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A PROBABILISTIC MODEL OF SOIL DETACHMENT BY SHALLOW TURBULENT FLOW

M. A. Nearing
ASSOC. MEMBER
ASAE

ABSTRACT

This study presents a simple probabilistically based mathematical model for the detachment of cohesive soil particles by shallow turbulent flows. The model is based on the concept that bursting events in turbulent flows cause the detachment of soil particles when the localized shear stress associated with the bursting event exceeds the local tensile strength on the soil surface. Both the shear stresses of the turbulent bursts and the local soil resistance were described in terms of probability density functions, and the overlap of the two probability functions represents the probability that a given bursting event causes detachment of particles. The model was applied with favorable results to detachment data from the controlled laboratory experiment of Nearing et al. (1991) for two soils, each with three aggregate size distributions. The model helps to explain why neither shear stress nor streampower adequately define detachment rates by flow, and it shows mathematically that 'critical hydraulic shear stress' is a stochastic, rather than deterministic variable. The model also explains that detachment occurs when average flow stresses are orders of magnitude less than soil strengths because burst events in the turbulent flow create much greater local shear levels than the averages, and detachment occurs only for those burst events where stress exceeds the local tensile strength of the soil.

KEYWORDS. Erosion, Modeling, Shallow turbulent flow.

INTRODUCTION

Relationships for detachment of soil by shallow surface flow of water are used in soil erosion models to estimate initiation and rates of erosion by scour in rills or micro-channels. The relationships for rill erosion are important because the models are usually quite sensitive to rill erosion rates (Nearing et al., 1990a) and their form has a major effect on relationships of slope length and steepness to soil loss (Nearing et al., 1990b). Most current models rely on a gross characteristic of flow, most commonly either hydraulic shear stress or streampower, to characterize the capability of the flow to detach soil (Foster, 1982; Rose, 1985; Hairsine, 1988). Often a threshold term, such as critical shear stress of flow, is used as an indicator of the initiation of scour, and hence the beginning of rill formation. The relative advantages and disadvantages of using streampower vs. using shear stress have been discussed (Lane et al., 1982), though there seems to be no clear evidence of an advantage of using one flow characteristic over the other.

It has been recognized that the mechanics of detachment of particles by flow is not a simple function of averages of a flow parameter such as shear stress or streampower. Grass (1970) showed that the initiation of movement of fine bed sand could be described as an overlap of two probability distributions, one which described the distribution of instantaneous bed shear stresses and another which described the resistance of the individual sand grains on the bed. Christensen (1975) discussed the concept of critical shear stress in terms of probabilities for the case of cohesionless sand. Christensen recognized that instantaneous bed shear stresses in turbulent flows could be much greater than average shear stress of the flow, and he derived a method of calculating the probability of erosion associated with the critical shear stress.

The boundary shear stresses in turbulent flow are highly localized in both space and time (Grass, 1982). The stochastic nature of the boundary shear stress is associated with turbulent 'bursts' which occur periodically in the flow, and with which are associated a large fraction of the Reynold's stress of flow. Cleaver and Yates (1973) and Croad (1981) have developed models based on the detachment of cohesive soil by these turbulent burst events. The model of Cleaver and Yates is a deterministic model, in which averages of the lift forces associated with the turbulent burst events were calculated and related to detachment rates of colloidal particles in a rotating cylinder device. Croad's (1981) model, reported by Raudkivi and Tan (1984), is also deterministic, and uses the concept of rate-process theory to characterize the resistance of the soil to detachment by flow.

Detachment of cohesive soil by flow has been related to soil shear strength in several studies (Lane, 1955; Lyle and Smerdon, 1965) and at least once to soil tensile strength (Nearing et al., 1991). Soil strength is on the order of kPa, while flow shear stresses which initiate detachment, i.e., critical shear stresses, are typically on the order of Pa. It is difficult to explain the three orders of magnitude difference between strength of the soil and stresses required to 'fail', or detach, the soil using a deterministic approach.

Nearing et al. (1991) recently conducted a controlled laboratory study of detachment of two soils in a hydraulic flume at low shear stress levels. The purpose of that study was to investigate the relationships between detachment rates by flow and some flow and soil characteristics under conditions where initiation of scour occurs. One objective

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The author is **M. A. Nearing**, Assistant Professor, Dept. of Agricultural Engineering, University of Georgia, Athens.

was to assess the relative usefulness of using flow shear stress or streampower (Bagnold, 1977) to estimate detachment rates under these conditions. The results were somewhat unexpected in that neither shear stress nor streampower uniquely related to detachment rate in the flume. In one series of tests, flow depth was held constant and slope was increased. In another series of tests slope was held constant and the effect of flow depth was assessed. Slope had a much greater influence than did flow depth on detachment rates. That is, an increase in shear stress or streampower by increasing the slope caused a much greater increase in detachment rate than did the same increase in shear stress or streampower by increasing flow depth. Also, the relationship between detachment rate and either shear stress or streampower were exponential. A linear or near-linear relationship between detachment and shear stress are most often assumed in modeling erosion.

The objective of this study was to develop a probabilistically based model for the detachment of cohesive soil by shallow surface flow of water. Reasons for developing the model included:

- To better understand the mechanisms of detachment of soil particles at shear stress levels normally considered at or near the critical "threshold" levels for particle entrainment.
- To understand why neither shear stress nor streampower uniquely defined detachment rates from the experimental study of Nearing et al. (1991), and why detachment rates were more sensitive to slope gradient than to flow depth.
- To better understand how soil particles are detached by shear flows where the shear stresses are orders of magnitude less than the tensile failure strengths of the soils.
- To understand why the detachment data from Nearing et al. (1991) showed an exponential relationship between detachment rates and flow shear stress or streampower at shallow flow depth and low slope.

MECHANICS OF DETACHMENT BY FLOW

Detachment of soil particles by turbulent flow of water from the surface of the soil is induced when the localized flow shear stresses of the flow exceed the local resistance to detachment of the soil particles. For turbulent flows, the local shear stresses can be much greater than the average flow shear stresses and more localized in both space and time. The detachment of the soil particles from the soil surface involves a tensile failure. The particles are pulled from the surface against the cohesive forces of the soil which act as the resisting forces to detachment. This resistance to detachment may be characterized in terms of the local tensile strength of the soil at the interface with the flowing water.

From these basic considerations of the mechanics of soil detachment, a probabilistic model may be hypothesized of the form:

$$e = F\,P\,M \qquad (1)$$

where

F (1/s*m^2) = temporal and spatial frequency of turbulent bursting 'events',

P = probability that the shear stress of the burst-event exceeds the local resistance to detachment and induces tensile failure,

M (kg) = average mass of soil detached per failure event, and

e (kg/s m^2) = is the detachment rate per unit area of soil surface.

FREQUENCY OF BURSTS (F)

If the streamwise and spanwise distances between the centers of the bursts are $\lambda 1$ and $\lambda 3$, respectively, then the spatial frequency of bursts, f_s, is by definition:

$$f_s = 1/(\lambda_1 \lambda_3) \qquad (2)$$

Croad (1981) (as reported by Raudkivi and Tan, 1984) used published data to estimate $\lambda 1$ and $\lambda 3$ as:

$$\lambda_1 = 500\nu / u_* \qquad (3a)$$

$$\lambda_3 = 100\nu / u_* \qquad (3b)$$

where

ν = kinematic viscosity of water, and

u_* = shear velocity.

These estimates for burst spacings are very close to the ones used by Cleaver and Yates (1973). Average shear velocity can be related to average flow velocity using Chezy depth vs. discharge relationships as:

$$u_* = u(f/8)^{1/2} \qquad (4)$$

where

f = Darcy-Weisbach friction factor, and

u = mean velocity of flow.

Equation 2 can be rewritten with appropriate substitutions for λ values, ν, and u_* as:

$$f_s = 20 * 10^8 (F/8)\, u^2 \qquad (5)$$

Temporal frequency of bursting events was studied by Rao et al. (1971) and Gordon (1975) and may be estimated as:

$$f_t = u / 5\delta \qquad (6)$$

where δ is the displacement thickness of the turbulent boundary layer (Hinze, 1975).

Multiplying the spatial and temporal frequencies gives the functional form for F in equation 1. Substituting u with the Chezy relationship for u as a function of flow depth, h, and bed slope, S, gives the equation:

$$F = 4 * 10^8\, C\, g(h\, S)^{3/2} / \delta \qquad (7)$$

where C is the Chezy coefficient [$C = (8g/f)^{1/2}$].

Examination of equation 7 indicates that the frequency term for the model represented by equation 1 should be approximately proportional to $(h\,S)^{3/2} / \delta$. A question arises with regard to the thickness of the turbulent boundary layer for the case of shallow flows. The studies of Nearing et al. (1991) were conducted in a hydraulic flume with laminar flow at the entrance of the flume. The flow at the entrance edge of the flume was split to eliminate the boundary layer at that point, thus, the thickness of the boundary layer at the entrance point to the channel was zero. Under those conditions, it is relatively easy to calculate the thickness of the turbulent boundary layer at the point 6 m down the flume where the testing station was located (Hinze, 1975; Olson, 1973). Under these conditions, the calculated displacement thickness of the boundary layer was on the order of 15 to 17 mm, which was greater than or on the order of the thickness of the flow depths used, and ranged from 5 to 20 mm. In the field situation, it would be expected that the turbulent boundary layer would be thicker than in the flume experiments. For shallow flows under consideration here, we make the assumption that the boundary layer thickness, δ, can only be as thick as the depth of flow, h, and, therefore, the frequency of turbulent bursting events, F, can be approximated as:

$$F = K_1 \, C h^{1/2} S^{3/2} \tag{8}$$

where K_1 is the linear coefficient between the h and S terms, and h is substituted for the δ in equation 7.
The Chezy coefficient is retained in the equation since C can be dependent on flow depth.

PROBABILITY OF A FAILURE EVENT (P)

If the shear stresses of turbulent bursting events and the resistance force, or tensile strength, of the soil under the bursts are described using probability density functions, then the overlap of the two distributions is the probability of a bursting event causing detachment. This concept is equivalent to the capacity-demand model used in reliability analysis (Harr, 1977) where the capacity is the resistance force and the demand is the shear stress of the burst event (fig. 1). For cases of normal or lognormal distributions, the region of overlap, or probability of failure, can be easily calculated. For the case of normal distributions, the probability of failure, P, is:

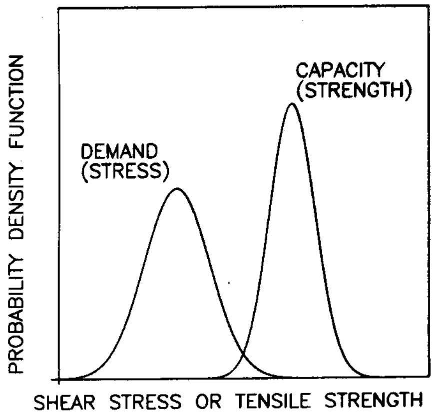

Figure 1–Schematic diagram showing the overlap of two probability distributions: one for shear stresses associated with turbulent burst-events; and one for the resistance of the soil to detachment forces.

$$P = 1/2 - \Psi \left(T - \tau_b\right) / \left(S_T^{\,2} + S_\tau^{\,2}\right) \tag{9}$$

where
Ψ = cumulative probability function of the standard normal variate,
T = mean value of the capacity term (which in our case is the soil resistance or tensile strength),
τ_b = the mean value of the demand (which in our case is burst-event shear stress),
S_T = standard deviation of the capacity, and
S_τ = standard deviation of the demand.

Harr (1977) gives the equation for P for the case of lognormal distributions, which also requires the means and standard deviations of the two probability density functions.

FINAL DETACHMENT EQUATION

Equation 1 for detachment by shallow surface flow may be rewritten with substitutions as:

$$\dot{e} = K\, C\, P h^{1/2} S^{3/2} \tag{10}$$

where the coefficient K incorporates the K_1 from equation 8 and M from equation 1, P is a function of T, τ_b, S_T, and S_τ as represented for the normal distribution form in equation 9.

METHODS

The flow detachment model was applied to the laboratory flume data of Nearing et al. (1991) for three aggregate size distributions of the Paulding and Russell soils. Flow depths in that study ranged from 5 to 20 mm and slopes from 0.5% to 2.0%.

The mean value for the shear stresses of the burst-events was estimated using the following equation:

$$\tau_b = 0.9\left(\Delta / \Delta_m\right)\left(\lambda_1 \lambda_3 / \zeta_1 \zeta_3\right)\tau \tag{11}$$

where
Δ = periodicity of burst events,
Δm = length of the burst,
ζ_1 and ζ_3 = streamwise and spanwise size of the area under the burst,
τ = average total shear stress of the flow, and
0.9 = coefficient which represents the total percentage of Reynold's stress of the flow which are related to the bursting events.

The ratio of $\Delta{:}\Delta_m$ was measured by Rao et al. (1971) and found to be a constant value of 2 over a wide range of flow Reynold's numbers. Values of ζ_1 and ζ_3 were estimated by Croad (1981) (as reported by Raudkivi and Tan, 1984). Using Croad's estimates, the ratio ($\lambda_1 \lambda_3 / \zeta_1 \zeta_3$) is on the order of 83. This ratio represents the inverse of the fraction of total area within which the burst event occurs. The coefficient of 0.9 was reported by Gordon (1975) and by Gupta and Kaplan (1972) on two sets of measured data. Using these estimates from the literature gives an estimate for the average shear stress of the burst-events as:

$$\tau_b = 150\tau \tag{12}$$

The shear stress associated with the localized bursting events is approximately 150 times greater than the average shear stress of flow.

The coefficient of variation of the instantaneous shear stresses, CV_τ, was estimated to be 0.4, which was taken from the results of the study of Grass (1970) on the relationship between turbulent events and the entrainment of fine bed sand. The standard deviation, S_τ was calculated from the coefficient of variation and the mean value of shear stress.

The mean value of the tensile strength was measured for each of the six soil materials by Nearing et al. (1991). The assumption was made that the measured tensile strengths under zero matric potential was a reasonable representation of the average resistance to tensile failure under the flow conditions of the flume experiment.

Chezy roughness coefficients were calculated for each of the flow conditions of the experiment by Nearing et al. (1991).

The constant, K, and the coefficient of variation of tensile strength, CV_T, (which is a direct function of S_T) needed to evaluate equation 10 were determined simultaneously by optimization to minimize differences between measured and predicted detachment rates. Since the values of detachment rate varied over orders of magnitude, and since the low erosion rates were considered important in characterizing the overlap of the probability distributions, the data was transformed to logarithmic space prior to optimization. This transformation gave a more equal weight in the analysis to all the detachment rate values. Each of the six detachment data sets contained 7 data points (each point representing the average of 6 to 8 experimental replications), and 2 parameters (K and CVT) in the model were optimized; therefore there were 5 degrees of freedom in each analysis.

The analyses were performed for both the case of normal and the case of lognormal distributions of shear stress and resistance forces.

RESULTS AND DISCUSSION

The case of normal distributions for shear stress and soil resistance resulted in a slightly better fit between measured and calculated detachment rates than did the lognormal case (Table 1). Plots of measured and calculated detachment rates for the Paulding soil are shown in figures 2, 3, and 4. Results for the Russell soil were similar.

The results of the model exhibited several trends which were consistent with trends found with the measured data, and which would not be readily apparent outside the framework of the probabilistic description herein presented. The calculated results showed a greater sensitivity to bed slope than to flow depth, which is consistent with the measured data. This fact is reflected in the calculated detachment rates because of the greater exponent for slope in equation 10, which results from the theorized effect of the shallow flow depths on the frequency of bursting events. Frequency of bursts is known to be a function of boundary layer displacement thickness, which obviously cannot be greater than depth of water flow. The frequency of burst-events in shallow flows has never been measured, and therefore the cause-effect relationship suggested here is a hypothesis, but the impact of the assumptions used in the equation is that the proposed model is consistent with the experimental results. Thus, the model provides a possible rationale for explaining why neither shear stress nor streampower uniquely define detachment rates at shallow flow depths that are associated with flow in rills. The fluid mechanics of shallow turbulent flows warrants further experimental investigation. Further research is also needed to assess the relationships between flow shear stress, flow depth, and bed slope for a wider range of conditions than those studied by Nearing et al. (1991).

The model was consistent with the measured data in that the plots of detachment rate vs. average shear stress of flow were non-linear, quasi-exponential in form for both the cases of constant depth with increasing slope and constant slope with increasing depth (figs. 2, 3, and 4). The exponential relationship between detachment rate and shear

TABLE 1. Results of the optimization procedure for fitting measured vs. calculated detachment rates*

Soil Type	Aggregate Sizes (m m)	Normal Distributions CV_T	Normal Distributions r^2	Lognormal Distributions CV_T	Lognormal Distributions r^2
Russell	0 - 1	0.182	0.97	0.945	0.92
	1 - 2	0.197	0.88	0.898	0.80
	2 - 4	0.210	0.94	0.883	0.90
Paulding	0 - 1	0.172	0.93	1.074	0.87
	1 - 2	0.213	0.98	0.929	0.92
	2 - 4	0.205	0.98	0.843	0.99

* For the six soil materials used in the Nearing, 1991, experiment.

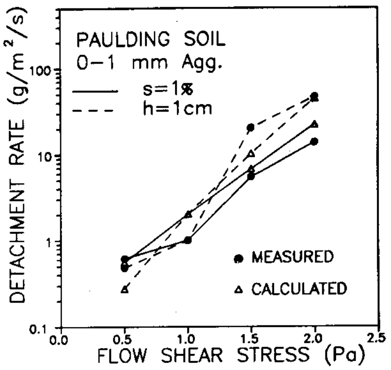

Figure 2–Measured and calculated detachment rates for the aggregates of the Paulding soil as a function of average flow shear stress.

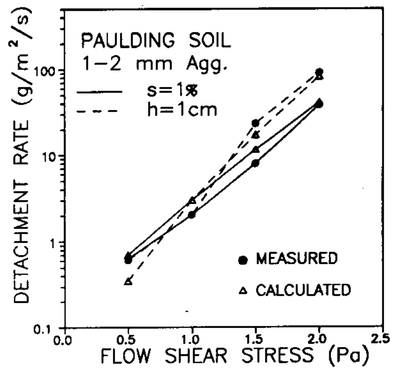

Figure 3–Measured and calculated detachment rates for the 1-2 mm aggregates of the Paulding soil as a function of average flow shear stress.

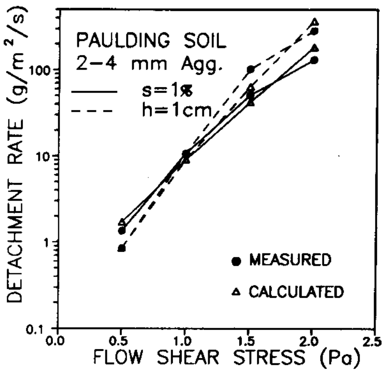

Figure 4–Measured and calculated detachment rates for the 2-4 mm aggregates of the Paulding soil as a function of average flow shear stress.

stress relates to the overlap of the two probability distributions. For small regions of overlap (i.e., for low P), the cumulative probability function (Ψ in eq. 9) is closely approximated by a negative exponential function of standard normal variate, which (from eq. 9) is approximately proportional to the difference between strength and stress terms. Since the burst stress term in equation 9 is proportional to the average stress term (eq. 12), this results in the overall exponential trend in the relationship of detachment rate vs. average flow shear stress.

The probabilistic model addresses an apparent inconsistency in terms of the orders of magnitude differences between soil strength and flow shear stresses. Soil strength is typically on the order of kPa, while flow shear stresses are typically of the order of Pa. In the Nearing et al. study, flow shear stresses ranged from 0.5 to 2.0 Pa, while tensile strengths ranged from 1.0 to 2.0 kPa, a difference in magnitude of 1000. The proposed model helps to explain how erosion occurs with this major difference between soil strength and flow stresses. Part of the answer is that local shear stresses associated with turbulent burst-events are much greater than the average flow shear stresses (eq. 12). The other part of the answer lies in the fact that it is not the average levels of shear and resistance which determine detachment rates, but rather it is the tails of the two distributions which dominate the process. Detachment does not occur for each turbulent burst-event, but only for those where stress exceeds strength, which may be only a small fraction (estimated by P) of the total number of burst events.

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