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Ching Y. Loh  
Taitech, Inc., Cleveland, Ohio

Sin-Chung Chang  
Glenn Research Center, Cleveland, Ohio

Xiao Y. Wang  
Taitech, Inc., Cleveland, Ohio

Philip C.E. Jorgenson  
Glenn Research Center, Cleveland, Ohio

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**Ching Y. Loh**  
Taitech, Inc.  
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**Xiao Y. Wang**  
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**Sin-Chung Chang**  
National Aeronautics and Space Administration  
Glenn Research Center  
Cleveland, Ohio 44135

**Philip C. E. Jorgenson**  
National Aeronautics and Space Administration  
Glenn Research Center  
Cleveland, Ohio 44135

## Abstract

A typical gap noise problem is considered in this paper using the new space-time conservation element and solution element (CE/SE) method. Implementation of the computation is straightforward. No turbulence model, LES (large eddy simulation) or a preset boundary layer profile is used, yet the computed frequency agrees well with the experimental one.

## 1 Introduction

Gap noise is an important issue in aerospace and auto industries. For example, modern automobiles are required to be comfortable and quiet, however, due to vortex shedding in high speed air flow over door gaps, unpleasant tonal noise may be generated. Strong oscillations occur in a feedback cycle in which the vortices shed from the upstream edge of the gap convect downstream and impinge on the other edge, generating acoustic waves that propagate upstream to excite new vortices. Numerical simulation of such a complicated process requires a scheme that can : (a) resolve acoustic waves with low dispersion and numerical dissipation, (b) handle nonlinear and discontinuous waves (e.g. shocks), and (c) have an effective (near field) non-reflecting boundary condition (NRBC). The new space time conservation element and solution element method, or CE/SE for short, is a numerical method that meets the above requirements [1-4]. A detailed description of the 2-D CE/SE Euler scheme can be found in [1,2], only a brief sketch is given here.

In nature, the new method may be categorized as a finite volume method, where the conservation element (CE) is equivalent to a finite control volume (or cell) and the solution element (SE) can be understood as the cell interface. However, due to its rigorous treatment of the fluxes and geometry, it is different from the existing schemes. As demonstrated in the previous papers [4-7], the CE/SE scheme features:

(1) space and time treated on the same footing, the integral equations of conservation laws are solved for with second order accuracy,

(2) high resolution, low dispersion and low dissipation,

(3) novel, truly multi-dimensional, simple but effective non-reflecting boundary condition,

(4) Straight forward implementation of computation, no numerical fix or parameter choice is needed;

(5) robust enough to cover a wide spectrum of compressible flow: from weak linear acoustic waves to strong, discontinuous waves (shocks), appropriate for linear and non-linear aeroacoustics.

Currently, the CE/SE scheme has been developed to such a stage that a 3-D unstructured CE/SE Navier-Stokes solver is already available. However, in the present paper, as a general introduction to the CE/SE method, only the 2-D unstructured Euler CE/SE solver is chosen as a prototype and is sketched in Section 2. Then application of the CE/SE scheme to the auto-door gap noise problem is depicted in Sections 3.

## 2 The 2-D Unstructured CE/SE Scheme

The CE/SE scheme can be used with either structured or unstructured grids. Here, the unstructured CE/SE is used as a prototype for introductory description.

### 2.1 Conservation Form of the Unsteady Euler Equations

Consider a dimensionless conservation form of the unsteady Euler equations of a perfect gas. Let  $\rho$ ,  $u$ ,  $v$ ,  $p$ , and  $\gamma$  be the density, streamwise velocity component, transversal velocity component, static pressure, and constant specific heat ratio, respectively. The 2-D Euler equations then can be written in the following vector form:

$$\mathbf{U}_t + \mathbf{F}_x + \mathbf{G}_y = \mathbf{0}, \quad (1)$$

where  $x$ ,  $y$ , and  $t$  are the streamwise and transversal coordinates and time, respectively. The conservative flow

variable vector  $\mathbf{U}$  and the flux vectors  $\mathbf{F}$  and  $\mathbf{G}$ , are given by:

$$\mathbf{U} = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{pmatrix},$$

with

$$U_1 = \rho, U_2 = \rho u, U_3 = \rho v, U_4 = p/(\gamma-1) + \rho(u^2 + v^2)/2;$$

$$F_1 = U_2, \quad F_3 = U_2 U_3 / U_1,$$

$$F_2 = (\gamma - 1)U_4 + [(3 - \gamma)U_2^2 - (\gamma - 1)U_3^2] / 2U_1,$$

$$F_4 = \gamma U_2 U_4 / U_1 - (\gamma - 1)U_2 [U_2^2 + U_3^2] / 2U_1^2,$$

$$G_1 = U_3, \quad G_2 = U_2 U_3 / U_1,$$

$$G_3 = (\gamma - 1)U_4 + [(3 - \gamma)U_2^2 - (\gamma - 1)U_3^2] / 2U_1,$$

$$G_4 = \gamma U_3 U_4 / U_1 - (\gamma - 1)U_3 [U_2^2 + U_3^2] / 2U_1^2.$$

By considering  $(x, y, t)$  as coordinates of a three-dimensional Euclidean space  $E_3$  and using Gauss' divergence theorem, it follows that Eq. (1) is equivalent to the following integral equation:

$$\oint_{S(V)} \mathbf{H}_m \cdot d\mathbf{S} = 0, \quad m = 1, 2, 3, 4, \quad (2)$$

where  $S(V)$  denotes the surface around a volume  $V$  in  $E_3$  and  $\mathbf{H}_m = (F_m, G_m, U_m)$ ,  $m = 1, 2, 3, 4$ .

## 2.2 Unstructured CE/SE

The CE/SE scheme is naturally adapted to unstructured triangle grids. Each triangle center ( $O$  in  $\triangle ABC$ ) and its 3 neighboring triangle centers ( $D, E, F$ ) form 3 conservation elements or CEs (quadrilateral cylinders -  $ADBO, BECO$  and  $CFAO$ ), as shown in Fig. 1. These triangle centers are the nodes where the unknowns  $\mathbf{U}$ ,  $\mathbf{U}_x$ ,  $\mathbf{U}_y$  are defined.

Assume that at the previous time level  $n$  (Fig. 1),  $\mathbf{U}$ , and its spatial derivatives  $\mathbf{U}_x$ ,  $\mathbf{U}_y$  are given at all the nodes (triangle centers), the CE/SE time marching is based on a 'tripod' mode, i.e.,  $\mathbf{U}$ ,  $\mathbf{U}_x$ ,  $\mathbf{U}_y$  at the new time level  $n + 1$  (e.g., the shaded circle  $O'$  in Fig. 1) are computed from their data at its surrounding neighboring nodes at time level  $n$  (e.g.,  $D$  in Fig. 1, note that  $O$  itself is not used).

During the time marching, the above flux conservation relation (2) in space-time is the *only* mechanism that transfers information between node points. A conservation element CE, or computational cell, is the finite volume to which (2) is applied. Discontinuities are allowed to occur in the interior of a conservation element and nowhere else. A solution element SE associated with

a node (e.g.,  $D, E, F$  in Fig. 1) is here a set of interface planes in  $E_3$  that passes through this node (e.g. planes  $DAA'D', DBB'D', DBOA$  associated with node  $D$ ). Within a given solution element  $SE(j, n)$ , where  $j, n$  are the node index, and time step respectively, the flow variables are not only considered continuous but are also approximated by linear Taylor expansions:

$$\mathbf{U}^*(x, y, t; j, n) = \mathbf{U}_j^n + (\mathbf{U}_x)_j^n (x - x_j) +$$

$$(\mathbf{U}_y)_j^n (y - y_j) + (\mathbf{U}_t)_j^n (t - t^n), \quad (3)$$

$$\mathbf{F}^*(x, y, t; j, n) = \mathbf{F}_j^n + (\mathbf{F}_x)_j^n (x - x_j) +$$

$$(\mathbf{F}_y)_j^n (y - y_j) + (\mathbf{F}_t)_j^n (t - t^n), \quad (4)$$

$$\mathbf{G}^*(x, y, t; j, n) = \mathbf{G}_j^n + (\mathbf{G}_x)_j^n (x - x_j) +$$

$$(\mathbf{G}_y)_j^n (y - y_j) + (\mathbf{G}_t)_j^n (t - t^n), \quad (5)$$

where  $j$  is the node index of  $D, E$  or  $F$ , the partial derivatives of  $\mathbf{F}$  and  $\mathbf{G}$  can be related to the corresponding one of  $\mathbf{U}$  by using the chain rule and  $\mathbf{U}_t$  can be obtained from (1). These Taylor expansions are used to accurately evaluating fluxes on  $SE$ 's e.g., on planes  $DAA'D', DBB'D'$  or  $DBOA$ .

In principle, the number of equations derived from these flux conservation laws (each CE provides 4 scalar equations and the total number of scalar equations is 12) matches the number of unknowns (here 12 scalar unknowns). All the unknowns are solved for based on these relations. No extrapolations (interpolations) across a stencil of cells are needed or allowed.

In practice, there is no need to solve a  $12 \times 12$  equation system. In the space-time  $E_3$  space, (2) is applied to the hexagon cylinder  $ADBECF - A'D'B'E'C'F'$ .  $\mathbf{U}$  at the hexagon center at the new time level is first evaluated and then,  $\mathbf{U}$  at the center  $O'$  of triangle  $A'B'C'$  is obtained by Taylor expansion. Since the evaluation of  $\mathbf{U}_x$  and  $\mathbf{U}_y$  involves application of artificial damping and some form of limiters (weighted average), the reader is referred to the original papers [1-3] of Chang *et al* for details.

## 2.3 Non-Reflecting Boundary Conditions

In the CE/SE scheme, non-reflecting boundary conditions (NRBC) are constructed so as to allow fluxes from the interior domain to smoothly exit to the exterior of the domain. Some variants of the NRBC frequently employed in CE/SE schemes are:

(1) Type I - 'steady NRBC':

For a ghost grid node  $(j, n)$  lying outside the domain at the top (or bottom) of the domain the NRBC requires that

$$(\mathbf{U}_x)_j^n = (\mathbf{U}_y)_j^n = 0,$$

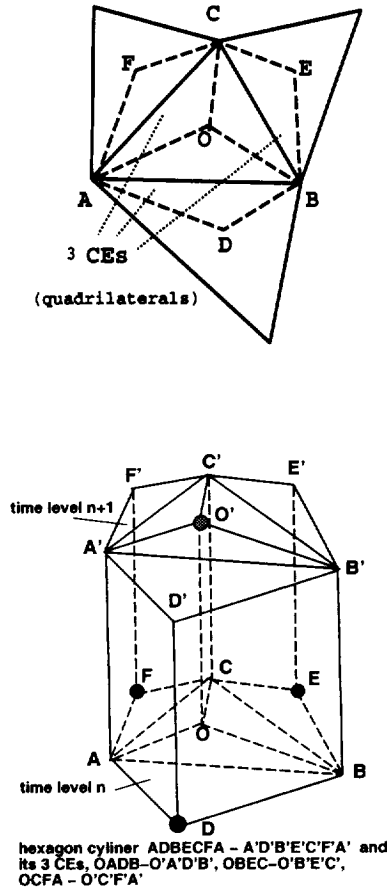


Figure 1: CE/SE unstructured grid, showing 3 CEs and the hexagon cylinder in  $E_3$ .

while  $U_j^n$  is kept fixed at the initially given steady boundary value.

(2) Type II - 'outflow NRBC':

At the downstream boundary, where there are substantial gradients in  $y$  direction, the NRBC requires that

$$(U_x)_j^n = 0, \quad U_j^n = U_{j'}^{n-1/2}, \quad (U_y)_j^n = (U_y)_{j'}^{n-1/2},$$

where  $j'$  is the index of an interior node closest to the boundary ghost node  $j$  and  $U_j^n$  and  $(U_y)_j^n$  are now defined by simple extrapolation from the interior. This NRBC is valid for either supersonic or subsonic flows. It should be noted that although these NRBC's bear similarity to those used in finite difference schemes, the role they play is very different. It can be shown that the above NRBC's allow fluxes to smoothly exit to the exterior of the domain.

### 3 Numerical results

Figure 2 illustrates the geometric configuration of the door gap noise problem, which represents a typical aeroacoustic feedback system, producing sustainable

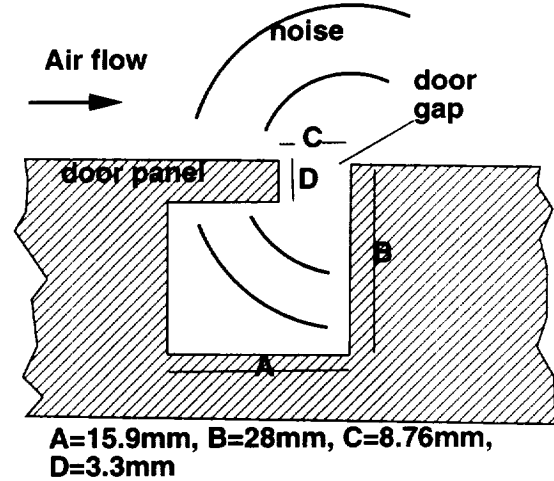


Figure 2: auto-door gap problem and the gap dimensions.

feedback cycle oscillations as a result of vortex shedding. The problem is given as a benchmark problem at the 3rd CAA (Computational Aeroacoustics) Workshop (NASA Glenn, Cleveland, OH, November, 1999) and the corresponding experimental data will be released later. The actual dimensions of the auto-door gap cavity are also prescribed in Fig. 2. The length scale is chosen to be twice the dimension  $C$  in Fig. 2, and the speed of sound and density of the ambient flow are respectively the scales for velocity and density. In the current computation, there are 48184 triangle elements for the unstructured mesh in the computational domain. These triangles are actually obtained by dividing a rectangular structured mesh cell into 4 pieces. The rectangular cell keeps a uniform size of  $\Delta x = 0.00625$  and  $\Delta y = 0.01136$  around the area of the gap and the interior of the cavity, but grows larger near the outer boundaries. At the gap and cavity solid walls, no slip boundary conditions are applied.

#### 3.1 High speed case ( $v = 50.9m/s$ )

The mean flow follows the  $x$ -direction with Mach number  $M = 0.1497$ , corresponding to the car speed of  $50.9m/s$ . Initially, ambient flow condition is imposed on the entire flow field. At the inflow, the  $M = 0.1497$  flow is imposed. At the top and bottom the Type I NRBC is used while at the outflow, the Type II NRBC is specified. The domain shown in Fig. 3 is exactly the computational domain, no buffer zone is used but still, the CE/SE NRBC works well. Fig. 3 is taken at time steps 4000, 14000, and 24000 (with  $\Delta t = 0.00125$ ), it shows isobar snapshots of the computed flow field at different stages. The generation of vortices and the generation of

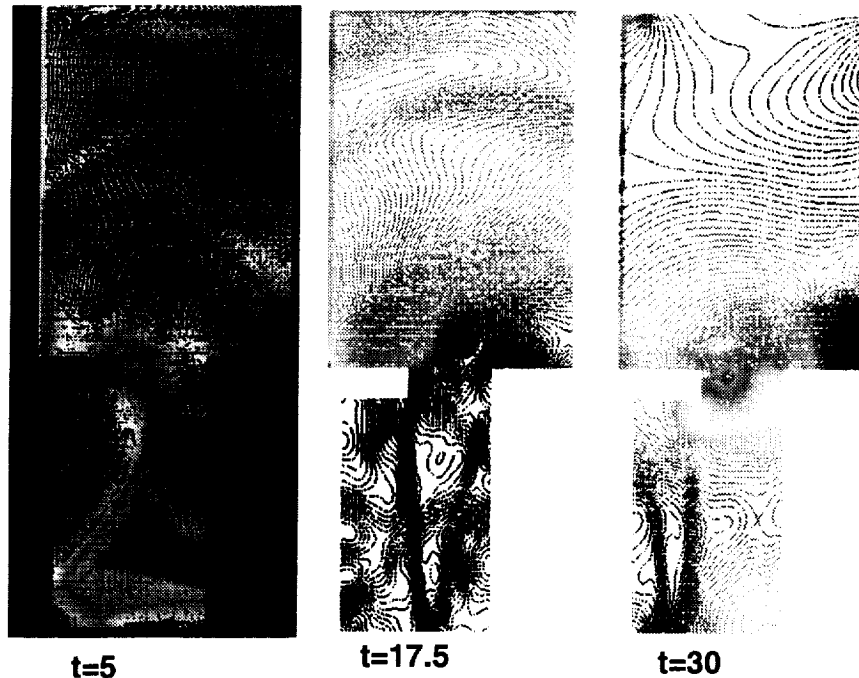


Figure 3: Isobar snapshots at different time steps.

(nonlinear) acoustic waves by vortex-gap edge impingement, both inside and outside of the cavity, are clearly displayed.

The computation is totally carried out for 225,000 time steps. Starting from time step 60,000, when the unsteady flow is considered as fully developed, the pressure history at the mid-point of the left wall of the cavity is recorded and provided later for FFT (fast Fourier transform) analysis.

Fig. 4 displays the PSD (power spectrum density) in log. scale for the time series. The  $x$ -axis denotes the reduced frequency – the Strouhal number  $St$ .  $St = 1$  is equivalent to a frequency of 19,406 Hz. It is observed that there are several different tone spikes extending to very high frequencies. The frequency at the lowest tone is about 1,839 Hz. Despite the coarse resolution in frequency domain, this agrees well with the experimental frequency data 1,824 Hz provided by the 3rd CAA Workshop [8].

Fig. 5 is an enlargement of the numerical Schlierens around the cavity at time step 225,000. Eddies are clearly displayed.

### 3.2 Low speed case ( $v = 26.8m/s$ )

In this case, all the conditions are the same as above except the mean flow Mach number is lower,  $M = .0788$ ,

corresponding to the car speed of  $v = 26.8m/s$ . For running at such a low Mach number, most of the numerical schemes need some form of preconditioning. However, this doesn't seem to bother the CE/SE scheme. Totally 390,000 time steps are run for this case and the PSD is shown in Fig. 6. The strongest (also the lowest) tone frequency is estimated to be about 1,911 Hz. Since this frequency is not far away from 1,824 Hz, while the car speed is almost reduced by half, it is guessed that the tone may come from the Helmholtz resonant frequency of the cavity.

## 4 Concluding Remarks

A typical vortex induced, self-excited tonal gap noise is simulated and compared to the experimental data. Through the 'difficult' problem, the capability of the new CE/SE scheme is demonstrated. Specifically,

1. the novel non-reflecting boundary conditions based on flux balance is simple, genuinely multi-dimensional, and easy to implement, it is effective even for near field boundaries;
2. the (2nd order) CE/SE scheme is robust, efficient and yields high resolution, low dispersion results similar to those of higher-order schemes.

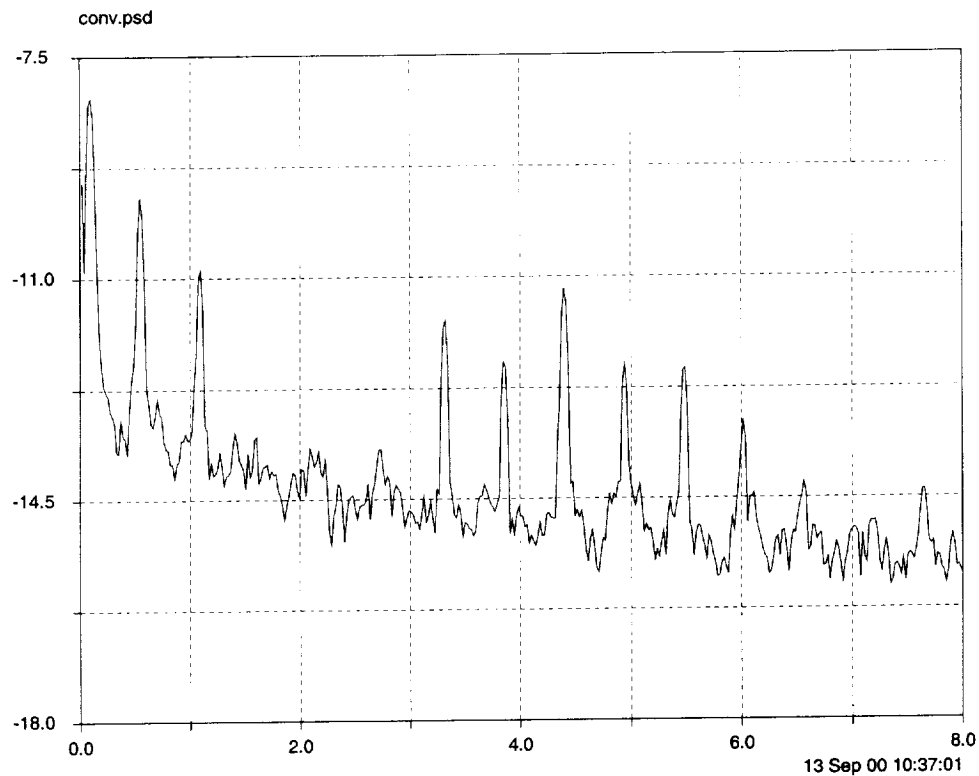


Figure 4: PSD at a node inside the door cavity; x-axis: Strouhal number, high speed case.

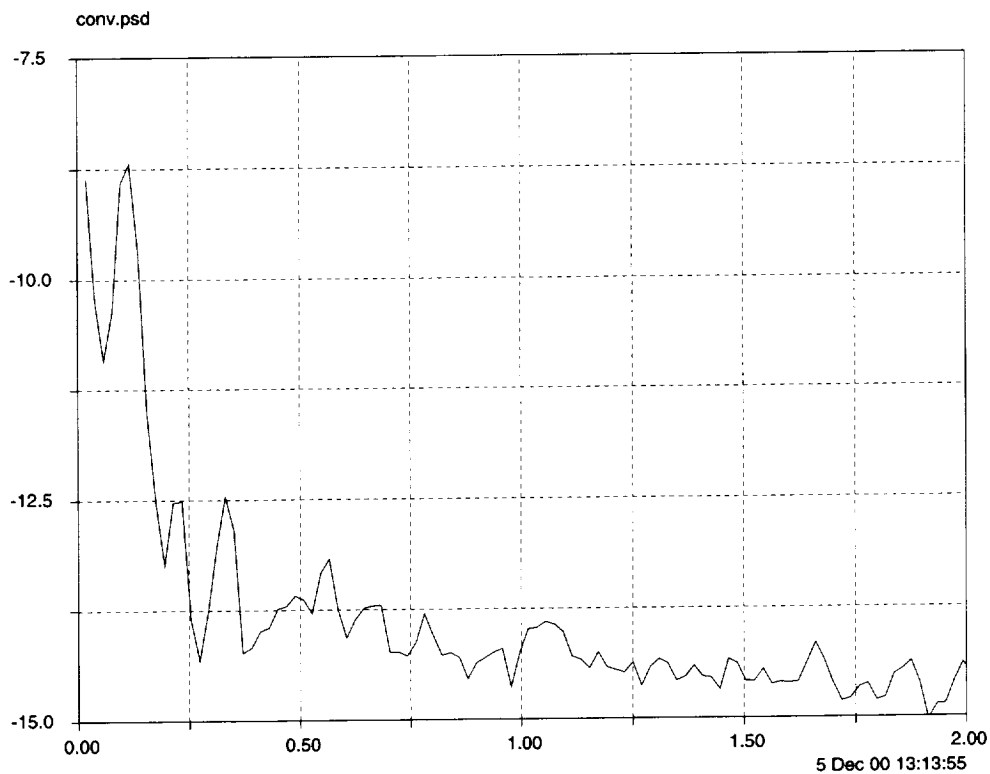


Figure 6: PSD at a node inside the door cavity; x-axis: Strouhal number, low speed case.

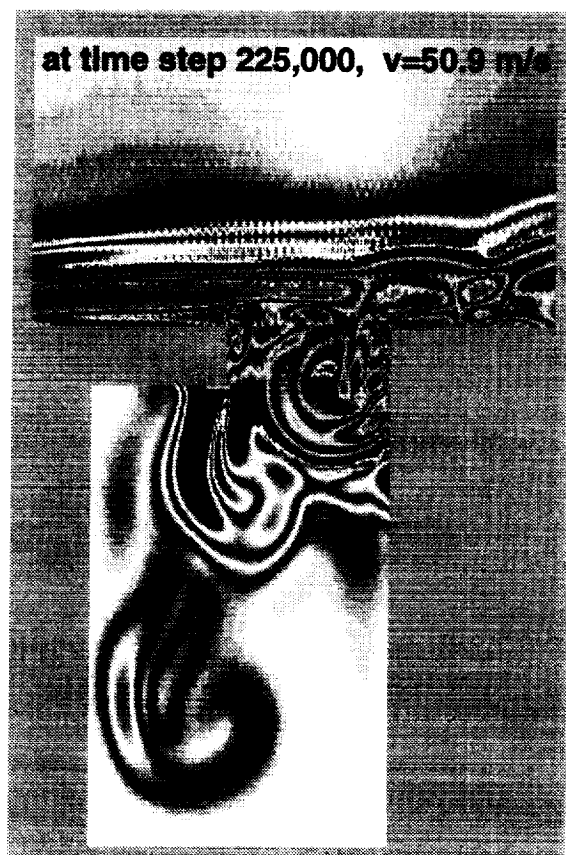


Figure 5: Numerical Schlierens in the cavity area.

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