6.0 - Chapter Introduction

In this chapter, you will learn to use moving averages to estimate and analyze estimates of contract cost and price.

Single Moving Average. If you cannot identify or you cannot measure an independent variable that you can use to estimate a particular dependent variable, your best estimate is often an average (mean) of past observations. The single moving average builds on this principle by defining the number of observations that you will consider. It assumes that the recent past is the best predictor of the future.

In a single moving average, data collected over two or more time periods (normally at least three) are summed and divided by the number of time periods. That average then becomes a forecast for future time periods. As data from a new time period is added, data from an earlier time period is dropped from the average calculation. For example, a 12-month moving average uses data from the most recent 12 months. A 6-month moving average uses data from the most recent six months. You must determine the appropriate number of time periods to consider in the analysis. You can use any time period, but monthly data is the most common.

Double Moving Average. If you believe that there is a trend in the data, you can use a double moving average. A trend in the data means that the observation values tend to either increase or decrease over time. A double moving average requires that you calculate a moving average and then calculate a second moving average using the averages from your first moving average as observations.

6-Step Procedure for Using Moving Averages. When using moving averages, you should use the following 6-step procedure in your analysis:

Step 1. Collect the time services data.

Step 2. Determine which moving average model to use.

- No time-series trend -- use a single moving average.
- Time-series trend -- use a double moving average.

Step 3. Develop 1-period forecasts using different averaging periods to compare with actual observations to evaluate accuracy.

Step 4. Evaluate 1-period forecast accuracy using mean absolute deviations (MADs) between forecasts and actual observations.

Step 5. Select the averaging period found to produce the most accurate results.

Step 6. Use the moving average in forecasting.

6.1 - Identifying Situations For Use

Situations for Use. You can use moving averages in any situation where you are attempting to forecast a variable and you cannot identify or you cannot measure an independent variable except time that appears to be related to changes in the variable. In contract pricing, moving averages are often used to:

- **Develop estimating rates and factors.** For example, most production operations involve substantial amounts of material. When material is used, there is normally some amount of scrap that can no longer be used for its intended purpose. This can include:
  - Waste from production operations (e.g., sheet metal left over after shapes have been cut from it).
  - Spoilage (e.g., material that has exceeded its useful shelf life, losses in storage, defective parts, etc.).
  - Defective parts (e.g., parts that fail inspection during the production process).
  - Material scrap rates are affected by a variety of factors including production methods, product design, and materials. Because the specific effect of these variables is difficult to identify and measure, scrap rates are commonly estimated using moving averages.
  - Rates may be calculated in either dollars or units of material and are commonly calculated in one of the following ways:

$$\frac{\text{Scrap Dollars}}{\text{Total Assembly Material Dollars}} \text{ or } \frac{\text{Scrap Units}}{\text{Total Assembly Material Units}}$$

$$\frac{\text{Scrap Dollars}}{\text{Material Dollars Purchased}} \text{ or } \frac{\text{Scrap Units}}{\text{Material Unit Purchased}}$$

- o In calculating such estimating factors, you should track the cost element being estimated (scrap) and the factor base separately over the averaging period. Then you can calculate the moving average rate by completing the rate calculation. However, if that is not possible, you can calculate the moving average using the scrap rates from each data period. The major disadvantage of the latter method is that periods of high and low production receive the same weight in analysis.

- **Estimate contractor sales volume.** A variety of factors affect sales volume (e.g., company products, the economy, Government spending, and many others). Estimates should include current contracts, known future contracts, other known sales, and currently unknown sales. One way of estimating currently unknown sales is with a moving average based on recent sales experience.

- **Estimate contract requirements**. Many contracts obligate the contractor to meet uncertain requirements. A requirements contract for a particular product may require the contractor to meet all Government demand during the contract period. A maintenance contract may require the contractor to respond to unscheduled service calls. In these and similar situations, a moving average can provide estimates of future requirements based on the recent past.

- **Estimate economic change.** A moving average can be used to estimate future economic change based on recent history. For example, wage rates and product price changes (index numbers) can be estimated using moving averages.

6.2 - Determining Which Moving Average Model To Use

General Criteria for Model Selection. There are several moving average models that can be used in contract pricing. The two most commonly used are the single moving average

and the double moving average. Your decision on which model to use will depend on whether the data indicate a trend (upward or downward) in the values of the dependent variable. If there is:

- No time-series data trend - use a single moving average.
- Time-series data trend - use a double moving average.

Methods to Determine If Data Indicate Dependent Variable Trend. There are three common methods you could use to determine whether or not there is trend in a data set: graphic analysis, regression analysis, and Spearman's rank correlation coefficient.

- **Graphic Analysis.** Graphic analysis entails plotting the data (either manually or by computer) and determining by visual inspection whether or not there is trend in the data. The problem with this technique is that it is not consistently accurate. It is particularly difficult to make a decision when there may or may not be a slight trend.
- **Regression Analysis.** Regression analysis entails the calculation of a least-squares-best-fit (LSBF) estimating equation using time as the independent variable and testing the significance of the slope using the T-test. Though this is an accurate technique, it is rather tedious even when done using a computer.
- **Spearman's Rank Correlation Coefficient**. Spearman's rank correlation coefficient, also known as the Rank Spearman (RANSP) test involves calculation of an RS value and comparing that value with a critical value obtained from a table. This is the test that is most commonly used, because it is accurate and relatively easy to calculate even when done manually. However, before using the RANSP test, assure that the following three conditions have been met:
    - You must have data from at least four observations.
    - You must not have reason to suspect a cyclical or seasonal effect.
    - You must not have reason to suspect that there is a change in trend direction (monotonic trend).

7-Step RANSP Test Process. Use the following 7-step procedure to complete the RANSP test:

Step 1. Put the data in table format in the order in which the values occurred and number the time periods. Assign a value of one (1) to the oldest time period from which data were obtained. Then assign time-period numbers (t) sequentially to each time period from which data were obtained.

Step 2. Create a new variable R_Y. R_Y is the rank of variable Y. Assign an R_Y of 1 to the smallest value of Y, an R_Y of 2 to the second smallest a 2, and so forth. In the event of a 2-way tie, sum the next two ranks available, divide by 2 and assign that quotient to both numbers. In the event of an n-way tie, sum the next n ranks available and divide by n.

Step 3. Create a new variable D. D is the difference between the t value for variable Y and the rank of variable Y:

$D = t - R_Y$

Step 4. Square each D value and sum the resulting values.

SD^2

Step 5. Calculate RS. Use the following formula:

$$RS = 1 - \frac{6\Sigma D^2}{n(n^2 - 1)}$$

RS	Interpretation
-1	Perfect Negative Trend
0	No Trend
1	Perfect Positive Trend

Step 6. Compare the absolute value of RS, with the critical or table value of RS (RS_{crit}). The absolute value of RS ($|RS|$) is the calculated value of RS without consideration of sign (i.e., absolute numbers have no sign -- positive or negative). In the absence of other guidance, use a significance level (a) of .10. A partial RS_{crit} table appears below. See Appendix for a complete RS_{crit} table.

Step 7. Select the appropriate moving average model. Use the following criteria:

If	Then
$|RS| > RS_{crit}$	Assume that there is trend in the data -- use double moving average.
$|RS| = RS_{crit}$	Indifference point -- use either single or double moving average. Normally, you would use a single moving average, because it is easier to use.
$|RS| < RS_{crit}$	Assume that there is no trend in the data -- use a single moving average.

RANSP Test Example 1

Assume that you have collected historical quarterly wage data and you want to determine if there is trend in the data.

Step 1. Put the data in table format in the order in which the values occurred and number the time periods.

Quarter (t)	Wage Rate (Y)
1	$12.50
2	$11.80
3	$12.85
4	$13.95
5	$13.30
6	$13.95
7	$15.00
8	$16.20
9	$16.10

Step 2. Create a new variable R_Y. R_Y is the rank of variable Y. The smallest value of Y receives a 1, the second smallest a 2, etc. In the event of an n-way tie, sum the next n ranks available and divide by n.

T	Y	R_Y
1	$12.50	2
2	$11.80	1
3	$12.85	3

4	$13.95	5.5
5	$13.30	4
6	$13.95	5.5
7	$15.00	7
8	$16.20	9
9	$16.10	8

Step 3. Create a new variable D. For each time period, D is the difference between the t value and the R_Y value for that time period:

$$D = t - R_Y$$

T	Y	R_Y	D
1	$12.50	2	-1.00
2	$11.80	1	1.00
3	$12.85	3	0.00
4	$13.95	5.5	-1.50
5	$13.30	4	1.00
6	$13.95	5.5	.50
7	$15.00	7	0.00
8	$16.20	9	-1.00
9	$16.10	8	1.00

Step 4. Square each D value and sum the resulting values (SD^2).

T	Y	R_Y	D	D^2
1	$12.50	2	-1.00	1.00
2	$11.80	1	1.00	1.00
3	$12.85	3	0.00	0.00
4	$13.95	5.5	-1.50	2.25
5	$13.30	4	1.00	1.00
6	$13.95	5.5	.50	.25
7	$15.00	7	0.00	0.00
8	$16.20	9	-1.00	1.00
9	$16.10	8	1.00	1.00
Total				7.50

Step 5. Calculate RS.

$$RS = 1 - \frac{6\Sigma D^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(7.50)}{9(9^2 - 1)}$$

$$= 1 - \frac{45}{9(80)}$$

$$= 1 - \frac{45}{720}$$

$$= 1 - .0625$$

$$= .9375$$

Step 6. Compare the absolute value of RS, with the critical or table value of RS (RS$_{crit}$). Since there is no guidance to the contrary, use a significance level of .10. Since there are nine observations, RS$_{crit}$ at that level of significance is .4667.

Partial Table¾ Critical Values of RS			
	Significance Level (a)		
N	.10	.05	.01
8	.5000	.6190	.8095
9	.4667	.5833	.7667
10	.4424	.5515	.7333

Step 7. Select the appropriate moving average model. With RS (.9375) greater than RS$_{crit}$ (.4667). Assume that there is a trend in the data. Use a double moving average.

RANSP Test Example 2. Again, assume that you have collected historical quarterly wage data and you want to determine if there is trend in the data.

Step 1. Put the data in table format in the order in which the values occurred and number the time periods.

Quarter (t)	Wage Rate (Y)
1	$12.70
2	$12.60
3	$12.00
4	$13.00
5	$12.10

6	$12.50
7	$12.80
8	$13.00
9	$12.85

Step 2. Create a new variable R_Y. R_Y is the rank of variable Y. The smallest value of Y receives a 1, the second smallest a 2, etc. In the event of an n-way tie, sum the next n ranks available and divide by n.

T	Y	R_Y
1	$12.70	5
2	$12.60	4
3	$12.00	1
4	$13.00	8.5
5	$12.10	2
6	$12.50	3
7	$12.80	6
8	$13.00	8.5
9	$12.85	7

Step 3. Create a new variable D.

$D = t - R_Y$

T	Y	R_Y	D
1	$12.70	5	-4.00
2	$12.60	4	-2.00
3	$12.00	1	2.00
4	$13.00	8.5	-4.50
5	$12.10	2	3.00
6	$12.50	3	3.00
7	$12.80	6	1.00
8	$13.00	8.5	-0.50
9	$12.85	7	2.00

Step 4. Square each D value and sum the resulting values (SD^2).

T	Y	R_Y	D	D^2
1	$12.70	5	-4.00	16.00

2	$12.60	4	-2.00	4.00
3	$12.00	1	2.00	4.00
4	$13.00	8.5	-4.50	20.25
5	$12.10	2	3.00	9.00
6	$12.50	3	3.00	9.00
7	$12.80	6	1.00	1.00
8	$13.00	8.5	-0.50	0.25
9	$12.85	7	2.00	4.00
			Total	67.50

Step 5. Calculate RS.

$$RS = 1 - \frac{6\Sigma D^2}{n(n^2 - 1)}$$
$$= 1 - \frac{6(67.5)}{9(9^2 - 1)}$$
$$= 1 - \frac{405}{9(80)}$$
$$= 1 - \frac{405}{720}$$
$$= 1 - .5625$$
$$= .4375$$

Step 6. Compare the absolute value of RS, with the critical or table value of RS (RS_{crit}). Since there is no guidance to the contrary, use a significance level of .10. With nine observations, RS_{crit} at that level of significance is .4667.

Partial Table-Critical Values Of RS			
	Significance Level (a)		
N	.10	.05	.01
8	.5000	.6190	.8095
9	.4667	.5833	.7667
10	.4424	.5515	.7333

Step 7. Select the appropriate moving average model. With RS (.4375) less than RS_{crit} (.4667). Assume that there is a no trend in the data. Use a single moving average.

6.3 - Evaluating And Using Single Moving Averages

Procedures for Selecting a Single Moving Average for Forecasting. The single moving average is designed to smooth random variation in the estimate. The more data periods you use to calculate a single moving average, the greater the smoothing affect. For example, a 12-period moving average will average out most random variation, because each observation is only one-twelfth of the average. However, a 12-period moving average will be slow to react to a true change in the variable that you are attempting to estimate. On the other hand, a three period moving average will react much faster, because one data point is one-third of the calculation instead of one-twelfth.

   No averaging period is best for forecasting in all circumstances. You must identify the best averaging period for each situation:

Step 1. Develop 1-period forecasts using different available periods so that you can compare forecasts with actual observations to evaluate accuracy.

- Use at least three periods of data in developing a moving average. You can calculate 3-period moving averages beginning in Period 3. You can calculate 4-period moving averages beginning in Period 4. For any value of n, you can calculate an n-period single moving average beginning in Period n.
- To conduct a meaningful evaluation of forecast accuracy, you must have at least two forecasts and actual data from the same periods for accuracy evaluation. As a result, the largest number of periods (n) that you can use for developing single moving averages is two less than the total number of observations.

Step 2. Evaluate 1-period forecast accuracy using mean absolute deviations (MADs) between forecasts and actual observations.

Step 3. Select the averaging period found to produce the most accurate results.

Calculations Required for Forecast Development. Develop 1-period forecasts using different available periods so that you can compare forecasts with actual observations to evaluate accuracy.

Step 1A. Calculate Single Moving Averages. Calculate single moving averages for available averaging periods using the following equation:

$$M1_{n,t} = \frac{Y_{n,t} + Y_{t-1} + \ldots + Y_{t-n+1}}{n}$$

Where:

$M_{n,t}$ = A single n-period moving average calculated in Period t

Y_t = An observation in Period t of the variable being forecast

n = The number of time periods in the moving average

Step 1B. Develop Forecasts Using Moving Averages. Once you calculate a moving average, you can use that average for forecasting.

$$FM1_{n,t,t+h} = M1_{n,t}$$

Where:

$FM1_{n,t,t+h}$ = A single, n-period, moving average forecast made in Period t for Period t+h

n = The number of periods in the moving average

t = The period in which the forecast is made

h = The horizon, the number of periods you are forecasting into the future

Developing 1-Period Forecasts for Example 2 Data.

Step 1A. Calculate Single Moving Averages. In the previous section, we determined that we should use a single moving average to forecast future wage rates from the data below. Here we will use the data to demonstrate the procedures for single moving average forecast development.

Quarter (t)	Wage Rate (Y)

1	$12.70
2	$12.60
3	$12.00
4	$13.00
5	$12.10
6	$12.50
7	$12.80
8	$13.00
9	$12.85

Note that we have observations from nine periods. That means that we can calculate 3-period, 4-period, 5-period, 6-period, and 7-period moving averages. With nine observations, we cannot evaluate forecasts based on a single moving average of more than seven (9 - 2) periods.

3-Period Single Moving Average			
Quarter (t)	Actual (Y)	å ^{3}Y	M1*
1	$12.70		
2	$12.60		
3	$12.00	$37.30	$12.43
4	$13.00	$37.60	$12.53
5	$12.10	$37.10	$12.37
6	$12.50	$37.60	$12.53
7	$12.80	$37.40	$12.47
8	$13.00	$38.30	$12.77
9	$12.85	$38.65	$12.88

 * Complete terminology for these moving averages is M13,t. To save space the term has been simplified to M1.

Step 1B. Develop Forecasts Using Moving Averages: Once we calculate a single moving average, we can use that average to develop a forecast. To evaluate the accuracy of each moving average, we forecast one period into the future so that we can compare the forecast with the actual Y value. For example, the moving average from Period 3, becomes the Forecast for Period 4.

3-Period Single Moving Average Forecast				
Quarter (t)	Actual (Y)	å ^{3}Y	M1	FM1**

1	$12.70			
2	$12.60			
3	$12.00	$37.30	$12.43	
4	$13.00	$37.60	$12.53	$12.43
5	$12.10	$37.10	$12.37	$12.53
6	$12.50	$37.60	$12.53	$12.37
7	$12.80	$37.40	$12.47	$12.53
8	$13.00	$38.30	$12.77	$12.47
9	$12.85	$38.65	$12.88	$12.77

** Complete terminology for the forecasts in this column is
FM13,t,t+1. To save space the term has been simplified to
FM1.

   We would develop the 4-period, 5-period, 6-period, and
7-period single moving average forecasts using the same
procedure.

4-Period Single Moving Average Forecast				
Quarter (t)	Actual (Y)	$\mathring{A}^4Y$	M1	FM1
1	$12.70			
2	$12.60			
3	$12.00			
4	$13.00	$50.30	$12.58	
5	$12.10	$49.70	$12.43	$12.58
6	$12.50	$49.60	$12.40	$12.43
7	$12.80	$50.40	$12.60	$12.40
8	$13.00	$50.40	$12.60	$12.60
9	$12.85	$51.15	$12.79	$12.60

5-Period Single Moving Average Forecast				
Quarter (t)	Actual (Y)	$\mathring{a}^5Y$	M1	FM1
1	$12.70			
2	$12.60			
3	$12.00			
4	$13.00			
5	$12.10	$62.40	$12.48	

6	$12.50	$62.20	$12.44	$12.48
7	$12.80	$62.40	$12.48	$12.44
8	$13.00	$63.40	$12.68	$12.48
9	$12.85	$63.25	$12.65	$12.68

6-Period Single Moving Average Forecast				
Quarter (t)	Actual (Y)	$\Sigma^6 Y$	M1	FM1
1	$12.70			
2	$12.60			
3	$12.00			
4	$13.00			
5	$12.10			
6	$12.50	$74.90	$12.48	
7	$12.80	$75.00	$12.50	$12.48
8	$13.00	$75.40	$12.57	$12.50
9	$12.85	$76.25	$12.71	$12.57

7-Period Single Moving Average Forecast				
Quarter (t)	Actual (Y)	$\Sigma^7 Y$	M1	FM1
1	$12.70			
2	$12.60			
3	$12.00			
4	$13.00			
5	$12.10			
6	$12.50			
7	$12.80	$87.70	$12.53	
8	$13.00	$88.00	$12.57	$12.53
9	$12.85	$88.25	$12.61	$12.57

   Calculations Required for Evaluating Forecast Accuracy.
Evaluate 1-period forecast accuracy using mean absolute
deviations (MADs) between forecasts and actual
observations.

     You can use several different statistics to measure the
accuracy of a moving average forecast: the range of the
error terms, the standard error of the forecast, or the
mean absolute deviation of the forecast (MADf). Of these
three options, the statistic which best combines the

qualities of ease of computation and utility is the MADf. As a result, it is the statistic most commonly used to evaluate moving average accuracy.

The MADf tells us on average how much, in absolute terms, actual values deviated from the forecasted value.

$$MAD_F = \frac{\Sigma |D|}{n}$$

Where:

MAD_F = The mean absolute deviation of the forecast

S = Summation of all the variables that follow the symbol

$|D|$ = The absolute value of the deviation (i.e., the difference,
without regard to sign) between the actual value which occurred and the value forecasted

$|D| = |Y - F|$

n = The number of deviations (Ds) computed

Evaluate forecast accuracy of single moving averages calculated using the data in Example 2.

3-Period Single Moving Average Evaluation					
t	Actual (Y)	M1	FM1	D	|D|
1	12.70				
2	12.60				
3	12.00	12.43			
4	13.00	12.53	12.43	0.57	0.57
5	12.10	12.37	12.53	-0.43	0.43
6	12.50	12.53	12.37	0.13	0.13
7	12.80	12.47	12.53	0.27	0.27
8	13.00	12.77	12.47	0.53	0.53
9	12.85	12.88	12.77	0.08	0.08
				Total Absolute Deviation	2.01
Mean Absolute Deviation = å |D| ÷ n = 2.01 ÷ 6 =					0.34

4-Period Single Moving Average Evaluation

t	Actual (Y)	M1	FM1	D	|D|
1	12.70				
2	12.60				
3	12.00				
4	13.00	12.58			
5	12.10	12.43	12.58	−0.48	0.48
6	12.50	12.40	12.43	0.07	0.07
7	12.80	12.60	12.40	0.40	0.40
8	13.00	12.60	12.60	0.40	0.40
9	12.85	12.79	12.60	0.25	0.25
				Total Absolute Deviation	1.60
Mean Absolute Deviation = å |D| ÷ n = 1.60 ÷ 5 =					0.32

5-Period Single Moving Average Evaluation

t	Actual (Y)	M1	FM1	D	|D|
1	12.70				
2	12.60				
3	12.00				
4	13.00				
5	12.10	12.48			
6	12.50	12.44	12.48	0.02	0.02
7	12.80	12.48	12.44	0.36	0.36
8	13.00	12.68	12.48	0.52	0.52
9	12.85	12.65	12.68	0.17	0.17
				Total Absolute Deviation	1.07
Mean Absolute Deviation = å |D| ÷ n = 1.07 ÷ 4 =					0.27

6-Period Single Moving Average Evaluation

t	Actual (Y)	M1	FM1	D	|D|
1	12.70				
2	12.60				
3	12.00				
4	13.00				
5	12.10				

6	12.50	12.48			
7	12.80	12.50	12.48	0.32	0.32
8	13.00	12.57	12.50	0.50	0.50
9	12.85	12.71	12.57	0.28	0.28
Total Absolute Deviation					1.10
Mean Absolute Deviation = å |D| ÷ n = 1.10 ÷ 3 =					0.37

7-Period Single Moving Average Evaluation					
t	Actual (Y)	M1	FM1	D	|D|
1	12.70				
2	12.60				
3	12.00				
4	13.00				
5	12.10				
6	12.50				
7	12.80	12.53			
8	13.00	12.57	12.53	0.47	0.47
9	12.85	12.61	12.57	0.28	0.28
Total Absolute Deviation					0.75
Mean Absolute Deviation = å |D| ÷ n = .75 ÷ 2 =					0.38

Selecting an Averaging Period. Select the averaging period found to produce the most accurate results.

Summary Of MAD Computations	
n	MAD$_F$
3	0.34
4	0.32
5	0.27
6	0.37
7	0.38

The lowest MAD$_F$ in this example was attained using a 5-period single moving average. Accordingly, you should select a 5-period single moving average for forecasting.

Use the Single Moving Average in Forecasting. Use the moving average with the lowest MAD_F for forecasting. Based on an evaluation of the data in Example 2, you should use the most recent 5-period single moving average to forecast for any future period. For example, the forecast for Period 13 would be $12.65.

The selection of the most accurate averaging period for forecast development is essential. Different averaging periods can produce substantially different forecasts. For example, using different averaging periods and the data in this example, you could have calculated a wide range of forecasts for Period 13.

Period 13 Forecast Comparison	
n	$FM1_{n,9,13}$
3	$12.88
4	$12.79
5	$12.65
6	$12.71
7	$12.61

Of these possibilities, the 5-period single moving average forecast, $12.65, appears to be the most reasonable.

6.4 - Evaluating And Using Double Moving Averages

Procedures for Selecting a Double Moving Average for Forecasting. The double moving average is designed to develop a forecast that smoothes random variation and projects any trend exhibited in the data. As with the single moving average, no averaging period is best for forecasting in all circumstances. You must identify the best averaging period for each situation:

Step 1. Develop 1-period forecasts using different available periods so that you can compare forecasts with actual observations to evaluate accuracy.

> o Normally, we use at least a three period double moving average. Since a double moving average is

a moving average of moving averages, you cannot
begin to calculate a 3-period double moving
average until Period 5. You can calculate 4-
period double moving averages beginning in Period
7. For any value of n, you can calculate an n-
period double moving average beginning in Period
2n - 1.

- o To conduct a meaningful evaluation of forecast
  accuracy, you must have at least two forecasts
  and actual data for the same period for accuracy
  evaluation. As a result, you must have 2n + 1
  data points in order to calculate a double moving
  average forecast and the related MAD_F.

**Step 2. Evaluate 1-period forecast accuracy using mean
absolute deviations (MADs) between forecasts and actual
observations.**

**Step 3. Select the averaging period found to produce the
most accurate results.**

Calculations Required for Forecast Development. Develop 1-
period forecasts using available averaging periods so that
you can compare forecasts with actual observations to
evaluate accuracy.

Step 1A. Calculate Double Moving Averages. Calculate double
moving averages for available averaging periods using the
following equation:

$$M2_{n,t} = \frac{M1_{n,t} + M1_{n,t-1} + \ldots + M1_{n,t-n+1}}{n}$$

Where:

$M2_{n,t}$ = An n-period double moving average calculated
in Period t

$M1_{n,t}$ = An n-period single moving average calculated
in period t

n = Number of periods in the moving average

Note: You must use the same value of n for calculating both
M1 and M2.

Step 1B. Develop Forecasts Using Moving Averages: Once you calculate a double moving average, you can use that average to develop a forecast.

$$FM2_{n,t,t+h} = A_{n,t} + B_{n,t} + h$$

Where:

$FM2_{n,t,t+h}$ = The, n-period, double moving average forecast made in period t for period t+h

$A_{n,t}$ = The intercept fpr an n-period double moving average forecast, calculated:

$$A_{n,t} = 2M1_{n,t} - M2_{n,t}$$

$B_{n,t}$ = The slope for an n-period double moving average forecast, calculated:

$$B_{n,t} = \frac{2}{n-1}\left(M1_{n,t} - M2_{n,t}\right)$$

n = The number of periods in the moving average

t = The period in which the forecast is made

h = The horizon, the number of periods you are forecasting into the future

Note: Depending on the value of n and the period in which the forecast is made, there is a unique intercept ($A_{n,t}$) and slope ($B_{n,t}$).

Developing 1-Period Forecasts for Example 1 Data.

Step 1A. Calculate Double Moving Averages. In the previous section, we determined that we should use a double moving average to develop a forecast from the data below. Here we will use the data to demonstrate the procedures for double moving average forecast development.

Quarter (t)	Wage Rate (Y)
1	$12.50
2	$11.80
3	$12.85
4	$13.95
5	$13.30
6	$13.95
7	$15.00
8	$16.20
9	$16.10

$\overset{3}{\text{å}}Y$	M1	$\overset{3}{\text{å}}M1$	M2
37.15	12.38		
38.60	12.87		
40.10	13.37	38.62	12.87
41.20	13.73	39.97	13.32
42.25	14.08	41.18	13.73
45.15	15.05	42.86	14.29
47.30	15.77	44.90	14.97

Step 1B. Develop Forecasts Using Moving Averages. Period 5 is the first period that we can develop a 3-period double moving average forecast, because that is the first period that we have the values of M1 and M2 that we need to make the forecast. In Period 5, we can make a forecast for Period 6 as follows:

$$A_{3,5} = 2M1_{3,5} - M2_{3,5}$$
$$= 2(13.37) - 12.87$$
$$= 26.74 - 12.87$$
$$= 13.87$$

$$B_{3,5} = \frac{2}{3-1}(M1_{3,5} - M2_{3,5})$$
$$= \frac{2}{3-1}(13.37 - 12.87)$$
$$= \frac{2}{2}(13.37 - 12.87)$$
$$= 13.37 - 12.87$$
$$= .50$$

$$FM2_{3,5,6} = A_{3,5} + B_{3,5}(h)$$
$$= 13.87 + .50(1)$$
$$= 13.87 + .50$$
$$= 14.37$$

This forecast developed for Period 6 and the forecasts using later data that were developed for Period 7, Period 8, and Period 9 are show in the table below.

	3-Period Double Moving Average Forecast					
t	Actual	M1	M2	A	B	FM2
1	12.50					
2	11.80					
3	12.85	12.38				
4	13.95	12.87				
5	13.30	13.37	12.87	13.87	0.50	
6	13.95	13.73	13.32	14.14	0.41	14.37
7	15.00	14.08	13.73	14.43	0.35	14.55
8	16.20	15.05	14.29	15.81	0.76	14.78
9	16.10	15.77	14.97	16.57	0.80	16.57

Forecasts developed using a 4-period moving average and the same procedures are shown In the following table. Note that only two forecasts can be made for comparison with actual observations.

4-Period Double Moving Average

Forecast						
t	Actual	M1	M2	A	B	FM2
1	12.50					
2	11.80					
3	12.85					
4	13.95	12.78				
5	13.30	12.98				
6	13.95	13.51				
7	15.00	14.05	13.33	14.77	0.48	
8	16.20	14.61	13.79	15.43	0.55	15.25
9	16.10	15.31	14.37	16.25	0.63	15.98

Calculations Required for Evaluating Forecast Accuracy.
Evaluate 1-period forecast accuracy using mean absolute
deviations (MADs) between forecasts and actual
observations. Here we use the same formula for calculating
MADf that we used in evaluating the accuracy of single
moving averages.

$$MAD_F = \frac{\Sigma |D|}{n}$$

Where:

MAD_F = The mean absolute deviation of the forecast

S = Summation of all the variables that follow the symbol

$|D|$ = The absolute value of the deviation (i.e., the
difference,
without regard to sign) between the actual value which
occurred and the value forecasted

$|D| = |Y - F|$

n = The number of deviations (Ds) computed

3-Period Double Moving Average Forecast Evaluation								
t	Actual	M1	M2	A	B	FM2	D	|D|
1	12.50							
2	11.80							
3	12.85	12.38						

4	13.95	12.37						
5	13.30	13.37	12.87	13.87	0.50			
6	13.95	13.73	13.32	14.14	0.41	14.37	-0.42	0.42
7	15.00	14.08	13.73	14.43	0.35	14.55	0.45	0.45
8	16.20	15.05	14.29	15.81	0.76	14.78	1.42	1.42
9	16.10	15.77	14.97	16.57	0.80	16.57	-0.47	0.47
					Total Absolute Deviation			2.76
Mean Absolute Deviation = å |D| ÷ n = 2.76 ÷ 4 =								0.69

4-Period Double Moving Average Forecast Evaluation

	Actual	M1	M2	t	B	FM2	D	|D|
1	12.50							
2	11.80							
3	12.85							
4	13.95	12.78						
5	13.30	12.98						
6	13.95	13.51						
7	15.00	14.05	13.33	14.77	0.48			
8	16.20	14.61	13.79	15.43	0.55	15.25	0.95	0.95
9	16.10	15.31	14.37	16.25	0.63	15.98	0.12	0.12
					Total Absolute Deviation			1.07
Mean Absolute Deviation = å |D| ÷ n = 1.07 ÷ 2 =								0.54

Select Averaging Period

Select the averaging period found to produce the most accurate results.

Summary Of MAD Computations

n	MAD$_F$
3	0.69
4	0.54

The lowest MADf in this example was attained using a 4-period double moving average. Accordingly, you should select a 4-period double moving average for forecasting.

Use a Double Moving Average in Forecasting. Use the moving average with the lowest MAD_F for forecasting. Based on our evaluation of the data in Example 1, we would use the 4-period double moving average for forecasting. For example our forecast for Period 13 [four periods (h) into the future] would be $18.77, calculated as follows:

$$A_{3,5} = 2M1_{4,9} - M2_{4,9}$$
$$= 2(15.31) - 14.37$$
$$= 30.62 - 14.37$$
$$= 16.25$$

$$B_{3,5} = \frac{2}{4-1}(M1_{4,9} - M2_{4,9})$$
$$= \frac{2}{4-1}(15.31 - 14.37)$$
$$= \frac{2}{3}(15.31 - 14.37)$$
$$= \frac{2}{3}(.94)$$
$$= .63$$

$$FM2_{3,5,6} = A_{4,9} + B_{4,9}(h)$$
$$= 16.25 + .63(4)$$
$$= 16.25 + 2.52$$
$$= 18.77$$

The selection of the most accurate averaging period for forecast development is essential. For example, using different averaging periods and the data in this example, we could have calculated two very different forecasts.

Forecast Comparison	
n	$FM2_{n,9,13}$
3	$19.77
4	$18.77

6.5 - Identifying Issues And Concerns

 Questions to Consider in Analysis. As you perform price
or cost analysis, consider the issues and concerns
identified in this section, whenever you use moving
averages.

- **Is a moving average the best choice for estimate
  development?**

When using a moving average, you assume that the trend
experienced over time is the best guide available to
forecast future variable values. If that assumption is not
correct, you should use another technique. Detailed
estimates that consider all the facts involved are normally
more defensible in negotiations than the result of any
estimating relationship. If an independent variable (other
than time) can be identified and measured, another
comparison technique may provide better results than moving
average analysis. For example, estimating parts demand
based on sales and usage data would probably produce better
results than an estimate based on use of a moving average.
A moving average can estimate price changes based on recent
periods but it cannot predict a turning point that will
alter the historical pattern.

- **Is the type of moving average selected appropriate for
  the situation?**

When there is a trend in the data, you should use a double
moving average. When there is no trend, you should use a
single moving average. If you use a single moving average
in a situation where a trend exists, your forecast will not
consider the trend.

- **Is the averaging period the best choice for the data?**

You should select the averaging period that provides the
best estimates when tested against actual observations.
Take special care in your analysis when the moving average
covers a large number or periods (e.g., 12 months).
Selection of an average that covers a large number of
periods is often appropriate because it dampens the effect
of random fluctuation. However, an average that considers a
large number of data points will also make it more

difficult to identify a trend in the data. Occasionally, an estimator will use a large number of periods to mask a trend in the data. When analyzing estimates made using a moving average, you should look at the raw data and consider appropriate alternative estimating procedures.

- ***How far into the future can you forecast?***

The farther into the future that you forecast, the greater the risk. Remember, that you cannot predict a change in an historical trend with a moving average.