NASA Technical Memorandum 102515

Composite Laminate Tailoring With Probabilistic Constraints and Loads

P.B. Thanedar and C.C. Chamis
Lewis Research Center
Cleveland, Ohio

Prepared for the
Energy Technology Conference and Exhibition
sponsored by the American Society of Mechanical Engineers
New Orleans, Louisiana, January 14–18, 1990

NASA

(NASA-TM-102515) COMPOSITE LAMINATE TAILORING WITH PROBABILISTIC CONSTRAINTS AND LOADS (NASA) 18 p CSCL 11D

N90-20138

Unclas
G3/24 0271799

COMPOSITE LAMINATE TAILORING WITH PROBABILISTIC STRENGTH CONSTRAINTS AND LOADS

P.B. Thanedar* and C.C. Chamis†
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

E-5182

SUMMARY

In this study, a reliability-based structural synthesis procedure has been developed to tailor laminates to meet reliability-based (ply) strength requirements and achieve desirable laminate responses. The main thrust of the paper is to demonstrate how to integrate the optimization technique in the composite laminate tailoring process to meet reliability design requirements. The question of reliability arises in fiber composite analysis and design because of the inherent scatter that is observed in the constituent (fiber and matrix) material properties during experimentation. Symmetric and asymmetric composite laminates subject to mechanical loadings are considered as application examples. These application examples illustrate the effectiveness and ease with which reliability considerations can be integrated in the design optimization model for composite laminate tailoring.

NOMENCLATURE

[A]	(3 x 3) composite axial stiffness submatrix
B	design variable vector; $(b_1, b_2, \ldots, b_k)$
[C]	(3 x 3) composite coupling stiffness submatrix
[D]	(3 x 3) composite flexural rigidity submatrix
ϕ	ply failure criterion via modified distortion energy
E	Young's modulus
ε	strain vector; $(\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy})$
F	single, real valued objective function to be minimized
G	shear modulus
M	applied moment vector; (M_{xx}, M_{yy}, M_{xy})
N	applied membrane load vector; (N_{xx}, N_{yy}, N_{xy})
S	strengths
t_i	thickness of the i^{th} ply

---

*National Research Council - NASA Research Associate at Lewis Research Center.

†Senior Aerospace Scientist, Associate Fellow AIAA.

χ curvature vector; $(\chi_{xx}, \chi_{yy}, \chi_{xy})$

θ_i ply orientation angle for the i^{th} ply

ρ_i weight density of the i^{th} ply

Subscripts:

f,m,c,l fiber, matrix, composite and ply

k volume ratio

L,U lower and upper bound, respectively

T,C tension, compression

11 direction along the fiber

22,33 directions transverse to the fiber

INTRODUCTION

There is a great potential for structural tailoring of high performance, lightweight structures made up of fiber composites. This is because of the great range of desirable properties of fiber composites and their tailoring capacity to individual design requirements (refs. 1 and 2). Structural tailoring is a powerful concept in that it provides the formalism to configure laminates with several significant advantages when compared to conventional isotropic materials.

Principal among these is the ability to tailor ply properties (material selection) and fiber orientation (configuration selection) to the given mechanical, thermal and hygral loads. Thus, a laminate is a building block that contributes to the overall thermostructural action of a component made out of composites. Hence, it is only logical to concentrate on tailoring of laminate properties and configuration as a first step in the synthesis of composite structures in order to meet reliability design requirements.

The idea of applying optimization techniques to automate the structural tailoring of composite components such as engine blades with the aid of computers is not new. The structural tailoring of engine blades (STAEBL) and structural tailoring of advanced turboprops (STAT) computer programs have been developed and routinely used at NASA Lewis Research Center (refs. 3 and 4). Almost all of the existing structural tailoring software is based on a deterministic approach. Thus it is not possible to directly account for the inherent composites properties scatter, in the currently available tailoring software. To be realistic, uncertainties in the composite material properties must be reflected in the analysis and design methods for structural and mechanical systems. Chamis (ref. 5) has underscored the importance and usefulness of probabilistic approach in the analysis of space propulsion systems to improve the reliability of engine components.

In general, the input data prone to uncertainties during structural analysis include material properties, boundary conditions, loading conditions and geometry of the system. In the present research, we shall focus our attention to uncertainties in the material properties and how to incorporate them in the laminate tailoring process. The most common and traditional approach to account for uncertainties in the design process is to introduce a factor of safety. But this approach does not quantify the reliability of the system design. Recently, probabilistic concepts in the form of Monte Carlo simulation have been applied to quantify the uncertainties in composite micromechanics by Chamis and coworkers (ref. 6). Such probabilistic composite micromechanics concepts suggest a viable approach towards integrating reliability considerations in laminate tailoring in the form of behavioral constraints and/or probabilistic load conditions as a first step.

The main objective of this research effort is to develop a tailoring (synthesis/optimization) procedure by combining a robust optimizer with composite mechanics and probabilistic constraints and/or loads. It is widely recognized that at the optimum design, one or more performance constraints are at their allowable values and any uncertainties in the given input data may for these constraints lead to an unsafe design. This is because the active performance constraints may become violated with variations in the input data. The procedure described herein, is applied to several generic sample cases to illustrate that fiber composite laminates can be tailored to meet design requirements with assured probability of survival.

PROBABILISTIC COMPOSITE MICROMECHANICS

The branch of composite mechanics that predicts ply material properties based on the constitutent properties, volume fraction and fiber orientation is known as composite micromechanics, and frequently incorporates the traditional mechanics of materials assumptions. As mentioned in the introduction, Chamis and coworkers (ref. 6) have developed a probabilistic approach to composite micromechanics using Monte Carlo simulation. Thus, scatter in the primitive variables namely constituent (fiber and matrix) elastic properties, strengths, and fiber volume ratio have been quantified at the ply level properties. A straightforward way of developing probabilistic analysis is to perform Monte Carlo simulation using deterministic composite micromechanics equations. Thus, fiber volume ratio, fiber and matrix elastic properties and strengths are considered to be independent random variables. The scatter in these independent random variables is quantified by defining the type of probability distribution function (PDF) e.g., normal, Weibull etc. and the related statistical quantities such as the mean and the coefficient of variation etc. The available experimental data usually provide the estimation of the mean value and the coefficient of variation.

The ply level properties such as moduli (longitudinal, transverse and shear), strengths (longitudinal tensile/compressive, transverse tensile/compressive and intralaminar shear) are considered to be dependent random variables. The results of the probabilistic analysis are presented in terms of (1) histogram or PDF, and (2) cumulative distribution function (CDF) of the ply level properties. Typical PDF and CDF curves for the longitudinal tensile strength ($S_{\ell 11T}$) of a carbon graphite fiber (AS)/intermediate modulus high strength (IMHS) epoxy matrix composite system ply (ref. 6) are shown in figure 1. Note that one can readily assess the probability of a random sample to be higher than a given value for $S_{\ell 11T}$ from the CDF curve. In this study, the longitudinal tensile/compressive, transverse tensile/compressive and intralaminar shear strengths of an AS/IMHS ply are assumed to be random variables and their probability distribution function curves for case 2 from reference 6 have been utilized. Table I gives the deterministic mean values of the strengths ($S_{\ell 11T}$, $S_{\ell 11C}$, $S_{\ell 22T}$, $S_{\ell 22C}$, $S_{\ell 12S}$) for a unidirectional AS/IMHS ply along with the strengths corresponding to 90 percent cumulative probability.

STRUCTURAL TAILORING SOFTWARE

The computer program IDESIGN (Interactive Design Optimization of Engineering Systems) is an interactive general purpose optimization software suitable for tailoring of laminate properties (ref. 7). The IDESIGN program has been widely used in static structural, dynamic, distributed parameter, shape optimization and numerous other application problems (refs. 8 and 9). In summary, IDESIGN program solves the following nonlinear programming problem (NLP) with an objective function (F) to be minimized subject to the equality and inequality constraints. Thus, the design optimization model, in the form of an NLP problem, is to find a k-dimensional design variable vector, **B** to:

$$\text{Minimize } F(\mathbf{B}) \text{ such that}$$

$$G(\mathbf{B}) \leq 0 \tag{1}$$

$$\mathbf{B}_L \leq \mathbf{B} \leq \mathbf{B}_U$$

Note that in the above NLP problem defined by equation (1), it is assumed that the functions F and G are continuous and differentiable. The design variables (**B**) are also continuous real variables. The Sequential Quadratic Programming (SQP) method is chosen as the optimization algorithm (refs. 10 and 12). The numerical data such as starting values and upper/lower bounds on the design variables, number of design variables and constraint , tolerances on constraint violation and convergence criterion are provided to the IDESIGN program through an input data file. The gradients of the objective and constraint functions are calculated by a forward finite difference scheme in the IDESIGN program. In every design iteration of the SQP method, the following quadratic programming (QP) subproblem is solved to get the search direction **d** at a design point **B**,

$$\text{Minimize } \{F(\mathbf{B}) + \nabla F T\mathbf{d} + 0.5\, \mathbf{d}^T H \mathbf{d}\}$$

$$\text{such that } G(\mathbf{B}) + \nabla G^T, \mathbf{d} \leq 0 \tag{2}$$

where $\nabla \mathbf{F}$ and $\nabla \mathbf{G}$ are gradient vectors of the objective and constraint functions; B_F is the approximation to the Hessian of the Lagrange function. The details about the QP subproblem and its solution can be found in Ref. 9. A suitable step size (α) needs to be found along the search direction **d** so that progress can be made towards the optimum solution. Note that the search direction (**d**) is a vector whereas the step size (α) is a scalar. Thus, the change in design is given as $\alpha \cdot \mathbf{d}$ and the recursive formula for the next design point is given as,

$$\mathbf{B}^{new} = \mathbf{B}^{old} + \alpha \cdot \mathbf{d} \quad (3)$$

The SQP method converges when the residue, $||\mathbf{B}^{new} - \mathbf{B}^{old}||$ becomes very small. Overall, IDESIGN software is interactive, user friendly and modular. Any application/analysis program can be easily coupled with the IDESIGN system.

The Integrated Composite Analyzer (ICAN), a stand-alone computer code, has been used to analyze and design multilayered fiber composite laminates using micromechanics equations and laminate theory (ref. 13). Input parameters of this user-friendly program include material system, fiber volume ratio, laminate configuration, fabrication factors, and environmental conditions. Output features include the practically most of the composite hygral, thermal and mechanical properties that are required to perform structural/stress analyses in service environments. As such, ICAN may be utilized as an effective tool for the preliminary design of composite structures (ref. 14) as shown in figure 2. In addition, ICAN has a resident data bank which houses the properties of a variety of constituent (fiber and matrix) materials with provisions to add new constituent materials as they become available. Note that ICAN program is converted into a subroutine for the IDESIGN software to perform synthesis of laminate properties. Thus, the ICAN code is utilized to evaluate the objective and constraint functions in equation (1). The overall flowchart for the computations performed in the proposed structural tailoring program is given in figure 3.

LAMINATE CONFIGURATION

An eight-ply laminate configuration made up of AS-IMHS composite material system is considered herein. Both symmetric and asymmetric laminate configurations are considered. The use and cure temperatures are both 70 °F with 0 percent moisture content. The individual ply orientation angle, ply thickness and fiber volume ratio for various plies in a laminate can be defined in the ICAN input data file.

PROBLEM DESCRIPTION

The laminate tailoring tasks considered herein can be grouped under two categories:

Category 1: The application examples in this category are deterministic in nature and are mainly selected to test the laminate tailoring approach proposed in the paper. Here, no hygral, thermal or mechanical loads are applied on the laminate. The eight-ply laminate is confined to be symmetric. The design variables are constituent (fiber and matrix) material properties, fiber volume ratio (k_f) and the ply angles (θ_i). Thus, the total number of design variables is 34. Only one composite material system AS-fiber in an intermediate-modulus-high-strength matrix (AS/IMHS) is considered in this example. There are no performance constraints.

The following laminate tailoring application examples were studied in category 1:

(1) To simultaneously minimize coefficients of thermal expansion (a) and maximize thermal conductivities (b) and maximize shear modulus (c). Note that a four-ply laminate is considered for this example. The classical optimization model cannot deal with multi-objective function problem. Thus, one needs to construct a composite objective function so as to minimize (a + 1/b + 1/c), where a, b, and c denote coefficient of thermal expansion, thermal conductivity, and shear modulus, respectively.

(2) To simultaneously maximize the elastic moduli along laminate structural axes x, y, and z. Thus, the composite objective function is to minimize

$$\left(\frac{E_{cxx}}{E_{\ell 11}} + \frac{E_{cyy}}{E_{\ell 22}} + \frac{G_{cxy}}{G_{\ell 12}}\right) \tag{4}$$

where ($E_{\ell 11}$), ($E_{\ell 22}$) and ($G_{\ell 12}$) are the corresponding values of moduli of a unidirectional laminate in the material axes (1, 2, and 3).

(3) To maximize the twisting stiffness (D_{33}) while constraining the coupling between bending and twisting represented by D_{13} and D_{23} terms in the flexural rigidity matrix (D) to zero. Note that the force displacement relationships for a composite laminate are given as:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & C \\ C & D \end{bmatrix} \begin{Bmatrix} \varepsilon_o \\ X \end{Bmatrix} \tag{5}$$

Thus, the problem is to maximize D_{33} such that $D_{13} = 0$ and $D_{23} = 0$.

(4) To minimize the coupling between bending and twisting actions by minimizing D_{13} and D_{23} terms of the flexural rigidity matrix in equation (5) simultaneously. This shall minimize the asymmetry in the laminate configuration. The objective function is to minimize ($D_{13} + D_{23}$).

(5) To maximize the bending as well as twisting stiffness of the laminates, or minimize the negative of the diagonal elements in the flexural rigidity matrix i.e., to minimize: ($- D_{11} - D_{22} - D_{33}$).

(6) To maximize the extensional stiffness of the laminate, or minimize the negative of the diagonal elements in the axial stiffness matrix (A) in equation (5) i.e., minimize: ($- A_{11} - A_{22} - A_{33}$).

(7) To minimize the shear stretch coupling effect in a laminate, or minimize the off diagonal elements of the axial stiffness matrix i.e., minimize: ($A_{13} + A_{23}$).

Category 2: The application examples in this category are probabilistic in nature. They represent point designs for specified cumulative probability of individual strenghts. Here, mechanical loads (axial forces and bending moments) are applied to the eight-ply composite laminate. The design variables include fiber volume ratios, ply orientation angles and ply thicknesses. Thus, the asymmetric laminate has 24 design variables whereas the symmetric laminate has 12 design variables. The constraints are imposed on the ply failure criteria such that the modified distortion energy (for combined stress failure) is always positive for each ply. Thus, there are eight constraints. The modified distortion energy (ϕ) is defined in the ICAN program as:

$$\phi = 1 - \left[\left(\frac{\sigma_{\ell 11\alpha}}{S_{\ell 11\alpha}}\right)^2 + \left(\frac{\sigma_{\ell 22\beta}}{S_{\ell 22\beta}}\right)^2 - k_{\ell 12}\frac{\sigma_{\ell 11\alpha}}{S_{\ell 11\alpha}}\frac{\sigma_{\ell 22\beta}}{S_{\ell 22\beta}} + \left(\frac{\sigma_{\ell 12s}}{S_{\ell 12s}}\right)^2\right] \qquad (6)$$

The ply failure criterion constraint is written as,

$$\phi > 0$$

The reader is referred to ICAN user's and programmer's manual for the details of equation (6). Note that the ply strengths S in equation (6) to be used are the values corresponding to 90 percent cumulative probability as given in table I. The corresponding probabilistic ply constitutent properties and any small deviations in the ply orientation angle are included in the scatter assumed for the fiber volume ratio, 0.45 to 0.55, which was used to develop the CDF curves for strength in figure 1. The objective function to be considered is to minimize the weight per unit area of the composite laminate i.e.,

$$\sum_{i=1}^{8} \rho_i t_i$$

where ρ_i and t_i are the weight density and the thickness of i^{th} ply, respectively.

The following upper and lower bounds are imposed on the fiber volume ratios, ply angles and thicknesses for all the plies,

$$\left.\begin{array}{l} -90° \leq \theta_i \leq 90° \\ 0.45 \leq (k_f)_i\ 0.55 \\ 0.005 \text{ in.} \leq t_i \leq 0.55 \text{ in.} \end{array}\right\} \quad i = 1,8$$

The initial ply stacking sequence for all the cases is (25/-25/25/-25/-25/25/-25/25), the fiber volume ratio and ply thickness is 0.45 and 0.01, respectively, for all the plies. The following example problems are studied under the probabilistic category:

(8) To minimize the weight of the eight-ply composite asymmetric laminate subject to the satisfaction of the ply failure criterion for all the plies where axial loads, N_{xx} = 3000, N_{yy} = 1000, and N_{xy} = 1000 lb/in., are applied. This example has 24 design variables and 8 behavioral constraints.

(9) This example is the same as example 8 except that the laminate is considered to be symmetric. Thus, there are 12 design variables and 8 constraints.

(10) To minimize the weight of the eight-ply symmetric composite laminate subject to the satisfaction of the ply failure criterion for all the plies where bending moments, M_{xx} = 100, M_{yy} = 100, and M_{xy} = 100 lb in./in., are applied.

(11) To minimize the weight of the eight-ply symmetric composite laminate subject to the satisfaction of the ply failure criterion for all the plies where axial loads, N_{xx} = 3000, N_{yy} = 1000, N_{xy} = 1000 lb/in., and bending moments, M_{xx} = 100, M_{yy} = 100, M_{xy} = 100 lb in./in., are applied.

RESULTS AND DISCUSSION

The results for the example problems in the first category (deterministic) are summarized in table II. As expected, the ply orientation angles and fiber volume ratio are the dominant design variables in the tailoring process. Note that less than 25 design cycles are required for these example problems to converge to the optimum designs. The final results obtained in the example problems agree with the intuitive reasoning guided by the physics of the problem and appear to be superior than what can be achieved manually by using parametric studies.

The ply orientation angle, fiber volume ratio and ply thickness for each ply for the examples in the second category are given in tables III to VI for example numbers 8 to 11, respectively. The initial and final values for the weight of the composite laminate is given in table VII. The ply stress constraints for the initial and final values are summarized in table VIII. Note that the starting design for all the example problems was infeasible because the initial design violates the probabilistic ply strength constraints.

The point probabilistic designs described in cases 8 to 11 can readily be extended to generate cumulative distribution functions for probability of failure versus weight. Three such examples are described below.

(1) Probabilistic distribution of the ply strengths ($S_{\ell 11t}$, $S_{\ell 11c}$, $S_{\ell 22t}$, $S_{\ell 12s}$) while loads are assumed to be deterministic.

(2) Probabilistic distribution of the loads on the laminate whereas the ply strengths are fixed to a predetermined level of probability of failure.

(3) Combined probabilistic load and strength.

In case (1), the ply strengths are selected for different probability levels of 50, 60, 70, 80, and 90 percent from the respective CDF curves as shown in figure 1 for longitudinal tensile strength ($S_{\ell 11t}$). The laminate weight per unit area is then optimized subject to each of these probabilistic strength constraints for the above mentioned probability levels. The optimum laminate weight corresponding to different strength reliability levels can then be plotted (fig. 4). Note that the laminate loads are assumed to be deterministic and thus fixed.

In case (2), the laminate loads are randomly selected for different probability levels between the range 80 to 120 percent from the a probability density curve. The mean values for the loads are N_{cxx} = 3000, N_{cyy} = 1000 and

N_{cxy} = 1000 lb in./in. Once again, the laminate weight is optimized subject to the strength constraints for a given probability and subject to the probabilistically varying loads. The optimum laminate weight corresponding to different load probability levels can then be plotted (fig. 4).

In case (3), both load and strengths are probabilistically described. The resulting curve is shown in figure 4. These three cases, collectively, demonstrate how optimization techniques can be routinely used to tailor laminates for probabilistic design requirements. The nearly flat curve for probabilistic load with fixed probability for strengths lends credence to point probability design. Stated differently, the optimizer will select the design variables to meet a specified reliability. For example, in a specific design, the structural reliability is assured after the probability for strength excedence has been selected. This is independent of the loading conditions and their respective scatter.

CONCLUSIONS AND SUMMARY

A procedure has been developed for the tailoring of composite laminates subject to probabilistic behavior constraints and/or loads. The probabilistic nature of the composite material properties such as ply strengths is considered by constraining the properties to a known probability level on the cumulative distribution function (CDF) curve. The CDF curves for the ply mechanical properties are obtained from probabilistic composite micromechanics. One of the main advantages of the procedure is that the probability of failure need not be evaluated in the laminate tailoring process. The example problems described demonstrate how to tailor composite laminates by accounting for uncertainties at the material and/or loads level. It is concluded from the example cases that laminate tailoring with probabilistic constraints and/or loads for point and/or distributed designs can be integrated in existing structural tailoring software for composites in a simple and straightforward manner. The results show that simultaneous consideration of probabilistic loads and strength constraints results in optimum designs with lowest reliability. Probabilistic strength constraints yield optimum designs with the highest reliability.

REFERENCES

1. Arora, J.S., 1989, Introduction to Optimum Design, McGraw-Hill Book Co., New York.

2. Arora, J.S. and Tseng, C.H., 1987, "User's Manual for IDESIGN Version 3.5," Optimal Design Laboratory, The University of Iowa, Iowa City, IA.

3. Brown, K.W., Harvey, P.R., and Chamis, C.C., 1987, "Structural Tailoring of Advanced Turboprops," 28th Structures, Structural Dynamics, and Materials Conference, Part 1, AIAA, NY, 1987, pp. 827-837.

4. Brown, K.W., Pratt, T.K., and Chamis, C.C., 1983, "Structural Tailoring of Engine Blades (STAEBL)," 24th Structures, Structural Dynamics, and Materials Conference, Part 1, AIAA, NY, 1983, pp. 79-88.

5. Chamis, C.C., 1986, "Probabilistic Structural Analysis Methods for Space Propulsion System Components," NASA TM-88861.

6. Gabrielle, G.A. and Beltracchi, T.J., 1987, "An Investigation of Pschenichnyi's Recursive Quadratic Programming Method for Engineering Optimization," Journal of Mechanisms, Transmissions and Automation in Design, Vol. 109, No. 2, pp. 248-253.

7. Ginty, C.A. and Endres, N.M, 1987, "Composite Space Antenna Structures: Properties and Environmental Effects," NASA TM-88859.

8. Haug, E.J. and Arora, J.S., 1979, Applied Optimal Design: Mechanical and Structural Systems, Wiley Interscience, New York.

9. Murthy, P.L.N. and Chamis, C.C., 1984, "ICAN: Integrated Composites Analyzer," NASA TM-83700.

10. Stock, T.A., Bellini, P.X., Murthy, P.L., and Chamis, C.C., 1988, "Probabilistic Composite Micromechanics," 29th Structures, Structural Dynamics, and Materials Conference, Part 3, AIAA, NY, 1988, pp. 1289-1298.

11. Vinson, J.R. and Sierakowski, R.L., 1986, The Behavior of Structures Composed of Composite Materials, Martinus Nijhoff, Doedrecht.

12. Thanedar, P.B., Arora, J.S., Tseng, C.H., Lim, O.K., and Park, G.J., 1987, "Performance of Some SQP Algorithms on Structural Design Problems," International Journal for Numerical Methods in Engineering, Vol. 23, No. 12, pp. 2187-2203,

13. Tsai, S.W., 1985, Composites Design 1985, Think Composites Co., Dayton, OH.

14. Vanderplaats, G.N. and Sugimoto, H., "Application of Variable Metric Methods to Structural Synthesis," Engineering Computations, Vol. 2, No. 2, pp. 96-100.

TABLE I. - PLY VALUES OBTAINED FROM DETERMINISTIC AND PROBABILISTIC ANALYSIS

Ply property	Deterministic, ksi	Probabilistic[a], ksi
$E_{\ell 11}$	15 750	---
$E_{\ell 22}$	1 065	---
$G_{\ell 12}$	516	---
$S_{\ell 11T}$	203	150
$S_{\ell 11C}$	165	85
$S_{\ell 22T}$	11.74	4
$S_{\ell 22C}$	27.41	9
$S_{\ell 12S}$	10.01	5.5

[a]The property values correspond to 90 percent cumulative probability on the cdf curves given in reference 6 for case 2.

TABLE II. - RESULTS FOR EXAMPLES IN CATEGORY I

Example	Design objective	Initial design	Final design
1	Minimize: coefficient of thermal expansion Maxmimize: thermal conductivity Maximize: shear modulus	$(25/-25)_s$ $k_f = 0.45$	$(30.9/-30.9)_s$ $k_f = 0.65$
2	Maximize: E_{c11}, E_{c22}, E_{c12}	$(25/-25/25/-25)_s$ $k_f = 0.45$	$(-13/13/-13/13)_s$ $k_f = 0.65$
3	Maximize: D_{33} $D_{13} = 0, D_{23} = 0$	$(25/-25/25/-25)_s$ $k_f = 0.45$	$(0/-83/68/-33)_s$ k_f = 0.[illegible]5
4	Minimize: D_{13}, D_{23}	(25/-25/25/-25)s $k_f = 0.45$	$(0/0/0/0)_s$ $k_f = 0.25$
5	Maximize: D_{11}, D_{22}, D_{33}	(25/-25/25/-25)s $k_f = 0.45$	(0/0/0/0)s $k_f = 0.65$
6	Maximize: A_{11}, A_{22}, A_{33}	$(25/-25/25/-25)_s$ $k_f = 0.45$	$(0/0/0/0)_s$ $k_f = 0.65$
7	Minimize: A_{13}, A23	$(25/-25/25/-25)_s$ $k_f = 0.45$	$(-90/90/-90/90)_s$ $k_f = 0.25$

TABLE III. - RESULTS FOR EXAMPLE 8[a]

$$\left[\text{Problem: minimize} \sum_{i=1}^{8} \rho_i t_i \text{ such that } \cdot \phi(I) \geq 0.01;\ I = 1,8\right]$$

$N_{xx} = 3000,\ N_{yy} = 1000,\ N_{xy} = 1000$

Ply	1	2	3	4	5	6	7	8
θ_i	26	-26	35	-35	-35	35	-26	-26
t_i	0.005	0.005	0.005	0.005	0.045	0.055	0.0055	0.0094
$(k_f)_i$	.55	.53	.45	.45	.45	.45	.45	.55

[a]Unsymmetric ply angle layup.

TABLE IV. - RESULTS FOR EXAMPLE 9

$$\left[\text{Problem: minimize} \sum_{i=1}^{8} \rho_i t_i \text{ such that } \cdot \phi(I) \geq 0.01;\ I = 1,8\right]$$

$N_{xx} = 3000,\ N_{yy} = 1000,\ N_{xy} = 1000$

Ply	1	2	3	4	5	6	7	8
θ_i	27	-27	35	-35	-35	35	-27	27
t_i	0.0068	0.0067	0.005	0.005	0.005	0.005	0.0067	0.0068
$(k_f)_i$	.53	.53	.45	.45	.45	.45	.53	.53

TABLE V. - RESULTS FOR EXAMPLE 10

[Problem: minimize $\sum_{i=1}^{8} \rho_i t_i$ such that $\cdot\ \phi(I) \geq 0.01;\ I = 1,8$]

$M_{xx} = 100,\ M_{yy} = 100,\ M_{xy} = 100$

Ply	1	2	3	4	5	6	7	8
θ_i	45	-25	25	-25	-25	25	-25	45
t_i	0.023	0.005	0.005	0.005	0.005	0.005	0.005	0.023
$(k_f)_i$	.45	.45	.45	.45	.45	.45	.45	.45

TABLE VI. - RESULTS FOR EXAMPLE 11

[Problem: minimize $\sum_{i=1}^{8} \rho_i t_i$ such that $\cdot\ \phi(I) \geq 0.01;\ I = 1,8$]

$N_{xx} = 3000,\ N_{yy} = 1000,\ N_{xy} = 1000;\ M_{xx} = 100,\ M_{yy}\ 100,\ M_{xy} = 100$

Ply	1	2	3	4	5	6	7	8
θ_i	45	-25	25	-25	-25	25	-25	45
t_i	0.027	0.017	0.005	0.005	0.005	0.005	0.017	0.027
$(k_f)_i$	.45	.55	.45	.55	.55	.45	.55	.45

TABLE VII. - INITIAL AND FINAL LAMINATE WEIGHT WITH PROBABILISTIC CONSTRAINTS

Example	Initial	Optimum
8	0.0042	0.0024
9	.0042	.0025
10	.0042	.0051
11	.0042	.0058

TABLE VIII. – LAMINATE TAILORING WITH PROBABILISTIC CONSTRAINTS INITIAL (I) AND FINAL (F) VALUES

(a) Example 8

Constraint	1		2		3		4		5		6		7		8	
	I	F	I	F	I	F	I	F	I	F	I	F	I	F	I	F
Ply stress, ksi:																
$\sigma_{\ell 11}$	45	2.9	45	1.7	45	11	45	16	45	69	45	10	45	13	45	14
$\sigma_{\ell 22}$	4.9	10	4.9	9.5	4.9	8.3	4.9	7.2	4.9	1.5	4.9	-3.6	4.9	-8.3	4.9	-9.5
$\sigma_{\ell 12}$	.52	1.3	.52	.75	.52	2.2	.52	2.5	.52	.05	.52	-.5	.52	-.79	.52	-.86
Combined stress failure criterion	-.26	1.2	-.26	5	-.26	.1	-.26	.5	-.26	.9	-.26	1.1	-.26	5	-.26	3.1

(b) Example 9

Constraint	1		2		3		4		5		6		7		8	
Ply stress, ksi:																
$\sigma_{\ell 11}$	45	90	45	90	45	67	45	68	45	68	45	67	45	90	45	90
$\sigma_{\ell 22}$	4.9	4	4.9	4	4.9	4.2	4.9	4.2	4.9	4.2	4.9	4.2	4.9	42	4.9	4.0
$\sigma_{\ell 12}$	.52	-2.3	.52	2.3	.52	-2.3	.52	2.3	.52	2.3	.52	-2.3	.52	2.3	.52	2.3
Combined stress failure criterion	-.26	.01	-.26	2.1	-.26	3.4	-.26	8.0	-.26	1.1	-.26	4.1	-.26	.01	-.26	0.5

(c) Example 10

Constraint	1		2		3		4		5		6		7		8	
Ply stress, ksi:																
$\sigma_{\ell 11}$	220	140	-84	-7	96	36	-17	-1.6	16	1.2	-96	-33	84	6.8	-220	-140
$\sigma_{\ell 22}$	41	.4	42	4.2	17.6	.4	8.4	.8	-8	-.8	-17	-.5	-42	-4.1	-41	-.5
$\sigma_{\ell 12}$	35	.3	-11.5	1.7	15	1.1	-2.3	.4	-2.3	-.2	-15	-1.1	15	-1.6	-35	-.4
Combined stress failure criterion	-134	.1	-125	2.1	-24	5	-4	1.2	-.16	1.9	-9	.3	-28	5	-57	1.8

(d) Example 11

Constraint	1		2		3		4		5		6		7		8	
Ply stress, ksi:																
$\sigma_{\ell 11}$	260	86	-39	-1.7	140	14	28	-.24	62	.24	-50	-13.9	130	1.7	-18	-86
$\sigma_{\ell 22}$	46	.5	47	2.6	22	2	13	.36	-3.4	-.3	-12	-.2	-37	-2.6	-36	-.55
$\sigma_{\ell 12}$	36	-93	-12	1.2	15.8	.4	-2.3	.16	1.7	-.16	-14	-.4	11	-1.2	-35	93
Combined stress failure criterion	-158	.1	-147	0.8	-34	1.2	-9.8	0.3	.41	1.9	-7.7	8.7	-24	1.1	-52	0.1

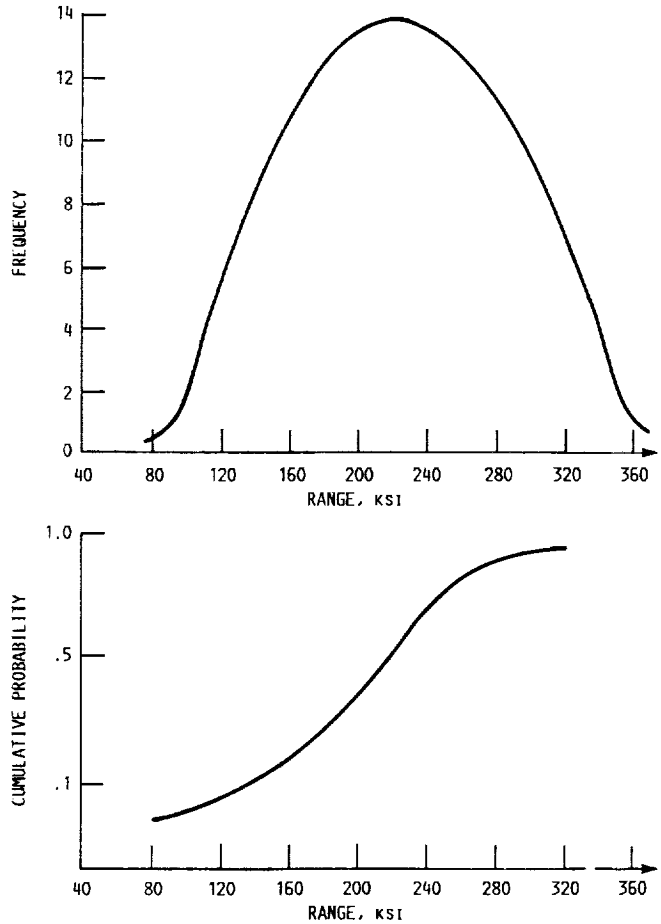

FIGURE 1. - PDF AND CDF CURVES FOR $S_{\ell 11t}$.

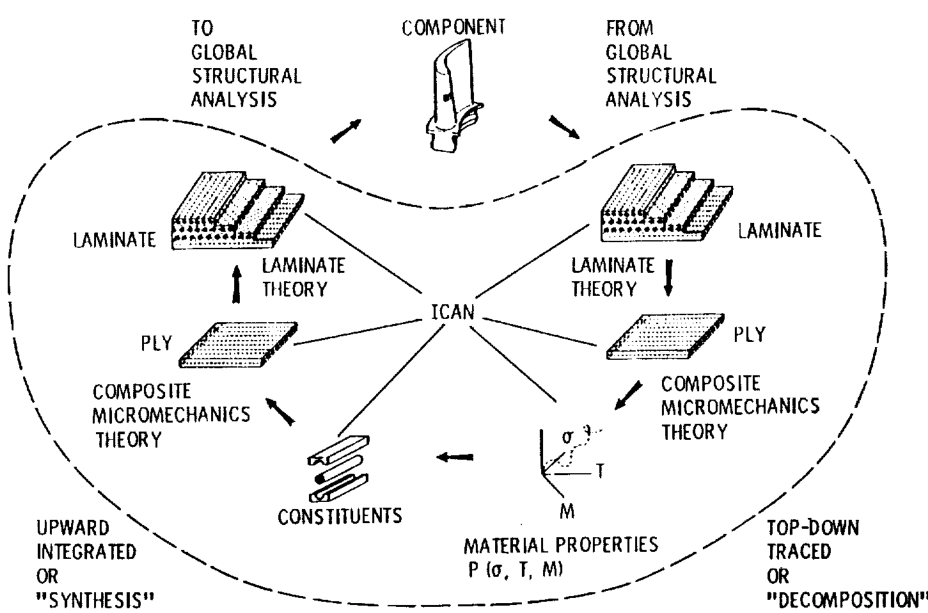

FIGURE 2. - ICAN: INTEGRATED COMPOSITES ANALYZER.

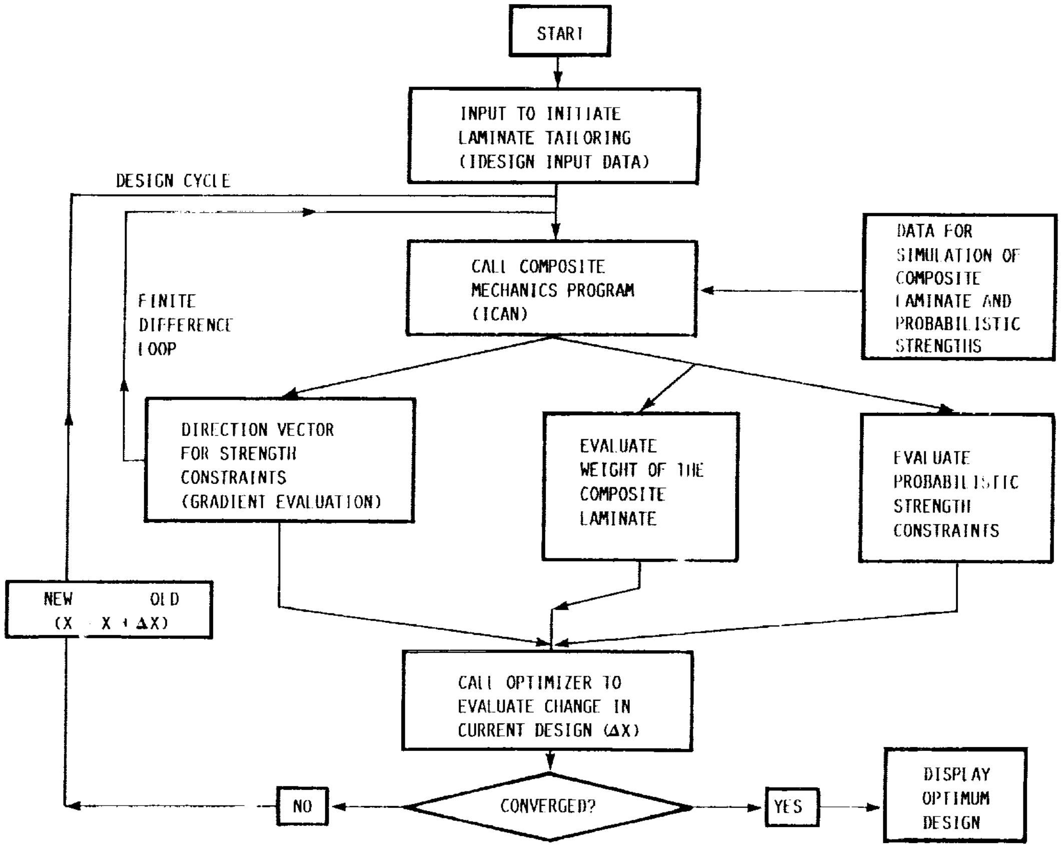

FIGURE 3. - FLOWCHART FOR COMPOSITE LAMINATE TAILORING WITH PROBABILISTIC CONSTRAINTS.

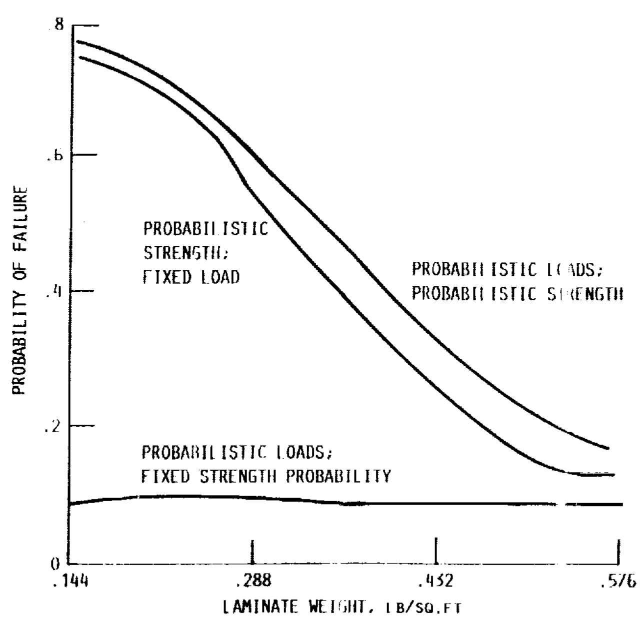

FIGURE 4. LAMINATE OPTIMUM WEIGHT FOR SPECIFIED RELIABILITY.

National Aeronautics and Space Administration

Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-102515		

4. Title and Subtitle

Composite Laminate Tailoring With Probabilistic Constraints and Loads

5. Report Date

6. Performing Organization Code

7. Author(s)

P.B. Thanedar and C.C. Chamis

8. Performing Organization Report No.

E-5182

10. Work Unit No.

505-63-1B

9. Performing Organization Name and Address

National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135-3191

11. Contract or Grant No.

13. Type of Report and Period Covered

Technical Memorandum

12. Sponsoring Agency Name and Address

National Aeronautics and Space Administration
Washington, D.C. 20546-0001

14. Sponsoring Agency Code

15. Supplementary Notes

Prepared for the Energy Technology Conference and Exhibition, sponsored by the American Society of Mechanical Engineers, New Orleans, Louisiana, January 14-18, 1990. P.B. Thanedar, National Research Council—NASA Research Associate at Lewis Research Center; C.C. Chamis, NASA Lewis Research Center.

16. Abstract

In this study, a reliability-based structural synthesis procedure has been developed to tailor laminates to meet reliability-based (ply) strength requirements and achieve desirable laminate responses. The main thrust of the paper is to demonstrate how to integrate the optimization technique in the composite laminate tailoring process to meet reliability design requirements. The question of reliability arises in fiber composite analysis and design because of the inherent scatter that is observed in the constituent (fiber and matrix) material properties during experimentation. Symmetric and asymmetric composite laminates subject to mechanical loadings are considered as application examples. These application examples illustrate the effectiveness and ease with which reliability considerations can be integrated in the design optimization model for composite laminate tailoring.

17. Key Words (Suggested by Author(s))

Fiber composites; Optimization; Stiffness; Conductivities; Thermal expansion; Symmetric; Asymmetric; Probabilistic strengths; Point probability; Probabilistic optimization

18. Distribution Statement

Unclassified – Unlimited
Subject Category 24

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price*
Unclassified	Unclassified	18	A03

NASA FORM 1626 OCT 86

*For sale by the National Technical Information Service, Springfield, Virginia 22161