Theoretical One-Dimensional Water to Soil Impact Pressures

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ABSTRACT

THEORETICAL equations which describe the pressures caused by one-dimensional impact of water onto soil surfaces were derived from basic mechanics. The pressures of impact on the soil skeleton and in the soil pores were shown to be functions of the densities and volume fractions of the soil pores and skeleton, a dynamic coupling coefficient, and the compressional wave velocities in the two soil fractions. Theoretical or empirical relationships between the dynamic parameters and soil matric potential, porosity, and degree of saturation were presented, and the total and effective vertical stresses and pore water pressures caused by impact were calculated as a function of those static soil properties. The calculated total vertical stresses were of the order of 0.05 to 0.16 the value for impact on rigid surfaces, and were largely a function of soil matric potential and porosity. The calculated pore water pressures were primarily dependent upon the degree of saturation. At high saturation levels and low matric suctions, calculated pore water pressure exceeded calculated effective vertical stress, indicating a liquified and unstable state for that soil condition. The results will be useful in providing input functions for numerical or analytical studies of soil response to waterdrop impacts.

INTRODUCTION

The theory for the one-dimensional impact of compressible liquids onto homogeneous solid surfaces is well developed (Adler, 1979; Springer, 1976). The vertical pressures of impact are a function of the relative velocity and the dynamic impedences (the products of material densities and compressional wave velocities) of the two materials. Furthermore, it has been shown that the actual pressure of liquid-solid impact for waterdrops and waterjets are of the same order of magnitude as the theoretical one-dimensional values (Adler, 1979). A similar theory for one-dimensional water to soil impact has not been developed. Such a theory would provide an estimate of the order of magnitude of the stresses imposed by waterdrop impact on soil surfaces which initiate erosion and cause surface sealing. The theory would also provide for a method of making parametric studies of soil properties and their effects on raindrop impact stresses and hence their effects on the detachability of soil particles by raindrops.

For a homogenous solid, the pressures generated by an impacting liquid are a function of the densities and compressional wave velocities of the two materials (Springer, 1976; Adler, 1979):

$$P_w = \rho_w C\, v/[1 + \rho_w C/\rho_t U_t] \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

where C is the velocity of a compressional wave in water, v is the relative velocity of the two materials at the time of impact, ρ_w is the density of the water, ρ_t is the density of the solid target, and U_t is the velocity of a compressional wave in the solid. Equation [1] was derived in detail by Springer (1976). For low impact velocities, such as those associated with natural rainfall on a stationary target, the velocity, C, of the induced compressional wave in the water is equal to the velocity of sound in water. In the case of a rigid solid $\rho_t U_t >> \rho_w C$ and P_w reduces to the classical "water hammer" pressure, P_h, (Adler, 1976)

$$P_h = \rho_w\, C\, v \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [2]$$

Numerical computations and laboratory experiments of waterdrops or jets on essentially rigid surfaces (i.e., where $\rho_t U_t >> \rho_w C$) have shown that the impact pressures were of the order of magnitude as that computed from equation [2] (Hwang and Hammitt, 1977; Johnson and Vickers, 1973; Rochester and Brunton, 1974; Rosenblatt et al., 1977).

Soils are not homogenous, and with impact of a liquid onto a soil two compressional waves are generated, a frame wave and a fluid wave (Biot, 1956). The frame wave is associated primarily with the soil skeleton and the fluid wave is associated primarily with the pore fluid, although coupled motion of the two phases may exist for both waves. Both the frame and fluid waves must be considered for a complete description of the one-dimensional soil impact problem.

Pressures generated by waterdrop and waterjet impact on rigid solid surfaces are of the order as those predicted from the theory of one-dimensional fluid-solid impact. Equations for the one-dimensional impact of water onto soil surfaces have not been developed. The purpose of this study was to develop those equations, evaluate required parameters using the best available knowledge, and calculate the theoretical one-dimensional impact pressures of water on soil surfaces for a range of soil conditions. The knowledge thereby developed will provide a basis for input to numerical or analytical schemes which analyze the response of various soils to waterdrop impact loading.

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IMPACT EQUATIONS

The derivation of the equations for the one-dimensional impact pressure of water onto a soil surface can be made by considering the stress differences across the compressional waves which propagate into the water and soil. Upon impact three conpressional waves are generated; one in the impacting water, the frame wave in the soil skeleton, and the fluid wave in the soil pores in the soil. The equations for the pressure differences across the compressional waves may be derived from jump conditions of continuum mechanics (Eringin, 1967):

$$P_w = \rho_w C (v - u_w) \quad \ldots\ldots [3]$$

$$P_p = \rho_p U_p u_p \quad \ldots\ldots [4]$$

$$P_f = \rho_f U_f u_f \quad \ldots\ldots [5]$$

where P_w is the pressure difference across the compressional wave in the impacting water, P_p is the pressure difference across the fluid wave, P_f is the pressure difference across the frame wave, ρ_p is the density of the material displaced by the fluid wave, ρ_f is the density of the material displaced by the frame wave, U_p is the velocity of the fluid wave, U_f is the velocity of the frame wave, u_w is the material velocity of the water, u_p is the material velocity of the pore fluid, u_f is the material velocity of the soil skeleton, and v is the velocity of the impacting water immediately before impact (Fig. 1). The material displacement across the interface between soil and water must be continuous, hence,

$$u_w = u_p = u_f = u \quad \ldots\ldots [6]$$

where u is equal to the velocity of the interface. Furthermore, the forces across the interface must be the same. Thus if β_f is the fraction of the soil in which the frame wave is propagated and β_p is the fraction of the soil in which the fluid wave is propagated, then

$$P_w = \beta_f P_f + \beta_p P_p \quad \ldots\ldots [7]$$

After eliminating u_w, u_p, u_f, P_p, and P_w and solving for P_f, the resultant equation is

$$P_f = \rho_w C v / [\beta_f + \beta_p(\rho_p U_p/\rho_f U_f + (\rho_w C/\rho_f U_f)] \quad \ldots [8]$$

The development of equation [8] is essentially analogous to that of equation [1] wherein the non-homogeneity of the soil material and the subsequent development of multiple compressional waves are considered.

P_f is the pressure of impact generated at the soil surface primarily on the soil frame or skeleton. An analogous equation may be similarly derived for the pressures which are generated primarily in the soil pores and are associated with the fluid wave. It is

$$P_p = \rho_w C v / [\beta_p + \beta_f(\rho_f U_f/\rho_p U_p) + (\rho_w C/\rho_p U_p)] \quad \ldots [9]$$

The deformation of strength of a soil mass may be analyzed either as a function of the total stress on the soil skeleton or as a function of the effective stress, which is the total stress on the soil skeleton minus the pore water pressure. In the case of water to soil impact P_f is the total stress on the soil skeleton and P_p is the pore water pressure. Thus for soil mechanical analyses it is useful to define the vertical effective stress for the case of water to soil impact, P_e, as

$$P_e = P_f - P_p \quad \ldots\ldots [10]$$

By combining equations [3], [4], [5], [6], [7], and [10] the effective stress P_e may also be given as

$$P_e = \rho_w C v/[\beta_f + (\rho_p U_p + \rho_w C)/(\rho_f U_f - \rho_p U_p)] \quad \ldots [11]$$

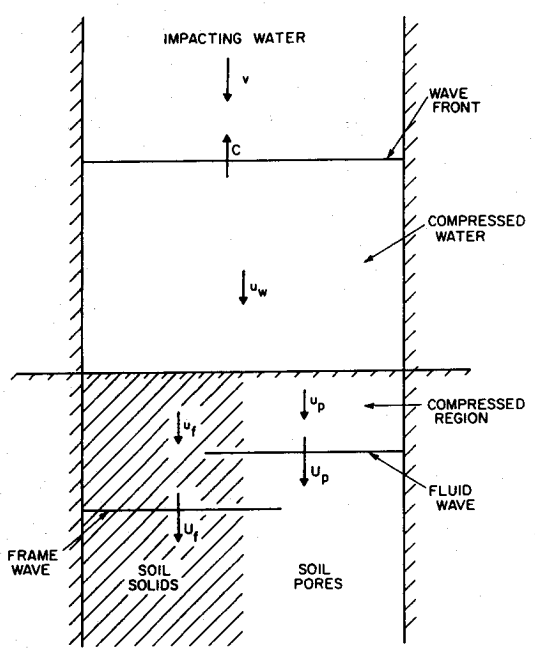

Fig. 1—Compressional waves generated by 1-D water to soil impact.

PARAMETERS

Equations [8], [9], and [11] provide a means for calculating the vertical stesses induced on a soil surface due to the one-dimensional impact of the water. The parameters, C, v, and ρ_w are assumed to be known for the system, hence the parameters ρ_p, ρ_f, β_p, β_f, U_p, U_f must be determined in order to calculate the pressures. As will be shown, these parameters are known to be functions of porosity, degree of saturation, and effective confining pressure of the soil. The pressures of water impact are therefore functions of these soil properties.

As will also be shown, a complete understanding of the necessary parameters for all soil conditions is lacking. As new knowledge relating soil properties to the dynamic parameters listed above becomes available, the pressures calculated from the impact equations will be more accurate. The relationship used herein for the parameters are based on best available knowledge.

In presenting relationships for calculating the parameters for the equations, the range of saturation values considered were restricted to S>0.85. This was done for two reasons. For purposes of understanding splash erosion phenomenon, relatively high values of saturation are of primary interest. Also, by restricting the range of saturation considered, less extrapolation of existing experimental data as well as theoretical concepts were necessary in evaluating the parameters. This is not

to imply that the validity of equations [8], [9], and [11] is restricted to high saturation levels.

The Density and Volume Fraction Parameters

Due to the coupled motion of the solid and pore phases of the soil for both the frame and fluid wave, the densities ρ_f and ρ_p must include a coupling coefficient ρ_{fp}. Biot (1956) provided the theoretical basis for defining the coupling coefficient. The defining equations for the dynamic density parameters, with slight modification to include the parameter S for a moderate degree of desaturation, are

$$\rho_f + 2\rho_{fp} + \rho_p = \rho \quad \ldots\ldots [12]$$

where ρ is the total mass density defined by

$$\rho = (1-f)\rho_s + S\, f\rho_w \quad \ldots\ldots [13]$$

where ρ_s is the solid mass density and ρ_w is the density of the pore water, f is the porosity, and S is the degree of saturation of the soil pores. Furthermore,

$$\rho_f = (1-f)\rho_s - \rho_{fp} \quad \ldots\ldots [14]$$

and

$$\rho_p = S\, f\rho_w - \rho_{fp} \quad \ldots\ldots [15]$$

Equations [14] and [15] come directly from Biot's (1956) definition of ρ_{fp} and equation [13]. The coupling coefficient, therefore, is a negative quantity, the magnitude of which represents the mass of pore water per unit total volume which is displaced by the frame wave, and also the mass per unit total volume of solids which is displaced by the fluid wave.

The value of the coupling coefficient for a saturated soil was measured experimentally by Hardin and Richart (1963). They determined a value of $-\rho_{fp}$ of approximately $0.4\ \rho_w f$. For dry soils, the coupling coefficient is zero, and the coupling for partially saturated soils is not known. For purposes of calculation herein and until better information is available, a linear relationship between ρ_{fp} and S in the range $0.85<S<1$ will be assumed, hence,

$$-\rho_{fp} = 0.4\ \rho_w\, S\, f \quad \ldots\ldots [16]$$

The dynamic density parameters may be calculated from equations [14] and [15], using equations [16] to calculate the dynamic coupling coefficient.

The parameters β_f and β_p are the volume fractions of the material which is displaced due to the passing of the compression waves. In the case of β_f for the frame wave, the soil skeleton of volume (1-f) is displaced and a portion of the pore water, $-\rho_{fp}/\rho_w$ is displaced. Hence,

$$\beta_f = (1-f) - \rho_{fp}/\rho_w \quad \ldots\ldots [17]$$

In the case of β_p, the volume of solids displaced is $-\rho_{fp}/\rho_s$, hence,

$$\beta_p = Sf - \rho_{fp}/\rho_s \quad \ldots\ldots [18]$$

The Wave Velocities

From the partial differential wave equation and elasticity theory it can be shown that

$$U_p^2 = B_p/\rho_p \quad \ldots\ldots [19]$$

where B_p is the bulk compressibility of the material with density ρ_p in which the wave of velocity U_p is propogating. The density ρ_p has been derived. The bulk compressibility, B_p, can be calculated from the equation

$$\frac{1}{B_p} = \left[\frac{\text{volume solids displaced}}{\text{total volume}}\right]\frac{1}{B_s} + \left[\frac{\text{volume voids}}{\text{total volume}}\right]\frac{1}{B_{aw}} \quad \ldots\ldots [20]$$

where B_s is the bulk compressibility of the solid phase and B_{aw} is the bulk compressibility of the combined air-water phase. B_{aw} can be computed from the equation

$$1/B_{aw} = (1-S)/B_a + S/B_w \quad \ldots\ldots [21]$$

where B_a is the bulk compressibility of air and B_w is the bulk compressibility of water. The ratio of volume of voids to total volume is f. The ratio of volume of solids displaced by the fluid wave to total volume is a function of the coupling coefficient and is equal to $-\rho_{fp}/\rho_s$. Hence, equation [20] may be rewritten as

$$1/B_p = (-\rho_{fp}/\rho_s)/B_s + f(1-S)/B_a + fS/B_w \quad \ldots\ldots [22]$$

The fluid wave velocity may be computed from equation [19] using equation [22] to calculate B_p. Equation [22] is similar to the "Wood" equation which was presented by Allen et al. (1980). The Wood equation, however, assumes complete coupling between the solid and fluid phases whereas equation [22] assumes a more realistic function (i.e., equation [16]) for the coupling coefficient. Also herein, the density parameter ρ_p was calculated as a function of S whereas in Allen et al. (1980) it was not. As a result we do not place the restriction on the range of validity of equation [19] as did Allen et al. (1980) on the Wood equation (i.e., $S>0.98$), but assume it to be valid for $0.85<S<1$.

The frame wave velocity, U_f, has been found empirically to be a function of confining pressure, σ, of the form (Hardin & Richart, 1963)

$$U_f = a\,\sigma^b \quad \ldots\ldots [23]$$

For saturated sands they found that $a = 66$ and $b = 0.37$ when σ is in kPa and U_f is in m/s. For dry sands they found $a = 93.5$ and $b = 0.306$. Moist material had values between the extremes. For purposes of calculation we assume here a linear relationship between a and S and between b and S. Hence, the empirical relationship

$$U_f = [93.5-27.5\ S]\ \sigma^{[0.306\ +\ 0.064\ S]} \quad \ldots\ldots [24]$$

was used to calculate the frame velocity U_f.

σ must be known in order to calculate U_f. The effective confining pressure, σ, for nearly saturated soils in equal to the soil matric suction (Towner, 1961). For partially saturated soils the relationship is more complex (Towner and Childs, 1972). For the calculations made here we are interested in the near saturated state and it was assumed that σ and soil water suction are essentially equivalent.

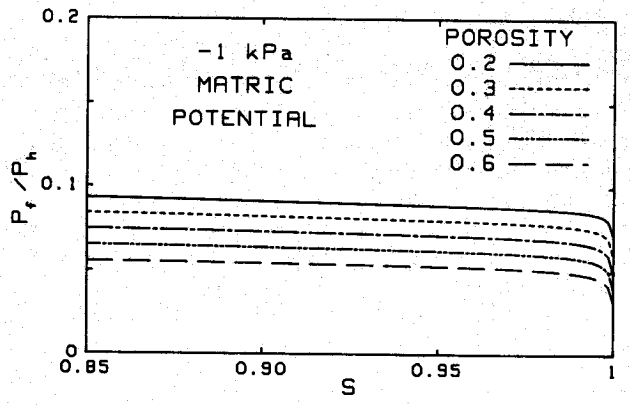

Fig. 2—Theoretical normalized impact pressures on the soil skeleton as a function of degree of saturation for five porosities at -1 kPa matric potential.

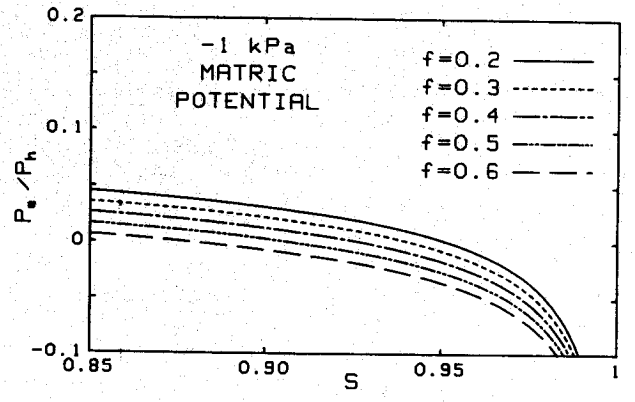

Fig. 4—Theoretical normalized vertical effective stresses as a function of degree of saturation for five porosities at -1 kPa matric potential.

This assumption is more restrictive for a sand than for a clay. Towner and Childs (1972) showed for a beach sand that effective stress and suction were equivalent to about 2 to 3 kPa, whereas Towner (1961) found effective stress and suction to be equivalent to at least 400 kPa for a pure kaolinite.

CALCULATIONS

P_f, P_p, and P_e were calculated as functions of S, f, and σ for a range of values. The pressures were normalized to the water hammer pressure, P_h (P_h for water impacting at a velocity of 10 m/s is 15 MPa). The density of the solid phase was assumed to be 2650 kg/m^3 and the density of water was assigned a value of 1000 kg/m^3. The bulk compressibility of air, B_a, was assigned a value of 789 kPa; the bulk compressibility of water, B_w, was assigned a value of 2.24 GPa; and the bulk compressibility of the solid phase, B_s, was assigned a value of of 36.9 GPa (for quartz).

Fig. 2, 3, and 4 show the relationship between P_f, P_p, and P_e and S for five porosities, f, at a soil matric potential of -1 kPa. The calculated values of P_p did not vary significantly as a function of f and hence were plotted as a single line. Figs. 5, 6, and 7 show the theoretical effects of soil matric potential (i.e., effective confining pressure) on the calculated pressures at f=0.40. The calculated values of P_f did not vary significantly as a function of S and hence were plotted only for S=0.90. Pressure was not plotted at values of matric potential greater than -0.5 kPa. Computation of the frame wave velocity at very low confining pressures is a questionable extrapolation of existing experimental data. The limiting value of -0.5 kPa was chosen arbitrarily.

DISCUSSION

The calculated theoretical vertical stresses on the soil skeleton, P_f, ranged 0.05 to 0.16 P_h over the range of soil characteristics used in the calculations. This suggests that the vertical pressures of impact of on soil surfaces are likely to be much less than the pressures of impact on rigid surfaces, which are of the order of P_h. For a soil with S=0.90 and matric potential, ψ, of -10 kPa, P_f was calculated to be 0.16 P_h. Another soil at S=0.90 and ψ=-0.5 kPa would have P_f equal to 0.05 P_h. P_f was lower at low suctions (Fig. 5). P_f also decreased as f increased (Fig. 2), but the differences were not as great as for the changes with ψ. P_f was not sensitive to S (Fig. 2).

The calculated pore water pressures were highly dependent upon the degree of saturation, S, as would be intuitively expected, and P_p increased to very high value as S approached 1 (Fig. 3). The effects of porosity and matric potential (Fig. 6) on P_p were negligible. At high saturations and low matric suctions, P_p was greater than P_f and hence the soil would liquify under those conditions (Holtz and Kovacs, 1981), which is a mechanically unstable soil condition. Liquification for ψ=-1kPa would occur at S≥0.875 for f=0.6 and at S≥0.953 for f=0.2. The liquifaction potential is greater,

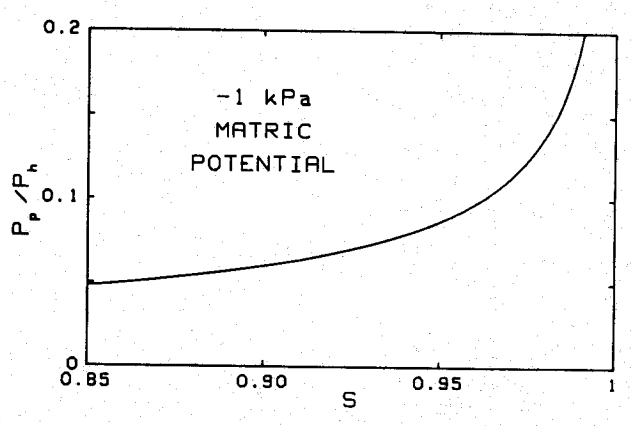

Fig. 3—Theoretical normalized pore water pressures as a function of degree of saturation at -1 kPa matric potential.

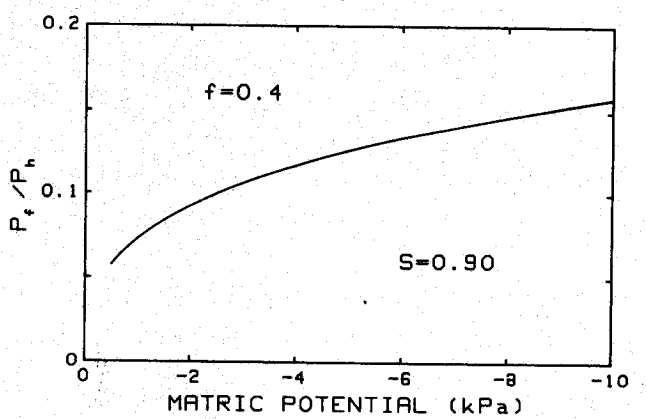

Fig. 5—Theoretical normalized impact pressures on the soil skeleton as a function of matric potential at 0.4 porosity.

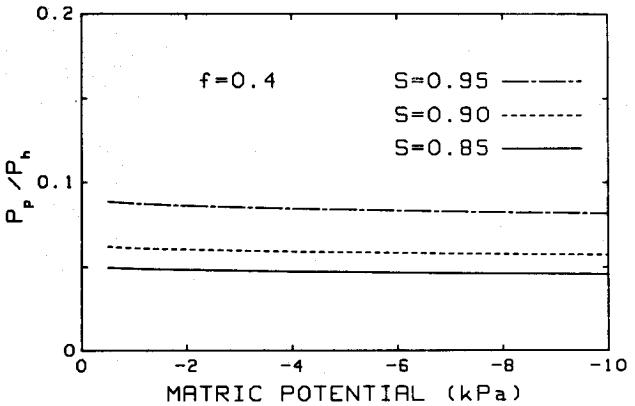

Fig. 6—Theoretical normalized pore water pressures as a function of matric potential for three degrees of saturation at 0.4 porosity.

therefore, for soils with higher porosity. At f=0.4 liquifaction would occur for $\psi > (-0.5$ to $-1.8)$ kPa for S between 0.85 and 0.95. This suggests that for near saturated soil conditions, when matric suction is low, the soil surface is highly unstable under raindrop impact.

Soil matric potential, degree of saturation, and porosity were treated as independently varying parameters in the calculations of impact pressures. Obviously they are interrelated under natural field conditions. If the relationship between matric suction and degree of saturation for a soil were given, for instance, the impact pressures could be calculated as functions of either variable using the theoretical equations presented herein.

This study raises basic questions regarding the definitions of terms related to soil erosion processes. The mechanics approach to problem solving is to relate some input function, often a stress function, to the observed material response, often deformation or failure. The function which relates the input to the material response is called a constitutive relation, which is material dependent and often restricted to a certain range of conditions. In the case of detachment of soil particles caused by raindrop impact, the mechanical input is related to the erosivity term used in erosion literature. The erodibility is the material dependent parameter which predicts the soil response to the erosivity term. It is essentially a constitutive relationship which is valid under certain restricted environmental conditions.

The erosivity term is normally considered, in soil erosion literature, to be exclusively rainfall dependent.

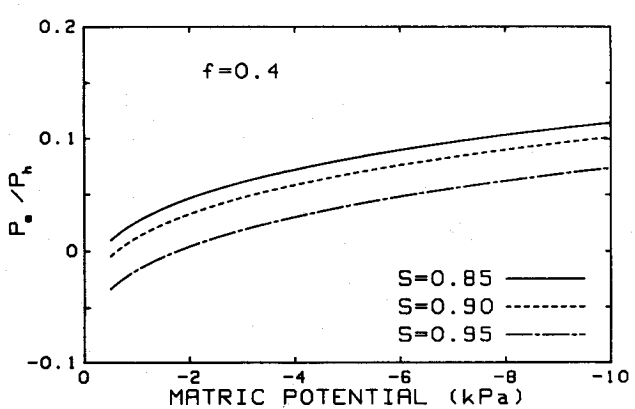

Fig. 7—Theoretical normalized vertical effective stresses as a function of matric potential for three degrees of saturation at 0.4 porosity.

However, from a fundamental perspective for the case of drop impact on a soil surface, it is apparent from this study that soil parameters may affect the loading function for water to soil impact. If this effect was incorporated into an erosion model it might be convenient to place it into either an erodibility term or into an erosivity term according to the stucture of the model. However, from a fundamental mechanics perspective it should be recognized that soil properties may affect the magnitudes of impact loading as well as the soil response to loading.

The data relating soil matric potential, porosity, and saturation to the dynamic parameters are limited. Nevertheless, the theory does provide some important information regarding impact pressures on soil surfaces:

1. The theory presents the governing equations for the pressures of impact, including those for total and effective stresses and pore water pressures.
2. The theory provides order of magnitude values for impact pressures. In the past, it has been assumed that measurements on rigid surfaces relate to the case of soils. The theory suggests that the actual pressures are an order of magnitude less than for rigid surfaces. Calculated vertical stresses were 0.05 to 0.16 the water hammer pressure.
3. The theory provides a method for assessing the relative effects of soil matric potential, porosity, and saturation on impact pressures on soil surfaces. The results indicated that the soil surface was subject to liquifaction at high saturations and low suctions. At ψ=-1 kPa, liquification may be expected to occur for S > [0.87 to 0.95].

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