

A nonparametric/parametric analysis of the Universal Soil Loss Equation

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Abstract

Due to its modest data demands and transparent model structure, the Universal Soil Loss Equation (USLE) remains the most popular tool for water erosion hazard assessment. However, the model has several shortcomings, two of which are likely to have prominent implications for the model results. First, the mathematical form of the USLE, the multiplication of six factors, easily leads to large errors whenever one of the input data is misspecified. Second, the USLE has a modest correlation between observed soil losses and model calculations, even with the same data that was used for its calibration. This raises questions about its mathematical model structure and the robustness of the assumed parameter values that are implicitly assigned to the model. This paper, therefore, analyzes if the USLE could benefit from mathematical model transformations that, on one hand, mitigate the impact of incorrect input factors and, on the other hand, result in a better fit between model results and observed soil losses. For the analysis, we revisit the original data set and consider the USLE factors as variables rather than their common interpretation as parameters. We first use both nonparametric and parametric techniques to test the robustness of the implicit parameter assignments in the USLE equation. Next, we postulate alternative mathematical forms and use parametric test statistics to evaluate parameter significance and model fit. A tenfold cross-validation of the model with the best fit tests the sensitivity of the parameters for inclusion or exclusion of the data. The results show that the

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USLE model is not very robust, however, only slight model improvements are obtained by drastic modifications of its functional form, thereby sacrificing the simple model structure that was intended by its designers.

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1. The USLE

1.1. The model

The USLE is the most widely used model for prediction of water erosion hazards and planning of soil conservation measures. It was adopted in 1958 by the Soil Conservation Service in the USA to make long-term assessments of soil losses under different cropping systems and land management practices. On the basis of a considerable experience with more than 10 000 plot years, 20 years later (Wischmeier and Smith, 1978), an updated equation was formulated which product form bears a resemblance to a Cobb–Douglas function with parameters of the value 1:

$$A = R \times K \times L \times S \times C \times P, \quad (1)$$

where A represents the soil loss, commonly expressed in tonnes $\text{ha}^{-1} \text{year}^{-1}$. R refers to the rainfall erosivity factor, calculated by the summation of the erosion index EI30 over the period of evaluation. EI30 is a compound function of the kinetic energy of a storm and its 30-min maximum intensity. The latter factor is defined as the greatest average rainfall intensity experienced in any 30-min period during a storm. It can be computed from automatic rain gauge charts by locating the greatest amount of rainfall in any 30-min period and then doubling this amount to get the same dimensions as normal rainfall intensity, i.e. rainfall per hour. K is the soil erodibility factor reflecting the susceptibility of a soil type to erosion. It is expressed as the average soil loss per unit of the R factor. L is an index of slope length, expressed as the ratio of the expected soil loss to that observed for a field of 22.6-m length. S is a slope gradient index, the ratio of the expected soil loss to that observed for a field of specified slope of 9%. C is an index for the protective coverage of canopy and organic material in direct contact with the ground. It is measured as the ratio of soil loss from land cropped under specific conditions to the corresponding loss from tilled land under clean-tilled continuous fallow conditions. Finally, the protective factor P represents the soil conservation operations or other measures that control the erosion, such as contour farming, terraces, and strip cropping. It is expressed as the ratio of soil loss with a specific support practice to the corresponding loss with up-and-downslope culture.

The simple structure of the USLE formula (Eq. (1)) makes it easy to formulate transparent policy scenarios by changing the land use types (C and P factors) under given ecological conditions (R , K , L , and S factors). This, together with the low data requirements compared with physical-based models, such as WEPP and EUROSEM, explains the

popularity of the USLE in small-scale water erosion studies at a continental (UNEP/RIVM/ISRIC, 1996b; Van der Knijff et al., 2000), nationwide (Van der Knijff et al., 1999; Schaub and Prasuhn, 1998; UNEP/RIVM/ISRIC, 1996a, 1997; Bissonnais et al., 1999), statewide (Hamlett et al., 1992), regional (Folley, 1998), and watershed level (Mellerowicz et al., 1994; Merzouk and Dhman, 1998; Young et al., 1987; Dostal and Vrana, 1998). The USLE is also popular in (nationwide) land evaluation studies where it is linked with rule-based procedures to determine the decrease in productivity (Kassam et al., 1991; Struif-Bontkes, 2001) or to estimate changes in nutrient balances (Smaling, 1993).

However, the USLE has some intrinsic model limitations which require attention. A nonparametric analysis of the original USLE data set (Keyzer and Sonneveld, 1998) reveals that data on higher soil losses is scarce and large errors can be expected for high rainfall data in combination with steep slopes and the lower and higher K values. This analysis also showed that observations are largely concentrated around the lower soil losses, where the model should give more reliable estimates. However, Nearing (1998) showed that small soil losses were consistently overestimated while the higher ones were underestimated. These results confirm earlier findings where the USLE model shows a rather poor statistical fit when used to explain annual soil losses ($R^2 = 0.57$) of the same data that were used for its calibration (Risse et al., 1993). Furthermore, the mathematical form of the USLE, the multiplication of six factors, leads to large errors whenever one of the factors is misspecified (Wischmeier, 1976). This raises questions about the model specification of the USLE and robustness of the implicit parameter assignments.

Therefore, an approach seems justified to investigate the impact of an equation transformation of the USLE model that, on one hand, could mitigate the impact of erroneous input factors and, on the other hand, result in a better fit between model results and observed soil losses.

1.2. Improving the USLE

Various attempts to improve the predictive capability of the USLE technology have been made in recent years, both along traditional and nontraditional avenues of inquiry. Along the traditional route, whereby we seek to improve the prediction capabilities of the model by focusing on better parameter estimations, the most extensive work is undoubtedly the Revised Universal Soil Loss Equation (RUSLE) (Renard et al., 1998). The changes from the USLE to the RUSLE generally fit into two categories: (a) incorporation of new or better data and (b) consideration of selected erosion processes. Recent data from the western United States was used to develop a new map for the rainfall erosivity factor, R . Obviously, this improvement and other similar improvements in the RUSLE that are based on new or better data will have impact only for regional or other applications for which the new data is relevant.

The incorporation of selected erosion processes into the RUSLE model has the potential for broader prediction improvements. Some of these improvements include functions for the seasonal variability in the soil erodibility factor, K ; slope length and steepness factors that are dependent on rill to interrill erosion ratios; inclusion of support practice, P , factors for subsurface drainage, rangelands, off-grade contouring, and strip-crop rotations; and the dependence of the contour P -factor on storm severity (Renard et al.,

1991, 1998). Perhaps the most extensive changes in the model are the inclusions of sub-factors in the cover-management factor, C , for the effects of prior land use, canopy cover, ground surface cover, surface roughness, and soil moisture. It is difficult to assess the improved function of the RUSLE as compared to the USLE because so many of the changes in the new model are targeted for specific applications. For example, improvements in the P -factor for subsurface drainage have significance only for the specific case of drained fields, and do not affect the application of the model elsewhere. The increased prediction capability of the RUSLE might best be stated in terms of the increase in the scope of application, rather than its increased prediction accuracy for cases in which the USLE was developed. It should also be noted that the RUSLE focuses on application within the United States, and adaptation and use elsewhere have as yet been limited. In one study using data that was collected on natural runoff plots located primarily in the eastern half of the United States, the RUSLE model did not outperform the USLE in its prediction accuracy (Tiwari et al., 2000).

Also, recently, two nontraditional approaches to improving the USLE have been undertaken. One involves the use of fuzzy logic-based modeling (Tran et al., 2002). In this case, a data set consisting of 1700 plot-years of information from 200 individual plots at 21 sites in the United States were used. These were data that was used, in part, to develop the USLE model. RUSLE parameters estimated from a previous study (Rapp, 1994) were listed with the measured soil loss data from each plot year. The data was divided into several subsets based on ranges of some of the RUSLE parameters, and then the measured soil loss data was related to the RUSLE parameters using the fuzzy logic approach. The results indicated that the new fuzzy logic-based model provided a better fit to the data than did RUSLE. Of course, in this case, the data used to develop the fuzzy logic-based model was the same as those used to evaluate the model. The same data was also used, in part, to develop the USLE that was precursor to the RUSLE. It is unclear if the relative rankings of the two models (fuzzy logic and RUSLE) that would remain the same were the models applied to other, independent data. Independent data from a significant number of locations is difficult to find.

A second nontraditional approach to improving the USLE was made by Licznar and Nearing (2003). This approach involved using artificial neural networks to predict soil loss from natural runoff plots. Data from 2879 erosion events from eight locations in the United States was used. In this case, neither USLE nor RUSLE parameters were considered, nor were the results directly compared the USLE or RUSLE results. However, the results were compared to the Water Erosion Prediction Project (WEPP) model (Flanagan and Nearing, 1995). We include a review of this work here because it used the same natural runoff plot data source as was used to develop the USLE and the fuzzy logic-based model discussed above, and it falls within the realm of essentially empirically based approaches to erosion modeling. Neural networks were created using commercially available software that related soil loss to 10 data elements: precipitation (mm), duration of precipitation (h), canopy cover (ND), interrill cover (ND), effective hydraulic conductivity (mm h^{-1}), adjusted interrill soil erodibility K_i (kg s m^{-4}) as computed by the WEPP model (Flanagan and Nearing, 1995), adjusted baseline rill erodibility K_r (kg s m^{-4}) as computed by the WEPP model (Flanagan and Nearing, 1995), number of days since last disturbance (day), slope steepness (m m^{-1}), and slope length (m). The neural network was

trained on a subset of the data, then applied to the remainder of the data set. Overall, the neural network performed as good as or better than the WEPP model in predicting soil loss.

The approach that we follow in this paper is to revisit the original USLE data set and interpret the individual factors as variables instead of their usual interpretation of parameters. This allows us to postulate alternative mathematical forms that accommodate the USLE factors to test if we can improve the model fit. We start the analysis by testing the robustness of the original mathematical form of the USLE using nonparametric reliability estimates and parametric test results. We will also use the flexible form of the nonparametric regression to evaluate the ‘best’ possible model fit with observed soil losses and compare these with the parametric model fits. Finally, for the model with the highest fit, we will test the sensitivity of the parameters for the inclusion or exclusion of the data by a tenfold cross-validation (Weiss and Kulowski, 1991). In this procedure, the data set is subdivided, at random, into 10 sets of about equal size. The model is estimated each time with nine subsets of the data, keeping the remaining 10% as an evaluation set.

This paper is organized as follows. In Section 2, we introduce the data set and the nonparametric and parametric technique that are used for the analysis. Section 3 presents the results and Section 4 concludes.

2. Data and methods

2.1. Data

We use a representative sample of the original data set to review relationships among observed soil loss, the model error, and explanatory variables. The data set was used by Risse et al. (1993) to evaluate model efficiency and comprises 1704 observations on annual soil loss, collected from 208 natural run-off plots at 22 experimental stations in the USA, during the period 1930–1980. Each variable of the USLE equation was determined on these sites.

2.2. Nonparametric estimation

The nonparametric estimation technique that is applied in this study yields a flexible form that evaluates the ‘best’ possible fit between observed soil losses and model results. Furthermore, we use statistics on the reliability of the estimate to evaluate the robustness of the assumed parameter values of the model. The nonparametric technique is effectuated by a kernel density regression (e.g. Bierens, 1987) according to the following stochastic model

$$\hat{y}(x) = \int y(x + \varepsilon)\psi(\varepsilon)d\varepsilon,$$

where y is the dependent variable (e.g. observed soil loss), x is a vector of explanatory variables and ε denotes measurements errors in x . The function $y(x + \varepsilon)$ is the unknown (erosion) function, and the regression takes the expected value of this function. For an infinite sample of observations spread evenly over the domain of x , it would be possible to

evaluate this expected value. However, with a finite sample of size S , the value of y can only be estimated, and for this, the kernel density regression uses the Nadaraya–Watson estimator:

$$\tilde{y}(x) = \sum_s P^s(x) y^s, \quad (2)$$

where P^s , at any point x with observations, indicates the probability of being the correct value of $\tilde{y}(x)$. This (Eq. (2)) is a probability weighted sample mean. The probabilities are computed on the basis of the distance of x^s from the given point x , attributing higher weight to nearby points. Weights are assigned using a postulated density function (the kernel) for ε whose spread is controlled by the window size parameter θ , which standard value is determined by Eq. (3),

$$\theta = \left(\frac{4}{n(d+2)} \right)^{\frac{1}{d+4}}, \quad (3)$$

with n the number of observations and d the number of exogenous variables (Silverman, 1986). The mollifier program supposes that all the elements of ε are independently and normally distributed. The mollifier also calculates several statistics that reflect the estimates' reliability. In this exercise, we use the partial derivative of the regression curve with respect to the explanatory variable x_k^t at data point x^t , as well as a measure of reliability for it. For this, it evaluates at every data point:

$$\frac{\partial \tilde{y}(x^t)}{\partial x_k^t} = \sum_s \frac{\partial P_s(x^t)}{\partial x_k^t} y^s. \quad (4)$$

From Eq. (4), we derive the probability of a wrong sign of the first derivative, the mathematical explanation of which is relegated to the Appendix A. A value of 0.5 indicates that on average, the slope information is uninformative, above 0.5 it has the wrong sign, and the more below 0.5, the more reliable the average slope.

2.3. Parametric techniques

The parametric technique used in this paper is the conventional regression techniques of ordinary least squares. We use t -scores to test robustness of assumed parameter values of the USLE factors and evaluate the correlation coefficient (R^2) between the observed soil losses and model results when alternative mathematical forms of the USLE are used.

3. Model robustness and modifications

3.1. Robustness

The robustness of the functional form is tested both in a nonparametric and a parametric way. The factor values of R , K , L , S , C , and P form here the independent variables and observed soil losses constitute the dependent variable.

The *nonparametric* test calculates the probability of a wrong sign of the first derivative of the individual parameters which provides a measure of significance of the associated variable with the dependent variable. Table 1 presents the probability of a wrong sign of the first derivative for the USLE factors. All factor values are close to 0.5, while the *P*-factor even exceeds this level, indicating a rather low association of the variables with soil loss.

However, there is no formal test statistic such as a χ^2 to relate this statistic to the measured one and hence the results have to be interpreted without a formal test. Therefore, we will also analyze the robustness of the USLE form with a parametric technique that allows for a more common and formal interpretation. The approach is to interpret the mathematical formula of the USLE as a Cobb–Douglas function with power coefficients of 1. We then evaluate the significance of added parameters α_i in Eq. (5) as:

$$\log A = \alpha_0 + \sum_{i=1}^5 (1 + \alpha_i) \log X_i, \quad (5)$$

where α_0 is a constant and α_i a parameter value that belongs to the vector X_i , which represents the USLE factors (*R*, *K*, *LS*, *C*, and *P*). The results of this estimation are given in Table 2.

All parameters are significant at the 1% level, except for the *K*-factor, which is highly insignificant. Parameter values of *K* and *LS* factors are almost zero indicating that the exponents are close to 1, approximating the assumed values in the USLE model. The estimated parameter values for *C*, *R*, and especially the *P*-factor, however, largely deviate from the assumed value of the power exponent and seem not to be very robust. Together with the results of the nonparametric analysis, this implies that possible model improvements can be obtained by estimating different parameter values for the individual USLE factors.

3.2. Improving the model fit

We start the exercise on model improvement by applying a nonparametric regression on the USLE factors and soil losses. The flexible form of the nonparametric model provides us with a ‘theoretical’ best fit between model results and observed soil losses. The R^2 of the nonparametric regression, again between factor values of *R*, *K*, *L*, *S*, *C*, and *P* as

Table 1
Results of the nonparametric regression on the USLE data set

Variable	Probability of wrong sign for 1st derivative
<i>R</i> -factor	0.47312
<i>LS</i> -factor	0.48187
<i>K</i> -factor	0.49746
<i>C</i> -factor	0.46322
<i>P</i> -factor	0.50082

Table 2
Results of the parametric regression on the USLE data set

Parameter (symbol Eq. (5))	Estimate
Constant (α_0)	0.3947
R (α_1)	0.1641*
K (α_2)	-0.0702*
LS (α_3)	0.0243*
C (α_4)	-0.1387*
P (α_5)	-0.6692**

* Parameter values were at the 0.0001 significance level.

** Parameter value at the 0.001 significance level.

independent variables and observed soil losses as dependent variable, shows a value of 0.76, which is, for the given window size, the best achievable fit between this model and the observed data that can be obtained. However, this nonparametric model has as disadvantage that while the model derived in this manner is highly flexible and elaborate, it only reflects theoretical restrictions through the choice of variables and not through properties of the functions, precisely because the moulding of the model into a particular shape is almost exclusively driven by the data. In the context of policy modeling, in this case on decisions of land management, this is considered undesirable and a parametric approach is therefore preferred.

We therefore aim at a better model fit by testing different analytical forms in which model calculations (A) and individual factor estimations (Xb) are taken as empirical data for independent variables, while observed soil losses (S) form the dependent variable. The selection of nonlinear deviations of individual factors is based on the results of the nonparametric analysis (Keyzer and Sonneveld, 1998). The functions and correlation coefficients are presented in Table 3. All parameter estimates were significant at the 0.01 level and the Cobb–Douglas function has the best fit. The parameter values of the variables (α_1 to α_5) are therefore similar as those in Table 2 minus the value one, while the value of the constant (α_0) remains the same. Analogous to the discussion of the results presented in Table 2, we note that most parameter values of the Cobb–Douglas function are close to unity approximating the original USLE exponents, while the intercept is near zero. The C and R components have a somewhat larger deviation, while the P -factor is an exception with a much lower value. The large deviation of the P -factor is most probably due to the small sizes of the run-off plot which are not suitable for the monitoring of soil

Table 3
USLE variables and assessments accommodated in mathematical forms and correlation coefficients

Name	Functional format	R^2
Linear	$S = a(A) + b$	0.5700
Quadratic	$S = a(A)^2 + b(A) + c$	0.5773
Polynomial	$S = a(A)^3 + b(A)^2 + c(A) + d$	0.5774
Cobb–Douglas	$\ln S = a + \sum_i b_i \ln X_i$	0.6035
Exponential I	$S = \exp(aA + b)$	0.5762
Exponential II	$S = \exp(a \sum_i b_i \ln X_i)$	0.4387
Mitscherlich–Baule	$S = a(1 - \exp(-\sum_i b_i X_i))$	0.5768

conservation practices. The R^2 of 0.6045 is only slightly higher than the USLE model fit ($R^2 = 0.57$).

3.3. Tenfold cross-validation

The estimated parameters of the Cobb–Douglas function are tested for their sensitivity to the inclusion or exclusion of observations by a tenfold cross-validation procedure. Fig. 1 presents the estimates.

All parameters retain their original sign, but vary in the deviations from their mean value. Parameters for rainfall erosivity, soil erodibility, and coverage factor exhibit minor fluctuations, and are highly significant throughout the estimation rounds. The topography index (*LS*-factor) is also highly significant but its value is less stable reporting maximum deviations from its mean value of 15%. The protection factor is most sensitive for the exclusion of the data and has a maximum of 100% deviation from its mean value and has the lowest significance of all parameters. The R^2 differs only with 0.01 or 0.02 from the 0.6035 value that was obtained for the entire data set. The results are sufficient to deduce that the parameters are stable except for the protection factor which shows a high sensitivity to the inclusion or exclusion of observations.

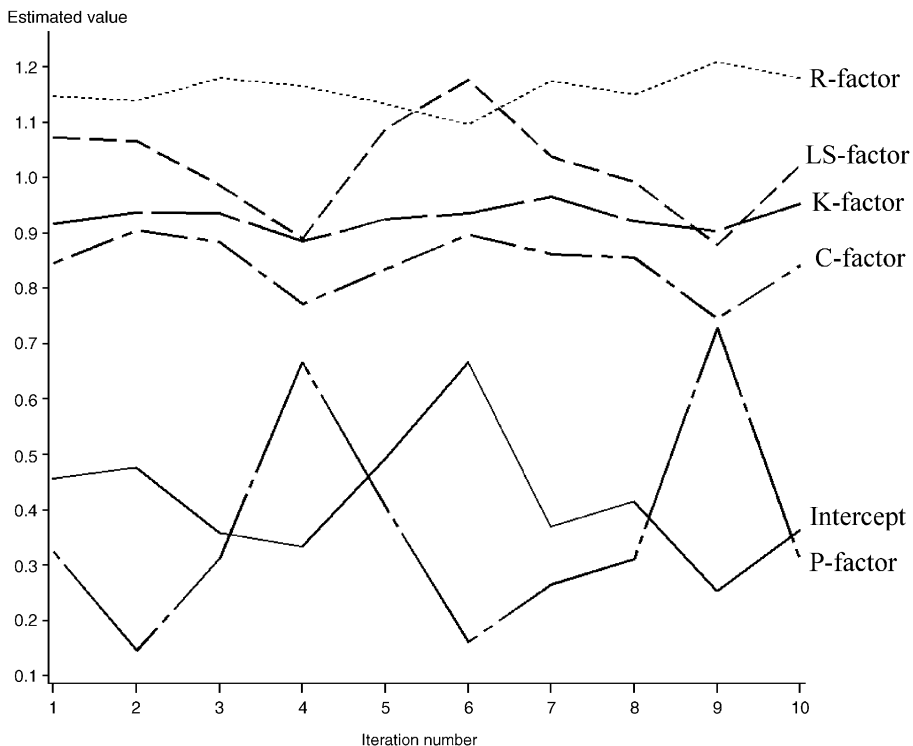


Fig. 1. Tenfold cross-validation.

We therefore conclude that, although the assumed parameter values of the parametric form of the USLE are not robust, postulating different parametric forms only slightly increases the robustness of the parameter values, but does not result in substantial improvements of the correlation with observed soil losses and goes at the expense of the simple USLE model structure.

4. Summary and conclusions

This paper investigates if the USLE could benefit from mathematical model transformations that, on one hand, mitigate the impact of incorrect input factors and, on the other hand, result in a better fit between model results and observed soil losses. For the analysis, the original USLE data set is revisited whereby we consider the individual USLE factors as variables rather than their common interpretation as parameters. We first use both nonparametric and parametric techniques to test the robustness of the implicit parameter assignments in the USLE equation. Next, we postulate alternative mathematical forms and use test statistics to evaluate parameter significance and model fit. A tenfold cross-validation of the model with the best fit evaluates the sensitivity of the parameters for inclusion or exclusion of the data.

Both the nonparametric and parametric results show that the parameter values of the USLE formula are not very robust. However, we could only obtain a slight model improvement in robustness of parameter estimates and model fit by modifying its functional form, thereby sacrificing the simple model structure that was intended by its designers (Wischmeier, 1976).

Further research should not aim at refinements of the functional format but, for example, concentrate on the inclusion of other explanatory variables like the IXEA index, proposed by Kinnell (1995). This dynamic index combines the summation of excess rainfall and rainfall kinetic energy and enhances the fit, especially when soils have high infiltration capacities.

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Appendix A. Statistical reliability of mollifier estimates

The mollifier program assesses the partial derivative of the regression curve as well as a measure of its reliability. For this, it calculates the first partial derivative of

$$y_{\theta}^s(x) = \sum_s y^s P_{\theta}^s(x) \quad (\text{A1})$$

to x_k at point x , where k represents an explanatory variable, at all data points x^t .

$$\frac{\partial \tilde{y}(x^t)}{\partial x_k} = \sum_s \frac{\partial P_\theta^s(x^t)}{\partial x_k} y^s. \quad (\text{A2})$$

Since $\sum_s \frac{\partial P_\theta^s(x^t)}{\partial x_k} = 0$, we can write

$$\frac{\partial \tilde{y}(x^t)}{\partial x_k} = \sum_s \frac{\partial P_\theta^s(x^t)}{\partial x_k} (y^s - y^t), \quad (\text{A3})$$

where y^t refers to the t th observation. As by definition, $\frac{\partial P_\theta^s(x^t)}{\partial x_k} = P_\theta^s \frac{\partial \log P_\theta^s(x^t)}{\partial x_k}$, and it follows that

$$\frac{\partial \tilde{y}(x^t)}{\partial x_k} = \sum_s P_\theta^s(x^t) \left[\frac{\partial \log P_\theta^s(x^t)}{\partial x_k} (y^s - y^t) \right]. \quad (\text{A4})$$

Let us now rewrite and interpret the term in square brackets.

$$\frac{\partial \log P_\theta^s(x^t)}{\partial x_k} = \frac{\partial \log \psi_s(x^t)}{\partial x_k} - \sum_{h=1}^S P_\theta^h(x^t) \frac{\partial \log \psi_h(x^t)}{\partial x_k} \quad (\text{A5})$$

Now, for a density $\psi_s(x^t) = \psi((x^s - x^t)/\theta)$ where ψ is a normal joint density with diagonal variance matrix and variance σ_k^2 around x^t , it follows that

$$\frac{\partial \log \psi_s(x^t)}{\partial x_k} = \frac{x_k^s - x_k^t}{\sigma_k^2}. \quad (\text{A6})$$

Hence, the term in square brackets can be rewritten as

$$\frac{\partial \tilde{y}(x^t)}{\partial x_k} = \sum_s P_\theta^s(x^t) [\zeta_k^s \delta_k^s], \quad (\text{A7})$$

where $\zeta_k^s = \frac{x_k^s - x_k^t}{\sigma_k^2} - \sum_h P_\theta^h(x^t) \frac{(x_k^h - x_k^t)}{\sigma_k^2}$ and $\delta_k^s = (y^s - y^t)$.

In other words, the term in square brackets is the contribution of observation s to the slope.

For given x^t , this enables us to define the probability of a positive sign for the slope as

$$P_k^+(x^t) = \sum_s P_\theta^s(x^t | \zeta_k^s \delta_k^s \geq 0). \quad (\text{A8})$$

Hence, the probability of a wrong sign can be calculated as

$$P_k^\#(x^t) = P_k^+(x^t), \text{ if } \frac{\partial \tilde{y}(x^t)}{\partial x_k} < 0, \text{ and } 1 - P_k^+(x^t), \text{ if } \frac{\partial \tilde{y}(x^t)}{\partial x_k} \geq 0. \quad (\text{A9})$$

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