SENSITIVITY ANALYSIS OF THE WEPP WATERSHED MODEL FOR RANGELAND APPLICATIONS
I: HILLSLOPE PROCESSES

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ABSTRACT. *Uncertainty in the hydrologic and soil erosion predictions of the WEPP watershed model due to errors in model parameter estimation is identified through a sensitivity analysis based on the Monte Carlo method. Hillslope component model sensitivities to model inputs for rangeland conditions are presented. Model sensitivities provide guidance in the collection of input data where the model is intended to simulate soil erosion. The results show that hydrologic and erosion predictions are very sensitive to attributes that define a storm event (amount, duration, and time to peak and intensity) and to the saturated hydraulic conductivity parameter. Sensitivity to critical shear stress in soil erosion predictions indicates that interrill flow is the dominant factor of sediment transport under consolidated, nontill managed soils.*

Keywords. *Soil erosion, Rangeland, Modeling, Sensitivity analysis, WEPP.*

The United States Department of Agriculture-Agricultural Research Service (USDA-ARS) Water Erosion Prediction Project (WEPP) watershed model is a process-based, distributed parameter model, designed to simulate the effects of management practices on erosion and sediment yield of cropland and rangeland watersheds. The WEPP represents a new erosion prediction technology based on concepts of stochastic weather generation, fundamental hydrology, soil physics, plant science, hydraulics, and erosion mechanics (Lane and Nearing, 1989).

The advantages of the WEPP watershed model over empirical soil erosion models are that erosion and hydrologic parameters can be directly calculated from soil characteristics. In addition, the model has the ability to:

- Estimate the spatial and temporal distributions of soil loss and sediment yield at any point on a hillslope within a watershed.
- Be used to explore the internal operation of sediment production systems as the model parameters are physically based.
- Be applied beyond the range of conditions for which they were validated (Stone et al., 1990).

The WEPP watershed model simulates all the major processes affecting erosion and sediment yield (i.e., rainfall, runoff, plant growth, tillage operations, grazing effects). It is made up of three major components: hillslope, channel, and impoundment. The hillslope component calculates erosion and deposition on rill and interrill flow areas. The channel component calculates erosion and deposition within concentrated flow areas which can be represented as permanent channels or ephemeral gullies. The impoundment component calculates deposition of sediment within terrace impoundments and stock tanks.

The WEPP simulates many watershed processes to improve the accuracy of model predictions; however, uncertainty exists in each of the model components. Under field applications, the model complexity and the natural variability of hydrologic parameters frequently induce uncertainties due to errors in parameter estimation. Sensitivity analysis is a common technique used to assess model uncertainty in relation to errors in parameter estimation.

Unfortunately, deterministic sensitivity analysis does not represent an adequate approach to deal with model uncertainty of complex, nonlinear models like WEPP which are subjected to large variances in hydrologic systems. Sensitivity analysis based on the Monte Carlo method provides a criterion by which to judge uncertainties in model predictions due to errors in parameter estimation when the system variability is represented in probabilistic terms (Zimmerman et al., 1990).

During the process of model development the sensitivity of the WEPP hillslope component was analyzed by Nearing et al. (1989) and Flanagan and Nearing (1991). However, no efforts have been made to evaluate the sensitivity of the entire watershed model that comprises multiple hillslopes and channels. So, it is necessary to identify the model uncertainty due to errors in parameter estimation for watershed applications.

This article presents the results of a sensitivity analysis of the WEPP watershed model for rangeland applications in predicting hydrologic and soil erosion variables on hillslopes.

SENSITIVITY OF COMPLEX HYDROLOGIC MODELS

Sensitivity analysis ranks model parameters based on their contribution to overall error in model predictions. It is a measure of model uncertainty because it indicates the

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Article was submitted for publication in October 1992; reviewed and approved for publication by the Soil and Water Div. of ASAE in August 1993. Presented as ASAE Paper No. 92-2018.

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expected errors in model prediction due to errors in model parameters. Because of the nature of the data and the stochastic effects of random measurements in most hydrologic applications, large variances in measurements are the rule, and deterministic sensitivity analysis may be less useful. Sensitivity analysis of time-varying and nonlinear models is difficult because of the complex or nonexistent analytical solutions of the model equations (Whitehead and Young, 1979; Gardner et al., 1981).

Deterministic models use a single value for each parameter to produce a single prediction. These models ignore the effect of imprecise parameter estimation and the system's natural variability. For any assessment situation, model parameters are best represented by a range (or frequency distribution) of values. This range translates into a range (or frequency distribution) of model predictions. To explicitly account for uncertainty in parameter estimation requires modeling approaches that are stochastic (i.e., probabilistic) rather than deterministic (Hoffman and Gardner, 1983).

Any real system contains natural variability, and, therefore, system behavior is most realistically represented as a frequency distribution of potential behavior. The distribution of variables describing system behavior is a result of the mathematical characteristics of the model and the distributions of the model parameters. The purpose of a stochastic sensitivity analysis is to assess the effect that a parameter has on an output variable over the range of parameter values that are likely to be exhibited (Gardner and O'Neill, 1983).

Garen and Burges (1981) stated that the utility of the output from watershed models can be greatly enhanced if it is accompanied by measurements of its accuracy. This is very important in model selection, decision-making, engineering design, data collection, and model refinement. However, trade-offs exist between model complexity and accuracy of parameters and input data. As models become more complex, data and parameter estimation requirements usually become greater. Adding complexity to a model may improve its ability to represent the behavior of a natural system, but the added complexity may increase uncertainty in model predictions. Eventually, adding complexity to a model is likely to increase uncertainty in model predictions to unacceptable levels (Kirchner, 1991).

Several uncertainty-error analysis techniques are commonly used to verify error propagation in hydrologic models such as first-order uncertainty analysis (Clifford, 1973) and the two-point estimate method (Rosenblueth, 1975). For complex nonlinear models that involve the use of time-dependent driving variables, the Monte Carlo simulation method gives the best responses in parameter uncertainty analysis (Whitehead and Young, 1979; Rubinstein, 1981; Scavia et al., 1981; Gardner, 1984; Kirchner, 1991).

Beven and Jakeman (1988) discussed the advantages of the Monte Carlo approach, in which a number of model runs are made using random selections of parameter values or initial boundary conditions. The Monte Carlo technique is not limited by the degree of nonlinearity of the model or by the degree of uncertainty or any assumptions about the form of the distributions from which the random selections are made. Any cross-correlations between the selected random variables can be preserved. The main disadvantage of this approach is the expense for computation in complex models since a large number of runs may be necessary before convergence of the predictive uncertainty estimates is achieved. The Monte Carlo method has been used to check the accuracy of approximations of first-order variance propagation in lake eutrophication (Scavia et al., 1981), ecology (Gardner et al., 1981), stream water quality (Burges and Lettenmaier, 1975), and watershed models (Garen and Burges, 1981).

THE WEPP HILLSLOPE COMPONENT

HYDROLOGY

In WEPP, the characteristics of runoff occurring on hillslopes as a result of rainfall events provide the basic information to model erosion by flowing water. During a rainfall event, water infiltrates the soil through a process regulated by soil characteristics. Infiltration is computed using the Green and Ampt equation (Green and Ampt, 1911) as modified by Chu (1978) for unsteady rainfall events, in which the soil surface can alternate between unponded and ponded conditions.

The WEPP model updates the infiltration parameters on a daily basis to account for temporal variations of soil moisture content, surface crusting, and vegetation cover by applying equations derived by Rawls et al. (1989). The accuracy of the infiltration parameters is fundamental to compute rainfall excess. During stages in which the rainfall intensity exceeds the infiltration rate, the portion of rainfall that does not infiltrate or becomes depression storage flows down slope. This overland flow is routed using the kinematic wave equations (Liggett and Woolhiser, 1967).

In general, the development of runoff calculations for the WEPP model is constrained by its compatibility with the erosion and deposition calculations. The erosion and deposition equations are solved for steady-state conditions. Therefore only the steady-state discharge, duration of runoff, and flow shear stress are needed. Thus, the hydrologic and hydraulic processes which affect erosion and deposition within a watershed are represented at a level of complexity compatible with those calculations which represent the erosion and deposition processes.

HILLSLOPE EROSION

The processes of detachment and deposition, shear stress, flow in rills and on interrill areas, and sediment transport capacity by flowing water, as described by Foster and Meyer (1972), serve as a prototype for the WEPP erosion prediction technology. The steady-state continuity equation for sediment is:

$$\frac{dG}{dx} = D_i + D_r \qquad (1)$$

with:

$$D_i = C_i K_i I_e^2 S_f G_e \left(\frac{R_s}{w} \right) \qquad (2)$$

$$D_r = C_r K_r \left(\tau - \tau_{cr} \right) \left(1 - \frac{G}{T_c} \right) \qquad (3)$$

where G is the sediment load (F L^{-1} T^{-1}), x is the distance along the slope (L), D_i is delivery rate of sediment from interrill areas (F L^{-2} T^{-1}), D_r is the rill detachment rate (F L^{-2} T^{-1}), C_i is the interrill canopy cover parameter (dimensionless), K_i is interrill soil erodibility parameter (F L^{-4} T^{-1}), I is the rainfall intensity (L T^{-1}), S_f is the interrill slope adjustment factor (dimensionless), G_e is the effective ground cover on interrill erosion (dimensionless), R_s is the spacing of the rills (L), w is the rill width (L), C_r is rill cover parameter (dimensionless), K_r is the rill soil erodibility parameter (T L^{-1}), τ is the average shear stress in the cross section (F L^{-2}), τ_{cr} is the critical shear stress required for detachment to occur (F L^{-2}), and T_c is the transport capacity of the flow (F L^{-1} T^{-1}). (Note: L represents length, T time, and F force for all variables).

Equation 2 describes delivery of soil particles detached by raindrop impact from interrill areas and transported in shallow flow to rills. Equation 3 describes the rate of soil particle detachment in rill flow areas due to shear stress by concentrated flow. Substituting from equations 2 and 3, when the transport capacity is greater than the sediment load, the sediment continuity equation is:

$$\frac{dG}{dx} = C_i K_i I^2 S_f G_e \left(\frac{R_s}{w}\right) + C_r K_r (\tau - \tau_{cr}) \left(1 - \frac{G}{T_c}\right) \quad (4)$$

in which the term $(1 - G/T_c)$ is considered a feedback term for rill detachment that reflects the fact that soil detachment rates in rills are a function of the sediment load in the flow relative to the capacity of the flow to transport sediment. When the sediment load of the flow is greater than its transport capacity, net deposition is included in the continuity equation as follows:

$$\frac{dG}{dx} = C_i K_i I^2 S_f G_e \left(\frac{R_s}{w}\right) + \left(\frac{\beta V_f}{q}\right) (T_c - G) \quad (5)$$

where β is a dimensionless deposition parameter equal to 0.5, V_f is the effective particle fall velocity (LT^{-1}), and q is the flow discharge per unit width (L^2 T^{-1}). Four hydrologic variables are required to drive the erosion model equations previously described: peak runoff, effective runoff duration, effective rainfall intensity, and effective rainfall excess duration. The linkage between these hydrologic variables and the erosion component was described by Foster et al. (1989) and Flanagan (1990).

SENSITIVITY ANALYSIS

OVERVIEW OF APPROACH

A sensitivity analysis based on the Monte Carlo method was performed to identify the relative importance of model parameters and error propagation of the WEPP single storm watershed model Version 91.2 when applied to rangeland conditions. This analysis constitutes a major step within the established procedures by the USDA-ARS to evaluate its developmental computer models with the purpose of identifying the sources of model uncertainty for field applications.

The importance of model parameters was determined as follows:

- Ten thousand model simulations with fixed rainfall parameters and 1,200 model simulations using observed rainfall events were performed by using the single event simulation mode of the WEPP watershed model.
- A multiple linear regression analysis was performed using the model inputs generated by the Monte Carlo method and model outputs.
- Model parameter uncertainty was assessed from the regression coefficients of the multiple linear equation. The major statistical assumptions considered for this analysis were:
    - Parameter sensitivity indices and the propagation of error can be approximated by the β of a normalized multiple linear equation containing the model parameters.
    - The model parameters included in a linear relationship are uncorrelated and can be generated randomly from appropriate probability distributions.
    - The linear model is able to assess unbiased estimates of sensitivity indices of model parameters of a complex nonlinear model when a large number of model simulations are performed.

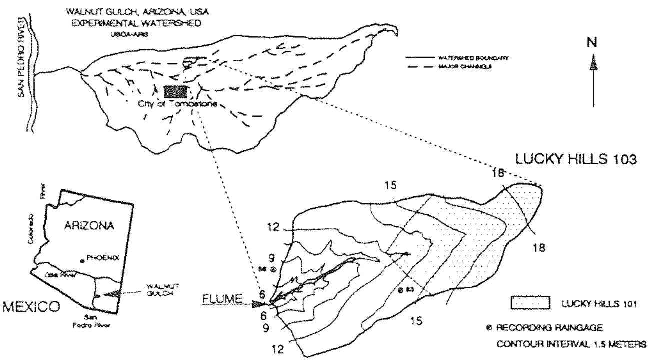
SCOPE OF THE ANALYSIS

The WEPP watershed model sensitivity for rangeland applications was based on climate, soils, and vegetation characteristics of Lucky Hills 103 (LH-103) watershed located at the Walnut Gulch Experimental Watershed operated by the USDA-ARS Southwest Watershed Research Center, in Tombstone, Arizona (fig. 1a). This watershed of 3.7 ha is assumed to be representative of millions of hectares of brush and grass rangelands found throughout the semiarid Southwest and is considered a transition zone between the Chihuahuan and Sonoran deserts (Simanton et al., 1985).

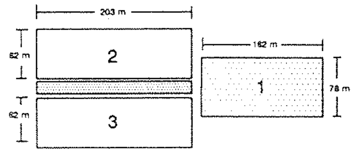
Rainfall is bimodally distributed, with average annual rainfall about 300 mm. Nearly 99% of the annual runoff occurs during the summer thunderstorm season of July to mid-September. LH-103 has well-drained calcareous soils with large percentages of rock and gravel on the surface. The vegetation is predominantly brushland type with a very low density of perennial grasses. The watershed representation and the hillslope configuration for the lateral areas, planes 2 and 3, of LH-103 utilized in WEPP for this analysis are illustrated in figures 1b and 1c, respectively. In addition, the average hillslope steepness and initial random roughness used in the simulations were taken as 11.3% and 0.25 m, respectively.

PARAMETER EQUATIONS

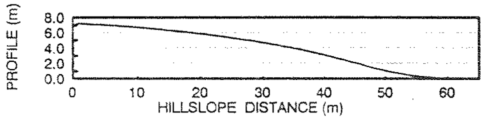
Probability distributions of model inputs represented by the model parameters and variables listed in table 1 were determined using the sample statistics of the watershed characteristics shown in table 2. These model inputs were utilized to determine the overall model error by varying them randomly and independently based on probability distributions according to the Monte Carlo method as illustrated in figure 2. Model predictors (dependent variables) that served as indicators of model sensitivity to random changes in model inputs included hydrologic



(a) Geographic location of Lucky Hills 103 watershed (LH-103).



(b) LH-103 representation in WEPP.



(c) Hillslope configuration in WEPP of lateral areas 2 and 3.

Figure 1–Geographic location, representation in WEPP, and hillslope configuration of lateral areas (2 and 3) of LH-103.

variables (runoff volume and peak runoff) and soil erosion variables (sediment detachment and sediment delivery).

The use of regression models to estimate probability distributions is justified since a few samples of K_i, K_r, τ_{cr}, K_s model parameters were available to estimate reliable distributions. The distributions of these model parameters were estimated by inputting random variates of the watershed characteristics into regression equations. Regression equations of erosion parameters for rangeland conditions were developed by Alberts et al. (1989) based on readily available soil characteristics. The interrill erodibility parameter is estimated as:

Table 1. Model parameters and input variables used for sensitivity analysis

Category	Description	Units
Rainfall	Depth (R)	(mm)
	Duration (D)	(min)
	Time to peak — storm duration ratio t_p	—
	Maximum intensity — average intensity ratio i_p	—
Soil	Interrill erosion parameter (K_i)	(kg s/m^4)
	Rill erosion parameter (K_r)	(s/m)
	Critical shear stress parameter (τ_{cr})	(Pa)
	Saturated hydraulic conductivity parameter (K_s)	(mm/h)
	Sand (Sa)	(%)
	Silt (Si)	(%)
	Clay (Cl)	(%)
	Organic matter (Om)	(%)
	Cation exchange capacity (CEC)	(meq)
	Bulk density (BD)	(g/cm^3)
	Rocks fragments	(%)
	Rock-gravel surface cover	(%)
	Initial saturation (Sat)	(mm/mm)
Vegetation	Litter biomass	kg/m^2
	Standing biomass	kg/m^2

Table 2. Characteristics of soil and vegetation at Lucky Hills 103, Walnut Gulch Experimental Watershed near Tombstone, AZ

	Characteristic	Units	Mean	Std. Dev.	C.V. (%)
Soil	Sand	(%)	55.12	5.059	9.1
	Silt	(%)	23.90	5.807	14.6
	Clay	(%)	21.00	2.928	13.9
	Organic matter	(%)	1.32	0.526	39.7
	Bulk density	(g/cm^3)	1.39	0.191	13.6
	CEC	(meq)	34.90	5.500	15.7
	Rock fragments	(%)	15.59	1.25	8.0
	Rock-gravel cover	(%)	10.75	1.34	12.5
	Initial saturation	(mm/mm)	0.53	0.150	28.3
Vegetation	Litter biomass	(kg/m^2)	0.0067	0.0164	244.0
	Standing biomass	(kg/m^2)	0.2276	0.3276	118.0

$$K_i = (1709 - 1765\ Sa - 645\ Si - 4557\ Om - 902\ \theta_{fc})\ 10^3 \qquad (6)$$

with R^2 as 0.94 and SE as 70.00

where
Sa = sand content (0 – 1)
Si = silt (0 – 1)
Om = organic matter in soil (0 – 1)
θ_{fc} = volumetric water content of the soil at 0.033 MPa (m^3/m^3)
R^2 = coefficient of determination
SE = standard error of estimate

The units of K_i and other model parameters are listed in table 1.

The rill erodibility parameter is calculated as:

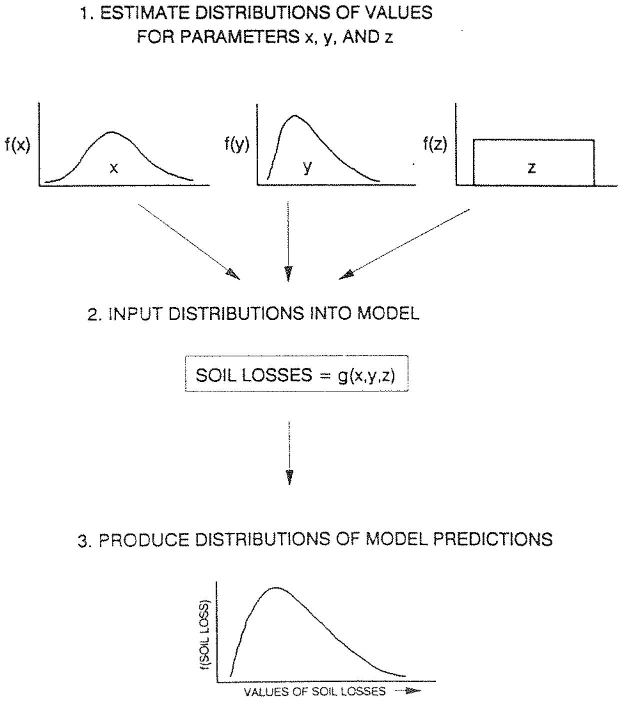


Figure 2–The concept of parameter uncertainty analysis based on the Monte Carlo method.

$$K_r = 0.0017 + 0.0024\ Cl - 0.0088\ Om - 0.00088\left(\frac{\rho_b}{1000}\right) - 0.00048\ R_i \qquad (7)$$

with $R^2 = 0.60$ and SE = 0.00028
where Cl is the clay content (0 – 1), ρ_b is the soil bulk density (kg/m^3), and R_i is the total root biomass (kg/m^2). The critical shear stress erodibility parameter is calculated as:

$$\tau_{cr} = 3.23 - 5.6\ Sa - 24.4\ OM + 0.9\left(\frac{\rho_b}{1000}\right) \qquad (8)$$

with $R^2 = 0.62$ and SE = 0.79

A frequency distribution for the baseline saturated hydraulic conductivity was obtained by applying equation 9 as proposed by Rawls et al. (1989):

$$K_s = 0.0002\ C^2 \frac{\eta_e^3}{(1.0 - \eta)^2}\left(\frac{0.001\ \rho_b}{\theta_r}\right) \qquad (9)$$

where K_s is the saturated hydraulic conductivity of the soil (m/s), η is the soil porosity, η_e is the effective soil porosity after correcting η for entrapped air at soil saturation, and θ_r is the residual volumetric water content (m/m). The parameter C is predicted from:

$$\begin{aligned} C = &- 0.17 + 18.1\ Cl - 69.0 Sa^2\ Cl^2 - 41.0\ Sa^2\ Si^2 \\ &+ 1.18\ Sa^2\left(\frac{\rho_b}{1000}\right)^2 + 6.9\ Cl^2\left(\frac{\rho_b}{1000}\right)^2 \\ &+ 49.0\ Sa^2\ Cl - 85.0\ Si\ Cl^2 \end{aligned} \qquad (10)$$

where all variables were previously defined. Factors that play a significant role in soil-water infiltration such as soil crusting, coarse fragments, frozen soil, macroporosity, and soil cover are used to adjust the baseline saturated hydraulic conductivity to an effective saturated hydraulic conductivity, K_e.

SENSITIVITY INDEX

Traditional sensitivity indices based on parameter variation around baseline values allow the determination of model response to one parameter at a time. For complex models such as WEPP in which there are interactions among parameters, it is necessary to perform a global sensitivity analysis to quantify the effects of each parameter on overall model uncertainty.

Regression methods show that the slope, b_i, of the regression of Y, the model prediction of interest, on a particular parameter x_i, is the least-squares estimate of the classic sensitivity index (Tomovic, 1963). If several parameters are simultaneously and independently varied, then the multiple regression of Y (runoff volume, sediment yield, etc) on all x (the selected model inputs), is:

$$Y = b_0 + b_1 x_1 + b_2 x_2 + \ldots + \ +b_n x_n \qquad (11)$$

where b represents regression coefficients. Normalized sensitivity indices (standardized coefficients), can be obtained for each variable in equation 11 by subtracting its mean and dividing by its estimated standard deviation. The normalized regression model is as follows:

$$\frac{Y - Y_m}{s_Y} = \beta_1 \frac{x_1 - x_{1m}}{s_{x_1}} + \beta_2 \frac{x_2 - x_{2m}}{s_{x_2}} + \ldots + \beta_n \frac{x_n - x_{nm}}{s_{x_n}} \qquad (12)$$

The standardized coefficients bear a close relationship to the estimated coefficients of the original unnormalized multiple regression model. It is not difficult to show that:

$$\beta_j = b_j \frac{s_{x_j}}{s_Y} \qquad (13)$$

where β_j is the normalized sensitivity index of parameter x_j, j = 1, 2,]. . . n. The standardized coefficient, β, adjusts the estimated slope parameter, b, by the ratio of the standard deviation of the model parameter (independent variable) to the standard deviation of the model output (dependent variable). A normalized sensitivity index of 0.7 means that one standard deviation change in the model parameter will lead to a 0.7 standard deviation change in the model prediction (Pindyck and Runbinfeld, 1991).

Because the proposed multiple linear regression model requires the variables to be independent for proper sensitivity coefficients, the Chezy-C and matric potential (N_s) parameters were not included in this sensitivity analysis since they are derived from parameters previously calculated by the model. Sensitivity coefficients for the effects of hillslope length and steepness are not presented herein. An analysis for the effects of shape, length, and steepness of the hillslopes on hydrologic and soil erosion variables computed by WEPP when applied to rangelands was presented by Parker (1991).

The selection and ranking of model parameters based on model sensitivity were obtained by performing a stepwise regression analysis of variable selection at the $\rho < 0.1$ level of statistical significance. Model sensitivities for the single storm mode of WEPP were obtained for variable rainfall conditions using the observed rainfall events, and for fixed rainfall conditions in which the rainfall properties were maintained constant during 10,000 simulations.

RESULTS

MODEL INPUTS

Model input random variates were generated by the WEPP watershed model modified for sensitivity analysis purposes. Statistics of the random variates as well as the best distribution fitted after 10,000 simulations are shown in table 3. Relative frequency distributions of rainfall events at LH-103 are shown in figure 3. Rainfall depth, rainfall duration, ratio of the time to peak over storm

Table 3. Statistics and fitted distribution for the random variates of model inputs

Parameter or Variable	Units	D.T.*	Statistics Mean	 S.D.†	Range of Test (min - max)
Rainfall					
Depth	(mm)	L	14.64	8.56	5.080 - 56.90
Duration	(h)	L	3.24	3.16	0.167 - 22.20
t_p	—	L	0.32	0.27	0.003 - 0.97
i_p	—	L	9.37	8.32	1.175 - 58.73
Soil					
Interrill erosion (K_i)	(kg s/m^4)	N	285,368.0	98,892.0	90,341.0 - 502,491.0
Rill erosion (K_r)	(s/m)	N	0.0008	0.0002	0.00003 - 0.00178
Critical shear stress (τ_{cr})	(Pa)	N	1.084	0.34	0.330 - 1.820
Sat. hydraulic cond. (K_s)	(mm/h)	L	4.311	4.87	0.301 - 34.441
Sand	(%)	N	55.12	5.06	37.510 - 85.240
Silt	(%)	N	23.90	5.80	0.000 - 47.180
Clay	(%)	N	21.00	2.93	10.000 - 31.350
Organic matter	(%)	N	1.32	0.52	0.440 - 2.170
CEC	(meq)	N	35.00	5.50	11.700 - 59.270
Bulk density	(g/cm^3)	N	1.39	0.19	1.060 - 1.680
Rock fragments	(%)	N	15.56	1.20	10.500 - 20.160
Initial saturation	(mm/mm)	L	0.55	0.15	0.200 - 0.950
Rock & gravel cover	(%)	L	10.75	1.20	6.440 - 15.210
Vegetation					
Litter biomass	(kg/m^2)	L	0.0067	0.016	0.0003 - 0.024
Standing biomass	(kg/m^2)	L	0.2276	0.327	0.0381 - 0.800

* Distribution type: N = normal, L = log-normal.

† Standard deviation.

duration (t_p), and the ratio of the maximum intensity over the average storm intensity (i_p) are the four rainfall characteristics required for the WEPP model. These rainfall characteristics fitted the log-normal distribution.

When the Monte Carlo method was applied the high variability of the soil characteristics and inaccuracy of the regression equations to calculate model parameters (Alberts et al., 1989) produced unrealistic random variates. To avoid this problem, parameter values were selected for the 90% interval of the distribution by rejecting values in the 5% tail areas. Because random variates of soil characteristics were generated assuming a normal distribution, the hillslope soil erosion parameters computed by equations 6, 7, and 8 using these characteristics are also normally distributed.

Infiltration of the soil matrix is described in the model by two major linked parameters from the Green-Ampt equation: the saturated hydraulic conductivity (K_s) and the average matric potential (N_s). The saturated hydraulic conductivity was adjusted to the effective hydraulic conductivity, K_e, by correcting the initial K_s value for soil macroporosity and surface cover factors. The best fit of these infiltration parameters (K_s, K_e, and N_s) and the initial soil water content variable was obtained using the log-normal distribution (fig. 4).

MODEL SENSITIVITY TO HYDROLOGIC VARIABLES ON HILLSLOPE AREAS

Model variables which control the hillslope erosion process are rainfall characteristics, runoff volume, and peak discharge. In this section we quantify the significance of these variables to the erosion computations considering; a) variable rainfall and b) fixed rainfall conditions.

Variable Rainfall. This analysis evaluated the effects of parameters that define a storm event on runoff volume and peak runoff predictions. Observed rainfall data were used for the analysis. Figures 5a and 5c show the magnitude of the events in relative frequency distributions for the simulated runoff volume and peak runoff. Both variables resembled the relative frequency distribution of rainfall (fig. 3). Large rainfall events produced large runoff amounts and high runoff peaks. Table 4 lists the normalized sensitivity indices of predicted runoff volume and peak runoff for the selected variables at $\rho < 0.1$ significance level.

Because the indices were obtained by regression procedures, a positive index means that an increase in the input variable increases the predicted model variable in proportion to the sensitivity index, and a negative index means that an increase in the input variable decreases the model prediction in proportion to the index.

Rainfall depth and duration, K_s, saturation, standing biomass, and litter had the greatest effects on runoff volume and peak runoff calculations. Rainfall depth had the strongest effect on runoff volume and peak runoff prediction. For every standard deviation error on the input rainfall depth, there is a corresponding error of 1.061 and 1.029 standard deviations in the predicted runoff volume and peak runoff, respectively.

Rainfall duration appeared with a negative sign, meaning that the longer the duration the less the runoff volume and lower the peak runoff. However, such an effect may have been related to the climatic conditions at Lucky Hills 103, since observed rainfall was utilized. Previous studies at this location have shown that most runoff is caused by short-duration, high-intensity storm events, whereas runoff volume from long duration events is significantly reduced by infiltration. Saturated hydraulic conductivity, standing biomass, litter, and bulk density had negative effects on both runoff volume and peak runoff, indicating that any increase in these variables reduces the predicted runoff volume and peak runoff on hillslopes.

Fixed Rainfall. To quantify the effects of hillslope characteristics on model sensitivity in high intensity–short

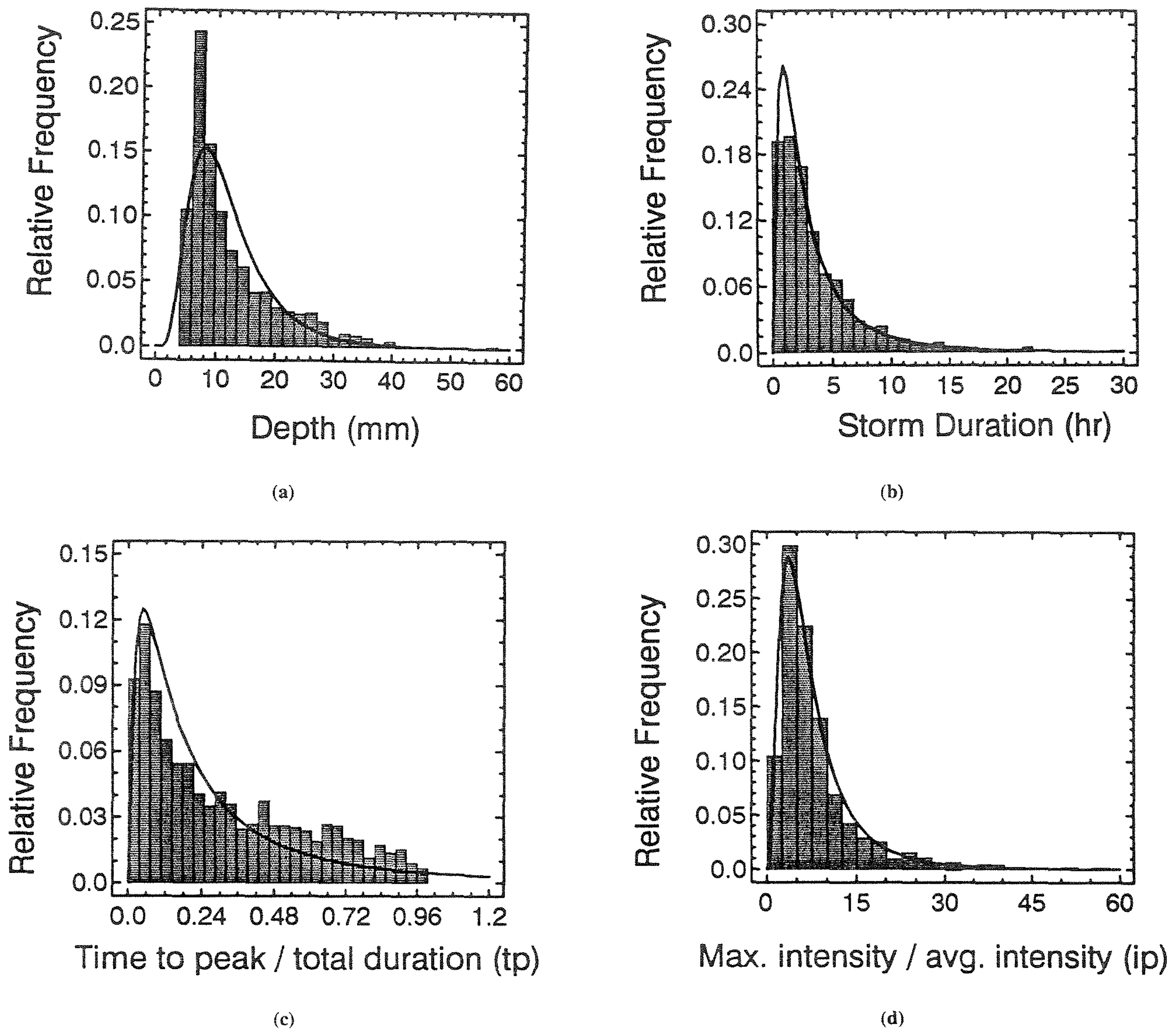


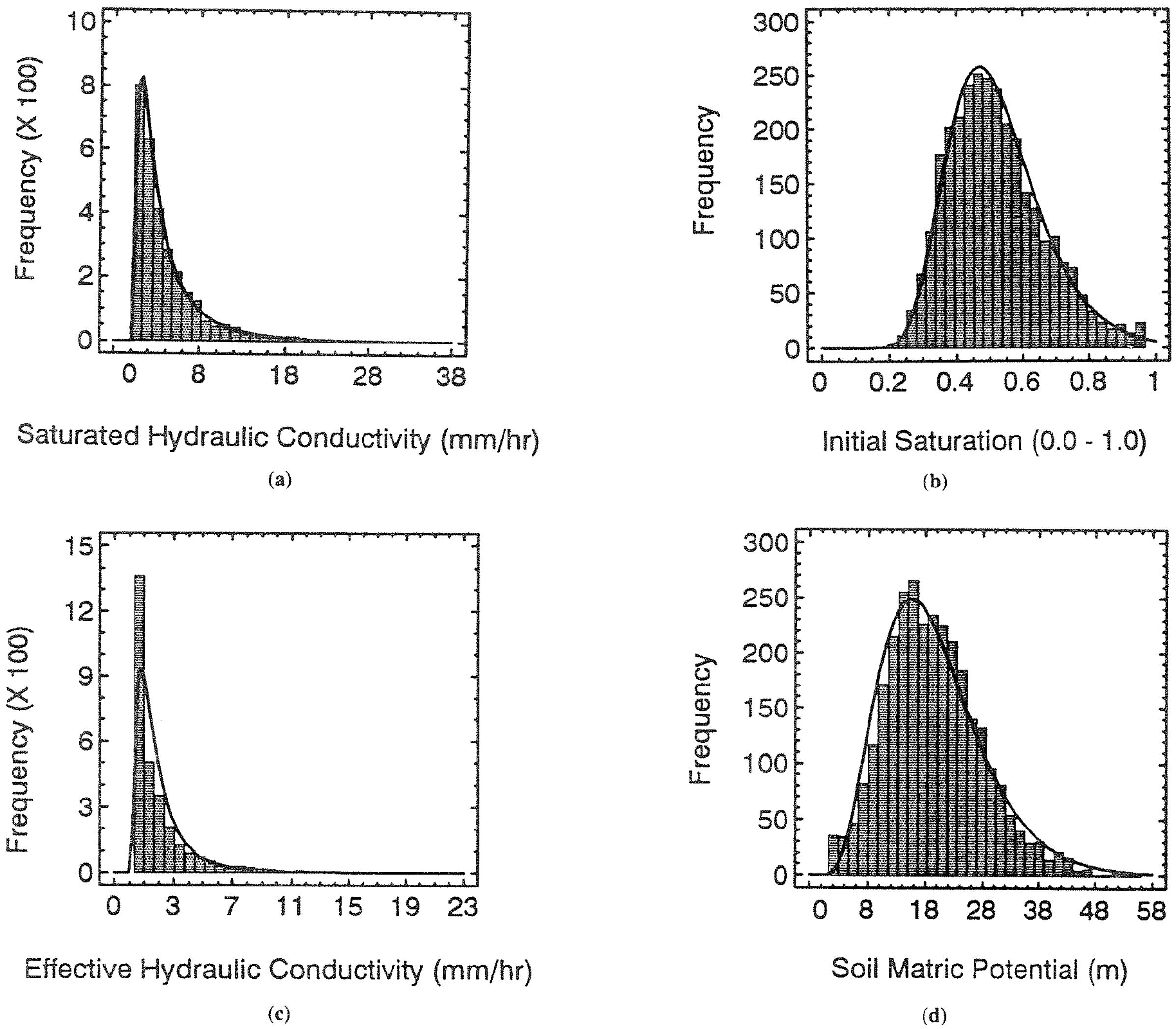
Figure 3–Relative frequency distributions of rainfall characteristics at Lucky Hills 103, Walnut Gulch Experimental Watershed near Tombstone, AZ.

duration storms, rainfall parameters were fixed at 50-mm depth, 1-h duration, with $t_p = 0.5$, and $i_p =$ to 1.37 for about 10,000 model simulations. Hillslope parameters and variables were varied randomly and independently from one model simulation to another according to random variate generation. Each simulation thus represents a unique scenario (a combination of parameters) of the site characteristics. The relative high fixed-rainfall depth was chosen to ensure that all WEPP model internal components would be activated during the fixed rainfall simulations.

Figures 5b and 5d show the relative frequency distributions of simulated runoff volume and peak for the fixed rainfall simulations. Except for a few scenarios, most simulations produced high runoff volumes and peaks compared to the varying rainfall simulations. The relative distribution of both the runoff volume and peaks are almost a mirror image of the variable rainfall runoff volume and peak relative distributions. There was a shift from the right skewed relative distribution under varying rainfall to the left skewed relative distributions under fixed rainfall. Figures 5a through 5d represent relative distributions of extreme cases of rainfall input. If intermediate rainfall depths (i.e., 10 to 50 mm) were used, there would probably be a progression from right to left of the skew of the relative distribution curves; with a normal distribution being found somewhere within the range of depths used.

Table 5 shows the model sensitivity to predict volume and the peak runoff with fixed rainfall characteristics. All hillslope variables selected by the stepwise regression procedure for the variable rainfall analysis were also selected when using fixed rainfall. However, rock cover appeared in the regression equation as a result of eliminating the rainfall characteristics from the analysis.

When rainfall characteristics were excluded from the regression equations the coefficient of determination decreased considerably. Runoff volume and peak discharge are very sensitive to saturated hydraulic conductivity. Only 68% of the total variation in runoff volume and peak runoff could be explained by the variables included in the regression model. The lower R^2 indicates that the βs of the



(a) (b) (c) (d)

Figure 4–Frequency distributions of infiltration parameters at Lucky Hills 103, Walnut Gulch near Tombstone, AZ.

linear relationship are no longer accurate indicators of the classical sensitivity values, although they are unbiased predictors of the response function to larger uncertainties in the parameters (Gardner, 1984). However, the low R^2 is a good indication that precipitation parameters are the most important input variables for the model. This result supports the findings of Osborn and Lane (1982), who pointed out that rainfall characteristics are fundamental inputs on watershed computer models to simulate runoff characteristics.

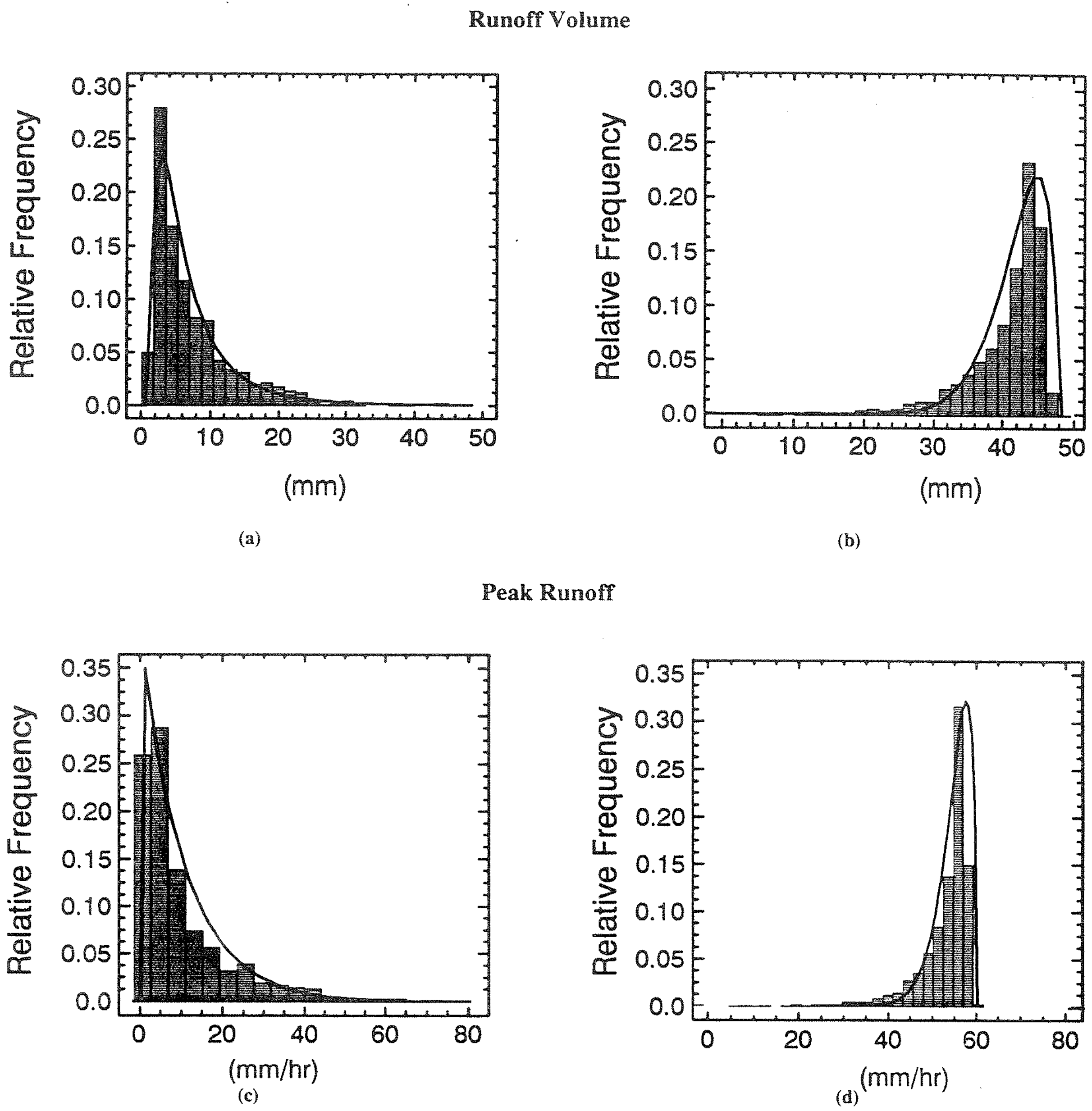
MODEL SENSITIVITY TO EROSION VARIABLES ON HILLSLOPE AREAS

The model sensitivity to soil erosion variables was evaluated for average total sediment detachment and delivery from hillslopes. Sediment detachment results include both rill and interrill detachment processes. The sediment delivery variable comprises the net effect of detachment and deposition processes occurring on a hillslope.

Variable Rainfall. Figures 6a and 6c show the relative frequency histograms and the fitted distribution for simulated sediment detachment and delivery with varied rainfall characteristics. Table 6 shows the model sensitivity indices for both variables. Hillslope sediment detachment predictions are sensitive to changes in rainfall characteristics (depth, duration, and i_p), but are not sensitive to t_p at $\rho < 0.1$ significance level. Increases in rainfall depth and i_p ratio resulted in increases in sediment detachment. Reduction in sediment detachment was predicted for larger storm durations. Again, it is important to remember that the more erosive storms in the southwestern United States are high-intensity, short-duration events.

Sediment detachment predictions are more sensitive to the critical shear stress parameter, τ_{cr}, than the interrill, K_i, and rill erodibility, K_r, parameters. This can be explained by observing that τ_{cr} is a threshold value that has to be overcome before rill erosion occurs. When τ_{cr} is larger, as on rangelands with highly consolidated soils, τ_{cr} represents a major parameter that controls rill erosion calculations in the model (eq. 3).

Other model sensitivities to sediment detachment predictions are shown for K_s, litter, biomass, and initial soil water saturation. This set of model inputs drives the


Runoff Volume

(a)

(b)

Peak Runoff

(c)

(d)

Figure 5–Relative frequency distributions of predicted runoff volume and peak runoff for variable rainfall and fixed rainfall characteristics.

Table 4. Sensitivity of hydrologic variables for storm-based WEPP simulation with variable rainfall characteristics

Parameter or Variable	Sensitivity of Runoff Volume (β)	F*†	Parameter or Variable	Sensitivity of Peak Runoff (β)	F
Depth‡	1.061	6862	Depth	1.029	3774
Duration‡	–0.297	462	Duration	–0.444	606
K_s	–0.255	483	K_s	–0.244	257
i_p	0.133	108	i_p	0.217	168
Saturation	0.123	118	Saturation	0.109	54
t_p	0.053	21	Biomass	–0.094	40
Litter	–0.043	15	t_p	0.080	28
Clay	0.037	10	Litter	–0.041	8
Biomass	–0.035	9	Clay	0.036	6
Bulk Density	–0.029	6	Bulk Density	–0.027	3
r-square	0.902			0.832	
Tot. Var.§	14			14	

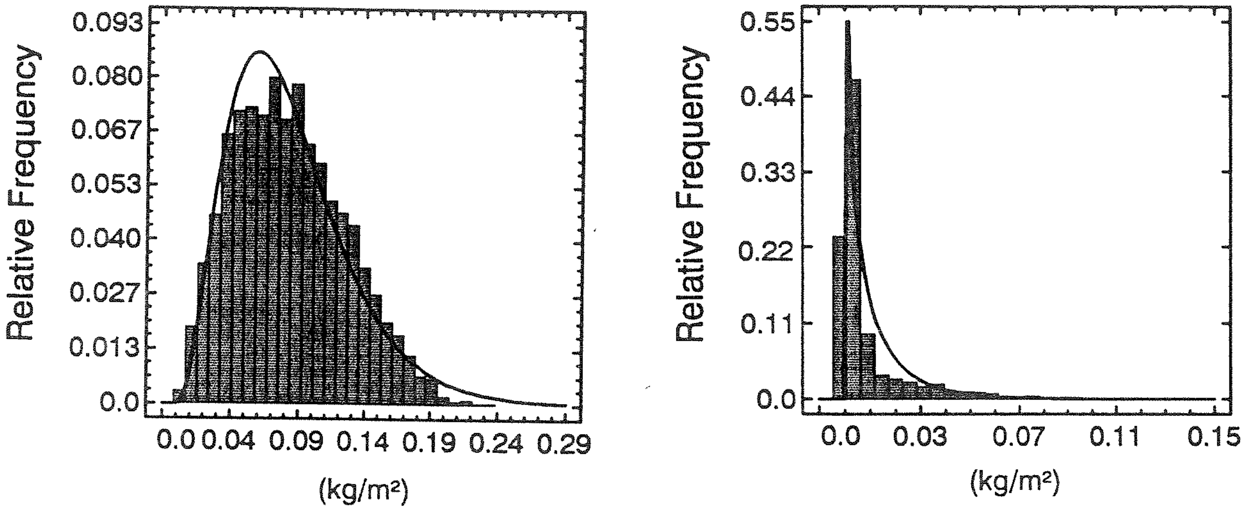
* F statistic.
† $\rho < 0.1$ significance level for staying in the model.
‡ Rainfall.
§ Variables included in the stepwise regression procedure.

Table 5. Sensitivity of hydrologic variables for storm-based WEPP simulation with fixed rainfall characteristics*

Parameter or Variable	Sensitivity of Runoff Volume (β)	F*†	Parameter or Variable	Sensitivity of Peak Runoff (β)	F
K_s	–0.750	6687	K_s	–0.716	6187
Saturation	0.239	680	Biomass	–0.310	1151
Biomass	–0.150	265	Saturation	0.207	518
Clay	0.090	96	Litter	0.102	127
Litter	–0.085	86	Clay	0.083	83
Bulk Density	–0.068	55	Bulk Density	–0.055	39
Rock Cover	–0.016	3	Rock Cover	–0.014	7
r-square	0.690			0.681	
Tot. Var.§	9			9	

* Rainfall: 50 mm, 1 h, $t_p = 0.50$, $i_p = 1.37$.
† F statistic.
‡ $\rho < 0.1$ significance level for staying in the model.
§ Variables included in the stepwise regression procedure.

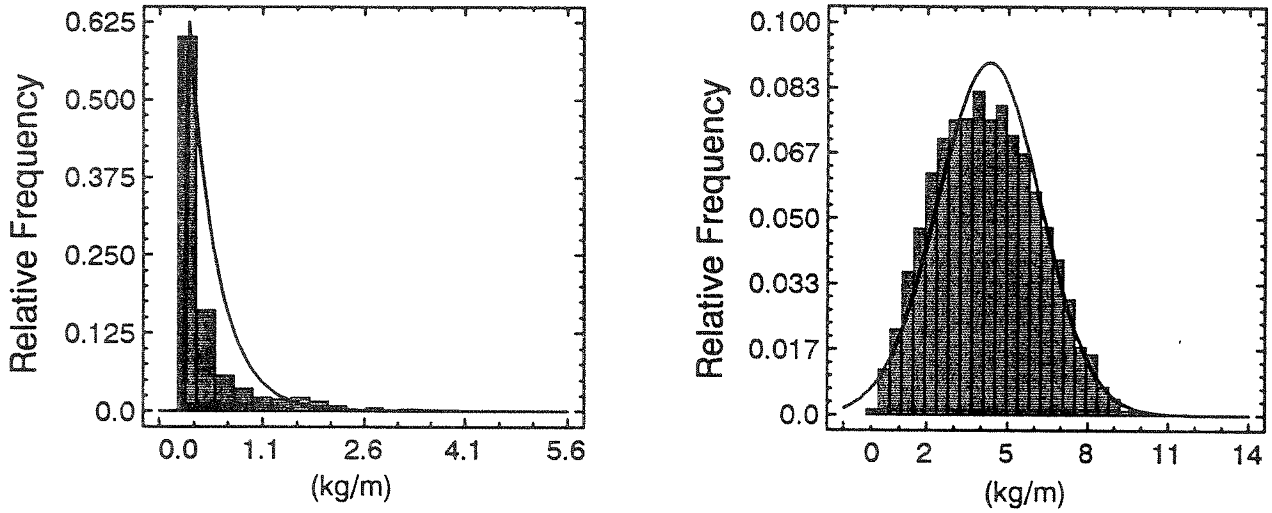
Sediment Detachment



(a)

(b)

Sediment Delivery



(c)

(d)

Figure 6–Relative frequency distributions of predicted soil erosion variables for variable rainfall and fixed rainfall characteristics.

processes of infiltration. Any decrease in K_s or increase in soil water content increases the potential for soil particles detachment by flowing water. Any increase in litter and biomass protects the soil particles from detachment.

Fixed Rainfall. Model sensitivity to sediment detachment and delivery was evaluated for the effects of a large rainfall event because most of the soil erosion is caused by large events. Figures 6b and 6d show the relative frequency histograms and fitted distributions for both sediment detachment and sediment delivery that resulted from 10,000 model simulations using the same fixed rainfall values used in the hydrology sensitivity analysis. Both output variables followed a log-normal distribution that is represented by the solid line of theoretical fit. Table 7 presents the sensitivity indices for the two erosion variables with constant rainfall characteristics.

When rainfall characteristics were fixed, both sediment detachment and delivery were more sensitive to changes in

Table 6. Sensitivity of soil erosion variables for storm-based WEPP simulation with variable rainfall characteristics

Parameter or Variable	Sensitivity of Runoff Volume (β)	F*†	Parameter or Variable	Sensitivity of Peak Runoff **(b)**	F
Depth‡	0.841	1188	Depth	0.911	1539
Duration‡	–0.352	179	Duration	–0.382	235
τ_{cr}	–0.229	110	K_s	–0.194	85
i_p	0.136	31	i_p	0.157	46
K_s	–0.144	41	τ_{cr}	–0.148	52
Litter	–0.141	42	Litter	–0.111	29
Biomass	–0.106	23	Biomass	–0.099	23
K_i	0.068	9	Saturation	0.066	10
K_r	0.062	8	t_p	0.055	7
Saturation	0.043	3	K_i	0.049	5
			K_r	0.036	3
r-square	0.636			0.681	
Tot. Var.§	17			17	

* F statistic.
† $\rho < 0.1$ significance level for staying in the model.
‡ Rainfall.
§ Variables included in the stepwise regression procedure.

Table 7. Sensitivity of soil erosion variables for storm-based WEPP simulation with fixed rainfall characteristics*

Parameter or Variable	Sensitivity of Runoff Volume (β)	F*†	Parameter or Variable	Sensitivity of Peak Runoff (β)	F
Litter	–0.547	7804	Litter	–0.564	9856
τ_{cr}	–0.483	6070	τ_{cr}	–0.466	6481
K_i	0.415	4481	K_i	0.412	5085
Biomass	–0.327	2778	Biomass	–0.332	3297
K_r	0.173	782	K_r	0.171	876
K_s	–0.096	241	K_s	–0.126	473
Rock Cover	–0.087	198	Rock Cover	–0.085	220
Saturation	0.030	23	Saturation	0.037	42
			Clay	0.011	4
r-square	0.887			0.870	
Tot. Var.§	12			12	

* Rainfall: 50 mm, 1 h, $t_p = 0.50$, $i_p = 1.37$.

† F statistic.

‡ $\rho < 0.1$ significance level for staying in the model.

§ Variables included in the stepwise regression procedure.

Table 8. Correlation matrix of variable rainfall parameters for LH-103

	Depth	Duration	i_p	t_p
Depth	1.00	0.380	–0.127	0.093
Duration		1.00	0.015	0.438
i_p			1.00	–0.138
t_p				1.00

Table 9. Correlation matrix of soil parameters for LH-103

	Organic Matter	CEC	Rock Frag-ments	Sand	Silt	Clay	Bulk Density
Organic Matter	1.0	0.881	–0.074	–0.021	–0.018	0.110	–0.158
CEC		1.0	–0.231	–0.165	0.236	0.148	0.373
Rock Fragments			1.0	–0.102	–0.025	0.241	0.918
Sand				1.0	–0.942	–0.953	0.171
Silt					1.0	0.805	–0.293
Clay						1.0	–0.042
Bulk Density							1.0

soil erodibility parameters. The critical shear stress parameter had the highest sensitivity index, followed by interrill erosion. Both types of output variables are intensified by any increase in soil water saturation.

Under large rainfall events WEPP is quite sensitive to litter and biomass because they affect both runoff and erosion calculations significantly. Litter is the most important variable affecting sediment detachment and delivery predictions. For the case of sediment delivery, for every standard deviation increase in litter, there is a decrease of 0.564 standard deviations in predicted sediment delivery.

VALIDITY OF THE APPROACH

The validity of this approach to assess model parameter uncertainty was analyzed by reviewing the:

- Adequacy of the sensitivity indices.
- Assumption of parameter independence.
- Validity of the regression model to evaluate a nonlinear model.
- Number of model simulations necessary to reach convergence.

Adequacy of the Sensitivity Index. The justification for using the β of a linear model for sensitivity indices is based on regression analysis. Regression methods show that slope, b_i, of the regression of Y, the model prediction of interest, on a particular parameter x_i, is the least-squares estimate of the classic sensitivity index (Tomovic, 1963). If several parameters are varied simultaneously, the sensitivity of the dependent variable Y to variations in the independent variables, x_i, is approximated by $\partial Y / \partial x_i = b_i$, the partial derivative of the equation with respect to each independent variable, x_i. Because the linear model comprises thousands of model simulations, β represents an averaged sensitivity for the explored range of parameter values (independent variables on the regression equation).

Parameter Independence. The parameters included in the multiple linear equations to obtain β were considered independent. Tables 8 and 9 present the correlation coefficients for the rainfall and soil parameters, respectively. Highly correlated parameters were not included in the same regression model. Because sand, silt and clay are highly correlated, only clay was used. Major concerns on parameter dependence involved the inclusion of rainfall depth and duration, and rainfall duration and t_p. The rest of the correlations were considered insignificant.

Validity of the Regression Model. The validity in using a linear regression model to assess the uncertainties in model parameters of a nonlinear model was checked by analysis of residuals. Normal probability and residual plots of runoff volume and sediment delivery are presented in figure 7. Nonnormality is evident and the variance of sediment delivery tends to increase for larger sediment delivery predictions. Nevertheless, this approach is able to identify unbiased estimates of parameter sensitivity which are useful during model development and data collection (Gardner, 1984). Further research is needed to determine the higher-order effects for nonlinear cases.

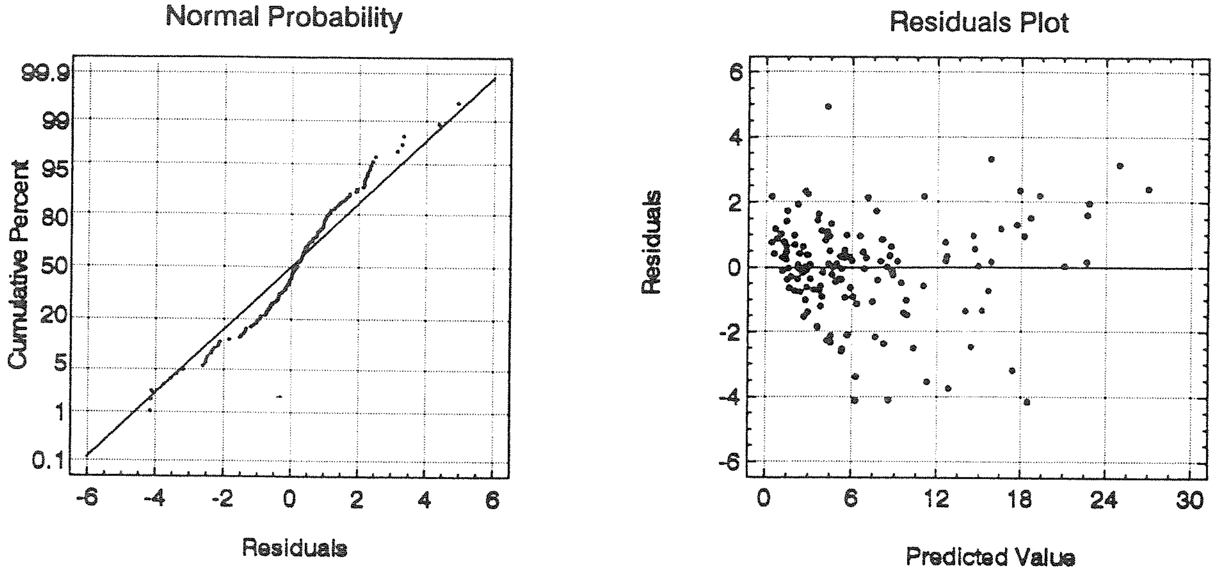
Number of Simulations. Figure 8 illustrates the changes in parameter sensitivity for different numbers of model simulations using variable rainfall. Using the set of simulations for the parameter sensitivity under variable rainfall, β stabilized at approximately 400 simulations. This indicates that the number of simulations performed for this analysis was satisfactory.

CONCLUSIONS

This analysis is a first attempt to identify the WEPP watershed model uncertainties by applying the Monte Carlo method for sensitivity analysis using information of a typical Southwestern semi-arid rangeland watershed. Uncertainty in the hydrologic and soil erosion predictions due to errors in model parameter estimation is identified.

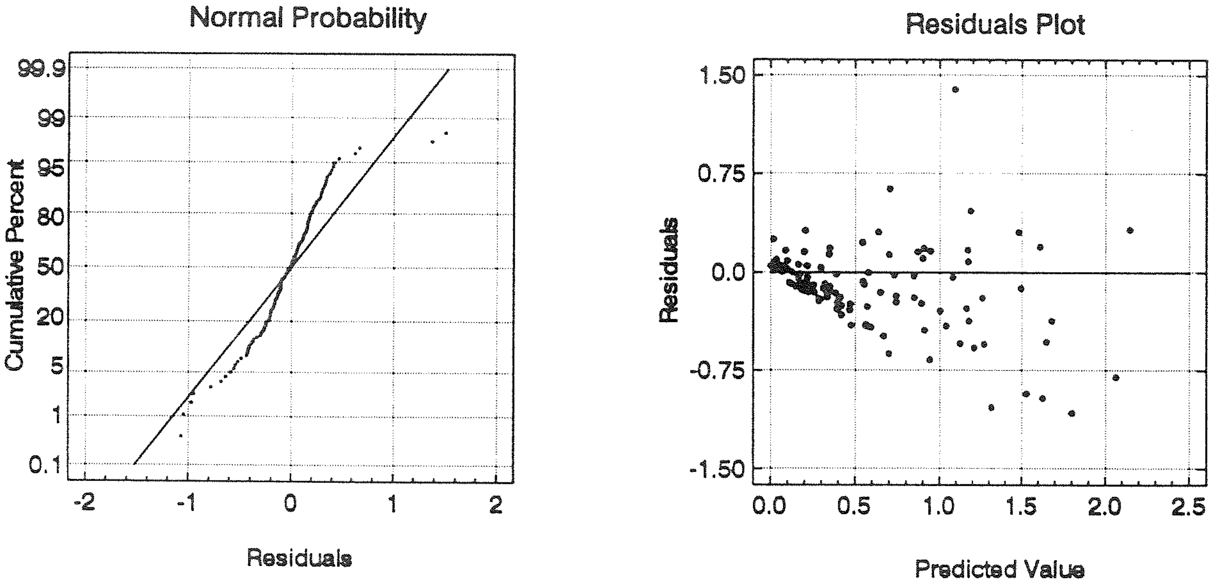
The results indicate that runoff volume and peak runoff predictions from hillslopes are very sensitive to rainfall characteristics (depth, duration, and i_p), and the parameters and variables that regulate infiltration (saturated hydraulic conductivity, initial soil water content, and standing biomass). The importance of rainfall characteristics on runoff predictions was confirmed by a reduction in the coefficients of determination of multiple linear regressions equations (0.90 to 0.69) when rainfall characteristics of storm events were fixed at 50 mm/h. Under fixed rainfall conditions the saturated hydraulic conductivity is the most important parameter in predicting runoff volume and peak runoff.

Runoff Volume for Variable Rainfall



(a) (b)

Sediment Delivery for Variable Rainfall



(c) (d)

Figure 7–Residual analysis plots for runoff and sediment delivery for variable rainfall.

Erosion predictions from hillslopes are highly sensitive to rainfall characteristics (depth, duration, and i_p ratio). Infiltration and soil cover parameters significantly affect sediment detachment and sediment delivery from hillslopes. Sediment detachment and sediment delivery are more sensitive to the critical shear stress parameter, τ_{cr}, than to the interrill, K_i, and rill, K_r, erosion parameters for both variable and fixed rainfall conditions. However, under fixed rainfall conditions both sediment detachment and sediment delivery predictions are more sensitive to the interrill erosion parameter, K_i, than to the rill erosion parameter, K_r. This supports the findings of Nearing et al. (1990), who mentioned that for places where no-till management factors are involved, as on rangelands, interrill erodibility is the dominant soil erosion factor.

The results presented herein show that because of the low coefficients of determination caused by the large variations of parameters, the sensitivity indices cannot be generalized. However, β is useful for this stage of model development when parameter uncertainties are still being determined. These sensitivity indices not only represent a warning signal of parameter uncertainty during this stage of model development, but they also provide guidance in data collection of model inputs. Ranking model parameters

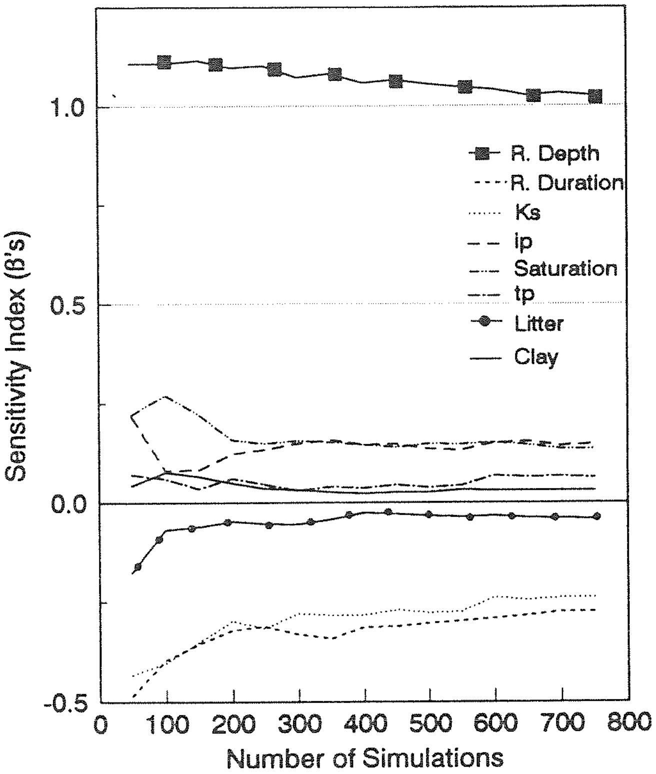


Figure 8–Change in the sensitivity index (β) with the number of model simulations.

according to model sensitivity helps the model user in deciding which parameters should be measured in the watershed and which parameters can be obtained from the relevant literature.

Finally, three major tasks are needed for a more extensive use of this methodology. First, it is necessary to incorporate nonlinear effects and interactions between parameters into the sensitivity indices. Second, because this sensitivity analysis included the characteristics of only one watershed the results cannot be generalized for multiple geographic locations or different land use conditions. If this approach is used, it should be realized that baseline conditions for a wider range of soils, topography, vegetation, and climate of rangeland watersheds are required. Third, although this study used version 91.5 of the WEPP watershed model, not the final version of the model, it is expected that the most sensitive parameters already identified will continue to be the most sensitive for future versions of the model when applied to rangeland watersheds. Only the magnitude of error in model predictions is expected to change.

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