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PONDING TIME FOR VARIABLE RAINFALL RATES

Runoff estimates during a storm depend crucially on the prediction of time to ponding t_p , under arbitrary rainfall patterns. On the basis of careful numerical calculations, Smith (1972) and Smith and Chery (1973) have proposed an accurate parametric model, yielding t_p with a better than 10% precision, such that

$$(r_p K_s^{-1} - 1)^{\beta-1} \int_0^{t_p} r dt = A \quad (1)$$

where r is the rainfall rate given as a function of time, r should not vary too rapidly near ponding time, so that r_p , the rate at the time of ponding, is well defined. The soil properties are defined by three parameters: A , which is essentially a linear function of the initial water content; θ_i , assumed to be uniform; β which is close to two; and K_s the conductivity at saturation, i.e. the final infiltration rate. One of the most important implications of equation (1) is that for the same r_p and the same initial conditions, ponding occurs when $\int_0^{t_p} r dt$ reaches a critical value, independent of the rainfall pattern.

An alternate expression is proposed in the following which has about the same accuracy as equation (1) for Smith's examples and is simpler to use, i.e., it requires one less parameter to be obtained by curve fitting. Furthermore Smith's model is an empirical one based on calculations for simple rainfall patterns, while the present one, although approximate, is derived from theoretical considerations and can be applied in principle for any rainfall pattern.

We have (see equation (A9) in the Appendix)

$$\ln^{-1} [r_p / (r_p - K_s)] \int_0^{t_p} r dt = B \quad (2)$$

The parameter B is the equivalent of A in equation (1), and it is shown in the Appendix that B is theoretically related to the sorptivity S , by

$$B = S^2 / 2K_s \quad (3)$$

It is not surprising that both K_s and S should enter the problem since they are the
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two fundamental soil properties which are usually measured in practice to define a soil, e.g. see Talsma (1969) and Talsma and Parlange (1972).

Basically $(r_p / K_s - 1)^{\beta-1}$ in equation (1) is replaced by $\ln^{-1} [r_p / (r_p - K_s)]$ in equation (2). The analytical dependence on r_p is different in each case, but more importantly the need for the empirical parameter β has now disappeared. Note that during the early stages of infiltration, with $r = St^{-1/2} / 2 \gg K_s$, equation (2) is an identity and thus t_p is indeterminate. This checks with the physical consideration that in this case the soil surface is saturated but without ponding. However, equation (1) does not have the same property except in the special case $\beta = 2$. For that special case, equations (1) and (2) then become identical when $r = St^{-1/2} \gg K_s$, if $A = B$. In general, Smith (1972) found that β was close to two; hence, even if not equal, A and B should have the same general behavior. For instance, it is shown in the Appendix that S^2 varies linearly with θ_i , equation (A8). Consequently, equation (3) shows that B , and hence A , should also vary linearly with θ_i . This is in full agreement with Smith's calculations.

The fundamental observation of Smith that $\int_0^{t_p} r dt$ is constant when r_p and θ_i are unchanged is also in agreement with equation (2). Consequently, to compare the prediction of equation (2) with Smith's calculations, as summarized by equation (1), it is sufficient to consider the case of r and θ_i constant. This comparison is important to check the precision of equation (2) for the cases considered numerically by Smith (1972), since his numerical results should indicate any soil behavior which differs significantly from assumptions on which this equation is based.

Table 1 gives the rainfall rates and the exact ponding times calculated by numerical simulation for six different soils (as in Smith 1972) and the corresponding values of B and A from equation (1). The values of

Table 1. Values of K_2 (cm/min), r_p (cm/min), t_p (min), for six soils from numerical simulation and corresponding values of B from equation (2), as well as β and the values of A from equation (1)

Poudre S.	r_p	0.339	0.423	0.508	0.635	0.762	0.931
$\beta = 1.92$	t_p	8.74	5.09	3.36	2.07	1.39	0.88
$K_s = 0.1397$	B	5.56	5.38	5.29	5.28	5.22	5.04
	A	4.09	4.13	4.15	4.20	4.18	4.04
Nickel G.S.L.	r_p	0.0847	0.127	0.1481	0.1693	0.1905	0.2117
$\beta = 2.0$	t_p	17.13	7.17	5.18	3.87	2.97	2.32
$K_s = 0.0267$	B	3.83	3.86	3.86	3.82	3.75	3.64
	A	3.15	3.43	3.49	3.50	3.48	3.40
Nibley S.C.L.	r_p	0.0635	0.0868	0.127	0.148	0.169	0.191
$\beta = 1.935$	t_p	30.12	16.48	6.92	4.94	3.80	3.02
$K_s = 0.0167$	B	6.31	6.36	6.22	6.10	6.18	6.28
	A	5.05	5.19	5.13	5.03	5.08	5.15
Colby S.L.	r_p	0.847	0.1270	0.1481	0.1693	0.1905	0.2117
$\beta = 1.96$	t_p	8.05	3.42	2.50	1.94	1.54	1.26
$K_s = 0.0085$	B	6.49	6.29	6.29	6.38	6.47	6.52
	A	5.66	5.50	5.49	5.56	5.63	5.66
Colby S.L. (constant ϕ)	r_p	0.0635	0.0847	0.1058	0.127	0.1693	0.3175
$\beta = 1.94$	t_p	11.8	6.85	4.33	2.92	1.57	0.483
$K_s = 0.0085$	B	5.23	5.50	5.49	5.38	5.18	5.68
	A	4.35	4.58	4.55	4.43	4.23	4.51
Muren C.	r_p	0.0847	0.127	0.1481	0.1693	0.1905	0.2138
$\beta = 1.89$	t_p	15.71	7.12	5.25	4.06	3.22	2.54
$K_s = 0.0095$	B	11.80	11.60	11.69	11.85	11.99	11.94
	A	8.44	8.52	8.49	8.51	8.53	8.40

t_p shown here differ slightly from those presented by Smith (1972), owing to improvements in numerical precision. The values of r_p and t_p are used to calculate the left-hand side of equation (2) which is then equal to B . This function should be constant and equal to $S^2/2K_s$ according to equation (3). However, equation (2) is based on an approximate theory; hence, we expect that B is not truly constant. Indeed, Table 1 shows that B varies slightly with r_p for each soil. However, its dependence on r_p is minimal. For instance, in the worst case, that of Poudre sand, B varies by less than 10%, which is comparable to the variation of A (see Table 1). Hence, even for that soil, taking $S^2/2K_s = 5.3$ cm in equation (2), t_p is predicted to within 5%. For Nickel G. S. L., Nibley S. C. L. and Colby S. L., B is near constant and taking $S^2/2K_s$ equal to 3.8, 6.2 and 6.4 cm, respectively, in

equation (2) yields t_p within 2%. The values of $S^2/2K_s$ for Colby S. L. (constant ϕ) are equal to 4.45 cm with a 4% precision. Finally, in the case of Muren C. taking $S^2/2K_s = 11.7$ cm in equation (2) yields t_p with a 2% precision.

In conclusion, equation (2) provides a precise correlation of Smith's calculations, which should be applicable to general rainfall patterns. The prediction of the ponding time requires a knowledge of the value of B , which was assumed here to be constant even for large variations of r_p . We are presently improving the model by an optimization technique (Parlange 1975) which takes into account the small variation of B with r_p .

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APPENDIX

Recently, an approximate model was presented which predicts the ponding time for constant r (Parlange 1972). When r is a given function of time rather than a constant, the model can be extended following the general procedure used by Parlange (1972) for constant r . To the lowest order, it is assumed that the water flux within the soil varies with time but not position. This assumption is exact when the diffusivity is a delta-function and is a good approximation for most soils which have a very rapidly varying diffusivity. Then,

$$K - D \partial\Theta/\partial z = r \tag{A1}$$

where K is the conductivity, and D is the diffusivity, which are functions of the water content Θ , and z is the distance of a point from the surface. Equation (A1) is easily integrated to yield,

$$z = \int_{\Theta}^{\Theta_1} D(\Theta) [r - K(\Theta)]^{-1} d\Theta, \tag{A2}$$

where Θ_1 is the water content at the surface and is an unknown function of time, and Θ_i is the initial water content. The dependence of Θ_1 on time is obtained from conservation of mass, or

$$-\int_0^t r dt = \int_{\Theta_i}^{\Theta_1} (\Theta - \Theta_i) (\partial z / \partial \Theta) d\Theta \tag{A3}$$

combining equations (A2) and (A3) yields

$$\int_0^t r dt = \int_{\Theta_i}^{\Theta_1} (\Theta - \Theta_i) D(\Theta) [r - K(\Theta)]^{-1} d\Theta \tag{A4}$$

In particular, the ponding time t_p is the time at which the water content Θ_1 reaches the value Θ_s , water content at saturation, or,

$$\int_0^{t_p} r dt = \int_{\Theta_i}^{\Theta_s} (\Theta - \Theta_i) D(\Theta) [r - K(\Theta)]^{-1} d\Theta \tag{A5}$$

Equation (A5) is rather formidable since it presumes a knowledge of the dependence of both D and K on Θ . However, Talsma et al. (1972) have shown that accurate results are obtained if one assumes that $D(\Theta - \Theta_i)$ and $dK/d\Theta$ have a similar dependence on Θ . Specifically, $(\Theta - \Theta_i)D$ is written as (Talsma et al. 1972)

$$(\Theta - \Theta_i)D = (dK/d\Theta)S^2/2K_s \tag{A6}$$

where K_s is the conductivity at saturation and

$$S^2 \approx 2 \int_{\Theta_i}^{\Theta_s} (\Theta - \Theta_i) D d\Theta \tag{A7}$$

As long as D varies rapidly with Θ , we can also write

$$S^2 \approx 2 (\Theta_s - \Theta_i) \int_0^{\Theta_s} D d\Theta \tag{A8}$$

which shows that S^2 depends linearly on Θ_i . Equation (A5) is then greatly simplified when equation (A6) is used and reduces to

$$\ln^{-1} [r_p / (r_p - K_s)] \int_0^{t_p} r dt = S^2 / 2K_s \tag{A9}$$