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ON ORBITAL ALLOTMENTS FOR GEOSTATIONARY SATELLITES

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Technical Report 718688-2
Grant NAG 3-159
November, 1986

National Aeronautics and Space Administration
Lewis Research Center
21000 Brookpark Rd.
Cleveland, Ohio 44135

NOTICES

50272-101

REPORT DOCUMENTATION PAGE	1. REPORT NO.	2.	3. Recipient's Accession No.
4. Title and Subtitle ON ORBITAL ALLOTMENTS FOR GEOSTATIONARY SATELLITES			5. Report Date November, 1986
			6.
7. Author(s) David J.A. Gonsalvez, Charles H. Reilly, Clark A. Mount-Campbell			8. Performing Organization Rept. No. 718688-2
9. Performing Organization Name and Address The Ohio State University ElectroScience Laboratory 1320 Kinnear Road Columbus, Ohio 43212			10. Project/Task/Work Unit No.
			11. Contract(C) or Grant(G) No. (C) (G) Grant NAG 3-159
12. Sponsoring Organization National Aeronautics and Space Administration Lewis Research Center 21000 Brookpark Rd. Cleveland, Ohio 44135			13. Type of Report & Period Covered Technical Report
			14.

15. Supplementary Notes

16. Abstract (Limit: 200 words)

The following satellite synthesis problem is addressed: communications satellites are to be allotted positions on the geostationary arc so that interference does not exceed a given acceptable level by enforcing conservative pairwise satellite separations. A desired location is specified for each satellite, and the objective is to minimize the sum of the deviations between the satellites' prescribed and desired locations.

Two mixed integer programming models for the satellite synthesis problem are presented. Four solution strategies, branch-and-bound, Benders' decomposition, linear programming with restricted basis entry, and a new switching heuristic, are used to find solutions to example synthesis problems. Computational results indicate the switching algorithm yields solutions of good quality in reasonable execution times when compared to the other solution methods.

It is demonstrated that the switching algorithm can be applied to synthesis problems with the objective of minimizing the largest deviation between a prescribed location and the corresponding desired location. Furthermore, it is shown that the switching heuristic can use no conservative, location-dependent satellite separations in order to satisfy interference criteria.

17. Document Analysis a. Descriptors

b. Identifiers/Open-Ended Terms

c. COSATI Field/Group

18. Availability Statement	19. Security Class (This Report)	21. No. of Pages
A. Approved for public release; distribution is unlimited.	Unclassified	150
	20. Security Class (This Page) Unclassified	22. Price

(See ANSI-Z39.18) See Instructions on Reverse

OPTIONAL FORM 272 (4-77)
(Formerly NTIS-35)
Department of Commerce

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CHAPTER I

INTRODUCTION

One of the major advances in the technology of communications has been the use of satellites in geostationary orbit as broadcast and relay stations for various types of communications signals. Arthur C. Clarke [1945] suggested using a chain of satellites in order to broadcast television programs over the entire earth; the orbit he suggested for these satellites has since come to be known as the geostationary orbit.

The geostationary orbit is located in the equatorial plane at a distance of 42000 km from the center of the Earth. The orbital period of a satellite in such an orbit is 24 hours; therefore, for a terrestrial observer the satellite appears to be fixed in the sky. When transmitting signals in the microwave spectrum, a transmitting station on the surface of the Earth has an effective broadcast range of perhaps 50 to 100 km, depending on the tower height and the terrain. On the other hand, a satellite in geostationary orbit can broadcast a signal beam that will cover up to one third of the globe.

In the past two decades, a number of nations, private companies, and multinational consortia of private companies have placed satellites in the geostationary orbit in order to improve their communication capabilities. Quite soon it was realised that the unique geostationary orbit and the usuable frequency spectrum are limited resources. Hence, by international agreement, it was decided that a series of World and Regional Administrative Radio Conferences (e.g., WARC-77, RARC-83, WARC-85, WARC-88), facilitated by the International Telecommunications Union (ITU), would generate allotments which would specify how these resources are to be distributed among the nations of the world.

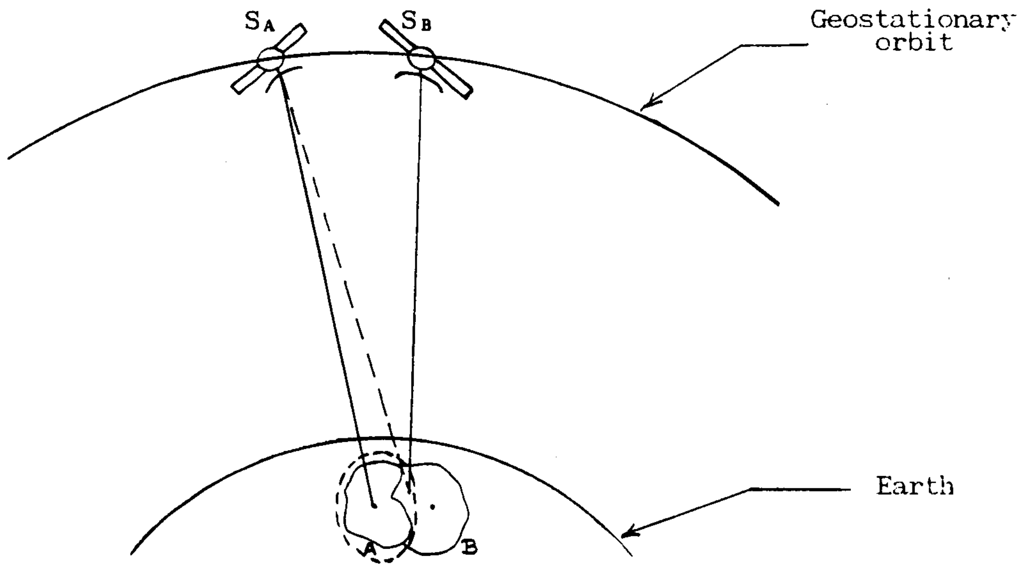
Problems in resource allocation are frequently solved by applying optimization techniques to appropriate mathematical programming models. The resource allocation problems associated with satellite deployment in the geostationary orbit can be modelled as mathematical programs. Consequently, the application of optimization techniques in the field of satellite system planning has aroused considerable interest in the international communications community.

The optimization problems associated with the distribution of the orbit/frequency spectrum resource are considered to be extremely difficult to solve. The complex interactions between the system geometry, the size, shape, and location of the areas (nations, portions of nations, or combinations of nations) being serviced, and the nature of electronic transmissions make developing and solving an optimization model of the entire system difficult. The system geometry, the service areas, and the equipment used for transmission and reception determine the strength of the signals from transmitting satellites that are received at earth station receivers in the service areas. Hence, these factors also determine the "adequacy" of the system.

For the Broadcasting Satellite Service (BSS), a scenario is defined as adequate if the resulting overall equivalent protection margins, calculated in decibels (dB) at suitable receiver locations, are all positive numbers. These margins are a complicated function of the desired signal strength, the strengths of the interfering signals, and the frequency separations between the carrier of the desired signal and those of the interfering signals (WARC ORB-85 [1985]). Wang [1986] has shown that this requirement is the same as requiring the ratio of the desired carrier signal to an equivalent interfering signal to exceed a threshold value. In the case of all satellites transmitting at the same frequency (only co-channel interference), the equivalent interfering signal power is the same as the aggregate interference power, i.e., the sum of the powers (in Watts) of all the interfering signals. In this case, the requirement for positive equivalent protection margins is satisfied if the ratio of the desired signal power C to the aggregate interference power I (aggregate C/I ratio) exceeds the required protection ratio.

No international agreement has been reached on the procedure to be adopted for the Fixed Satellite Service (FSS), but the same type of requirement appears applicable for the down-link calculation. The problem addressed here is the optimization of FSS orbital allotments on the basis of only down-link considerations.

In Figure 1 a simplified system is shown. There are two service areas A and B with their boundaries indicated by continuous curves. Each has its own satellite, denoted by S_A and S_B, respectively. A signal broadcast from S_A to service area A has some desired strength at



———	desired signal path
- - -	interfering signal path
A , B	- service areas
S_A , S_B	- satellites broadcasting to service areas A and B respectively

Figure 1. A Simplified Satellite System and Interference Geometry

all points on the dotted ellipse surrounding A. The strength of the signal intensifies as one moves toward the center of the ellipse. Since a portion of the ellipse covers parts of service area B, if satellite S_B were transmitting to country B at the same frequency and polarization as S_A, then there would be considerable interference from S_A in country B.

The reduction of such spillover interference to as low a level as possible, or at least to a level that satisfies some desired protection ratio, is an essential part of the optimization problems associated with satellite communications in the geostationary orbit. Simultaneously, requests for satellite orbit locations and frequency channel assignments that are made by the administrations of service areas have to be satisfied to the greatest extent possible.

The positioning of satellites, the allocation of frequencies, and the assignment of polarizations, have a major economic impact on most nations of the world. It is surprising that, although the installation of a system of broadcast satellites can cost upward of a billion dollars, there have been only a few efforts to optimize the orbit locations of these satellites and the frequency assignments.

In this research, the focus is on one of the optimization problems associated with the allotment of communications satellites, namely, the satellite location problem. This problem is defined here as the minimization of the deviation of assigned satellite locations from given desired locations, subject to meeting the required protection ratio. Four methods for solving this problem are presented : mixed integer programming, Benders' decomposition, a restricted basis entry procedure, and a switching heuristic. Solutions are obtained for some real problems using the four methods, and the performance of the four solution techniques is evaluated. A parallel is drawn between the formulation of the satellite location problem and some classes of scheduling problems.

In Chapter 2, the satellite location problem is formulated as a nonlinear mathematical programming problem. The concept of minimum required satellite separation, which is essential to the development of the formulations and solution methods described in this manuscript, is presented in Chapter 2. A review of other satellite allotment problems and associated solution methods which have appeared in the literature is provided in Chapter 2.

The four solution methods are discussed in detail in Chapter 3, along with a review of past work with similar techniques. In the same chapter, other mathematical programming formulations for the satellite location problem are presented. These formulations avoid the nonlinearities in the formulation given in Chapter 2 through the use of variables restricted to integer values or through the implementation of complementary variables (pairs of variables, one of which always has to be zero).

The specific details of the implementation of the four solution methods in computer codes are presented in Chapter 4. Experimental results obtained using the solution methods on a set of test problems are also discussed in the fourth chapter.

In Chapter 5, the versatile nature of the fourth solution method, the switching heuristic, is explored. Its application to alternate formulations of the satellite location problem with different objective functions or with feasible arc limitations is considered. The ability of the heuristic to allow for minimum required orbital separations that vary with satellite location is also discussed in the fifth chapter. Experimental results for these variations are presented.

Chapter 6 consists of a summary of the research and its contributions to the area of satellite system synthesis and the field of Operations Research. Recommendations for future work in the application of the switching heuristic to problems related to the satellite location problem are also presented in the same chapter.

CHAPTER II

PROBLEM DESCRIPTION AND BACKGROUND

In Broadcasting Satellite Service (BSS) and Fixed Satellite Service (FSS) system synthesis, a common objective is to provide every user with signals that are adequately protected from interference by other users (Christensen [1981], WARC [1985]). There are two basic ways that a system can be designed to accomplish this. The first way adequate protection can be achieved is by providing sufficient physical separation in orbit between the satellites that interfere with one another. The second option is to provide sufficient frequency discrimination through the allocation of adequately separated channels to possible interferers. The allotment procedure, therefore, has to specify orbit locations and frequency channel assignments for satellites so as to achieve the goal that every desired signal is adequately protected from interference. Constraints, such as the satellite locations having to be within the arc visible from the corresponding earth stations and the channel assignments having to be made from a given frequency spectrum, are also part of the satellite system synthesis. In addition to meeting the above criteria the system design will have to satisfy, to the greatest extent possible, various requirements that might be requested by the governments of the countries involved. Otherwise, there would be little hope of reaching international agreement on any allotment scheme.

The amount of separation that is required between satellites and between frequency channel assignments to provide the required protection from interference is a function of the design parameters of the satellite and the receiving earth station. Since various nations have vastly differing economic and technological capabilities, technical design parameters are largely left to the discretion of the individual administrations, subject to meeting certain standards agreed to by all the participating nations. At present, the technical design parameters for individual administrations are not available as variables that can be adjusted to optimize the system design. For the purposes of this research, a fixed and known level of technology for all satellites and earth stations is assumed.

The problem under consideration is the satellite location problem. In essence, this problem involves the assignment of satellite locations to administrations so as to meet signal adequacy criteria, subject to visible arc and elevation angle requirements. Frequency allocation is not treated as part of the problem -- the assumption is either that the full frequency bandwidth is assigned to all administrations or that the frequency assignments are pre-specified.

If the system synthesis problem can be solved through the assignment of satellite locations only, then that solution is preferable to one which assigns both satellite locations and frequencies. WARC ORB-85 [1985] tentatively recommended to the second session of the conference that each ITU member should receive at least one allotment consisting of an orbital position with which a bandwidth of 800 MHz is associated in certain up-link and down-link bands totalling 1600 MHz of bandwidth. This would appear to imply full spectrum use in the case of at least these allotments. Hence the assumption about full bandwidth frequency assignments is reasonable.

There has been a considerable amount of work done in the area of frequency allocation, mostly in the area of ground based radio communication. The satellite location problem is the focus of this research, partly because it has not been studied as extensively as the frequency allocation problem and partly because of the computational advantages associated with the solution strategies which can be used to solve it.

2.1 Mathematical Programming Formulation of the Satellite Location Problem

In the course of this research, several mathematical programming techniques are applied in order to solve the satellite location problem. In Figure 2, a basic mathematical formulation of the problem is presented. Variables and parameters are defined along with the formulation in Figure 2. Later, this formulation is modified and built upon. The nature of the problem, the assignment of satellite locations satisfying the signal adequacy criteria, will remain the same throughout this manuscript.

$$\text{MINIMIZE} \sum_{j=1}^{n} (X_j^+ + X_j^-) \qquad (2.1)$$

subject to

$$X_j + X_j^+ - X_j^- = d_j \qquad j=1,\ldots n \qquad (2.2)$$

$$| X_i - X_j | \geq \Delta S_{ij} \qquad i=1,\ldots n-1 \quad j=i+1,\ldots n \qquad (2.3)$$

$$E_j \leq X_j \leq W_j \qquad j=1,\ldots n \qquad (2.4)$$

$$X_j, X_j^+, X_j^- \geq 0 \qquad j=1,\ldots n \qquad (2.5)$$

where

X_j = actual location of satellite j

X_j^+ , X_j^- = distance between assigned and desired locations of satellite j to the east and west, respectively

d_j = desired location for satellite j

ΔS_{ij} = minimum orbital separation required between satellites i and j

E_j , W_j = eastern and western limits, respectively, on feasible locations for satellite j

n = number of satellites

$| a |$ = absolute value of a

Figure 2. Mathematical Formulation of the Satellite Location Problem

The objective function (2.1) used here and for most of the research described herein is the minimization of the sum of the deviations of the assigned locations of satellites from given desired locations. From now on, this objective will be referred to as minimizing the total deviation.

Constraints (2.2) measure the deviations of the assigned locations from the corresponding desired locations. Constraints (2.3) enforce the required minimum satellite separations ($\triangle S$) between pairs of satellites. Constraints (2.4) require a satellite to be located within a given feasible orbital arc. Constraints (2.5) are non-negativity constraints on the decision variables.

The absolute value of the difference between two decision variables appears in constraints (2.3). As a result the program is nonlinear. The measurement of this absolute value in terms of the decision variables is what makes the problem difficult to solve in the context of mathematical programming. In Chapter 3, alternate nonlinear and mixed integer programming models for the same problem are presented.

This model also uses the concept of the minimum required orbital separation between pairs of satellites ($\triangle S$). This concept simplifies mathematical programming formulations for the satellite location problem considerably, since without it all the complex calculations to determine interferences would have to be included in the formulation. In a sense, the calculation of $\triangle S$ acts like a preprocessor allowing the mathematical programming techniques to focus on the optimization rather than on the demanding interference computations. Levis et al.

[1983a] provide a formulation that includes the interference calculations, and the extreme complexity of that formulation as compared to the simplicity of formulations using $\triangle S$ is easily observable. In the next section, the satellite separation concept is discussed in more detail.

Finally, it should be noted that all satellite locations are required to be non-negative. This implies a rescaling of parameters from the conventional negative and positive longitude system (-180° to +180°). Here, the easternmost boundary among all the feasible arcs is designated as 0 degrees, and all other longitudes are correspondingly adjusted.

2.2 The concept of minimum required satellite separation ($\triangle S$)

The minimum separation required between a pair of satellites i and j, in order to provide adequate co-channel signal interference protection to each from the other at ground stations on the boundaries of their respective service areas, is denoted by $\triangle S_{ij}$. By using these separation values for all pairs of satellites, the nonlinearities and trigonometric functions arising from the system geometry and the antenna and frequency discrimination functions can be avoided in the mathematical programming formulations.

Christensen [1981] and Ito et al. [1979] have devised solution methods for satellite synthesis problems which use the concept of minimum orbital separation matrices. In this research, the separation concept as developed by Wang [1986] is used. Wang defines the minimum required orbital separation between two satellites i and j as

$$\triangle S_{ij} = \max_{k \in K} \{ \triangle \phi_{ijk} \} \tag{2.6}$$

where K corresponds to a set of locations equally spaced in the common feasible arc of satellites i and j, $\triangle\phi_{ijk}$ is the required separation between satellites i and j when the separation is centered at location k, and this separation is such that the signals of both satellites are sufficiently protected from a single entry co-channel interference level which has been selected as likely to result in meeting aggregate interference protection ratio requirements. If the feasible arcs for the two satellites do not overlap, the required separation is calculated with the center of the separation being the midpoint between the closest endpoints of the two feasible arcs. The iterative procedure for calculating $\triangle S$ is described in Wang [1986].

There are two issues that have to be considered with the definition of the minimum required orbital separation given above.

1. As indicated earlier, $\triangle S_{ij}$ is the maximum of $\triangle\phi_{ijk}$ over the feasible arcs of satellites i and j. A functional relationship can be established between $\triangle\phi_{ijk}$ and the longitude in the center of the separation between the satellites i and j, and for most pairs of satellites the shape of the function is as shown in Figure 3.

It can be seen that the variation in $\triangle\phi$ is not great when the satellites move from a position directly over the service areas. The increase in $\triangle\phi$ is more rapid at low elevation angles. Occasionally the function is concave rather than convex - this occurs when the

Figure 3. Variation in $\Delta\phi$ with Satellite Location

service areas being served by the satellites are separated by latitude but not by longitude (Yamamura and Levis [1985]).

In the mathematical programming formulations developed for the satellite location problem, the maximum required separation value $\triangle S$ is used. As a result, the solutions obtained will always satisfy single entry interference requirements, but might be conservative in that the actual separations between satellites may exceed the separation required to meet the signal protection ratio criterion for the assigned satellite locations. The case might also arise that while the actual problem has a feasible solution our formulations might be unable to find it, owing to the usage of the conservative maximum separation $\triangle S$. In Chapter 5, some comparison studies in the use of $\triangle \phi_{ijk}$'s in place of $\triangle S_{ij}$'s are discussed.

2. The calculation of $\triangle S$ values considers only single entry co-channel interference, and, therefore, any solution found may not meet the equivalent margin requirements for aggregate multi-channel protection. This problem can be alleviated to some extent by increasing the single entry protection ratios so as to make it likely that the required aggregate multi-channel protection will be achieved.

2.3 Literature Review

In this section, previous work done on the problems of satellite location and frequency allocation is reviewed.

At the preparatory seminar for the 1983 RARC (Regional Administrative Radio Conference) in Ottawa, Canada, in 1981, the Canadian delegation presented a method for generating BSS allotments. The overall objective was to assign orbital locations, frequencies and polarizations to service areas so that the capacity of the spectrum/orbit resource was optimally used, subject to protection ratio and technical constraints. The method was a combination of manual synthesis and automated heuristic procedures. Christensen [1981] provides a comprehensive review of the entire system and its development. The system was based on the ideas of Chouinard and Vachon [1981] and Nedzela and Sidney [1981]. It involved an initial generation of service area clusters by the user from which transmission discrimination and minimum required orbital separation matrices were generated. Channels and polarizations were manually assigned and the plan was checked for satisfaction of protection ratio requirements. At this point, the user had several choices - a neighbourhood search, manual assignment changes, generation of new cluster configurations, or termination. The system was tested on some BSS problems with limited success. No results of applications to actual problems are available.

Levis et al. [1983a] formulated a nonlinear programming model for BSS synthesis and developed a gradient search procedure that was applied to some small BSS problems. They attempted to assign orbit locations and frequencies so as to minimize the worst protection ratio violation. Reilly et al. [1986] implemented a cyclic coordinate method for the same model, and performed an experiment to assess the performance of the two methods on synthesis problems. They found that the cyclic coordinate method outperformed the gradient search technique for the set of BSS test problems used in the experiment.

Ito et al. [1979] proposed two methods for the optimization of satellite locations in the geostationary orbit. These methods have been applied to small problems with some success and led to the development of ORBIT-II, an orbit spacing minimization program [1984]. They formulate the problem as a nonlinear programming problem which attempts to minimize the length of the orbital arc occupied by the satellites to be positioned, subject to meeting single entry and aggregate interference criteria. The first solution method uses a penalty function algorithm which is commonly used in nonlinear programming, and the second uses successive linear approximation of non-linear constraints.

In the literature on satellite system synthesis, the approaches presented by Christensen and Ito et al. are the only methods that have actually been applied to the satellite location and frequency allocation problems. No computational experience is reported for problems involving more than 10 satellites with any of these methods.

Ottey et al. [1986] have proposed the use of several optimization techniques on a set of variations of the satellite location and frequency allocation problems - the variations mainly being in the objective function. They do not indicate whether any attempt at actual implementation of the methods they propose has been made.

In actual scenario generation, there has been considerable use of interactive synthesis methods. These methods essentially consist of an experienced system designer generating a scenario which is then evaluated using an analysis program. The designer can vary the scenario based on the results of the analysis. This method was used in generating the BSS scenario for Region 2 (North, South and Central America) at RARC '83.

The frequency allocation portion of the problem has received considerable attention in the literature, since it is similar to the frequency allocation problems of land-based radio broadcasting.

Cameron [1977] considered the problem of assigning a single channel to each service area so as to minimize the number of channels required, subject to restrictions on co-channel assignments to interfering service areas. He observed that the problem could be formulated as a graph colouring problem, and proposed solving it by solving a sequence of minimum cardinality set covering problems. Baybars [1982] formulated the same problem as a 0-1 programming problem and added additional constraints to restrict adjacent channel assignments. He used graph theoretic results to establish bounds on the solution and presented empirical results for some small problems.

Zoellner [1973] investigated frequency assignment strategies under the condition that the assigner does not possess prior information about successive cases that will require assignment. Zoellner and Beall [1977] assume complete knowledge of all the assignment cases that must be accommodated. In both studies node colouring order based assignment procedures are used.

Levis et al. [1983b] indicate alternate set covering formulations for the frequency assignment problem. They also provide a formulation for multiple channel assignments.

Mathur et al. [1985] develop integer programming models for the frequency assignment problem with the constraints that intermodulation interference is maintained within desirable limits. They also consider the incremental problem of adding to an existing network. The context of their work is ship-to-ship transmissions, but the models can be applied to satellite transmissions.

Mizuiki et al. [1984] propose a method of evaluating co-channel interference in terms of the margin with respect to the minimum required carrier-to-interference protection ratio. They generate an interference matrix and use it to allocate frequencies with an assignment problem approach.

For analysis purposes, a program such as SOUP [1983] (Spectrum Orbit Utilization Program) can be used to evaluate scenarios. The program's features make it extremely useful in analysing a solution generated by a synthesis procedure. SOUP is also capable of measuring the impact of using satellite separations based on single entry co-channel interference, e.g. $\triangle S$, to find solutions to synthesis problems where aggregate interference levels have to be below threshold values.

CHAPTER III

THE FOUR SOLUTION TECHNIQUES

In this chapter, four solution techniques for the satellite location problem are presented. These techniques are mixed integer linear programming, Benders' decomposition, linear programming with restricted basis entry, and a switching heuristic. In each case, the corresponding mathematical programming formulations are discussed, and past applications of the technique, and variations thereof, that have appeared in the literature are reviewed. The specific implementations and modifications that are used in this research are also mentioned. The minimization of total deviation is the objective function used for the satellite location problem throughout this chapter.

3.1 Mixed Integer Linear Programming

Mixed integer linear programming, hereafter referred to as mixed integer programming or MIP, is a widely used technique in the field of mathematical programming. Garfinkel and Nemhauser [1972] provide a comprehensive summary of most of the common classes of problems that can be solved using this method and an in-depth review of several implementation strategies. The most common strategy used for problems with no special structure is that of enumeration, which is usually implemented using some version of a branch and bound technique.

The satellite location problem, as formulated in Figure 2, is not an MIP formulation since it is not linear and none of the decision variables are restricted to integer values. To convert that formulation into a mixed integer program the non-linear constraints are replaced with a combination of integer variables and linear constraints. This MIP formulation is given in Figure 4. The non-linear constraints (2.3) of the formulation in Figure 2 are replaced by the constraints (3.3) and (3.4) and the binary variables denoted by Y_{ij}.

The objective function in the MIP (3.1) is identical to that in the first formulation (2.1). Constraints (3.2), (3.5), and (3.6) are identical to constraints (2.2), (2.4), and (2.5), respectively, in Figure 2. Together, constraints (3.3), (3.4), and (3.7) enforce the minimum required orbital separation between pairs of satellites.

An advantage of using an MIP model is that the optimum solution is always found if the given problem is feasible. The drawback is that the computational requirements for solving the problem tend to grow exponentially as the problem size (in terms of the number of integer variables) increases. By applying this technique to small problems a standard, the optimum solution, is obtained against which the performance of heuristic methods can be compared.

The model shown in Figure 4 can be reformulated into a more elegant model by rescaling. The advantage of this reformulation (Figure 5) lies in the fact that the constraint matrix consists of

$$\text{MINIMIZE} \quad \sum_{j=1}^{n} \left(X_j^+ + X_j^- \right) \tag{3.1}$$

subject to

$$X_j + X_j^+ - X_j^- = d_j \qquad j=1,\ldots n \tag{3.2}$$

$$X_i - X_j + (e-w-\triangle S_{ij})\, Y_{ij} \geq (e-w) \qquad \begin{matrix} i=1,\ldots n-1 \\ j=i+1,\ldots n \end{matrix} \tag{3.3}$$

$$-X_i + X_j - (e-w-\triangle S_{ij})\, Y_{ij} \geq \triangle S_{ij} \qquad \begin{matrix} i=1,\ldots n-1 \\ j=i+1,\ldots n \end{matrix} \tag{3.4}$$

$$E_j \leq X_j \leq W_j \qquad j=1,\ldots n \tag{3.5}$$

$$X_j ,\ X_j^+ ,\ X_j^- \geq 0 \qquad j=1,\ldots n \tag{3.6}$$

$$Y_{ij} = 0 \text{ or } 1 \qquad \begin{matrix} i=1,\ldots n-1 \\ j=i+1,\ldots n \end{matrix} \tag{3.7}$$

where

X_j , X_j^+ , X_j^- , d_j , $\triangle S_{ij}$, E_j , W_j , n are as defined in Figure 2.

$e = \min \{E_j\}$; $w = \max \{W_j\}$; $j=1,\ldots n$

$$Y_{ij} = \begin{cases} 1 & \text{if satellite } i \text{ is located to the west of satellite } j \\ 0 & \text{otherwise} \end{cases}$$

Figure 4. 0-1 Mixed Integer Formulation of the Satellite Location Problem.

$$\text{MINIMIZE} \quad \sum_{j=1}^{n} (X_j^{+} + X_j^{-}) \tag{3.8}$$

subject to

$$X_j + X_j^{+} - X_j^{-} = d_j \qquad j=1,\ldots n \tag{3.9}$$

$$X_i - X_j + Y_{ij} \leq 1 - \triangle S_{ij} \qquad i=1,\ldots n-1,\; j=i+1,\ldots n \tag{3.10}$$

$$X_i - X_j + Y_{ij} \geq \triangle S_{ij} \qquad i=1,\ldots n-1,\; j=i+1,\ldots n \tag{3.11}$$

$$E_j \leq X_j \leq W_j \qquad j=1,\ldots n \tag{3.12}$$

$$X_j , X_j^{+} , X_j^{-} \geq 0 \qquad j=1,\ldots n \tag{3.13}$$

$$Y_{ij} = 0 \text{ or } 1 \qquad i=1,\ldots n-1,\; j=i+1,,,,n \tag{3.14}$$

where

X_j , X_j^{+} , X_j^{-} , d_j , $\triangle S_{ij}$, E_j , W_j , n are as defined in Figure 2 and scaled by $\{(w - e) + \max \triangle S\}$

$e = \min \{E_j\}$; $w = \max \{W_j\}$; $j=1,\ldots n$

(e and w are computed before scaling)

$Y_{ij} = \begin{cases} 1 & \text{if satellite } i \text{ is located to the east of satellite } j \\ 0 & \text{otherwise} \end{cases}$

Figure 5 . 0-1 Mixed Integer Formulation of the Satellite Location Problem.
(Constraint matrix contains only +1,-1 or 0)

elements that are either +1,-1 or 0. This permits the constraint matrix (often called the "A" matrix) to be compactly stored, and thus allows for a more efficient handling of the basis. In mathematical terms, the two models are equivalent.

3.2 Benders' Decomposition

Benders [1962] developed a method for decomposing large scale mathematical programming problems into "master" and "sub" problems, which, when solved in an iterative fashion, yield the optimal solution to the original problem. The advantages over solving the original problem as a whole are that both the master problem and subproblem are smaller and easier to solve and may posess some special structure which facilitates their economical solution.

Benders' decomposition can be summarized as follows : A master problem and a subproblem are derived from the original problem. A master problem is solved, yielding a solution which defines a subproblem. Next, a subproblem is solved or determined to be infeasible. Dual solutions or extreme rays then define one or more constraints or "cuts" for the master problem. These cuts are added to the master problem. The new master problem is solved and the iterative process continues. Finite convergence of the algorithm follows from the finite number of possible constraints. The algorithm must terminate, either with the information that the original problem is infeasible or unbounded, or with the optimal solution, in a finite number of iterations.

The concepts involved will become clearer as the development of the Benders' decomposition model for the satellite location problem progresses. An attractive feature of this method is the availability of upper and lower bounds on the optimal objective function value. At each iteration in the solution of a minimization problem, the upper bound is the best solution found among all the subproblem solutions in the current and previous iterations. The lower bound is the optimal solution to the current master problem. A test for optimality is the condition that these bounds are equal.

In solving mixed integer programs with Benders' decomposition, it is often the case that the best decomposition strategy occurs with the master problem being a pure integer program containing the integer variables and the pure integer constraints. The subproblem is then a linear program in the continuous variables and the remaining constraints, the integer variables being held fixed at the values assigned to them in the optimal solution of the preceding master problem.

Benders' decomposition has often been used in practice. In most cases, some special structure in the master problem or subproblem is exploited and problems that were computationally intractable with respect to implicit enumeration techniques are solved quite efficiently.

Sherali and Adams [1984] apply Benders' decomposition to a set of discrete location allocation problems. They consider a master problem in which they relax some of the integrality constraints and then solve it using a partial enumeration scheme. The subproblem reduces to a transportation problem. Geoffrion and Graves [1974] designed a multi-commodity distribution system using this approach. Their problem serves as a classic example of Benders' decomposition with the original problem decomposing into a set of transportation problems when the binary variables are held fixed. Federgruen and Zipkin [1984] addressed a combined vehicle routing and inventory allocation problem with a version of Benders' decomposition.

Mount-Campbell et al. [1986] have formulated the satellite location problem as a mixed integer program. Their formulation is shown in Figure 6. The objective is the minimization of total deviation. In this formulation, satellites are assigned to positions in an ordering; the locations of the positions are simultaneously determined. Constraints (3.16) insure that there is only one satellite assigned to each position, and constraints (3.17) insure that each satellite is assigned to some position. If satellite i is assigned to position j, then the distance of position j from the desired location of satellite i is measured by constraint (3.18). The minimum required orbital separations between pairs of satellites are enforced by constraints (3.19). Constraints (3.19) also ensure that positions with a high index are west of positions with a low index. This formulation is a relaxation of the problems formulated in Figures 2 and 4 because of the absence of boundaries on the feasible arc for each satellite location.

$$\text{MINIMIZE} \quad \sum_{j=1}^{n} (Y_j^{+} + Y_j^{-}) \qquad (3.15)$$

subject to

$$\sum_{i=1}^{n} X_{ij} = 1 \qquad j=1,\ldots n \qquad (3.16)$$

$$\sum_{j=1}^{n} X_{ij} = 1 \qquad i=1,\ldots n \qquad (3.17)$$

$$Y_j + Y_j^{+} - Y_j^{-} = \sum_{i=1}^{n} d_i X_{ij} \qquad j=1,\ldots n \qquad (3.18)$$

$$Y_{j+k} - Y_j \geq \Delta S_{ih} (X_{ij} - (1-X_{h,j+k})) \qquad \begin{matrix} j=1,\ldots n-1; \\ k=1,\ldots n-j; \\ i=1,\ldots n; \\ h=1,\ldots n \end{matrix} \qquad (3.19)$$

$$Y_j , Y_j^{+} , Y_j^{-} \geq 0 \qquad j=1,\ldots n \qquad (3.20)$$

$$X_{ij} = 0 \text{ or } 1 \qquad i=1,\ldots n; \; j=1,\ldots n \qquad (3.21)$$

where

n = number of satellites and satellite orbit positions

Y_j = actual longitude of satellite orbit position j

Y_j^{+} (Y_j^{-}) = westward (eastward) distance of an orbit position from the desired location of the satellite in that position.

X_{ij} = 1 if satellite i is assigned to position j, 0 else

ΔS_{ih} = required separation between satellites i and h

d_i = desired location for satellite i

Figure 6. 0-1 Mixed Integer Program for the Satellite Location Problem (Used in Benders' Decomposition)

Mount-Campbell et al. suggest using Benders' decomposition to solve the formulation given in Figure 6. The form of the master problem and the subproblem are indicated in Figures 7 and 8, respectively. The procedure used to decompose the problem (Figure 6) is a classic illustration of Benders' decomposition. The integer variables and the corresponding constraints (3.16),(3.17) and (3.21) are retained in the master problem while the continuous variables and the constraints (3.18),(3.19) and (3.20) make up the linear subproblem. There is an interesting physical interpretation of the decomposition in this case - the master problem orders the satellites, and given the ordering, the subproblem assigns locations to positions while enforcing required orbital separation between pairs of satellites.

The dual of the subproblem is shown in Figure 9. The dual variables are used in constructing the Benders' constraint for the master problem as indicated in equation (3.26). The constraint matrix in the dual of the subproblem is unimodular. Even though the number of variables in the dual can be large for actual problems, a majority of them can be discarded because a large number of the corresponding primal constraints are redundant, once an ordering is fixed. The dual of the subproblem, therefore, is easily solvable using standard linear programming methods.

MINIMIZE Z

subject to

$$Z \geq \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^{q} X_{ij} + c^{q} \qquad q=1,\ldots S \qquad (3.22)$$

$$\sum_{i=1}^{n} X_{ij} = 1 \qquad j=1,\ldots n \qquad (3.23)$$

$$\sum_{j=1}^{n} X_{ij} = 1 \qquad i=1,\ldots n \qquad (3.24)$$

$$X_{ij} = 0 \text{ or } 1 \qquad i=1,\ldots n; j=1,\ldots n \qquad (3.25)$$

where

S = number of Benders' constraints generated so far

Z = "dummy" variable

X_{ij} is as defined in Figure 6

a_{ij}^{q} = coefficient of X_{ij} in the $q^{\underline{th}}$ Benders' constraint obtained by collecting coefficient terms from the expanded constraint 3.26

c^{q} = collected constant terms for the $q^{\underline{th}}$ Benders' constraint

The Benders' constraint before collection of coefficients is :

$$Z \geq \sum_{i=1}^{n} \sum_{j=1}^{n} u_{j}^{q} d_{i} X_{ij} + \sum_{j=1}^{n-1} \sum_{k=1}^{n-j} \sum_{i=1}^{n} \sum_{h=1}^{n} \Delta S_{ih} (X_{ij} - (1 - X_{h,j+k})) w_{jkih}^{q} \qquad (3.26)$$

d_{i} and ΔS_{ih} are as defined in Figure 6

$$w_{jkih}^{q} = \begin{cases} v_{jk}^{q} & \text{if } X_{ij} = 1 \text{ and } X_{h,j+k} = 1 \text{ in the previous} \\ \text{assignment} \;; & 0 \text{ else} \end{cases} \qquad (3.27)$$

u_{j}^{q} and v_{jk}^{q} are the dual variables from the $q^{\underline{th}}$ subproblem (See Figure 9)

Figure 7. The Master Problem in Benders' Decomposition (at the $q^{\underline{th}}$ iteration)

$$\text{MINIMIZE} \quad \sum_{j=1}^{n} (Y_j^+ + Y_j^-) \tag{3.28}$$

subject to

$$Y_j + Y_j^+ - Y_j^- = K_j^q \qquad j=1,\ldots n \tag{3.29}$$

$$Y_{j+k} - Y_j \geq Q_{jk}^q \qquad j=1,\ldots n-1; k=1,\ldots n-j \tag{3.30}$$

$$Y_j \,,\, Y_j^+ \,,\, Y_j^- \geq 0 \qquad j=1,\ldots n \tag{3.31}$$

where

K_j^q = Right hand side of (3.18) with X_{ij} given by master problem

Q_{jk}^q = Right hand side of (3.19) with X_{ij} given by master problem

Y_j , Y_j^+ , Y_j^- are as defined in Figure 6

Figure 8. The Subproblem (Primal) in Benders' Decomposition
(at the $q^{\underline{th}}$ iteration)

$$\text{MAXIMIZE} \quad \sum_{j=1}^{n} K_j^q u_j + \sum_{j=1}^{n-1} \sum_{k=1}^{n-j} Q_{jk}^q v_{jk} \qquad (3.32)$$

subject to

$$u_1 - \sum_{j=1}^{n-1} v_{1j} \leq 0 \qquad (3.33)$$

$$u_j + \sum_{k=1}^{j-1} v_{k,j-k} - \sum_{k=1}^{n-j} v_{jk} \leq 0 \qquad j=2,\ldots n-1 \qquad (3.34)$$

$$u_n + \sum_{k=1}^{n-1} v_{k,n-k} \leq 0 \qquad (3.35)$$

$$-1 \leq u_j \leq 1 \qquad j=1,\ldots n \qquad (3.36)$$

$$v_{jk} \geq 0 \qquad j=1,\ldots n-1;\ k=1,\ldots n-j \qquad (3.37)$$

where

u_j = the dual variable corresponding to the $j^{\underline{th}}$ constraint in the set of constraints (3.29) in the primal subproblem (Figure 8)

v_{jk} = the dual variable corresponding to the $jk^{\underline{th}}$ constraint in the set of constraints (3.30) in the primal subproblem (Figure 8)

K_j^q, Q_{jk}^q are as defined in Figure 8

Figure 9. The Subproblem (Dual) in Benders' Decomposition (at the $q^{\underline{th}}$ iteration)

3.2.1 On Solving the master problem

The master problem (Figure 7) is a pure integer program except for the single continuous variable z. At each iteration one constraint of type (3.22), a Benders' constraint, is added to the master problem. This constraint is generated from the dual variable values of the solution to the subproblem at the previous iteration. In the absence of these constraints, the master problem reduces to an assignment problem, to which a solution can be found extremely efficiently. Therefore, in order to take advantage of this fact, the Benders' constraints are taken into the objective function using Lagrangean relaxation as shown in Figure 10. Fisher [1981] and Shapiro [1979] discuss the concept of Lagrangean relaxation and present thorough expositions of the subject.

The relaxed master problem has an additional set of variables- the Lagrangean multipliers. The determination of these multipliers is not easy; in Appendix A, a master problem for which optimal Lagrangean multipliers do not exist is shown.

Since a Lagrangean relaxation of the master problem is being considered, for any feasible set of Lagrangean multipliers, the objective function in the relaxation is a lower bound on the optimal solution to the master problem. Therefore, determining Lagrangean multipliers such that the Lagrangean objective function is maximised ensures the tightest possible lower bound on the solution to the master problem, and perhaps even the optimal solution. Since the optimal solution to the master problem is not obtained in most cases, the lower

MINIMIZE $$Z - \sum_{q=1}^{S} L^{q} \left(Z - \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^{q} X_{ij} - c^{q}\right) \tag{3.38}$$

subject to

$$\sum_{i=1}^{n} X_{ij} = 1 \qquad j=1,\ldots n \tag{3.39}$$

$$\sum_{j=1}^{n} X_{ij} = 1 \qquad i=1,\ldots n \tag{3.40}$$

$$X_{ij} = 0 \text{ or } 1 \qquad i=1,\ldots n; j=1,\ldots n \tag{3.41}$$

where

S = number of Benders' constraints generated so far

Z, a_{ij}^{q}, c^{q}, X_{ij} are as defined in the master problem (Figure 7)

L^{q} = Lagrangean multiplier for the $q^{\underline{th}}$ Benders' constraint
(The Lagrangean multipliers are non-negative and sum to unity)

Figure 10. The Lagrangean Relaxation Formulation of the Master Problem (at the $q^{\underline{th}}$ iteration)

bound obtained is substituted in place of the master problem optimal solution, as the lower bound on the optimum solution to the original problem.

In the following sections, the different methods that are used in trying to determine the Lagrangean multipliers are presented. All the methods are heuristic methods in the sense that none of them can guarantee finding the multipliers that yield the optimum solution to the master problem. In section 3.2.2, a method of computing upper and lower bounds on the optimum solution to a master problem is introduced. The upper bound corresponds to a feasible solution to the master problem and can be used if the heuristics cannot find a better solution.

3.2.1.1 The fixed interval line search (FILS)

In this method, a line search is performed at fixed intervals over the range [0,1] for the Lagrangean multiplier value corresponding to the latest Benders' constraint. All the previous Benders' constraints are combined into a single constraint. In creating this combined constraint, it is assumed that the Lagrangean multipliers of the previous constraints remain in the same proportion to one another in this Benders' iteration as they were at the end of the last Benders' iteration. This assumption is made with the expectation that relationships between Lagrangean multipliers remain reasonably constant, allowing the search procedure to concentrate on the determination of the value of the newest multiplier and to avoid a search for all the multipliers at each iteration.

The fixed interval line search is performed as follows :

STEP 1 : Combine all the previous Benders' constraints into a single constraint. In combining these constraints the coefficients in a constraint are weighted by the value of the Lagrangean multiplier corresponding to that constraint at the previous Benders' iteration, i.e., the contribution of each previous constraint to the "combined constraint" is proportional to its Lagrangean multiplier value at the previous iteration.

STEP 2 : Construct a Lagrangean relaxation of the master problem with the new Benders' constraint and the combined constraint. All the previous constraints are discarded in this relaxation except for their contributions to the combined constraint. Assign a multiplier "L" to the new constraint and a multiplier "1-L" to the combined constraint.

STEP 3 : Solve this Lagrangean relaxation of the master problem as an assignment problem, for values of L going from 1.0 to 0.0 in equal steps, to find that value of L which provides the tightest lower bound on the optimum solution to the master problem.

STEP 4 : Determine the actual master problem objective function value for each of the assignments generated in step 3.

STEP 5 : If any of the objective function values calculated in step 4 is between the bounds on the optimal solution to the master problem , then return to the Benders' procedure with the solution to the relaxation whose value in the master problem is closest to the lower bound on the master problem. Update the lower bound on the original problem with the largest Lagrangean objective function value found if the lower bound is thereby improved. If no solution within the bounds is found and the best assignment solution found has already been used to generate a Benders' cut, terminate the Benders' procedure or use another method to find a solution to the master problem.

3.2.1.2 The Golden Section Search Method (GSLS)

A golden section search is a sequential search strategy for problems where the objective is to determine the maximum (or minimum) of a unimodal function of one variable over a given range. When these conditions are met, it is an optimal search strategy.

The procedure in this method is as follows :

STEP 1 : Same as STEP 1 in FILS.

STEP 2 : Same as STEP 2 in FILS.

STEP 3 : Perform a Golden section search to determine that value of "L" which maximizes the solution of the Lagrangean relaxation. Simmons [1975] describes the implementation of the standard Golden section search technique used here.

STEP 4 : Same as STEP 4 in FILS.

STEP 5 : Same as STEP 5 in FILS

3.2.1.3 The Rounded Linear Programming Solution (RLPS)

This method does not use Lagrangean multipliers - instead the master problem is solved as a linear program, i.e., with the integrality constraints relaxed. The solution is rounded to give an integer assignment which is taken as the solution to the master problem.

STEP 1 : Set up the master problem with all the Benders' constraints explicitly stated. Relax the integrality constraints.

STEP 2 : Solve the linear program created in step 1.

STEP 3 : Round the continuous solution obtained in step 2 to an integer assignment as follows:

a. Determine the largest variable value and identify the corresponding row and column.

b. Set the value of this variable to 1.

c. Set all other variables in this row and in this column to 0.

d. Remove this row and column from further consideration.

Repeat a, b, c, and d until an assignment is generated. If all non-zero values are zeroed out before an assignment is obtained, complete the assignment based on the order of increasing desired locations.

STEP 4 : Return to the Benders' procedure with this assignment as the solution to the master problem.

The solution to the linear programming relaxation of the master problem is a lower bound on the optimum solution to the master problem, and hence, it is also a lower bound on the optimum solution to the original problem and is used as such.

3.2.1.4 The Subgradient Method (SGRD)

Subgradient optimization is frequently used as a method of solving problems involving Lagrangean relaxation. Held et al. [1974] describe the technique which had been discussed by Held and Karp [1971] in the context of the traveling salesman problem. Held et al. showed that subgradient optimization is effective in approximating the maximum of piecewise linear concave functions. Poljak [1967] discusses the theoretical aspects of the algorithm and presents results on the rates of convergence. Shapiro [1979] and Fisher [1981] provide excellent reviews of subgradient optimization in the context of Lagrangean relaxation.

In this method all the Lagrangean multipliers are considered explicitly - no proportionality assumptions about the multipliers of previous constraints are made.

Let Z be the optimal solution to the master problem (Figure 7), and let $Z_D(L)$ be the optimal solution of the Lagrangean relaxation of the master problem (Figure 10), where L is the vector of Lagrangean multipliers. As mentioned earlier, $Z_D(L) \leq Z$ (Fisher [1981]) and maximizing $Z_D(L)$ with an appropriate choice of L yields the tightest lower bound on Z.

$$\text{Let } Z_D = \max_{L} Z_D(L) \qquad (3.42)$$

The set of feasible solutions to the relaxation can be represented as $\{\underline{x}^t, \; t=1,\ldots T\}$. The value of T is finite, but it can be extremely large. Then

$$Z_D = \max \; w \qquad (3.43)$$

subject to

$$w \leq y - \sum_{q \in S} L^q \left(y - \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^q x_{ij}^t - c^q \right) \quad t=1,\ldots T \qquad (3.44)$$

$$\sum_{q \in S} L^q = 1 \; ; \qquad L^q \geq 0 \quad q \in S \; ; \qquad (3.45)$$

The function $Z_D(L)$ is continuous and concave and is the lower envelope of a finite family of linear functions. The function $Z_D(L)$ is non-differentiable at any L' where the Lagrangean problem has multiple optima. A vector g is a subgradient of $Z_D(L)$ at L' if

$$Z_D(L) \leq Z_D(L') + g(L - L') \quad \text{for all } L \qquad (3.46)$$

The function $Z_D(L)$ is subdifferentiable everywhere. The vector whose components are indicated in (3.47) is a subgradient of $Z_D(L)$ at any L for which $\underline{x}^t$ solves the Lagrangean relaxation.

$$-y + \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^q x_{ij}^t + c^q \qquad q=1,\ldots,|S| \qquad (3.47)$$

Convex combinations of these subgradients are also subgradients. A solution L^* is optimal (i.e. it is the vector of Lagrangean multipliers for the solution Z_D) if and only if 0 is a subgradient of $Z_D(L)$ at L^*.

The subgradient method can be stated as follows : Given an initial set of Lagrangean multipliers L_0, generate a sequence $\{ L_k \}$ with

$$L_{k+1} = L_k + t_k\,(-y_k + A\,\underline{x}_k + \underline{c}) \tag{3.48}$$

where

$\underline{x}_k$ = the optimal solution to the Lagrangean relaxation with multipliers L_k,

$$t_k = \text{scalar step size} = \frac{d_k\,(Z^* - Z_D(L_k))}{\|-y_k + A\underline{x}_k + \underline{c}\,\|^2}, \tag{3.49}$$

d_k = scalar such that $0 \leq d_k \leq 2$, $d_0 = 2$, and

Z^* = upper bound on $Z_D(L)$ generated using heuristics

In practice, d_k is halved whenever $Z_D(L)$ fails to increase in a given number of iterations. Even though the algorithm theoretically converges to the optimum value of L, practical limitations necessitate termination on a prespecified iteration limit. The values for t_k and d_k given above, and the use of an iteration limit, are suggested by both Fisher [1981] and Shapiro [1979].

3.2.1.5 The Adjustment Method

In this section, a method of finding Lagrangean multipliers based on a weighted adjustment technique is introduced. The aim here is to find a solution that lies within the bounds on the optimal solution value to the master problem, and thereby, avoid premature termination of the Benders' procedure.

STEP 1 : Set k = 0 and initialize the Lagrangean multipliers L_{ik} for all $i \in S$ where S is the set of indices for Benders' constraints.

STEP 2 : Solve the Lagrangean relaxation, an assignment problem, with the multipliers L_{ik}.

STEP 3 : Evaluate each Benders' constraint with the optimal assignment as ZB_i, $i=1,\ldots,|S|$

STEP 4 : Compute the "mean constraint value" (ZM) and the "range" (ZR) as follows :

$$ZM = (ZB_1 + ZB_2 + \ldots + ZB_s) / S \tag{3.50}$$

$$ZR = \max (ZB_i) - \min (ZB_i) \; . \tag{3.51}$$

STEP 5 : Update Lagrangean multipliers as follows :

$$L_{i,k+1} = L_{ik} * (1-WF) + ((ZB_i - ZM) / ZR) * WF \tag{3.52}$$

where WF = given weighting factor. (WF = 0.1 in experiments).

STEP 6 : Normalize the multipliers $L_{i,k+1}$ so that the multipliers sum to 1.

STEP 7 : Set k = k + 1 . If k < maximum iterations go to step 2, otherwise return to Benders' procedure with best solution found. (There is no guarantee that this solution will lie between the bounds on the optimal solution to the master problem).

This method is designed principally for finding a solution to the master problem that lies within the bounds on the optimal solution, rather than for yielding an extremely tight lower bound. An attempt to achieve this is made by weighting the Lagrangean multipliers in such a manner that the contribution of each Benders' constraint when taken into the Lagrangean objective function is approximately equal. There is no guarantee that the method will find a solution to the master problem which lies within the bounds on the optimal solution.

3.2.2 Generation of bounds on the optimal solution to the master problem

In this section, a method to determine upper and lower bounds on the optimum solution to the master problem at any iteration in the Benders' procedure is developed.

For the ease of exposition, consider the case where the master problem (Figure 7) has two Benders' constraints. Suppose two assignment problems are solved, the first with objective function $a^1x + c^1$ and the second with objective function $a^2x + c^2$ (i.e. the right hand sides of the first and second Benders' constraints respectively). Let the optimal assignments to the two problems be x^1 and x^2, respectively. Let

$$A^{11} = a^1x^1 + c^1 \quad , \quad A^{21} = a^1x^2 + c^1 \quad ,$$

$$A^{22} = a^2x^2 + c^2 \quad , \text{ and } \quad A^{12} = a^2x^1 + c^2 \qquad (3.53)$$

By optimality $A^{11} \leq A^{21}$ and $A^{22} \leq A^{12}$. Without loss of generality assume $A^{11} < A^{22}$. Then the three possible cases can be plotted on a line diagram as follows :

Case 1 : A^{11} *————————¦ A^{12}

A^{22} *————————¦ A^{21}

Case 2 : A^{11} *—————————¦ A^{12}

A^{22} *———¦ A^{21}

Case 3 : A^{11} *————————————¦ A^{12}

A^{21} ¦————* A^{22}

increasing function value ———>

These are the only three possible cases when $A^{11} < A^{22}$. The lower bound on the objective function value of the master problem in all three cases is A^{22}. If A^{22} is not the lower bound then there exists some assignment which yields an objective function value for the second problem that is less than A^{22}. But this violates the fact that A^{22} is the optimal solution value to the assignment problem with the second objective function.

The upper bound is given by the minimum of the two right endpoints in each case, i.e., A^{12} in case 1, A^{21} in case 2, and A^{22} in case 3. These are obviously upper bounds because in each case there is an assignment that satisfies all the Benders' constraints. If the upper and lower bounds are equal, as in case 3, then we have the optimum solution to the master problem.

This result can be extended to the general situation with $|S|$ Benders' constraints.

Define $$A^{kq} = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^{q} x_{ij}^{k} + c^{q} \qquad q,k \in S \tag{3.54}$$

where x_{ij}^{k} is the optimal solution to the assignment problem with objective function given by

$$\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^{k} x_{ij} + c^{k}$$

A lower bound on the master problem is given by

$$LB_z = \max_{q \in S} \{ A^{qq} \} \tag{3.55}$$

An upper bound on the optimal solution to the master problem is given by

$$UB_z = \min_{m,n \in S} \left\{ A^{mn} \;\middle|\; \begin{array}{ll} A^{mq} \le A^{mn} & \text{for all } q \in S \text{ , } q \ne n \\ A^{rq} \ge A^{mn} & \text{for some } q \in S \text{ and for all } r \in S \end{array} \right\} \tag{3.56}$$

Proof : Let the lower bound as defined by equation 3.55 be A^{pp}. If A^{pp} is not a lower bound on the optimal solution to the master problem, then there is some optimal solution to the master problem, say, $\underline{x}^{k}$ such that the right hand side of constraint p evaluated with this vector is A^{kp}, and $A^{kp} < A^{pp}$. But, by definition, A^{pp} is the optimum solution to the assignment problem with the right hand side of constraint p as its objective function. Therefore, no solution $\underline{x}^{k}$ such that $A^{kp} < A^{pp}$ exists, and A^{pp} is a valid lower bound on the optimal solution to the master problem.

Evaluated at $\underline{x}^{*}$, the right hand side of every Benders' constraint has a value less than or equal to UB_z as defined in equation 3.56. By definition, $\underline{x}^{*}$ is an assignment. Therefore, a feasible solution vector ($\underline{x}^{*}$) which yields a master problem objective function value equal to UB_z exists. Hence, UB_z is an upper bound on the optimal solution to the master problem.

Calculation of these bounds involves solving one assignment problem at each iteration q (assuming one Benders' constraint added per iteration) and making 2(q-1) functional evaluations. It also involves storing the indices of the variables in the assignment at each iteration.

3.3 Linear Programming with Restricted Basis Entry

In this section, the initial non-linear mathematical programming model (Figure 2) is reformulated as an "almost" linear program with a set of complementarity constraints. These constraints are non-linear, but through some modifications in the simplex method for linear programming they can be enforced. However, there is now no guarantee of optimality, or even of feasibility, upon termination of the algorithm. Ignizio [1984] has used complementarity constraints in goal programming approaches to minimal interference scheduling problems. He claims that the method is an extremely efficient heuristic. He does not indicate how the method was implemented, other than saying that a restricted basis entry rule was used.

In Figure 11, a formulation of the satellite location problem as an "almost" linear program with complementarity constraints is shown. The objective function (3.57) is the same as that in Figure 2, i.e., minimization of the total deviation. Constraints (3.58) evaluate the deviation of assigned locations from desired locations. Constraints (3.59),(3.60),(3.63), and (3.64) together enforce the minimum required separation between pairs of satellites, constraints (3.64) being the complementarity constraints. Constraints (3.61) establish the

$$\text{MINIMIZE} \quad \sum_{j=1}^{n} \left(X_j^+ + X_j^- \right) \tag{3.57}$$

subject to

$$X_j + X_j^+ - X_j^- = d_j \qquad j=1,\ldots n \tag{3.58}$$

$$X_i - X_j - P_{ij} + N_{ij} = 0 \qquad i=1,\ldots n-1,\; j=i+1,\ldots n \tag{3.59}$$

$$P_{ij} + N_{ij} \geq \triangle S_{ij} \qquad i=1,\ldots n-1,\; j=i+1,\ldots n \tag{3.60}$$

$$E_j \leq X_j \leq W_j \qquad j=1,\ldots n \tag{3.61}$$

$$X_j \,,\, X_j^+ \,,\, X_j^- \geq 0 \qquad j=1,\ldots n \tag{3.62}$$

$$P_{ij} \,,\, N_{ij} \geq 0 \qquad i=1,\ldots n-1,\; j=i+1,\ldots n \tag{3.63}$$

$$P_{ij} \cdot N_{ij} = 0 \qquad i=1,\ldots n-1,\; j=i+1,\ldots n \tag{3.64}$$

where

X_j, X_j^+, X_j^-, E_j, W_j, d_j, $\triangle S_{ij}$, n are as defined in Figure 2.

P_{ij} (N_{ij}) = degrees west (east) of satellite j that satellite i is located.

Figure 11. "Almost" Linear Programming Formulation of the Satellite Location Problem with Complementarity Constraints

boundaries on the feasible arc for each satellite. Constraints (3.62) are the non-negativity constraints.

The satellite location problems that are formulated in Figures 2, 4, and 11 are equivalent - the differences lie in the mathematical programming formulations. The absolute value constraints (2.3) in Figure 2 are expressed as a combination of integer variables and linear constraints in Figure 4, and are now expressed as a combination of continuous variables and linear and complementarity constraints in Figure 11.

In the implementation of the formulation in Figure 11, constraints (3.64) are not explicitly specified. Rather they are enforced through a restricted basis entry procedure within the simplex method. At each iteration, only those variables whose complements would be nonbasic after a change of basis are considered for entry into the set of basic variables. Pivots in which the entering non-basic variable is the complement of the leaving basic variable are permitted. Through the application of these rules, the complementarity constraints are implicitly enforced. Non-basic variables always have value zero, and one variable of a complementary pair is always forced to be non-basic.

3.4 The Switching Method

In this section, a heuristic switching method for the solution of the satellite location problem is introduced. This technique has been developed because it is likely that in most optimal solutions to problems of this type, the prescribed location of a satellite will not be far from its desired location. Therefore, if one starts with the satellites ordered by their desired locations and consider perturbations in this order, there is a reasonably good chance of finding the ordering that yields the optimum solution or at least an ordering that yields a very good solution.

To apply this method, a systematic determination of orderings to be considered is required. In the interests of efficiency, the examination of previously considered orderings should be avoided. A further requirement is a procedure for efficiently determining the solution to the satellite location problem for a given ordering. A switching technique which permutes small subsets of adjacent satellites, combined with a linear programming procedure which solves the satellite location problem for any given permutation of the satellites, fits these requirements.

A basic outline of the method is as follows :

1. Determine an ordering of the satellites.
2. Solve a linear program with the ordering from step 1 to obtain a solution to the satellite location problem. (The LP minimizes total deviation while ensuring that minimum separation requirements are met.)
3. Repeat steps 1 & 2 for a different ordering.

If there are n satellites there are n! possible orderings and it is obvious that all possible orderings cannot be evaluated for large values of n, for example, n > 10, in reasonable amounts of computer time. This method considers orderings in which the satellites are likely to remain in the vicinity of their desired locations. The total number of orderings considered is but a tiny fraction of all possible orderings.

A linear programming formulation for the satellite location problem with a specified ordering of the satellites is shown in Figure 12. The objective function is the minimization of total deviation. Constraints (3.66) measure the deviation of each assigned location from the corresponding desired location. Constraints (3.67) ensure that the minimum required orbital separation between all pairs of satellites is met. Since index k is always greater than index j, position k and the corresponding satellite S(k) is to the west of position j and the satellite in position j, S(j). Since an ordering is specified, absolute values, nonlinear constraints, or integer variables are not needed to enforce the minimum required separation.

In Figure 13, the dual of the linear program presented in Figure 12 is shown. The constraints in the dual formulation refer only to orbit positions. All references to satellites occupying specific positions occur only in the objective function. Therefore, a feasible solution to the dual problem is feasible irrespective of the satellite ordering; it may not be optimal, but it will be feasible. It is this property that is exploited to efficiently examine orderings.

From duality theory in linear programming, it is known that at optimality the primal and dual objective function values are identical, and that if the primal is solved, the dual solution can also be obtained and vice-versa (Bazaraa and Jarvis [1977]). Therefore, no information is lost by solving the dual problem. Instead, a great deal

$$\text{MINIMIZE} \quad Z = \sum_{j=1}^{n} (Y_j^+ + Y_j^-) \tag{3.65}$$

subject to

$$Y_j + Y_j^+ - Y_j^- = d_i \qquad i=S(j);\ j=1,\ldots n \tag{3.66}$$

$$Y_k - Y_j \geq \triangle S_{ih} \qquad \begin{array}{l} i=S(j);\ j=1,\ldots n-1 \\ h=S(k);\ k=j+1,\ldots n \end{array} \tag{3.67}$$

$$Y_j\ ,\ Y_j^+\ ,\ Y_j^- \geq 0 \qquad j=1,\ldots n \tag{3.68}$$

where

n = number of satellites and orbit positions

Y_j = the actual location of orbit position j

Y_j^+ , Y_j^- = the deviation to the east and west respectively, of the prescribed location of position j from the desired location of the satellite in position j

S(j) = the satellite currently occupying position j

d_i = desired location of satellite i

$\triangle S_{ih}$ = required orbital separation between satellites i and h

Figure 12. Primal Linear Programming Formulation for the Satellite Location Problem with Satellites in a Given Ordering (Used in the Switching Heuristic)

$$\text{MAXIMIZE} \quad Z = \sum_{j=1}^{n} d_i u_j + \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} \Delta S_{ih} v_{jk} \tag{3.69}$$

subject to

$$u_j + \sum_{k=1}^{j-1} v_{kj} - \sum_{k=j+1}^{n} v_{jk} \leq 0 \qquad j=1,\ldots n \tag{3.70}$$

$$-1 \leq u_j \leq 1 \qquad j=1,\ldots n \tag{3.71}$$

$$v_{jk} \geq 0 \qquad j=1,\ldots n-1; k=j+1,\ldots n \tag{3.72}$$

where

u_j = dual variable corresponding to $j^{\underline{th}}$ constraint in set (3.66) in the primal formulation (Figure 12)

v_{jk} = dual variable corresponding to $jk^{\underline{th}}$ constraint in set (3.67) in the primal formulation (Figure 12)

n, d_i, ΔS_{ih} are as defined in the primal formulation (Figure 12)

Figure 13. Dual Linear Programming Formulation for the Satellite Location Problem with Satellites in a Given Ordering. (Used in the Switching Heuristic)

is gained in this case. In the dual, there are only n explicit constraints as opposed to the primal where there could be as many as $0.5n^2 + n$ constraints. On the other hand, in the dual, there are many more variables than in the primal, but in general, many variables are easier to handle than many constraints in linear programming.

The switching heuristic can be summarized as follows :

STEP 1 : Create an initial ordering O_1 by ordering satellites in increasing order of desired locations.

STEP 2 : Solve the dual linear program to optimality with this ordering O_1.

STEP 3 : Specify a starting subgroup size, say k, and a maximum subgroup size, say k^*.

STEP 4 : If $k > k^*$,stop. Otherwise evaluate all k! permutations of the satellites in positions 1 to k, with the satellites in positions k+1 to n not changing position. The locations of all satellites may be changed. Each evaluation involves reoptimizing the dual after making the necessary changes in the objective function. Update the ordering to the best one found so far - the one which minimizes the objective function value.

STEP 5 : Repeat step 4 with subgroups 2 to k+1, 3 to k+2,...., and n-k+1 to n, updating the ordering at the end of the examination of each subgroup.

STEP 6 : If no improved solution is found, increase k by 1 and go to step 4. If an improved solution is found, go to step 4 with k unchanged.

The performance of this procedure can be considerably improved by introducing a type of fathoming procedure in the reoptimization of the dual. Initially, the dual is optimized with the ordering by desired location. The optimal solution is the initial incumbent solution. Once a switch is made, since the basis for the dual is still feasible, the new objective function value for this feasible solution can be determined. (The objective function coefficients are the only parameters changed in making a switch). If the value of the new objective function is greater than the current incumbent solution value, the switch is discarded since it cannot yield a better solution on reoptimization. If the new objective function value is less than the current incumbent solution value, reoptimization is required to determine whether a better solution can be reached. After each primal iteration in the reoptimization of the dual problem, an updated objective function value is obtained since the dual solution is primal feasible. If, after any iteration, this updated objective function value exceeds that of the current incumbent solution value, the reoptimization is terminated and the switch discarded from consideration. If the reoptimization is completed, a new incumbent solution has been found.

By using the fathoming procedure described above, the dual (or primal) does not have to be solved from scratch for each permutation. At most, a few primal pivots with the dual problem have to be performed for the majority of switches, since a large portion of the solution vector is unchanged when the switches made are on satellites in the vicinity of one another.

The implementation issues are discussed in the following chapter. Extensions of this method to include feasible arcs are presented in Chapter 5. Feasible arcs can introduce unboundedness in the dual, and correspondingly, infeasibility in the primal for some or all orderings. In the absence of feasible arcs, the primal and the dual are always feasible and bounded.

Switching or interchange techniques have been widely used in the past as heuristics for a large variety of problems. A seminal paper in the area is the one by Reiter and Sherman [1965]. They propose the use of an intelligent search based on the nature of the problem combined with a random search to improve the probability of escaping from being trapped at local optima. Lin and Kernighan [1973] use an interchange heuristic in an effective procedure for generating near optimum solutions to traveling salesman problems. Their work is an extension of an earlier paper by Lin [1965]. Eilon et al. [1971], Cassidy and Bennett [1972], and Wren and Holliday [1972] all use variations of interchange heuristics in their work on vehicle routing problems. Federgruen and Zipkin [1984] use an interchange heuristic as one approach to solving combined vehicle routing and inventory allocation problems.

CHAPTER IV

EMPIRICAL STUDY OF THE FOUR SOLUTION TECHNIQUES

In the last chapter, four solution techniques for the satellite location problem were presented. In this chapter, the computer implementation of these methods is discussed. The methods are tested on a set of problems generated from real data and the performance of each method is evaluated in terms of final solution value and the CPU time required to reach that solution. Some of the difficulties involved in the implementation of these methods and the steps taken to overcome them are also mentioned.

4.1 Implementation of the Mixed Integer Programming Method.

General purpose mixed integer programming packages are available commercially. The mixed integer programs formulated in Figures 4 and 5 can be solved by any such code. Since no commercial code was available, a package developed by Martin and Gonsalvez [1981] is used in the implementation of this method. The package solves mixed integer programs using the branch and bound technique of implicit enumeration. Branching directions are chosen through the evaluation of pseudo-costs, which estimate the improvement or degradation in the objective function value involved in following a particular path. The concept of pseudo-costs was introduced by Benichou et al. [1971]. Gonsalvez [1983] provides a brief description of the code and some of its features.

In trying to solve the satellite location problem as formulated in Figure 4 with this computer code, there were several instances of premature termination due to accuracy check problems. The accuracy check problems appear to be caused by the large disparity between the values of the constraint matrix coefficients (the differences ranging to four orders of magnitude). These accuracy problems provided the motivation for the reformulation of the model in Figure 4 which appears in Figure 5.

4.2 Implementation of the Benders' Decomposition Method

A flow chart of the Benders' decomposition procedure, as applied to the satellite location problem, is shown in Figure 14. The diagram highlights the major modules of the computer program and the relationships between them.

The initialization portion of the code consists of the input of the problem data : the service areas, the minimum required separation matrix, and the desired locations for the satellites. An initial assignment, in which the satellites are ordered by increasing desired location, is generated.

Once an assignment (i.e., an ordering) is specified there are a large number of redundant separation constraints in the subproblem (i.e., constraints of type (3.30) in Figure 8). These redundant constraints are identified by the procedure described in Appendix B.

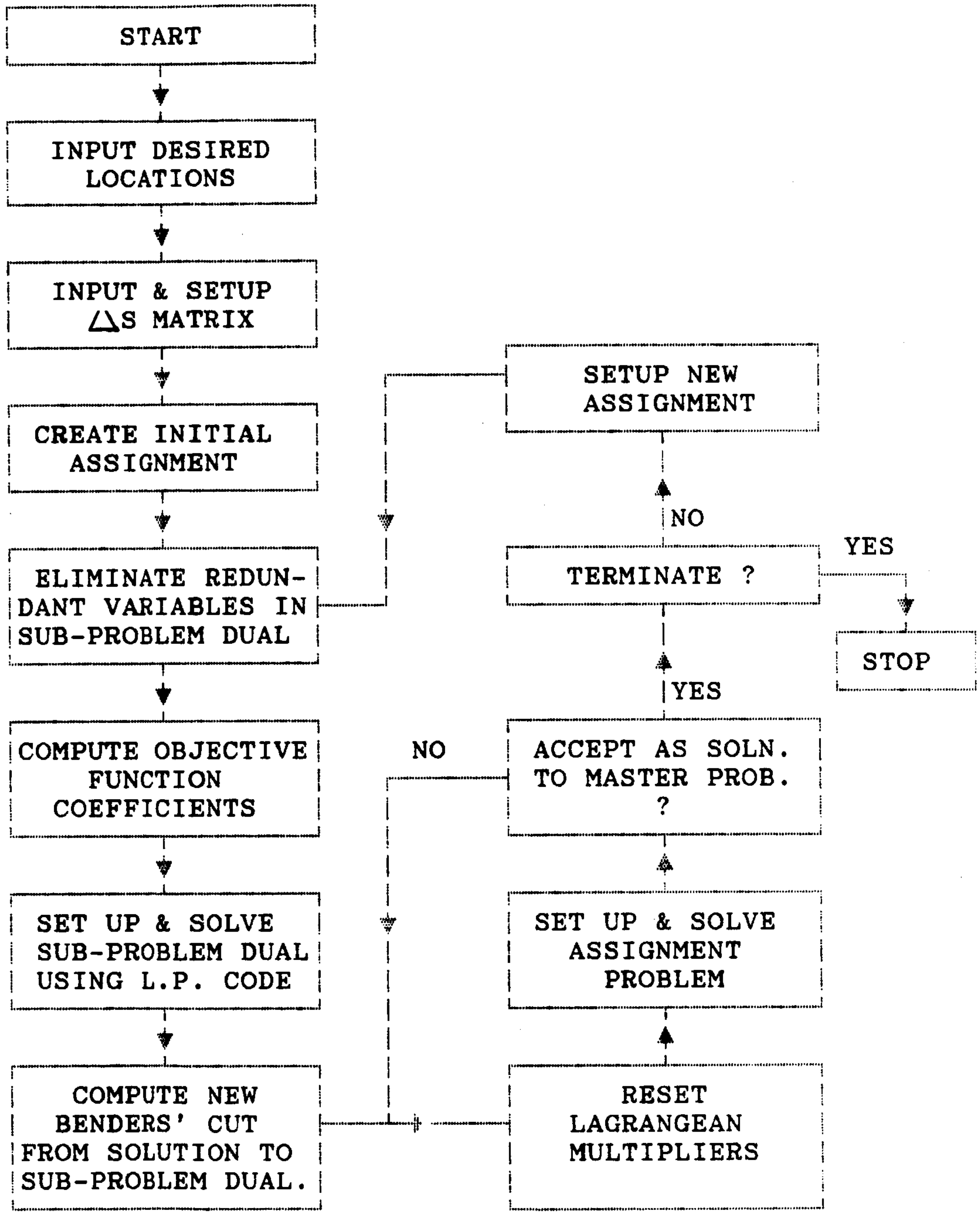


Figure 14. Flow Chart for Benders' Decomposition as Applied to the Satellite Location Problem.

Redundant constraints in the primal formulation of the subproblem appear as variables in the subproblem's dual and these variables can be discarded without affecting the optimum solution.

After identifying and discarding the variables corresponding to redundant primal constraints, the subproblem's dual is set up in a form acceptable to the linear programming code used by means of a matrix generator. The subproblem dual is then solved. The linear programming code used is PROFOR developed by Martin [1979].

The Benders' constraint is constructed from the dual variables in the optimum solution to the subproblem as indicated in equation (3.26). A general description of the generation of Benders' cuts is presented in Lasdon [1970].

The constraints are stored using a variation of sparse storage techniques. The assignment from which the current subproblem was generated is stored (this requires n storage elements where n is the number of satellites in the problem under consideration). The solution vector for the "u" variables in the dual formulation (Figure 9) is stored (n storage elements). The $u_j d_i$ portion of the coefficient for X_{ij} in the Benders' constraint (3.26) can be easily obtained from the above two sets of stored values - d_i being the desired location of the satellite in position j (given by the assignment) and u_j being directly obtained from the solution vector. Finally, only those X_{ij} that were equal to 1 in the previous assignment have $\triangle S_{ih}\, w_{jkih}$ coefficients (see equation 3.27) in the current master problem. These are computed and stored (n storage elements). The coefficients of the variables in each Benders' constraint are thus compactly stored using 3n storage elements, even though there are n^2 variables in each constraint and most of these variables have non-zero coefficients. Without such a storage technique for the Benders' constraints, the storage requirements for large problems would be so great as to necessitate storing the majority of constraints in external memory.

The new Benders' constraint is added to the master problem. Upper and lower bounds on the optimum solution to the master problem are generated using the procedure described in section 3.2.2. An assignment which yields an objective function value within these bounds is termed an "acceptable" solution to the master problem.

The master problem is now solved using one of the five heuristic methods described in sections 3.2.1.1 to 3.2.1.5. In the experiments presented in this chapter, the particular method used at each iteration is selected at random. If the selected method fails to find an acceptable solution to the master problem, then the next method in the sequence {FILS, GSLS, RLPS, SGRD, ADJ1} is used. If all five methods fail to find a solution within the bounds on the optimum solution to the current master problem, the procedure terminates. The procedure also terminates if the lower bound (lower bound on the current master problem) and the upper bound (best subproblem solution obtained so far) on the original problem are equal, in which case the solution obtained is the optimal solution. If an acceptable solution to the master problem is found, the subproblem corresponding to this assignment is generated and a new iteration started.

Four of the heuristics for solving the master problem are based on Lagrangean relaxation. They repeatedly solve assignment problems in the process of finding a solution to the master problem. The code used to solve these assignment problems is an adaptation of a code developed by Burkhard and Derigs [1980], and is based on the shortest augmenting path method.

The rounded linear program solution heuristic does not perform well, in terms of computation time, on larger problems and hence was not used in the set of experiments described in this chapter. The constraint matrix for the master problem is extremely dense (i.e. few coefficients are zero) and hence even solving it as a linear program, with the integrality constraints relaxed, can be extremely time consuming. As the number of Benders' constraints increases with successive iterations, the situation becomes worse. For example, in a problem with 12 satellites, 76 iterations were performed in 1 minute of CPU time with the other four heuristics but only 14 iterations could be performed in 1 minute when RLPS was included.

It was intended to first solve the master problem as a linear program, relaxing the integrality constraints, and then use the shadow prices on the Benders' constraints as starting values for the Lagrangean multipliers in the subgradient method and in the adjustment method. Since solving the linear program has proved impractical in terms of computation time, these multipliers are initialized to the values they had at the termination of the previous iteration. These values are then multiplied by a factor of 0.9 and the multiplier of the added constraint is arbitrarily set to 0.1 in order to obtain a feasible set of initial multiplier values for the current iteration.

4.3 Implementation of the Restricted Basis Entry Method

Enforcing restricted basis entry in a simple form requires only minor modifications to any standard linear programming code. In the implementation described in this section, a linear programming package (PROFOR) developed by Martin [1979] is used. The following modifications are made to the linear programming package. At each iteration, after the selection of the non-basic variable to enter the basis, a check is made to ensure that the complement of the selected variable will not be in the basis at the completion of the pivot. This check is performed only if the entering non-basic variable posesses a complement (i.e., the check is performed only for the P_{ij} and N_{ij} variables as defined in Figure 11). If a pivot will result in both complements being in the basis simultaneously, thus allowing a complementarity constraint to be violated, then a new entering variable is selected from the list of candidate non-basic variables. If none of the non-basic variables in the set of candidates can enter the basis, then the procedure terminates.

The above procedure may terminate at infeasible or non-optimal solutions. No attempt is made to influence the choice of pivot elements, other than the step described above which prevents complements from being in the basis at the same time. Standard selection procedures for the entering non-basic variable and the leaving basic variable are used. For a discussion of pivot procedures and selection of the row and column for a pivot in the simplex method for linear programming, the reader is referred to Bazaraa and Jarvis [1977].

4.4 Implementation of the Switching Technique

The steps involved in generating an efficient computer program for the switching heuristic are described in this section. The heuristic is described in section 3.4. There are two distinct parts of the program : generation of an ordering and the solution of the linear program that provides the optimal locations for satellites given an ordering. A flow chart of the modules in the program and their interrelationships is given in Figure 15.

The linear program can be solved by any available linear programming code. It is preferable to use one that stores the inverse of the basis in a product form, since this allows an efficient generation of feasible starting bases for the linear programs, as is described later in this section. The linear programming code used is PROFOR which was developed by Martin [1979]. This program is based on the revised simplex method and uses the product form of the inverse for basis storage. Both the revised simplex method and the product form of the inverse are discussed in Bazaraa and Jarvis [1977].

Even though the individual linear programs generated by the switching heuristic are easily solved, solving each linear program from the basis corresponding to the origin to the optimal solution is not computationally practical, when the number of orderings to be

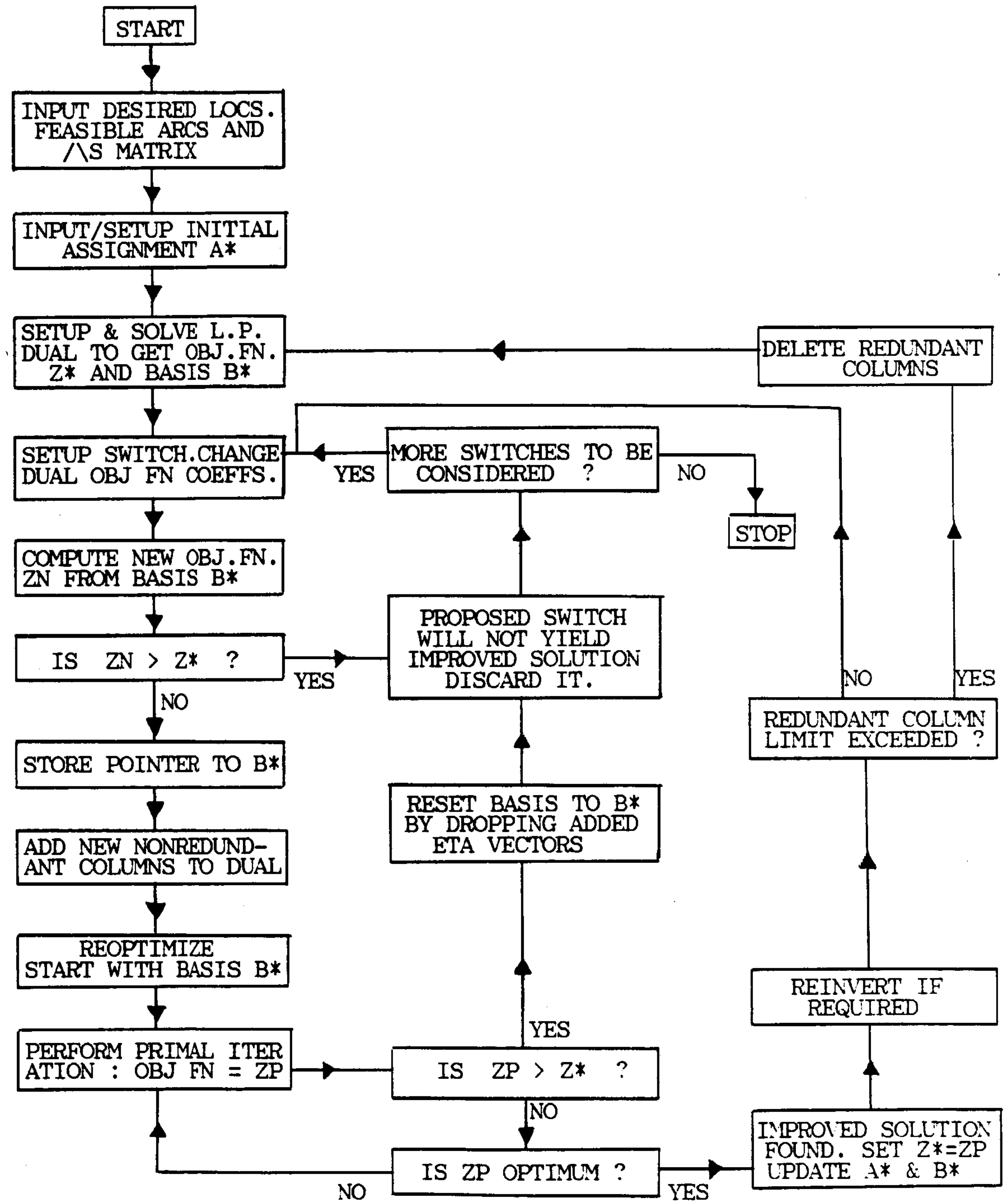


Figure 15. Flow Chart for the Switching Heuristic

considered through the switching method is taken into account. Each linear program is not solved starting from the origin, rather the optimal basis for the linear program corresponding to the ordering before the switch is used as a starting basis. This basis is still feasible after the switch since only the objective function coefficients are changed in the dual formulation (Figure 13). It is unlikely that a major portion of the solution changes after a switch since only the satellites in the current subgroup change positions in the ordering. Hence, this choice of a starting basis is likely to be very close to an optimal basis for the linear program corresponding to the new ordering.

Suppose the initial ordering is R. The linear program for the dual formulation (Figure 13) is generated for this ordering and it is solved to optimality. Let S be the optimal solution vector, Z^* the optimal objective function value, and B^* the optimal basis. Since Z^* is the optimal dual objective function value, by the theory of duality the optimal objective function value for the primal problem with satellite ordering R is identical. Though the dual linear program is examined and solved, it is the primal objective function of minimizing total deviation that is the actual goal and this corresponds to driving Z^* as low as possible.

A switch is made in the ordering R, resulting in a new ordering R1. The basis B^* is unlikely to be optimal for the dual linear program corresponding to the new ordering R1, but it is still feasible. Therefore, the solution vector S is also feasible and is used to evaluate the new objective function (say Z^n). If Z^n is greater than or equal to Z^*, then ordering R1 cannot yield a better solution than ordering R, since Z^n can only increase when the linear program corresponding to R1 is optimized. Reoptimization is not required and ordering R1 can be discarded.

On the other hand, if Z^n is less than Z^*, then there is a possibility that R1 might be a better ordering than R with respect to the objective function. The linear program corresponding to R1 is reoptimized using the basis B^* as the initial basis. At each iteration in the solution of the linear program, the current objective function value (Z^p) is computed. Since the dual formulation is a maximization problem, Z^p has to increase at each iteration. If Z^p becomes greater than or equal to Z^* at any iteration, then the linear program can be terminated and the ordering R1 discarded. The linear programming code uses the product form of the inverse in which the inverse is stored as a set of vectors called "eta vectors"; one eta vector for each pivot. The inverse of the basis B^* is regenerated by dropping the eta vectors that were added to the inverse in the optimization of the linear program corresponding to R1.

The final possibility is that the reoptimization with ordering R1 is completed, and Z^p at this point is less than Z^*. Therefore, the new ordering R1 is an improvement on the old ordering R. The ordering R1 is now called R, the current basis is called B^*, Z^* is set equal to Z^p, and the new solution vector takes the place of S. A new switch is selected and the procedure is repeated.

During the reoptimization, all reinversions of the basis are suppressed. This is done to allow an easy regeneration of the basis B^*

by dropping eta vectors when a switch does not improve on the current best solution.

Once an ordering is specified there are a large number of constraints of type (3.67) in the primal linear program which are implicitly satisfied through other separation constraints. These constraints in the primal formulation are termed "redundant" constraints, and the corresponding variables in the dual formulation are called "redundant" variables. In the context of the dual problem, a basis, in which all redundant variables are nonbasic, exists for any feasible solution and in particular for the optimal solution. Therefore dropping redundant variables from the linear program dual does not effect the solution obtained. The number of variables in each of the linear programs generated by the switching heuristic is thus reduced substantially; often this reduction is on the order of 90 %. The procedure used for identifying the redundant variables is described in Appendix B. For the solution of the dual linear program corresponding to the initial ordering, all redundant variables among the "v" variables are identified and removed. The "u" variables are never redundant.

After a switch is made, a dual variable that was redundant before the switch may no longer be redundant. A minimum required separation, which was previously implicitly satisfied through the other enforced separations, now has to be explicitly enforced after the switch. A new variable has to be added to the linear program dual, and this is done by adding a column to the constraint matrix. Since this variable was previously redundant, it is nonbasic with respect to the existing basis. The existing basis still provides a feasible starting basis for the linear program dual after the switch.

It is also possible that a nonredundant variable becomes redundant after a switch, and can be removed from the problem. However, removing a variable is not as straightforward as adding a variable. If the variable to be removed is currently nonbasic, it can be deleted through a rearrangement of column indices and corresponding changes in the basis pointers. If the variable is in the basis, then a pivot has to be performed to make it nonbasic before it can be dropped. Neither situation lends itself to an efficient implementation. In this implementation, these redundant variables are left in the problem until their number exceeds a given limit, at which point they are all removed.

The switching heuristic as stated in section 3.4 specifies that all permutations of satellites within a subgroup are explicitly examined. This leads to repeated examination of some orderings as successive subgroups are considered. By considering only permutations of a subgroup in which the last satellite in the subgroup does not occupy the last position, repeated examinations of the same ordering are avoided. All orderings that would have been considered previously are still examined. A proof of these statements is given in Appendix C.

4.5 Generation of the test problems

A set of seven test problems was created in order to compare the performances of the four techniques. These scenarios were selected to represent actual satellite location problems. For each scenario, nations which are in geographical proximity to one another constitute the service areas. Most of the satellites in a scenario are unable to occupy the same geostationary orbit location without causing unacceptable interference to the neighbouring service areas. Therefore, the nations included in each scenario have to arrive at some mutually satisfactory satellite location plan. The seven scenarios are described in Appendix D, the names of the nations in a scenario, the desired orbital location for the satellites, and the worst case minimum required separation matrices (ΔS matrices) being given.

Each service area in a scenario is specified by a set of test points. These test points, denoted by geographical longitude and latitude, are usually at the boundaries of the nation. The polygon formed by joining these test points should envelope the country, thereby ensuring that, if interference levels at the test points are acceptable, then interference levels are acceptable throughout the service area. The test points for each country in the scenarios were determined using an atlas, with the criterion that the polygon formed by the test points covered the entire country.

In these experiments, elliptical satellite signal beams are assumed throughout, while Earth station beams are circular in cross-section. It is, therefore, necessary to compute the ellipse that will cover the polygon representing a service area most efficiently. This ellipse is termed the minimum ellipse and is computed using the minimum ellipse program developed by Akima [1983]. The minimum ellipse is specified by the length of its major and minor axes, the beam center in longitude and latitude coordinates, and an orientation angle. These ellipse parameters and the corresponding minimum ellipse itself change as the satellite location is shifted. Therefore, minimum ellipses are computed at 2° intervals over the entire feasible arc for each service area.

Once the test points have been specified and the minimum ellipses determined, the minimum required orbital separation between each pair of satellites is calculated. This required separation ($\triangle\phi$), which is required in order to satisfy the specified threshold C/I ratio, varies as the satellite locations change. The separation is computed at 4° intervals over the entire common feasible arc for each pair of satellites. The largest value among these separations is denoted as the $\triangle$S value (worst case minimum required separation) for that pair of satellites. A complete $\triangle$S matrix which contains $\triangle$S values for all pairs of satellites is computed for each scenario.

All the input that is required to define a scenario for any of the four solution techniques is now available :

1. the service area names
2. the number of satellites for each service area
3. the desired location for each satellite
4. the $\triangle$S matrix

4.6 Discussion of experimental results

The four solution techniques, mixed integer programming (MIP), Benders' decomposition, the restricted basis entry method (RBE), and the switching heuristic were applied to each of the seven test problems. In every case, the objective was to minimize the total deviation. Feasible arc restrictions were not imposed for any of the runs. The final solution obtained and the total CPU time taken for each of the twenty-eight runs are presented in Table 1.

Some restrictions on the lengths of individual runs were imposed. Maximum time limits of 300 seconds and 60 seconds were set for the Benders' procedure and the RBE method, respectively. The Benders' procedure also had a maximum iteration limit of 150 iterations.

Overall, the switching heuristic outperforms the other three techniques both in terms of final solution value and in terms of total time taken. The switching heuristic finds the best solution for six out of seven scenarios and in the seventh case arrives at a solution that is within 10% of the best solution. With regard to solution time, the switching heuristic is again the best alternative in six out of seven scenarios.

The MIP method is guaranteed to find an optimal solution if one exists. However, with the satellite location problem, the method runs into considerable difficulties with accuracy checks, often resulting in premature termination. This may be due to the sensitivity of the code used, or due to the structure of MIP model of the satellite location problem. For only two of the seven scenarios does the method find the

TABLE 1. COMPARISON OF THE PERFORMANCE OF THE FOUR SOLUTION METHODS

Objective Function : Minimize total deviation
(without feasible arc limitations)

SCENARIO (# Sat.)		METHODS			
		M.I.P	BENDERS'	RES. BASIS	SWITCHING
S.AMERICA (13)	SOLN.	21.92 o	44.39 c	52.84	21.92
	TIME	83	109	8	9
E.EUROPE (12)	SOLN.	53.46 a	51.63 c	110.01	41.48
	TIME	13	95	9	5
W.EUROPE (12)	SOLN.	31.40 o	47.13 c	81.25	34.21
	TIME	161	95	8	4
S.E.ASIA (10)	SOLN.	24.25 a	29.84 c	80.85	23.29
	TIME	20	68	3	2
N.AFRICA (10)	SOLN.	38.72 a	24.49 c	***	24.49
	TIME	110	66	60	2
EUROPE (26)	SOLN.	212.56 a	230.18 b	***	150.69
	TIME	301	300	60	71
NCS AMERICA (26)	SOLN.	*** a	106.30 b	***	40.56
	TIME	125	300	60	87

All times measured in CPU seconds on an IBM 3081-D

o - optimum solution
a - termination due to accuracy checks
b - termination due to time limit
c - termination due to iteration limit
*** - no feasible solution found at termination

optimum solution. In the remaining five scenarios, the program is terminated by excessive accuracy checks with nonoptimal solutions.

The Benders' procedure did not perform as well as the switching heuristic or the MIP method. Only in one case does it provide the best solution, and in this case the switching heuristic reached the same solution in 2.4 seconds compared to the 66 seconds taken by the Benders' procedure. It was hoped that the Benders' procedure would converge to near optimal solutions quickly. The procedure converges, but only slowly when compared to the switching heuristic. The solution times for Benders' procedure are of the same order of magnitude as those taken by the MIP method.

Among the four techniques, the restricted basis entry method converges to the worst solutions. Though it quickly arrives at solutions for four of the smaller problems, it fails to converge for two of the larger problems with 26 service areas and one smaller problem with 10 service areas.

From the results of these experiments, it is clear that the switching heuristic is an excellent method for solving the satellite location problem in terms of the "goodness" of the solution, the solution time, and in reliability.

CHAPTER V

SWITCHING HEURISTICS FOR VARIATIONS IN THE SATELLITE LOCATION PROBLEM

The switching heuristic, for solving the satellite location problem where the objective function is the minimization of the total deviation, was introduced in Chapter 3. The comparison study in Chapter 4 showed that the switching heuristic outperformed the other three solution methods that were applied to the set of seven test problems, both in terms of the quality of the solution obtained and in the time needed to reach that solution.

Variations in the switching heuristic are introduced in this chapter. These variations extend the applicability of the switching heuristic to different satellite location models. A new objective function, the minimization of maximum deviation of the prescribed location of a satellite from its desired location, is discussed. The addition of feasible arc constraints to the satellite location problem is considered. Even though these constraints were specified in the initial formulations in Figures 2,4, and 5, they were not enforced in the experiment discussed in Chapter 4.

In the discussion of the minimum required separation matrices in Chapter 2, it was mentioned that solutions obtained using $\triangle S$ values (worst case minimum required separation) might be conservative. The use of $\triangle\phi$ (the minimum required separation for the current location of the satellites) in place of $\triangle S$ is investigated in this chapter. Application of the switching heuristic to a set of test problems first with $\triangle S$, and then with $\triangle \phi$, shows that this change improves the objective function value significantly. The switching heuristic can easily accomodate $\triangle \phi$ in place of $\triangle S$, but the other three solution methods have to undergo extensive modification and significant growth in the number of decision variables in order to implement the change.

In the final section of this chapter, the application of the switching heuristic to a "real world" scenario consisting of 28 service areas and 59 satellites is described. The results obtained confirm the initial conclusions made about the effectiveness of the heuristic.

Although the variations on the satellite location problem discussed in this chapter can theoretically be solved using any of the other three solution methods, with some reformulation of the models, only the switching heuristic is used in the experiments with these problems. This decision is made based on the excellent performance of the heuristic as evidenced by the experimental results presented in Chapter 4.

5.1 Minimization of the maximum deviation of prescribed locations from the corresponding desired locations.

Until now, the objective function for the satellite location problem has been the minimization of total deviation. Other objective functions for the satellite location problem might be appropriate, and one such objective function is the minimization of the maximum deviation between an assigned location and the corresponding desired location (hereafter referred to as the minimization of maximum deviation). Since this objective function ensures that the largest deviation is minimized, it tries to prevent any one satellite from having a large deviation so that the other satellites may have small deviations, a situation which may occur when the objective function is the minimization of total deviation.

Although the other solution methods can also be applied to the satellite location problem with this objective function, only the switching heuristic is used in this case. The switching heuristic that is applied is the same as the one used with the objective function of minimization of the total deviation. Changing the objective function to the minimization of maximum deviation affects only the linear programming formulations associated with the switching heuristic and does not require any changes in the implementation of the heuristic itself.

The new primal linear programming formulation is shown in Figure 16. The objective function (5.1), the minimization of the maximum deviation, is acheived through the minimization of a single "dummy" variable which represents the maximum deviation of an assigned location from the corresponding desired location. Constraints (5.2) and (5.3) together ensure that Z is the maximum deviation over all satellites. For any satellite to the east of its desired location, constraint (5.2) ensures that Z is at least as large as the deviation between prescribed and desired locations. Constraints (5.3) perform the same task for

$$\text{MINIMIZE} \quad Z \tag{5.1}$$

subject to

$$Y_j + Z \geq d_i \qquad i=S(j);j=1,\ldots n \tag{5.2}$$

$$Y_j - Z \leq d_i \qquad i=S(j);j=1,\ldots n \tag{5.3}$$

$$Y_k - Y_j \geq \Delta S_{ih} \qquad \begin{matrix} i=S(j);j=1,\ldots n-1 \\ h=S(k);k=j+1,\ldots n \end{matrix} \tag{5.4}$$

$$Y_j \geq 0 \qquad j=1,\ldots n \tag{5.5}$$

$$Z \geq 0 \tag{5.6}$$

where

n = number of satellites and orbit positions

Y_j = the prescribed location for orbit position j

Z = the maximum deviation from any desired location

S(j) = the satellite currently in orbit position j

d_i = the given desired location for satellite i

ΔS_{ih} = the required separation between satellites i and h

Figure 16. The Primal Formulation for the Satellite Location Problem with the Objective of Minimizing the Maximum Deviation and for a Given Ordering.

satellites located to the west of their desired locations. Constraints (5.4) ensure that the minimum required separations are met for all pairs of satellites. Constraints (5.5) and (5.6) are non-negativity constraints on the decision variables.

The dual linear programming formulation corresponding to the primal formulation in Figure 16, is shown in Figure 17. Constraints (5.8) correspond to the "y" variables in the primal and constraint (5.9) corresponds to the "Z" variable in the primal. The constraint matrix is still defined solely by orbit positions, all satellite dependent parameters appearing only in the objective function. Therefore, the implementation of the switching heuristic is exactly the same as it was when the objective function was the minimization of total deviation. The presentation of experimental results with this objective function is deferred until Section 5.3.

5.2 Feasible arc constraints for the satellite location problem

The portion of the geostationary orbit that is visible from every test point in a service area is called the visible arc for that service area. A satellite can be located only in the visible arc corresponding to its service area. Further, satellite locations usually have to be assigned so that a satellite is at least some specified minimum angle of elevation above the horizon for all the test points in its service area. The portion of the geostationary orbit within which the satellite(s) of a service area can be positioned taking into account

$$\text{MAXIMIZE} \quad Z = \sum_{j=1}^{n} d_i u_j + \sum_{j=1}^{n} d_i w_j + \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} \Delta S_{ih} \, v_{jk} \tag{5.7}$$

subject to

$$u_j + w_j + \sum_{k=1}^{j-1} v_{kj} - \sum_{k=j+1}^{n} v_{jk} \leq 0 \qquad j=1,\ldots n \tag{5.8}$$

$$\sum_{j=1}^{n} u_j - \sum_{j=1}^{n} w_j \leq 1 \tag{5.9}$$

$$u_j \geq 0 \tag{5.10}$$

$$w_j \leq 0 \tag{5.11}$$

$$v_{jk} \geq 0 \tag{5.12}$$

where

n, d_i, ΔS_{ih} are as defined in Figure 16

u_j = the dual variable corresponding to the $j^{\underline{th}}$ constraint in the set of primal constraints (5.2)

w_j = the dual variable corresponding to the $j^{\underline{th}}$ constraint in the set of primal constraints (5.3)

v_{jk} = the dual variable corresponding to the $jk^{\underline{th}}$ constraint in the set of primal constraints (5.4)

Figure 17. The Dual Formulation for the Satellite Location Problem with the Objective of Minimizing the Maximum Deviation for a Given Ordering

these limitations and other requirements (if any) is called the feasible arc for that service area. These constraints on the assignment of satellite locations were specified as simple upper and lower bounds on the decision variables corresponding to assigned satellite locations in the formulations given in Figures 2,4 and 5.

In the experiment comparing the four solution methods, constraints that would force satellites to be located within their respective feasible arcs were not enforced. The objective function considered in that experiment, the minimization of total deviation, tends to place satellites within their feasible arcs, since the specified desired location for each satellite is at or very near to the center of the feasible arc for each satellite. The same is true when the objective function is the minimization of maximum deviation. However, with either objective, there is no guarantee that all satellites will be placed in their respective feasible arcs in any solution obtained. Constraints that explicitly force satellites to be placed in their specified feasible arcs are included in all the experiments described in this chapter.

In the primal formulations (Figures 12 and 16), irrespective of the objective function, the feasible arc constraints are imposed through simple lower and upper bound constraints on the decision variables corresponding to satellite locations. The constraints to be added to the primal linear programming formulations are indicated below.

$$y_j \geq L_i \qquad i=S(j);j=1,\ldots n \qquad (5.13)$$

$$y_j \leq H_i \qquad i=S(j);j=1,\ldots n \qquad (5.14)$$

where

y_j = the assigned location of orbit position j

S(j) = the satellite in position j in the given ordering

n = the number of satellites

L_i = the lower limit on the feasible arc for satellite i

H_i = the upper limit on the feasible arc for satellite i

The corresponding dual formulations after the addition of the feasible arc constraints are given in Figures 18 and 19. The dual formulation in Figure 18 corresponds to the objective function of minimizing the total deviation, and the formulation in Figure 19 to the objective function of minimizing the maximum deviation. The changes in these two formulations, from their counterparts in which feasible arc limits were not imposed (Figures 13 and 17 respectively), amount to the addition of two sets of variables. These "p" and "q" variables correspond to the lower and upper bound constraints, respectively, that are placed on the decision variables in the primal formulations.

The effect that the addition of feasible arc limits has on the switching heuristic is that feasible solutions to the problem may not exist for any given ordering, while feasible solutions for any possible ordering always exist for the formulations without feasible arc constraints. Infeasibility in the primal linear program corresponds to unboundedness in the dual linear program. To allow for dual unboundedness, the implementation of the switching heuristic has to be modified.

The most direct implementation of feasible arc limits would be to terminate the optimization of the dual problem when an indication of

$$\text{MAXIMIZE} \quad Z = \sum_{j=1}^{n} K_j u_j + \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} Q_{jk} v_{jk} + \sum_{j=1}^{n} L_j p_j + \sum_{j=1}^{n} H_j q_j \qquad (5.15)$$

subject to

$$u_j + \sum_{k=1}^{j-1} v_{kj} - \sum_{k=j+1}^{n} v_{jk} + p_j + q_j \le 0 \qquad j=1,\ldots n \qquad (5.16)$$

$$-1 \le u_j \le 1 \qquad j=1,\ldots n \qquad (5.17)$$

$$v_{jk} \ge 0 \qquad j=1,\ldots n-1; k=j+1,\ldots n \qquad (5.18)$$

$$p_j \ , \ -q_j \ge 0 \qquad j=1,\ldots n \qquad (5.19)$$

where

n, K_j, Q_{jk}, u_j, v_{jk} are as defined in Figure 13.

L_j = lower limit of feasible arc for the satellite in position j

H_j = upper limit of feasible arc for the satellite in position j

p_j = dual variable corresponding to constraint (5.13) i.e. the constraint on the lower limit of feasible arc in the primal

q_j = dual variable corresponding to constraint (5.14) i.e. the constraint on the upper limit of feasible arc in the primal

Figure 18. The Dual Formulation for the Satellite Location Problem with Feasible Arcs - The Objective Function is the Minimization of Total Deviation

$$\text{MAXIMIZE} \quad Z = \sum_{j=1}^{n} d_j u_j + \sum_{j=1}^{n} d_j w_j + \sum_{j=1}^{n} L_j p_j + \sum_{j=1}^{n} H_j q_j$$

$$+ \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} \Delta S_{ih} v_{jk} \qquad (5.20)$$

subject to

$$u_j + w_j + p_j + q_j + \sum_{k=1}^{j-1} v_{kj} - \sum_{k=j+1}^{n} v_{jk} \leq 0 \qquad j=1,\ldots n \qquad (5.21)$$

$$\sum_{j=1}^{n} u_j - \sum_{j=1}^{n} w_j \leq 1 \qquad (5.22)$$

$$u_j \ , \ -w_j \geq 0 \qquad j=1,\ldots n \qquad (5.23)$$

$$v_{jk} \geq 0 \qquad j=1,\ldots n-1; k=j+1,\ldots n \qquad (5.24)$$

$$p_j \ , \ -q_j \geq 0 \qquad j=1,\ldots n \qquad (5.25)$$

where

n, d_i, ΔS_{ih}, u_j, w_j, v_{jk} are as defined in Figure 17

L_j, H_j, p_j, q_j are as defined in Figure 18

Figure 19. The Dual Formulation for the Satellite Location Problem with Feasible Arcs - The Objective Function is the Minimization of Maximum Deviation

unboundedness is obtained, and discard the ordering corresponding to the dual problem under consideration. This would be similar to the termination of reoptimization based on the objective function value which is described in Section 4.4.

In the linear programming code used, termination due to unboundedness leaves the basis in an undefined state. As a result, the basis corresponding to the previous feasible solution cannot be regenerated. This difficulty is avoided by introducing artificial upper and lower bounds on the "p" and "q" variables, respectively, which were previously unbounded. Restricting the values that the "p" and "q" variables can assume eliminates the possibility of the dual problem being unbounded for any ordering of satellites. If the artificial bounds on the "p" and "q" variables allow the variables a range much larger than the entire feasible arc, then no feasible solutions to the primal problem are eliminated from consideration. This method of avoiding unboundedness in the dual is used in the experiments described in this chapter. All orderings are dual feasible and bounded, but some have very high objective function values (those corresponding to infeasible primal solutions), and these are quickly fathomed once a solution corresponding to a primal feasible solution is found.

5.3 Experimental results for the satellite location problem with different objective functions and feasible arc constraints.

For the seven scenarios described in Appendix C, the associated satellite location problems with limits on the feasible arcs are solved using the switching heuristic, first with the objective of minimizing the total deviation and then with the objective of minimizing the maximum deviation.

The effect of the subgroup size on the performance of the switching heuristic is examined. As mentioned in Chapter 3, the subgroup size is the number of satellites that are permuted at a time. Subgroup sizes of 2,3,4, and 5 are considered as is increasing the subgroup size by one, from 2 to 5, during the execution of the algorithm. When the subgroup size is limited to a single value, the switching heuristic only considers subgroups of that size and terminates when no improved solutions can be found. With the increasing subgroup size option, if the heuristic cannot find an improved solution with the current subgroup size, it increases the subgroup size by one and continues. Termination occurs when the given maximum subgroup size is reached and no improved solution can be found.

The results for the computer runs made with the objective function of minimizing total deviation and with limits on feasible arcs are presented in Table 2. The results for the runs with the objective function of minimizing maximum deviation and with limits on feasible arcs are presented in Table 3. The layouts for the two tables are identical. The name of the scenario and the number of satellites in it

TABLE 2. SWITCHING HEURISTIC RUNS ON THE SEVEN SCENARIOS WITH THE OBJECTIVE OF MINIMIZING TOTAL DEVIATION AND WITH LIMITS ON FEASIBLE ARCS.

SCENARIO (# SAT)	SUB GROUP SIZE	FINAL SOLUTION VALUE	TOTAL TIME TAKEN	# OF MAJOR ITERS.	# OF FEAS. SOLNS.	TIME TO A FEAS. SOLN.
S.AMERICA	2	27.23	0.7	3	0	---
	3	22.14	1.8	3	0	---
(13)	4	30.94	3.7	2	1	1.9
	5	30.94	16.1	2	1	8.1
	INCR.	27.01	10.7	8	4	2.5
E.EUROPE	2	41.48	0.8	5	13	0.1
	3	47.85	0.9	3	9	0.1
(12)	4	47.85	2.7	3	12	0.1
	5	47.85	7.0	2	14	0.1
	INCR.	41.48	6.7	8	13	0.1
W.EUROPE	2	31.71	0.6	3	0	---
	3	37.63	1.0	3	0	---
(12)	4	47.08	4.6	5	6	1.3
	5	41.77	18.0	5	6	4.2
	INCR.	43.26	7.4	8	1	1.4
S.E.ASIA	2	62.00	1.6	3	0	---
	3	54.41	2.3	4	7	0.2
(10)	4	42.61	4.3	4	7	0.3
	5	42.61	11.0	3	12	0.4
	INCR.	42.61	12.4	12	16	0.4
N.AFRICA	2	24.63	1.6	2	0	---
	3	59.44	2.1	3	7	0.2
(10)	4	54.56	3.9	3	6	0.4
	5	41.42	13.3	3	14	1.4
	INCR.	41.42	12.7	10	10	0.3

TABLE 2. (contd.) SWITCHING HEURISTIC RUNS ON THE SEVEN SCENARIOS WITH THE OBJECTIVE OF MINIMIZING TOTAL DEVIATION AND WITH LIMITS ON FEASIBLE ARCS.

SCENARIO (# SAT)	SUB GROUP SIZE	FINAL SOLUTION VALUE	TOTAL TIME TAKEN	# OF MAJOR ITERS.	# OF FEAS. SOLNS.	TIME TO A FEAS. SOLN.
EUROPE	2	239.10	3.7	6	0	---
	3	193.00	9.0	5	0	---
(26)	4	200.29	43.1	7	7	27.4
	5	160.96	89.5	3	6	30.5
	INCR.	204.58	81.7	14	5	6.9
NCS AMER.	2	59.29	3.9	3	1	1.7
	3	50.75	10.8	4	13	2.7
(26)	4	55.78	41.9	5	15	7.1
	5	63.22 t	120.0	3	32	32.9
	INCR.	46.44	55.0	9	8	1.7

All times are in CPU seconds on an IBM 3081-D

t - termination due to run time limit of 2 minutes

TABLE 3. SWITCHING HEURISTIC RUNS ON THE SEVEN SCENARIOS WITH THE OBJECTIVE OF MINIMIZING MAXIMUM DEVIATION AND WITH LIMITS ON FEASIBLE ARCS.

SCENARIO (# SAT)	SUB GROUP SIZE	FINAL SOLUTION VALUE	TOTAL TIME TAKEN	# OF MAJOR ITERS.	# OF FEAS. SOLNS.	TIME TO A FEAS. SOLN.
S.AMERICA	2	5.00	1.1	3	0	---
	3	5.52	2.0	3	0	---
(13)	4	5.52	7.0	3	0	---
	5	4.90	18.3	2	1	9.7
	INCR.	4.68	9.5	8	1	3.1
E.EUROPE	2	8.14	1.3	5	7	0.2
	3	8.51	1.4	3	8	0.2
(12)	4	7.98	3.0	3	10	0.2
	5	7.23	6.1	2	13	0.2
	INCR.	7.23	5.1	9	11	0.2
W.EUROPE	2	11.13	1.4	3	0	---
	3	10.16	1.4	3	0	---
(12)	4	7.57	2.7	3	2	1.3
	5	7.41	9.6	3	2	4.0
	INCR.	7.41	18.2	10	11	2.5
S.E.ASIA	2	13.48	1.7	2	2	0.2
	3	12.96	1.9	2	3	0.2
(10)	4	12.78	2.7	2	4	0.3
	5	12.78	7.5	2	4	0.5
	INCR.	12.78	7.2	6	5	0.2
N.AFRICA	2	4.35	1.7	2	0	---
	3	11.07	2.2	2	1	0.5
(10)	4	11.07	3.4	2	1	0.8
	5	10.19	9.9	2	2	2.1
	INCR.	11.07	8.3	6	1	0.6

TABLE 3. (contd.) SWITCHING HEURISTIC RUNS ON THE SEVEN SCENARIOS WITH THE OBJECTIVE OF MINIMIZING MAXIMUM DEVIATION AND WITH LIMITS ON FEASIBLE ARCS.

SCENARIO (# SAT)	SUB GROUP SIZE	FINAL SOLUTION VALUE	TOTAL TIME TAKEN	# OF MAJOR ITERS.	# OF FEAS. SOLNS.	TIME TO A FEAS. SOLN.
EUROPE	2	17.39	7.7	6	0	---
	3	15.63	16.5	7	0	---
(26)	4	14.53	30.2	5	4	22.3
	5	14.34	85.7	3	6	41.5
	INCR.	14.01	109.8	17	9	24.1
NCS AMER.	2	8.78	9.7	6	1	7.0
	3	7.45	6.5	2	2	3.8
(26)	4	7.43	17.0	2	5	10.6
	5	5.70 t	120.0	3	21	40.3
	INCR.	6.00	51.6	11	6	7.0

All times are in CPU seconds on an IBM 3081-D

t - termination due to run time limit of 2 minutes are stated in the first column. The subgroup size is given in the second column. The final solution obtained, the time taken to reach termination, and the number of major iterations performed, appear in the third, fourth, and fifth columns, respectively. The number of feasible solutions found and the time that elapsed before the first feasible solution was found are shown in the sixth and seventh columns.

From Tables 2 and 3, it is clear that the switching heuristic with increasing subgroup size and the heuristic with a fixed subgroup size of 5, consistently outperform the alternatives with regard to solution value. Out of the fourteen problems (seven each with minimization of total deviation and minimization of maximum deviation), the switching heuristic with increasing subgroup size found the best solution in ten cases. The heuristic with fixed subgroup size of 5 found the best solution in nine cases. The other three alternatives together could find the best solution only three times. In some problems, two or more of the five strategies found the best solution. The heuristic with increasing subgroup size performs better than the heuristic with subgroup size equal to 5 in terms of computation time, requiring less CPU time for eleven of the fourteen problems.

In order to determine how close the solutions found by the switching heuristic are to the optimal solution, an attempt was made to solve all fourteen problems using the mixed integer programming method (MIP). The results are presented in Tables 4 and 5. The MIP finds the optimum solution for five out of the fourteen problems, the remaining nine being terminated due to accuracy checks or limits on run time. For nine problems, the MIP provided feasible solutions. A comparison

TABLE 4. COMPARISON OF SWITCHING SOLUTIONS TO MIP SOLUTIONS
MINIMIZE TOTAL DEVIATION (WITH FEASIBLE ARCS)

PROBLEM	M.I.P			SWITCHING				
(# SAT)	OPTIMUM SOLN.	TIME	# OF NODES	BEST SOLN.	METHOD (K=)	TIME	SOL. W/ INCR. K	TIME
S.AMER (13)	27.01	121	442	27.01	INCR.	11	27.01	11
E.EUR (12)	43.87 @	160	500	41.48	2	1	41.48	7
W.EUR (12)	41.77 t	300	1500	41.77	5	18	43.26	7
SE.ASIA (10)	41.61	207	1827	42.61	4	4	42.61	12
N.AFRIC (10)	41.42	197	1460	41.42	INCR.	13	41.42	13
EUROPE (26)	--- t	600	--	160.96	5	90	204.6	82
NCS AM. (26)	--- t	600	--	50.44	INCR.	55	50.44	55

NOTES :

@ - not proven optimum, termination due to accuracy checks

t - not proven optimum, termination due to time limit

All times in CPU seconds on an IBM 3081-D.

The MIP runs were made with REAL*16 (128 bits) arithmetic. All switching runs were made with REAL*8 (64 bits) arithmetic. On average REAL*16 is 3 times slower than REAL*8. (Accuracy check problems necessitated the use of REAL*16 for the MIP runs.)

TABLE 5. COMPARISON OF SWITCHING SOLUTIONS TO MIP SOLUTIONS
MINIMIZE MAXIMUM DEVIATION (WITH FEASIBLE ARCS)

PROBLEM (# SAT)	M.I.P			SWITCHING				
	OPTIMUM SOLN.	TIME	NODES	BEST SOLN.	METHOD (K=)	TIME	SOL. W/ INCR. K	TIME
S.AMER (13)	4.68	252	1045	4.68	INCR.	9	4.68	9
E.EUR (12)	7.08 t	300	1200	7.23	INCR.	5	7.23	5
W.EUR (12)	7.33 t	300	1300	7.41	5	10	7.41	18
SE.ASIA (10)	12.78	269	1793	12.78	4	3	12.78	7
N.AFRIC (10)	--- @	9	--	10.19	5	10	11.07	8
EUROPE (26)	--- @	325	--	14.01	INCR.	110	14.01	110
NCS AM. (26)	--- @	62	--	5.70	5	120	6.00	52

NOTES :

@ - not proven optimum, termination due to accuracy checks

t - not proven optimum, termination due to time limit

All times in CPU seconds on an IBM 3081-D.

The MIP runs were made with REAL*16 (128 bits) arithmetic. All switching runs were made with REAL*8 (64 bits) arithmetic. On average REAL*16 is 3 times slower than REAL*8. (Accuracy check problems necessitated the use of REAL*16 for the MIP runs.)

with the best solutions generated by the switching heuristic for these nine problems shows that the switching solution is the same as the MIP solution in five cases. The switching algorithm found an optimal solution in four of these cases. In one case, the switching heuristic finds a solution better than the prematurely terminated MIP solution. For the remaining three cases, the solutions generated by the heuristic are within 2.5% of those found by the MIP.

In Tables 4 and 5, the solutions obtained using the switching heuristic with increasing subgroup size are indicated. For eight of the nine problems for which feasible solutions have been found by MIP, the switching solutions are the best solutions obtained by the switching method.

These results indicate that the switching heuristic consistently provides optimal or near optimal solutions for the satellite location problems with limits on feasible arcs and for the objective functions studied.

5.4 On using $\triangle\phi$ in place of $\triangle S$

In all the experiments discussed so far, $\triangle S$ (the worst case minimum required separation) has been used as the separation that is required between pairs of satellites. The effect of using $\triangle\phi$ (the minimum required separation based on the current location of the satellites) instead of $\triangle S$ is now examined.

In the implementation of this change with the switching heuristic, $\Delta\phi$ matrices, spaced at specified intervals from the easternmost limit to the westernmost limit of the feasible arcs, are used. In the experiments, an interval spacing of 10° is used.

The minimum required separations are redefined at the beginning of each iteration using the $\Delta\phi$ matrices. (An iteration consists of Steps 4 and 5 in the description of the heuristic in Section 3.4). The midpoint between the current prescribed satellite locations for each pair of satellites is found. The maximum of the two $\Delta\phi$ values for this pair of satellites that are nearest to the calculated midpoint becomes the current minimum required separation for the pair. For example, if satellites i and j are currently located at 11° and 14° respectively, the midpoint is at 12.5°. Matrices for $\Delta\phi$ are available at 10° and 20°, since the matrices are available at 10° intervals. The current minimum required separation is set to the maximum of the two $\Delta\phi$ values for satellites i and j in these two matrices. As the iteration progresses and a new subgroup of satellites is considered, the minimum required separations for all satellite pairs within the subgroup are reset using the above procedure.

At the initial ordering, no prescribed satellite locations are available. The ΔS values are used for the minimum required separations, and the dual linear program is solved to optimality. With the satellite locations obtained from the optimal solution, the minimum required separation values are reset as described in the previous paragraph. The linear program corresponding to the initial ordering is solved again, this time with the revised separation values. The optimal solution obtained is taken as the starting solution in the switching heuristic.

This is only one possible implementation of $\triangle\phi$ matrices with the switching heuristic. There are several other ways in which these matrices could be used in place of $\triangle S$, for example, every time a switch is made all minimum required separations could be reset. This implementation offers the benefits of using the $\triangle\phi$ matrices with only a limited amount of time spent in resetting the minimum required separations and in table lookups.

In the experiment to compare the effects of using $\triangle\phi$ in place of $\triangle S$, seven test problems (see Appendix C) are used. These seven scenarios are the same as those used in the previous experiments. Two problems are solved for each scenario, one where the objective is the minimization of the total deviation and the other with the objective of minimizing of the maximum deviation. For all fourteen problems, limits on feasible arcs are included. The switching heuristic with increasing subgroup size as implemented in Chapter 4, together with the changes described in this chapter, is applied to each of the fourteen problems using $\triangle\phi$ matrices to generate the minimum required separations. The solutions for the same set of problems in which $\triangle S$ is used, are already available from the previous experiment (See Tables 4 and 5).

The results for the seven problems with the objective function of minimizing total deviation are shown in Table 6. For the seven problems with the objective function of minimizing maximum deviation, the results are presented in Table 7. The final solution, the time to termination, and the number of major iterations performed are given for

TABLE 6. COMPARISON BETWEEN SWITCHING WITH $\triangle\phi$ AND SWITCHING WITH $\triangle S$

Objective : Minimize total deviation
(with feasible arc restrictions)

SCENARIO	SWITCHING WITH $\triangle\phi$ (INCR. K 2 TO 5)			SWITCHING WITH $\triangle S$ (INCR. K 2 TO 5)			WITH $\triangle S$ BEST SOLN. KNOWN
	SOLUTION	TIME	ITER.	SOLUTION	TIME	ITER.	
S.AMERICA	17.04	9	8	27.01	11	8	27.01 o
E.EUROPE	31.74	12	13	41.48	7	8	41.48
W.EUROPE	23.71	11	7	43.26	7	8	41.77
S.E.ASIA	34.75	8	10	42.61	12	12	41.61 o
N.AFRICA	26.24	6	8	41.42	13	10	41.42 o
EUROPE	98.68 t	120	17	204.58	82	14	160.96
NCS.AMER.	28.77	81	17	50.44	55	9	50.44

All times are in CPU seconds on an IBM 3081-D

o - proven optimum solution

t - terminated due to time limit

ITER = number of major iterations in the Switching heuristic

TABLE 7. COMPARISON BETWEEN SWITCHING WITH $\triangle\phi$ AND SWITCHING WITH $\triangle S$

Objective : Minimize maximum deviation
(with feasible arc restrictions)

SCENARIO	SWITCHING WITH $\triangle\phi$ (INCR. K 2 TO 5)			SWITCHING WITH $\triangle S$ (INCR. K 2 TO 5)			WITH $\triangle S$ BEST SOLN. KNOWN
	SOLUTION	TIME	ITER.	SOLUTION	TIME	ITER.	
S.AMERICA	3.00	9	9	4.68	9	8	4.68 o
E.EUROPE	5.71	13	11	7.23	5	9	7.08
W.EUROPE	6.12	33	14	7.41	18	10	7.33
S.E.ASIA	10.95	10	9	12.78	7	6	12.78 o
N.AFRICA	11.35	11	8	11.07	8	6	10.19
EUROPE	10.98	104	21	14.01	110	17	14.01
NCS.AMER.	3.82	88	10	6.00	52	11	5.70

All times are in CPU seconds on an IBM 3081-D

o - proven optimum solution

ITER = number of major iterations in the Switching heuristic the computer runs made for both the $\triangle\phi$ and $\triangle S$ situations. The last column contains the best solution known for the problem when $\triangle S$ is used. This solution was generated either through mixed integer programming or with the switching heuristic.

Using $\triangle\phi$ yields reductions of more than 16% in the objective function value in every case, from the best solution with $\triangle S$, when the objective is the minimization of total deviation. The smallest percentage reduction is 16% (the S.E.ASIA scenario), and the largest is 43% (the NCS. AMER. scenario). When the objective is the minimization of maximum deviation, the percentage reductions in objective function value range from 14% to 36% in the six cases where improvements are obtained. In one instance, the best solution using $\triangle S$ is 10% better than the solution obtained using $\triangle\phi$.

There is a tendency for runs made with $\triangle\phi$ to take more time and more iterations than runs with $\triangle S$ when the objective is the minimization of maximum deviation. No obvious trend with regard to time or number of iterations appears when the objective is minimization of total deviation, although the two larger problems (26 satellites) are more time consuming and require more iterations when $\triangle\phi$ is used. The gains in the objective function values are significant enough when $\triangle\phi$ is used in place of $\triangle S$, that increases in solution time and number of iterations are of minor importance.

The conclusion drawn from this experiment is that it is definitely preferable to use $\triangle\phi$ in place of $\triangle S$ in satellite location problems. When $\triangle\phi$ is used, the heuristic is more likely to find feasible solutions. However, a feasible solution obtained using $\triangle S$ values is likely to have larger C/I margins than a feasible solution to the same problem obtained when $\triangle\phi$ values are used. The use of $\triangle\phi$ in place of $\triangle S$ does not appreciably degrade the performance of the switching heuristic with regard to computation time.

5.5 Application of the Switching heuristic to the Region 2 scenario

The experiments described in Sections 4.6, 5.3, and 5.4 were conducted using scenarios which ranged in size from 10 to 26 service areas and satellites. In this section, the switching heuristic is applied to a scenario which has 28 service areas and 59 satellites. Several of the 28 service areas have more than one satellite. This scenario was provided by NASA Lewis Research Center. It is representative of satellite synthesis problems for Region 2, the nations in the western hemisphere. Satellite allotments in the Fixed Satellite Service (FSS) for Region 2 is one of the major goals of the World Administrative Radio Conference scheduled for mid 1988 (WARC'88).

The scenario for Region 2 is described in Appendix E. The names of the 28 service areas, the number of satellites associated with each service area, the desired locations and the limits on the feasible arcs for service areas are given. The $\triangle S$ matrix is also shown.

In this experiment, two versions of this scenario are considered. In the first version (VER.1), all 25 satellites belonging to service area USA have the same desired location, 96°. The minimum required separation between two satellites belonging to USA is 2°, irrespective of where the satellites are located. Hence, in any feasible solution, at best the 25 USA satellites could be spaced at 2° intervals about the desired location 96°, ranging in position from 72° to 120°. Therefore, in the second version of the scenario (VER.2), the desired locations for the 25 USA satellites are given desired locations ranging from 72° to 120° at 2° intervals. The two versions of the scenario are the same in all other respects.

In Table 8, the results of four computer runs made with this scenario are presented. In each case, the switching heuristic with increasing subgroup size (2 to 5) was used. The objective function throughout the experiment was the minimization of total deviation. Limits on the feasible arcs for service areas were enforced in all four cases. Two runs were made with each version of the scenario - the first run being made with $\triangle S$ and the second with $\triangle \phi$.

The final solution values obtained in the runs made with VER.1 are not directly comparable with the values obtained with VER.2, owing to the differences in the desired locations for the satellites belonging to the USA.

The only feasible solution that was found occurred when VER.2 was used with $\triangle \phi$. The remaining three runs terminated with infeasible solutions, although in every one of these three cases, only one or two satellites were positioned outside their respective feasible arcs. The combination of spacing the desired locations for satellites belonging to the same service area and using $\triangle \phi$ instead of $\triangle S$ improves the performance of the switching heuristic considerably over the case where

TABLE 8. RESULTS OF RUNS WITH THE REGION 2 SCENARIO

VERSION OF SCENARIO	$\triangle$S OR $\triangle\phi$	FINAL SOLN.	TOTAL TIME TAKEN	# OF MAJOR ITER.	SOLN. FEAS. ?	FIRST FEAS. SOLN.	TIME TO FEAS.
VER.1	$\triangle$S	759.59	1200	26	NO	----	---
VER.1	$\triangle\phi$	792.91	1200	44	NO	----	---
VER.2	$\triangle$S	488.19	710	27	NO	----	---
VER.2	$\triangle\phi$	404.19	1200	34	YES	430.32	335

All times are in CPU seconds on an IBM 3081-D

Runs taking 1200 seconds were terminated by limits on run time the same desired locations and ΔS are used. Each of the factors by itself does not dramatically change the solution obtained.

CHAPTER VI

RECOMMENDATIONS AND CONCLUSIONS

A major portion of this research has been focused on the application of the switching heuristic to satellite location problems. The objective functions that were used were the minimization of total deviation and the minimization of maximum deviation from given desired locations. In an effort to indicate directions for future research with the switching heuristic, other problems to which the switching heuristic can be applied, with appropriate changes in its implementation, are discussed in this chapter. Two applications are related to satellite system synthesis, while a third is associated with the more general area of job sequencing and scheduling. This manuscript concludes with a summary of the research performed and the contribution it makes to the fields of satellite system synthesis and Operations Research.

6.1 The allocation of arc segments to service areas

The problem concentrated on throughout this manuscript has been the assignment of orbit locations to individual satellites. These assigned locations can be said to be point assignments in the geostationary orbit. An alternate synthesis strategy is the allocation of a portion of the geostationary orbit, i.e. an arc segment, to every service area in a given scenario. In this case, the administration of each service area is at liberty to place as many satellites in as many locations as it wishes, as long as all the satellites are located within the arc segment allocated to that service area. It is up to each individual administration to resolve interference problems between its own satellites.

The allocation of arc segments differs from the assignment of satellite locations in that arc segments are assigned to <u>service areas</u> in the former case, while specific orbit locations are assigned to <u>satellites</u> in the latter. In the allocation of arc segments the system design has to allow for an administration placing a satellite anywhere in its assigned arc. Hence, in the allocation procedure, interference calculations have to be made based on the worst possible situation. This corresponds to separating the nearest edges of two arc segments allocated to two service areas by the minimum separation required between satellites that belong to those service areas.

If, in the scenario to be examined, a large proportion of the service areas have multiple satellites, then the number of decision variables in the arc segment approach is likely to be less than the number of decision variables in the point location approach. The term "decision variables" is used here in a generic sense and not with regard to any actual implementation. Another advantage of the arc segmentation approach over the point location approach is that the former gives much more freedom to the individual administrations in the actual location of satellites, and in the addition of satellites at a future date. The assignment of specific locations to satellites is a more rigid strategy in that satellites cannot be added at a later date without the re-evaluation of the entire initial plan.

It is possible that in real problems, involving all the national administrations in one or more continents, solutions with the arc segment allocation approach will contain a large number of infinitesimal arc lengths. If arc segment allocation problems are hard to solve, the point location strategy might be used to determine the feasiblity of a particular scenario.

A possible objective function for the arc segment allocation problem is the maximization of the smallest arc segment assigned to any service area. One set of constraints in the problem consists of the limits on feasible arcs, which constrain the arc segment assigned to a service area to be within the feasible arc for that service area. The other constraints that have to be satisfied are the minimum required separations between satellites.

A primal formulation for the linear program corresponding to the arc segment allocation problem described above is shown in Figure 20. The formulation is for the situation when the ordering among the arc segments is given (i.e., a formulation that can be used with the switching heuristic). This formulation corresponds to the primal formulation used when the ordering of the satellites was given (Figure 12). In this formulation, arc segments are ordered by their respective midpoints.

MINIMIZE $-Z$ (6.1)

subject to

$$Y_k - Y_j - Z \geq \Delta S_{ih} \qquad \begin{matrix} j=1,\ldots,n-1; & k=j+1,\ldots,n \\ i=S(j) \quad ; & h=S(k) \end{matrix} \qquad (6.2)$$

$$Y_j - 0.5\,Z \geq L_i \qquad i=S(j);\ j=1,\ldots n \qquad (6.3)$$

$$Y_j + 0.5\,Z \leq H_i \qquad i=S(j);\ j=1,\ldots n \qquad (6.4)$$

$$Z \geq 0 \qquad (6.5)$$

where

Y_j = the midpoint of the arc segment in position j in the ordering of arc segments

Z = the minimum arc segment length that is assigned

L_i = the lower limit of the feasible arc for the arc segment for service area i

H_i = the upper limit of the feasible arc for the arc segment for service area i

$S(j)$ = the service area whose arc segment is currently in position j

n = the number of service areas

ΔS_{ih} = the minimum required separation between satellites of service area i and service area h

Figure 20. The Primal Formulation of the Linear Program for the Arc Segment Allocation Problem with a Given Ordering of the Arc Segments.

The objective function (6.1) is the minimization of "-Z", thereby maximizing "Z", the minimum arc segment. The primal is set up as a minimization problem so that the dual is a maximization problem, thereby retaining a similar structure to that of previously considered dual formulations used with the switching heuristic.

Constraints (6.2) ensure that minimum required separations are met. In the formulation in Figure 20, a satellite is assumed to exist at the mid-point of every arc segment. Satellites in the arc segments i and h, in positions j and k respectively, are separated by a distance equal to the sum of $\triangle S_{ih}$ and Z. Therefore, if satellites were positioned at locations (Y_j + 0.5Z) and (Y_k - 0.5Z) in the arc segments corresponding to i and h respectively, the minimum required separation would still be satisfied. Appropriate $\triangle\phi$'s can be used in place of $\triangle S$'s. Constraints (6.3) and (6.4) ensure that assigned arc segments do not fall outside the limits on corresponding feasible arcs. If $\triangle S_{ih}$ is equal to 0 in any constraint of type (6.2), the variable Z is dropped from that constraint, the constraint then being defined by the inequality (6.6).

$$Y_k - Y_j \geq 0 \tag{6.6}$$

The solution obtained by solving this formulation assigns arc segments of equal length (i.e. equal to Z) to all service areas.

The dual formulation for the problem is shown in Figure 21. The constraint matrix refers only to the positions in an ordering. All references to a particular arc segment and its satellite appear only in the objective function. Therefore, the switching heuristic can be

$$\text{MAXIMIZE} \quad \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} \Delta S_{ih} v_{jk} + \sum_{j=1}^{n} L_i p_j + \sum_{j=1}^{n} H_i q_j \tag{6.7}$$

subject to

$$\sum_{k=1}^{j-1} v_{kj} - \sum_{k=j+1}^{n} v_{jk} + p_j + q_j \leq 0 \qquad j=1,\ldots n \tag{6.8}$$

$$-\sum_{j=1}^{n-1} \sum_{k=j+1}^{n} v_{jk} - 0.5 \sum_{j=1}^{n} p_j + 0.5 \sum_{j=1}^{n} q_j \leq -1 \tag{6.9}$$

$$v_{jk} \geq 0 \qquad j=1,\ldots,n-1;\ k=j+1,\ldots,n \tag{6.10}$$

$$p_j \ , \ -q_j \geq 0 \qquad j=1,\ldots n \tag{6.11}$$

where

n, ΔS_{ih} , L_i , H_i are as defined in Figure 20

v_{jk} = the dual variable corresponding to the $jk^{\underline{th}}$ constraint in the constraint set (6.2) in the primal

p_j = the dual variable corresponding to the $j^{\underline{th}}$ constraint in the constraint set (6.3) in the primal

q_j = the dual variable corresponding to the $j^{\underline{th}}$ constraint in the constraint set (6.4) in the primal

Figure 21. The Dual Formulation of the Linear Program for the Arc Segment Allocation Problem with a Given Ordering of the Arc Segments.

implemented, with some modification, as described in Sections 3.4, 4.4, and 5.2.

In making a switch in the ordering of arc segments, it might be necessary to drop a v_{jk} variable in constraint (6.9), if a constraint of type (6.2) in the primal becomes one of type (6.6) after the switch. If a constraint of type (6.6) in the primal becomes one of type (6.2) after the switch, then a v_{jk} variable has to be added in constraint (6.9) in the dual. When a v_{jk} variable is dropped from the dual formulation, the basis might become infeasible and this condition has to be allowed for in the heuristic.

This definition and implementation of the arc segmentation problem is only one of many that might be possible. There is no empirical evidence that the switching heuristic performs satisfactorily on this formulation of the arc segmentation problem. The sole purpose here is to indicate problems relevant to satellite system synthesis that might be solved using the switching heuristic.

6.2 The explicit allocation of frequency channels

The allocation of frequency channels to service areas, as part of the system synthesis, has not been explicitly considered in the solution methods considered so far. The solution techniques are based on the assumption that either the full frequency spectrum is required by all users, or that channel assignments are prespecified. If channel assignments are prespecified, then these frequency allocations can be taken into account in the computation of the $\triangle S$ and $\triangle \phi$ matrices for that scenario. In some situations, these assumptions might not be acceptable. Tentative methods by which frequency allocation can be included in satellite system synthesis with the switching heuristic are suggested in this section.

One method, suggested by Reilly [1986b], is to fix the frequency channels, and apply the switching heuristic to determine the satellite locations. Then keeping the satellite locations fixed, the switching heuristic can be used to make other frequency assignments that improve the objective function value. When the heuristic terminates in the latter phase, the whole procedure can be repeated. This method requires the determination of the functional relationship between the separation required in frequency ($\triangle f$) and the separation required in orbit location ($\triangle S$) in order to achieve a desired protection ratio.

An alternate method might involve defining a surrogate variable that represents a particular orbit location and frequency combination, and then formulating the location and frequency assignment problem in terms of these surrogate variables. The switching heuristic or a similar technique could be applied to the reformulated model in the surrogate variables.

An initial attempt at defining the relationship between $\triangle f$ and $\triangle S$ for particular pairs of service areas has been made by Buyukudura [1986].

6.3 On the possible application of the switching heuristic to classes of job scheduling problems.

The satellite location problem, as defined in Figure 2, is similar to some categories of job scheduling problems. For example, the scheduling of jobs with the objective of meeting given due dates as closely as possible, with processing and setup times for each job, is closely related to the satellite location problem where the objective function is the minimization of maximum deviation. The due dates in the scheduling problem correspond to the desired locations in the satellite problem, the processing and setup times correspond to the minimum required separations. The decision variables, the assigned locations in the satellite problem, are the job completion dates in the scheduling problem. The processing and setup times required for a job added to the date a job becomes available for processing gives the earliest possible completion date for the job, and this corresponds to the lower limit on the feasible arc for a satellite in the satellite location problem. If tardiness is not permitted in the scheduling problem, then the due date is the upper limit on the job completion date, and this corresponds to the upper limit on the feasible arc in the satellite location problem.

If the formulation in Figure 2 represented a job scheduling problem as described above, then the variable X_j would represent the completion date of job j, d_j would be the due date for job j, E_j the earliest possible completion date for the job, Wj the due date added to the maximum tardiness allowed. The $\triangle S_{ij}$ would represent the sum of processing and setup time for job j if it were scheduled after job i. With this definition, $\triangle S_{ij}$ is not the same as $\triangle S_{ji}$. However, when the switching heuristic is used, the ordering is always specified before any dual formulation is solved. Therefore, the $\triangle S$ values to be used are known. If the job scheduling problem is such that set up times occur only between adjacent jobs (i.e. non-adjacent jobs do not interfere with one another), then eliminating redundancies in the separation matrix will result in a primal formulation with n-1 separation constraints and the corresponding dual variables will also number n-1, n being the number of jobs.

The switching heuristic has not been applied to job scheduling problems of the type described above. The intention here is to indicate the applicability of the switching heuristic in a more general context with the expectation that the heuristic will perform as well in the area of job scheduling as it did in satellite system synthesis.

6.4. Conclusions

In this manuscript, the satellite location problem in satellite system synthesis has been examined. Four solution techniques have been considered as candidate solution methods for the problem. An experimental study was carried out on a set of test problems with the four methods. The empirical results indicated that the switching heuristic consistently outperformed the other three methods.

Variations in the satellite location problem were explored and the corresponding modifications required in the switching heuristic were developed. Experimental results with these variations indicate that the switching heuristic is well suited to these variations as well as the original problem. Implementation of the switching heuristic on a large real problem demonstrated that the heuristic can provide acceptable solutions for such problems.

This research has been an intensive study of the satellite location problem. The switching method developed, an interchange heuristic coupled with an efficient implementation of linear programming duality, has provided an effective tool for satellite system systhesis. The techniques that were developed can be applied in other areas, notably in job scheduling and sequencing. Finally, it has been demonstrated that complex problems do not always require complex solution techniques; a judicious combination of an appropriate formulation of the problem and a simple heuristic can be very effective.

APPENDIX A

A DEMONSTRATION THAT LAGRANGEAN MULTIPLIERS FOR THE OPTIMUM SOLUTION TO THE BENDERS' MASTER PROBLEM MAY NOT EXIST

Consider a satellite location problem in which three satellites are to be assigned locations. In the corresponding Benders' master problem (Figure 7), we have three satellites (indexed by i's) and three orbit positions (indexed by j's). The constraints (3.23), (3.24) and (3.25) in Figure 7 define a 3X3 assignment problem. Assume that there are three Benders' constraints, e,f, and g, and that the coefficients of the variables in these three constraints are as follows :

	VARIABLES								
	X_{11}	X_{12}	X_{13}	X_{21}	X_{22}	X_{23}	X_{31}	X_{32}	X_{33}
CONSTRAINTS									
e	100	0	0	0	1	0	0	0	1
f	99	1	101	101	99	1	1	101	99
g	99	101	1	1	99	101	101	1	99

The six possible assignments (denoted by $A^1,\dots,A^6$) for this problem are defined below. The decision variables in parentheses are set to 1, and all the other decision variables are set to 0 in order to obtain the assignments.

$A^1 = (X_{11}, X_{22}, X_{33})$

$A^2 = (X_{21}, X_{32}, X_{13})$

$A^3 = (X_{31}, X_{12}, X_{23})$

$A^4 = (X_{11}, X_{32}, X_{23})$

$A^5 = (X_{21}, X_{12}, X_{33})$

$A^6 = (X_{31}, X_{22}, X_{13})$

By evaluating each constraint with each possible assignments, we get the following table. The solution to the master problem with each assignment is the maximum of all the values in the column corresponding to that assignment.

	ASSIGNMENTS					
	A^1	A^2	A^3	A^4	A^5	A^6
CONSTRAINT e	102	0	0	100	1	1
CONSTRAINT f	297	303	3	201	201	201
CONSTRAINT g	297	3	303	201	201	201
SOLN. VALUE IN MASTER PROBLEM	297	303	303	201	201	201

The assignments A^4, A^5, and A^6 provide alternate optimal solutions to the master problem. If A^5 is to be an optimum solution to the Lagrangean relaxation of the master problem (Figure 10), then the solutions provided by all other assignments must not be any better than the solution provided by A^5. This condition is represented by the following set of inequalities, (A.1) to (A.5).

$$102 L_e + 297 L_f + 297 L_g \geq L_e + 201 L_f + 201 L_g \qquad (A.1)$$

$$0\,L_e + 303\,L_f + 3\,L_g \geq L_e + 201\,L_f + 201\,L_g \tag{A.2}$$

$$0\,L_e + 3\,L_f + 303\,L_g \geq L_e + 201\,L_f + 201\,L_g \tag{A.3}$$

$$100\,L_e + 201\,L_f + 201\,L_g \geq L_e + 201\,L_f + 201\,L_g \tag{A.4}$$

$$1\,L_e + 201\,L_f + 201\,L_g \geq L_e + 201\,L_f + 201\,L_g \tag{A.5}$$

(L_i = the Lagrangean multiplier corresponding to constraint i.)

Adding (A.2) and (A.3), we obtain

$$0\,L_e + 306\,L_f + 306\,L_g \geq 2\,L_e + 402\,L_f + 402\,L_g \tag{A.6}$$

The constraints on the Lagrangean multipliers in the Lagrangean relaxation are

$$L_e + L_f + L_g = 1 \tag{A.7}$$

$$L_e\,,\ L_f\,,\ L_g \geq 0 \tag{A.8}$$

Clearly the system of equations and inequalities denoted by (A.6),(A.7) and (A.8) is inconsistent. Therefore, there does not exist a set of Lagrangean multipliers for which the solution to the Lagrangean relaxation is the same as the optimal solution to the master problem , A^5. The above exercise can be repeated for the other optimal assignments A^4 and A^6. For this example, there does not exist a set of Lagrangean multipliers for which the solution to the Lagrangean relaxation is an optimal solution to the master problem.

APPENDIX B

IDENTIFICATION OF REDUNDANT $\triangle S$ VALUES ONCE THE SATELLITE ORDERING IS SPECIFIED

Once an ordering of the satellites is specified in the satellite location problem, the resulting problem is a linear program. Further, a large number of the minimum required separation values ($\triangle S$) become redundant. The removal of these redundant values from the problem reduces its size considerably, thereby reducing the computation time required to solve the linear program. The reduction occurs in the number of constraints in the primal problems (Figures 8,12, and 16) and in the number of variables in the dual problems (Figures 9,13, and 17). The method used to identify the redundant $\triangle S$ values is described in this appendix.

Define S_i as the satellite in position i in the given ordering. The minimum required separation between satellites S_i and S_j is denoted by $\triangle S_{ij}$, and $\triangle M$ denotes the maximum value among all the $\triangle S_{ij}$. Let n be the number of satellites in the problem under consideration.

Consider a satellite S_j in position j. The $\triangle S$ values that are redundant between S_j and satellites in positions k (k=j+1,...n) need to be determined.

Define

$$\triangle T(j+1) = \triangle S_{j,j+1} \tag{B.1}$$

$$\triangle T(k) = \max \{ \triangle T(h) + \triangle S_{hk} , \triangle S_{jk} \} \quad j+1 \leq h \leq k-1; \quad k=j+2,...n \tag{B.2}$$

By defining $\triangle T(k)$ as indicated in equations B.1 and B.2, the pairwise separations that are required between all the satellites S_j, S_{j+1}, ..., S_k as well as the current ordering of satellites are used in calculating the minimum separation that will occur between satellites S_j and S_k in any solution that satisfies the minimum separation requirements with this ordering. The three conditions for identifying redundant $\triangle S_{jk}$ values for the current ordering are the following.

1. If $\triangle S_{jk} = 0$ and $k \neq j+1$ then $\triangle S_{jk}$ is redundant since no minimum separation is required between satellites S_j and S_k. (If $k = j+1$, then $\triangle S_{jk}$ is not redundant since the constraint corresponding to this j,k pair has to be included in the formulation in order to ensure that the given ordering is not violated).

2. If $\triangle S_{jk} < \triangle T(k)$ then $\triangle S_{jk}$ is redundant. For the current ordering, in any solution that satisfies the minimum required separations for all pairs of satellites between satellite S_j and satellite S_k, satellites S_j and S_k will be separated by at least $\triangle T(k)$. Hence $\triangle S_{jk}$ is redundant.

3. If $\triangle T(k) > \triangle M$ then $\triangle S_{jh}$ is redundant for $h=k+1,\ldots n$. By definition, $\triangle T(h) \geq \triangle T(k)$ for $h=k+1,\ldots n$. Since $\triangle T(h) \geq \triangle T(k) > \triangle M \geq \triangle S_{jh}$, by the second condition above, $\triangle S_{jh}$ is redundant for $h=k+1,\ldots,n$.

The above rules follow from the fact that if the sum of $\triangle S_{ab}$ and $\triangle S_{bc}$ is greater than $\triangle S_{ac}$, then $\triangle S_{ac}$ is always enforced when the satellites are in the order a,b,c. Therefore an explicit constraint on the minimum required separation between satellites a and c is unnecessary.

Application of this procedure prior to solving the linear program results in a much smaller problem and correspondingly reduced computational time in the Benders' subproblem phase and also in the switching technique.

APPENDIX C

ON AVOIDING THE REPETITION OF SATELLITE ORDERINGS IN A MAJOR ITERATION

The procedure used to restrict any particular satellite ordering to being evaluated only once in a major iteration of the switching heuristic is presented as a theorem in this appendix. The reduction of the number of evaluations performed in each major iteration with this procedure is calculated.

Let n = number of satellites and number of orbit positions

k = number of satellites in a subgroup

$S(j)$ = the satellite in position j

m = minor iteration

A minor iteration, m, consists of the evaluation of all satellite orderings resulting from the permutation of satellites in positions $\{ m, m+1,\dots,m+k-1 \}$, the satellites in positions $\{1,\dots,m-1\}$ and in positions $\{m+k,\dots,n\}$ remaining in their respective positions. A major iteration consists of the set of minor iterations $\{1,2,\dots,n-k+1\}$.

<u>Theorem C.1 :</u>

If in each minor iteration m ($m=2,\dots,n-k+1$), only those satellite orderings are considered where $S(m+k-1)$ in minor iteration $m-1$ is <u>not</u> in position $m+k-1$, then every satellite ordering considered in a major iteration is different from all the other orderings considered in that major iteration.

Proof :

Consider a minor iteration m. The satellites S(1) ,..., S(m-1) , S(m+k) ,..., S(n) do not switch positions in this minor iteration. Therefore, in minor iterations 1 to m for this major iteration, the same satellite has been in position m+k for all the satellite orderings examined. Let this satellite be Z. Now consider minor iteration m+1. The satellites being switched in this minor iteration are those in positions m+1,...,m+k. Since all satellite orderings examined so far have had Z in position m+k, any satellite ordering that does not have Z in position m+k has not been evaluated in this major iteration. By definition, all satellite orderings considered in this minor iteration, m+1, do not have Z in position m+k. Therefore all satellite orderings considered in minor iteration m+1 are distinct from those considered previously in this major iteration. Being permutations, they are distinct among themselves. Extending the argument to all minor iterations completes the proof.

Theorem C.2 :

If the procedure indicated in Theorem C.1 is used, then only repetitions of orderings previously evaluated in the major iteration are eliminated from consideration.

Proof :

Let Z be as defined in the proof for Theorem C.1. In iteration m+1, the orderings that are eliminated from consideration using the procedure of Theorem C.1 are those with Z in position m+k. The proof is accomplished by showing that all these orderings that are eliminated have been previously considered in the current major iteration.

At iteration m+1 consider an ordering with Z in position m+k. Assume it has not been previously examined in this major iteration. Let this satellite ordering be Q = S(1) ,..., S(m+k-1) , Z , S(m+k+1) ,..., S(n). By definition of the procedure, this ordering must have been evaluated in minor iteration m, except if S(m+k-1) had been the last satellite in the subgroup at the beginning of iteration m. A recursion of the argument for iterations m-1,...,1 shows that if Q has not been considered previously, then some permutations of the satellites in positions 1,...,k at the start of the major iteration have not been examined at minor iteration 1. This contradicts the stated procedure which is that at the first minor iteration all orderings which have permutations of satellites in positions 1,...,k and other satellites fixed in position must be considered. Therefore the assumption that Q has not been examined before is false, and the proof is complete.

<u>Calculation of the reduction in the number of evaluations when Theorem C.1 is implemented.</u>

A. Prior to the implementation of Theorem C.1

Number of evaluations per minor iteration = $k! - 1$

Total number of minor iterations = $n - k + 1$

Number of evaluations per major iteration = $(k!-1)(n-k+1)$

B. After the implementation of Theorem C.1

Number of evaluations eliminated per minor iteration = $(k-1)!$

Number of evaluations per minor iteration = $k! - 1 - (k-1)!$

Number of evaluations in minor iteration 1 = $k! - 1$

Number of evaluations per major iteration

$$= (k!-1) + (k! - 1 - (k-1)!)(n-k)$$

C. Reduction in the number of evaluations is $(k-1)!\,(n-k)$

APPENDIX D

THE SEVEN TEST SCENARIOS

The seven test scenarios that are used in the experiments in Chapters 4 and 5 are described in this appendix. The scenarios were generated for service areas which are actual nations and for realistic situations. The test points that define the service areas were taken from an atlas, and chosen such that the polygon formed by joining the test points covered the corresponding service area. The minimum ellipses were calculated using the computer program developed by Akima [1981]. The $\triangle S$ matrices were obtained using the computer program developed by Wang [1986]. The electrical system characteristics (e.g., antenna discrimination patterns, Earth station antenna gains, channel bandwidth) used in the $\triangle S$ calculations are those used by Wang for FSS applications.

For each scenario the names of the countries in the scenario are given along with a four character code for the name. The desired location and the limits on the feasible arc for each satellite are specified. These values are defined in degrees of longitude relative to a stated longitude so that they are always non-negative.

The worst case minimum required separation for every pair of satellites is specified as an element of the $\triangle S$ matrix. The separation is given in degrees of longitude. The $\triangle S$ matrix is symmetric about the diagonal and hence only elements above the diagonal are stated.

SCENARIO 1 : S.AMERICA (13 SATELLITES - 12 SERVICE AREAS)

S/A NAME	CODE	DESIRED LOCATION	LIMITS ON FEASIBLE ARC	
ARGENTINA	ARG1	10.00	5.00	15.00
BOLIVIA	BOL1	17.50	12.50	22.50
BRAZIL	BRZ1	0.00	0.00	10.00
CHILE	CHL1	25.00	20.00	30.00
COLOMBIA	CLM1	25.00	20.00	30.00
ECUADOR	ECD1	30.00	25.00	35.00
GUYANA	GUY1	7.50	2.50	12.50
PARAGUAY	PRG1	7.50	2.50	12.50
PERU	PRU1	27.50	22.50	32.50
SURINAM AND FRENCH GUIANA	SFG1	5.00	0.00	10.00
URUGUAY	URG1	7.50	2.50	12.50
VENEZUELA	VEN1	15.00	10.00	20.00
BRAZIL	BRZ2	15.00	10.00	20.00

Desired locations and feasible arcs are defined relative to 60° W.lon.

Note : The service area BRAZIL has two satellites BRZ1 and BRZ2.
The two service areas SURINAM and FRENCH GUIANA share a satellite SFG1.

THE ΔS MATRIX :

	ARG1	BOL1	BRZ1	CHL1	CLM1	ECD1	GUY1	PRG1	PRU1	SFG1	URG1	VEN1	BRZ2
ARG1	----	4.33	4.92	4.66	0.54	0.47	0.49	4.41	1.41	0.47	4.55	0.51	4.92
BOL1	----	----	4.71	4.39	0.63	0.52	0.48	4.45	4.84	0.46	0.99	0.52	4.71
BRZ1	----	----	----	4.33	5.13	3.14	5.02	4.59	5.29	4.35	4.61	4.24	6.00
CHL1	----	----	----	----	0.53	0.48	0.48	3.97	4.34	0.44	4.13	0.52	4.33
CLM1	----	----	----	----	----	4.19	2.36	0.49	4.46	1.44	0.35	4.91	5.13
ECD1	----	----	----	----	----	----	0.09	0.00	4.69	0.00	0.00	1.35	3.14
GUY1	----	----	----	----	----	----	----	0.00	0.51	4.57	0.00	4.72	5.02
PRG1	----	----	----	----	----	----	----	----	1.46	0.00	2.46	0.00	4.59
PRU1	----	----	----	----	----	----	----	----	----	0.44	0.45	1.45	5.29
SFG1	----	----	----	----	----	----	----	----	----	----	0.00	3.86	4.35
URG1	----	----	----	----	----	----	----	----	----	----	----	0.00	4.61
VEN1	----	----	----	----	----	----	----	----	----	----	----	----	4.24

SCENARIO 2 : E.EUROPE (12 SATELLITES - 12 SERVICE AREAS)

S/A NAME	CODE	DESIRED LOCATION	LIMITS ON FEASIBLE ARC	
FINLAND	FIN1	24.00	9.00	39.00
BULGARIA	BUL1	25.00	10.00	40.00
ROMANIA	ROM1	25.00	10.00	40.00
GREECE	GRC1	27.00	12.00	42.00
ALBANIA	ALB1	30.00	15.00	45.00
POLAND	POL1	30.00	15.00	45.00
HUNGARY	HUN1	30.50	15.50	45.50
YUGOSLAVIA	YUG1	31.00	16.00	46.00
CZECHOSLOVAKIA	CZH1	33.00	18.00	48.00
SWEDEN	SWD1	33.00	18.00	48.00
AUSTRIA	AUS1	37.00	22.00	52.00
E.GERMANY	EGR1	38.00	23.00	53.00

Desired locations and feasible arcs are defined relative to 50° E.lon.

THE $\triangle$S MATRIX :

	FIN1	BUL1	ROM1	GRC1	ALB1	POL1	HUN1	YUG1	CZH1	SWD1	AUS1	EGR1
FIN1	----	0.00	0.85	0.12	0.00	2.49	0.96	0.35	1.28	4.82	0.92	1.46
BUL1	----	----	4.62	4.60	4.46	1.96	3.75	4.57	2.71	0.52	3.15	1.91
ROM1	----	----	----	3.38	3.63	3.84	4.60	4.43	4.11	1.25	4.12	3.18
GRC1	----	----	----	----	4.59	1.23	2.59	4.23	1.52	0.38	2.21	1.21
ALB1	----	----	----	----	----	1.29	2.44	4.69	1.53	0.22	2.73	1.26
POL1	----	----	----	----	----	----	4.53	3.47	4.55	3.89	3.92	4.64
HUN1	----	----	----	----	----	----	----	4.68	4.64	1.74	4.48	3.57
YUG1	----	----	----	----	----	----	----	----	4.11	1.19	4.62	3.30
CZH1	----	----	----	----	----	----	----	----	----	2.63	4.60	4.41
SWD1	----	----	----	----	----	----	----	----	----	----	1.76	3.95
AUS1	----	----	----	----	----	----	----	----	----	----	----	4.10

SCENARIO 3 : W.EUROPE (12 SATELLITES - 12 SERVICE AREAS)

S/A NAME	CODE	DESIRED LOCATION	LIMITS ON FEASIBLE ARC	
ITALY	ITL1	38.00	30.00	48.00
NORWAY	NOR1	28.00	28.00	28.00
DENMARK	DEN1	40.00	30.00	50.00
W.GERMANY	WGR1	41.00	31.00	51.00
SWITZERLAND	SWZ1	41.50	31.50	51.50
NETHERLANDS	NTH1	45.00	35.00	55.00
BELGIUM	BLG1	45.50	35.50	55.50
FRANCE	FRA1	47.00	37.00	57.00
UNITED KINGDOM	UK_1	52.50	42.50	60.00
SPAIN	SPN1	55.00	45.00	60.00
IRELAND	IRL1	57.50	47.50	60.00
PORTUGAL	POR1	58.50	48.50	60.00

Desired locations and feasible arcs are defined relative to 50° E.lon.

THE $\triangle$S MATRIX :

	ITL1	NOR1	DEN1	WGR1	SWZ1	NTH1	BLG1	FRA1	UK_1	SPN1	IRL1	POR1
ITL1	----	0.82	1.46	4.60	4.76	3.58	4.04	4.96	1.96	3.83	1.99	1.31
NOR1	----	----	3.86	2.95	0.98	2.15	1.32	1.00	3.05	0.39	0.40	0.00
DEN1	----	----	----	4.77	1.50	4.04	3.30	3.38	3.65	0.94	1.34	0.00
WGR1	----	----	----	----	4.66	4.64	4.51	4.65	3.77	2.20	2.37	1.19
SWZ1	----	----	----	----	----	3.13	3.63	4.43	2.85	3.23	2.10	1.91
NTH1	----	----	----	----	----	----	4.46	4.67	4.57	1.84	2.93	1.25
BLG1	----	----	----	----	----	----	----	4.81	4.14	2.31	3.01	1.45
FRA1	----	----	----	----	----	----	----	----	4.38	4.48	3.98	3.03
UK_1	----	----	----	----	----	----	----	----	----	1.42	4.60	1.18
SPN1	----	----	----	----	----	----	----	----	----	----	1.29	4.45
IRL1	----	----	----	----	----	----	----	----	----	----	----	0.86

SCENARIO 4 : S.E.ASIA (10 SATELLITES - 10 SERVICE AREAS)

S/A NAME	CODE	DESIRED LOCATION	LIMITS ON FEASIBLE ARC	
PHILIPPINES	PHP1	8.00	0.00	23.00
TAIWAN	TWN1	9.00	0.00	24.00
INDONESIA	IDN1	11.50	0.00	26.50
VIETNAM	VTM1	24.00	9.00	30.00
CAMBODIA	CMB1	25.00	10.00	30.00
LAOS	LAO1	26.00	11.00	30.00
MALAYSIA	MLY1	20.00	5.00	30.00
CHINA	CHN1	25.00	10.00	30.00
THAILAND	THL1	28.00	13.00	30.00
BURMA	BRM1	30.00	15.00	30.00

Desired locations and feasible arcs are defined relative to 130° E.lon.

THE $\triangle$S MATRIX :

	PHP1	TWN1	IDN1	VTM1	CMB1	LAO1	MLY1	CHN1	THL1	BRM1
PHP1	----	2.21	4.06	1.73	1.70	1.30	4.31	1.99	1.03	0.45
TWN1	----	----	0.36	1.82	0.03	1.05	0.00	3.50	0.94	0.29
IDN1	----	----	----	3.84	3.50	1.06	4.64	0.64	4.20	3.14
VTM1	----	----	----	----	4.59	4.67	3.94	4.17	4.74	4.29
CMB1	----	----	----	----	----	4.59	2.72	1.37	4.58	4.10
LAO1	----	----	----	----	----	----	0.77	4.11	4.75	4.57
MLY1	----	----	----	----	----	----	----	0.52	4.61	1.75
CHN1	----	----	----	----	----	----	----	----	4.07	4.71
THL1	----	----	----	----	----	----	----	----	----	4.84

SCENARIO 5 : N.AFRICA (10 SATELLITES - 10 SERVICE AREAS)

S/A NAME	CODE	DESIRED LOCATION	LIMITS ON FEASIBLE ARC	
LIBYA	LBY1	7.50	0.00	22.50
NIGER	NGR1	17.50	2.50	30.00
MALI	MAL1	29.00	14.00	30.00
MOROCCO	MOR1	30.00	15.00	30.00
MAURITANIA	MAU1	30.00	25.00	30.00
SUDAN	SUD1	0.00	0.00	8.00
EGYPT	EGP1	0.00	0.00	5.00
CHAD	CHD1	6.00	0.00	21.00
TUNISIA	TNS1	15.50	0.50	30.00
ALGERIA	ALG1	23.00	8.00	30.00

Desired locations and feasible arcs are defined relative to 30° E.lon.

THE $\triangle$S MATRIX :

	LBY1	NGR1	MAL1	MOR1	MAU1	SUD1	EGP1	CHD1	TNS1	ALG1
LBY1	----	4.71	1.43	1.10	0.55	4.66	4.56	4.67	4.99	4.88
NGR1	----	----	4.93	0.89	2.41	1.57	1.38	4.79	0.67	4.58
MAL1	----	----	----	4.67	4.88	0.50	0.47	1.56	1.16	4.60
MOR1	----	----	----	----	5.04	0.39	0.34	0.42	2.23	5.08
MAU1	----	----	----	----	----	0.46	0.39	0.47	0.94	4.42
SUD1	----	----	----	----	----	----	4.27	4.81	0.45	0.55
EGP1	----	----	----	----	----	----	----	3.50	1.37	1.32
CHD1	----	----	----	----	----	----	----	----	1.21	3.27
TNS1	----	----	----	----	----	----	----	----	----	4.62

SCENARIO 6 : EUROPE (26 SATELLITES - 26 SERVICE AREAS)

S/A NAME	CODE	DESIRED LOCATION	LIMITS ON FEASIBLE ARC	
USSR	USR1	10.00	0.00	18.00
FINLAND	FIN1	24.00	9.00	39.00
BULGARIA	BUL1	25.00	10.00	40.00
ROMANIA	ROM1	25.00	10.00	40.00
GREECE	GRC1	27.00	12.00	42.00
ALBANIA	ALB1	30.00	15.00	45.00
POLAND	POL1	30.00	15.00	45.00
HUNGARY	HUN1	30.50	15.50	45.50
YUGOSLAVIA	YUG1	31.00	16.00	46.00
CZECHOSLOVAKIA	CZH1	33.00	18.00	48.00
SWEDEN	SWD1	33.00	18.00	48.00
AUSTRIA	AUS1	37.00	22.00	52.00
E.GERMANY	EGR1	38.00	23.00	53.00
ITALY	ITL1	38.00	30.00	48.00
NORWAY	NOR1	28.00	28.00	28.00
DENMARK	DEN1	40.00	30.00	50.00
W.GERMANY	WGR1	41.00	31.00	51.00
SWITZERLAND	SWZ1	41.50	31.50	51.50
NETHERLANDS	NTH1	45.00	35.00	55.00
BELGIUM	BLG1	45.50	35.50	55.50
FRANCE	FRA1	47.00	37.00	57.00
UNITED KINGDOM	UK_1	52.50	42.50	60.00
SPAIN	SPN1	55.00	45.00	60.00
IRELAND	IRL1	57.50	47.50	60.00
PORTUGAL	POR1	58.50	48.50	60.00
ICELAN	ICL1	70.00	55.00	75.00

Desired locations and feasible arcs are defined relative to 50° E.lon.

THE $\triangle$S MATRIX :

	USR1	FIN1	BUL1	ROM1	GRC1	ALB1	POL1	HUN1	YUG1	CZH1	SWD1	AUS1	EGR1
	ITL1	NOR1	DEN1	WGR1	SWZ1	NTH1	BLG1	FRA1	UK_1	SPN1	IRL1	POR1	ICL1
USR1	----	4.50	3.73	4.89	3.04	1.50	4.56	4.92	3.45	4.86	4.43	3.65	3.70
	1.37	4.51	3.57	2.91	1.22	1.96	1.32	0.55	0.54	0.48	0.37	0.42	0.00
FIN1	----	----	0.00	0.85	0.12	0.00	2.49	0.96	0.35	1.28	4.82	0.92	1.46
	0.43	4.39	2.00	1.23	0.00	0.41	0.32	0.45	0.50	0.25	0.00	0.00	0.00
BUL1	----	----	----	4.62	4.60	4.46	1.96	3.75	4.57	2.71	0.52	3.15	1.91
	3.77	0.00	1.09	2.16	2.10	1.52	1.39	1.21	0.00	0.39	0.00	0.00	0.00
ROM1	----	----	----	----	3.38	3.63	3.84	4.60	4.43	4.11	1.25	4.12	3.18
	3.09	0.53	2.18	3.17	2.16	2.40	2.03	1.34	0.36	0.47	0.00	0.00	0.00

	USR1	FIN1	BUL1	ROM1	GRC1	ALB1	POL1	HUN1	YUG1	CZH1	SWD1	AUS1	EGR1
GRC1	—	—	—	—	—	4.59	1.23	2.59	4.23	1.52	0.38	2.21	1.21
ALB1	—	—	—	—	—	—	1.29	2.44	4.69	1.53	0.22	2.73	1.26
POL1	—	—	—	—	—	—	—	4.53	3.47	4.55	3.89	3.92	4.64
HUN1	—	—	—	—	—	—	—	—	4.68	4.64	1.74	4.48	3.57
YUG1	—	—	—	—	—	—	—	—	—	4.11	1.19	4.62	3.30
CZH1	—	—	—	—	—	—	—	—	—	—	2.63	4.60	4.41
SWD1	—	—	—	—	—	—	—	—	—	—	—	1.76	3.95
AUS1	—	—	—	—	—	—	—	—	—	—	—	—	4.10
EGR1	—	—	—	—	—	—	—	—	—	—	—	—	—
ITL1	—	—	—	—	—	—	—	—	—	—	—	—	—
NOR1	—	—	—	—	—	—	—	—	—	—	—	—	—
DEN1	—	—	—	—	—	—	—	—	—	—	—	—	—
WGR1	—	—	—	—	—	—	—	—	—	—	—	—	—
SWZ1	—	—	—	—	—	—	—	—	—	—	—	—	—
NTH1	—	—	—	—	—	—	—	—	—	—	—	—	—
BLG1	—	—	—	—	—	—	—	—	—	—	—	—	—
FRA1	—	—	—	—	—	—	—	—	—	—	—	—	—
U_K1	—	—	—	—	—	—	—	—	—	—	—	—	—
SPN1	—	—	—	—	—	—	—	—	—	—	—	—	—
IRL1	—	—	—	—	—	—	—	—	—	—	—	—	—
POR1	—	—	—	—	—	—	—	—	—	—	—	—	—

	ITL1	NOR1	DEN1	WGR1	SWZ1	NTH1	BLG1	FRA1	UK_1	SPN1	IRL1	POR1	ICL1
GRC1	3.95	0.12	0.38	1.33	2.32	1.11	1.22	1.27	0.27	0.51	0.00	0.00	0.00
ALB1	4.67	0.00	0.00	1.98	2.83	1.34	1.56	1.54	0.24	1.06	0.00	0.00	0.00
POL1	3.45	2.28	4.45	4.34	2.92	3.46	2.77	2.99	1.41	1.20	0.45	0.00	0.00
HUN1	4.02	0.90	2.34	3.85	2.98	3.07	3.11	2.95	1.09	1.71	0.25	0.00	0.00
YUG1	4.53	0.52	1.33	4.25	4.19	3.19	3.56	3.55	1.33	0.82	1.02	0.00	0.00
CZH1	4.29	1.33	3.05	4.52	3.73	4.19	3.44	3.75	1.83	1.99	1.04	0.21	0.00
SWD1	1.20	4.59	4.61	3.89	1.35	2.18	1.53	1.39	1.64	0.51	0.37	0.17	0.00
AUS1	4.86	1.24	2.21	4.72	4.46	3.48	3.65	4.20	2.07	2.66	1.52	0.62	0.00
EGR1	3.09	2.58	4.65	4.65	3.34	4.46	3.80	4.11	2.73	1.38	1.29	0.82	0.00
ITL1	—	0.82	1.46	4.60	4.76	3.58	4.04	4.96	1.96	3.83	1.99	1.31	0.27
NOR1	—	—	3.86	2.95	0.98	2.15	1.32	1.00	3.05	0.39	0.40	0.00	0.00
DEN1	—	—	—	4.77	1.50	4.04	3.30	3.38	3.65	0.94	1.34	0.00	0.00
WGR1	—	—	—	—	4.66	4.64	4.51	4.65	3.77	2.20	2.37	1.19	0.19
SWZ1	—	—	—	—	—	3.13	3.63	4.43	2.85	3.23	2.10	1.91	0.00
NTH1	—	—	—	—	—	—	4.46	4.67	4.57	1.84	2.93	1.25	0.00
BLG1	—	—	—	—	—	—	—	4.81	4.14	2.31	3.01	1.45	0.00
FRA1	—	—	—	—	—	—	—	—	4.38	4.48	3.98	3.03	0.72
U_K1	—	—	—	—	—	—	—	—	—	1.42	4.60	1.18	2.97
SPN1	—	—	—	—	—	—	—	—	—	—	1.29	4.45	0.32
IRL1	—	—	—	—	—	—	—	—	—	—	—	0.86	2.05
POR1	—	—	—	—	—	—	—	—	—	—	—	—	0.00

SCENARIO 7 : NCS AMERICA (26 SATELLITES - 26 SERVICE AREAS)

S/A NAME	CODE	DESIRED LOCATION	LIMITS ON FEASIBLE ARC	
USA	USA1	46.00	31.00	50.00
MEXICO	MEX1	48.00	33.00	50.00
CANADA	CAN1	50.00	38.00	50.00
SURINAM & FRENCH GUIANA	SFG1	4.50	0.00	19.50
CARIBBEAN	CRB1	0.00	0.00	15.00
BRAZIL	BRZ1	4.00	0.00	19.00
GUYANA	GUY1	9.00	0.00	24.00
PARAGUAY	PRG1	8.00	0.00	23.00
URUGUAY	URG1	5.50	0.00	20.50
ARGENTINA	ARG1	13.50	0.00	28.50
VENEZUELA	VEN1	16.50	1.50	31.50
BOLIVIA	BOL1	13.00	0.00	28.00
CHILE	CHL1	21.00	6.00	36.00
COLOMBIA	CLM1	24.50	9.50	39.50
PERU	PRU1	25.00	10.00	40.00
ECUADOR	ECD1	28.00	13.00	43.00
CUBA	CUB1	29.50	14.50	44.50
COSTA RICA	CTR1	34.00	19.00	49.00
NICARAGUA	NCG1	35.00	20.00	50.00
HONDURAS	HND1	36.00	21.00	50.00
BELIZE	BLZ1	38.50	23.50	50.00
EL SALVADOR	SLV1	39.00	24.00	50.00
GUATEMALA	GTM1	40.00	25.00	50.00
HAITI	HTI1	23.50	8.50	38.50
JAMAICA	JMC1	27.00	12.00	42.00
PANAMA	PNR1	30.00	15.00	45.00

Desired locations and feasible arcs are defined relative to 50° W.lon.

THE $\triangle$S MATRIX :

	USA1	MEX1	CAN1	SFG1	CRB1	BRZ1	GUY1	PRG1	URG1	ARG1	VEN1	BOL1	CHL1
	CLM1	PRU1	ECD1	CUB1	CTR1	NCG1	HND1	BLZ1	SLV1	GTM1	HTI1	JMC1	PNR1
USA1	----	4.39	4.77	0.47	4.64	0.65	0.48	0.47	0.42	0.53	0.51	0.51	0.53
	0.53	0.53	0.49	3.75	0.43	0.46	0.47	0.98	0.38	0.77	0.43	0.39	0.46
MEX1	----	----	0.55	0.43	3.91	0.62	0.47	0.00	0.00	0.00	0.54	0.35	0.43
	3.49	0.53	0.49	4.23	1.86	3.79	4.04	4.20	3.88	4.40	2.08	3.32	0.94
CAN1	----	----	----	0.33	0.52	0.59	0.36	0.35	0.30	0.42	0.41	0.41	0.41
	0.50	0.44	0.37	0.51	0.32	0.38	0.44	0.38	0.24	0.47	0.44	0.40	0.36

	USA1	MEX1	CAN1	SFG1	CRB1	BRZ1	GUY1	PRG1	URG1	ARG1	VEN1	BOL1	CHL1
	CLM1	PRU1	ECD1	CUB1	CTR1	NCG1	HND1	BLZ1	SLV1	GTM1	HTI1	JMC1	PNR1
SFG1	----	----	----	----	1.07	4.35	4.57	0.00	0.00	0.47	3.86	0.46	0.44
	1.44	0.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CRB1	----	----	----	----	----	1.70	2.71	0.00	0.00	0.49	4.30	0.44	0.52
	4.11	0.53	0.46	4.77	1.15	2.96	2.95	2.20	1.07	1.42	4.43	4.57	1.41
BRZ1	----	----	----	----	----	----	5.02	4.59	4.61	4.92	4.24	4.71	4.33
	5.13	5.29	3.14	0.48	0.43	0.46	0.47	0.39	0.37	0.46	0.42	0.37	1.06
GUY1	----	----	----	----	----	----	----	0.00	0.00	0.49	4.72	0.48	0.48
	2.36	0.51	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PRG1	----	----	----	----	----	----	----	----	2.46	4.41	0.00	4.45	3.97
	0.49	1.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
URG1	----	----	----	----	----	----	----	----	----	4.55	0.00	0.99	4.13
	0.35	0.45	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ARG1	----	----	----	----	----	----	----	----	----	----	0.51	4.33	4.66
	0.54	1.41	0.47	0.00	0.37	0.34	0.29	0.00	0.00	0.08	0.00	0.00	0.41
VEN1	----	----	----	----	----	----	----	----	----	----	----	0.52	0.52
	4.91	1.45	1.35	0.49	1.86	1.38	0.95	0.37	0.68	0.41	1.07	1.09	3.31
BOL1	----	----	----	----	----	----	----	----	----	----	----	----	4.39
	0.63	4.84	0.52	0.00	0.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.34
CHL1	----	----	----	----	----	----	----	----	----	----	----	----	----
	0.52	4.41	0.48	0.45	0.41	0.46	0.42	0.36	0.31	0.40	0.41	0.37	0.43
CLM1	----	----	----	----	----	----	----	----	----	----	----	----	----
	----	4.46	4.19	1.52	3.56	4.83	3.60	1.84	2.94	2.45	1.87	2.74	4.45
PRU1	----	----	----	----	----	----	----	----	----	----	----	----	----
	----	----	4.69	0.48	0.43	0.46	0.45	0.39	0.37	0.44	0.40	0.34	0.44
ECD1	----	----	----	----	----	----	----	----	----	----	----	----	----
	----	----	----	0.00	0.42	0.38	0.00	0.00	0.00	0.00	0.00	0.00	0.97
CUB1	----	----	----	----	----	----	----	----	----	----	----	----	----
	----	----	----	----	0.00	1.27	1.82	2.21	0.95	1.79	4.43	3.56	0.00
CTR1	----	----	----	----	----	----	----	----	----	----	----	----	----
	----	----	----	----	----	4.55	2.67	1.25	2.53	2.13	0.00	0.51	4.56
NCG1	----	----	----	----	----	----	----	----	----	----	----	----	----
	----	----	----	----	----	----	4.38	3.12	3.79	3.39	1.36	2.28	3.46
HND1	----	----	----	----	----	----	----	----	----	----	----	----	----
	----	----	----	----	----	----	----	4.41	4.50	4.25	1.52	2.33	1.54
BLZ1	----	----	----	----	----	----	----	----	----	----	----	----	----
	----	----	----	----	----	----	----	----	3.10	4.56	1.14	1.64	0.74
SLV1	----	----	----	----	----	----	----	----	----	----	----	----	----
	----	----	----	----	----	----	----	----	----	4.70	0.80	1.26	1.42
GTM1	----	----	----	----	----	----	----	----	----	----	----	----	----
	----	----	----	----	----	----	----	----	----	----	0.85	1.68	1.29
HTI1	----	----	----	----	----	----	----	----	----	----	----	----	----
	----	----	----	----	----	----	----	----	----	----	----	3.90	0.00
JMC1	----	----	----	----	----	----	----	----	----	----	----	----	----
	----	----	----	----	----	----	----	----	----	----	----	----	0.00

APPENDIX E

THE REGION 2 SCENARIO

The introductory paragraphs of Appendix D apply here also, with the following exception. It has been assumed that, in order to accomodate 25 U.S.A satellites, the U.S.A. Earth station antenna diameters have been increased so as to allow 2° satellite separations of these satellites without exceeding the single-entry interference threshold. However, the $\triangle S$ and $\triangle \phi$ calculations involving U.S.A. and other administrations are made with Wang's parameters, i.e., 4.5 m (dia.) Earth station antennas with a half power beam width of 1.17 degrees. With respect to interference between the U.S.A. and other administrations, the calculation is therefore overly conservative.

In the $\triangle S$ matrix, for service areas with more than one satellite, the minimum required separation to avoid self interference is given as the diagonal element of the matrix.

SCENARIO : REGION 2 (59 SATELLITES - 28 SERVICE AREAS)

S/A NAME	CODE	# OF SATS.	DESIRED LOCATION	LIMITS ON FEASIBLE ARC	
USA	USA1	25	96.00	62.00	130.00
MEXICO	MEX1	3	102.00	50.00	154.00
CANADA	CAN1	3	103.00	88.00	118.00
SURINAM & FRENCH GUIANA	SFG1	2	54.00	0.00	122.00
CARIBBEAN	CRB1	1	73.00	16.00	130.00
BRAZIL	BRZ1	3	55.00	4.00	106.00
GUYANA	GUY1	1	56.00	0.00	122.00
PARAGUAY	PRG1	1	58.00	0.00	122.00
URUGUAY	URG1	1	56.00	0.00	120.00
ARGENTINA	ARG1	1	68.00	14.00	122.00
VENEZUELA	VEN1	1	68.00	14.00	122.00
BOLIVIA	BOL1	1	62.00	0.00	126.00
CHILE	CHL1	1	69.00	14.00	124.00
COLOMBIA	CLM1	1	69.00	14.00	124.00
PERU	PRU1	1	74.00	10.00	138.00
ECUADOR	ECD1	1	78.00	10.00	146.00
CUBA	CUB1	1	80.00	16.00	144.00
COSTA RICA	CTR1	1	84.00	16.00	152.00
NICARAGUA	NCG1	1	85.00	18.00	152.00
HONDURAS	HND1	1	86.00	20.00	152.00
BELIZE	BLZ1	1	89.00	20.00	158.00
EL SALVADOR	SLV1	1	89.00	20.00	158.00
GUATEMALA	GTM1	1	90.00	22.00	158.00
HAITI	HTI1	1	73.00	6.00	140.00
JAMAICA	JMC1	1	78.00	10.00	146.00
PANAMA	PNR1	1	80.00	12.00	148.00
BAHAMAS	BAH1	1	76.00	0.00	152.00
TRINIDAD	TRD1	1	70.00	0.00	140.00

Desired locations and feasible arcs are defined relative to 0° lon.

THE $\triangle$S MATRIX :

	USA1	MEX1	CAN1	SFG1	CRB1	BRZ1	GUY1	PRG1	URG1	ARG1	VEN1	BOL1	CHL1	CLM1	PRU1	ECD1	CUB1	CTR1	NCG1	HND1	BLZ1	SLV1	GTM1	HTI1	JMC1	PNR1	BAH1	TRD1
USA1	2.00	4.39	4.77	0.47	4.64	0.65	0.48	0.47	0.42	0.53	0.51	0.51	0.53	0.53	0.53	0.49	3.75	0.43	0.46	0.47	0.98	0.38	0.77	0.43	0.39	0.46	3.78	0.36
MEX1		4.13	0.55	0.43	3.91	0.62	0.47	0.00	0.00	0.00	0.54	0.35	0.43	3.49	0.53	0.49	4.23	1.86	3.79	4.04	4.20	3.88	4.40	2.08	3.32	0.94	4.35	0.36
CAN1			4.17	0.33	0.52	0.59	0.36	0.35	0.30	0.42	0.41	0.41	0.41	0.50	0.44	0.37	0.51	0.32	0.38	0.44	0.38	0.24	0.47	0.44	0.40	0.36	0.40	0.14
SFG1				4.25	1.07	4.35	4.57	0.00	0.00	0.47	3.86	0.46	0.44	1.44	0.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.18
CRB1						1.70	2.71	0.00	0.00	0.49	4.30	0.44	0.52	4.11	0.53	0.46	4.77	1.15	2.96	2.95	2.20	1.07	1.42	4.43	4.57	1.41	4.32	4.90
BRZ1						3.91	5.02	4.59	4.61	4.92	4.24	4.71	4.33	5.13	5.29	3.14	0.48	0.43	0.46	0.47	0.39	0.37	0.46	0.42	0.37	1.06	0.37	1.30
GUY1								0.00	0.00	0.49	4.72	0.48	0.48	2.36	0.51	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.65
PRG1									2.46	4.41	0.00	4.45	3.97	0.49	1.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
URG1										4.55	0.00	0.99	4.13	0.35	0.45	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ARG1											0.51	4.33	4.66	0.54	1.41	0.47	0.00	0.37	0.34	0.29	0.00	0.00	0.08	0.00	0.00	0.41	0.00	0.36
VEN1												0.52	0.52	4.91	1.45	1.35	0.49	1.86	1.38	0.95	0.37	0.68	0.41	1.07	1.09	3.31	0.53	4.88
BOL1													4.39	0.63	4.84	0.52	0.00	0.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.34	0.00	0.00
CHL1														0.52	4.41	0.48	0.45	0.41	0.46	0.42	0.36	0.31	0.40	0.41	0.37	0.43	0.20	0.34
CLM1															4.46	4.19	1.52	3.56	4.83	3.60	1.84	2.94	2.45	1.87	2.74	4.45	0.49	1.41
PRU1																4.69	0.48	0.43	0.46	0.45	0.39	0.37	0.44	0.40	0.34	0.44	0.32	0.34

	USA1	MEX1	CAN1	SFG1	CRB1	BRZ1	GUY1	PRG1	URG1	ARG1	VEN1	BOL1	CHL1	CLM1	PRU1	ECD1	CUB1	CTR1	NCG1	HND1	BLZ1	SLV1	GTM1	HTI1	JMC1	PNR1	BAH1	TRD1
ECD1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.00	0.42	0.38	0.00	0.00	0.00	0.00	0.00	0.00	0.97	0.00	0.16
CUB1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.00	1.27	1.82	2.21	0.95	1.79	4.43	3.56	0.00	4.40	0.00
CTR1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4.55	2.67	1.25	2.53	2.13	0.00	0.51	4.56	0.31	1.23
NCG1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4.38	3.12	3.79	3.39	1.36	2.28	3.46	0.99	0.79
HND1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4.41	4.50	4.25	1.52	2.33	1.54	1.52	0.00
BLZ1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3.10	4.56	1.14	1.64	0.74	2.87	0.00
SLV1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4.70	0.80	1.26	1.42	0.70	0.00
GTM1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.85	1.68	1.29	1.73	0.00
HTI1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3.90	0.00	3.45	0.00
JMC1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.00	2.45	0.00
PNR1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.00	2.30
BAH1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.00

LIST OF REFERENCES

AKIMA,H. [1981]
"A Method for Determining the Minimum Elliptical Beam of a Satellite Antenna." NTIA Report 81-88, U.S. Department of Commerce.

BAYBARS,I. [1982]
"Optimal Assignment of Broadcasting Frequencies." Eur. J. Opns. Res. 9, pp 257-263.

BAZARAA,M.S., and J.J.JARVIS. [1977]
Linear Programming and Network Flows. John Wiley and Sons, New York.

BENDERS,J.F. [1962]
"Partitioning Procdeures for solving mixed variables programming problems." Numer. Math. 4, pp 238-252.

BENICHOU,M., J.M.GAUTHIER, G.HENTGES, and G.RIBIERE. [1977]
"The Efficient Solution of Large Scale LP Problems - Some Algorithmic Techniques and Computational Results." Math. Program. 13, pp 280-322.

BURKHARD,E., and U.DERIGS. [1980]
"Assignment and Matching Problems - Solutions with FORTRAN programs." Lecture Notes in Economics and Mathematics Series, Springer Verlag.

BUYUKUDURA,M. [1986]
Private Communication.

CAMERON,H. [1977]
"The Solution of the Graph Coloring Problem as a Set Covering Problem." IEEE Trans. Electromagnetic Compatibility 19, No.3.

CASSIDY,P., and H.BENNETT. [1972]
"TRAMP - A Multi-depot Vehicle Scheduling System." Opnl. Res. Quart. 23, pp 151-163.

CHOUINARD,G., and M.VACHON. [1981]
"A Synthesis Procedure of a plan by computer using an assignment method and inductive selection." ITU/CITEL/DOC-Canada, Preparatory Seminar for 1983 RARC, Ottawa, Canada.

CHRISTENSEN,J. [1981]
"BSS CAPS - A System Description." ITU/CITEL/DOC - Canada, Preparatory Seminar for 1983 RARC, Ottawa, Canada.

CLARKE,A.C. [1945]
Private document, May 1945. Reprinted in Spaceflight Vol.10, No.3, pp 85-86.

EILON,S., C.WATSON-GANDY, and N.CHRISTOFIDES. [1971]
Distribution Management: Mathematical Modelling and Practical Analysis. Hafner Publishing Co., New York.

FEDERGRUEN,A., and P.ZIPKIN. [1984]
"A Combined Vehicle Routing and Inventory Allocation Problem." Opns. Res. 32, No.5, pp 1019-1037.

FISHER,M.L. [1981]
"The Lagrangian Relaxation Method for Solving Integer Programming Problems." Mgmt. Sci. 27, pp 1-18.

GARFINKEL,R.S., and G.L.NEMHAUSER. [1972]
Integer Programming. John Wiley & Sons, New York. pp 108-130.

GEOFFRION,A.M., and G.W.GRAVES [1974]
"Multicommodity Distribution System Design by Benders' Decomposition." Mgmt. Sci. 20, pp 822-844.

GONSALVEZ,D.J.A. [1983]
"Computational Evaluation of State of the Art Techniques for Mixed Integer Programming." M.S. Thesis, Dept. of Industrial and Systems Engineering, The Ohio State University, Columbus, Ohio, 43210.

HELD,M., and R.M.KARP. [1971]
"The traveling salesman problem and minimum spanning trees - part II". Math. Program. 1, pp 6-25.

HELD,M., P.WOLFE, and H.P.CROWDER. [1974]
"Validation of Subgradient Optimization". Math. Program. 6, pp 62-88.

IGNIZIO,J. [1984]
"A Generalized Goal Programming Approach to the Minimal Interference, Multicriteria N * 1 Scheduling Problem." IIE Trans. 16, pp 316-322.

ITO,Y., T.MIZUNO, and T.MURATANI. [1979]
"Effective Utilization of Geostationary Orbit through Optimization." IEEE Trans. on Communications, Vol COM-27, No. 10, pp 1551-1558.

LASDON,L.S. [1970]

Optimization Theory for Large Systems. Macmillan Publishing Co., Inc. New York.

LEVIS,C.A., C.H.MARTIN, D.J.GONSALVEZ, and C.W.WANG. [1983a]
"Engineering Calculations for Communications Satellite Systems Planning." Interim Report 713533-3, The ElectroScience Laboratory, The Ohio State University, Columbus, Ohio 43212, on Grant No. NAG 3-159 for NASA/Lewis Research Center, 21000 Brookpark Road, Cleveland, Ohio 44135.

LEVIS,C.A., C.H.MARTIN, C.W.WANG, and D.J.GONSALVEZ. [1983b]
"Engineering Calculations for Communications Satellite Systems Planning." First Summary Report 713533-2, The ElectroScience Laboratory, The Ohio State University, Columbus, Ohio 43212, on Grant No. NAG 3-159 for NASA/Lewis Research Center, 21000 Brookpark Road, Cleveland, Ohio 44135.

LIN,S. [1965]
"Computer Solutions of the Traveling Salesman Problem." Bell Systems Technical Journal 44, pp 2245-2269.

LIN,S., and B.W.KERNIGHAN. [1973]
"An Effective Heuristic Algorithm for the Traveling Salesman Problem." Opns. Res. 21, pp 498-516.

MARTIN,C.H. [1979]
"PROFOR - A Product Form of the Inverse Implementation of the Revised Simplex Method for Linear Programming." Dept. of Industrial and Systems Engineering, The Ohio State University, Columbus, Ohio, 43210.

MARTIN,C.H. and D.J.A.GONSALVEZ [1981]
"ENUMCODE - An Enumeration Code for Mixed Integer Programming." Dept. of Industrial and Systems Engineering, The Ohio State University, Columbus, Ohio, 43210.

MATHUR,K., H.M.SALKIN, K.NISHIMURA, and S.MORITO. [1985]
"Applications of Integer Programming in Radio Frequency Management". Mgmt. Sci. V.31, No.7, pp 829-839.

MOUNT-CAMPBELL,C.A., C.H.REILLY, and D.J.GONSALVEZ. [1986]
"A Mixed, 0-1 Integer Linear Programming Formulation of the FSS Synthesis Problem using Minimum Required Pair-wise Separations". Working Paper, Dept. of Industrial and Systems Engineering, The Ohio State University, Columbus, Ohio 43210.

MIZUIKI,T., Y.ITO, and T.MURATANI. [1984]
"Optimization of Frequency Assignment." Presented at the International Conference on Communications, Amsterdam. May 1984.

NEDZELA,M., and J.SIDNEY. [1981]

"A Synthesis Algorithm for Broadcasting Satellite Service." ITU/CITEL/DOC-Canada, Preparatory Seminar for 1983 RARC, Ottawa, Canada.

ORBIT-II. [1984]
"The Orbit Spacing Minimizer - User's Manual." Kokusai Denshin Denwa Co.,Ltd., Tokyo, Japan.

OTTEY,H.R., T.M.SULLIVAN, and F.S.ZUSMAN. [1986]
"Optimization Techniques Applied to Spectrum Management for Communications Satellites." A Collection of Technical Papers, AIAA Eleventh Communication Satellite Systems Conference, San Diego, California, March 1986, pp 199-210.

POLJAK,B.T. [1967]
"A General Method for Solving Extremum Problems". Soviet Math. Doklady, Vol. 8, pp 593-597.

REILLY,C.H., C.A.MOUNT-CAMPBELL, D.J.GONSALVEZ, C.H.MARTIN, C.A.LEVIS, and C.W.WANG. [1986]
"Broadcasting Satellite Service Synthesis Using Gradient and Cyclic Coordinate Search Procdeures." A Collection of Technical Papers, AIAA Eleventh Communication Satellite Systems Conference, San Diego, California, March 1986, pp 237-245.

REILLY,C.H. [1986b]
Private Communication.

REITER,S., and G.SHERMAN. [1965]
"Discrete Optimizing." J. Soc. Indust. Appl. Math., Vol. 13, No. 3, pp 864-889.

SHAPIRO,J.F. [1979]
"A Survey of Lagrangian Techniques for Discrete Optimization". Ann. Discrete Math. Vol 5, pp 113-138.

SHERALI,H., and W.ADAMS. [1984]
"A Decomposition Algorithm for a discrete location allocation problem." Opns. Res. 32, No.4, pp 878-900.

SIMMONS,D.M. [1975]
Nonlinear Programming for Operations Research. Prentice-Hall,Inc., Englewood Cliffs, N.J. Chapter 5.

SOUP V. [1983]
Spectrum Orbit Utilization Program Version 5. ORI Inc., Silversprings, Maryland.

WANG,C. [1986]
"Optimization of Orbital Assignment and Specification of Service Areas in Satellite Communications." Ph.D dissertation. Dept. of

Electrical Engineering, The Ohio State University, Columbus, Ohio, 43210.

WARC ORB-85 [1985]
WARC on the Use of the Geostationary-Satellite Orbit and the Planning of Space Services Utilizing It. Report to the Second Session of the Conference. ITU, Geneva, August/September 1985.

WREN,A., and A.HOLLIDAY. [1972]
"Computer Scheduling of Vehicles from One or More Depots to a Number of Delivery Points." Opnl. Res. Quart. 23, pp 333-344.

YAMAMURA,Y., and C.A.LEVIS. [1985]
"Calculation of allowable orbital spacings for the fixed satellite service." Technical Report 716548-1, The ElectroScience Laboratory, The Ohio State University, Columbus, Ohio 43212, on Grant No. NAG 3-159, for NASA/Lewis Research Center, 21000 Brookpark Road, Cleveland, Ohio 44135.

ZOELLNER,J.A. [1973]
"Frequency Assignment Games and Strategies." IEEE Trans. on Electromagnetic Compatibility, Vol. EMC-15, pp 191-197.

ZOELLNER,J.A, and C.L.BEALL. [1977]
"A breakthrough in spectrum conserving frequency assignment technology." IEEE Trans. on Electromagnetic Compatibility, Vol. EMC-19, pp 313-319.