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INNOVATIVE DESIGN OF COMPOSITE STRUCTURES: THE USE OF CURVILINEAR FIBER FORMAT TO IMPROVE BUCKLING RESISTANCE OF COMPOSITE PLATES WITH CENTRAL CIRCULAR HOLES

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ABSTRACT

This paper explores the gains in buckling performance that can be achieved by deviating from the conventional straightline fiber format and considering the situation whereby the fiber orientation in a layer, or a group of layers, can vary from point to point. The particular situation studied is a simply supported square plate with a centrally located hole loaded in compression. By using both a sensitivity analysis and a gradient-search technique, fiber orientation in a number of regions of the plate are selected so as to increase the buckling load relative to baseline straightline designs. The sensitivity analysis is used to determine which regions of the plate have the most influence on buckling load, and the gradient search is used to find the design that is believed to represent the absolute maximum buckling load for the conditions prescribed. Convergence studies and sensitivity of the final design are discussed. By examining the stress resultant contours, it is shown how the curvilinear fibers move the load away from the unsupported hole region of the plate to the supported edges, thus increasing the buckling capacity. The tensile capacity of the improved buckling design is investigated, and it is shown that both tensile capacity and buckling capacity can be improved with the curvilinear fiber concept.

BACKGROUND AND INTRODUCTION

Conventional design philosophies for fiber-reinforced composite structures are based on the idea of using multiple layers of fibers embedded in a matrix, the fibers in each layer being straight and aligned in a particular direction. Though each layer may have its own unique fiber orientation, the idea of allowing the fiber orientation within a layer to vary from point to point has not been seriously considered. Fabrication techniques and analysis procedures have precluded this form of fiber usage, herein referred to as a curvilinear format, and have emphasized the use of fibers in a straight line. Contemporary fiber handling techniques, however, make the issue of a curvilinear format less of an obstacle. This paper explores the potential gain in buckling resistance by using the curvilinear format with a plate containing a centrally located circular hole and subjected to a uniaxial compressive load. This work is considered a follow-on to earlier work [1] directed at improving the tensile load capacity of a plate with a centrally located circular hole by using the curvilinear fiber concept. Of interest are hole sizes of a structural level, e.g., windows, access holes, etc., as opposed to holes of the size used for mechanical fasteners, e.g., in bolted joints.

In that previous work, using the maximum strain criterion, the tensile capacity of a plate with a stacking arrangement denoted as $(\pm 45/C_6)_S$ was shown to have a load capacity greater than several equal-weight conventional straightline designs. The notation 'C' in the stacking sequence signifies a layer in the laminate that has a fiber orientation that varies from point to point within the plate. In the above laminate, then, the central 12 layers have the curvilinear format. In the tension study the fiber orientations within the layers with the curvilinear format were determined by aligning the fibers in those layers with the principal stress directions in those layers. This should not be confused with aligning the fibers with principal stress directions of the laminate. Principal stress directions of a laminate are somewhat meaningless.

Ideally, with the curvilinear format the fiber directions would vary continuously from point to point. For the tension problem the fiber directions would everywhere be aligned with the principal stress directions of the particular layers. However, a finite-element discretization was used in the analysis of the tension problem. Thus instead of having the fiber directions varying continuously, they were assumed to be constant within each element of the finite-element model, but they varied from element to element. With 120 elements in a quarter plate model, this approximation was not a detriment. Since principal stress directions depend on the fiber orientations, aligning the fibers with the principal stress directions required iteration. Usually within 4 or 5 iterations the fibers were aligned with the principal stress directions in each element and the performance of the curvilinear designs was compared with conventional straightline format designs. Figure 1 shows the fiber orientations of the curvilinear layers that resulted from the iteration process. For this curvilinear case, a $(\pm 45/0_6)_S$ is considered the straightline counterpart. The tensile strength of the curvilinear $(\pm 45/C_6)_S$ was shown to have capacity 1.26 times greater than this orthotropic straightline counterpart. Relative to a baseline quasi-isotropic design, the curvilinear design was 1.60 times stronger in tension. The baseline quasi-isotropic, the curvilinear, and the orthotropic straightline counterpart designs were all predicted to fail due to fiber tension near the net-section hole edge. Though the curvilinear design had the same failure mode, it was shown that the primary effect of the curvilinear design was to move the load around the hole in a more effective manner than the straightline designs did. Material near the hole edge, where the stresses were highest in all designs, was not stressed as highly in the curvilinear design as in the other designs. Hence the tensile load capacity was greater. Similar conclusions were drawn regarding other curvilinear designs, e.g., $(\pm 45/C_2)_{2S}$ vs. $(\pm 45/0_2)_{2S}$.

The increased tension capacity of the curvilinear designs could well come at the expense of a degradation in buckling load. Thus the buckling loads of the baseline quasi-isotropic, curvilinear, and orthotropic straightline counterpart designs were computed and compared. In particular, the buckling loads for a uniform stress loading and simple support boundary conditions were examined. The baseline quasi-isotropic design exhibited the highest buckling load, due to the fact that half layers were in a $\pm 45°$ orientation. The curvilinear design buckling loads were generally 5-15% less than the baseline quasi-isotropic design, and at least as high as the orthotropic straightline counterparts. Thus the study concluded that though the improved tensile performance was at the expense of buckling performance, the penalty was no worse, and in fact less, than other orthotropic straightline designs.

The studies related to buckling immediately raised the question: Can buckling performance be improved by utilizing the curvilinear concept? How many layers should have the curvilinear format? What is the basis for comparison to measure improvements in buckling resistance? Is the tensile strength of these designs degraded? It seems certain that working with the principal stress directions is not the approach to take. However, as in the principal stress direction approach to improving tensile strength, it seems logical to consider the fiber orientation at each point in the plate (or, in the context of the analysis, in each finite-element) as a variable that can be used to control buckling load. The principal question is: How do we determine these fiber orientations so as to result in the maximum buckling load? As alluded to above, of equal importance is the influence of these fiber orientations on the tensile load. Improved buckling performance could well come at the expense of degraded tensile capacity. This present paper addresses improvement in buckling performance but also evaluates the tensile capacity of the improved buckling designs.

The approach taken in this buckling improvement study is similar to the approaches used in optimization studies, namely, sensitivity studies and finding the conditions that lead to a maximum of a performance measure. The sections that follow focus on these issues and provide details of the approach. In particular, the next section briefly outlines the method of analysis. The section after that further discusses the method of analysis and presents several designs that represent an improvement over a baseline quasi-isotropic design, a design, as mentioned above, that is quite good for buckling resistance. Convergence of the designs and insight into the reasons behind the improvements in buckling capacity are presented. The tensile load capacities of the curvilinear improved buckling designs are then discussed. The paper concludes with a recommended design that not only shows an improved buckling capacity, but also shows improved tensile performance. Follow-on activities are discussed at the close of this paper.

METHOD OF ANALYSIS

The finite-element method is again used throughout this buckling study. The presence of the hole requires a prebuckling analysis for determining the inplane stress resultants. This is accomplished with an 8-node isoparametric element. Within an element the fiber orientation is constant. The buckling analysis, of course, uses the prebuckling stress resultants to compute the level of those resultants that causes the plate to buckle. The finite element used for the buckling analysis is also an 8-node element with a constant fiber orientation.

The material properties used in the study represent a typically available polymer matrix graphite composite, and are the same as used in ref. 1 [2]. The properties are:

$$\begin{gathered} E_1 = 138\text{ GPa }(20.0\text{ Msi})\,;\ E_2 = 8.96\text{ GPa }(1.30\text{Msi}) \\ G_{12} = 7.10\text{ GPa }(1.03\text{ Msi})\,;\ \nu_{12} = 0.30 \\ \text{layer thickness} = 0.125\text{ mm }(0.005\text{ in.}) \end{gathered} \tag{1}$$

The failure strains used to evaluate tensile load are [2]:

$$\begin{gathered} \text{tensile failure in fiber direction}:\ 10.5\text{x}10^{-3} \\ \text{compressive failure in fiber direction}:\ 10.5\text{x}10^{-3} \\ \text{tensile failure perpendicular to fiber}:\ 5.8\text{x}10^{-3} \\ \text{compression failure perpendicular to fiber}:\ 23.0\text{x}10^{-3} \\ \text{shear failure in plane of layer}:\ 13.1\text{x}10^{-3} \end{gathered} \tag{2}$$

SENSITIVITY STUDIES

With a sensitivity study, as it applies here, the fiber orientation at each point is considered as the design variable. For a finite-element representation, wherein the fiber orientation is constant within each element, element fiber orientations are considered the design variables. Here, the sensitivity of the buckling load to the design variables is examined. In the previous study to improve tensile capacity, 120 elements were used. If this many elements were used for the buckling study, there would be 120 design variables. Determining the sensitivity of the buckling load to each of the 120 fiber orientations would be an impossible task. Hence, for the buckling study, less fiber orientations are used. This, in effect, prescribes that a given fiber orientation be fixed over a larger region of the plate than if 120 fiber orientations were used. Though 120 finite-elements could be used to compute the lowest buckling load, far less elements produce the same result, and obviously require far less computation. A convergence study, to be discussed, indicated that no more than 18 elements in a quarter plate are necessary to accurately compute the lowest buckling load. As a first step at studying sensitivity, then, 18 different fiber orientations in a quarter plate are considered. Though it is well known that the presence of the D_{16} and D_{26} terms prevents quarter-plate symmetry [3], a quarter-plate analysis provides a good estimate of the effectiveness of the curvilinear format for improving buckling resistance. A schematic of the quarter plate divided into 18 regions is illustrated in fig. 2. The numbering scheme for the regions is shown in the figure. Here only square plates are studied, in particular, square plates with a hole diameter to width ratio, W/D = 3.0. Specifically, the plate dimensions considered are chosen for purposes of possible small-scale verification experiments. The plate length and width are 254 mm (10 in.) and the hole diameter is

84.7 mm (3.33 in.). Sixteen layer plates are studied. The plate is assumed to be simply supported on all four edges. The loading used is actually a uniform displacement along opposite edges rather than the uniform stress loading discussed in the INTRODUCTION. The displacement along the right edge, $\Delta/2$, is illustrated in fig. 2. A displacement loading more closely represents experimental fixturing used in buckling studies. The laminate used to begin this sensitivity study is a $(\pm 45/0_6)_S$, fiber orientation being measured counterclockwise from the +x axis. Shown in fig. 2 is the fact that the sensitivity study is based on allowing the 0° fibers in one of the 18 regions of the plate to vary (in the figure, region 15). In this one region the laminate could be considered a $(\pm 45/\theta_6)_S$ laminate. The sensitive study examines the influence of this variable fiber orientation on the buckling load by computing the buckling load as a function of θ. This variation is allowed on a region-by-region basis. For each region there is a fiber orientation that results in the greatest buckling load. The sensitivity of the buckling load to the fiber orientation in each region is illustrated in fig. 3. In this figure the buckling load is normalized by the buckling load of the $(\pm 45/0_6)_S$ laminate. Hence, for each region, when $\theta = 0°$, the normalized buckling load, $\overline{P}_{cr}$, is unity because the laminate is actually the original $(\pm 45/0_6)_S$ laminate. (In absolute terms, the buckling load of the $(\pm 45/0_6)_S$ laminate is 23.9 kN/m (136.5 lb/in). The buckling load reported is the average N_x along the uniformly displaced boundary.)

By studying fig. 3, two important facts can be learned. First, it is clear that the buckling load is not sensitive to the fiber orientation in some regions of the plate. For example, along the centerline, regions 1, 2, and 3, fiber orientation is not overly important. By contrast, around the hole, regions 7, 10, 13, and 16, improvements in the buckling load can be made by changing the fiber orientation. In a similar vein, the sensitivity study indicates that the buckling load can be degraded be changing the fiber orientation in regions 9, 12, 15, and 18.

As a second point, fig. 3 indicates that if effects are additive, there can be significant improvements in buckling resistance if fiber orientations are chosen properly in the various regions of the plate. To that end, if a plate design is considered that uses the orientation in each region that results in the maximum buckling load, a considerable increase in buckling resistance is achieved. Such a design is illustrated in fig. 4, the orientation in each region being the orientation from fig. 3 that results in the maximum buckling load when the fiber orientations were varied one region at a time. The buckling load is 2.23 times the buckling load of the original $(\pm 45/0_6)_S$ laminate, indeed a significant improvement. Relative to the baseline quasi-isotropic case, the buckling load of the sensitivity design is 2.26 times greater. (It should be noted that for a uniform displacement loading of a plate simply supported on all four sides, the buckling load of a quasi-isotropic laminate is about 1% less than the buckling load of a $(\pm 45/0_6)_S$ laminate. This is opposed to the stress-loaded simply-supported case discussed in the INTRODUCTION, in relation of ref. 1, where the quasi-isotropic laminate was 15% more resistant to buckling. Hence the buckling loads being discussed in this buckling improvement study can be considered as being normalized either by the quasi-isotropic baseline, or by the original $(\pm 45/0_6)_S$ straightline orthotropic design.)

Though the sensitivity study results in a design with improved buckling resistance, the approach may be somewhat misleading. In reality, the fiber orientation in each region that maximizes the buckling load more than likely depends on the fiber orientation in other regions. That is, the regions interact. In the sensitivity study it was assumed that the fiber orientations in the other regions were 0°, the only nonzero fiber orientation being the one in the region being studied. To allow for interaction between regions, the fiber orientation in each of the 18 regions is considered an unknown, and a gradient-search [4] technique is used to find the combination of orientations in the 18 regions that maximizes the buckling load. As used here, there is no guarantee the technique will find the absolute maximum. The maximum in the neighborhood of the initial angles given to the gradient-search algorithm is what the technique will find. This concern in mind, if the gradient-search technique is applied, the design illustrated in fig. 5 results. This design has a buckling load 2.96 times the baseline quasi-isotropic case, an improvement over the sensitivity analysis design. Comparing figs. 4 and 5 it can be seen that the sensitivity analysis and the gradient-search method result in similar fiber orientations around the hole, but somewhat different orientations away from the hole.

One of the first concerns that comes to mind when examining either fig. 4 or 5 is the issue of manufacturing. The change in fiber orientations between adjacent regions is large, for example, in fig. 5 between regions 7 and 8, or between regions 10 and 11. Such discontinuities in fiber direction are not possible for even the most advanced manufacturing methods, and they would lead to low material strength. Dividing the plate into more regions may reduce the discontinuity of fiber orientations between regions. And with more design variables, the improvement in buckling load could be expected to increase. As stated above, however, the computational effort becomes quite large with more regions. The approach taken here is to use less design variables, or other geometric arrangements of fiber angles. One variation of the geometric arrangement of regions is shown in fig. 6, the plate being divided into 6 radially oriented regions. There are still 18 finite elements in the discretization but there are only 6 different fiber angles. Figure 7 illustrates the sensitivity of the buckling load to the fiber orientations in these radially oriented regions. Except for region 1, the region along the centerline of the plate, and the region that corresponds to regions 1,2, and 3 in the 18-region scheme, each region has an influence on the buckling load. Depending on the sense of the fiber orientation and the particular region, it appears that even with just six regions, fiber orientation in each region can be used to improve buckling capacity. The sensitivity analysis design based on the results of fig. 7 is shown in fig. 8. The buckling load for this design is 1.83 times greater than the buckling load for the baseline quasi-isotropic laminate. The design resulting from the gradient-search technique is shown in fig. 9. The gradient-search design, with its factor of 1.85 over the baseline quasi-isotropic laminate, is essentially the same as the sensitivity design. These designs, where the fibers are grouped along radial regions, are referred to herein as a $(\pm 45/RG_6)_S$ designs, RG denoting radially-grouped. Compared to the factor of 2.96 for the gradient-search 18-region design, the buckling resistance of the radially-grouped design is not as great. However, this near factor of 2 in improvement is significant, and the design is much more manufacturable. With the design of fig. 9, the specific fiber angles in each region can be achieved by employing a smooth continuously changing fiber path.

To determine if the improved buckling designs are practical, in addition to being manufacturable, it is important to study the tensile strength of the designs. Before doing this, however, the issue of design convergence, and insight into the mechanisms by which the curvilinear designs enhance buckling resistance should be addressed.

CONVERGENCE

As mentioned previously, using 120 finite elements in the analysis would make the magnitude of the numerical computation insurmountable. The gradient-search technique, and other schemes like it, rely on repeated computations to determine the magnitude and direction of the local gradients in design space. As the solution proceeds, searching for the maximum buckling load, the prebuckling and buckling computations are repeated many, many times. Here, the 18 region breakdown was not chosen from finite-element considerations, but rather to strike a balance between not having too many regions, and allowing, at least initially, for some fiber orientation variations both radially and circumferentially. Obviously more than 18 finite elements could be used and still retain only 18 distinct fiber orientation regions. Likewise, for the radially-grouped design, 6, 12, 18, 24, etc. elements could be used and still maintain the six distinct fiber orientation regions. As stated earlier, for the six region gradient-search design of fig. 9, 18 elements were used. It is important to ask that if more than 18 elements were used in the finite-element analysis of the six distinct regions, would the fiber orientations in each region be the same as when 18 elements were used. To study this issue, the 18 element mesh was refined by subdividing the existing 18 elements and re-computing the 6 fiber orientations from the gradient-search scheme. The results using two mesh refinements are shown in Table 1, the two mesh refinements being shown in fig. 10. The two different refinements each use 36 element, 36 elements being the limit of the size of the eigenvalue problem that could be solved with the computer being used. It is clear the design has converged by using only 18 elements.

As a matter of interest, the convergence characteristics of the buckling loads for the two straightline designs being used as a comparison are shown in Table 2. Here it is seen that by using 18 elements, the error in the buckling loads for both cases is less than 1% different than the value obtained using 50 elements. Another issue of concern is how the final design angles as computed by the gradient-search technique depend on the starting values given the search algorithm. One logical set of starting values are those as determined by the sensitivity study. Other logical sets are all angles being set at 0°, or 90°, or random values. Here 0° in each region was the starting point for the search. Other starting values resulted in the same final angles, and the same buckling load. Some starting values resulted in other final angles in the regions, and consequently other, but lower, buckling loads. Care must be exercised in using search techniques, particularly when the design space has variables that do not strongly influence the result, e.g., in this case the fiber angles along the horizontal centerline (region 1 in fig. 6). In this study, enough calculations were made to be convinced that the design of fig. 9 represents an absolute extremum.

Table 1

Convergence of the $(\pm 45/RG_6)_S$ Curvilinear Design[1]

No. of Elements	Converged Fiber Angles in Each Region						Buckling Load[2]
	1	2	3	4	5	6	
18	-83.0	-63.0	-43.6	-28.9	-16.4	-5.0	1.0000
36a	-83.4	-63.2	-43.3	-28.7	-16.3	-5.0	1.0032
36b	-82.7	-63.5	-43.8	-29.0	-16.4	-5.1	1.0024

[1] W/L = 1.0, W/D = 3.0, quarter-plate analysis.

[2] Normalized by 18 element mesh result, 43.64 kN/m (249.2 lb/in.).

Table 2

Convergence of Buckling Analysis of Two Straightline Designs[1]

No. of Elements	Buckling Load	
	$(\pm 45/0/90)_{2S}$ [2]	$(\pm 45/0_6)_S$ [3]
4	0.9778	0.9607
8	0.9926	0.9825
12	0.9955	0.9905
18	0.9963	0.9927
36	1.0000	1.0000
50	1.0000	1.0000

[1] W/L = 1.0, W/D = 3.0, quarter-plate analysis.

[2] Normalized by the 50 element mesh results, 23.66 kN/m (135.1 lb/in.).

[3] Normalized by the 50 element mesh results, 24.08 kN/m (137.5 lb/in.).

SENSITIVITY OF FINAL DESIGN

In addition to knowing that the design has converged and is the absolute best design, it is important to know the sensitivity of the final design to variations in fiber orientation in each region. This is important for two reasons: First, if the buckling load is indeed a maximum, as a check, a sensitivity analysis should indicate that any variation in fiber orientation in any of the six regions causes the buckling load to decrease. Second, if for manufacturing or other practical reasons it is necessary to alter the fiber orientation in a particular region, e.g., along the centerline, a sensitivity analysis of the final design will indicate the impact of this alteration on the buckling load. Figure 11 shows the results of a sensitivity analysis for the $(\pm 45/RG_6)_S$ design of fig. 9. Since the buckling load decreases for any variation of fiber orientation, it can be said that the design of fig. 9 represents at least a local maximum in buckling load. Here it is claimed to be an absolute maximum. Also, it appears from fig. 9 that the region along the centerline, region 1, is least critical in determining buckling load. This is particularly interesting. The same conclusion was drawn from fig. 7, the sensitivity analysis of a $(\pm 45/0_6)_S$ laminate. Also evident from both figs. 7 and 11 is the fact that in region 6, any deviation from a slightly negative ($-5°$) orientation is quite detrimental to the buckling load.

PREBUCKLING STRESS RESULTANT CONTOURS

The primary reason behind the improved buckling capacity of the curvilinear designs is illustrated in figs. 12 and 13. In these figures the contours of stress resultant N_x, and the distribution of N_x across the loaded edge and across the net section for both a quasi-isotropic laminate and the $(\pm 45/RG_6)_S$ design are shown. Several interesting observations can be made. First consider the quasi-isotropic case, fig. 12. Since the displacement in the x direction is specified to be uniform along the loaded end, the stress resultant N_x is not uniform. (Here the results are normalized so the <u>average</u> value of N_x along the end is unity.) Because of the hole, the stress resultant on the loaded end is lower near the centerline. This is a well-known effect. At the net section, the stress resultant is high at the hole edge, and low at the outside edge. This is caused by the stress concentration effect of the hole, another well-known effect. For the conditions shown, the normalized stress resultant is about 3.5 at the hole edge, and attains a minimum value just less than unity at the outside edge. Considering the curvilinear design fig. 13, the stress resultant along the loaded end is also low near the centerline, as in the quasi-isotropic case. However, unlike the quasi-isotropic case, the stress resultant at the net section is high at the hole edge, reaches a minimum about one-half a hole radius away from the hole edge, and then increases to a fairly high value at the outside edge. At the hole edge the normalized stress resultant is about 3, it drops to a minimum of 1, and then increases to 2 at the outside edge. Also reflecting this is the fact that the stress resultant contours are increasing in value toward the plate edge for the $(\pm 45/RG_6)_S$ design, while they are decreasing in value toward the plate edge for the quasi-isotropic design. This characteristic of the net-section stress resultant indicates that by allowing the fiber orientation to vary from point to point, the load is directed away from the hole, which is unsupported, toward the outside edge, which is supported. In a sense, if the fibers are allowed to orient themselves to be most beneficial, they arrange themselves to direct the load to the supported region of the plate, namely the edges. This makes sense and provides impetus for seriously considering in any design process the idea of not a priori fixing fiber orientation. Hopefully an automated design tool will aid in determining the orientation that is most beneficial to the structure. What is also interesting is that the stress contours are more dense near the outside edge for the curvilinear design than for the quasi-isotropic design. That this is the tendency of the curvilinear designs is illustrated in fig. 14. Here the contours and stress resultant distributions for the 18 region $(\pm 45/C_6)_S$ case are shown. Indeed, given the chance, the fibers will do their best to move the load away from the unsupported region of the plate and unload the center portion of the plate.

Examination of the stress resultant distributions and stress contours for a uniform stress loading on the end (as opposed to the uniform displacement loading) leads to a similar conclusion. What the characteristics of the stress resultants show is that although the bending stiffnesses of the laminate in the various directions are important, i.e., the out-of-plane portion of the buckling problem, the prebuckling stress distribution, i.e., the inplane portion of the problem, is as important, if not more so.

TENSILE STRENGTH CONSIDERATIONS

Since in a normal operating environment both compressive and tensile loads are present, it is of paramount importance to evaluate the tensile strength of the designs that improve the buckling load. For a tensile load the stresses of concern are the prebuckling stresses with the sign reversed. Thus a tensile analysis is available as part of the buckling analysis.

The principal material direction strains ε_1, ε_2, and γ_{12} in each layer are computed and the maximum strain criterion applied using the strains in eq. 2. The strains as computed at the finite-element Gauss points are used and failure load levels, failure modes, and failure locations determined. (The prebuckling compressive strains are quite small compared to the failure strains and thus it is clear compressive loading causes buckling rather than material failure.)

Table 3 summarizes the tensile failure loads of the laminates discussed so far, in addition to a laminate to be discussed. For completeness, buckling loads are included in the table. In the table both the buckling load and the tensile failure load are normalized by the respective load for the quasi-isotropic laminate. For the curvilinear designs, only the designs determined by the gradient-search method are included. For the tensile failure, the failure mode and the location of this failure are indicated. As a reference, the buckling load of a 10 in. by 10 in. 16 layer displacement-loaded quasi-isotropic simply-supported laminate with a 3.33 in. hole is calculated to be 6.00 kN (1365 lbs) and the tensile failure load is 105 kN (23.6 kips). Tensile failure for this laminate is predicted to occur due to fiber failure at the net-section hole edge.

Table 3

Normalized Buckling and Tensile Load Capacities of Laminates

Laminate	Buckling Load[1]	Tensile Load[2]	Tensile Failure Details
quasi-isotropic	1.00	1.00	fiber failure at hole edge
$(\pm 45/0_6)_S$	1.01	1.27	fiber failure at hole edge
$(\pm 45/C_6)_S$	2.97	-	not considered[3]
$(\pm 45/RG_6)_S$	1.85	0.62	shear at loaded edge
$(\pm 45/0/90/RG_4)_S$	1.30	1.35	fiber failure at hole edge

[1] Normalized by the load for a quasi-isotropic laminate, 23.6 kN/m (135.6 lb/in.).

[2] Normalized by the load for a quasi-isotropic laminate, 413.3 kN/m (2360 lb/in.).

[3] Since this design was judged to be unrealistic, it was not pursued.

Immediately obvious in Table 3 is the fact that the $(\pm 45/RG_6)_S$ design, a manufacturable design, is about 40% weaker in tension than the baseline quasi-isotropic laminate, and considerably weaker in tension than the $(\pm 45/0_6)_S$ design. Failure is predicted to occur in shear at the loaded edge. Specifically, referring to figs. 6 and 9, failure is predicted to occur along the loaded edge of region 3. In this region, due to the fiber orientation of $-44°$, the laminate is effectively a $(+45/-45_7)_S$ laminate. This laminate, while strong in inplane shear in the plate coordinate system, is extremely weak in inplane shear in a coordinate system rotated at 45° to the plate coordinate system. Thus the plate fails at the loaded edge at a relatively low load level. Needed is reinforcement in the 0° and/or 90° direction in the plate coordinate system to overcome this weakness in shear. In this regard, four of the curvilinear layers are re-oriented so as to form a $(\pm 45/0/90/RG_4)_S$ laminate. Effectively, the $(\pm 45/0/90/RG_4)_S$ laminate is a 'sandwich', the curvilinear layers being sandwiched between two unsymmetric quasi-isotropic laminates. The fiber orientations in the curvilinear layers as found by the gradient-search technique for this laminate are not quite the same as in the $(\pm 45/RG_6)_S$ laminate. As can be seen from Table 3, the buckling load is not as high as the $(\pm 45/RG_6)_S$ design but it is higher than the quasi-isotropic baseline case. There are fewer curvilinear fibers to transfer the load away from the unsupported central portion of the plate and so reduced buckling resistance is expected. However, the tensile strength is significantly improved, and in fact, exceeds the baseline quasi-isotropic laminate. The improvement in tensile capacity relative to the quasi-isotropic case is due presumably to the load being transferred around the hole more effectively, as discussed in the INTRODUCTION regarding results from the previous study [1]. Hence, the problem of reduced tensile capacity has been eliminated by a slight redesign. Clearly, if tensile capacity is not a problem, the $(\pm 45/RG_6)_S$ laminate is the one to consider. However, the $(\pm 45/0/90/RG_4)_S$ laminate indicates that even with the curvilinear format, solutions to particular problems are possible.

FURTHER DISCUSSION

The results obtained to date are quite encouraging regarding the potential for utilizing the curvilinear format. An examination of other support conditions, e.g., clamped, uniform stress loading, etc. indicate similar gains in performance can be realized by considering a curvilinear design. At this point a comment should be made regarding the results discussed here. In this study the curvilinear layers were clustered into one group. For straight fibers it is known that clustering leads to a reduction in transverse tensile and shear strength in the principal material system. It is not clear this will be the case for clustered curvilinear layers. It may be. If it is, then, for example the 0's and 90's in the $(\pm 45/0/90/RG_4)_S$ laminate can be distributed differently. This will not influence the prebuckling stress state. Redistribution of the 0's and 90's will influence the bending stiffnesses. This will influence the buckling load and until specific cases are studied, it is not known to what degree. The point to be made, however, is that curvilinear designs result in performance increases and should be considered. If there are problems with a particular laminate, solutions are possible, as they are with straightline designs.

FINAL COMMENTS

Though the number of regions that can have unique fiber angles, and the number and arrangements of the layers within those regions would appear to be infinite, there is a practical limit to what can be fabricated. Tow placement devices must work with smooth fiber trajectories, and the fiber trajectories in one area of the plate have to be able to blend in a continuous fashion with the fiber trajectories in other areas of the plate. In addition, there are volume fraction considerations. Thus there is not an endless array of possibilities. Some other possibilities have been considered but have not been thoroughly investigated. For example, using circumferentially-grouped regions rather than radially-grouped regions could lead to high buckling loads. Orienting the fibers in spiral fashion around the hole is easier from a manufacturing consideration, and it leads to improved buckling resistance. And what of post-buckling? The buckled plates must eventually fail. It is important to know whether their post-buckling strength is improved or degraded. Material failure will most likely occur in the post-buckled range and this must be investigated. These areas are currently under investigation and results will be reported on at a later date.

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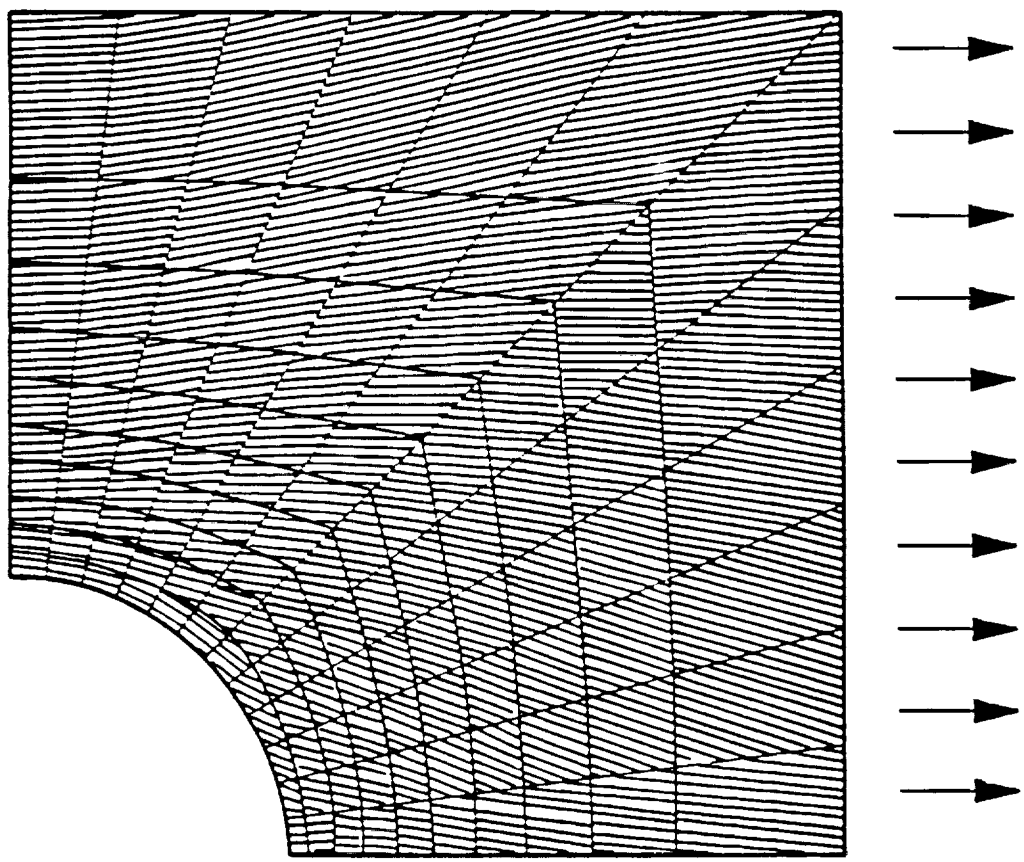


Fig. 1 **Fiber orientations in the curvilinear layers of the ($\pm$ 45/C_6)$_s$ tensile design. (ref 1.)**

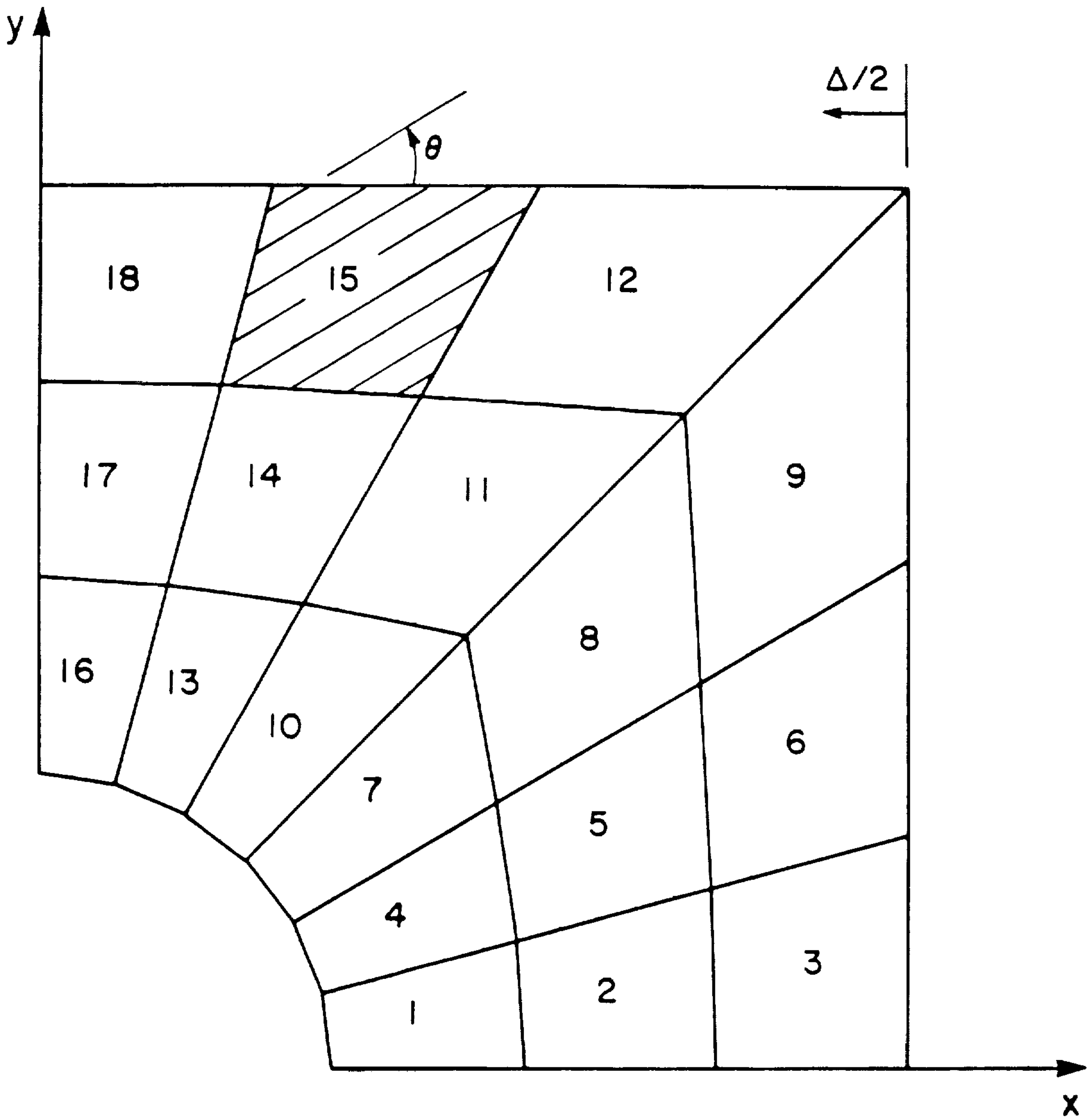


Fig. 2 Quarter plate divided into 18 regions.

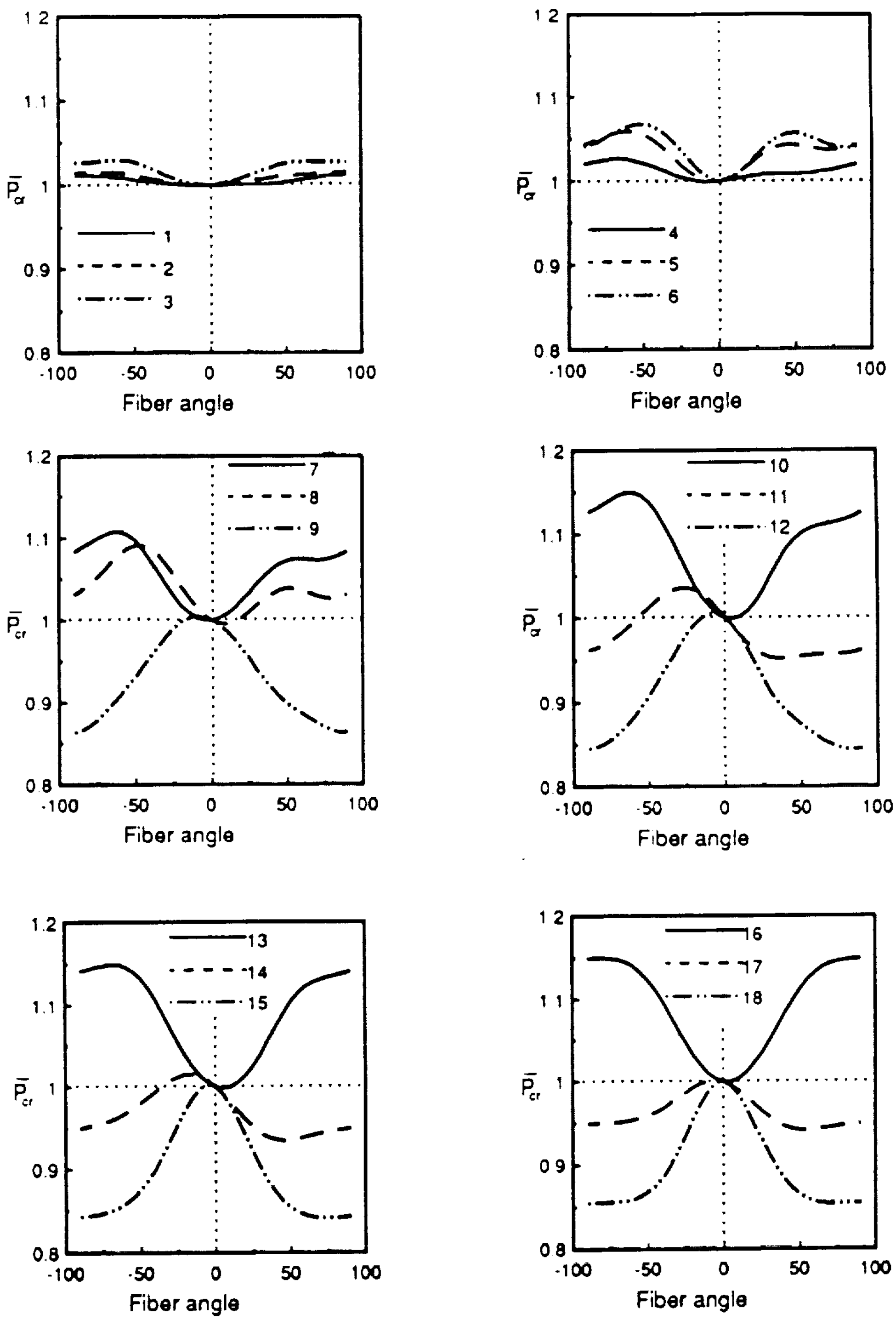


Fig. 3 **Sensitivity of buckling load to fiber angle in each of the 18 regions of a ($\pm$ 45/0_6)$_s$ laminate.**

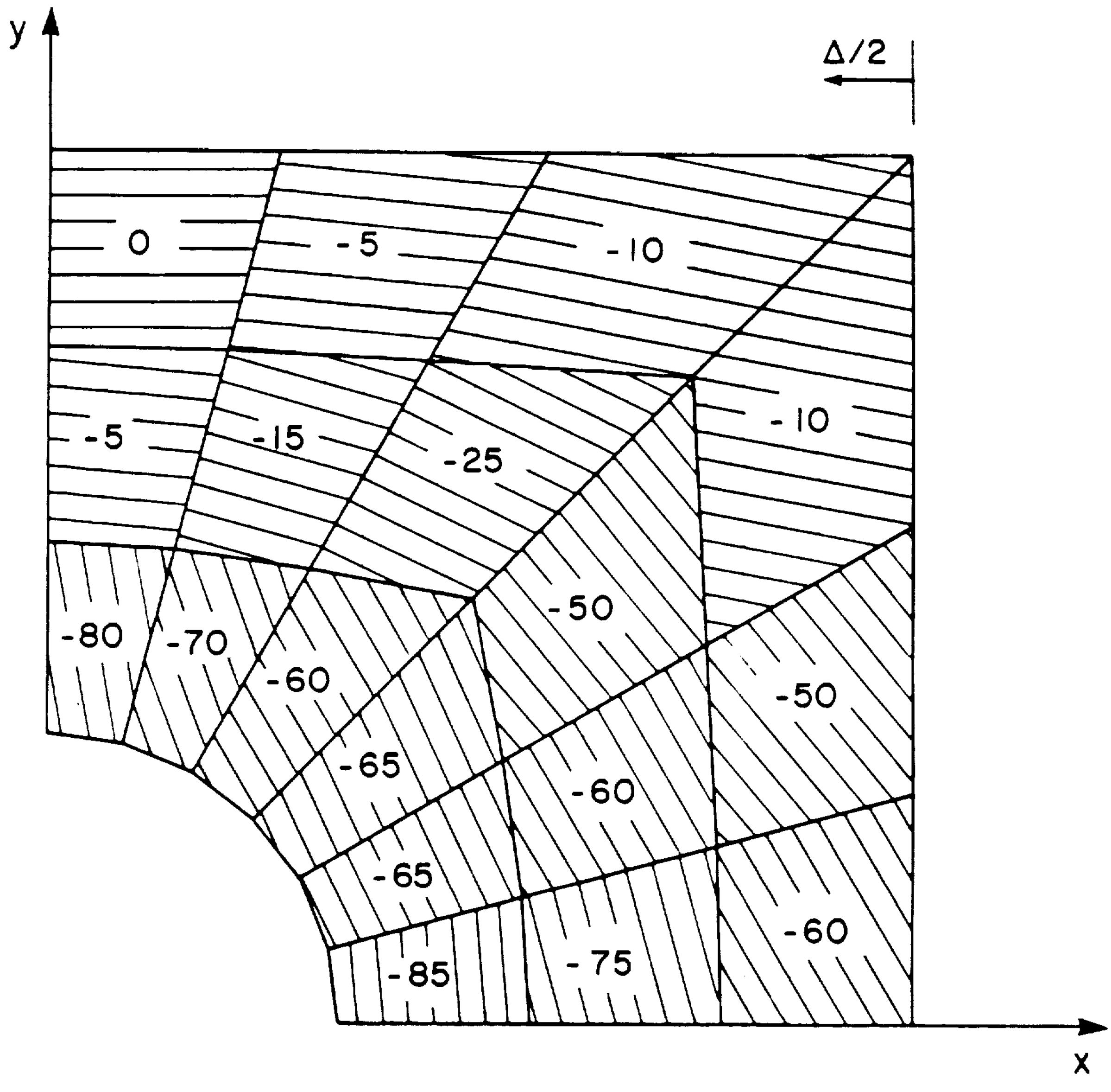


Fig. 4 **Fiber angles in the curvilinear layers that maximize buckling load in a ($\pm 45/C_6)_s$ plate, sensitivity design with 18 regions (buckling load 2.26 times buckling load of baseline quasi-isotropic design).**

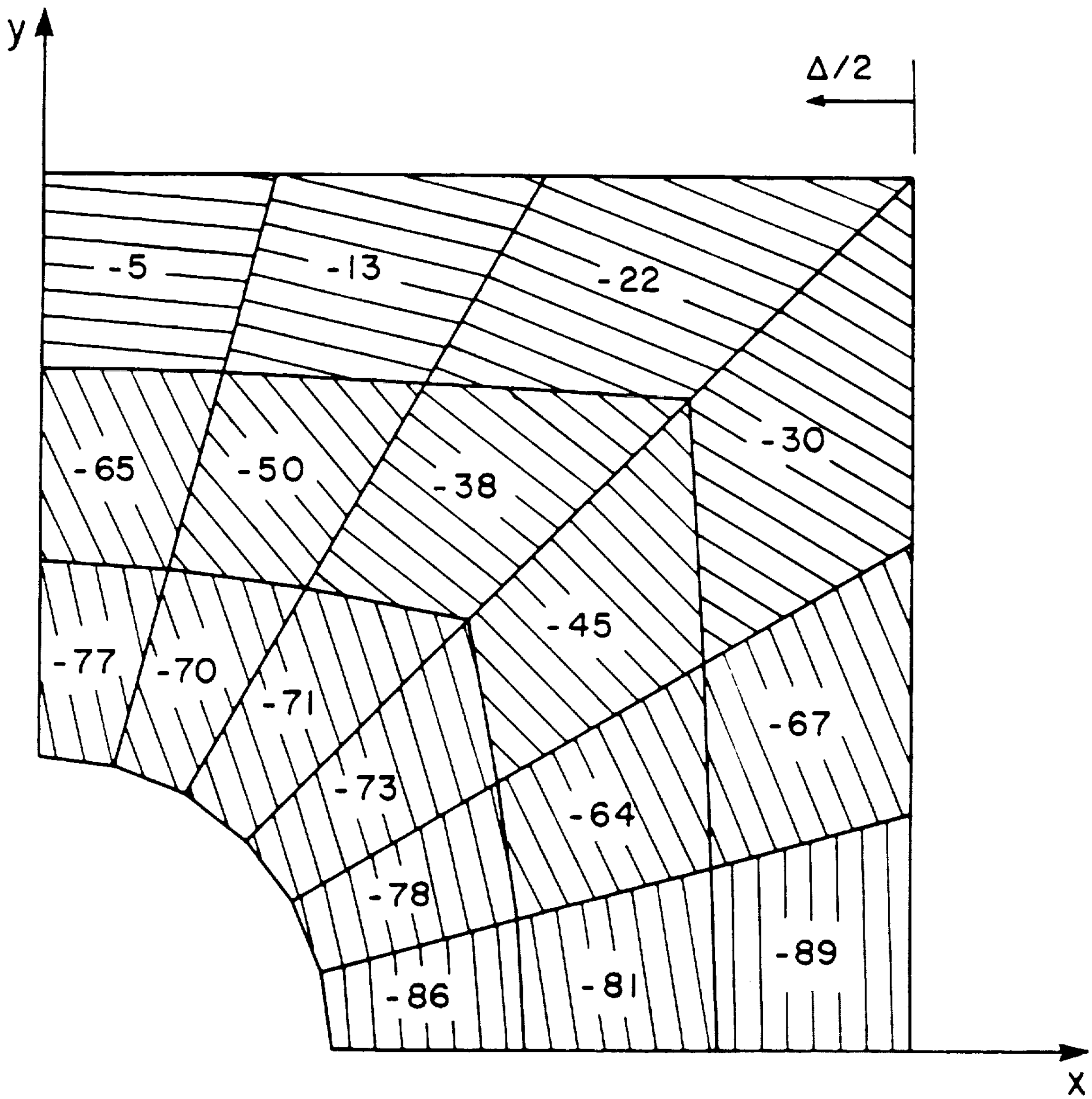


Fig. 5 **Fiber angles in the curvilinear layers that maximize buckling load in a $(\pm 45/C_8)_s$ plate, gradient-search design with 18 regions (buckling load 2.96 times buckling load of baseline quasi-isotropic design).**

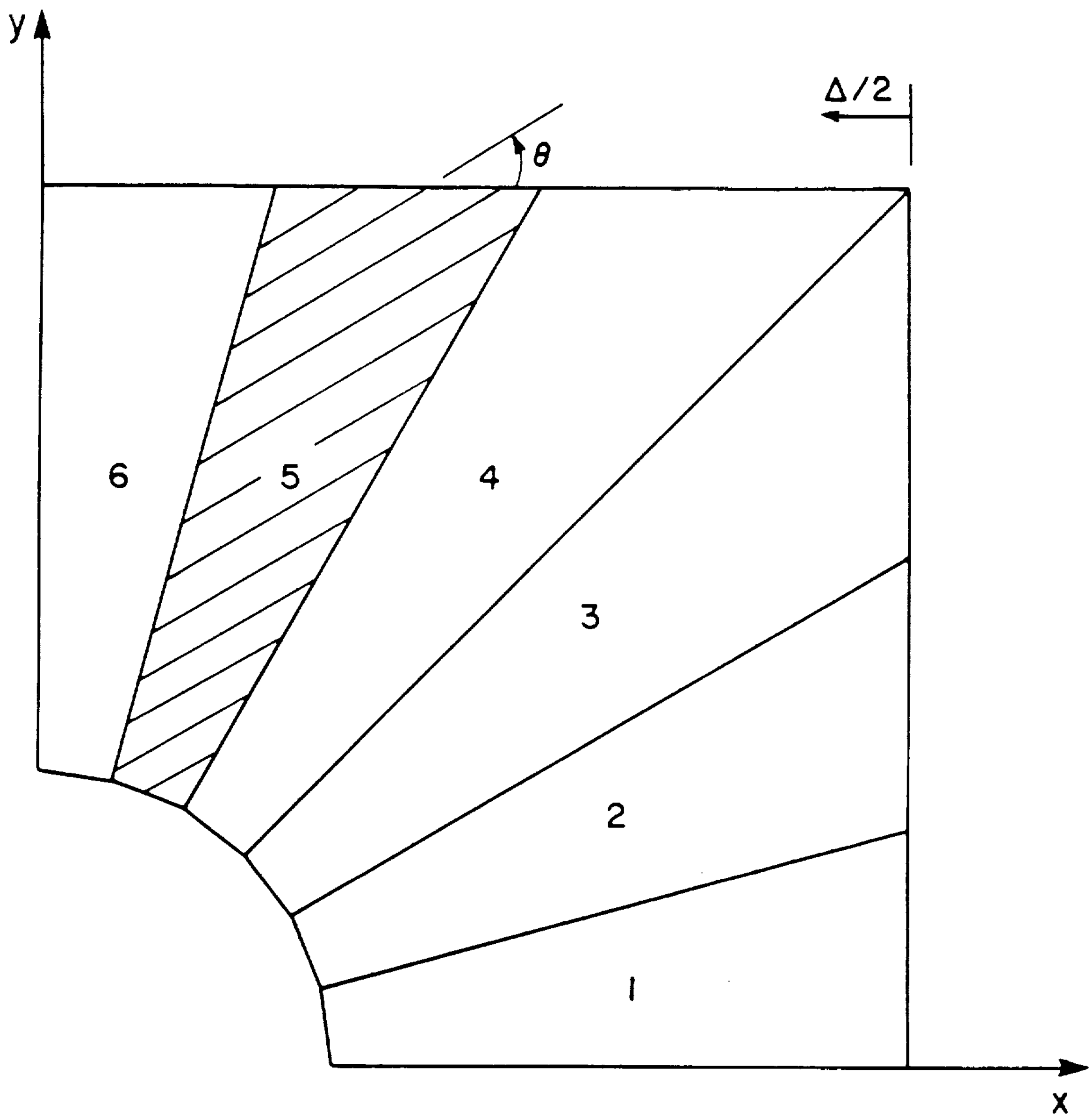


Fig. 6 Quarter plate divided into 6 radial regions.

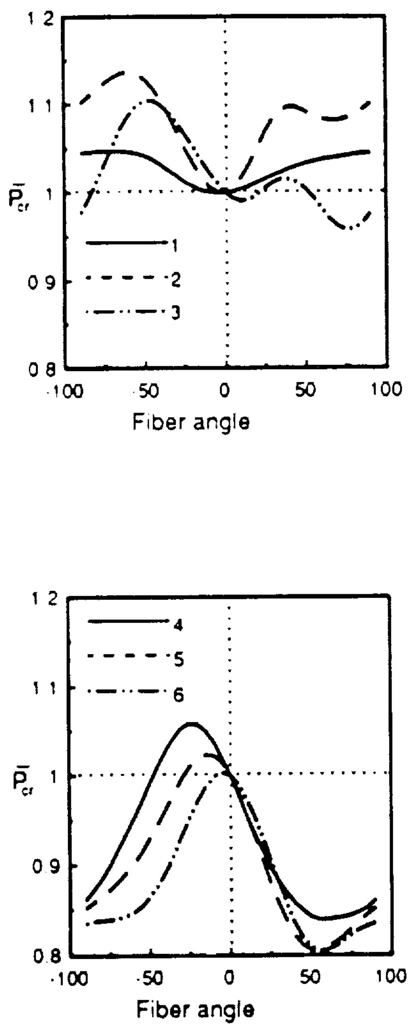


Fig. 7 **Sensitivity of buckling load to fiber angle in each of the 6 regions of a $(\pm 45/0_6)_s$ laminate.**

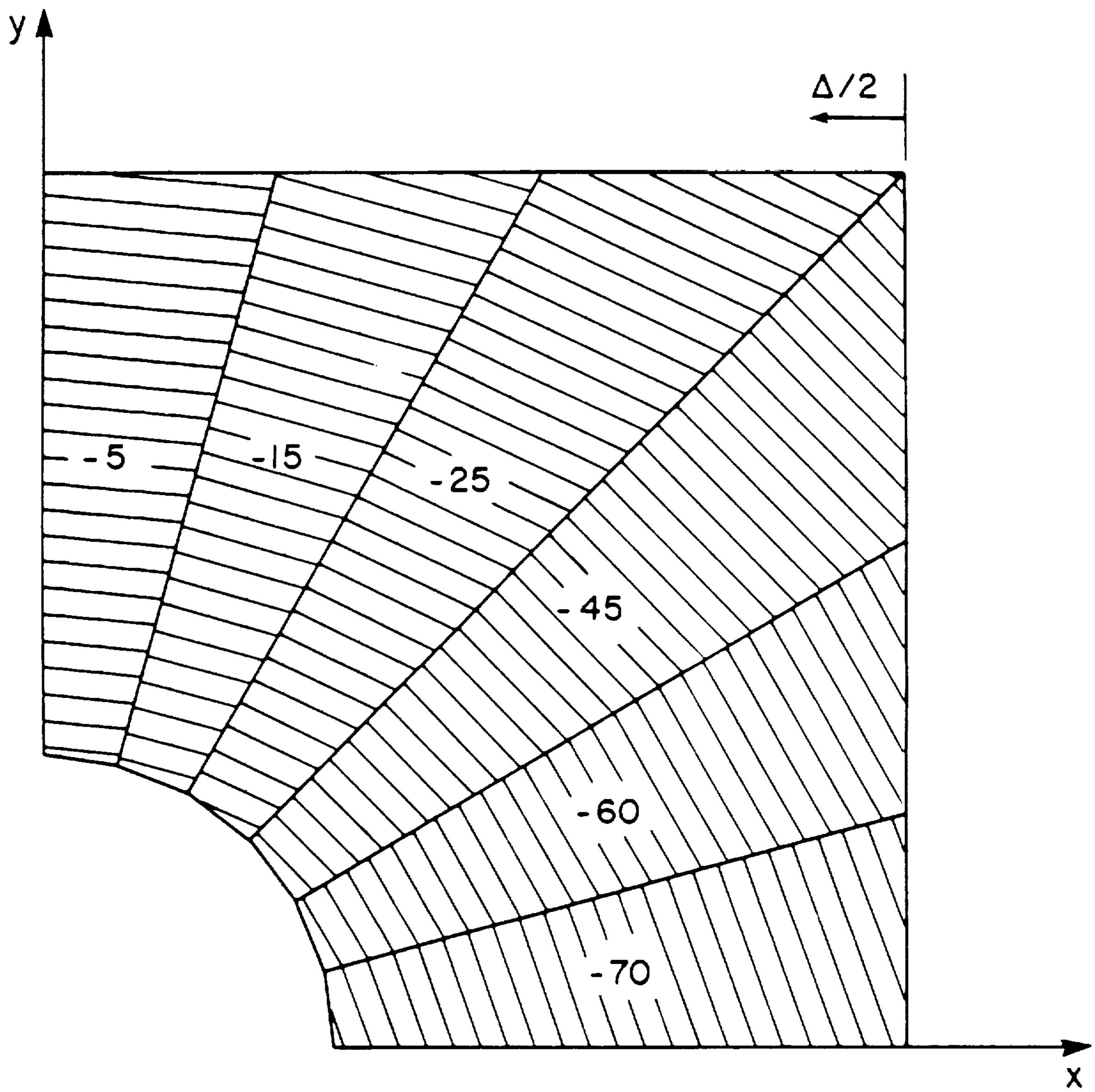


Fig. 8 Fiber angles in the curvilinear layers that maximize buckling load in a $(\pm 45/RG_6)_s$ plate, sensitivity design with 6 regions (buckling load 1.83 times buckling load of baseline quasi-isotropic design).

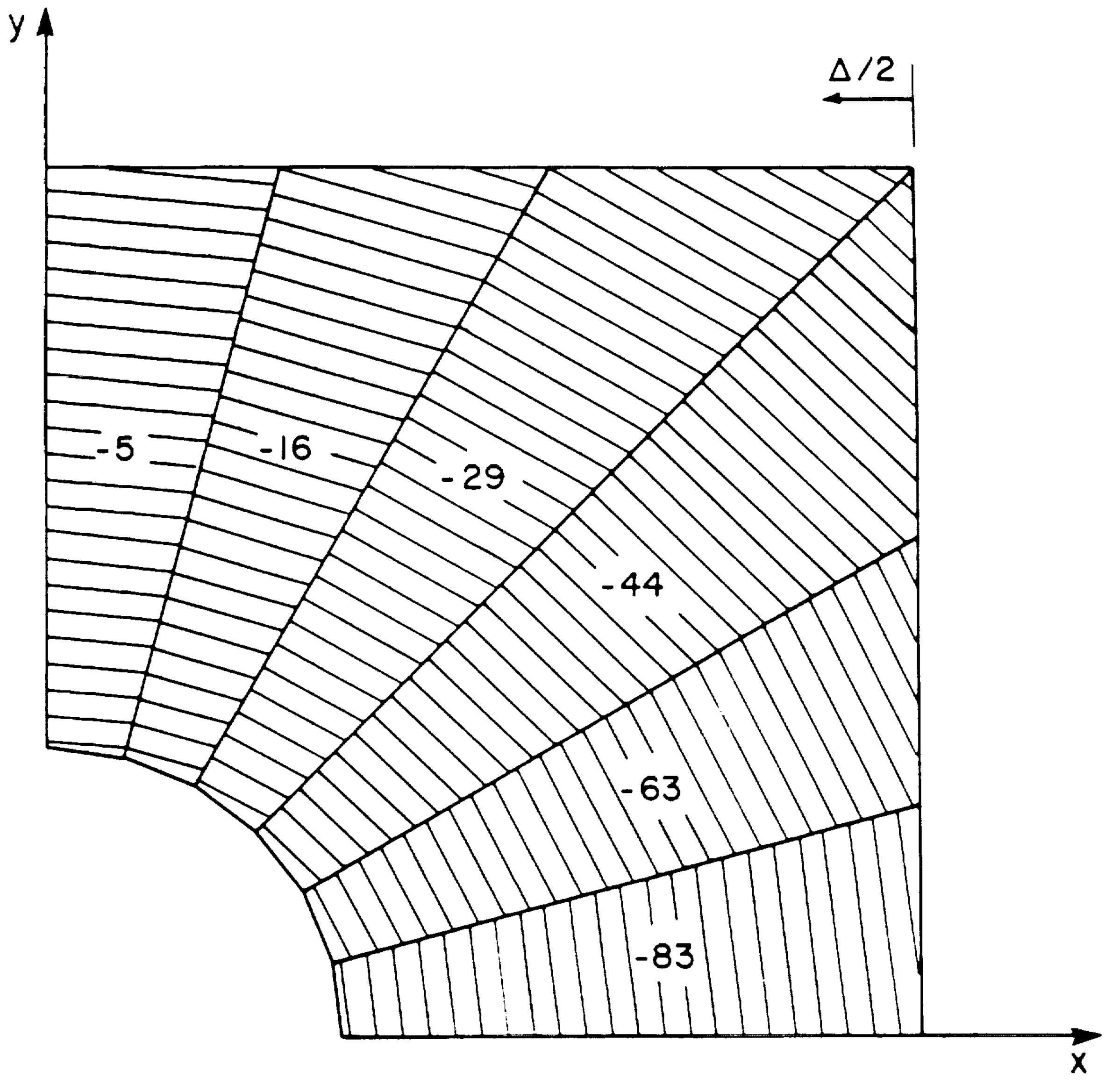


Fig. 9 Fiber angles in the curvilinear layers that maximize buckling load in a ($\pm 45/RG_6)_s$ plate, gradient-search design with 6 regions (buckling load 1.85 times buckling load of baseline quasi-isotropic design).

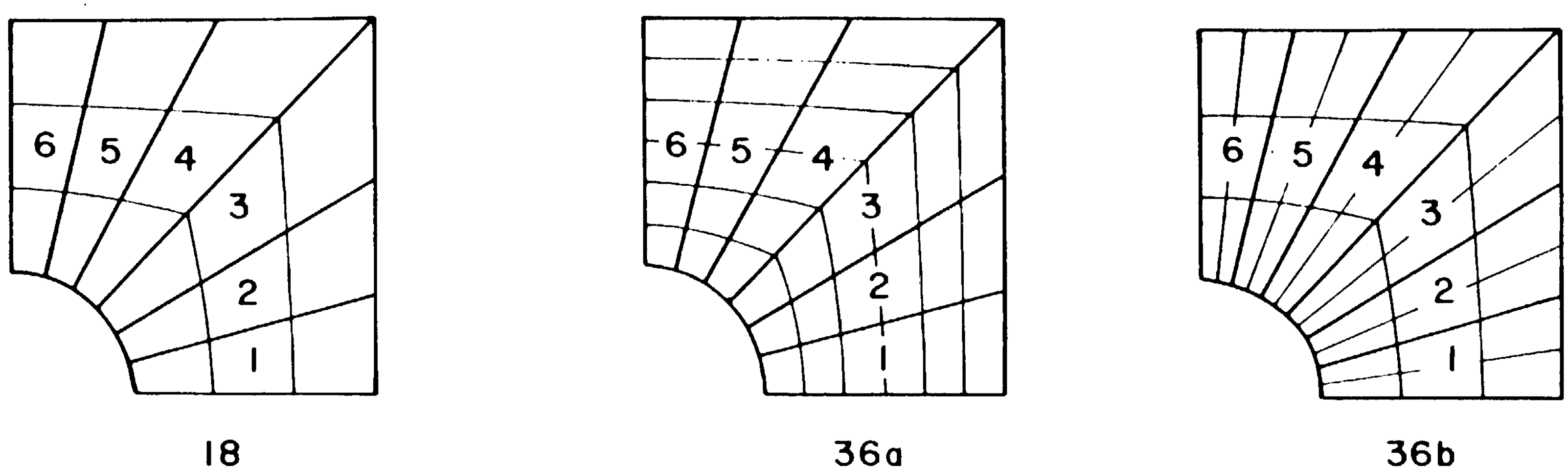
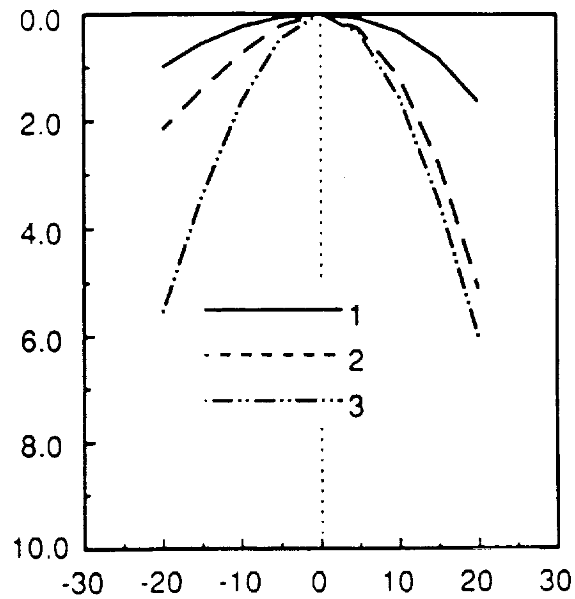


Fig. 10 Finite-element meshes used in study of convergence of the $(\pm 45/RG_6)_s$ curvilinear design (see Table 1).



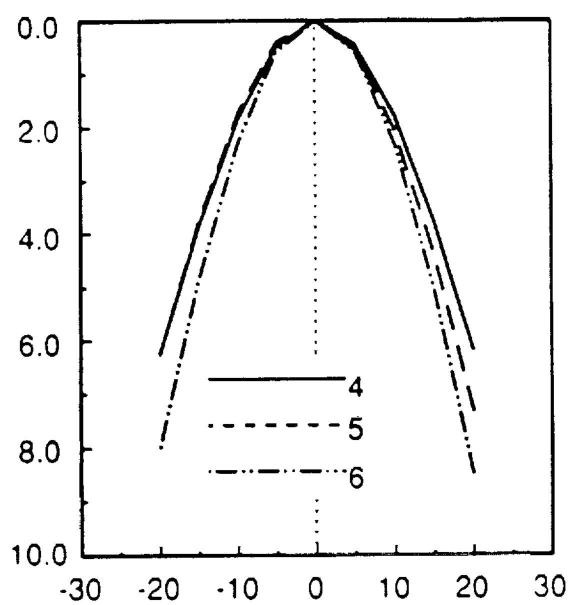


Fig. 11 Reduction in buckling load for small deviations in final design angles of ($\pm$ 45/RG_6)$_s$ design.

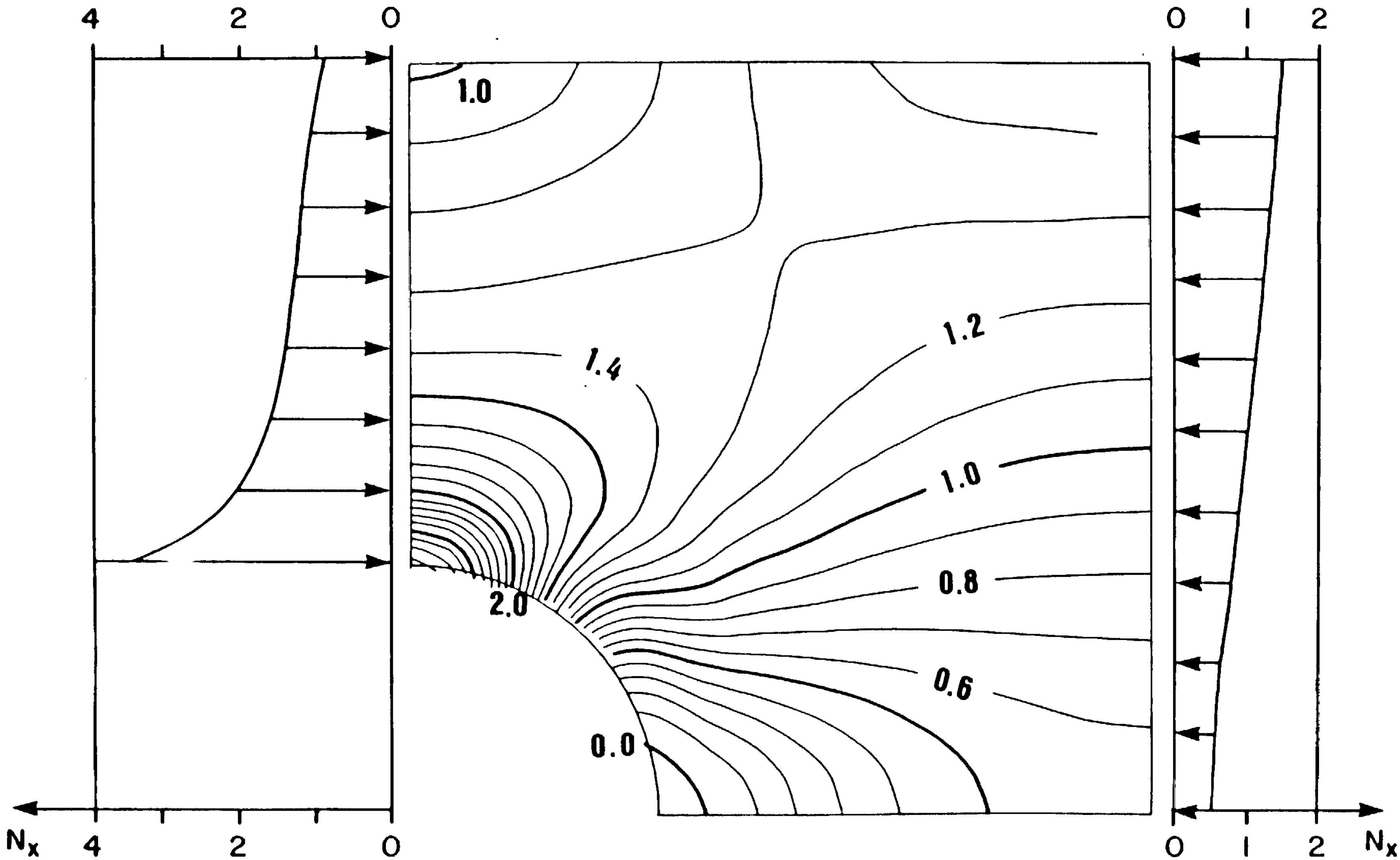


Fig. 12 Prebuckling stress resultant contours, quasi-isotropic design.

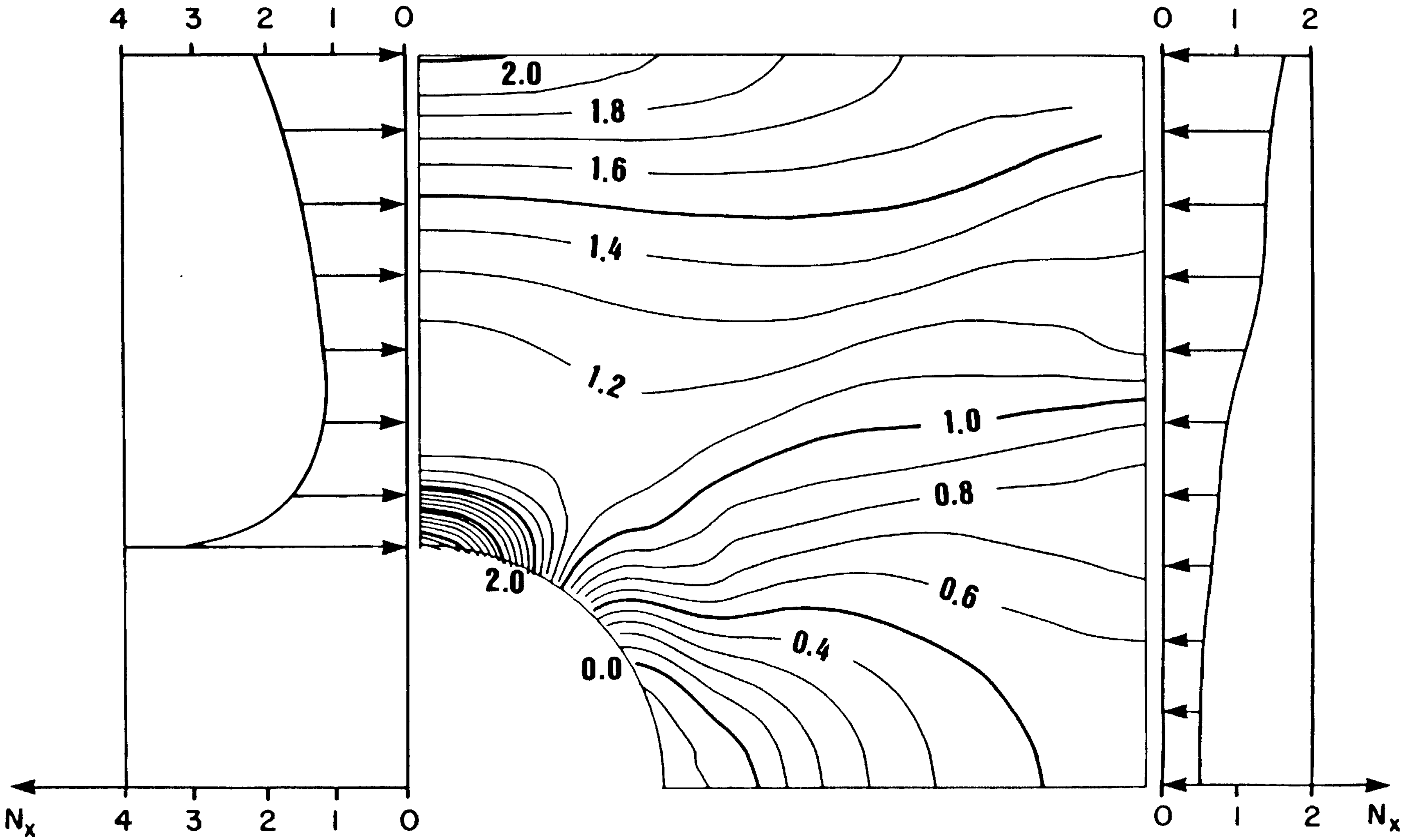


Fig. 13 Prebuckling stress resultant contours, $(\pm 45/RG_6)_S$ design.

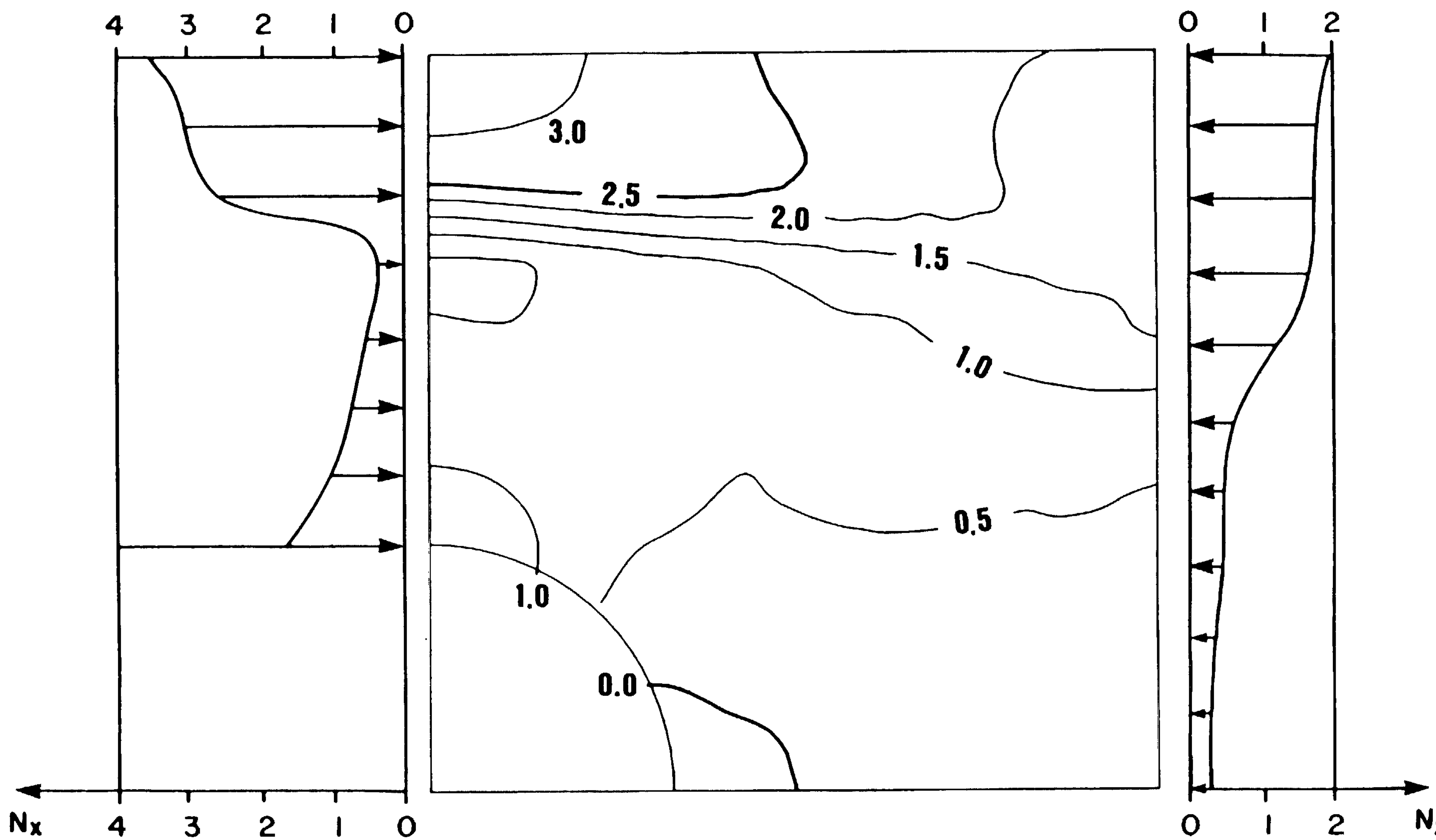


Fig. 14 Prebuckling stress resultant contours, $(\pm 45/C_6)_s$ design.