## 7.0 - Chapter Introduction

In this chapter, you will learn improvement curve concepts and their application to cost and price analysis.

*Basic Improvement Curve Concept*. You may have learned about improvement curves using the name learning curve analysis. Today, many experts feel that the term learning curve implies too much emphasis on learning by first-line workers. They point out that the theory is based on improvement by the entire organization not just first-line workers. Alternative names proposed for the theory include: improvement curve, cost-quantity curve, experience curve, and others. None have been universally accepted. In this text, we will use the term improvement curve to emphasize the need for efforts by the entire organization to make improvements to reduce costs.

Just as there are many names for the improvement curve, there are many different formulations. However, in each case the general concept is that the resources (labor and/or material) required to produce each additional unit decline as the total number of units produced over the item's entire production history increases. The concept further holds that decline in unit cost can be predicted mathematically. As a result, improvement curves can be used to estimate contract price, direct labor-hours, direct material cost, or any other recurring contract cost.

*Improvement Curve History*. The improvement curve is based on the concept that, as a task is performed repetitively, the time required to perform the task will decrease. Management planners have followed that element of the concept for centuries, but T. P. Wright pioneered the idea that improvement could be estimated mathematically. In February 1936, Wright published his theory in the *Journal of Aeronautical Sciences* as part of an article entitled "Factors Affecting the Cost of Airplanes." Wright's findings showed that, as the number of aircraft produced in sequence increased, the direct labor input per airplane decreased in a regular pattern that could be estimated mathematically.

During the mobilization for World War II, both aircraft companies and the Government became interested in the theory. Among other considerations, the theory implied that a fixed amount of labor and equipment could be expected to produce larger and larger quantities of defense products as production continued.

After World War II, the Government engaged the Stanford Research Institute (SRI) to study the validity of the improvement curve concept. The study analyzed essentially all World War II airframe direct labor input data to determine whether there was sufficient evidence to establish a standard estimating model. The SRI study validated a mathematical model based on the World War II findings that could be used as a tool for price analysis. However, that model was slightly different than the one originally offered by Wright.

Since World War II, the improvement curve concept has been used by Government and industry to aid in pricing contracts. Over the years, the improvement curve has been used as a contract estimating and analysis tool in a variety of industries including: airframes, electronics systems, machine tools, shipbuilding, missile systems, and depot level maintenance of equipment. Improvement curves have also been applied to service and construction contracts where tasks are performed repetitively.

*Identifying Basic Improvement Curve Theories*. Since 1936, many different formulations have been proposed to explain and estimate the improvement that takes place in repetitive production efforts. Of these, the two most popular are the unit improvement curve and the cumulative average improvement curve

- **Unit Improvement Curve.** The unit improvement curve is the model validated by the post-World War II SRI study. The formulation is also known by two other names: Crawford curve, after one of the leaders of the SRI research; and Boeing curve, after one of the firms that first embraced its use.
    - Unit curve theory can be stated as follows:

**As the total volume of units produced doubles the cost per unit decreases by some constant percentage.**

- The constant percentage by which the costs of doubled quantities decrease is called the rate of learning. The term "slope" in the improvement curve analysis is the difference between 100 percent and the rate of improvement. If the rate of improvement is 20 percent, the improvement curve slope is 80 percent (100 percent - 20 percent). The calculation of slope is described in detail later in the chapter.

   - Unit curve theory is expressed in the following equation:

$$Y = AX^B$$

Where:

Y = Unit cost (hours or dollars) of the $X^{th}$ unit

X = Unit number

A = Theoretical cost (hours or dollars) of the first unit sometimes called $t_1$.

B = Constant that is related to the slope and the rate of change of the improvement curve. It is calculated from the relationship:

$$B = \frac{\text{Logarithm of the Slope}}{\text{Logarithm of 2}}$$

In calculating B, the slope MUST be expressed in decimal form rather than percentage form. Then B will be a negative number, leading to the decreasing property stated above.

- **Cumulative Average Improvement Curve.** The cumulative average improvement curve is the model first introduced by Wright in 1936. Like the unit improvement curve, the cumulative average curve is also known by two other names: Wright Curve, after T.P. Wright; and Northrop Curve, after one of the firms that first embraced its use.
   - Cumulative average theory can be stated as follows:

**As the total volume of units produced doubles the average cost per unit decreases by some constant percentage.**

   - As with the unit improvement curve, the constant percentage by which the costs of doubled quantities decrease is called the rate of improvement. The slope of the improvement curve analysis is the difference between 100 percent and the rate of learning. However, the rate of improvement and the slope are measured using cumulative averages rather than the unit values used in unit improvement curve analysis.

- Unit curve theory is expressed in the following equation:

$Y = AX^B$

Where:

$\bar{Y}$ = Cumulative average unit cost (hours or dollars) of units through the $X^{th}$ unit

All other symbols have the same meaning used in describing the unit improvement curve.

- **Curve Differences.** Note that the only difference between definitions of the unit improvement curve and the cumulative average improvement curve theories is the word **average**. In the unit curve, unit cost is reduced by the same constant percentage. In the cumulative average curve, the cumulative average cost is reduced by the some constant percentage.
  - The most significant practical difference between the two different formulations is found in the first few units of production. Over the first few units, an operation following the cumulative average curve will experience a much greater reduction in cost (hours or dollars) than an operation following a unit curve with the same slope. In later production, the reduction in cost for an operation following a cumulative average curve will be about the same as an operation following a unit curve with the same slope.
  - Because of the difference in early production, many feel that the unit curve should be used in situations where the firm is fully prepared for production; and the cumulative average curve should be used in situations where the firm is not completely ready for production. For example, the cumulative average curve should be used in situations where significant tooling or design problems may NOT be completely resolved. In such situations, the production of the first units will be particularly inefficient so improvement should be rapid as problems are resolved.

- In practice, firms typically use one formulation regardless of differences in the production situation. Most firms in the airframe industry use the cumulative average curve. Most firms in other industries use the unit curve.

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## 7.1 - Identifying Situations For Use

*Situations for Use*. The improvement curve cannot be used as an estimating tool in every situation. Situations that provide an opportunity for improvement or reduction in labor hours per unit are the types of situations that lend themselves to improvement curve application. Use of the improvement curve should be considered in situations where there is:

- **A high proportion of manual labor.** It is more difficult to reduce the labor input when there is limited labor effort, the labor effort is machine paced, or individual line workers only touch the product for a few seconds.

- **Uninterrupted production.** As more and more units are produced the firm becomes more adept at production and the labor hour requirements are reduced. If supervisors, workers, tooling, or other elements of production are lost during a break in production, some improvement will also likely be lost.

- **Production of complex items.** The more complex the item the more opportunity there is to improve.

- **No major technological change.** The theory is based on continuing minor changes in production and in the item itself. However, if there are major changes in technology, the benefit of previous improvement may be lost.

- **Continuous pressure to improve.** The improvement curve does not just happen; it requires management effort. The management of the firm must exert continuous pressure to improve. This requires investment in the people and equipment needed to obtain improvement.

*Factors that Support Improvement*. As you examine situations that appear to have potential for improvement curve application, consider management emphasis on the following factors affecting the rate of improvement:

- **Job Familiarization By Workers.** As noted earlier, many feel that this element has been overemphasized over the years. Still, workers do improve from repetition and that improvement is an important part of the improvement curve.

- **Improved Production Procedures.** As production continues, both workers and production engineers must constantly be on the lookout for better production procedures.

- **Improved Tooling and Tool Coordination.** Part of the examination of production procedures must consider the tooling used for production. Tooling improvements offer substantial possibilities for reduction of labor requirements.

- **Improved Work Flow Organization.** Improving the flow of the work can substantially reduce the labor effort that does not add value to the product. Needless movement of work in progress can add significant amounts of labor effort.

- **Improved Product Producibility.** Management and workers must constantly consider product changes that will make the product easier to produce without degrading the quality of the final product.

- **Improved Engineering Support.** The faster production problems can be identified and solved, the less production labor effort will be lost waiting for problem resolution.

- **Improved Parts Support.** As production continues, better scheduling should be possible to eliminate or significantly reduce worker time lost waiting for supplies. In addition, production materials more appropriate for production can be identified and introduced to the production process.

**7.2 - Analyzing Improvement Using Unit Data And The Unit Theory**

*Unit Theory Application*. In this text, we will only consider application of the unit improvement curve in making initial contract pricing estimates. There are many texts that address other improvement curve theories (e.g., cumulative average improvement curves), as well as many advanced issues such as the effects of contract changes, breaks in production, and retained learning.

*Improvement Illustration*. To illustrate the effect of the unit curve, assume that the first unit required 100,000 labor-hours to produce. If the slope of the curve is 80 percent slope, the following table demonstrates the labor-hours required to produce units at successively doubled quantities.

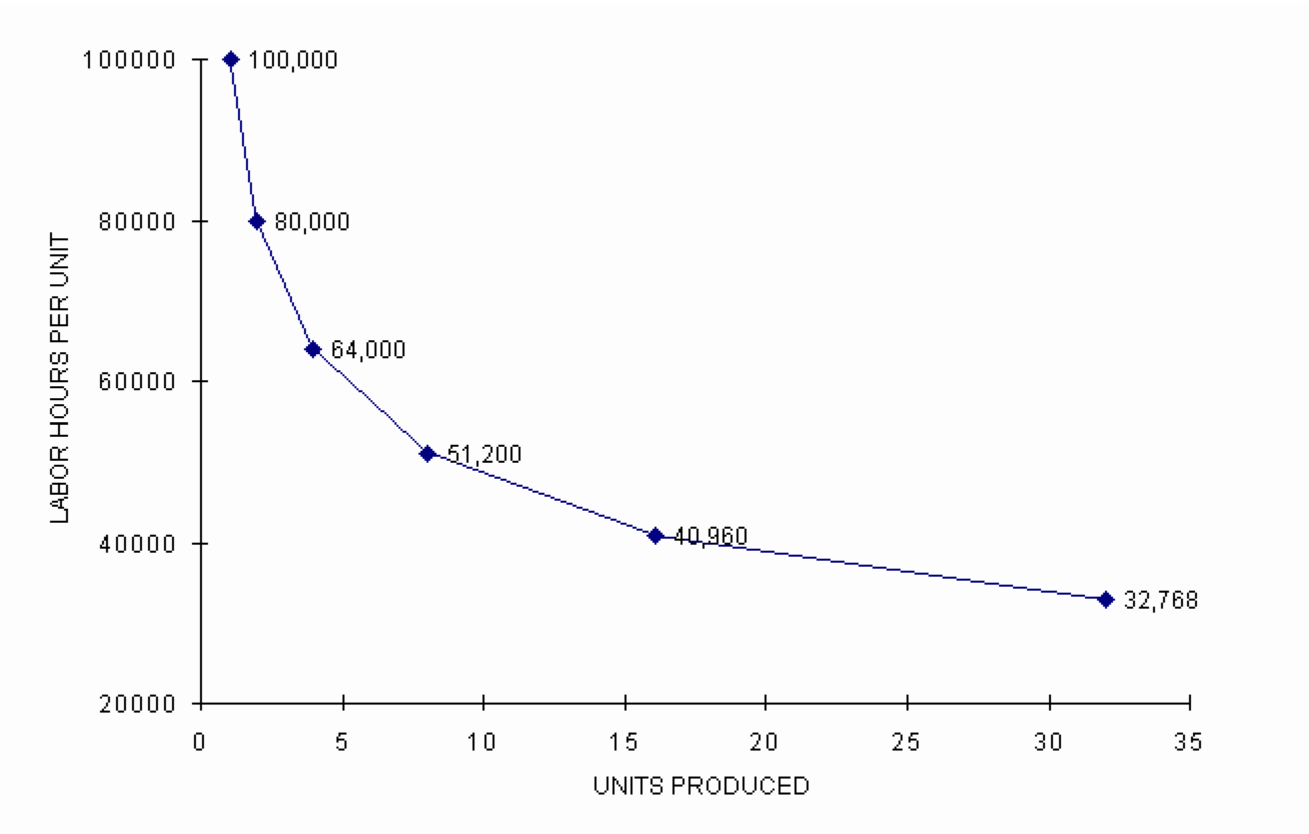
| Units Produced | LABOR-HOURS Per Unit at Doubled Quantities | Difference in LABOR-HOURS Per Unit at Doubled Quantities | Rate of Improvement (%) | Slope of Curve (%) |
|---|---|---|---|---|
| 1 | 100,000 | | | |
| 2 | 80,000 | 20,000 | 20 | 80 |
| 4 | 64,000 | 16,000 | 20 | 80 |
| 8 | 51,200 | 12,800 | 20 | 80 |
| 16 | 40,960 | 10,240 | 20 | 80 |
| 32 | 32,768 | 8,192 | 20 | 80 |

Obviously, the amount of labor-hour reduction between doubled quantities is not constant. The number of hours of reduction between doubled quantities is constantly declining. However, the rate of change or decline remains constant (20 percent).

Also note that the number of units required to double the quantity produced is constantly increasing. Between Unit #1 and Unit #2, it takes only one unit to double the quantity produced. Between Unit #16 and Unit #32, 16 units are needed.

*Graphing the Data*. Graphing the unit improvement curve demonstrates the relationship between the total units produced and unit cost.

- **Rectangular Coordinate Graph.** A labor-hour graph of this data drawn on ordinary graph paper (rectangular coordinates) becomes a curve as shown in the graph below. On this graph, equal spaces represent equal amounts of change. When thinking of numbers in terms of their absolute values, this graph presents an accurate picture, but it is difficult to make an accurate prediction from this curve.

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The graph is a curve because the number of hours of reduction between doubled quantities is constantly declining and an increasing number of units are required to double the quantity produced. Note that most of the improvement takes place during the early units of production. The curve will eventually become almost flat. The number of production hours could become quite small but it will never reach zero.

- **Log-Log Graph.** To examine the data and make predictions using unit improvement curve theory, we need to transform the data to logarithms. One way of making the transformation is through the use of log-log graph paper also known as full-logarithmic graph paper.

*Log-Log Paper.* There are several special elements that you must consider when using log-log graph paper.

- There is already a scale indicated on both the horizontal and vertical axes. Note that there are no zeros. Values can approach zero but never reach it.
- The scale only goes from "1" to "1". Each time the number scale goes from "1" to "1", the paper depicts a cycle. Each "1" moving up on the vertical axis or to the right on the horizontal axis is 10 times the "1" before it. You should mark the actual scale you are using in the margin of the log-log paper before starting to plot points.
  - In improvement curve analysis, always graph the number of the unit produced on the horizontal axis. Assign the first "1" on the left of the page a value of 1 representing the first unit produced. The second "1" is 10. The third "1" is 100. The fourth "1" is 1,000.
  - Always graph the cost in hours or dollars on the vertical axis. The scale will change depending on the data being graphed. The first "1" can be .001, .01, 1, 100, 1,000 or any other integral power of 10. Whatever the value assigned to the first "1," the next "1" is 10 times more, and the next one 10 times more than that. To determine the scale to be used:
  - Estimate the largest number to be plotted or read on the Y axis. This figure will probably be the theoretical cost of the first unit. For example, suppose this is 60,000 hours.
  - Determine the next integral power of ten above this number (e.g., the next integral power above 60,000 is 100,000).
  - Assign this value to the horizontal line at the top of the upper cycle on the Y axis. The horizontal line at the top of the next lower cycle must then represent 10,000 of the same units, and the line at the bottom of the lower cycle represents 1,000.

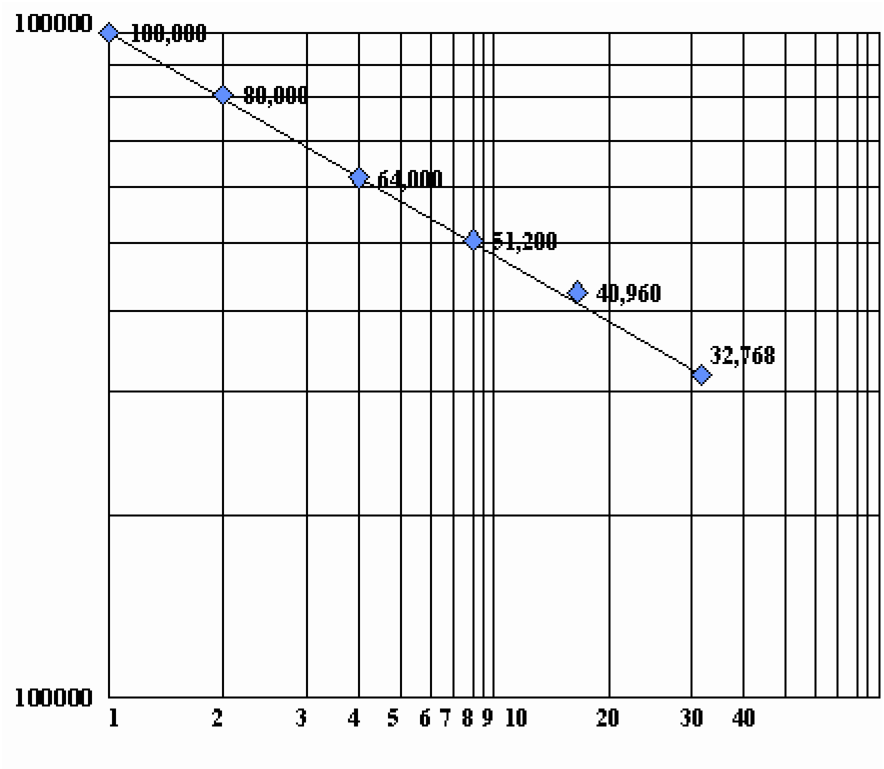
- On log-log graph paper, the distances between numbers on each axis are equal for equal percentage changes. For example, the distance between "1" and "2" is the same as between "4" and "8;" both represent a 100 percent increase.

*Log-Log Graph.* You can obtain surprisingly accurate results from a log-log graph, but your accuracy greatly depends on your graphing technique.

- Always use a sharp pencil.
- Make points plotted on the paper as small as possible and the lines as narrow as possible.
- When the smallest possible point has been marked on the paper, it may easily be lost sight of or confused with a blemish in the paper. To avoid this, draw a small ring around the point. Circles, triangles, and squares are also used to identify points which belong to different sets of data.
- Exercise great care in drawing a line. If it is supposed to go through a point, it should pass exactly through it, not merely close to it.

A graph of the data described in the example above forms a perfectly straight line when plotted on log-log paper. That is, a straight line passes exactly through each of the points. A straight line on log-log paper indicates that the rate of change is constant.

Since improvement curve theory assumes continuing improvement at a constant rate, the straight line becomes an excellent estimating tool. Assuming that improvement will continue at the same rate, the line can be extended to estimate the cost of future units.

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*Calculating the Theoretical Cost of Unit #1*. When we discuss improvement curves, we normally describe them in terms of the theoretical value for Unit #1 and the slope of the curve. With these two values, you can use graph paper, tables, or computer programs to estimate the cost of future units.

The value of Unit #1 is referred to as a **theoretical value** ($T_1$), because in most cases you will not know the actual cost of Unit #1. Instead, $T_1$ is the value indicated by the line-of-best-fit. On a graph, it is the point where the line-of-best-fit and the vertical line representing Unit #1 intersect. (**Remember:** The graph of the improvement curve always begins with Unit #1.)

*Estimating the Slope*. The term "slope" as used for improvement curves is a mathematical misnomer. It cannot be related to the definition of slope in a straight line on rectangular coordinates. Instead, the slope of an improvement curve is equal to 100 minus the rate of improvement.

- **Calculating Slope from Available Information.** You can calculate the slope of a curve, by dividing the unit cost ($Y_X$) at some unit (X) into the unit cost ($Y_{2X}$) at twice the quantity (2X) and multiplying the resulting ratio by 100.

$$\text{Slope} = 100\left(\frac{Y_{2X}}{Y_X}\right)$$

- **Graphic Measurement.** You can measure the slope of an improvement curve drawn on log-log paper by reading a cost ($Y_X$) at any quantity, X; reading a cost ($Y_{2X}$) at any quantity, 2X; dividing the second value by the first; and multiplying by 100. For example, if the number of hours to make Unit #5 is 70 and the number of hours to make Unit #10 is 50, the slope of the improvement curve is:

$$\begin{aligned}\text{Slope} &= 100\left(\frac{Y_{10}}{Y_5}\right)\\ &= 100\left(\frac{50}{70}\right)\\ &= 71.4 \text{ percent}\end{aligned}$$

- **Slope Research Data.** Labor activities will experience different rates of improvement.
  - The post-war SRI study revealed that many different slopes were experienced by different firms, sometimes by different firms manufacturing the same products. In fact, manufacturing data collected from the World War II aircraft manufacturing industry had slopes ranging from 69.7 percent to almost 100 percent. These slopes averaged 80 percent.
  - Research by DCAA in 1970 found curves ranging from less than 75 percent to more than 95 percent. The average slope was 85 percent.

- **Slope Selection and Verification.** Unfortunately, information on industry average curves is frequently misapplied by practitioners who use them as a standard or norm. Because each situation is different, you should select a slope based on your analysis of the situation and not on general averages. The order of preference in slope selection is:

- A curve developed from data pertaining to the production of the same product (as we did above).
- The median percentage from a group of curves for items having some similarity to the end item.
- The median percentage from the product category in which the item would most likely be included.

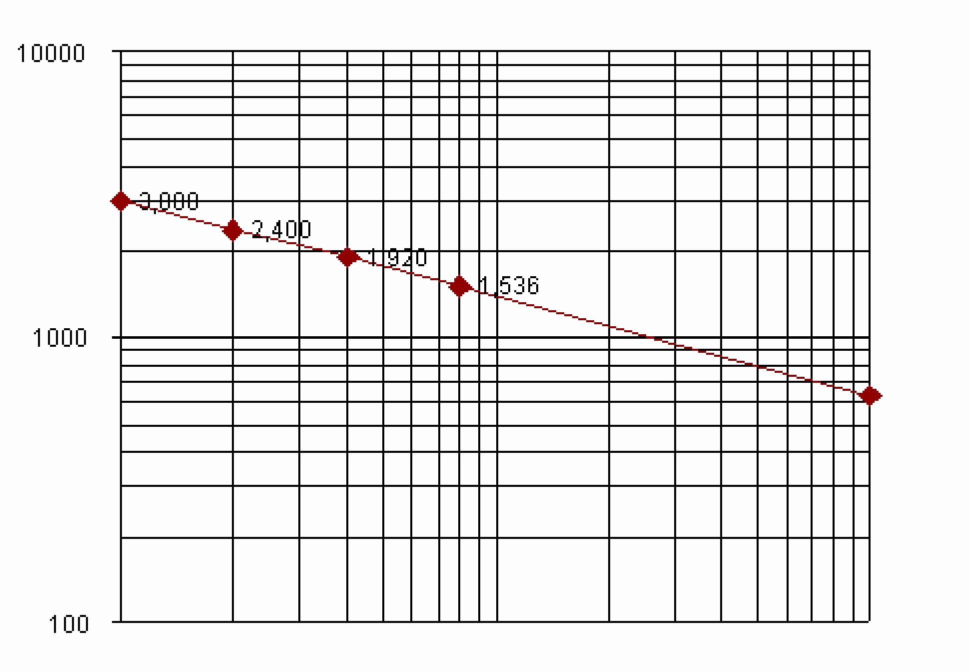
*Estimating the Cost/Price of Future Units*. The primary purpose for estimating an improvement curve is to predict the cost of future production. The prediction is based on the assumption (not always true) that the past is a good predictor of the future. In terms of the unit improvement curve theory, this assumption means that the unit cost (hours or dollars) of doubled quantities will continue to decrease by the same constant percentage.

Using a graph, you can predict future costs by drawing a line-of-best-fit through the historical data graphed on log-log paper and extending it through the unit for which you wish to make a cost estimate. Estimate cost using the Y value (cost) at the point were the two lines intersect.

For example, suppose we had the following unit cost data:

| Unit Number | Hours |
|---|---|
| 1 | 3,000 |
| 2 | 2,400 |
| 4 | 1,920 |
| 8 | 1,536 |

Plotting the data on log-log paper, you will observe a straight line with an 80 percent slope.




If you extend the line-of-best-fit to Unit #100, you can estimate the cost of Unit #100. As you can see from the graph, the extended line reveals an estimated cost of approximately 680 hours for Unit #100.

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**7.3 - Analyzing Improvement Using Lot Data And Unit Theory**

*Accounting System Data*. Use of the improvement curve is dependent on available cost data. The relevant accounting or statistical record system must be designed to make relevant data available for analysis. Costs, such as labor-hours per unit or dollars per unit, must be identified with the unit of product.

**Note:** It is preferable to use labor-hours rather than dollars since the dollars contain an additional variable, the effect of inflation or deflation, which the labor-hours do not contain.

Typically accounting systems do not record the cost of individual units. If the firm uses a job-order cost accounting, costs are accumulated on the job order in which the number of units completed are specified and costs are cut-off at the completion of the units. Process cost accounting also yields costs identified with end-item units. In this case, however, the costs are usually assigned to equivalent units produced over a period of time rather than actual units.

*Average Unit Cost*. To use unit improvement curve theory, you must be able to estimate the cost of a particular unit. Given lot or period costs, the only unit cost that we know is the average cost for the lot or period. However, we have a method for using average costs in improvement curve analysis.

For example, given the following data, we must be able to estimate the cost of an additional 40 units.

| **Lot Number** | **Lot Size (Units)** | **Lot Total Labor-Hours (Cost)** | **Lot Average Labor Hours (Cost)** |
|---|---|---|---|
| 1 | 6 | 40,800 | 6,800 |
| 2 | 9 | 40,500 | 4,500 |
| 3 | 15 | 52,500 | 3,500 |

*Calculating a Lot Plot Point for Graphic Analysis*. To graph the lot average unit cost, we must select a corresponding unit number. If we assume that costs go down during the lot, the average cost should occur at the middle of the lot - the lot mid-point. One problem is that the true lot mid-point (the unit where the average cost is incurred) depends on the slope of the improvement curve. Unfortunately, the slope of the curve also depends on the placement of the lot mid-point. The iterative process required to calculate the true lot mid-point for each lot is too cumbersome for manual computation. As a result, we use the following rules of thumb for graphic analysis:

- **For All Lots After The First Lot,** calculate the lot mid-point by **dividing the number of units in the lot by two**. Then add the resulting number to all the units produced prior to the lot to determine where the unit falls in the continuing improvement curve.

For example, what would be the plot point for a lot made up of units 91 through 100. There are 10 units in the lot, so the middle of the lot would be 5 (10 ÷ 2 = 5). Adding 5 to the 90 units produced prior to the lot, we find that the plot point would be 95.

- **For a First Lot of Less Than 10,** follow the same procedure that you follow for all lots after the first lot. Of course, the lot plot point will equal the lot mid-point because no units will have been produced prior to the first lot.

- **For a First Lot of 10 or More,** calculate the lot mid-point by **dividing the number of units in the lot by three.** This adjustment is necessary to compensate for the rapid decline in cost that takes place in the first lot of production.

Given the data above, use a table similar to the following, to calculate the necessary lot plot points and lot average hours:

| Lot No. | Lot Size | Cumulative Units | Lot Mid-Point | Lot Plot Point |
|---|---|---|---|---|
| 1 | 6 | 6 | 3.0 | 3.0 |
| 2 | 9 | 15 | 4.5 | 10.5 |
| 3 | 15 | 30 | 7.5 | 22.5 |

You can then use this information to estimate the cost of lots that have not yet been produced. For example, suppose you wanted to estimate the cost of a Lot #4 of 40 units to be produced after the 40 units described above. The final row of the table would be:

| | | | | |
|---|---|---|---|---|
| 4 | 40 | 70 | 20 | 50 |

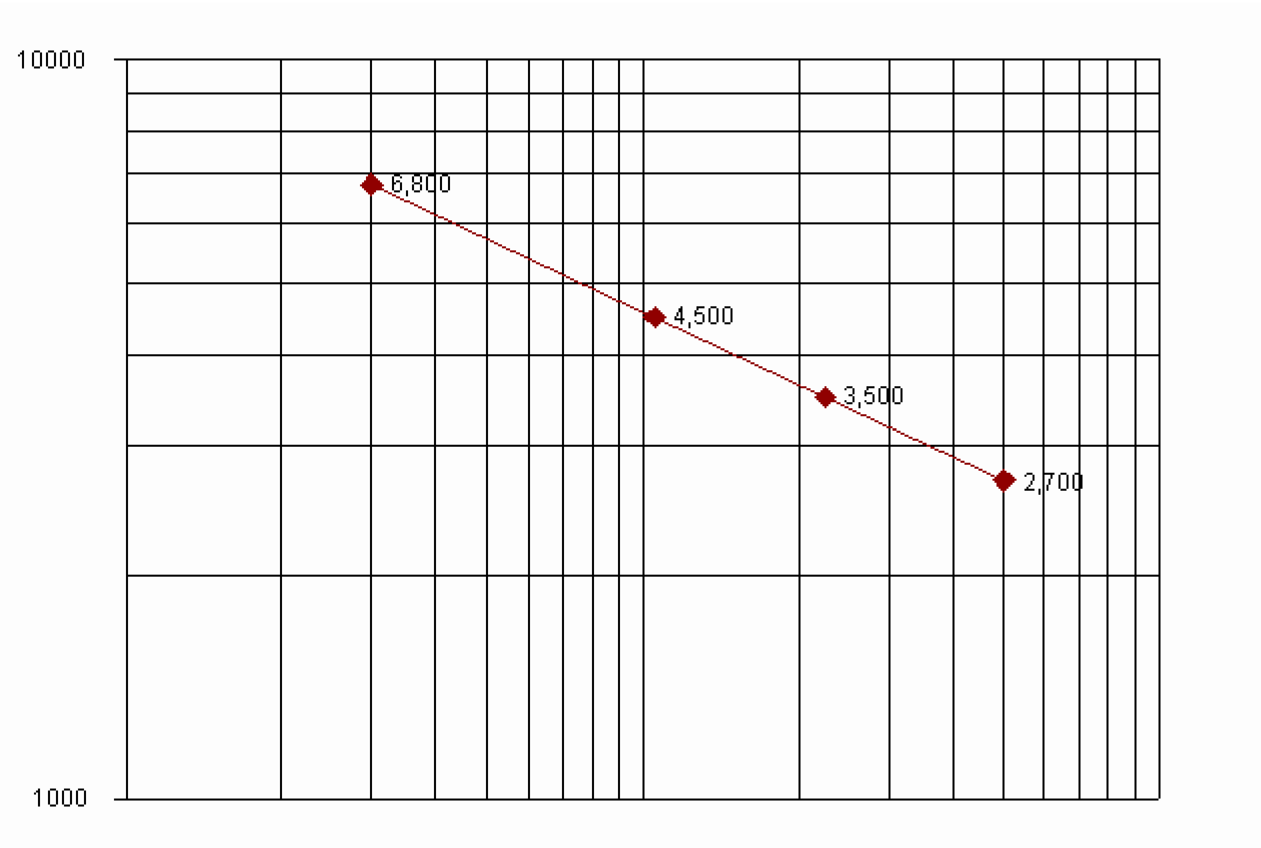
For this example, the lot plot point for Lot #4 would be at Unit #50. You would estimate the average unit cost for the lot using the cost of Unit #50.

You can combine the calculation for the lot average unit cost and the lot plot point into a single table, as shown below:

| Lot No. | Lot Size | Cumulative Units | Lot Mid-Point | Lot Plot Point | Lot Average Hours | Lot Total Hours |
|---|---|---|---|---|---|---|
| 1 | 6 | 6 | 3.0 | 3.0 | 6,800 | 40,800 |
| 2 | 9 | 15 | 4.5 | 10.5 | 4,500 | 40,500 |
| 3 | 15 | 30 | 7.5 | 22.5 | 3,500 | 52,500 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 4 | 40 | 70 | 20.0 | 50.0 | | |

*Plotting Data on a Log-Log Graph*. Plot the average lot cost data (Y) at the corresponding lot plot point (X) on log-log paper and for an improvement curve. Extend the improvement curve through Unit #50, the lot plot point for Lot #4.




On the Y axis, the lot average cost at Unit #50 is approximately 2,700 labor hours. With this information, you can estimate the cost of Lot #4 at 108,000 labor hours (i.e., 2,700 labor hours x 40 units).

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**7.4 - Fitting A Unit Curve**

*General Points to Consider*. Throughout this chapter, we have assumed that all data fit a perfectly straight line. Unfortunately, most data do not fall exactly on a straight line. You need to be able to identify a trend and fit data to that trend. You can visually fit a line using graphic analysis, but most lines-of-best-fit are developed using regression analysis.

Whatever method of analysis you use to fit an improvement curve, if a data point is a significant distance away from the trend set by other data points, look into the cause of the deviation. If your analysis indicates that the data point is not comparable with the rest of the data for some reason, consider adjusting or eliminating the data point from your analysis. However, never eliminate a data point from your analysis simply because it does not fit the apparent trend set by the remaining data.

*Graphic Analysis*. When visually fitting a straight line, graph the data then draw the line to minimize the distance between the straight line and the data points. Normally, you should give more weight to the larger lots as you fit the straight line.

When fitting a straight line on ordinary graph paper, you know that the line-of-best-fit must go through the average of the X values ($\bar{X}$) and the average of the Y values ($\bar{Y}$). When fitting a line-of-best-fit through improvement curve data on log-log paper, you have no similar fixed reference point. **Without this fixed reference point, even skilled analysts can arrive at very different lines.**

*Regression Analysis*. Normally, you can obtain more accurate results using regression analysis and a log-log transformation. Using the logarithmic values of X and Y instead of the actual values, the equation of the unit improvement curve ($Y = AX^B$) becomes:

$$\text{Log } Y = \text{Log } A + B(\text{Log } X)$$

The new equation describes a straight line ($Y = A + BX$) relationship. After this transformation, you can use regression analysis to fit a straight line to the data.

Improvement curve regression analysis programs differ in several ways including:

- **Use of True Lot Mid-Point.** In addition to the accuracy gained from using regression analysis, most improvement curve programs use the true lot mid-point rather than the rule-of-thumb calculations described earlier in this section for graphic analysis. The greatest effect of using the true lot mid-point is in the first lot. Examples of the differences between the rule-of-thumb and true lot mid-points are depicted in the following table:

| Selected First Lot Mid-Points | | | | |
|---|---|---|---|---|
| | | True-Lot Mid-Points | | |
| **Units in First Lot** | **Rule-of-Thumb** | **70% Curve** | **80% Curve** | **90% Curve** |
| 2 | 1.00 | 1.37 | 1.39 | 1.4 |
| 10 | 3.33 | 3.95 | 4.17 | 4.36 |
| 100 | 33.33 | 28.65 | 32.36 | 35.43 |
| 1,000 | 333.33 | 258.15 | 304.43 | 340.67 |
| 10,000 | 3333.33 | 2,495.48 | 3,002.85 | 3,384.18 |

Differences in calculating the lot mid-point will affect the results of the improvement curve analysis by the placement of the data points for analysis.

- **Method of Regression.** Not all improvement curve analysis programs use the same mathematical model for regression analysis. For example, some analysis programs assign a weight to each lot based on the lot size, while others do not. Software using unweighted regression considers all lots (large and small) equally. When weights are assigned to each lot based on lot size, larger lots receive more analysis consideration than smaller lots.

- **Measures of Fit.** Regardless of the regression model used to develop the line-of-best-fit, virtually all regression analysis software will provide measures of the line's goodness of fit.
    - The primary goodness of fit measure is the coefficient of determination ($r^2$) for the equation. As described in the chapter on "Using Regression Analysis," the coefficient of determination indicates the portion of variation in Y is explained by the regression line (e.g., an $r^2$ of .94 indicates that 94 percent of the variation in Y is explained by the relationship between X and Y).

- Many improvement curve analysis programs also provide the T-test for significance of the regression equation.

- **Graphic Analysis Capability.** Many regression analysis programs provide a capability to graph the data and the regression line. For most analysts, this display is one of the strongest tools for identifying anomalies in the data that affect the value of the regression analysis as an estimating tool.

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## 7.5 - Estimating Using Unit Improvement Curve Tables

*Estimating Choices* (DCAM App F). Once the cost of Unit #1, in hours or dollars, and the slope of the improvement curve have been established, we can develop estimates of future costs in several ways. You could graph the data on log-log paper and read your estimates from the graph or substitute the values into the improvement curve equation. Many analysts use an improvement curve table such as the partial table shown below.

| Partial Improvement Curve Table | | | | | | |
|---|---|---|---|---|---|---|
| | 79 Percent | | 80 Percent | | 81 Percent | |
| **Unit** | **Cum Total** | **Unit** | **Cum Total** | **Unit** | **Cum Total** | **Unit** |
| 1 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 | 1.000000 |
| 2 | 1.790000 | 0.790000 | 1.800000 | 0.800000 | 1.810000 | 0.810000 |
| 3 | 2.478245 | 0.688245 | 2.502104 | 0.702104 | 2.526065 | 0.716065 |
| 4 | 3.102345 | 0.624100 | 3.142104 | 0.640000 | 3.182165 | 0.656100 |
| 5 | 3.680837 | 0.578492 | 3.737741 | 0.595637 | 3.795233 | 0.613068 |
| 6 | 4.224550 | 0.543713 | 4.299424 | 0.561683 | 4.375245 | 0.580012 |
| 7 | 4.740494 | 0.515944 | 4.833914 | 0.534490 | 4.928703 | 0.553458 |
| 8 | 5.233533 | 0.493039 | 5.345914 | 0.512000 | 5.460144 | 0.531441 |
| 9 | 5.707214 | 0.473681 | 5.838864 | 0.492950 | 5.972892 | 0.512748 |
| 10 | 6.164223 | 0.457009 | 6.315374 | 0.476510 | 6.469477 | 0.496585 |
| 11 | 6.606656 | 0.442433 | 6.777485 | 0.462111 | 6.951880 | 0.482403 |
| 12 | 7.036189 | 0.429533 | 7.226831 | 0.449346 | 7.421690 | 0.469810 |
| 13 | 7.454188 | 0.417999 | 7.664747 | 0.437916 | 7.880206 | 0.458516 |
| 14 | 7.861784 | 0.407596 | 8.092339 | 0.427592 | 8.328507 | 0.448301 |
| 15 | 8.259928 | 0.398144 | 8.510538 | 0.418199 | 8.767503 | 0.438996 |
| 16 | 8.649429 | 0.389501 | 8.920138 | 0.409600 | 9.197970 | 0.430467 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 17 | 9.030982 | 0.381553 | 9.321821 | 0.401683 | 9.620576 | 0.422606 |
| 18 | 9.405190 | 0.374208 | 9.716181 | 0.394360 | 10.035902 | 0.415326 |
| 19 | 9.772580 | 0.367390 | 10.103736 | 0.387555 | 10.444457 | 0.408555 |
| 20 | 10.133617 | 0.361037 | 10.484944 | 0.381208 | 10.846691 | 0.402234 |
| 21 | 10.488713 | 0.355096 | 10.860211 | 0.375267 | 11.243003 | 0.396312 |
| 22 | 10.838235 | 0.349522 | 11.229900 | 0.369689 | 11.633750 | 0.390747 |
| 23 | 11.182513 | 0.344278 | 11.594336 | 0.364436 | 12.019252 | 0.385502 |
| 24 | 11.521844 | 0.339331 | 11.953813 | 0.359477 | 12.399798 | 0.380546 |
| 25 | 11.856497 | 0.334653 | 12.308597 | 0.354784 | 12.775651 | 0.375853 |
| 26 | 12.186716 | 0.330219 | 12.658929 | 0.350332 | 13.147049 | 0.371398 |
| 27 | 12.512724 | 0.326008 | 13.005031 | 0.346102 | 13.514210 | 0.367161 |
| 28 | 12.834725 | 0.322001 | 13.347104 | 0.342073 | 13.877334 | 0.363124 |
| 29 | 13.152906 | 0.318181 | 13.685335 | 0.338231 | 14.236605 | 0.359271 |
| 30 | 13.467440 | 0.314534 | 14.019894 | 0.334559 | 14.592192 | 0.355587 |

*Improvement Curve Table Estimates*. Improvement curve tables are an expansion of the $X^B$ portion of the basic unit improvement curve equation, Y = A $X^B$. The result is recorded as a decimal fraction, which is typically calculated to six or eight decimal places. There is a different table value for each unit and slope.

- **Unit Estimate.** To estimate the price or cost for a specific unit, you can simply multiply the theoretical cost or Unit #1 by the appropriate unit factor for the desired unit and slope.

$$Y_U = T_1 \times F_U$$

Where:

$Y_U$ = Unit cost estimate

$T_1$ = Theoretical cost of Unit #1

$F_U$ = Unit cost factor for the unit

For example, if $T_1$ is 2,000 labor hours and the improvement curve slope 80 percent, your estimate should be 762.4 labor hours:

$$\begin{aligned} Y_U &= T_1 \times F_U \\ &= 2{,}000 \times .381208 \\ &= 762.4 \text{ labor hours} \end{aligned}$$

- **Lot Estimate.** To estimate the price of a particular lot, you can use the cum total factors. You could estimate the cum total cost for all units after the proposed lot is completed and then subtract the estimated cum total cost of all units prior to the proposed lot The difference is the estimated cost for the proposed lot.

$$Y_L = (T_1 \times F_{C2}) - (T_1 \times F_{C1})$$

Where:

$Y_L$ = Lot cost estimate

$T_1$ = Theoretical cost of Unit #1

$F_{C2}$ = Cumulative cost factor for all production through the proposed lot

$F_{C1}$ = Cumulative cost factor for all production prior to the proposed lot

For ease of calculation, this equation may be rewritten as:

$$Y_L = T_1 (F_{C2} - F_{C1})$$

For example, if $T_1$ is 4,000 labor hours and the improvement curve slope is 80 percent, your estimate for Units #15 to #25 should be 15,192.24 labor hours, calculated as follows:

$$\begin{aligned} Y_L &= T_1 (F_{C2} - F_{C1}) \\ &= 4{,}000\ (12.308597 - 8.092339) \\ &= 4{,}000\ (4.216258) \\ &= 16{,}865.032 \text{ (rounded to 16,865 labor hours)} \end{aligned}$$

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## 7.6 - Identifying Issues And Concerns

*Questions to Consider in Analysis*. As you perform price or cost analysis, consider the issues and concerns identified in this section, whenever you use an improvement curve.

- ***Is improvement curve analysis used when the contract effort involves:***
  - A significant amount of manual labor in the contract?
  - Uninterrupted production?
  - Production of complex items?
  - No major technological change?
  - Or should involve, continuous pressure to improve?
- ***Is improvement curve use adequately documented?***

Documentation should include:

  - A statement describing the improvement curve theory used in developing the estimate.
  - A summary of related cost data for the product being purchased and any similar products.
  - A description of how available data were used in estimating the theoretical cost of Unit #1 and the slope of the curve.
  - A statement on how the improvement curve estimate was used in price or cost analysis.

- ***Does the documentation provide a valid base for estimate development?***

Like CERs, improvement curves are a form of comparison estimate. Unless you are satisfied that the historical data provide a valid base for the use of an improvement curve, estimates based on the curve should be suspect.

- ***Was improvement curve theory properly applied?***

Verify the application of the improvement curve to the data available. When a contractor proposes a cumulative average curve, consider both the unit and cumulative average curves.

  - The unit curve may provide more reasonable results than a cumulative average curve.
  - The cumulative average curve may conceal significant fluctuations in per unit labor hours.

- ***Did any improvement curve analysis isolate costs associated with contract changes and production interruptions?***

Changes and production interruptions can both affect estimate accuracy. Identify and consider their effect in your analysis.

  - Random fluctuations around an improvement curve line-of-best-fit should be expected. However, if costs increase or decrease dramatically, you should suspect that the actual costs have been affected by a contract change or a break in production. When you suspect that actual costs are affected by a contract change or break in production, contact the cognizant auditor and Government technical personnel for assistance in your analysis.
  - An offeror might overstate the impact of an interruption in production -- contending that the interruption has been so long that it will have to start from scratch. However, improvements in unit costs result in part from such factors as better product design, tooling, work methods, and work layout. If these were properly documented, some of the improvement should carry over to the new effort-regardless of the length of the interruption or turnover of personnel.

- ***Does the improvement curve analysis project continued improvement?***

Occasionally, an offeror will propose "negative learning." In other words, as more units are produced, the cost per unit increases. Do not accept the negative learning argument. If something has significantly changed, consider starting a new curve with a new first unit value and slope.

- ***Does the improvement curve estimate include the costs of rework and repair?***

The effort for rework and repair may or may not be included in the costs projected with the improvement curve. Therefore, you need to determine if these costs are included in the projected costs before allowing any add-on factors for rework or repair.