Determining Erodibility Parameters from Rangeland Field Data for a Process-Based Erosion Model

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ABSTRACT

The forms of the sediment continuity equation used in process-based erosion prediction models do not allow for direct analytical solution for soil erodibility parameters for the case when performing rills is not practical or desirable. Therefore, erodibility parameters cannot be calculated from rangeland field erosion data using simple regression unless simplifying assumptions are made which may compromise the accuracy of the parameter estimates. This study developed an optimization technique for determining erodibility parameters from rangeland field data for use in a three parameter steady-state erosion model, and evaluated the method's effectiveness. Model response to variation of the erodibility parameters was also evaluated. Estimated erodibility parameter values characterized the measured data adequately. Model response surface shapes did not indicate problems in terms of parameter identifiability, strong interdependence or sensitivity.

INTRODUCTION

The trend in the development of erosion prediction technology is towards using mathematical descriptions of fundamental erosion mechanics (Meyer and Wischmeier, 1969; Foster and Meyer, 1972; Shirley and Lane, 1978; and Foster et al., 1981). The erosion process is conceptualized as being divided into rill and interrill components with a rate parameter, K_r, and a threshold parameter, τ_c, for characterizing rill detachment; and a rate parameter, K_i, for characterizing interrill detachment. The sediment load equation which models steady-state erosion, such as the one described by Lane et al. (1987), cannot be solved analytically and, therefore, the erodibility parameters for the equation cannot be obtained from a direct solution of the erosion equation. Another technique must be used to determine erodibility parameter values from erosion data for a given experimental site and treatment.

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Laflen et al. (1987) outlined a procedure for determining soil erodibility parameters for cropland soils from field rainfall simulation experiments. The method entails adding clear water inflow at different discharge rates to the upper end of preformed rills and making flow measurements within the rill to calculate an average hydraulic shear stress for each inflow rate. They proposed to use linear regression to determine the slope of the relationship between rill detachment rate and hydraulic shear, which they suggested is the rill erodibility rate parameter, K_r. They then proposed to determine the threshold parameter, τ_c, from the constant of the regression equation. Such techniques are not possible for conditions where preforming of rills or channels is not desirable.

Using preformed rills for erosion experiments on rangeland is not desirable because the disturbance caused in forming the rills destroys the natural conditions of the site (Simanton et al., 1987). Evaluation of erodibility parameters from plot data on rangeland requires assumptions of channel hydraulics and flow partitioning between rill and interrill areas. The assumptions and solution techniques used in evaluating model parameters from field data on range sites should be the same as those used in the intended model.

Optimization techniques can be used to evaluate erodibility parameters for erosion models from field data (Blau et al., 1988). This method has several advantages over other methods for evaluating erodibility parameters. The optimization procedure uses the model to solve the erosion equations; therefore, simplifying assumptions, such as neglecting effects of partial filling of transport capacity on detachment rates and neglecting changes in hydraulic shear with downslope distance, need not be made. Rangeland erosion results should also be better represented by using the model to evaluate erodibility parameters for the reasons discussed above.

Blau et al. (1988) used an optimization procedure to identify erosion model parameters for the dynamic erosion model of Shirley and Lane (1978). Blau et al. (1988) optimized two parameters; one a sediment transport term, and the other a rill detachment rate term. The model did not include a threshold term for rill detachment. In that study, the model was relatively insensitive to the rill detachment rate term.

The purpose of our study was to develop and evaluate an optimization method for determining erodibility parameters from field data for use in modern process based erosion prediction technology, such as that described by Foster and Land (1987). The model uses a steady-state sediment continuity equation as a basis for calculating sediment loads, hence the parameters evaluated in this study are for the steady-state equation.

The proposed method may be used to optimize simultaneously for all three erodibility parameters, or if the interrill parameter, K_i, is known, the two rill erodibility parameters alone may be determined. Thus, the method may be used to evaluate parameters from experiments designed specially for determination of rill erosion parameters or from large plot data, such as from the rangeland experiments described by Simanton et al. (1987), wherein all three erodibility parameters must be evaluated simultaneously. The erosion equations that are used are outlined, response surfaces for hypothetical parameter values are presented, the optimization scheme is described, and examples using field data are presented.

EROSION EQUATIONS

The erodibility parameters to be optimized are for the equations outlined by Lane et al. (1987). Those equations are presented here. The steady-state continuity equation for sediment in overland flow is

$$dG(x)/dx = D_i + D_f(x) \quad [1]$$

where G (kg/s m) is sediment load (a function of x, distance downslope, D_i (kg/s m^2) is interrill detachment (considered constant with x), and D_f (kg/s m^2) is detachment or deposition by flow in the channel. Only detachment parameters are being considered here, and deposition equations will not be presented; although deposition may and should be considered if a portion of the field data to be optimized was collected under conditions where deposition occurred. Sediment load is controlled primarily by transport capacity when deposition is active. Therefore, erodibility parameters cannot be estimated from field data where deposition occurred.

For the case of detachment, rate of rill flow detachment, $D_f(x)$, is expressed as (for the case that $\tau(x) > \tau_c$)

$$D_f(x) = K_r(\tau(x) - \tau_c)\,(1 - G(x)/T_c(x)) \quad [2]$$

where

$\tau(x)$ (Pa) = shear stress in the rill acting on the soil
$T_c(x)$ (kg/s m) = transport capacity in the rill
K_r (s/m) = the erodibility rate parameter for the soil
τ_c (Pa) = threshold or critical hydraulic shear of the soil.

The value of D_f is taken to be zero until shear exceeds critical shear stress of the soil. Shear stress in the rill is computed at the lower end of the slope by assuming a rectangular channel cross section. The channel width was estimated using the relationship between width and discharge given by Lane and Foster (1980). Depth of flow in the channel was calculated using the Darcy-Weisbach flow equation. Transport capacity was computed at the bottom of the slope from the modified Yalin equation (Foster, 1982). (Note that equation (8.55) in this reference is incorrect—the exponent on Sg should be −0.4 instead of 0.4.) The equations of Foster et al. (1985) were used to compute the fractions of the particle size classes of primary clay, primary silt, primary sand, small aggregates and large aggregates as a function of soil texture and organic matter.

Shear stress in the rills was calculated from the equation

$$\tau(x) = (P\, s(x)\, x)^{2/3} \quad [3]$$

where

s = slope gradient
P = a constant given by

$$P = (\rho\, g)^{3/2}\, R / C \quad [4]$$

where

ρ = density of water, g
g = the gravity constant
C = the Chezy discharge coefficient
R = rainfall excess.

The value of Chezy C was estimated from a set of field rill data from the study of Laflen et al. (1987), and is a function of soil type. This is an important point in terms of evaluating rangeland erosion data. The methodology for measuring friction factors, which are used to calculate C, in channel areas on rangelands has not been developed. Assumptions that are used in the model to predict erosion rates on rangeland must be the same as those used to derive the erodibility parameters for the model such that the parameters are compatible with the model.

The transport relationship, $T_c(x)$, was simplified to (Foster and Meyer, 1972)

$$T_c(x) = B\, \tau(x)^{3/2} \quad [5]$$

where the parameter, B, was calibrated from the computation of T_c at the bottom of the slope as discussed above. Substituting equation [3] into equation [5], $T_c(x)$ was described as

$$T_c(x) = B\, P\, s(x)\, x \quad [6]$$

Slope was considered constant for the plots.

For the case of interrill detachment, the detachment rate, D_i, is represented in the model by an equation of the form

$$D_i = K_i\, I^2 \quad [7]$$

where

K_i (kg s/m^4) = interrill erodibility parameter
I (m/s) = rainfall intensity
D_i = considered to be constant with distance, x.

The erosion model solves equation [1] for G(x). The form of the differential equation is given by substituting equations [3] and [6] for $\tau_c(x)$ and $T_c(x)$, respectively, into equation [2] for $D_f(x)$, which is then substituted into equation [1]. Equation [1], with appropriate substitutions, is a first order linear differential equation of the form

$$dG/dx = K_i I^2 + K_r([P\ s\ x]^{2/3} - \tau_c)\ (1 - G(x)\ /\ [B\ P\ s\ x]) \quad \ldots\ldots\ldots\ldots\ldots [8]$$

The model solves equation [8] numerically.

Inflow of clear water must be added incrementally to the upper end of field erosion plots in order to obtain a range of shear stresses on the plots, or alternatively, a wide range of intensities must be applied to the plots. For the case of added inflow an effective slope length is calculated which is equivalent to that required to produce the measured runoff at each inflow level. The value of G(x) in the numerical solution of equation [8] is assigned to zero through the calculations until the downslope distance of the effective slope length is coincident with the upper end of the actual field plot. This in effect simulates the case of added inflow to the upper end of the plots. The boundary condition for solving equation [8] is: G(x=upper plot end)=0.

MODEL RESPONSE

An objective function describes error in a model's predictions. The objective function may be a least squares difference between predicted and measured field values of sediment load. Optimization involves minimizing such an objective function for a set of field data. The objective function may also be used to characterize the sensitivity of the model to variation of input parameters. Given a known optimum set of parameter values (K_i, K_r, τ_c), a sum of squares difference between the calculated sediment loads at the optimum point and the calculated sediment loads at points away from the optimum can be calculated. A contour map of the least squares objective function in the parameter space is a response surface.

The potential success of parameter identification and optimization for a model depends upon the shape of the response surface of the objective function (Blau, 1988). The shape of the response surface also gives an indication of the sensitivity of the model to the input parameters. Elongation of the contours may cause difficulties in finding objective function minima with optimization techniques. Elongation parallel to a given axis indicates insensitivity of the model to the parameter of that axis. Elongation at the 45 deg angle to the axis indicates interdependence of the parameters. The most favorable response surface shape with regards to parameter sensitivity and independence is circular.

Seven sets of synthetic data were generated with the model to evaluate the potential of optimizing the model for the erodibility parameters. Table 1 lists the parameters for the seven sets. The data generated were for the case of a 10-m-long plot with rainfall intensity varying from 30 to 180 mm/h and runoff ranging from 15 to 90 1/min/m width. Six synthetic data points were generated for each data set. Contours of the objective function, which was the least squares difference between sediment loads calculated at the known optimum point and those calculated for erodibility values away from the known optimum, were plotted for all seven data sets. Results for case 1, shown in Fig. 1, were typical for the seven data sets. In plotting these surfaces, one of the parameters was held constant while the other two were varied around the optimum values.

TABLE 1. Model Input and Optimization Results for Synthetic Erosion Data

Case	actual parameter values			2 parameter optimization		3 parameter optimization		
	K_r	τ_c	K_i	K_r	τ_c	K_r	τ_c	K_i
	s/m	Pa	kg s/m^4 *10^{-6}	s/m	Pa	s/m	Pa	kg s/m^4
1	.025	3.0	2.5	.0250	3.004	.0248	2.988	2.504
2	.025	3.0	1.0	.0250	3.004	.0253	3.008	0.969
3	.025	3.0	5.0	.0250	3.004	.0257	3.047	5.009
4	.025	0.5	2.5	.0250	0.514	.0250	0.494	2.501
5	.005	3.0	2.5	.0049	2.963	.0054	3.169	2.488
6	.050	3.0	2.5	.0500	3.004	.0491	2.962	2.513
7	.025	6.0	2.5	.0252	6.008	.0240	5.967	2.529

The response surfaces of the objectives function were relatively well behaved (Fig. 1), although some elongation of the surface was evident. The elongation of the response surface indicated a dependence between the parameters. For each synthetic data set, the response function remained low in a valley of increasing K_r and τ_c (Fig. 1a); the increase in K_r was offset by an increase in τ_c in terms of sediment load prediction. Similar dependence existed between K_r and K_i (Fig. 1b), but was not usually as evident between K_i and τ_c for the synthetic data (Fig. 1c). The dependency between erodibility parameters can be physically interpreted. From the rill detachment equation, equation [2], we see that for a given value of shear stress, $\tau(x)$, the rill detachment rate can be held constant if Kr is increased and τ_c is increased. For a set of two or more shear stresses a unique optimum set of Kr and τ_c exists, but the general shape of the response function shows the general dependence between the parameters and the resultant +45-deg response surface. For Kr and Ki the response surface elongation is −45 deg, which is indicative of the fact that increasing Kr and increasing Ki both result in increased soil detachment. In other words, an increase in Kr tends to offset a decrease in Ki in terms of calculated soil loss.

The shape of the model response surfaces and the dependency of the erodibility terms indicate the need for a relatively large range of output values and flow conditions to obtain the erodibility parameters. Three or even two erodibility parameters cannot be determined from an experimental condition with only one rainfall intensity, discharge rate, slope gradient and slope length. Greater numbers of flow conditions in an experiment result in greater numbers of degrees of freedom for determining the parameter values and subsequently less error in parameter estimates which might be caused by a dependency between parameters.

Response surfaces were plotted to evaluate model response for field data. A data set was taken from a rangeland site from the study described by Simanton et al. (1987). It was collected on a silty clay loam soil in Meeker, Colorado, in 1987. The response surfaces for that data set were plotted in Fig. 2. For this data set K_r and τ_c appeared to be relatively independent as did K_i and K_r (Figs. 2a and 2b). The parameters K_i and τ_c, however, appeared to have some dependency, as shown in Fig. 2c. A plot of K_i vs. τ_c for a second field plot at the same Meeker site (Fig. 3), however, did not show as strong a dependency.

Overall the response surfaces in the erodibility parameter space were considered to be relatively well behaved. Local minima were not evident, which

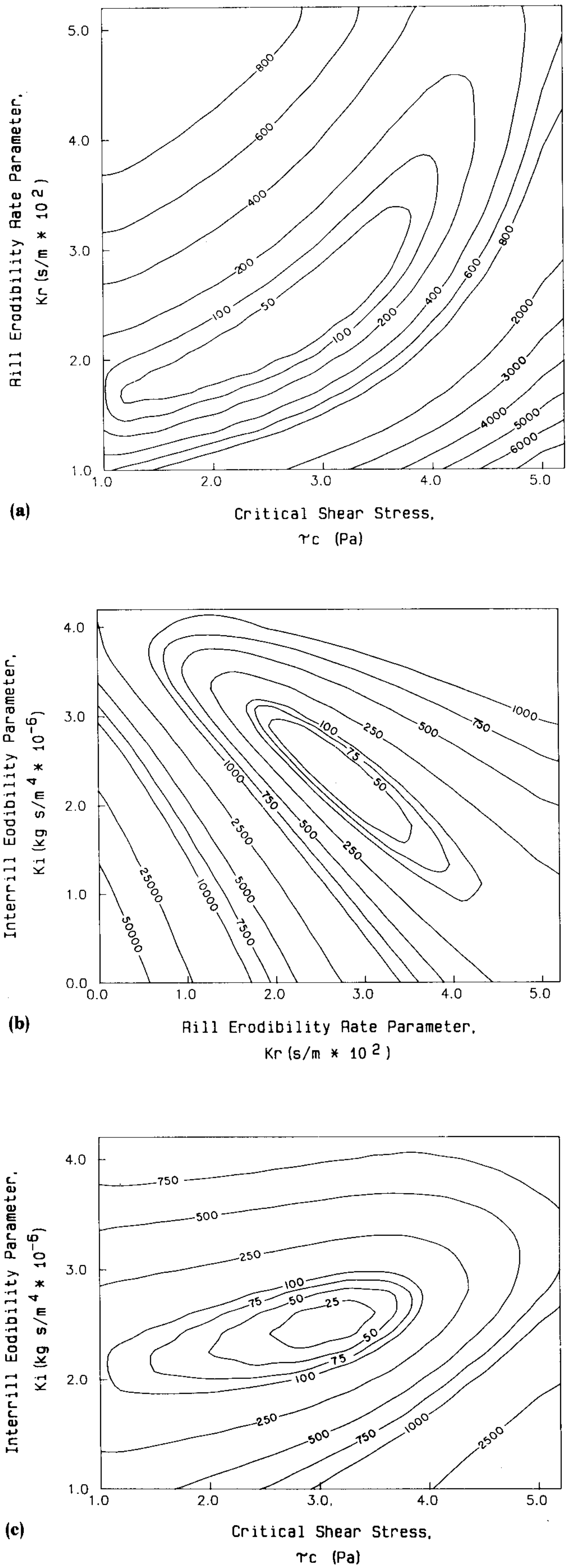

Fig. 1—Response surfaces for synthetic data of case 1 for (a) K_r vs. τ_c, (b) K_i vs. K_r and (c) K_i vs. τ_c.

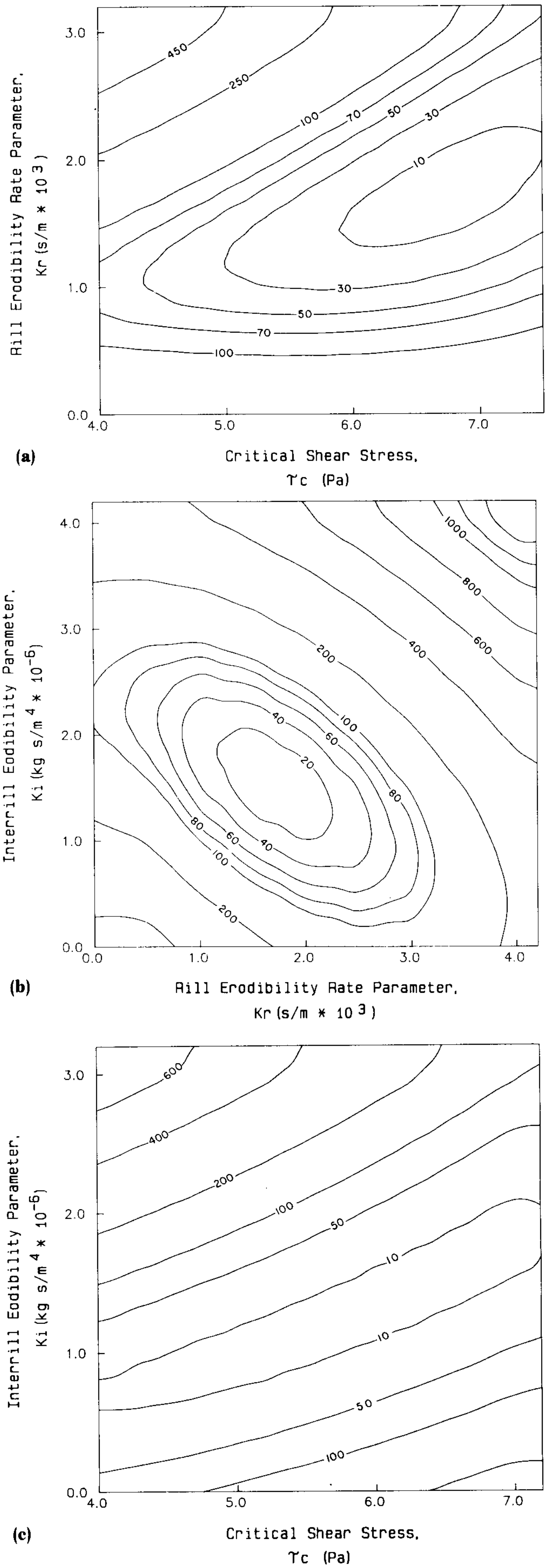

Fig. 2—Response surfaces for data from Meeker test site, plot no. 1, for (a) K_r vs. τ_c, (b) K_i vs. K_r and (c) K_i vs. τ_c.

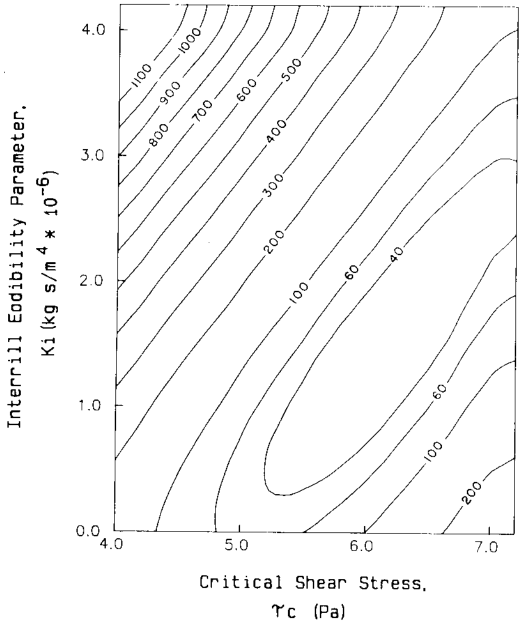

Fig. 3—Response surface for data from Meeker test site, plot no. 2, for K_I vs. τ_c.

coincides with the results of Blau et al. (1988). Dependencies between parameters were not as great as those found for the model studied by Blau et al. (1988). The fact that the optimization procedure worked well in finding response surface minima, as will be shown below, further indicated that the model is a viable description of the steady-state erosion process.

OPTIMIZATION

An optimization method was developed to obtain erodibility parameter values from sediment load data for the steady-state erosion model. The method for optimizing two parameters, K_r and τ_c is: (a) choose initial (best guess) values of K_r and τ_c as input, (b) the program then uses the erosion model to calculate sediment loads for an array of K_r and τ_c values around the central values, (c) the least squares objective function is calculated for each point on the array, (d) the minimum of the function is found, and (e) the central K_r and τ_c values are reset to correspond to the minimum point of the array. Then the program calculates a new array of the objective function around the new central values with a finer grid mesh and finds the minimum.

The process is repeated for successively finer grids. If the minimum of the objective function is found on the boundary of one of the grids, the central values are readjusted to that point on the grid, but the grid is not made finer. This is necessary when searching for the minimum value when it is located in a "valley" on the response surface, since the line of minimum objective function in the valley may pass between points of the grid set by the program. The accuracy of the method is dependent upon the number of times that the grid size is reduced and the mesh made finer. Experience in using the method indicated that a good optimum set of parameter values could be obtained after five reductions in grid size.

The process for optimizing for three parameters is the same as for optimizing two, except that a three-dimensional grid is used instead of a two-dimensional grid.

TABLE 2. Results of Parameter Optimization for Field Data Sets

Data Set	Site Location	Soil Texture	K_r	τ_c	K_i	Coefficient of Determination	Number of Samples
			s/m $*10^3$	Pa	kg s/m^4 $*10^{-6}$	r^2	n
Meeker 1	CO	sic	1.58	6.61	1.58	0.98	6
Meeker 2	CO	sic	5.31	6.24	1.95	0.98	6
Cuba 1	NM	fsl	0.78	2.12	0.22	0.99	6
Cuba 2	NM	fsl	0.52	1.38	0.27	0.89	6
Woodward 1	OK	l	5.59	4.65	0.025	0.99	5
Woodward 2	OK	l	3.66	5.91	0.16	0.95	5
Chickasha 1	OK	sl	0.12	1.69	0.30	0.99	5
Chickasha 2	OK	sl	0.55	2.16	0.48	0.86	6
Cottonwood 1	SD	sic	0.75	0.22	0.93	0.93	6
Cottonwood 2	SD	sic	1.24	2.06	0.57	0.97	7

The optimization technique was tested with the synthetic data. The results (Table 1) indicated no problem in determining the actual model parameter values using the optimization technique.

The model was optimized for ten sets of field data from five range sites described by Simanton et al. (1987). The results of the field data optimizations are presented in Table 2. Differences in estimated erodibility parameters for the two data sets of each site were generally within a normal range of variability for erodibility measurements, and were also generally lower than variabilities between sites. The exception was for the kr values of the Meeker site. This may be attributable to the fact that the Meeker site had a large τ_c value and that only a portion of the flow shear stress levels exceeded critical shear stress of the soil. The experimental procedure for determining Kr must include several levels of shear which exceed critical shear of the soil in order to obtain the best estimates of erodibility parameters for the soil at the site. Also, for four of the sites the data set with the higher Kr also showed either a higher critical shear or lower Ki, again showing some dependency between the parameters as discussed previously.

Figure 4 is a plot of predicted vs. measured sediment load results using the parameters derived from the optimization procedure for each range site. There were no apparent problems in using the optimization technique to obtain erodibility values from the data used in this study, and the model represented the data adequately.

SUMMARY

A method was outlined for optimizing rangeland field erosion data to obtain erodibility parameters for a steady-state erosion model with three erodibility terms. Using optimization to obtain the erodibility parameters ensures that they are derived using the same assumptions and solution techniques which the model uses, ie., it ensures that the parameters are compatable with the model. Important assumptions must be made with regards to channel friction factors, which are not readily measurable, as well as widths and spacings of channel areas. These factors have a significant influence on calculated shear stresses. The optimization technique provides a method for deriving the three erodibility parameters simultaneously for experiments where preformed rills or channels are not desirable. The

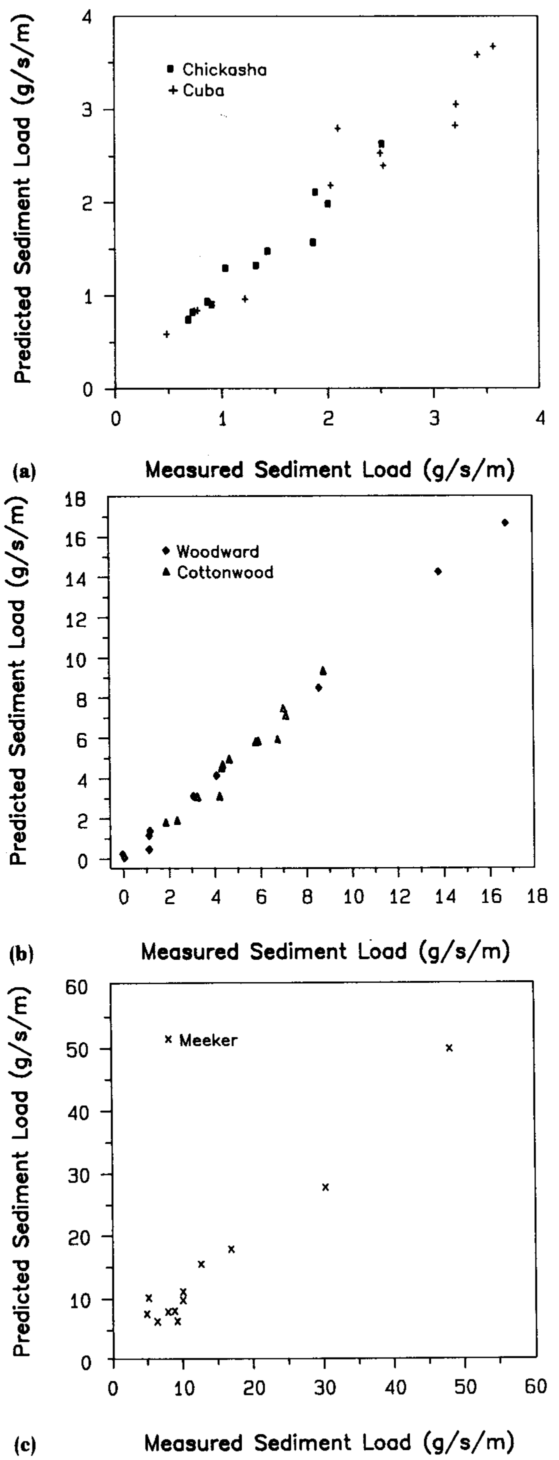

Fig. 4—Predicted vs. measured sediment load for (a) Chickasha and Cuba, (b) Woodward and Cottonwood, and (c) Meeker experimental sites.

method presented here calculated erodibility parameter values which characterized the measured data adequately. This result has important implications. On many rangeland experiments, detachment by surface flow is not apparent in the form of incised rills, yet the rill-interrill model appears to represent the data.

The response surfaces of the model output in the parameter space were relatively well behaved and did not indicate a problem in obtaining the erodibility parameters or in interpreting them within the context of the model. Local minima on the response surface were not found. Some interdependence of parameters was evident, but the dependence was not found to cause problems in identifying erodibility parameters or to put into question the validity of the model structure with respect to its parameterization. Thus, optimization procedures can be used to successfully identify erodibility parameters for rangeland sites for use in process based erosion prediction models.

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