ARS-SCS Watershed Modeling Workshop, Tucson, AZ
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KGR-1

AN ANALOG COMPUTER MODEL OF RUNOFF FROM A SEMIARID WATERSHED [1/]

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INTRODUCTION

A cooperative effort between the staff of the Southwest Watershed Research Center and the Utah Water Research Laboratory at Utah State University has produced an analog computer model of the surface runoff process which seems to describe the hydrologic conditions of a semiarid watershed very well.

The model includes provisions for the spatial variation of hydrologic elements such as rainfall, slope, channel size, soil, and vegetation within the watershed. The surface runoff process involves both overland flow and channel flow with the equations of unsteady flow (continuity and momentum) adapted to route the excess rainfall over pervious surfaces and thereby allow infiltration to continue as long as water is available in detention storage.

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Conceptual Model Description

This paper is a portion of a more detailed report by Amisial, et al.[3]. The runoff process considered in the model is that portion of the hydrologic cycle shown inside the dotted lines in Figure 1. Contrary to a general model, the runoff model described here is restricted to the following situations:

1. Short-duration storm events during which interflow and groundwater flow play no part.
2. Watersheds in which infiltration water does not reappear as surface flow within the watershed.

Only a portion of the rainfall occurring over a watershed appears as runoff in the channel at the watershed outlet. Most of the rainfall is accounted for by losses in the watershed. Some of the precipitation in the early portion of a storm is intercepted by vegetation, and some is stored in depressions on the land surface. This water subsequently evaporates or infiltrates to the soil profile. The remaining portion of the precipitation (precipitation excess) is the effective precipitation which produces overland and channel flow and is depleted by infiltration and channel seepage. Thus, the model being described has three phases in the surface runoff process:

1. The phase in which an effective precipitation is produced
2. The overland flow phase during which water flows over the land surface toward an established channel
3. The channel flow phase in which water collected from phase 2 flows in a channel system and ultimately produces an outflow hydrograph at the watershed outlet.

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[3] Amisial, R. A., Riley, J. P., Renard, K. G., and Israelsen, E. K. Analog computer solution of the unsteady flow equations and its use in modeling the surface runoff process. Utah State University Technical Report No. , 1970.

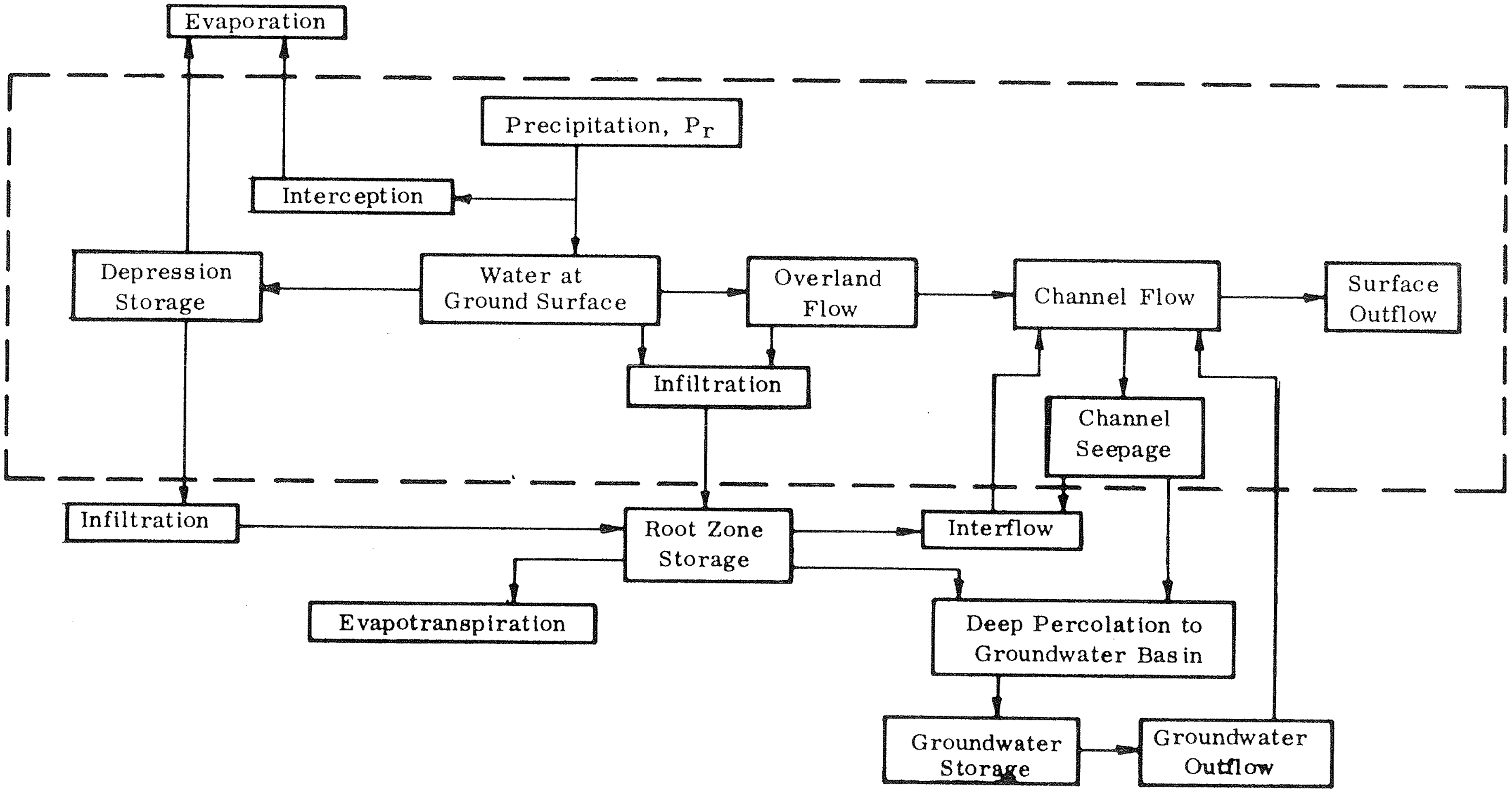


Figure 1.--Schematic diagram of the runoff cycle.

The watershed to be modeled was divided into Z subzones on the basis of the physiography. Each subzone, which is in fact a small drainage basin, is replaced by an equivalent basin having the same surface area as the subzone and composed of two identical rectangular sloping planes transected by a main channel (Fig. 2), (Wooding, 1965)[4/]. The rectangular planes over which the overland flow occurs, have a width equal to the length of the main stream channel within the subzone, and a slope which is an average between the land slope and the slopes of smaller tributaries of the subzone. The portion of channel in the equivalent subzone is assumed to be a straight channel having the same characteristics (width, length, and average slope) as the corresponding segment of the prototype channel. The channel is fed on both sides by outflow from the planes and from upstream by runoff from the preceding subzone.

Effective Precipitation

A flow chart for the surface runoff model is shown in Figure 3. Point rainfall values from recording rain gages in the watershed are used to input a weighted subzone precipitation using a digital computer program capable of integrating the point measurements in time and space[5/]. The program uses interpolation techniques to determine isohyetal lines for given time intervals (Δt). Elemental rainfall volumes are then computed as:

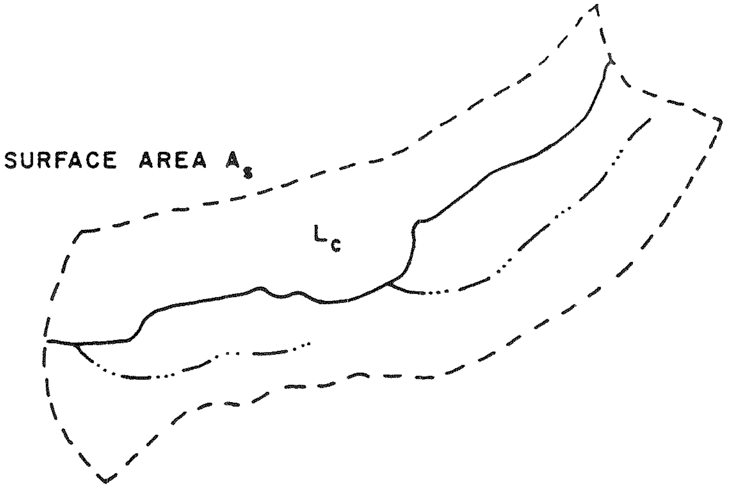
$$V_i = a_i \frac{P_i + P_{i+1}}{2} \qquad (1)$$

where: a_i is the elemental area

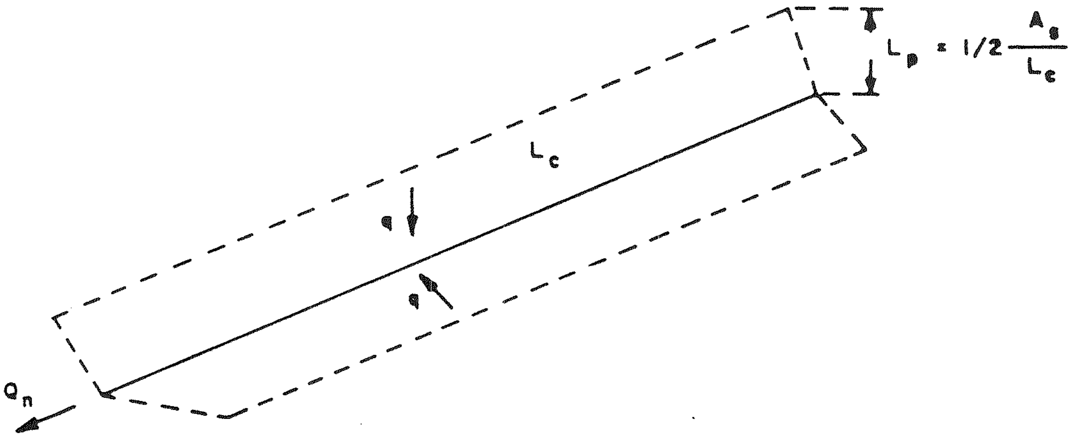
P_i and P_{i+1} are depths of rainfall on adjacent isohyetal lines.

[4/] Wooding, R. A. A hydraulic model for the catchment-stream problem. J. Hydrol. 3:254-267, North Holland Publishing Co., Amsterdam, 1965.

[5/] Kwan, J. Y., Riley, J. P., and Amisial, R. A. A digital computer program to plot isohyetal maps and calculate volumes of precipitation. Symposium: The Use of Analog and Digital Computers in Hydrology, IASH 1(80):240-248, 1968.



a. NATURAL SUBZONE



b. EQUIVALENT SUBZONE

FIGURE 2. SKETCHES SHOWING NATURAL SUBZONE 3 AND ITS EQUIVALENT SUBZONE.

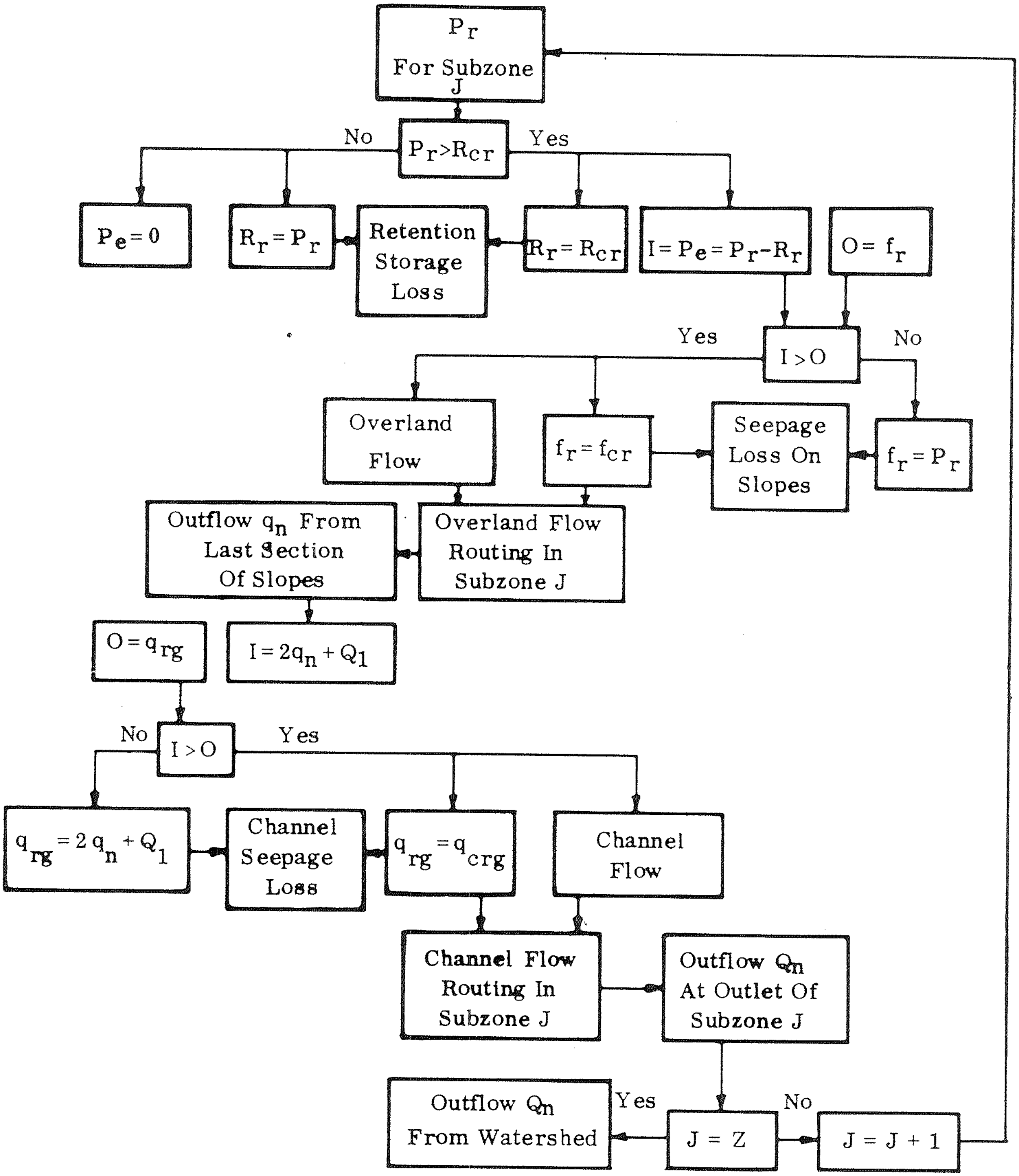


Figure 3.--Flow chart for the surface runoff model.

If the elemental area is bounded by one isohyetal line and a portion of the subzone boundary, rainfall values are determined by interpolation at several points along the portion of the boundary. The rainfall depth over the area is then computed as the average between the value on the isohyetal line and the average of the values at the chosen points along the boundary.

The rainfall rate over the subzone during the time period, Δt, is computed by summing the volumes over all elemental areas comprised in the subzone and dividing by the subzone area and time.

$$P_r = \frac{\Sigma_i V_i}{\Delta t \, \Sigma_i a_i} \tag{2}$$

Losses due to the combined effects of depression storage and vegetation interception are termed retention. Once the vegetative cover becomes thoroughly wetted and the surface depressions are filled, additional retention losses become very small. In general, the retention losses can be expected to be relatively high at the beginning of the storm event, becoming negligible as the event progresses. An exception is the case of an initially wet watershed where the retention losses can be assumed to be equal to zero. It is assumed that the maximum rate at which rainfall is lost to retention storage, R_{cr}, is given by the following expression:

$$R_{cr}(t) = k_r(R_{cs} - R_s(t)) \tag{3}$$

in which

R_{cs} is the retention storage capacity of vegetation and land surface,

k_r is a constant less than unity,

and

R_s = amount of rainfall in retention storage and is given as

$$R_s(t) = \int_o^t R_r \, dt \tag{4}$$

The actual retention rate, R_r, is given by the following equations:

$$R = O \quad , \text{ if } P_r = O$$

$$R_r = P_r \quad , \text{ if } O < P_r < R_{cr}$$

$$R_r = R_{cr} \quad , \text{ if } P_r \geq R_{cr}$$

The effective rainfall rate for each subzone of the model is then obtained as

$$P_e = P_r - R_r \tag{5}$$

Obviously, there will be no water available for surface runoff and infiltration until the rainfall rate exceeds the retention capacity rate.

Overland Flow

The rectangular sloping planes of the equivalent subzones of the model (Fig 2-b) are treated as wide, open channels, and the discharge per unit width (q_n) at the downstream edge is computed from the unsteady flow equations. They are:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} + O - I = O \tag{6}$$

and

$$\frac{\partial V}{\partial t} + \frac{1}{2} \frac{\partial V^2}{\partial x} + \frac{g}{B} \frac{\partial A}{\partial x} = g(S_o - S_f + G) \tag{7}$$

where

V = average velocity
x = distance
g = acceleration of gravity
Sf = friction slope
S_o = bed slope

$$B = \frac{\partial A}{\partial y}$$

$$G = \frac{1}{B} \left(\frac{\partial A}{\partial a}\right)\left(\frac{\partial a}{\partial x}\right)$$

O = unit rate of loss due to seepage
I = Unit rate of lateral inflow

The O term is the infiltration term in the overland flow computation and the (q_{crg}) seepage capacity rate in channel flow, while the I term is effective precipitation in the overland flow and the (q_n) or discharge per unit flow portion of the channel flow model.

In using these equations, the following assumptions are made:

1. The fluid is incompressible.
2. Acceleration is in the x-direction, being negligible in the other directions
3. Pressure distribution is assumed to be hydrostatic because the curvature of streamlines is small and the vertical velocity component is negligible. Thus, the pressure-distribution coefficient is equal to unity.
4. Shear stress (τ_o) is uniform over the perimeter of channel for an incremental distance, Δx
5. Lateral inflow and outflow rates are uniform for a particular incremental channel distance.
6. The momentum flux of the lateral inflow or outflow are ignored
7. The energy and momentum coefficients (α and β) are assumed to be equal to unity
8. The angle between bed and horizontal surface (Θ) is small so that

   $\sin \Theta \approx S_o \equiv$ slope of the bed
9. The effects of resistance to flow in unsteady conditions are the same as for steady flow.

An implicit differential-difference system (Fig. 4) was used to solve the equations on the analog computer. In operational form, the equations programmed were:

$$\frac{dA_j}{dt} = I - O - \frac{Q_{j+1}}{2\Delta x} + \frac{Q_{j-1}}{2\Delta x} \tag{8}$$

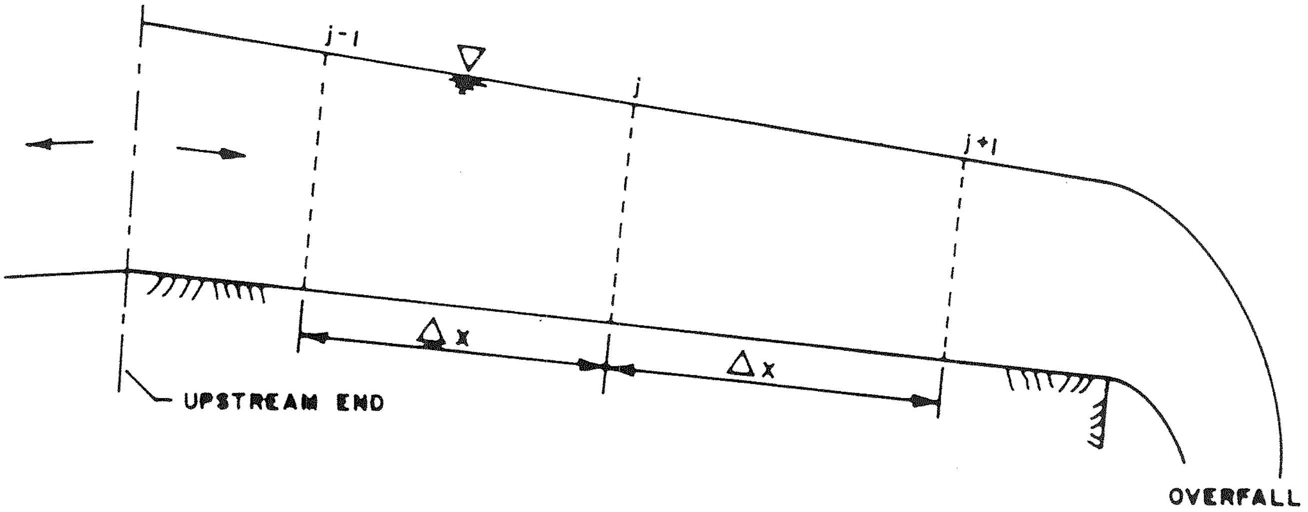


FIGURE 4. THE FLOW PROFILE FINITE INCREMENT PROCEDURE ADOPTED FOR THE MODEL.

$$\frac{dV_j}{dt} = gS_o + gG - gKV_j^2 - \frac{g}{2B\Delta x}\left(A_{j+1} - A_{j-1}\right) - \frac{1}{4\Delta x}\left(V_{j+1}^2 - V_{j-1}^2\right) \qquad (9)$$

$$Q_j = A_j V_j \qquad (10)$$

The slope of the energy line was taken as KV^2 instead of $K|V|V$, because the flow velocity is always positive.

The Horton form of the infiltration curve represented by the following equation was used:

$$f_{cr} = f_m + (f_o - f_m)\, e^{-k_f t} \qquad (11)$$

where f_{cr} = infiltration capacity rate

f_m = minimum value of infiltration rate

f_o = maximum value of infiltration rate

k_f = constant less than 1

t = real time

e = natural logarithim.

Because the infiltration rate on the planes is dependent on the effective rainfall rate and its relation to the infiltration capacity rate, boundary conditions had to be established. If t_o denotes the time at which the rainfall starts, t_e the time at which the effective rainfall rate P_e exceeds the infiltration capacity rate f_{cr}, and t_s the time at which the overland flow stops, then the actual infiltration rate per unit area can be defined by the by the expressions:

$t_o < t < t_e \quad , \quad f_r = P_e$

$t_e < t < t_s \quad , \quad f_r = f_{cr}$

where f_r is the actual infiltration rate.

Note: Readers are referred to the original, more detailed report for discussions of initial and boundary conditions as well as discussions of the mechanics of the analog computer.

Channel Flow

In the case of overland flow, the depth of flow was very small so that its influence on the infiltration rate was assumed to be negligible. The infiltration capacity curve of Horton can be looked upon as the seepage capacity rate curve under conditions of insignificant depth, but with increasing depth as is the case for channel flow, the seepage may be described by Darcy's Law. The seepage capacity rate, F_{cr}, was therefore assumed to be a function of Darcy's Law and an exponential decay in the form of the Horton infiltration curve:

$$F_{cr} = f_{cr} + cy \qquad (12)$$

in which

f_{cr} is given by Equation 11, with parameter adjustments for channel alluvium

y is the depth of channel flow

c is the constant which depends on the soil permeability and the distance of the water table from the ground surface.

The seepage capacity rate per unit length of channel is given by:

$$q_{crg} = BF_{cr} \qquad (13)$$

in which B represents the average width of channel.

If t_o represents the starting time of the storm, t_a the time at which the channel inflow $2q_n$ (flow from either side of equivalent planes) exceeds the seepage capacity rate q_{crg}, and t_s the time at which channel runoff ceases, then the actual seepage rate q_{rg} per unit length of channel can be obtained as

$$t_o < t < t_a \qquad q_{rg} = 2q_n$$

$$t_a < t < t_s \qquad q_{rg} = q_{crg}$$

Again, attention should be called to the fact that seepage will continue at capacity rate until water is no longer available in the channel.

(The reader is again referred to the Amisial report for a more detailed discussion of the initial and boundary conditions as well as conditions at a stream junction.)

MODEL REGULATION AND VERIFICATION

Subwatershed 11 of the Walnut Gulch Experimental Watershed was selected to test and verify the surface runoff model developed. This subbasin, with a drainage area of 2,035 acres, is situated in the northeastern portion of Walnut Gulch (Fig. 5). The predominant soil complex in the area is the Bernardino-Hathaway association with small areas of the Camoro soil in the alluvial swales [6]. Vegetation of this watershed is dominated by black grama and curly mesquite with only limited amounts of brush, primarily along the channel banks.

The channels of this subwatershed contain three main branches (Fig. 6). The middle channel, which traverses the entire length of the watershed, is 4.4 miles long with an average slope of about 2%. This channel contains a stock pond (instrumented with a water-level recorder to provide information about its contribution to the watershed runoff). It contributes runoff to the area downstream only during times when the pond is full. The north channel, with a length of about 2 miles, also has a slope of about 2% and enters the middle channel about 3,000 feet above the supercritical-depth flume. The south channel, which enters the center channel about 1,000 feet above the outlet flume, has a slope of slightly over 2% and is 3.6 miles long.

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[6] Renard, K. G. The hydrology of semiarid rangeland watersheds. U. S. Dept. Agr. Pub. ARS-41-162, 1970.

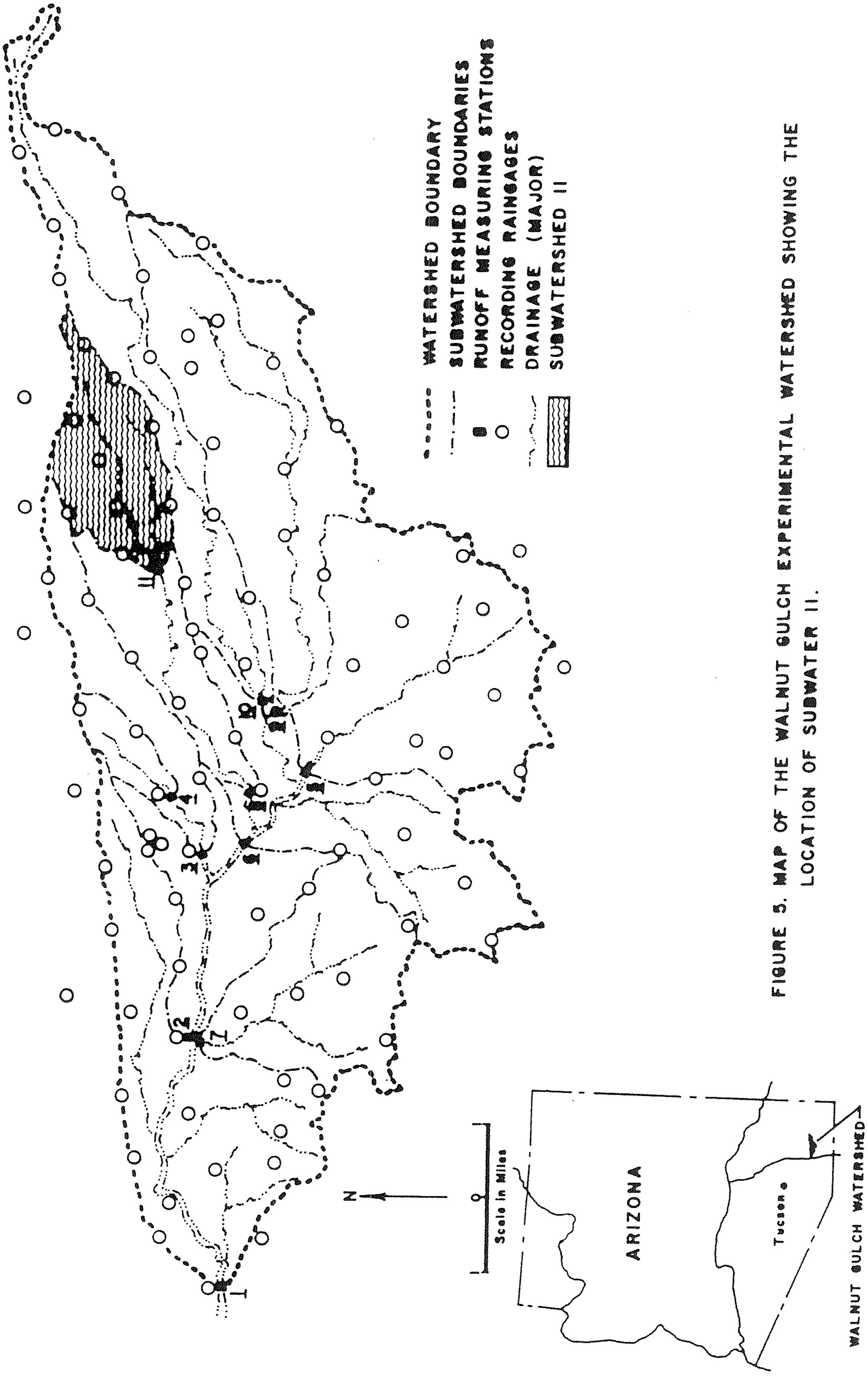

FIGURE 5. MAP OF THE WALNUT GULCH EXPERIMENTAL WATERSHED SHOWING THE LOCATION OF SUBWATER 11.

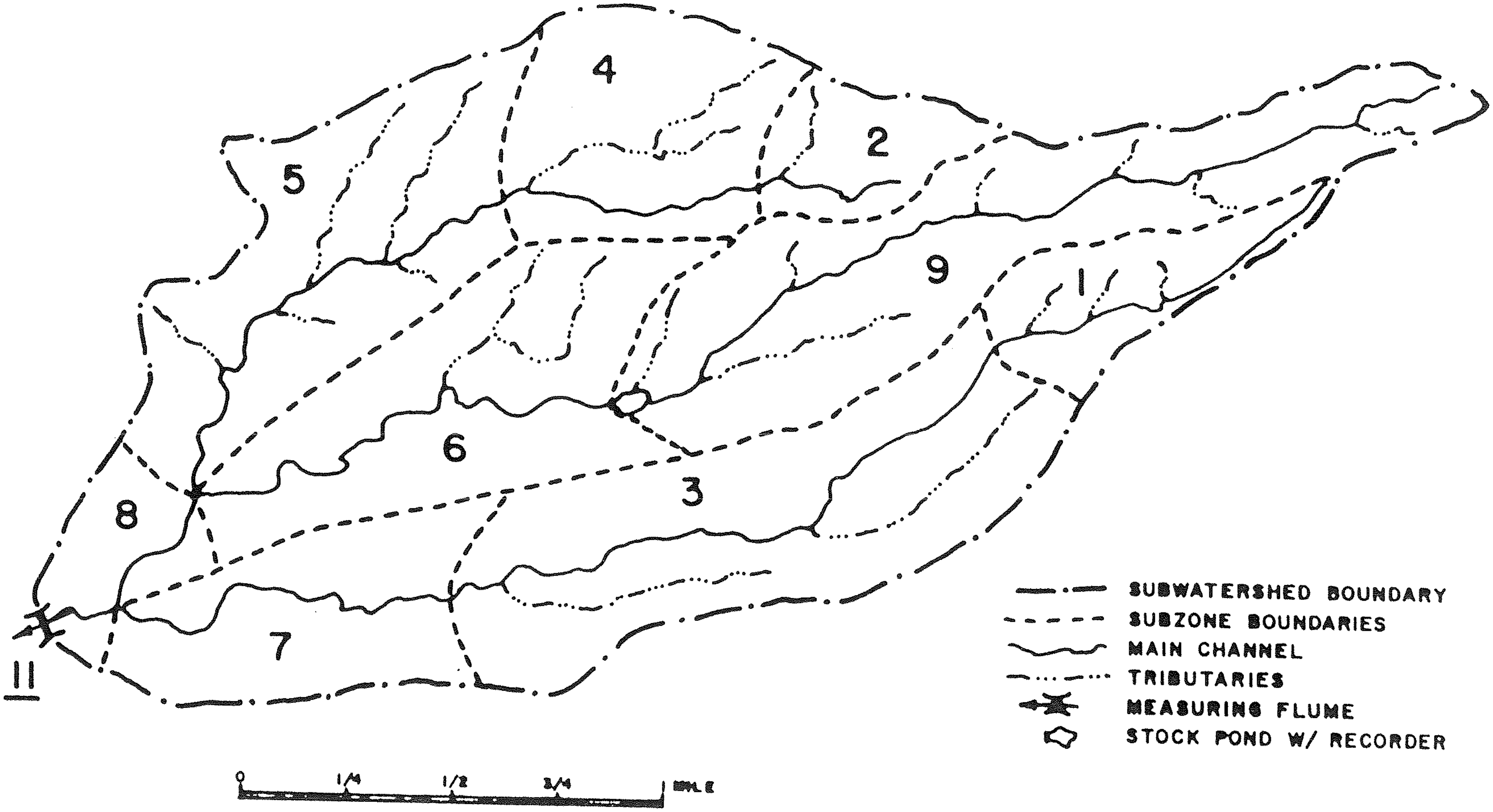

FIGURE 6. SUBWATERSHED II OF WALNUT GULCH EXPERIMENTAL WATERSHED, AS DIVIDED IN 9 SUBZONES.

The channel bed material is composed of coarse material which can be described by a log-normal size distribution. The geometric mean particle size in most segments of the channel is greater than 2 mm., with a geometric standard deviation of about 3.5 mm. The channels have the potential to absorb large quantities of the runoff because of high porosity and because they are dry prior to most flow events.

Precipitation in this subwatershed is measured by seventeen recording gages in and adjacent to the watershed. The subwatershed, which was divided into 9 subzones, thus had approximately 1 recording gage per subzone with eight additional gages defining the precipitation pattern around the subwatershed boundary.

The runoff model described involves the use of a number of parameters which must be fitted to a particular watershed by determining numerical values for the parameters. The parameters can be divided into two types, namely, the function parameters and the condition parameters.

Function parameters

The function parameters are those watershed characteristics which are constant with respect to time, such as length, width, and slope of the planes and channels. The function parameters pertaining to the plane are the width, length, and slope of the plane. The channel dimensions and slope are also considered as function parameters.

Condition parameters

The condition parameters are those parameters which vary with time within a given watershed and usually cannot be obtained by direct measurement. They are generally dependent upon surface and moisture conditions of the watershed. The retention rate accounts for losses due to interception and depression storage. The retention rate for the watershed model is determined when the constants R_{cs} and k_r, the rainfall rate and the watershed moisture conditions, are known.

The model distinguishes an infiltration value for the plane and another for the channel within the subzone. This distinction allows for the great difference in permeability between the channel bed material and that of the land surfaces. The constants required in the model are the maximum infiltration capacity rate, f_o, the minimum infiltration capacity rate, f_m, the time constant, k_f, and the constant c applied in the computation of channel seepage loss.

Within the analog computer program a different roughness coefficient may be used for each plane and channel section depending upon the condition of the soil surface and the meandering and irregularities of the channels.

Model verification was accomplished by fitting and regulating the parameters to coincide with the measured outputs for measured inputs. The model was regulated by a method of data adjustment involving the fitting of the condition parameters to a set of data under a particular set of criteria. The runoff events selected for study in this initial endeavor were those of July 20 and 29, 1966. Computed and measured outputs for these two storms are shown in Figures 7 and 8.

Adjustment or fitting of the condition parameter values is performed with a trial and error procedure. The approximations of individual parameters are refined after each computation until a close fit is obtained between the computed and observed hydrographs. Efforts were made to match the principal characteristics of the computed and measured hydrographs in the following priority:

1. Hydrograph peak discharge
2. Time to peak
3. Volume of runoff

The fitted value of the condition parameters for the overland flow and channel flow portions of the runoff model are shown in Table 1. Because of the uniformity of the vegetal cover, soil type, and channel alluvium within this subwatershed, one set of condition parameters was assumed to be indicative of the overland flow and another set for

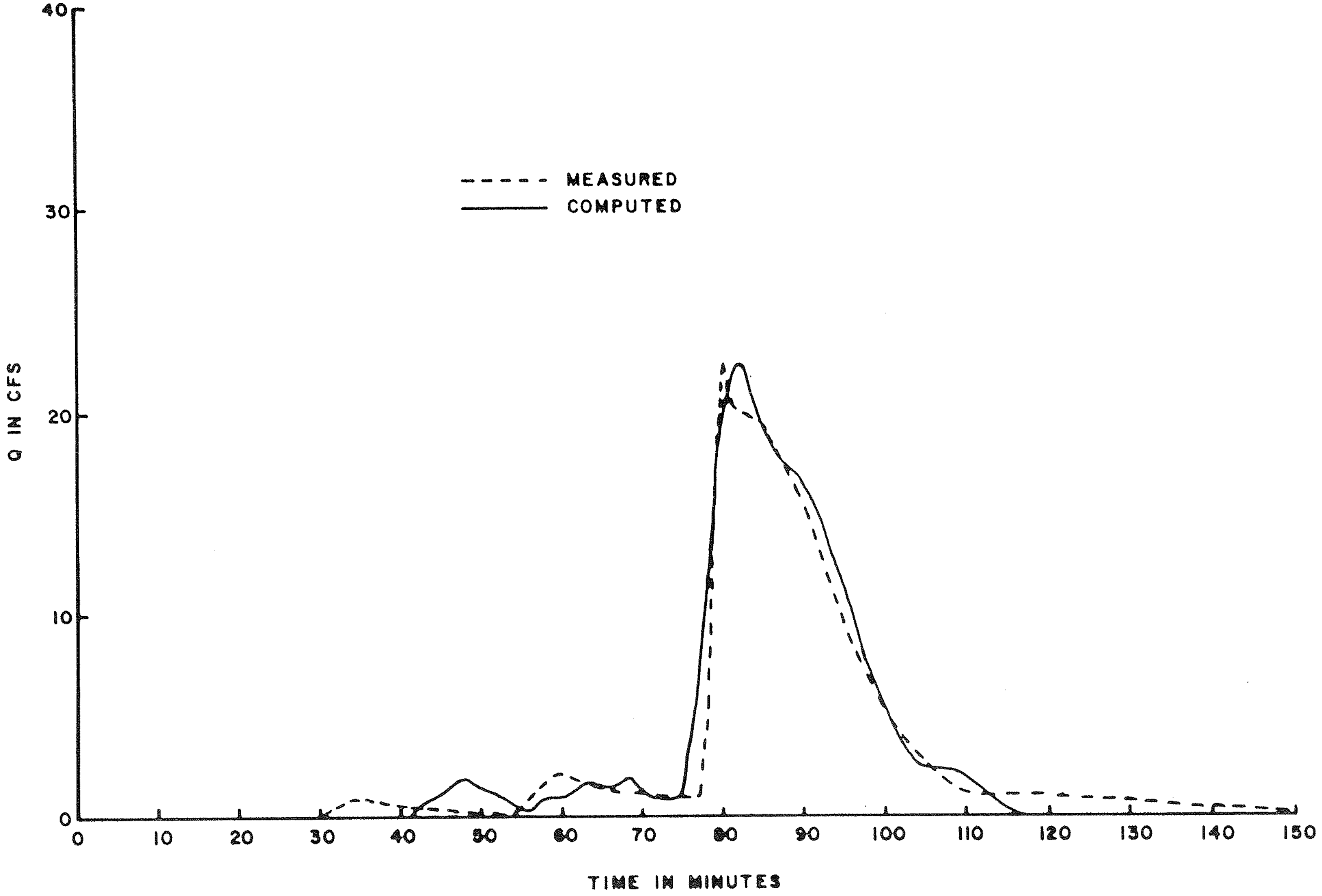

FIGURE 7. OUTFLOW FROM SUBWATERSHED II FOR THE EVENT OF JULY 20,1966.

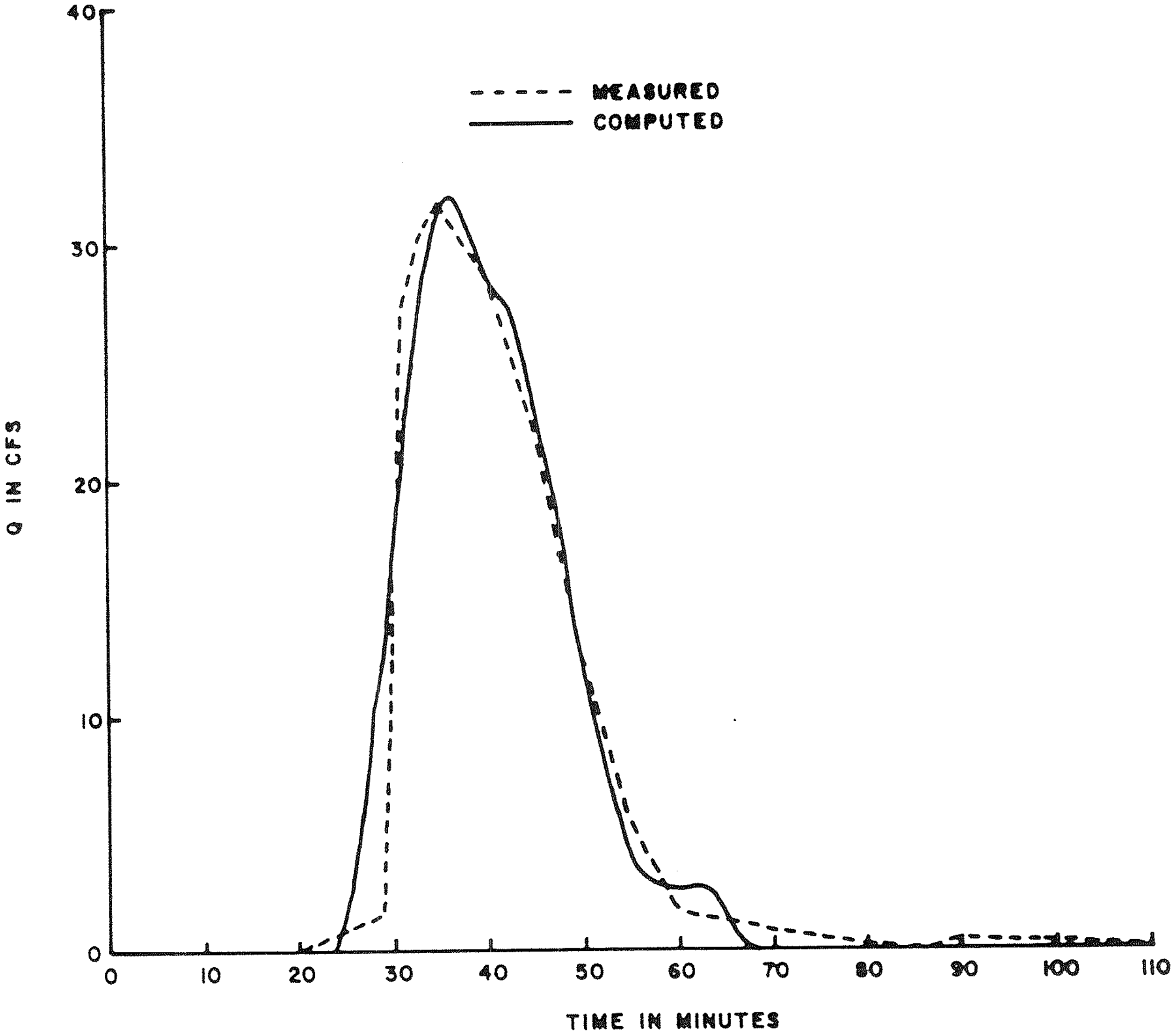

FIGURE 8. OUTFLOW FROM SUBWATERSHED II FOR THE EVENT OF JULY 29,1966.

TABLE 1.--Fitted values of constants in the runoff model

		Fitted values	
Constants	Units	Overland flow	Channel flow
R_{cs}	inches	0.15	---
k_r	---	1.0	---
f_m	inches/hr.	0.18	1.8
f_o	inches/hr.	1.8	4.2
k_f	---	0.60	0.04
c	Sec.^{-1}	---	.10
K	$\text{Sec.}^2/\text{ft.}^2$	0.093	.03

the channel flow. The infiltration values for the channel alluvium were assumed to be high because of the loose, coarse material. A higher roughness coefficient was assumed for the planes because the grass clumps and the erosion pavement relief are great in relation to the flow depths. These values were determined by solving the mathematical model for the July 20 storm. The fitted values were then used to predict the storm of July 29. The agreement between the computed and measured hydrograph appears to be very satisfactory.

The analog model was capable of producing graphs of precipitation, retention, and infiltration, as well as the hydrograph (Fig. 9). From these graphs, actual water lost to infiltration and retention, as well as the input and output water, can be computed. With a water budget for each subzone, the principle of conservation of mass for the subwatershed can be readily checked. For the areas checked, the error was found to be less than 1%.

The results of computations showing the distribution of the water to various portions of the model are shown in Table 2 for the storms of July 20 and July 29, 1966. Storm losses for July 20 are considerably larger than for the storm of July 29 because of lower antecedent moisture conditions on the earlier storm.

MODEL SENSITIVITY

The analog equipment used to solve the runoff equations was designed so that an outflow hydrograph could be obtained at three interior points in the model: at the downstream end of a unit width plane; at the end of a subzone; and at the outlet of the watershed.

To provide an idea of the magnitude of change in the computed overland flow and channel flow hydrographs caused by changes in the condition parameters, each parameter was systematically varied (the other parameters were kept constant), and the corresponding hydrograph was recorded with an automatic plotter. The responses of the overland flow model and the subzone channel flow to parameter variation were determined for subzone 2 (see sample in Figure 10).

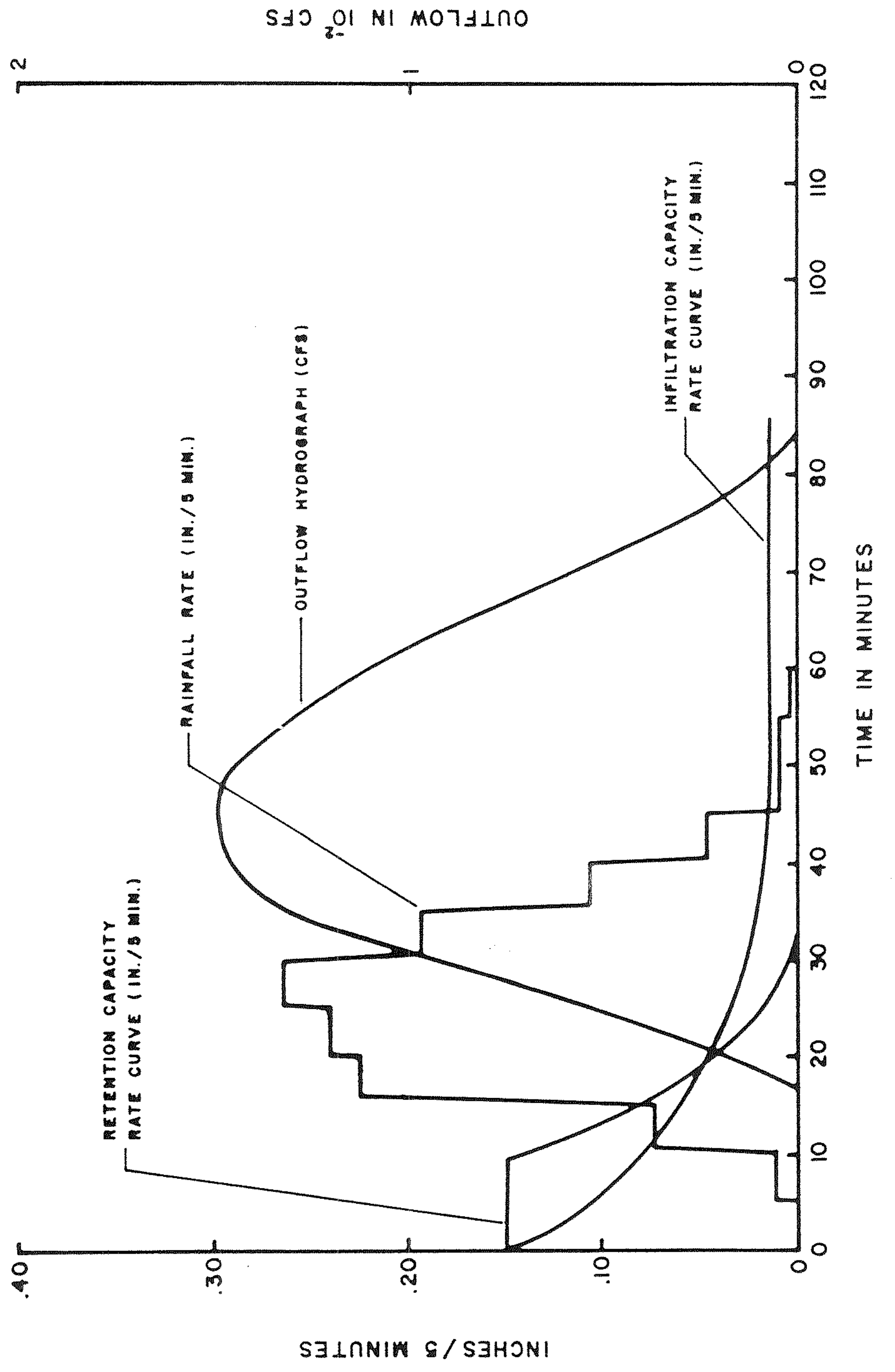

FIGURE 9. INPUT AND OUTPUT DATA FOR THE PLANE OF SUBZONE 2 USED IN THE CHECK ON THE CONSERVATION OF MASS PRINCIPLE, JULY 20,1966.

TABLE 2.--Distribution of watershed losse sfor the runoff events of July 20 and 29, 1966 as computed from the model inputs and outputs

	Losses to infiltration and retention storage on land surface		Losses to channel seepage		Losses to blind drainage (rainfall - subzone 9
Date of runoff event	Total vol.	Vol. per sq. mi. of land	Total vol.	Vol. per mi. of channel	Total vol.
	(acre-ft.)	(acre-ft/mi^2)	(acre-ft.)	(acre-ft/mi)	(acre-ft.)
7/20/66	114.18	42.80	36.14	4.84	40.47
7/29/66	37.06	13.90	9.86	1.32	12.9

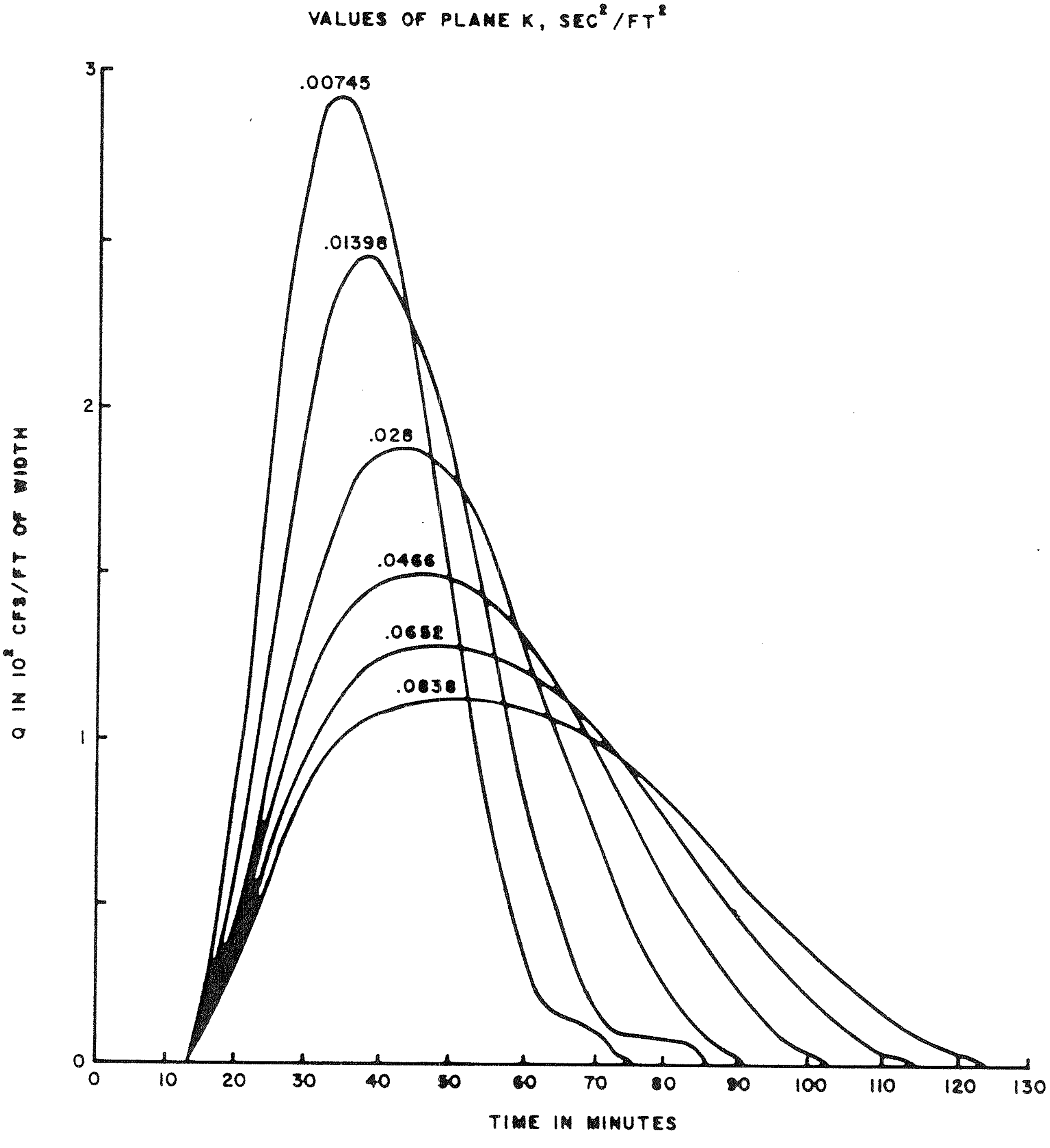

FIGURE 10. OVERLAND FLOW HYDROGRAPH FOR SUBZONE 2 AS AFFECTED BY CHANGES IN THE PLANE ROUGHNESS COEFFICIENT, K, JULY 20, 1966.

The hydrographs were subsequently integrated to obtain the flow volume. Figures 11 and 12 show the value of the runoff volume, peak discharge, and the time of rise (time from beginning of rainfall to peak discharge) as affected by variation in the condition parameter values.

As would be expected, the runoff volume from the plane decreased as the retention-storage capacity, R_{cs}, increased (Fig. 11). The time of rise changed very little, and the peak discharge decreased as the value of R_{cs} increased. For the same retention storage capacity, the amount of water lost to retention storage increases with decreasing k_r values. The effect of the plane roughness coefficient (K) on the flow hydrograph was quite dramatic and changed the hydrograph characteristics more than the other parameters. Although variations in the value of this term had essentially no effects on the runoff volume, the hydrograph peak, as well as the time of rise and flow duration, changed appreciably.

With the value of k_f held constant at 0.60, the characteristics of the hydrograph are significantly affected by changes in the minimum infiltration rate (f_m), while they experience little alteration when the initial infiltration rate (f_o) is modified. This is not the case for low values of k_f, and whether or not variation of f_m affects the hydrograph more than variation in f_o, depends on the value of k_f. An increased amount of infiltration loss with decreasing values of k_f was observed, which suggests that the maximum infiltration capacity rate plays an important role when the time constant k_f is small, while the minimum infiltration capacity rate is predominant for large values of k_f.

The channel flow hydrograph can be altered by changes in all the condition parameters of the model. Summaries of the changes in the hydrograph characteristics created by varying the plane condition parameters are shown in Figure 12. Figure 13 shows hydrograph characteristic changes caused by varying the channel condition parameters.

The effects of the retention terms (R_{cs} and k_r) were quite large on the runoff volume and peak discharge, but time of rise seemed somewhat independent of these terms. An interesting

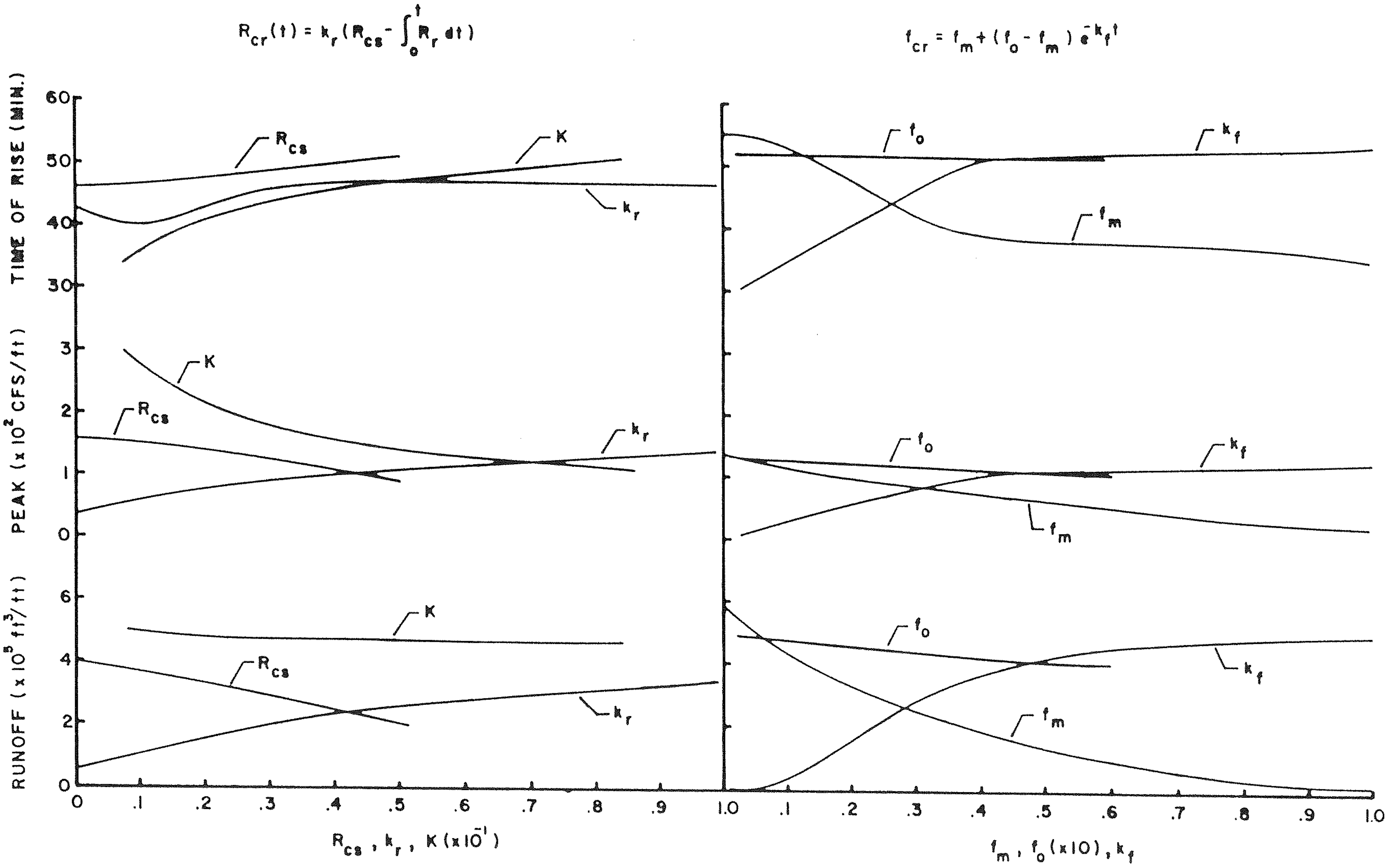

FIGURE II. OVERLAND FLOW HYDROGRAPH CHARACTERISTICS AS AFFECTED BY CHANGES IN VALUE OF PLANE VARIABLES.

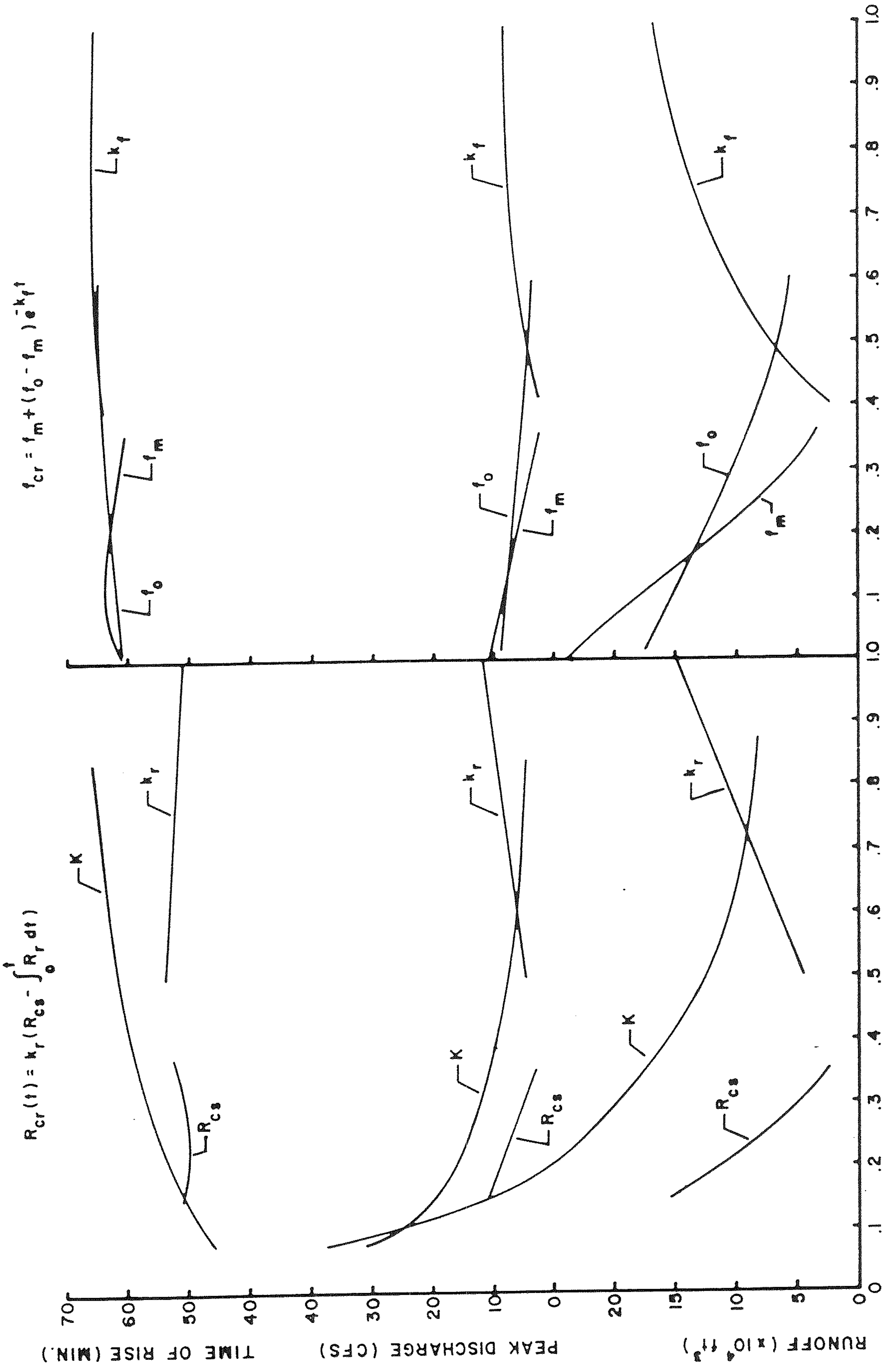

FIGURE 12. CHANNEL FLOW HYDROGRAPH CHARACTERISTICS AS AFFECTED BY CHANGES IN VALUE OF PLANE VARIABLES.

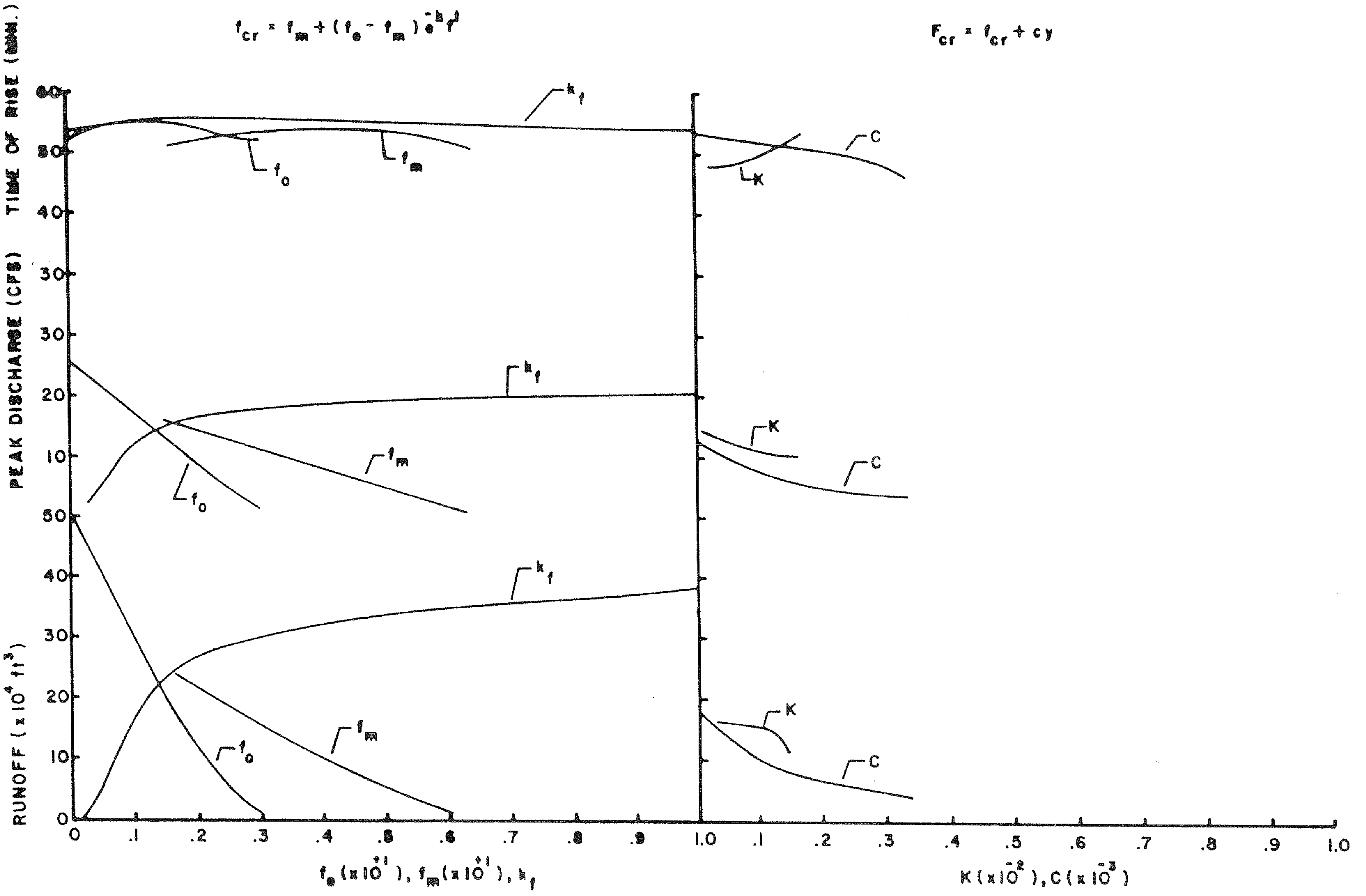

FIGURE 13. CHANNEL FLOW HYDROGRAPH CHARACTERISTICS AS AFFECTED BY CHANGES IN VALUE OF CHANNEL VARIABLES.

observation was that the time at which flow ceased for changes in R_{cs} was almost invariant for the overland flow while for the channel flow, increasing R_{cs} shortened the time at which the flow ceased.

The manner in which the channel flow model responds to changes in the plane and channel roughness coefficient is quite different. The chief difference between these two coefficients on the hydrograph characteristics for the range of values tested was in the hydrograph time of rise. The channel K has little effect on the hydrograph time of rise, whereas the plane K affects it significantly. Likewise, the plane K affected the runoff volume and peak discharge more than did the channel K.

The importance of the value of k_f in determining the relative weight of the initial and final infiltration values was again demonstrated. The hydrographs showed a greater sensitivity to changes in the plane f_m than to changes in the channel f_m. The hydrographs were also more sensitive to variation of channel f_o than to variation of the plane f_o.

The depth coefficient in the channel seepage term changed the hydrograph by decreasing the volume and peak discharges for increases in the value of c. The time of rise of the channel hydrograph appears to decrease only slightly with increasing values of c.

SUMMARY

1. The runoff model developed seems to describe the runoff process and provides a realistic outflow hydrograph after appropriate juggling of the condition parameters. The mathematical model of the surface runoff is flexible, and can be adapted to different flow conditions found on a watershed. It also presents the advantage of yielding directly such hydrograph characteristics as the time of rise without resorting to empirical formulas for this time delay.

2. Consideration of both overland and channel flow in the model allows computation of the water loss by retention and storage on the land surfaces and the channel seepage losses (transmission losses).

3. Subdivision of the watershed into subzones helps to describe the spatial precipitation distribution as well as the spatial distribution of both the function and condition parameters. Subdivision of the watershed also provides additional model calculation comparison points when the hydrograph is available at the outlet of subzones.

4. The results of sensitivity analysis or responses of the hydrograph to changes in parameter values agrees well with what might be experiences from field observations. The resistance term (K) of the overland and channel flow terms appears to have the largest effect on the computed hydrograph peak.

5. Additional checks on the runoff hydrograph from subzones or from the overland flow planes would be most desirable. All the condition parameters are now adjusted simultaneously to provide a computed hydrograph to match the observed hydrograph. Data for comparing the overland flow portion would provide an independent adjustment of first the overland flow and finally the channel flow.

APPENDIX A

List of Symbols

Symbol	Description	Unit
a_i	Elemental subarea in precipitation determination	$feet^2$
A	Cross-sectional area of the flow	$feet^2$
A_s	Watershed surface area	$feet^2$
B	Top width of cross-sectional area of flow	feet
c	Constant applied in channel seepage	Sec^{-1}
F_{cr}	Channel seepage capacity rate at any time	$in/hr/ft^2$
f_{cr}	Infiltration capacity rate at any time	in/hr
f_m	Minimum value of f_{cr}	in/hr
f_o	Maximum value of f_{cr}	in/hr
g	Acceleration due to gravity	ft/sec^2
I	Unit rate of lateral inflow into channel	ft^2/sec
j	Subscript indicating section number	
J	Subzone number	
K	Channel or plane roughness coefficient	sec^2/ft^2
k	Constant less than unity in Horton equation	

Symbol	Description	Unit
k_r	Constant ≤ 1 in retention capacity rate equation depending upon surface characteristics	
O	Unit rate of outflow loss due to seepage	ft^2/sec
P_e	Effective precipitation rate	in/hr
P_r	Actual precipitation rate	in/hr
Q	Total rate of flow at any section of channel	ft^3/sec
q	Rate of flow per unit width	$ft^3/sec/ft$
q_n	Rate of outflow at end of equivalent overland flow plane	$ft^3/sec/ft$
q_{crg}	Capacity rate of seepage loss per unit length of channel	in/hr/ft
q_{rg}	Actual rate of seepage loss from channel	in/hr
R_{cr}	Retention capacity rate	in/hr
R_{cs}	Retention storage capacity of vegetation and land surface	in/hr
R_r	Actual rate at which precipitation is entering retention storage	in/hr
R_s	Amount of precipitation in retention storage	inches
S_f	Energy grade line	ft/ft
S_o	Bed or land surface slope	ft/ft

Symbol	Description	Unit
t	Time	minutes or hours
V	Average velocity of flow	ft/sec
v_i	Volume of precipitation in elemental area between adjacent isohyetal	in-ft^2
x	Distance along plane or along channel	ft.
y	Flow depth	ft.
Z	Number of subzones	
γ_o	Bed shear	lb/ft^2
θ	Angle between channel bottom and horizontal	
$\alpha + \beta$	Energy and momentum coefficients	