Puerto Rico, 12 percent of the grade 4 student responses were rated "Correct," and 24 percent of the students gave responses that were rated "Partial."

Sample Question 2

Number of Students	Ms. King's Class	Mr. West's Class	Ms. Chang's Class
	20	25	28

In each class listed above, the students are lining up with a partner to walk to lunch. Which class will have one child with no other child for a partner?

Answer: _____

Explain your choice.

Percentage of students in each response category at grade 4: 2007

	Puerto Rico	Nation
Correct	12	56
Partial	24	19
Incorrect	52	23
Omitted	10	2

NOTE: Detail may not sum to totals because a small percentage of responses that did not address the assessment task are not shown.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample question 3 addresses the ratios and proportional reasoning subtopic. This subtopic includes questions about ratios, proportions, and percentages.

Sample question 4 addresses the number sense subtopic. This subtopic includes questions about place value, ordering and comparing numbers, and translating between different representations of numbers. The focus of this subtopic is on rational numbers and includes scientific notation and absolute value.

Sample Multiple-Choice Question

Sample question 3 asks students to compute a percentage in a real-world context. This question was included in a section that allowed the use of a calculator. The framework objective measured in this question is to solve problems involving percentages (including percent increase and decrease, interest rates, tax, discount, tips, or part/whole relationships).

In Puerto Rico, 28 percent of grade 8 students answered this question correctly (choice D). Some common errors that may lead to the incorrect answer choices for this question are given below:

- Interpreting the decimal representation of 91% (i.e., 0.91) as the percentage (choice A)

- Dividing the whole (57) by the part (52) and not converting to a percentage (choice B)

- Interpreting the score as the percentage (choice C)

- Finding the difference between the values (57 − 52 = 5) and subtracting this as a percentage from 100% (choice E)

Percentage of students in each response category at grade 8: 2007

	Puerto Rico	Nation
Choice A	27	12
Choice B	9	4
Choice C	19	5
Choice D	28	62
Choice E	15	16
Omitted	1	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Question 3

Tammy scored 52 out of 57 possible points on a quiz. Which of the following is closest to the percent of the total number of points that Tammy scored?

- A   0.91%
- B   1.10%
- C   52%
- D   91%
- E   95%

Sample Constructed-Response Question

Sample question 4 is a short constructed-response question that asks students to express the sum of three fractions as a decimal, where each fraction has a denominator that is a power of 10. This question was included in a section that allowed the use of a calculator. This is a conceptual question about place value, measuring the framework objective of using place value to model and describe integers and decimals.

Student responses for this question were rated using the following two-level scoring guide:

Correct — A response of 0.777

Incorrect — All incorrect responses

In Puerto Rico, 7 percent of the grade 8 student responses were rated "Correct."

Percentage of students in each response category at grade 8: 2007

	Puerto Rico	Nation
Correct	7	45
Incorrect	80	52
Omitted	13	3

NOTE: Detail may not sum to totals because a small percentage of responses that did not address the assessment task are not shown.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Question 4

Add the numbers

$\frac{7}{10}$, $\frac{7}{100}$, and $\frac{7}{1,000}$.

Write this sum as a <u>decimal</u>.

_____

Measurement

Measuring is the process by which numbers are assigned to describe the world quantitatively. The 2007 NAEP mathematics framework includes measurement attributes such as capacity, weight or mass, time, and temperature, as well as the geometric attributes of length, area, and volume.

Assessment questions on measurement at grade 4 focus on customary units such as inch, quart, pound, and hour, and common metric units such as centimeter, liter, and gram, as well as the geometric attribute of length. At grade 8, the emphasis is on the use of square units for measuring area and surface area, cubic units for measuring volume, degrees for measuring angles, and rates. More emphasis is placed on area and angle measurements than on linear measurements.

Subtopics in the measurement content area are

- **Measuring physical attributes**
- **Systems of measurement**

Student Results

In Puerto Rico, the averages of the question scores in measurement were 0.26 at grade 4 and 0.23 at grade 8 (figures 5 and 6). These scores were lower on average than the scores for public school students in the nation.

Results by gender for this content area varied by grade in Puerto Rico. While the average of the question scores in measurement did not differ significantly between male and female fourth-graders in Puerto Rico, the score for male eighth-graders was higher than that for their female peers. In the nation, the score was higher for male students than for female students at both grades.

The next few pages contain sample questions from the measurement content area in the 2007 NAEP mathematics assessment.

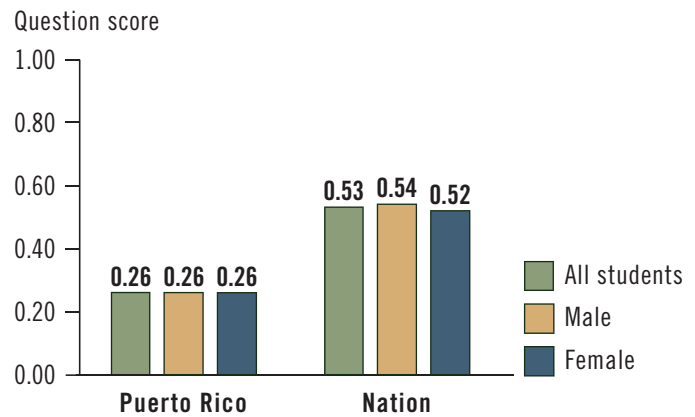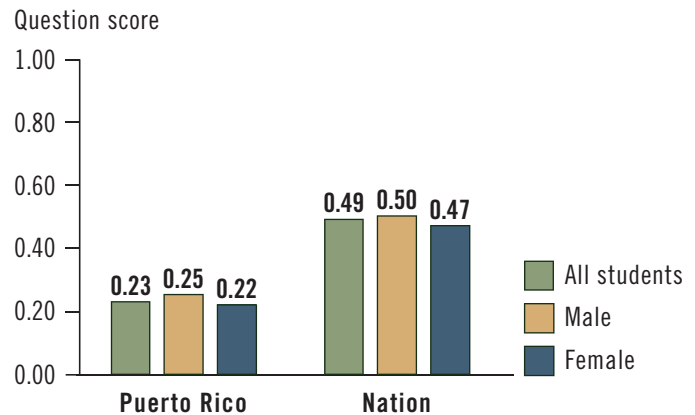**Figure 5.** Average of the question scores in NAEP mathematics for measurement at grade 4, by gender: 2007

Question score

Figure 6. Average of the question scores in NAEP mathematics for measurement at grade 8, by gender: 2007

Question score

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample question 5 addresses the systems of measurement subtopic. This subtopic includes questions about appropriate units of measurement, appropriate sizes of measurements in problem situations, conversions within the same measurement system, and determining when highly accurate measurements are important.

Sample question 6 addresses the measuring physical attributes subtopic. This subtopic includes questions about identifying attributes that can be measured; comparing objects or estimating the size of an object with respect to a given attribute such as length, time, or temperature; using appropriate measurement instruments; and solving problems involving the perimeter of plane figures or the area of squares and rectangles.

Grade 4

Sample Multiple-Choice Question

Sample question 5 asks students to identify the measurement that could be the length of a pencil. The framework objective for this question asks students to select or use the appropriate type of unit for the attribute being measured such as length, time, or temperature.

In Puerto Rico, 55 percent of grade 4 students answered this question correctly (choice D). The incorrect answer choices for the question are inappropriate measurements or units of measurement.

Percentage of students in each response category at grade 4: 2007

	Puerto Rico	Nation
Choice A	22	5
Choice B	9	1
Choice C	12	5
Choice D	55	89
Omitted	2	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Question 5

Which of the following could be the length of the pencil you use in school?

- Ⓐ 6 feet
- Ⓑ 6 pounds
- Ⓒ 6 ounces
- ⬤ 6 inches

Sample Constructed-Response Question

Sample question 6 is a two-step problem about carpeting a room. The first step in solving the problem is to determine the amount of carpet needed to cover the floor of a room (180 square feet), and the second step is to compute the cost of the carpet for this room (180 × $2.60 = $468). This question was included in a section that allowed the use of a calculator. The framework objective of solving problems involving the area of squares and rectangles is measured in this question.

Student responses for this question were rated using the following three-level scoring guide:

Correct — A response that had the correct area of the floor (180 square feet) and the correct cost of the carpet ($468)

Partial — A response that had the correct area of the floor but did not have the correct cost of the carpet

OR

A response that did not have the correct area of the floor but had the correct cost of the carpet

OR

A response that did not have the correct area of the floor but correctly computed the cost of the carpet based on this incorrect area

Incorrect — All incorrect responses

In Puerto Rico, 2 percent of the grade 4 student responses were rated "Correct," and 5 percent of the responses were rated "Partial."

Sample Question 6

Mark's room is 12 feet wide and 15 feet long. Mark wants to cover the floor with carpet. How many square feet of carpet does he need?

Answer: _____ square feet

The carpet costs $2.60 per square foot. How much will the carpet cost?

Answer: $_____

Percentage of students in each response category at grade 4: 2007

|	Puerto Rico	Nation
Correct	2	9
Partial	5	30
Incorrect	87	58
Omitted	6	3

NOTE: Detail may not sum to totals because a small percentage of responses that did not address the assessment task are not shown.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample questions 7 and 8 address the measuring physical attributes subtopic. This subtopic includes questions about comparing objects or estimating the size of an object with respect to a measurement attribute such as length, angle, weight, or mass; using appropriate measurement instruments; solving problems involving the perimeter or area of plane figures; and solving problems involving the volume or surface area of solids.

Grade 8

Sample Multiple-Choice Question

Sample question 7 asks students to identify the measurement that could be the area of a typical classroom floor. This question was included in a section that allowed the use of a calculator. The framework objective measured in this question is about estimating the size of an object with respect to a given measurement attribute (e.g., area).

In Puerto Rico, 47 percent of grade 8 students answered this question correctly (choice B). The incorrect answer choices for the question are inappropriate measurements or inappropriate units of measurement.

Percentage of students in each response category at grade 8: 2007

	Puerto Rico	Nation
Choice A	22	13
Choice B	47	68
Choice C	10	13
Choice D	7	2
Choice E	12	3
Omitted	2	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Question 7

Of the following, which is the best estimate for the area of a typical classroom floor?

- (A) 700 feet
- (B) 700 square feet
- (C) 700 cubic feet
- (D) 700 yards
- (E) 700 square yards

Sample Constructed-Response Question

Sample question 8 involves reasoning about the units on a scale. The figure on the left shows that two tick marks on the scale represent $\frac{1}{2}$ pound. Therefore, each tick mark represents $\frac{1}{4}$ pound, so the total weight of the two apples on the right is $\frac{7}{4}$ pounds or $1\frac{3}{4}$ pounds. The framework objective measured in this question is to compare objects with respect to length, area, volume, angle measurement, weight, or mass.

Student responses for this question were rated using the following two-level scoring guide:

Correct   A response of $\frac{7}{4}$ or $1\frac{3}{4}$

Incorrect   All incorrect responses

In Puerto Rico, 6 percent of the grade 8 student responses were rated "Correct."

Percentage of students in each response category at grade 8: 2007

	Puerto Rico	Nation
Correct	6	48
Incorrect	90	51
Omitted	4	1

NOTE: Detail may not sum to totals because a small percentage of responses that did not address the assessment task are not shown.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Question 8

Both figures below show the same scale. The marks on the scale have no labels except the zero point.
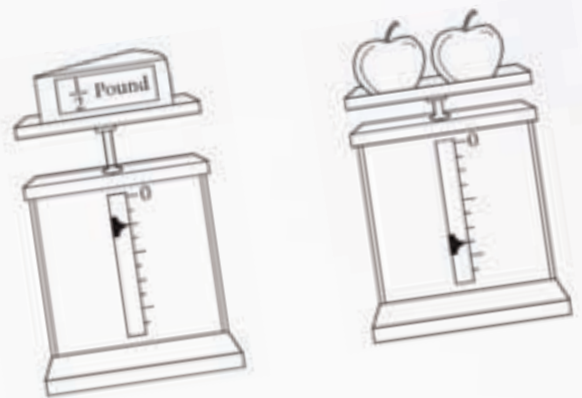
The weight of the cheese is $\frac{1}{2}$ pound. What is the total weight of the two apples?

Total weight of two apples = _____ pounds.

Geometry

School geometry roughly mirrors the historical development of geometry, which began as a practical collection of rules for calculating lengths, areas, and volumes of common shapes. This expanded over time to include the study of structures in space and ideas of symmetry and transformation.

Students at grade 4 are expected to be familiar with a library of simple figures and their attributes, both in the plane and in space. At grade 8, students are expected to be familiar with the properties of plane figures, especially parallel and perpendicular lines, angle relations in polygons, cross sections of solids, and the Pythagorean theorem.

Subtopics in the geometry content area are

- Dimension and shape
- Transformation of shapes and preservation of properties
- Relationships between geometric figures
- Position and direction
- Mathematical reasoning

Student Results

In Puerto Rico, the averages of the question scores in the geometry content area were 0.39 at grade 4 and 0.24 at grade 8 (figures 7 and 8). Both scores were lower than those for public school students in the nation.

The patterns in results by gender for geometry were similar in Puerto Rico and the nation. Among fourth- and eighth-graders in both Puerto Rico and the nation, the average of the question scores in geometry for male students was not significantly different from that of female students.

The next few pages contain sample questions from the geometry content area in the 2007 NAEP mathematics assessment.

Figure 7. Average of the question scores in NAEP mathematics for geometry at grade 4, by gender: 2007
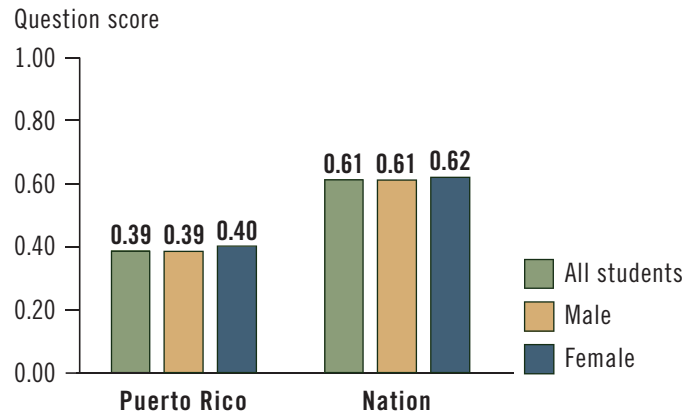
Question score

Figure 8. Average of the question scores in NAEP mathematics for geometry at grade 8, by gender: 2007
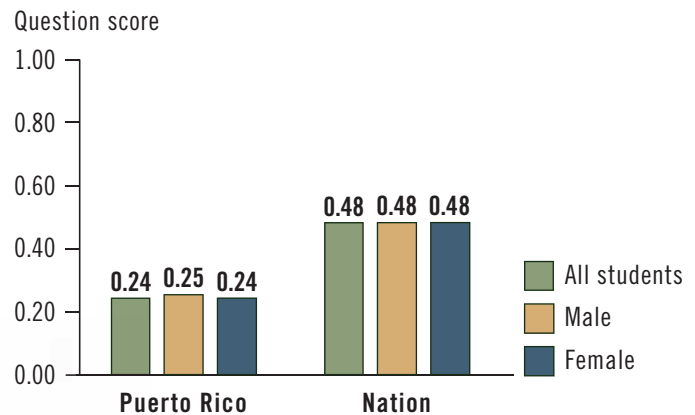
Question score

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample question 9 addresses the dimension and shape subtopic. This subtopic includes questions about identifying, describing, or drawing geometric figures in the plane; identifying or informally describing real-world objects using geometric shapes; and attributes of two- and three-dimensional shapes.

Sample question 10 addresses the mathematical reasoning subtopic. This subtopic includes questions about distinguishing objects in a collection that satisfy a given geometric definition.

Sample Multiple-Choice Question

Sample question 9 asks students to determine the number of right angles contained in a figure representing the path from a student's house to the school. The framework objective measured in this question is to identify or draw angles and other geometric figures in the plane.

In Puerto Rico, 42 percent of grade 4 students answered this question correctly (choice C).

Percentage of students in each response category at grade 4: 2007

	Puerto Rico	Nation
Choice A	22	34
Choice B	20	21
Choice C	42	41
Choice D	12	2
Omitted	3	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Question 9

The picture shows Rachel's path to school. How many right angle turns does Rachel make to get to school?

- (A) Two
- (B) Three
- Five
- (D) Seven

Sample Multiple-Choice Question

Sample question 10 is a reasoning question that presents four figures that have two different attributes: color and shape. The question presents information about both the color (the figure is shaded) and the shape (the figure is not a triangle) of a specific figure, requiring the student to select a rectangle that is shaded (figure D). The framework objective for this question asks students to distinguish which objects in a collection satisfy a given geometric definition and sometimes to explain their choices.

In Puerto Rico, 69 percent of grade 4 students answered this question correctly (choice D).

Percentage of students in each response category at grade 4: 2007

	Puerto Rico	Nation
Choice A	10	5
Choice B	11	3
Choice C	5	1
Choice D	69	90
Omitted	4	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.
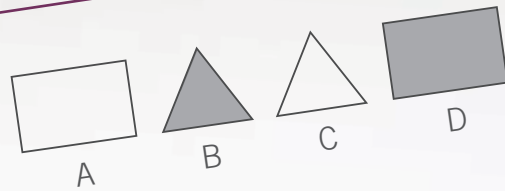
Sample Question 10

Melissa chose one of the figures above.

- The figure she chose was shaded.
- The figure she chose was <u>not</u> a triangle.

Which figure did she choose?

Ⓐ A
Ⓑ B
Ⓒ C
⬤ D

Sample question 11 addresses the position and direction subtopic. This subtopic includes questions about relative positions of points and lines including midpoints, parallel and perpendicular lines, and points of intersection; cross sections of solids; and the representation of geometric figures in a rectangular coordinate plane.

Sample question 12 addresses the subtopic of relationships between geometric figures. This subtopic includes questions about the properties of and relationships between geometric shapes in two and three dimensions; properties of and relationships between parallel or intersecting lines; and the Pythagorean theorem.

Sample Multiple-Choice Question

Sample question 11 is based on the definitions of line, ray, line segment, and the intersection of geometric figures. The framework objective measured in this question addresses how to describe the intersection of two or more geometric figures in the plane (e.g., intersection of a circle and a line).

In Puerto Rico, 29 percent of grade 8 students answered this question correctly (choice A). Some misconceptions represented by the incorrect answer choices for this question are given below:

- The union of rays PQ and QP is line PQ (choice B).
- Point P is a point on ray QP (choice C), and point Q is a point on ray PQ (choice D).
- Rays PQ and QP have no points in common, so the intersection is the empty set (choice E).

Percentage of students in each response category at grade 8: 2007

	Puerto Rico	Nation
Choice A	29	40
Choice B	57	41
Choice C	4	3
Choice D	1	1
Choice E	8	14
Omitted	1	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.
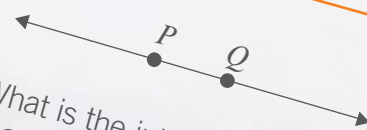
Sample Question 11

What is the intersection of rays PQ and QP in the figure above?

- (A) Segment PQ
- (B) Line PQ
- (C) Point P
- (D) Point Q
- (E) The empty set

Sample Constructed-Response Question

Sample question 12 is presented in a real-world setting and requires students to analyze the relationship between the radius of the tennis balls in a can and the minimum height of the can. To answer the question, students need to recognize that the minimum height of the can is 3 times the diameter of each ball, and that the diameter of each ball is 2 × 3 = 6 centimeters. Therefore, the minimum height of the can is 18 centimeters. This question was included in a section that allowed the use of a calculator. The framework objective for this question is to represent problem situations with simple geometric models to solve mathematical or real-world problems.

Student responses for this question were rated using the following three-level scoring guide:

Correct   A response that had the correct height of the can (18 centimeters) and a correct diagram or complete explanation

Partial   A response that had the correct height of the can (18 centimeters), with an incomplete, incorrect, or missing diagram or explanation

OR

A response that had a correct diagram or complete explanation, with an incorrect height or no height for the can

Incorrect   All incorrect responses

In Puerto Rico, 1 percent of the grade 8 student responses were rated "Correct," and 4 percent of the responses were rated "Partial."

Sample Question 12

Three tennis balls are to be stacked one on top of another in a cylindrical can. The radius of each tennis ball is 3 centimeters. To the nearest whole centimeter, what should be the minimum height of the can?

Explain why you chose the height that you did. Your explanation should include a diagram.

Percentage of students in each response category at grade 8: 2007

	Puerto Rico	Nation
Correct	1	17
Partial	4	20
Incorrect	53	49
Omitted	38	13

NOTE: Detail may not sum to totals because a small percentage of responses that did not address the assessment task are not shown.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Data Analysis and Probability

Data analysis is the process of collecting, organizing, summarizing, and interpreting data. This is the heart of the discipline called statistics. In the context of data analysis, probability can be thought of as the study of potential patterns in outcomes that have not yet been observed.

At grade 4, the data analysis and probability content area focuses on students' understanding of how data are collected and organized, how to read and interpret various representations of data, and basic concepts of probability. At grade 8, the emphasis is on students' ability to use a variety of techniques for organizing and summarizing data (including tables, charts, and graphs), to analyze statistical claims, and to use more formal terminology related to probability and data analysis.

Subtopics in the data analysis and probability content area are

- Data representation
- Characteristics of data sets
- Experiments and samples
- Probability

Student Results

In Puerto Rico, the averages of the question scores in data analysis and probability were 0.21 at grade 4 and 0.24 at grade 8 (figures 9 and 10). These scores were lower on average than those for the nation.

At grade 4, the average of the question scores in data analysis and probability did not differ significantly between male and female students in Puerto Rico. At grade 8, the score for female students in Puerto Rico was higher than that for their male peers. There was no significant difference in the scores between male and female students in the nation at either grade 4 or grade 8.

The next few pages contain sample questions from the data analysis and probability content area in the 2007 NAEP mathematics assessment.

Figure 9. Average of the question scores in NAEP mathematics for data analysis and probability at grade 4, by gender: 2007
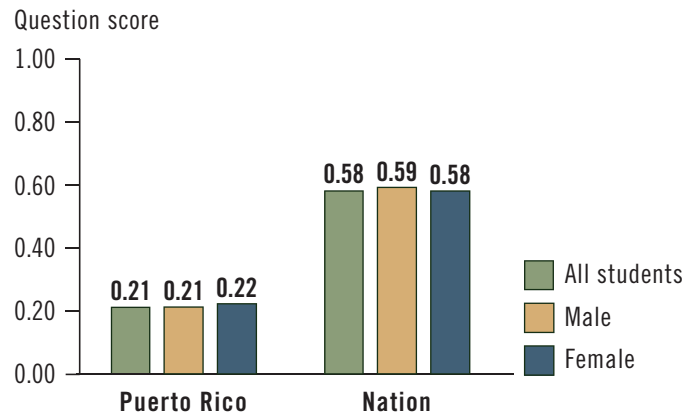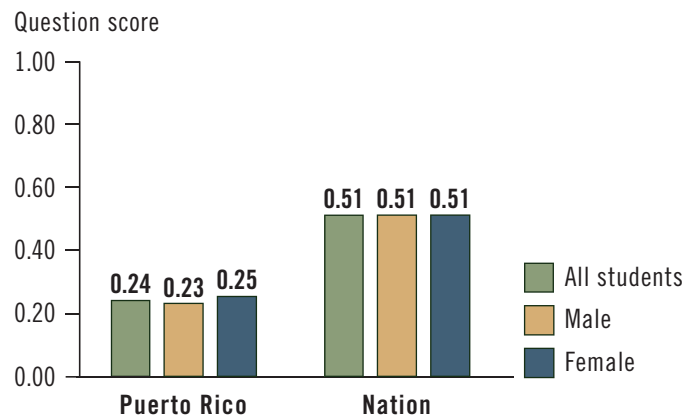

Figure 10. Average of the question scores in NAEP mathematics for data analysis and probability at grade 8, by gender: 2007

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Grade 4

Sample question 13 addresses the data representation subtopic. This subtopic includes questions about data presented in pictographs, bar graphs, circle graphs, line graphs, line plots, tables, and tallies.

Sample question 14 addresses the probability subtopic. This subtopic includes questions about informal probabilistic thinking and counting or representing the outcomes of given events.

Sample Multiple-Choice Question

Sample question 13 presents a pictograph together with a legend to interpret the symbols in the graph. The question asks about the total number of students represented by the symbols, where a full symbol represents 10 students and each partial symbol represents 5 students. Since there are 9 full symbols and 2 partial symbols, there are a total of 100 students represented in the graph. The framework objective measured in this question is about solving problems by estimating and computing within a single set of data.

In Puerto Rico, 35 percent of the grade 4 students answered this question correctly (choice B). The incorrect answer choices for the question may result from errors in counting and/or interpreting the symbols.

Percentage of students in each response category at grade 4: 2007

	Puerto Rico	Nation
Choice A	30	8
Choice B	35	84
Choice C	10	3
Choice D	20	4
Omitted	4	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Question 13

	4th Graders in Smith School
Car	𝅘𝅥𝅮 𝅘𝅥𝅮 𝅘𝅥𝅮 𝅘𝅥
Bus	𝅘𝅥𝅮 𝅘𝅥𝅮 𝅘𝅥𝅮 𝅘𝅥𝅮 𝅘𝅥𝅮
Walk	𝅘𝅥𝅮 𝅘𝅥
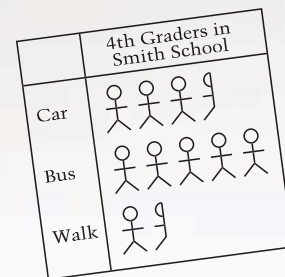
𝅘𝅥𝅮 = 10 Students

𝅘𝅥 = 5 Students

The pictograph shows how all the 4th graders at Smith School get to school. According to the pictograph, how many 4th graders attend Smith School?

- (A) 95
- (B) 100
- (C) 105
- (D) 110

Sample Constructed-Response Question

Sample question 14 asks students to label a picture of a spinner so the chance of landing on blue will be twice the chance of landing on red. Since the spinner has 9 sections, a correctly labeled spinner will have 6 blue sections and 3 red sections. This extended constructed-response question also asks students to explain how they obtained the answer. The framework objective measured in this question is to use informal probabilistic thinking to describe chance events (i.e., likely and unlikely, certain and impossible).

Student responses for this question were rated using the five-level scoring guide shown on the right. In Puerto Rico, less than 1 percent of the grade 4 student responses were rated "Extended," and 1 percent were rated "Satisfactory."

Sample Question 14

Luis wants to make a game spinner in which the chance of landing on blue will be twice the chance of landing on red. He is going to label each section either red (R) or blue (B).

Show how he could label his spinner.
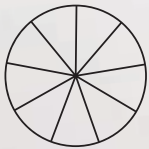
Number of blues: — Number of reds: —
Explain how you found your answer.

Percentage of students in each response category at grade 4: 2007

	Puerto Rico	Nation
Extended	#	8
Satisfactory	1	11
Partial	2	13
Minimal	2	3
Incorrect	80	59
Omitted	15	5

Extended A response with a correctly labeled spinner (6 blue sections and 3 red sections), the correct number of blues (6) and reds (3) on the answer lines, and a correct and complete explanation

Satisfactory A response with a correctly labeled spinner and the correct number of blues and reds on the answer lines, with an incomplete explanation

OR

A response with a correct and complete explanation indicating 6 blues and 3 reds, but the spinner was not labeled, or the number of blues and reds was incorrect or not given on the answer lines

Partial A response with a correctly labeled spinner and/or the correct number of blues and reds on the answer lines, but the explanation was incorrect or not given

OR

A response with a correct and complete explanation that did not have the correct number of blues and reds on the answer lines and did not have a correctly labeled spinner

Minimal A response that labeled the spinner or indicated on the answer lines that there were more blues than reds on the spinner, with an explanation related to chance

OR

A response with 6 reds and 3 blues on the spinner or on the answer lines, with an explanation related to chance

OR

A response that explained a process of 2 reds to 1 blue consistent with the labeling on the spinner

Incorrect All incorrect responses

Rounds to zero.
NOTE: Detail may not sum to totals because a small percentage of responses that did not address the assessment task are not shown.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample question 15 addresses the data representation subtopic. This subtopic includes questions about data presented in histograms, line graphs, scatterplots, box plots, circle graphs, stem and leaf plots, frequency distributions, tables, and bar graphs.

Sample question 16 addresses the subtopic of characteristics of data sets. This subtopic includes questions about statistical measures that describe data sets, such as the mean, median, mode, range, interquartile range, and standard deviation; the effect of outliers; and scatterplots.

Grade 8

Sample Multiple-Choice Question

Sample question 15 asks students to identify an appropriate graphical representation of a data set of recorded temperatures over time. This question was included in a section that allowed the use of a calculator. The framework objective measured in this question is to determine whether information is represented effectively and appropriately (histograms, line graphs, scatterplots, circle graphs, and bar graphs) given a graph or a set of data.

In Puerto Rico, 56 percent of grade 8 students answered this question correctly (choice C). The incorrect answer choices for the question are various types of data graphs.

Percentage of students in each response category at grade 8: 2007

	Puerto Rico	Nation
Choice A	13	4
Choice B	20	4
Choice C	56	84
Choice D	3	3
Choice E	7	5
Omitted	1	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Question 15

Which of the following types of graph would be best to show the change in temperature recorded in a city every 15 minutes over a 24-hour period?

(A) Pictograph
(B) Circle graph
⬤ Line graph
(D) Box-and-whisker plot
(E) Stem-and-leaf plot

Sample Constructed-Response Question

Sample question 16 is set in a real-world context and requires students to reason about measures of central tendency—in this case, the mean and the median. A correct answer for this question requires an understanding that 10 customers on Day 4 is an outlier for this set of data and therefore skews the mean. This question was included in a section that allowed the use of a calculator. The framework objective for this question asks students to calculate, use, or interpret mean, median, mode, or range.

Student responses for this question were rated using the following three-level scoring guide:

Correct A response that stated the median best represents the typical number of customers, with an explanation indicating that one number (10) that is very different from the others has an effect on the mean but not on the median

Partial A response that stated the median best represents the typical number of customers, with an incomplete, incorrect, or missing explanation

OR

A response that stated the mean best represents the typical number of customers, with an explanation indicating that one number (10) that is very different from the others has an effect on the mean but not on the median

Incorrect All incorrect responses

In Puerto Rico, less than 1 percent of the grade 8 student responses were rated "Correct," and 20 percent of the responses were rated "Partial."

Sample Question 16

The table below shows the number of customers at Malcolm's Bike Shop for 5 days, as well as the mean (average) and the median number of customers for these 5 days.

Number of Customers at Malcolm's Bike Shop

Day 1	100
Day 2	87
Day 3	90
Day 4	10
Day 5	91
Mean (average)	75.6
Median	90

Which statistic, the mean or the median, best represents the typical number of customers at Malcolm's Bike Shop for these 5 days?

Explain your reasoning.

Percentage of students in each response category at grade 8: 2007

	Puerto Rico	Nation
Correct	#	5
Partial	20	21
Incorrect	49	68
Omitted	30	6

Rounds to zero.
NOTE: Detail may not sum to totals because a small percentage of responses that did not address the assessment task are not shown.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Algebra

The concepts of algebra provide students with a powerful tool for understanding mathematics. A main focus in the study of algebra is representation, for example, using variables, functions, and coordinate geometry. Symbolic algebra easily and efficiently provides methods to represent problems and solve equations.

At grade 4, students are assessed in their under-standing of algebraic representation, patterns, and rules; graphing points on a line or a grid; and using symbols to represent unknown quantities. Much of the emphasis of the algebra content area at grade 4 is on recognizing, describing, and extending patterns and rules. At grade 8, the emphasis is on students' understanding of patterns and functions; algebraic representations; algebraic expressions, equations, and inequalities; and linearity, including slope.

Subtopics in the algebra content area are

• Patterns, relations, and functions
• Algebraic representations
• Variables, expressions, and operations
• Equations and inequalities

Student Results

In Puerto Rico, the averages of the question scores in algebra were 0.25 at grade 4 and 0.23 at grade 8 (figures 11 and 12). Both scores were lower than those for the nation.

At both grades in Puerto Rico, the average of the question scores in algebra for male students was not significantly different from that of female students. Results by gender for students in the nation varied by grade, where the score for male students was higher than the score for female students at grade 8 but showed no significant difference at grade 4.

The next few pages contain sample questions from the algebra content area in the 2007 NAEP mathematics assessment.

Figure 11.   Average of the question scores in NAEP mathematics for algebra at grade 4, by gender: 2007
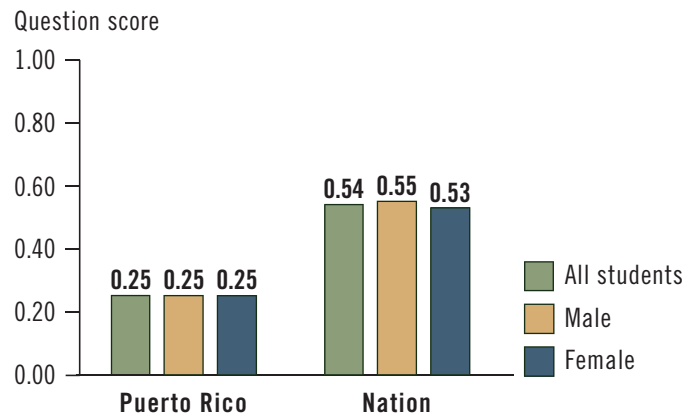
Question score

Figure 12.   Average of the question scores in NAEP mathematics for algebra at grade 8, by gender: 2007
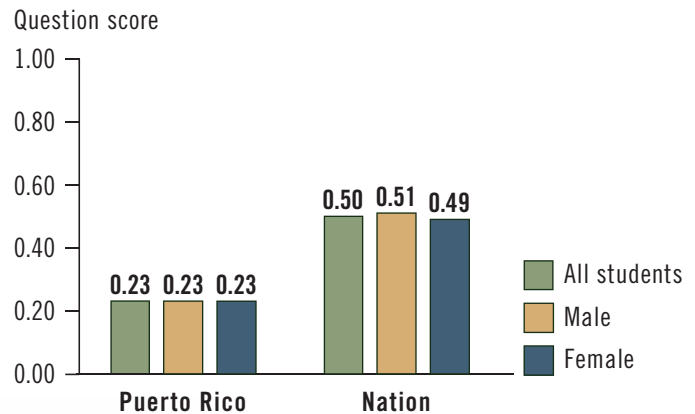
Question score

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Grade 4

Sample question 17 addresses the patterns, relations, and functions subtopic. This subtopic includes questions about recognizing, describing, and extending patterns and rules.

Sample question 18 addresses the variables, expressions, and operations subtopic. This subtopic includes questions about representing unknown quantities with symbols and expressing simple mathematical relationships in number sentences.

Sample Multiple-Choice Question

Sample question 17 describes a pattern and asks students to identify a number that will be included in the sequence generated by the pattern. The first three terms of the sequence are given (14, 26, 38), and applying the rule gives the next two terms in the sequence: 50 and 62. The framework objective measured in this question is to extend or find a missing term in a pattern or sequence given a description.

In Puerto Rico, 27 percent of grade 4 students answered this question correctly (choice D). The incorrect answer choices for the question may result from incorrectly computing subsequent terms of the sequence.

Percentage of students in each response category at grade 4: 2007

	Puerto Rico	Nation
Choice A	36	30
Choice B	13	4
Choice C	11	7
Choice D	27	55
Omitted	14	3

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Question 17

14, 26, 38, ___ , ___

The numbers in the pattern above are increasing by 12. Which of these numbers is part of the pattern?

(A) 52
(B) 58
(C) 60
● 62

Sample Multiple-Choice Question

Sample question 18 presents a balanced scale with cubes weighing 3 pounds and a cylinder weighing N pounds. The question asks for a number sentence (equation) that models the situation. This question was included in a section that allowed the use of a calculator. The framework objective measured in this question addresses expressing simple mathematical relationships using number sentences.

In Puerto Rico, 56 percent of the grade 4 students answered this question correctly (choice A). Some misconceptions represented by the incorrect answer choices for this question are given below:

- The equation indicates the number of cubes on the right side of the scale instead of the total weight (12) of these cubes (choice B).

- The equation indicates the number of cubes on the left side of the scale instead of the total weight (6) of these cubes (choice C).

- The equation indicates the number of cubes on both sides of the scale instead of the total weights of these cubes (choice D).

Sample Question 18

The weights on the scale above are balanced. Each cube weighs 3 pounds. The cylinder weighs N pounds. Which number sentence best describes this situation?

- $6 + N = 12$
- B $6 + N = 4$
- C $2 + N = 12$
- D $2 + N = 4$

Percentage of students in each response category at grade 4: 2007

	Puerto Rico	Nation
Choice A	56	79
Choice B	16	5
Choice C	17	9
Choice D	9	5
Omitted	1	2

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample question 19 addresses the equations and inequalities subtopic. This subtopic focuses on questions about linear equations and inequalities, graphs of lines, and using common formulas.

Sample question 20 addresses the algebraic representations subtopic. This subtopic includes questions about analyzing, interpreting, and translating among different representations (symbolic, graphical, tabular, verbal, and pictorial) of a linear relationship; representing points in a rectangular coordinate system; and recognizing common nonlinear relationships in meaningful contexts.

Grade 8

Sample Multiple-Choice Question

Sample question 19 presents the equation of a line in the *xy*-coordinate plane and asks students to identify the graph of the equation. The framework objective for this question is about interpreting relationships between symbolic linear expressions and graphs of lines by identifying and computing slope and intercepts (e.g., know in $y = ax + b$, that a is the rate of change and b is the vertical intercept of the graph).

In Puerto Rico, 13 percent of the grade 8 students answered this question correctly (choice A). The incorrect answer choices for the question are other lines in the plane.

Percentage of students in each response category at grade 8: 2007

	Puerto Rico	Nation
Choice A	13	25
Choice B	19	15
Choice C	23	15
Choice D	25	31
Choice E	17	13
Omitted	3	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Question 19

Which of the following is the graph of the line with equation $y = -2x + 1$?

Sample Multiple-Choice Question

Sample question 20 asks students to identify whether given algebraic expressions are equivalent to the expression 2x for all values of x. This question was included in a section that allowed the use of a calculator. The framework objective measured in this question is to translate between different representations of linear expressions using symbols, graphs, tables, diagrams, or written descriptions.

In Puerto Rico, 8 percent of grade 8 students correctly identified whether or not all of the three expressions are equivalent to 2x. The percentage of students who correctly answered each part of the question is shown below. The correct answer for parts (a) and (b) of this question is "Yes," and the correct answer for part (c) is "No."

	Puerto Rico	Nation
Part (a)		
Correct	76	90
Incorrect	16	8
Omitted	8	2
Part (b)		
Correct	24	40
Incorrect	57	54
Omitted	18	6
Part (c)		
Correct	40	65
Incorrect	38	28
Omitted	21	6

NOTE: Detail may not sum to totals because of rounding.

Percentage of students in each response category at grade 8: 2007

	Puerto Rico	Nation
Three correct	8	30
Two correct	34	38
One correct	48	28
None correct	7	2
Omitted	3	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Sample Question 20

Consider each of the following expressions. In each case, does the expression equal 2x for all values of x?

Fill in one oval to indicate YES or NO for each expression.

		Yes	No
(a)	2 times x	⬤	◯
(b)	x plus x	⬤	◯
(c)	x times x	◯	⬤

Technical Notes and Data Appendix

Sampling and Participation

Samples of schools and students in Puerto Rico were selected to be representative of their jurisdiction. To ensure unbiased samples, NCES and the Governing Board established participation rate standards that states and jurisdictions were required to meet in order for their results to be reported. School participation rates for the original sample needed to be at least 85 percent to meet reporting requirements.

In Puerto Rico, approximately 2,800 students from 100 public schools at each grade (4 and 8) participated in the 2007 NAEP mathematics assessment. School and student participation information for Puerto Rico and the nation is presented in table A-1. The school participation rate for Puerto Rico was 100 percent at each grade, and student participation rates were 96 percent for grade 4 and 94 percent for grade 8. For public schools in the nation, the school participation rates were 100 percent at both grades, and student participation rates were 95 percent at grade 4 and 92 percent at grade 8.

Table A-1. School and student participation rates in NAEP mathematics for public school students in Puerto Rico and the nation, by grade: 2007

Grade	School participation		Student participation	
	Number of schools participating	Weighted percent	Number of students assessed	Weighted percent
Grade 4				
Puerto Rico	100	100	2,800	96
Nation	7,300	100	189,800	95
Grade 8				
Puerto Rico	100	100	2,800	94
Nation	6,400	100	147,300	92

NOTE: The numbers of schools and students are rounded to the nearest hundred.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Translation

The 2007 NAEP mathematics assessment was translated into Spanish specifically for use in Puerto Rico. The translation process was designed to produce an assessment that was psychometrically equivalent to the English version of the assessment administered to students in the national sample. Rather than a word-by-word translation from English to Spanish, the process began with the translation of the assessment by a team of bilingual assessment specialists, and included thorough reviews by editorial and fairness reviewers, mathematics teachers designated by the Puerto Rico Department of Education, and Spanish language content experts. During the translation and review processes, mathematics textbooks used in Puerto Rico were consulted to ensure plausible contexts and accurate mathematical terminology. In addition, NCES carried out an independent translation verification review process to ensure the adequacy of wording in the context of Puerto Rico. Reviews at different stages provided valuable perspectives of how students in Puerto Rico might interpret specific wording and respond to certain contexts.

For students who are English-speaking Spanish language learners, a bilingual version of the assessment specifically designed for Puerto Rico was offered with English and Spanish versions of questions presented on facing pages.

Accommodations and Exclusions

Testing accommodations, such as extra testing time or individual rather than group administration, are provided in the NAEP assessments for students with disabilities (SD) who could not fairly and accurately demonstrate their abilities without modified test administration procedures. In addition, accommodations are offered for students in the national sample identified as English language learners (ELL). For the Spanish-language version of the NAEP mathematics assessment in Puerto Rico, where the majority of students are Spanish speakers, accommodations were offered to students identified as Spanish language learners (SLL). Less than 1 percent of students in Puerto Rico were identified as SLL in 2007. Almost all of these SLL students were assessed with the bilingual accommodation, in which both English and Spanish versions of assessment questions were provided.

Even with the availability of accommodations, there still remains a portion of students excluded from the NAEP assessment due to jurisdictions' policies and practices regarding the identification and inclusion of SD and/or ELL students (SLL in Puerto Rico). The percentages of public school students with disabilities excluded and accommodated in Puerto Rico and the nation are presented in table A-2. More information

about NAEP's policy on inclusion of special-needs students is available at http://nces.ed.gov/nationsreportcard/about/inclusion.asp.

Reporting Puerto Rico Results on the NAEP Scale

In 2005, average scale score results for Puerto Rico were reported on the NAEP scale and provided the opportunity of investigating the validity of those results (see more details in the *Technical Report on the NAEP Mathematics Assessment in Puerto Rico: Focus on Statistical Issues*). Although average scale score results were also reported for the 2003 NAEP mathematics assessment in Puerto Rico, changes in the translation of the questions between 2003 and 2005 prevented direct comparisons between the results for these two years.

As with all NAEP data, the results for Puerto Rico went through extensive quality control procedures, including checking for consistency between changes over time in the average of the question scores and changes in the average scale score. Specifically, changes in question-level performance over time generally predict changes in scale scores. While the relationship is found to be consistent for other states and jurisdictions participating in NAEP, the pattern was found to be different in Puerto Rico. Therefore, the tentative scale score results for Puerto Rico in 2007 were determined not to accurately reflect student performance. As such, only the average of the question score results from the 2007 assessment are presented in this report. NCES is continuing to investigate ways to make meaningful comparisons between the performance of students in Puerto Rico and students in the nation.

Table A-2. **Students with disabilities identified, excluded, and accommodated in NAEP mathematics as a percentage of all public school students in Puerto Rico and the nation, by grade: 2007**

Grade	Identified	Excluded	Assessed without accom-modations	Assessed with accom-modations
Grade 4				
Puerto Rico	20	#	1	19
Nation	14	3	3	8
Grade 8				
Puerto Rico	14	1	1	12
Nation	13	4	2	6

\# Rounds to zero.
NOTE: Less than 1 percent of students in Puerto Rico were identified as Spanish language learners (SLL) in 2007. Almost all of these SLL students were assessed with the bilingual accommodation. Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

The Question Score

The question score provides a way of showing how students perform on multiple-choice and constructed-response questions. For a multiple-choice question or a constructed-response question that is scored either "Correct" or "Incorrect," the question score is the percentage of correct responses expressed as a decimal. For a constructed-response question in which students could earn partial credit if they did not have a completely correct response, the question score is computed by adding the percent of students receiving full credit to a fraction of the percent of students receiving partial credit.

An example of computing the question score for a constructed-response question is provided below for the grade 4 sample question in this report that asks students to identify a property of odd numbers. Responses to this question were scored "Correct," "Partial," or "Incorrect." For Puerto Rico, 12 percent of the students gave a fully correct answer, and an additional 24 percent of the students gave a partial answer. The question score for this question was computed as: $12 + \frac{1}{2}(24) = 24$ or 0.24 when expressed as a decimal. The partial results were weighted by $\frac{1}{2}$ because there were two levels of credit ("Correct" and "Partial") for the question. Partial responses to a question with four levels of credit ("Extended," "Satisfactory," "Partial," and "Minimal") would receive weights of $\frac{3}{4}$ (Satisfactory), $\frac{1}{2}$ (Partial), and $\frac{1}{4}$ (Minimal). The fractions applied to partial responses are derived from the reciprocal of the number of credit levels for the question.

The question scores for both multiple-choice and constructed-response questions take into account those students who answered the question incorrectly, as well as those who reached the question but did not attempt to answer it. However, students who did not reach the question are not included in the calculation of the question score. A student is considered to have not reached a question when neither that question nor any subsequent question in the test section has been answered.

Tables A-3 and A-4 list the NAEP 2007 mathematics questions for grades 4 and 8 that were released to the public after the assessment. They are organized by content area and increasing order of difficulty for students in Puerto Rico. For comparison purposes, the question score for each question is also presented for public school students in the nation (excluding Puerto Rico). Significant differences between question scores for the nation and Puerto Rico are noted.

Table A-3.   Question scores on selected NAEP mathematics questions at grade 4, by content area: 2007

Number properties and operations	Puerto Rico	Nation
Order four-digit numbers from smallest to largest	0.49	0.83*
Identify place value representation of a number	0.47	0.87*
Find a sum based on place value	0.45	0.81*
Determine problem easiest to solve by mental math	0.43	0.70*
Identify fraction modeled by picture	0.32	0.80*
Find sum of numbers represented by base ten model	0.30	0.67*
Use place value to recognize a number	0.29	0.42*
Identify odd and even numbers	0.29	0.76*
Recognize property of odd numbers	0.24	0.65*
List fractions equivalent to given fractions	0.24	0.60*
Solve story problem involving addition and subtraction	0.22	0.36*
Compare unit fractions to solve a problem	0.21	0.41*
Use place value to determine amount of increase	0.20	0.61*
Interpret calculator output in a context [1]	0.19	0.29*
Find total quantity based on place value	0.19	0.58*
Determine what fraction of a figure is shaded [1]	0.19	0.78*
Solve a story problem involving subtraction [1]	0.17	0.73*
Solve story problem involving multiple operations [1]	0.13	0.21*
Solve story problem requiring multiple operations	0.12	0.36*
Identify all numbers having a certain factor [1]	0.08	0.37*
Calculate total cost using sales tax table [1]	0.06	0.35*
Use unit costs to solve a problem [1]	0.02	0.17*

Measurement	Puerto Rico	Nation
Determine attribute being measured from picture	0.64	0.75*
Recognize a reasonable measurement for an object	0.55	0.89*
Convert a quantity of pints to quarts [1]	0.47	0.65*
Estimate length of rope [1]	0.43	0.78*
Solve problem involving time [1]	0.31	0.49*
Identify best unit for measuring an object	0.27	0.40*
Find area of square with inscribed triangle	0.26	0.48*
Read a speedometer to solve a problem	0.19	0.63*
Find figure with same perimeter as given figure	0.15	0.37*
Explain how to find perimeter of a given shape	0.08	0.53*
Determine cost to carpet a room [1]	0.04	0.24*

Geometry	Puerto Rico	Nation
Reason to identify figure based on description	0.69	0.90*
Locate an object in a picture [1]	0.59	0.91*
Find number of right angles in a picture	0.42	0.41
Identify 3-D shape resulting from folding paper [1]	0.42	0.65*
Determine number of blocks used to build a figure	0.27	0.56*
Outline different squares in a figure	0.18	0.47*
Outline different triangles in a figure	0.11	0.38*
Outline non-rectangular, four-sided shape in a figure	0.01	0.13*

Data analysis and probability	Puerto Rico	Nation
Read information from a pictograph	0.35	0.84*
Complete a bar graph	0.29	0.79*
Determine probability of a specific outcome	0.21	0.64*
Identify color with highest chance of being chosen	0.17	0.56*
Identify picture representing greatest probability [1]	0.14	0.45*
Add data to a bar graph	0.06	0.42*
Label a spinner to satisfy a given condition	0.03	0.24*

Algebra	Puerto Rico	Nation
Identify number sentence that models balanced scale [1]	0.56	0.79*
Identify expression that represents a scenario	0.47	0.81*
Find missing figure in a pattern of shapes	0.37	0.45*
Identify number that would be in a pattern	0.27	0.55*
Relate input to output from a table of values [1]	0.23	0.19*

* Significantly different ($p < .05$) from Puerto Rico.
[1] This question was included in a section that allowed the use of a calculator.
NOTE: Regular type denotes a constructed-response question. *Italic* type denotes a multiple-choice question. The question score is expressed as a fraction of the maximum possible score and ranges from 0.00 to 1.00.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

Table A-4.   Question scores on selected NAEP mathematics questions at grade 8, by content area: 2007

Number properties and operations	Puerto Rico	Nation	Data analysis and probability	Puerto Rico	Nation
Use place value to write a number	0.70	0.81*	*Determine most appropriate graph for a situation* [1]	0.56	0.84*
Determine fraction of figure shaded	0.67	0.89*	*Find total cost based on unit price for a quantity*	0.38	0.80*
Find total charges given operating costs [1]	0.42	0.67*	*Identify relationship in scatterplot* [1]	0.26	0.61*
Estimate time given rate and distance	0.40	0.71*	Explain which survey is better [1]	0.25	0.68*
Identify number line representing given situation [1]	0.35	0.58*	*Determine an expected outcome*	0.20	0.59*
Convert raw points to a percentage [1]	0.28	0.62*	Use graph to identify incorrect written statements [1]	0.12	0.39*
Solve a story problem involving costs and profit [1]	0.28	0.52*	*Find number of combinations satisfying a condition*	0.12	0.18*
Divide to find greatest whole quantity [1]	0.28	0.61*	Explain which statistic is best in given situation [1]	0.10	0.16*
Identify number rounded to nearest hundred	0.25	0.58*	Determine probability of a given event [1]	0.01	0.33*
Divide large numbers in a given context [1]	0.19	0.50*			
Analyze a mathematical relationship [1]	0.15	0.42*	**Algebra**		
Determine distance given rate and time	0.12	0.36*	Solve system of linear equations given in context	0.56	0.73*
Interpret number expressed in scientific notation [1]	0.11	0.33*	*Recognize equivalent expressions* [1]	0.47	0.65*
Convert sum of fractions to decimal number using place value [1]	0.07	0.45*	*Determine an expression to model a scenario*	0.43	0.72*
Identify fractions listed in ascending order [1]	0.04	0.49*	*Identify point on a graph with specified coordinates*	0.33	0.78*
			Solve an algebraic equation [1]	0.32	0.82*
Measurement			*Use algebra to solve a story problem*	0.31	0.46*
Determine dimensions that give greatest volume	0.59	0.75*	Complete a table and write an algebraic expression [1]	0.30	0.55*
Recognize unit associated with specific attribute [1]	0.47	0.68*	*Determine equation relating sales and profit* [1]	0.24	0.53*
Solve story problem involving milliliters and liters [1]	0.27	0.36*	*Evaluate an expression for a specific value* [1]	0.19	0.79*
Estimate side length of a square given area [1]	0.22	0.49*	*Identify the graph of a linear equation*	0.13	0.25*
Approximate the length of the radius of a circle	0.13	0.19*	*Use formula to solve a problem* [1]	0.11	0.49*
Determine value of marks on scale	0.06	0.48*	*Convert temperature from Fahrenheit to Celsius* [1]	0.09	0.35*
			Extend pattern of shapes and find perimeter [1]	0.04	0.29*
Geometry					
Identify shape formed after folding paper [1]	0.32	0.41*			
Recognize shape formed by overlapping figures [1]	0.31	0.53*			
Determine measure of angle in triangle [1]	0.31	0.54*			
Describe region of intersection of two rays	0.29	0.40*			
Find angle measure of minor arc of a circle [1]	0.28	0.34*			
Identify type of triangle from picture	0.23	0.44*			
Use similarity of right triangles to solve problem [1]	0.08	0.12*			
Determine if given figure is parallelogram and explain	0.05	0.26*			
Assemble given shapes and determine total area [1]	0.05	0.28*			
Find container height given dimensions of contents [1]	0.03	0.27*			

* Significantly different ($p < .05$) from Puerto Rico.
[1] This question was included in a section that allowed the use of a calculator.
NOTE: Regular type denotes a constructed-response question. *Italic* type denotes a multiple-choice question. The question score is expressed as a fraction of the maximum possible score and ranges from 0.00 to 1.00.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2007 Mathematics Assessment.

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SUGGESTED CITATION

Dion, G.S., Kuang, M., and Dresher, A.R. (2008). *The Nation's Report Card: Mathematics 2007 Performance of Public School Students in Puerto Rico—Focus on the Content Areas* (NCES 2009–451). National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education, Washington, D.C.

CONTENT CONTACT

Emmanuel Sikali
202-502-7419
emmanuel.sikali@ed.gov