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# STATISTICAL ANALYSIS OF 59 INSPECTED SSME HPFTP TURBINE BLADES (UNCRACKED AND CRACKED)


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Science and Engineering Directorate



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16. ABSTRACT

This technical memorandum presents the numerical results of statistical analysis of the test data of Space Shuttle Main Engine high pressure fuel turbopump second-stage turbine blades, including some with cracks. Several statistical methods use the test data to determine the application of differences in frequency variations between the uncracked and cracked blades.

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TECHNICAL MEMORANDUM

# STATISTICAL ANALYSIS OF 59 INSPECTED SSME HPFTP TURBINE BLADES (UNCRACKED AND CRACKED)

## I. INTRODUCTION

This report presents the numerical results of statistical analysis of the test data of 59 Space Shuttle Main Engine high pressure fuel turbopump second-stage turbine blades, including some with cracks. Several statistical methods use the test data to determine the application of differences in frequency variations between the uncracked and cracked blades.

## II. TEST DATA

The test data comprise 59 HPFTP turbine blades brought in for test measurements of natural frequency in terms of kilohertz. The metallurgical material properties of the turbine blades are cast and directionally solidified. The data of modal analysis for the 59 blades are presented in Table 1 for use in the statistical analysis to obtain numerical data for comparisons of blade-to-blade frequency variations.

The frequency range of the test data is tabulated below for five vibrational modes:

| Mode | Frequency (kHz) Low | High | Percent kHz |
|---|---|---|---|
| 1 | 3.5315 | 3.6065 | 2.1 |
| 2 | 10.2750 | 11.0750 | 7.8 |
| 3 | 14.1500 | 14.8000 | 4.6 |
| 4 | 18.0000 | 19.0800 | 6.0 |
| 5 | 21.7550 | 24.0300 | 10.5 |

From a sample of the 59 blades, 10 blades have cracks. The blade inspection explanations for downstream shank cracks are summarized in Table 2. Also, it has been assumed that the tenth blade may not have sustained the crack; therefore, the number of 9 cracked blades have been included in the statistical analysis to determine the differences between the blades involved.

## III. ANALYSIS OF STATISTICAL PROPERTIES

In the case of frequency differences not being used, the mathematical statistics for the variance, mean, standard deviation and coefficient of variation have been computed for five different groups; namely, 59 uncracked and cracked turbine blades, 50 uncracked blades, 49 uncracked blades, 10 cracked blades, and 9 cracked blades. One of the solution techniques, the variance, is represented as an average of squared

TABLE 1. MODAL ANALYSIS, 59 SSME HPFTP SECOND STAGE TURBINE BLADES

| Blade No. | Frequency (kHz) | | | | |
|---|---|---|---|---|---|
| | First | Second | Third | Fourth | Fifth |
| N223 | 3.5562 | 10.725 | 14.450 | 18.930 | 23.930 |
| N224 | 3.5687 | 10.755 | 14.600 | 18.530 | 22.730 |
| N114 | 3.5875 | 10.850 | 14.650 | 18.580 | 22.205 |
| P328 | 3.5437 | 10.450 | 14.450 | 18.075 | 22.050 |
| M819 | 3.5625 | 10.325 | 14.150 | 18.405 | 22.305 |
| N230 | 3.5687 | 11.075 | 14.500 | 18.580 | 23.530 |
| N95 | 3.5500 | 10.525 | 14.550 | 18.430 | 23.005 |
| N23 | 3.5812 | 10.700 | 14.650 | 18.175 | 22.425 |
| N98 | 3.5625 | 10.600 | 14.500 | 18.280 | 22.305 |
| P119 | 3.5562 | 10.650 | 14.575 | 18.380 | 22.800 |
| P93 | 3.5812 | 10.850 | 14.650 | 18.730 | 22.080 |
| P129 | 3.5750 | 10.800 | 14.500 | 18.805 | 23.005 |
| P318 | 3.5500 | 10.675 | 14.600 | 18.380 | 22.805 |
| N123 | 3.5687 | 10.675 | 14.550 | 18.655 | 23.080 |
| N410 | 3.5562 | 10.850 | 14.450 | 18.980 | 23.755 |
| P224 | 3.5815 | 10.575 | 14.675 | 18.405 | 22.555 |
| P925 | 3.5562 | 10.875 | 14.700 | 18.480 | 23.205 |
| P912 | 3.5750 | 10.575 | 14.700 | 18.250 | 22.325 |
| Q313 | 3.5750 | 10.550 | 14.575 | 18.200 | 22.050 |
| N213 | 3.5812 | 10.700 | 14.675 | 18.555 | 22.705 |
| P13 | 3.5562 | 10.700 | 14.425 | 18.555 | 22.905 |
| N12 | 3.5750 | 10.650 | 14.525 | 18.150 | 22.250 |
| P12 | 3.5625 | 10.700 | 14.500 | 18.505 | 22.880 |
| P311 | 3.5687 | 10.575 | 14.650 | 18.555 | 22.855 |
| N730 | 3.5812 | 10.500 | 14.625 | 18.405 | 22.805 |
| N118 | 3.5562 | 10.575 | 14.800 | 18.680 | 22.905 |
| P910 | 3.5750 | 10.525 | 14.650 | 18.480 | 22.455 |
| N226 | 3.5625 | 11.000 | 14.600 | 19.055 | 24.030 |
| Q31 | 3.5315 | 10.275 | 14.400 | 18.000 | 22.075 |
| P324 | 3.5812 | 10.400 | 14.550 | 18.280 | 22.230 |
| N216 | 3.5437 | 10.825 | 14.600 | 18.805 | 22.880 |
| N215 | 3.5437 | 10.550 | 14.550 | 18.380 | 22.505 |

TABLE 1. (Concluded)

| Blade No. | Frequency (kHz) | | | | |
|---|---|---|---|---|---|
| | First | Second | Third | Fourth | Fifth |
| N912 | 3.5500 | 10.525 | 14.450 | 18.255 | 22.780 |
| N130 | 3.5750 | 10.425 | 14.425 | 18.255 | 22.755 |
| P211 | 3.5750 | 10.575 | 14.500 | 18.555 | 22.630 |
| P932 | 3.5687 | 10.625 | 14.550 | 18.150 | 22.100 |
| N120 | 3.5625 | 10.600 | 14.300 | 18.255 | 22.355 |
| P117 | 3.5687 | 10.825 | 14.550 | 18.755 | 23.455 |
| N11 | 3.5875 | 10.750 | 14.500 | 18.100 | 22.975 |
| P320 | 3.5625 | 10.850 | 14.475 | 18.455 | 22.930 |
| P322 | 3.5812 | 10.300 | 14.350 | 19.080 | 23.580 |
| N24 | 3.5625 | 10.475 | 14.725 | 18.580 | 22.430 |
| N227 | 3.5625 | 10.950 | 14.500 | 18.880 | 23.805 |
| M327 | 3.5687 | 10.550 | 14.550 | 18.480 | 22.955 |
| N22 | 3.5625 | 10.600 | 14.500 | 18.255 | 22.555 |
| N324 | 3.6065 | 10.350 | 14.777 | 18.580 | 21.755 |
| Q311 | 3.5687 | 10.450 | 14.550 | 18.380 | 22.755 |
| P130 | 3.5812 | 10.550 | 14.650 | 18.630 | 23.330 |
| Q318 | 3.5687 | 10.450 | 14.400 | 18.405 | 22.455 |
| N21 | 3.5562 | 10.700 | 14.425 | 18.405 | 23.255 |
| P14 | 3.5625 | 10.750 | 14.475 | 18.330 | 22.680 |
| M32 | 3.5625 | 10.975 | 14.550 | 18.780 | 23.830 |
| N217 | 3.5687 | 10.725 | 14.575 | 18.280 | 22.505 |
| N28 | 3.5625 | 10.575 | 14.575 | 18.150 | 22.625 |
| M414 | 3.5500 | 10.275 | 14.500 | 18.150 | 22.575 |
| N129 | 3.5750 | 10.700 | 14.650 | 18.430 | 22.555 |
| P232 | 3.5500 | 10.575 | 14.600 | 18.330 | 22.530 |
| P122 | 3.5687 | 10.725 | 14.550 | 18.455 | 22.430 |
| P713 | 3.5500 | 10.550 | 14.475 | 18.555 | 22.480 |

Source: Rockwell International Corp.

TABLE 2. SHANK CRACK SUMMARY
HPFTP INSPECTION NO. 2410

| Blade Position | Blade No. | Downstream Shank Inspection Results |
|---|---|---|
| 4 | N98 | 2 flakes and looks cracked (very tight) |
| 5 | P129 | Cracked and flaked, very tight, sharp junction |
| 6 | P318 | Looks cracked at junction |
| 10 | P925 | Very tight crack at junction |
| 20 | P713 | Cracked, very tight at junction |
| 21 | N224 | Could have very tight crack, not very clear |
| 23 | N95 | Very small flake out, may lead to crack |
| 40 | N24 | 3 flakes out, may lead to cracks, both sides of junction |
| 52 | P14 | Looks cracked at junction |
| 56 | M414 | Flake out of machine surface, may lead to crack |

Inspection date: 7-21-84
Source: Rockwell International Corp.

deviations from the sample mean, $\bar{x}$, and is expressed in kilohertz$^2$. The mean, $\bar{x}$, one of the measures of central tendency, is an average of the frequencies for each vibrational mode and is defined as a ratio of sum of frequencies and number of frequencies. The standard deviation is a measure of dispersion about the sample mean, $\bar{x}$, and is expressed in kilohertz. One way to measure the degree of dispersion is with the standard deviation, which is a square root of the variance. The coefficient of variation expresses group variability in terms relative to the central tendency of that group and is the percentage of standard deviation of the group mean. The computations involved are represented in tabular form in Table 3.

Table 3 shows that the variance in each of the five groups is largest for the fifth mode. The frequency distribution for each mode is plotted in Figures 1 through 5. Figure 5 explains the largest variance. The first mode has the least variance for all groups. Although the numerical results of previous studies are lacking, coefficient of variation calculations apparently indicate that each mode for all five groups does not have adequacy to represent the overall variability of the frequency.

Table 4 summarizes the comparison of the two groups; namely, 50 uncracked blades versus 9 cracked blades and 49 uncracked blades versus 10 cracked blades in terms of the variance ratio. The statistic $F = s_M^2/s_m^2$ is a value of a random variable having the F distribution with $n_M-1$ and $n_m-1$ degrees of freedom. $s_M^2$ represents the larger of the two sample variances and $s_m^2$ the smaller. The critical values, which are to be exceeded if significant differences exist between two groups of uncracked and cracked blades, are obtained from the appropriate tables of

## TABLE 3. ANALYSIS OF STATISTICAL PROPERTIES

| Mode | Variance ($s^2$) | Mean ($\bar{x}$) | Standard Deviation, (s) | Coefficient of Variation (%) |
|---|---|---|---|---|
| 59 Blades (Uncracked and Cracked) | | | | |
| 1 | 0.00017565 | 3.56598475 | 0.01325340 | 0.3717 |
| 2 | 0.03230514 | 10.63483051 | 0.17973632 | 1.6901 |
| 3 | 0.01283446 | 14.54494915 | 0.11328929 | 0.7789 |
| 4 | 0.06099942 | 18.46728814 | 0.24698060 | 1.3374 |
| 5 | 0.26088222 | 22.75745763 | 0.51076630 | 2.2444 |
| 50 Uncracked Blades | | | | |
| 1 | 0.00018646 | 3.56711400 | 0.01365519 | 0.3828 |
| 2 | 0.03493265 | 10.62900000 | 0.18690279 | 1.7584 |
| 3 | 0.01361336 | 14.54054000 | 0.11667629 | 0.8024 |
| 4 | 0.06808061 | 18.46400000 | 0.26092262 | 1.4131 |
| 5 | 0.29407723 | 22.76090000 | 0.54228888 | 2.3826 |
| 49 Uncracked Blades | | | | |
| 1 | 0.00018412 | 3.56746327 | 0.01356917 | 0.3804 |
| 2 | 0.03299639 | 10.63622449 | 0.18164907 | 1.7078 |
| 3 | 0.01386203 | 14.54136735 | 0.11773712 | 0.8097 |
| 4 | 0.06740295 | 18.47040816 | 0.25962079 | 1.4056 |
| 5 | 0.29946918 | 22.76469388 | 0.54723777 | 2.4039 |
| 10 Cracked Blades | | | | |
| 1 | 0.00007978 | 3.55874000 | 0.00893187 | 0.2510 |
| 2 | 0.03214556 | 10.62800000 | 0.17929182 | 1.6870 |
| 3 | 0.00836806 | 14.56250000 | 0.09147708 | 0.6282 |
| 4 | 0.03331222 | 18.45200000 | 0.18251636 | 0.9891 |
| 5 | 0.08239000 | 22.72200000 | 0.28703658 | 1.2632 |
| 9 Cracked Blades | | | | |
| 1 | 0.00007914 | 3.55971111 | 0.00889613 | 0.2499 |
| 2 | 0.01885694 | 10.66722222 | 0.13732059 | 1.2873 |
| 3 | 0.00887153 | 14.56944444 | 0.09418879 | 0.6465 |
| 4 | 0.02480903 | 18.48555556 | 0.15750882 | 0.8521 |
| 5 | 0.08968750 | 22.73833333 | 0.29947871 | 1.3171 |

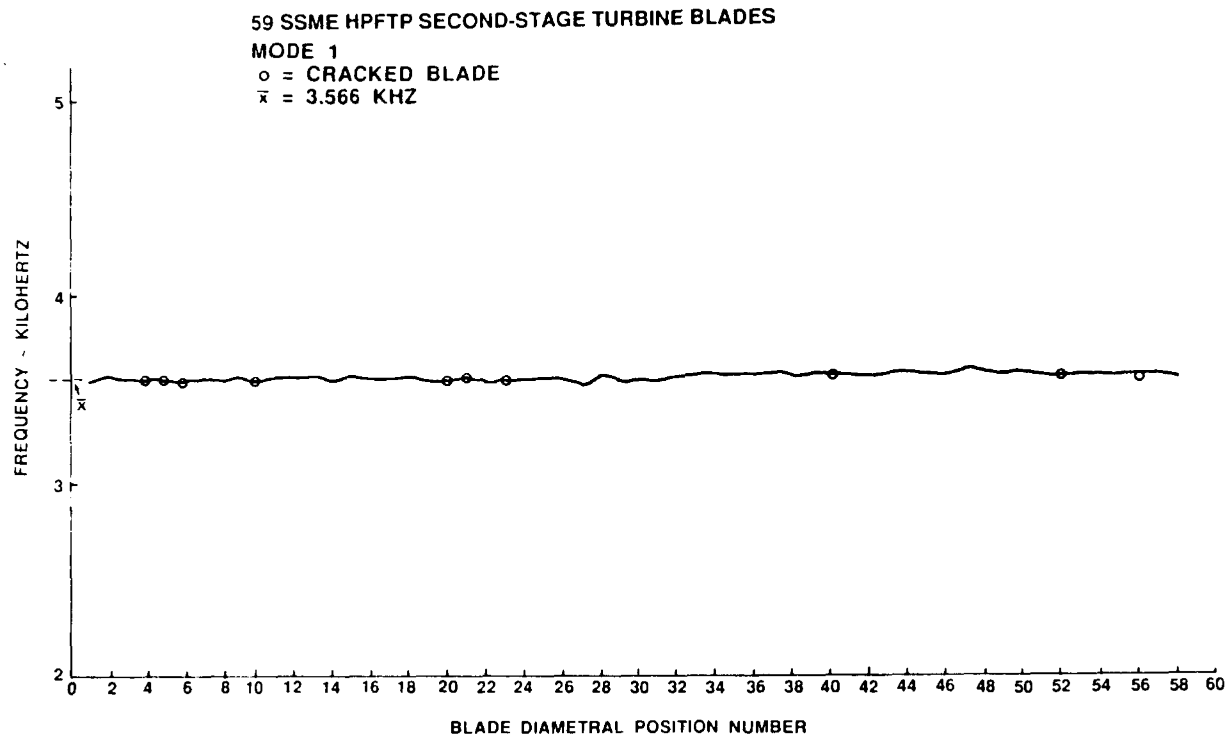


Figure 1. Frequency distribution of mode 1.

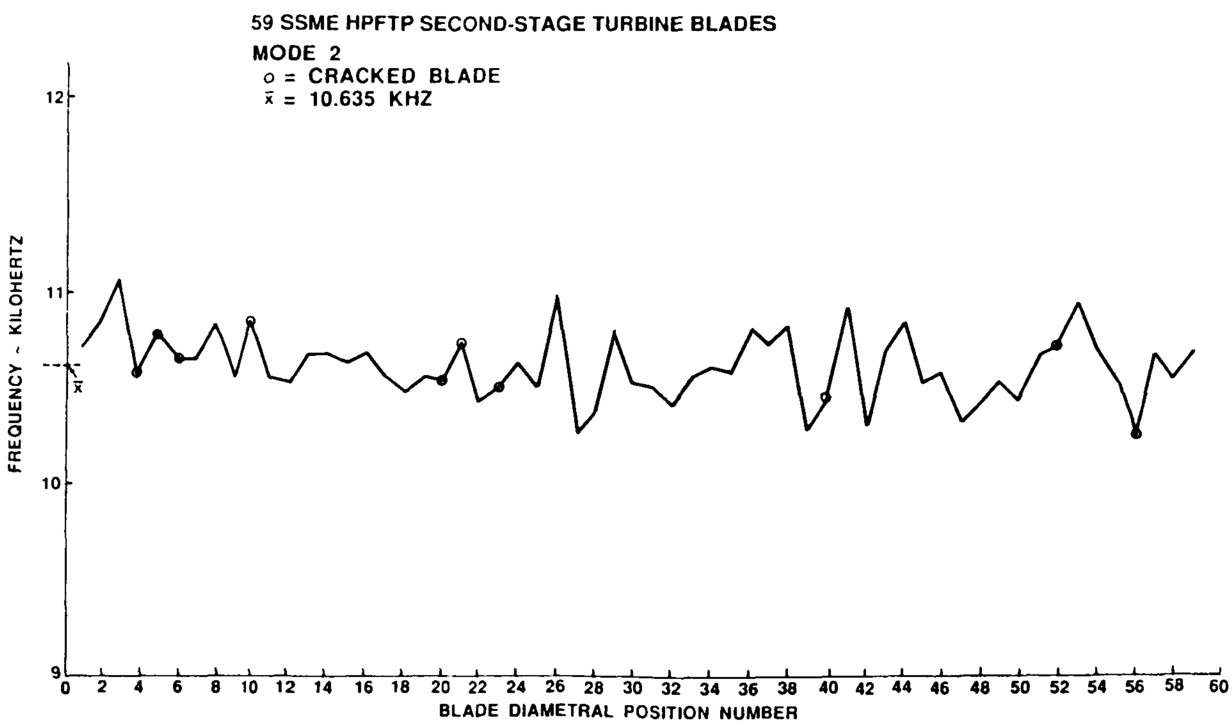


Figure 2. Frequency distribution of mode 2.

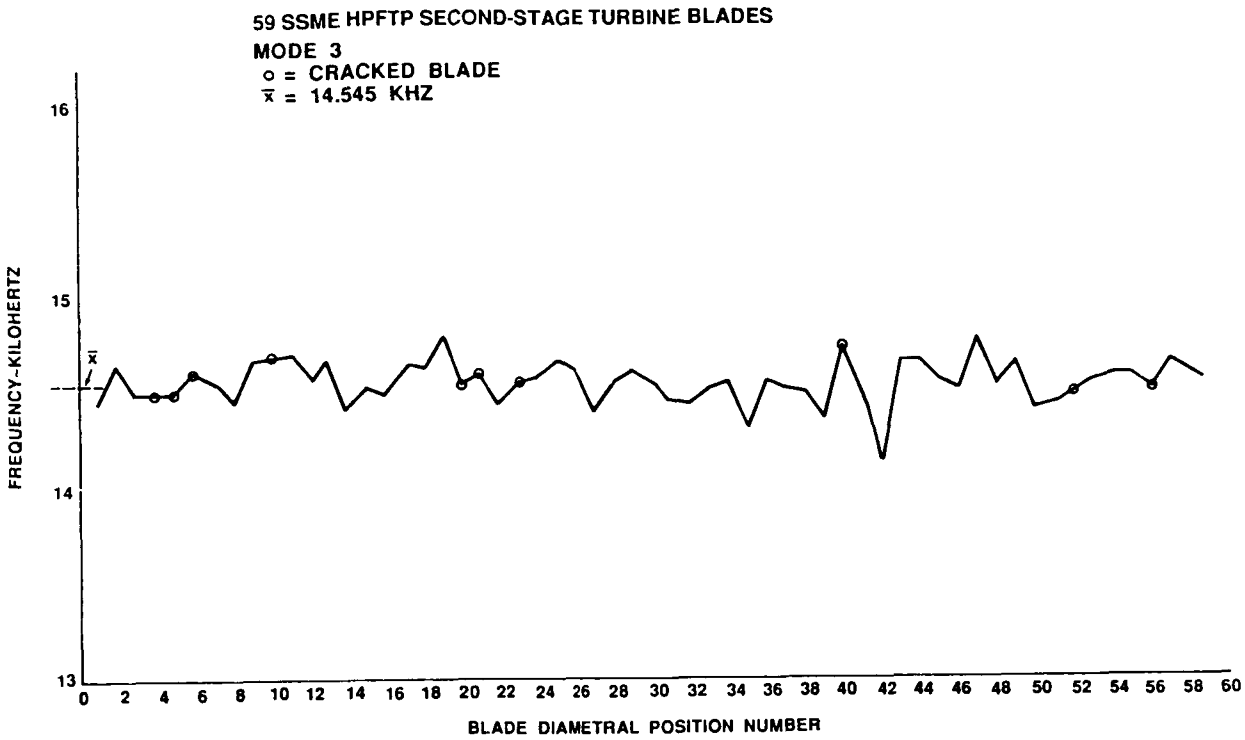


Figure 3. Frequency distribution of mode 3.

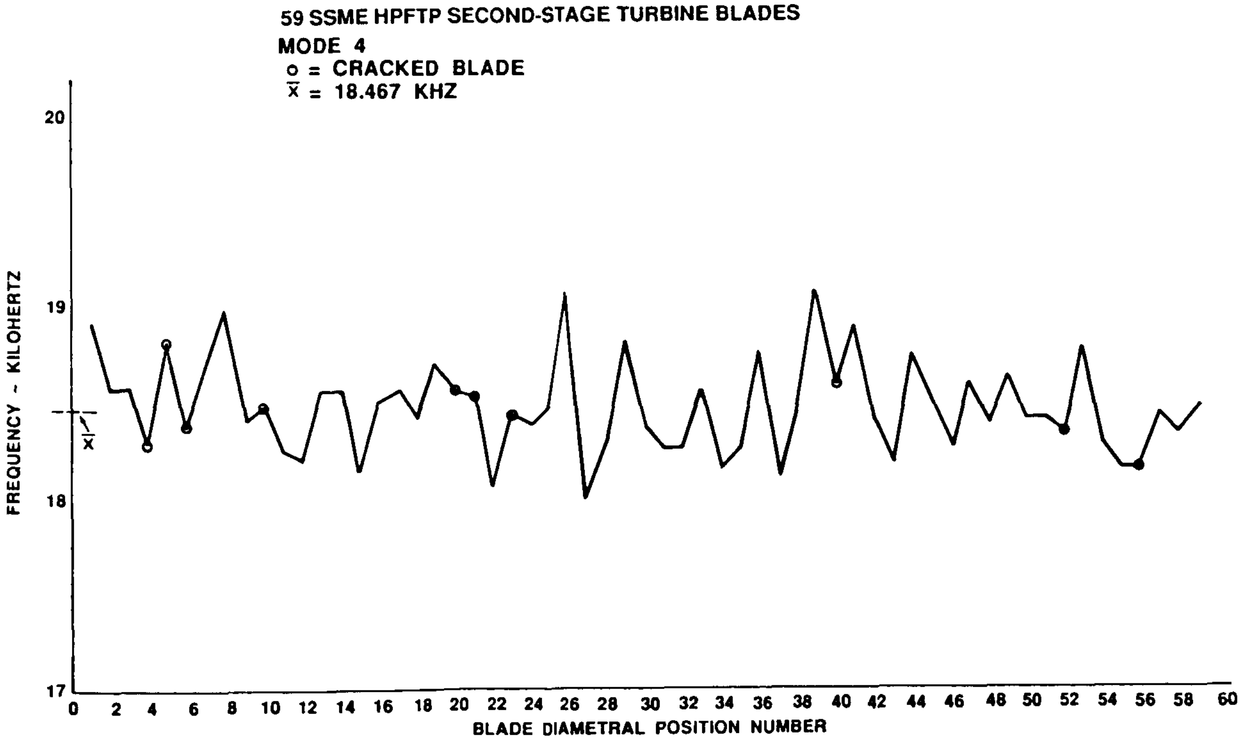


Figure 4. Frequency distribution of mode 4.

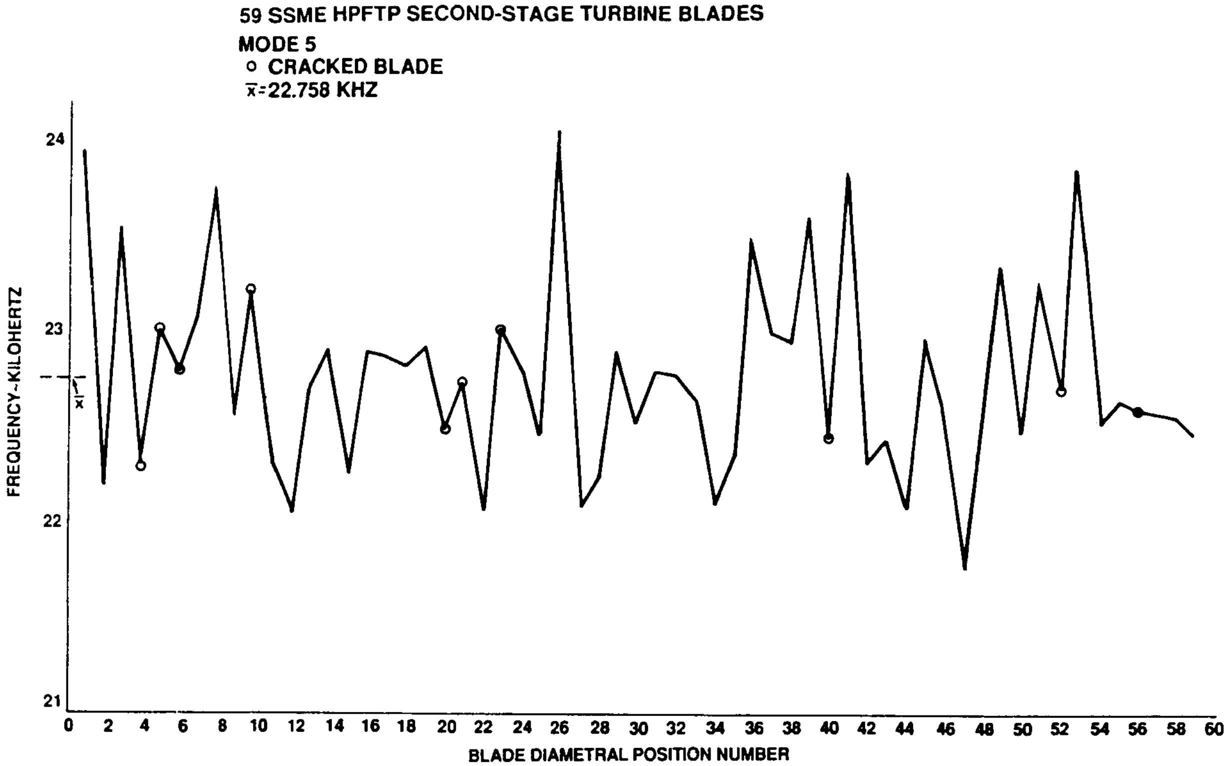


Figure 5. Frequency distribution of mode 5.

TABLE 4. VARIANCE RATIOS

| Mode | $s^2$ Ratio | F | $F_{0.01}(49,8)$ | Null Hypothesis |
|---|---|---|---|---|
| 50 Uncracked Blades versus 9 Cracked Blades | | | $F_{0.01}(49,8)$ | |
| 1 | $s^2_1\ 50u/s^2_1\ 9c$ | 2.3561 | 5.0795 | Not rejected |
| 2 | $S^2_2\ 50u/s^2_2\ 9c$ | 1.8525 | | Not rejected |
| 3 | $s^2_3\ 50u/s^2_3\ 9c$ | 1.5345 | | Not rejected |
| 4 | $s^2_4\ 50u/s^2_4\ 9c$ | 2.7442 | | Not rejected |
| 5 | $s^2_5\ 50u/s^2_5\ 9c$ | 3.2789 | | Not rejected |
| 49 Uncracked Blades versus 10 Cracked Blades | | | $F_{0.01}(48,9)$ | |
| 1 | $s^2_1\ 49u/s^2_1\ 10c$ | 2.3078 | 4.5340 | Not rejected |
| 2 | $s^2_2\ 49u/s^2_2\ 10c$ | 1.0265 | | Not rejected |
| 3 | $s^2_3\ 49u/s^2_3\ 10c$ | 1.6565 | | Not rejected |
| 4 | $s^2_4\ 49u/s^2_4\ 10c$ | 2.0234 | | Not rejected |
| 5 | $s^2_5\ 49u/s^2_5\ 10c$ | 3.6348 | | Not rejected |

F-distribution on the number of degrees of freedom. Accordingly, the test statistics for a 99 percent confidence level are found to be as follows:

$$F_{0.01}\ (49,8) = 5.0795 \text{ for the 50 versus 9 case}$$

and

$$F_{0.01}\ (48,9) = 4.5340 \text{ for the 49 versus 10 case.}$$

Null hypothesis for equality of each of both cases is not rejected since the differences are not so highly significant.

## IV. COMBINATIONS ANALYSIS FOR BLADES

For five vibrational modes, there are ten possibilities, according to the $\binom{5}{2}$ combinations formula. The variance ratios for ten possibilities of vibrational modes taken two at a time have been computed and summarized in Table 5 for 9 cracked blades, Table 6 for 10 cracked blades, Table 7 for 49 uncracked blades, Table 8 for 50 uncracked blades, and Table 9 for 59 uncracked and cracked blades. Examination of Figures 1 through 5 shows that through Tables 5 through 9 first vibrational mode has more vice effects than other vibrational modes, using the 99 percent confidence level. Two-tail test at the 1 percent level of significance is achieved by taking whichever of the two variance estimates is the larger as the numerator and comparing the ratio with the $F_{0.01}$ value. The critical values of the F statistic using the 0.02 level of significance are tabulated in Tables 5 through 9 with rejection and nonrejection of null hypothesis.

## V. PROBABILITIES

The following table summarizes the probability that one or more through nine or more turbine blades have cracks in a sample of 59 turbine blades when the probability that any one of the blades will sustain a crack is 0.16:

| Number of Blades with Cracks | Probability (%) |
|---|---|
| 1 or more | 100.00 |
| 2 or more | 99.96 |
| 3 or more | 99.75 |
| 4 or more | 98.98 |
| 5 or more | 96.94 |
| 6 or more | 92.66 |
| 7 or more | 85.33 |
| 8 or more | 74.75 |
| 9 or more | 61.66 |

There is 13.5 percent probability that exactly 49 turbine blades have no cracks in a sample of 59 turbine blades. There is a zero probability that no crack is found among the 59 blades.

TABLE 5. COMBINATIONS ANALYSIS FOR 9 CRACKED BLADES

| Possibility | $s^2$ Ratio | F | $F_{0.01}(8,8)$ | Null Hypothesis |
|---|---|---|---|---|
| 1 | $s_2^2/s_1^2$ | 238.2732 | 6.030 | Rejected |
| 2 | $s_3^2/s_1^2$ | 112.0992 | | Rejected |
| 3 | $s_4^2/s_1^2$ | 313.4828 | | Rejected |
| 4 | $s_5^2/s_1^2$ | 1133.2765 | | Rejected |
| 5 | $s_2^2/s_3^2$ | 2.1256 | | Not rejected |
| 6 | $s_4^2/s_2^2$ | 1.3156 | | Not rejected |
| 7 | $s_5^2/s_2^2$ | 4.7562 | | Not rejected |
| 8 | $s_4^2/s_3^2$ | 2.7965 | | Not rejected |
| 9 | $s_5^2/s_3^2$ | 10.1096 | | Rejected |
| 10 | $s_5^2/s_4^2$ | 3.6151 | | Not rejected |

TABLE 6. COMBINATIONS ANALYSIS FOR 10 CRACKED BLADES

| Possibility | $s^2$ Ratio | F | $F_{0.01}(9,9)$ | Null Hypothesis |
|---|---|---|---|---|
| 1 | $s_2^2/s_1^2$ | 402.9276 | 5.3500 | Rejected |
| 2 | $s_3^2/s_1^2$ | 104.8892 | | Rejected |
| 3 | $s_4^2/s_1^2$ | 417.5510 | | Rejected |
| 4 | $s_5^2/s_1^2$ | 1032.7150 | | Rejected |
| 5 | $s_2^2/s_3^2$ | 3.8415 | | Not rejected |
| 6 | $s_4^2/s_2^2$ | 1.0363 | | Not rejected |
| 7 | $s_5^2/s_2^2$ | 2.5630 | | Not rejected |
| 8 | $s_4^2/s_3^2$ | 3.9809 | | Not rejected |
| 9 | $s_5^2/s_3^2$ | 9.8458 | | Rejected |
| 10 | $s_5^2/s_4^2$ | 2.4733 | | Not rejected |

TABLE 7. COMBINATIONS ANALYSIS FOR 49 UNCRACKED BLADES

| Possibility | $s^2$ Ratio | F | $F_{0.01}(48,48)$ | Null Hypothesis |
|---|---|---|---|---|
| 1 | $s_2^2 / s_1^2$ | 179.2113 | 1.9769 | Rejected |
| 2 | $s_3^2 / s_1^2$ | 75.2880 | | Rejected |
| 3 | $s_4^2 / s_1^2$ | 366.0816 | | Rejected |
| 4 | $s_5^2 / s_1^2$ | 1626.4891 | | Rejected |
| 5 | $s_2^2 / s_3^2$ | 2.3803 | | Rejected |
| 6 | $s_4^2 / s_2^2$ | 2.0427 | | Rejected |
| 7 | $s_5^2 / s_2^2$ | 9.0758 | | Rejected |
| 8 | $s_4^2 / s_3^2$ | 4.8624 | | Rejected |
| 9 | $s_5^2 / s_3^2$ | 21.6036 | | Rejected |
| 10 | $s_5^2 / s_4^2$ | 4.4430 | | Rejected |

TABLE 8. COMBINATIONS ANALYSIS FOR 50 UNCRACKED BLADES

| Possibility | $s^2$ Ratio | F | $F_{0.01}(49,49)$ | Null Hypothesis |
|---|---|---|---|---|
| 1 | $s_2^2 / s_1^2$ | 187.3466 | 1.9628 | Rejected |
| 2 | $s_3^2 / s_1^2$ | 73.0096 | | Rejected |
| 3 | $s_4^2 / s_1^2$ | 365.1218 | | Rejected |
| 4 | $s_5^2 / s_1^2$ | 1577.1599 | | Rejected |
| 5 | $s_2^2 / s_3^2$ | 2.5661 | | Rejected |
| 6 | $s_4^2 / s_2^2$ | 1.9489 | | Not rejected |
| 7 | $s_5^2 / s_2^2$ | 8.4184 | | Rejected |
| 8 | $s_4^2 / s_3^2$ | 5.0010 | | Rejected |
| 9 | $s_5^2 / s_3^2$ | 21.6021 | | Rejected |
| 10 | $s_5^2 / s_4^2$ | 4.3195 | | Rejected |

TABLE 9. COMBINATIONS ANALYSIS FOR 59 BLADES (UNCRACKED AND CRACKED)

| Possibility | $s^2$ Ratio | F | $F_{0.01}(58,58)$ | Null Hypothesis |
|---|---|---|---|---|
| 1 | $s_2^2/s_1^2$ | 183.9177 | 1.8560 | Rejected |
| 2 | $s_3^2/s_1^2$ | 73.0684 | | Rejected |
| 3 | $s_4^2/s_1^2$ | 347.2782 | | Rejected |
| 4 | $s_5^2/s_1^2$ | 1485.2389 | | Rejected |
| 5 | $s_2^2/s_3^2$ | 2.5171 | | Rejected |
| 6 | $s_4^2/s_2^2$ | 1.8882 | | Rejected |
| 7 | $s_5^2/s_2^2$ | 8.0756 | | Rejected |
| 8 | $s_4^2/s_3^2$ | 4.7528 | | Rejected |
| 9 | $s_5^2/s_3^2$ | 20.3267 | | Rejected |
| 10 | $s_5^2/s_4^2$ | 4.2768 | | Rejected |

Using the test data for computation of hypergeometric distribution, probabilities are evaluated for the number of cracks in a sample of 59 turbine blades at inspection intervals. A 14.21 percent probability is attained that the first 20 samples at inspection intervals include only 5 blades with cracks. Moreover, a 20.93 percent probability is achieved for the first 40 samples which contain 8 cracked blades.

The probabilities calculated for each vibrational mode that a random variable having the standard normal distribution will produce values of frequency between $x_s$ (the smallest) and $x_1$ (the largest) are given in the following table:

| Mode | $x_s$ (kHz) | $x_1$ (kHz) | $P(x_s < x < x_1)$ (%) |
|---|---|---|---|
| 1 | 3.5315 | 3.6065 | 99.46 |
| 2 | 10.2750 | 11.0750 | 97.01 |
| 3 | 14.1500 | 14.8000 | 98.76 |
| 4 | 18.0000 | 19.0800 | 96.40 |
| 5 | 21.7550 | 24.0300 | 96.86 |

## VI. HISTOGRAM OF VIBRATIONAL FREQUENCIES

A graphical presentation of the shape of the distribution function representing the vibrational frequencies of the 59 uncracked and cracked turbine blades is shown in Figure 6 for each of five vibrational modes, using the data from Table 1. The width increments in the scale for fractional group-internal boundaries remain the same for

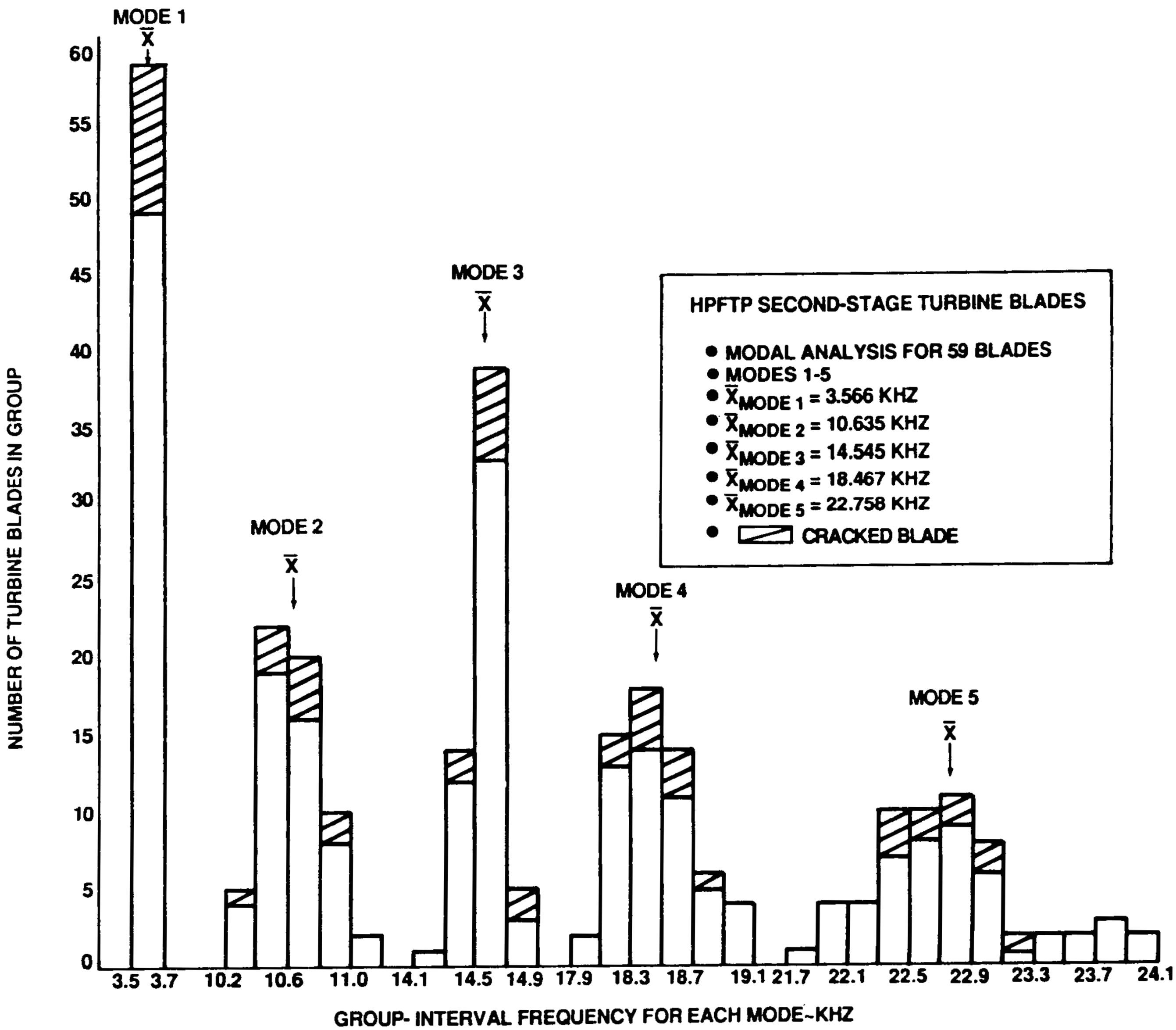


Figure 6. Histogram of frequency for each mode.

all modes. The relative frequency of a group is the empirical probability that a random observation from the population will fall into that group. For example, in the Mode 3 group, the relative frequency of the group interval 14.5 to 14.7 in Figure 6 is 39/59 and, therefore, the empirical probability that a random observation falling in this interval is 39/59. For the same group interval, the relative frequency of cracked blades is 6/59. Figure 6 shows most of the cracked blades in the sample mean neighborhood.

The histogram in Figure 7 presents a total frequency distribution for a combination of five vibrational modes. The base of the rectangle corresponds to the group interval width and the height to that group's frequency of turbine blades. The empirical probability for the cracked blades in the group interval 69.200 to 70.199 is 7/59.

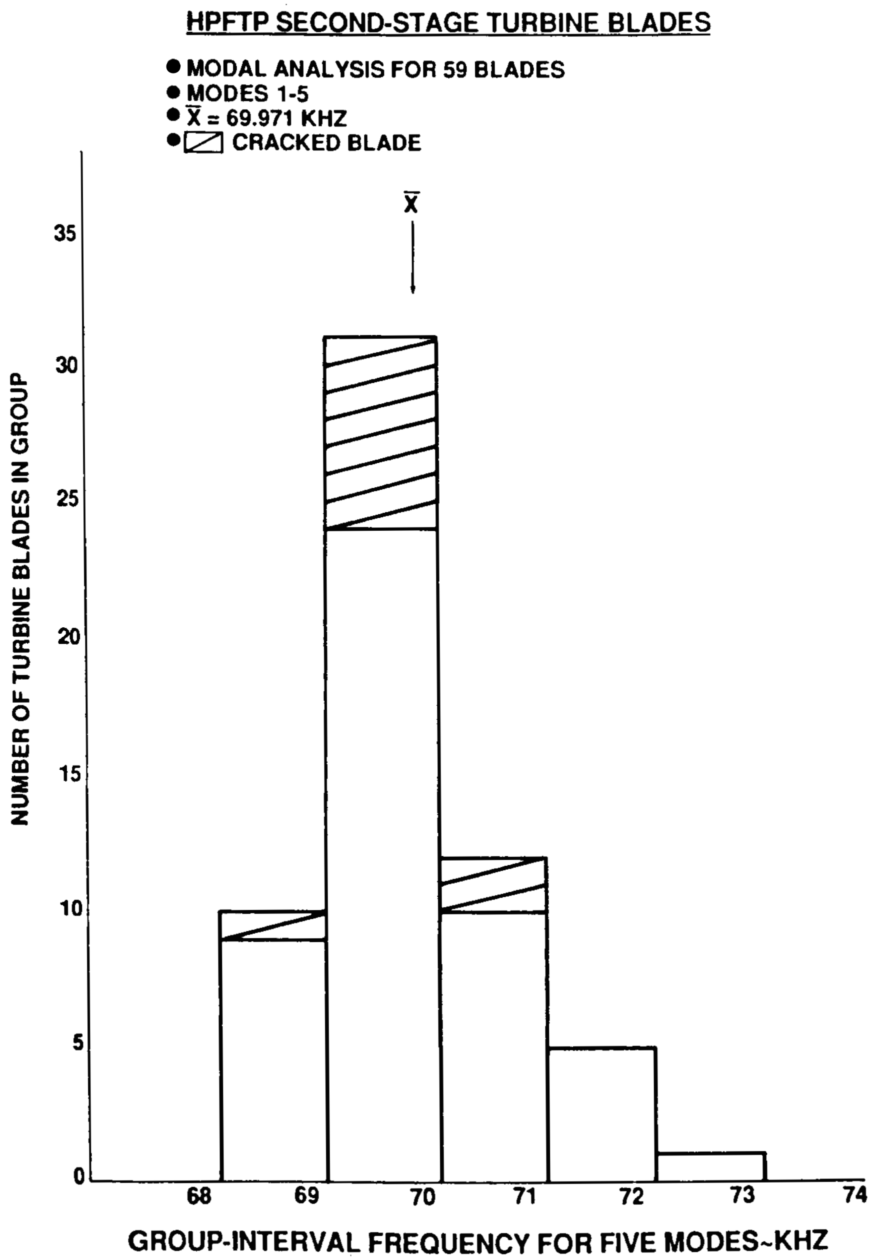


Figure 7. Histogram of total frequency for five modes.

## VII. METHOD OF FREQUENCY DIFFERENCES

The mathematical properties of the variance, mean, standard deviation, and coefficient of variation have been used in the statistical analysis for five different groups, based on the method of frequency differences. The five groups involved in the analysis consist of 59 uncracked and cracked blades, 50 uncracked blades, 49 uncracked blades, 10 cracked blades and 9 cracked blades. The data of frequency differences computed are shown in tabular form in Table 10. Table 11 provides the numerical results of the variance, sample mean, standard deviation and coefficient of variation for those five groups. The variance predicts the distribution of variates. The best overall measure of dispersion is standard deviation which indicates the amount of variability about the sample mean. Computations of the coefficient of variation used to express the standard deviation as the percentage of the sample mean show the large numerical values for the four other groups. Again, no previous studies are available to determine how significant the measure is with respect to the amount of variation. For example, in the 50-uncracked-blade group, the second vibrational mode has the standard deviation expressed as 18877.5 percent of the mean. All sample means are equal or nearly equal to zero. The numberical data of two groups of 50 uncracked blades versus 9 cracked blades and 49 uncracked blades versus 10 cracked blades are summarized in Table 12. Based on the larger and the smaller of two variances, the test statistics for a 99 percent confidence level are given below:

For the 50 versus 9 case:

$$F_{0.01}(49,8) = 5.0795$$

and

$$F_{0.01}(8,49) = 2.9135$$

and for the 49 versus 10 case:

$$F_{0.01}(48,9) = 4.5340$$

and

$$F_{0.01}(9,48) = 2.8220$$

Both cases do not have the null hypothesis rejected since the F values all do not exceed the $F_{0.01}$ values; therefore, the differences are statistically not significant.

## VIII. TEST REPEATABILITY ANALYSIS FOR P13 BLADE

Test repeatability analysis, using statistical inference techniques, is made of a particular turbine blade, serial number P13, which occupies a diametrical position number 14. The frequency data for the P13 blade, which was tested 20 times, are taken in an assembly of readings for five vibrational modes from Table 13 for the statistical analysis. Test measurements of the frequency always produce some

TABLE 10. FREQUENCY DIFFERENCES FOR 59 TURBINE BLADES

| Blade No. | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 |
|---|---|---|---|---|---|
| 1 | -0.02190 | -0.06250 | -0.15000 | 0.41250 | 1.61250 |
| 2 | 0.02505 | -0.05000 | 0.17500 | -0.17500 | -1.52500 |
| 3 | -0.00630 | 0.35000 | -0.07500 | 0.15000 | 1.27500 |
| * 4 | -0.00935 | -0.33750 | 0.0 | -0.41250 | -0.96250 |
| * 5 | 0.01875 | 0.16250 | -0.05000 | 0.47500 | 0.45000 |
| * 6 | -0.02185 | -0.06250 | 0.07500 | -0.35000 | -0.23750 |
| 7 | 0.01560 | -0.08750 | 0.02500 | -0.02500 | -0.20000 |
| 8 | -0.01890 | 0.22500 | -0.16250 | 0.45000 | 0.93750 |
| 9 | 0.02530 | -0.28750 | 0.10000 | -0.32500 | -0.92500 |
| *10 | -0.02205 | 0.30000 | 0.01250 | 0.15250 | 0.76500 |
| 11 | 0.00940 | -0.13750 | 0.06250 | -0.09000 | -0.30250 |
| 12 | -0.00310 | -0.08750 | -0.11250 | -0.20250 | -0.46500 |
| 13 | 0.01560 | 0.07500 | 0.17500 | 0.17750 | 0.22750 |
| 14 | -0.02190 | 0.02500 | -0.17500 | 0.20250 | 0.42750 |
| 15 | 0.01565 | -0.05000 | 0.06250 | -0.38000 | -0.64250 |
| 16 | -0.00935 | 0.08750 | -0.08750 | 0.15250 | 0.32750 |
| 17 | -0.00315 | -0.02500 | 0.08750 | 0.10000 | 0.01250 |
| 18 | 0.01875 | -0.07500 | -0.10000 | -0.21250 | -0.07500 |
| 19 | -0.00940 | 0.05000 | 0.25000 | 0.20000 | 0.26250 |
| *20 | -0.01245 | -0.11500 | -0.22500 | -0.05000 | -0.33750 |
| *21 | 0.02185 | 0.25500 | 0.13750 | 0.21500 | 0.46500 |
| 22 | -0.01565 | -0.19000 | -0.12500 | -0.40500 | -0.81750 |
| *23 | 0.00005 | -0.02500 | 0.03750 | 0.20250 | 0.58000 |
| 24 | -0.00630 | 0.12500 | -0.02500 | -0.07500 | 0.07000 |
| 25 | 0.01565 | -0.30000 | 0.06250 | -0.23750 | -0.96000 |
| 26 | 0.00925 | 0.60000 | 0.07500 | 0.81500 | 1.76500 |
| 27 | -0.04035 | -0.42500 | -0.17500 | -0.66750 | -1.05500 |
| 28 | 0.04360 | -0.15000 | 0.05000 | -0.12250 | -0.24750 |
| 29 | -0.01875 | 0.35000 | 0.05000 | 0.47500 | 0.51250 |
| 30 | -0.00315 | -0.12500 | 0.02500 | -0.15000 | -0.32500 |
| 31 | -0.00935 | 0.03750 | -0.03750 | -0.06250 | 0.15000 |
| 32 | 0.01250 | -0.12500 | -0.05000 | -0.15000 | 0.05000 |
| 33 | 0.00315 | 0.05000 | 0.01250 | 0.35250 | 0.20250 |
| 34 | -0.00005 | 0.03750 | 0.15000 | -0.25500 | -0.39250 |
| 35 | -0.00620 | -0.12500 | -0.25000 | -0.19750 | -0.42250 |
| 36 | -0.00630 | 0.15000 | 0.15000 | 0.57750 | 0.79000 |
| 37 | 0.02190 | -0.08750 | -0.01250 | -0.50500 | -0.21750 |
| 38 | -0.02185 | 0.32500 | 0.05000 | -0.13500 | -0.34750 |
| 39 | 0.01870 | -0.36250 | -0.25000 | 0.56250 | 0.90000 |
| *40 | -0.00935 | -0.15000 | 0.30000 | -0.40000 | -1.26250 |
| 41 | 0.0 | 0.55000 | 0.06250 | 0.38750 | 1.43750 |
| 42 | -0.00935 | -0.50000 | -0.42500 | -0.12250 | -0.81000 |
| 43 | 0.00935 | 0.11250 | 0.25000 | -0.39250 | 0.23250 |
| 44 | 0.00625 | 0.22500 | 0.05000 | 0.40250 | -0.61000 |
| 45 | -0.00315 | -0.17500 | -0.02500 | -0.01250 | 0.63750 |
| 46 | -0.02510 | 0.15000 | -0.16350 | -0.27500 | 0.20000 |
| 47 | 0.04090 | -0.17500 | 0.25200 | 0.26250 | -0.90000 |
| 48 | -0.02515 | 0.0 | -0.16350 | -0.22500 | 0.21250 |
| 49 | 0.01250 | 0.10000 | 0.17500 | 0.23750 | 0.72500 |
| 50 | 0.0 | -0.17500 | -0.13750 | -0.11250 | -0.83750 |

TABLE 10 (Concluded)

| Blade No. | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 |
|---|---|---|---|---|---|
| 51 | -0.00940 | 0.10000 | -0.01250 | 0.03750 | 0.68750 |
| *52 | 0.00315 | -0.08750 | -0.01250 | -0.26250 | -0.86250 |
| 53 | -0.00310 | 0.23750 | 0.02500 | 0.47500 | 1.23750 |
| 54 | 0.00620 | -0.05000 | 0.01250 | -0.18500 | -0.72250 |
| 55 | 0.00315 | 0.07500 | 0.03750 | -0.06500 | 0.08500 |
| *56 | -0.01875 | -0.36250 | -0.11250 | -0.14000 | -0.01500 |
| 57 | 0.02500 | 0.27500 | 0.10000 | 0.19000 | 0.00250 |
| 58 | -0.02185 | -0.13750 | 0.0 | -0.11250 | 0.03750 |
| 59 | 0.01560 | 0.07500 | 0.02500 | -0.17500 | -0.80000 |

*Blade has a crack

TABLE 11. ANALYSIS OF STATISTICAL PROPERTIES FOR FREQUENCY DIFFERENCES

| Mode | Variance ($s^2$) | Mean ($\bar{x}$) | Standard Deviation (x) | Coefficient of Variation (%) |
|---|---|---|---|---|
| 59 Blades (Uncracked and Cracked) | | | | |
| 1 | 0.00030045 | 0.0 | 0.01733354 | |
| 2 | 0.04927220 | 0.0 | 0.22197342 | |
| 3 | 0.01923653 | 0.0 | 0.13869580 | |
| 4 | 0.09764698 | 0.0 | 0.31248517 | |
| 5 | 0.54040216 | 0.0 | 0.73512050 | |
| 50 Uncracked Blades | | | | |
| 1 | 0.00031172 | 0.00062500 | 0.01765559 | 2824.8944 |
| 2 | 0.05131562 | 0.00120000 | 0.22652952 | 18877.4600 |
| 3 | 0.01928227 | -0.00550000 | 0.13886060 | - 2524.7382 |
| 4 | 0.09795208 | 0.00860000 | 0.31297297 | 3639.2206 |
| 5 | 0.54141028 | 0.02805000 | 0.73580587 | 2623.1938 |
| 49 Uncracked Blades | | | | |
| 1 | 0.00031023 | 0.00102041 | 0.01761346 | 1726.1160 |
| 2 | 0.04957267 | 0.00862245 | 0.22264921 | 2582.2036 |
| 3 | 0.01944059 | -0.00331633 | 0.13942952 | - 4204.3319 |
| 4 | 0.09952332 | 0.01163265 | 0.31547317 | 2711.9631 |
| 5 | 0.55265026 | 0.02892857 | 0.74340451 | 2569.7935 |
| 10 Cracked Blades | | | | |
| 1 | 0.00024822 | -0.00500000 | 0.01575488 | - 315.0976 |
| 2 | 0.05075618 | -0.04225000 | 0.22529132 | - 533.2339 |
| 3 | 0.01993229 | 0.01625000 | 0.14118177 | 868.8109 |
| 4 | 0.09414278 | -0.05700000 | 0.30682695 | - 538.2929 |
| 5 | 0.50824176 | -0.14175000 | 0.71291075 | - 502.9253 |
| 9 Cracked Blades | | | | |
| 1 | 0.00025298 | -0.00347222 | 0.01590548 | - 458.0781 |
| 2 | 0.04285625 | -0.00666667 | 0.20701751 | - 3105.2611 |
| 3 | 0.02012153 | 0.03055556 | 0.14185037 | 464.2375 |
| 4 | 0.10495382 | -0.04777778 | 0.32396577 | - 678.0679 |
| 5 | 0.56954063 | -0.15583333 | 0.75467915 | - 484.2861 |

TABLE 12. VARIANCE RATIOS FOR FREQUENCY DIFFERENCES

| Mode | $s^2$ Ratio | F | $F_{0.01}(49,8)$ | $F_{0.01}(8,49)$ | Null Hypothesis |
|---|---|---|---|---|---|
| 50 Uncracked Blades versus 9 Cracked Blades | | | | | |
| 1 | $s^2_{1\,50u}/s^2_{1\,9c}$ | 1.2322 | 5.0795 | | Not rejected |
| 2 | $s^2_{2\,50u}/s^2_{2\,9c}$ | 1.1974 | 5.0795 | | Not rejected |
| 3 | $s^2_{3\,9c}/s^2_{3\,50u}$ | 1.0435 | | 2.9135 | Not rejected |
| 4 | $s^2_{4\,9c}/s^2_{4\,50u}$ | 1.0715 | | 2.9135 | Not rejected |
| 5 | $s^2_{5\,9c}/s^2_{5\,50u}$ | 1.0520 | | 2.9135 | Not rejected |
| 49 Uncracked Blades versus 10 Cracked Blades | | | | | |
| | | | $F_{0.01}(48,9)$ | $F_{0.01}(9,48)$ | |
| 1 | $s^2_{1\,49u}/s^2_{1\,10c}$ | 1.2498 | 4.5340 | | Not rejected |
| 2 | $s^2_{2\,10c}/s^2_{2\,49u}$ | 1.0239 | | 2.8220 | Not rejected |
| 3 | $s^2_{3\,10c}/s^2_{3\,49u}$ | 1.0253 | | 2.8220 | Not rejected |
| 4 | $s^2_{4\,49u}/s^2_{4\,10c}$ | 1.0572 | 4.5340 | | Not rejected |
| 5 | $s^2_{5\,49u}/s^2_{5\,10c}$ | 1.0874 | 4.5340 | | Not rejected |

TABLE 13. SINGLE BLADE MODAL ANALYSIS
BLADE NO. P13, TESTED 20 TIMES
TEST FREQUENCY RANGE: 0-25 kHz

| Test No. | Frequency (kHz) | | | | |
|---|---|---|---|---|---|
| | First | Second | Third | Fourth | Fifth |
| 1 | 3.5562 | 10.750 | 14.525 | 18.430 | 22.905 |
| 2 | 3.5437 | 10.725 | 14.500 | 18.480 | 22.905 |
| 3 | 3.5500 | 10.725 | 14.475 | 18.505 | 22.905 |
| 4 | 3.5562 | 10.800 | 14.575 | 18.555 | 22.930 |
| 5 | 3.5750 | 10.725 | 14.375 | 18.455 | 22.855 |
| 6 | 3.5625 | 10.775 | 14.525 | 18.580 | 22.955 |
| 7 | 3.5587 | 10.775 | 14.550 | 18.555 | 22.980 |
| 8 | 3.5687 | 10.750 | 14.550 | 18.555 | 22.980 |
| 9 | 3.5437 | 10.700 | 14.500 | 18.530 | 22.930 |
| 10 | 3.5625 | 10.775 | 14.525 | 18.580 | 22.980 |
| 11 | 3.5562 | 10.800 | 14.525 | 18.580 | 22.955 |
| 12 | 3.5625 | 10.750 | 14.525 | 18.555 | 22.955 |
| 13 | 3.5687 | 10.800 | 14.525 | 18.605 | 22.955 |
| 14 | 3.5625 | 10.775 | 14.525 | 18.555 | 22.955 |
| 15 | 3.5750 | 10.800 | 14.575 | 18.580 | 22.980 |
| 16 | 3.5375 | 10.650 | 14.350 | 18.480 | 22.855 |
| 17 | 3.5562 | 10.750 | 14.450 | 18.555 | 22.930 |
| 18 | 3.5625 | 10.750 | 14.475 | 18.530 | 22.930 |
| 19 | 3.5625 | 10.725 | 14.550 | 18.505 | 22.930 |
| 20 | 3.5625 | 10.750 | 14.500 | 18.580 | 22.955 |

Source: Rockwell International Corp.

variations when the tests are repeated 20 times in the blade case. One of the possible causes for the variability is that the frequency being measured would show significant variations, due to changes in the testing process over the time interval required to make the measurements. Additionally, the accumulation of random errors in the measuring system would produce a variation that must be examined in relation to the magnitude of the measured frequency.

In order to obtain the improved values of sample variances of the P13 blade, 20 blades are selected randomly from a group of 59 turbine blades for computations of the statistical properties to check the average frequency levels. Selection of 20 blades at random is repeated ten times. The improved values of variances are averaged over 10 times. Improved $s_B^2$ values, after corrected for test error through the repeatability analysis, are obtained. As a result of this repeated procedure, the data are shown in Table 14.

TABLE 14. IMPROVED ANALYSIS

| Mode | $s^2_{A_{20}}$ randomly | $s^2_{P13/20}$ times | Improved $s_B^2$ | F |
|---|---|---|---|---|
| 1 | 0.000160432 | 0.00009622 | 0.000064212 | 1.4985 |
| 2 | 0.029544390 | 0.00144079 | 0.028103600 | 20.5057 |
| 3 | 0.010909987 | 0.00339474 | 0.086690524 | 2.2138 |
| 4 | 0.061002809 | 0.00224342 | 0.058759389 | 26.1919 |
| 5 | 0.236317631 | 0.00137336 | 0.234944271 | 171.0726 |

Differences for Modes 2, 4, and 5 are highly significant since their ratios exceed $F_{0.01}(19,19) = 3.0307$. Repeatability of the same computational procedure by 20 times yields very small changes in the values so the 10-time repeated procedure is acceptable.

Using the original data of frequency, the mathematical statistics for four measures of dispersion are computed for the blade, P13, which was tested 20 times. The statistical properties are summarized in Table 15 for five vibrational modes.

TABLE 15. ANALYSIS OF STATISTICAL PROPERTIES FOR P13 BLADE

| Mode | Variance ($s^2$) | Mean ($\bar{x}$) | Standard of Deviation (s) | Coefficient of Variation (%) |
|---|---|---|---|---|
| 1 | 0.00009622 | 3.559165 | 0.009809179 | 0.2756 |
| 2 | 0.00144079 | 10.752500 | 0.037957740 | 0.3530 |
| 3 | 0.00339474 | 14.505000 | 0.058264397 | 0.4017 |
| 4 | 0.00224342 | 18.537500 | 0.047364755 | 0.2555 |
| 5 | 0.00137336 | 22.936250 | 0.037058872 | 0.1616 |

Based on 20-time testing, the standard deviation is large for the sample of Mode 3 with Mode 4 as the next larger one. However, the computations of coefficient of variation determine Mode 2 to be the second largest percentage of the sample mean after Mode 3. Although Mode 3 is more variable than other modes, the group selection is inadequate to represent the overall variability of the variable.

An upper $3\sigma$ prediction limit has been determined for the P13 blade that was tested 20 times and is shown below for five vibrational modes:

| Mode | Mean ($\bar{x}$) | Standard Deviation (s) | Sample Size | Upper Prediction ($\bar{x}$ + Ks) |
|---|---|---|---|---|
| 1 | 3.5592 | 0.009809 | 20 | 3.5940 |
| 2 | 10.7525 | 0.037958 | 20 | 10.8872 |
| 3 | 14.5050 | 0.058264 | 20 | 14.7118 |
| 4 | 18.5375 | 0.047365 | 20 | 18.7056 |
| 5 | 22.9362 | 0.037059 | 20 | 23.0678 |

The upper prediction limit, with $3\sigma$ equivalent to 99.87 percent, represents an estimate of the percentage point of order P of a probability distribution. This percentage point defines a point on the probability distribution below which P = 100 percent of the data points would be expected to fall. K = 3.55 is a calculated value for a sample size 20 involving one-sided $t_\alpha$ which is the point exceeded with probability P.

The 95 percent confidence interval limits for standard deviation have been computed, using the data from Table 15, and are summarized below:

| Mode | Interval Estimate |
|---|---|
| 1 | $0.007459824 < \sigma < 0.014326620$ |
| 2 | $0.028866640 < \sigma < 0.055438493$ |
| 3 | $0.044309735 < \sigma < 0.085097016$ |
| 4 | $0.036020620 < \sigma < 0.069177740$ |
| 5 | $0.028183056 < \sigma < 0.054125668$ |

The calculations of the statistic $s_{59}^2/s_{13}^2$ to determine whether the groups of 59 blades and P13 blade differ in variability yield the results in Table 16. The critical F at the 0.02 level of significance with 59-1 degrees of freedom for the numerator and 20-1 degrees of freedom for the denominator is 2.6860. The Mode 1 result indicates that the groups do not differ in variability. The results for 59 blades for Modes 2-5 are significantly more variable than for P13 blade.

Table 17 represents the data of the ratios of larger and smaller sample variances for five vibrational modes, based on the ten possible outcomes for the 20-time-tested P13 turbine blade. $F_{0.01}(19,19) = 3.0307$, using the confidence level $\alpha = 0.02$, is obtained for the two-tail test comparison with the F-statistic values. With the Mode 1 variances as the denominator, the null hypothesis is rejected; more vice effect is produced for Mode 1 frequency measurements.

TABLE 16. VARIANCE RATIOS FOR 59 BLADES VERSUS P13 BLADE

| Mode | $s^2$ Ratio | F | $F_{0.01}(58,19)$ | Null Hypothesis |
|---|---|---|---|---|
| 59 Turbine Blades versus 20-Time-Tested P13 Blade | | | | |
| 1 | $s_{1\ 59}^2/s_{1\ P13/20t}^2$ | 1.8255 | 2.6748 | Not rejected |
| 2 | $s_{2\ 59}^2/s_{2\ P13/20t}^2$ | 22.4218 | | Rejected |
| 3 | $s_{3\ 59}^2/s_{3\ P13/20t}^2$ | 3.7807 | | Rejected |
| 4 | $s_{4\ 59}^2/s_{4\ P13/20t}^2$ | 27.1904 | | Rejected |
| 5 | $s_{5\ 59}^2/s_{5\ P13/20t}^2$ | 189.9591 | | Rejected |

TABLE 17. COMBINATIONS ANALYSIS FOR P13 BLADE TESTED 20 TIMES

| Possibility | $s^2$ Ratio | F | $F_{0.01}(19,19)$ | Null Hypothesis |
|---|---|---|---|---|
| 1 | $s_2^2/s_1^2$ | 14.9739 | 3.0307 | Rejected |
| 2 | $s_3^2/s_1^2$ | 35.2810 | | Rejected |
| 3 | $s_4^2/s_1^2$ | 23.3155 | | Rejected |
| 4 | $s_5^2/s_1^2$ | 14.2731 | | Rejected |
| 5 | $s_3^2/s_2^2$ | 2.3562 | | Not rejected |
| 6 | $s_4^2/s_2^2$ | 1.5571 | | Not rejected |
| 7 | $s_2^2/s_5^2$ | 1.0491 | | Not rejected |
| 8 | $s_3^2/s_4^2$ | 1.5132 | | Not rejected |
| 9 | $s_3^2/s_5^2$ | 2.4718 | | Not rejected |
| 10 | $s_4^2/s_5^2$ | 1.6335 | | Not rejected |

## IX. TWO-WAY ANALYSIS OF VARIANCE FOR P13

The two-way classification analysis of variance is performed to determine the effects of five different vibrational modes on 20 tests for the single P13 blade. The frequency measurements were repeated 20 times for the vibrational modes with the results shown in Table 13. Table 18 shows the appropriate sums of squares, degrees of freedom, mean squares, and F ratio.

TABLE 18. ANOVA SUMMARY FOR P13

| S.O.V. | D.O.F. | S.O.S. | M.S. | F |
|---|---|---|---|---|
| Modes | 4 | 4404.8184 | 1101.2046 | 1381219.1534 |
| Tests | 19 | 0.1018 | 0.0054 | 6.7222 |
| Error | 76 | 0.0606 | 0.0008 | |
| Total | 99 | 4404.9808 | | |

Since $F_{0.01}(4,76) = 3.621$ is exceeded by calculated $F_{modes}$ and since $F_{tests}$ exceeds $f_{0.01}(19,76) = 2.153$, there are significant effects, due to differences in the frequency readings for five vibrational modes.

From Table 18, a construction of a 0.99 confidence interval for $\sigma$ is made for a variance of 0.00079727 as a preliminary estimate of $\sigma$, resulting in, for 76 degrees of freedom:

$$0.023160367 < \sigma < 0.035295415 \quad .$$

## X. ONE-WAY ANALYSIS OF VARIANCE FOR BLADES

The one-way classification analysis of variance, one of the statistical inference techniques, considers the vibrational modes as a single source of variability for verification of the test hypothesis with the observation that each vibrational mode has a different, independent frequency population. Each sample of vibrational modes has the same number of observations. The numerical ANOVA results are summarized in Table 19 for five different groups of turbine blades.

The table depicts the source of variation in the first column, the degree of freedom in the second column, and the sum of squares in the third column. The fourth column is the mean square which is obtained by dividing the corresponding sum of squares by its degrees of freedom. The last column shows the F-statistic which is used to determine existence of significant differences between the vibrational modes. The critical values of the 99th percentile of F-statistic, which are to be exceeded if significant differences exist, are obtained from appropriate tables of the F distribution and are shown in the above table. Since the F-statistic exceeds the critical F-statistic in each group, the null hypothesis is rejected at the 0.01 level of significance, meaning that all vibrational modes are not obtaining consistent results. The two-way classification analysis of variance is made for the same five groups.

TABLE 19. ANOVA SUMMARY FOR BLADES

| S.O.V. | D.O.F. | S.S. | M.S. | F |
| --- | --- | --- | --- | --- |
| 59 uncracked and cracked blades | | | | |
| Modes | 4 | 12811.2342 | 3202.8086 | 43611.5967 |
| Error | 290 | 21.2974 | 0.0734 | |
| Total | 294 | 12832.5316 | | |
| $F_{0.01}(4,290) = 3.428$ | | | | |
| 50 uncracked blades | | | | |
| Modes | 4 | 10859.0652 | 2714.7663 | 33035.1699 |
| Error | 245 | 20.1336 | 0.0822 | |
| Total | 249 | 10879.1988 | | |
| $F_{0.01}(4,245) = 3.440$ | | | | |
| 49 uncracked blades | | | | |
| Modes | 4 | 10645.2606 | 2661.3152 | 32148.1136 |
| Error | 240 | 19.8679 | 0.0828 | |
| Total | 244 | 10665.1285 | | |
| $F_{0.01}(4,240) = 3.442$ | | | | |
| 10 cracked blades | | | | |
| Modes | 4 | 2165.9911 | 541.4978 | 17322.8720 |
| Error | 45 | 1.4067 | 0.0313 | |
| Total | 49 | 2167.3978 | | |
| $F_{0.01}(4,45) = 3.760$ | | | | |
| 9 cracked blades | | | | |
| Modes | 4 | 1952.1891 | 488.0473 | 17148.0351 |
| Error | 40 | 1.1384 | 0.0285 | |
| Total | 44 | 1953.3275 | | |
| $F_{0.01}(4,40) = 3.830$ | | | | |

The results are similar to the one-way analysis conclusion and are not presented herein. The two-way ANOVA computational procedure treats the frequency measurements pertaining to the number of vibrational modes distributed over the number of turbine blades.

## XI. CONFIDENCE LIMITS

The statistical analysis is performed to measure the empirical confidence limits within which the sample standard deviation can be expected to occur about the desired percentage of the time. Using a random sample of 59, the chi-square distribution with n-1 degrees of freedom becomes involved to obtain the inequality formula for the variance. Construction is made of a confidence interval for each turbine-blade group with the results provided in Table 20, based on the one-way ANOVA data.

TABLE 20. 95 PERCENT CONFIDENCE LIMITS

| Turbine-Blade Group | 95 Percent Confidence Limits |
|---|---|
| 59 uncracked and cracked | $0.25058930 < \sigma < 0.29505226$ |
| 50 uncracked | $0.26333113 < \sigma < 0.31457802$ |
| 49 uncracked | $0.26407509 < \sigma < 0.31605348$ |
| 10 cracked | $0.14636546 < \sigma < 0.22334109$ |
| 9 cracked | $0.13819571 < \sigma < 0.21661929$ |

This table shows that the probability is 95 percent that the true value of standard deviation lies between those lower and upper confidence limits for each group.

## XII. TEST OF RANDOMNESS

The test of randomness has been performed by means of Monte Carlo simulation to generate pseudo-random numbers to the 60 Bernoulli trials involving sequences of events which deviate from expectation under randomness for 59 turbine blades. The results of Monte Carlo simulation are depicted in Figure 8. Trial "O" represents the test data derived from Table 1. Each trial contains sequences of two symbols of uncracked blade and cracked blade for a group of 59 blades. The black symbol represents a cracked blade and the white symbol represents an uncracked blade. A run comprises a succession of identical symbols between different symbols. Specifically, for example in the 17th trial, the sequence contains 25 runs with 19 cracked blades and 40 uncracked blades.

Table 21 is a summary with computational procedure of test of randomness to test null hypothesis that the sequence of inspections is random. It shows a number of adjacent cracked blades in each trial. During the first 60 trials, only one four-adjacent cracked blade outcome has been found. Also, only 13 three-adjacent blade occurrences have constituted a random sample of size 59 from a continuous distribution. Table 21 indicates that it is not unusual to have many two-adjacent cracked and three-adjacent cracked blades. The total number of runs in a sequence of a

TABLE 21. TEST OF RANDOMNESS SUMMARY

| Bernoulli Trial No. | No. of Cracked Blades | No. of Uncracked Blades | No. of Runs | Cracked Blades | | | | $\mu_u$ | $\sigma$ | z | Randomness |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Single | Two Adjacent | Three Adjacent | Four Adjacent | | | | |
| 0 | 10 | 49 | 15 | 5 | 1 | 1 | 0 | 17.610 | 2.114 | -1.234 | Yes |
| 1 | 8 | 51 | 16 | 8 | 0 | 0 | 0 | 14.831 | 1.749 | 0.669 | Yes |
| 2 | 10 | 49 | 21 | 10 | 0 | 0 | 0 | 17.610 | 2.114 | 1.603 | Yes |
| 3 | 9 | 50 | 14 | 5 | 2 | 0 | 0 | 16.254 | 1.936 | -1.164 | Yes |
| 4 | 12 | 47 | 21 | 8 | 2 | 0 | 0 | 20.119 | 2.444 | 0.361 | Yes |
| 5 | 11 | 48 | 18 | 7 | 2 | 0 | 0 | 18.898 | 2.284 | -0.393 | Yes |
| 6 | 9 | 50 | 15 | 5 | 2 | 0 | 0 | 16.254 | 1.936 | -0.648 | Yes |
| 7 | 7 | 52 | 14 | 7 | 0 | 0 | 0 | 13.339 | 1.553 | 0.426 | Yes |
| 8 | 13 | 46 | 23 | 9 | 2 | 0 | 0 | 21.271 | 2.595 | 0.666 | Yes |
| 9 | 12 | 47 | 19 | 7 | 1 | 1 | 0 | 20.119 | 2.444 | -0.458 | Yes |
| 10 | 10 | 49 | 18 | 7 | 1 | 0 | 0 | 17.610 | 2.114 | 0.184 | Yes |
| 11 | 12 | 47 | 19 | 7 | 1 | 1 | 0 | 20.119 | 2.444 | -0.458 | Yes |
| 12 | 10 | 49 | 17 | 7 | 0 | 1 | 0 | 17.610 | 2.114 | -0.289 | Yes |
| 13 | 9 | 50 | 14 | 5 | 2 | 0 | 0 | 16.254 | 1.936 | -1.164 | Yes |
| 14 | 6 | 53 | 10 | 4 | 1 | 0 | 0 | 11.780 | 1.348 | -1.320 | Yes |
| 15 | 12 | 47 | 24 | 12 | 0 | 0 | 0 | 20.119 | 2.444 | 1.588 | Yes |
| 16 | 10 | 49 | 17 | 6 | 2 | 0 | 0 | 17.610 | 2.114 | -0.289 | Yes |
| 17 | 19 | 40 | 25 | 7 | 3 | 2 | 0 | 26.763 | 3.316 | -0.532 | Yes |
| 18 | 8 | 51 | 15 | 6 | 1 | 0 | 0 | 14.831 | 1.749 | 0.097 | Yes |
| 19 | 13 | 46 | 17 | 4 | 3 | 1 | 0 | 21.271 | 2.595 | -1.646 | Yes |
| 20 | 10 | 49 | 19 | 8 | 1 | 0 | 0 | 17.610 | 2.114 | 0.657 | Yes |
| 21 | 9 | 50 | 13 | 4 | 1 | 1 | 0 | 16.254 | 1.936 | -1.681 | Yes |
| 22 | 9 | 50 | 16 | 7 | 1 | 0 | 0 | 16.254 | 1.936 | -0.131 | Yes |
| 23 | 10 | 49 | 17 | 6 | 2 | 0 | 0 | 17.610 | 2.114 | -0.289 | Yes |
| 24 | 14 | 45 | 21 | 8 | 1 | 0 | 1 | 22.356 | 2.738 | -0.495 | Yes |
| 25 | 15 | 44 | 23 | 8 | 2 | 1 | 0 | 23.373 | 2.871 | -0.130 | Yes |
| 26 | 12 | 47 | 23 | 10 | 1 | 0 | 0 | 20.119 | 2.444 | 1.179 | Yes |
| 27 | 11 | 48 | 21 | 9 | 1 | 0 | 0 | 18.898 | 2.284 | 0.920 | Yes |
| 28 | 5 | 54 | 11 | 5 | 0 | 0 | 0 | 10.152 | 1.134 | 0.747 | Yes |
| 29 | 5 | 54 | 11 | 5 | 0 | 0 | 0 | 10.152 | 1.134 | 0.747 | Yes |
| 30 | 10 | 49 | 19 | 8 | 1 | 0 | 0 | 17.610 | 2.114 | 0.657 | Yes |

TABLE 21. (Concluded)

| Bernoulli Trial No. | No. of Cracked Blades | No. of Uncracked Blades | No. of Runs | Cracked Blades | | | | $\mu_u$ | $\sigma$ | z | Randomness |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Single | Two Adjacent | Three Adjacent | Four Adjacent | | | | |
| 31 | 9 | 50 | 17 | 7 | 1 | 0 | 0 | 16.254 | 1.936 | 0.385 | Yes |
| 32 | 10 | 49 | 17 | 6 | 2 | 0 | 0 | 17.610 | 2.114 | -0.289 | Yes |
| 33 | 9 | 50 | 16 | 7 | 1 | 0 | 0 | 16.254 | 1.936 | -0.131 | Yes |
| 34 | 10 | 49 | 16 | 6 | 2 | 0 | 0 | 17.610 | 2.114 | -0.762 | Yes |
| 35 | 8 | 51 | 17 | 8 | 0 | 0 | 0 | 14.831 | 1.749 | 1.240 | Yes |
| 36 | 16 | 43 | 26 | 10 | 3 | 0 | 0 | 24.322 | 2.996 | 0.560 | Yes |
| 37 | 9 | 50 | 16 | 7 | 1 | 0 | 0 | 16.254 | 1.936 | -0.131 | Yes |
| 38 | 11 | 48 | 17 | 6 | 1 | 1 | 0 | 18.898 | 2.284 | -0.831 | Yes |
| 39 | 9 | 50 | 15 | 5 | 2 | 0 | 0 | 16.254 | 1.936 | -0.648 | Yes |
| 40 | 10 | 49 | 19 | 8 | 1 | 0 | 0 | 17.610 | 2.114 | 0.657 | Yes |
| 41 | 6 | 53 | 11 | 4 | 1 | 0 | 0 | 11.780 | 1.348 | -0.578 | Yes |
| 42 | 6 | 53 | 11 | 4 | 1 | 0 | 0 | 11.780 | 1.348 | -0.578 | Yes |
| 43 | 7 | 52 | 11 | 3 | 2 | 0 | 0 | 13.339 | 1.553 | -1.506 | Yes |
| 44 | 13 | 46 | 21 | 8 | 1 | 1 | 0 | 21.271 | 2.595 | -0.104 | Yes |
| 45 | 10 | 49 | 17 | 6 | 2 | 0 | 0 | 17.610 | 2.114 | -0.289 | Yes |
| 46 | 13 | 46 | 27 | 13 | 0 | 0 | 0 | 21.271 | 2.595 | 2.207 | No |
| 47 | 7 | 52 | 15 | 7 | 0 | 0 | 0 | 13.339 | 1.553 | 1.069 | Yes |
| 48 | 8 | 51 | 17 | 8 | 0 | 0 | 0 | 14.831 | 1.749 | 1.240 | Yes |
| 49 | 11 | 48 | 21 | 9 | 1 | 0 | 0 | 18.898 | 2.284 | 0.920 | Yes |
| 50 | 11 | 48 | 19 | 8 | 0 | 1 | 0 | 18.898 | 2.284 | 0.044 | Yes |
| 51 | 8 | 51 | 13 | 5 | 0 | 1 | 0 | 14.831 | 1.749 | -1.046 | Yes |
| 52 | 8 | 51 | 17 | 8 | 0 | 0 | 0 | 14.831 | 1.749 | 1.240 | Yes |
| 53 | 12 | 47 | 18 | 6 | 3 | 0 | 0 | 20.119 | 2.444 | -0.867 | Yes |
| 54 | 15 | 44 | 26 | 11 | 2 | 0 | 0 | 23.373 | 2.871 | 0.915 | Yes |
| 55 | 13 | 46 | 21 | 7 | 3 | 0 | 0 | 21.271 | 2.595 | -0.104 | Yes |
| 56 | 16 | 43 | 27 | 10 | 3 | 0 | 0 | 24.322 | 2.996 | 0.894 | Yes |
| 57 | 7 | 52 | 13 | 7 | 0 | 0 | 0 | 13.339 | 1.553 | -0.218 | Yes |
| 58 | 5 | 54 | 9 | 3 | 1 | 0 | 0 | 10.152 | 1.134 | -1.016 | Yes |
| 59 | 8 | 51 | 17 | 8 | 0 | 0 | 0 | 14.831 | 1.749 | 1.240 | Yes |
| 60 | 10 | 49 | 17 | 6 | 2 | 0 | 0 | 17.610 | 2.114 | -0.289 | Yes |

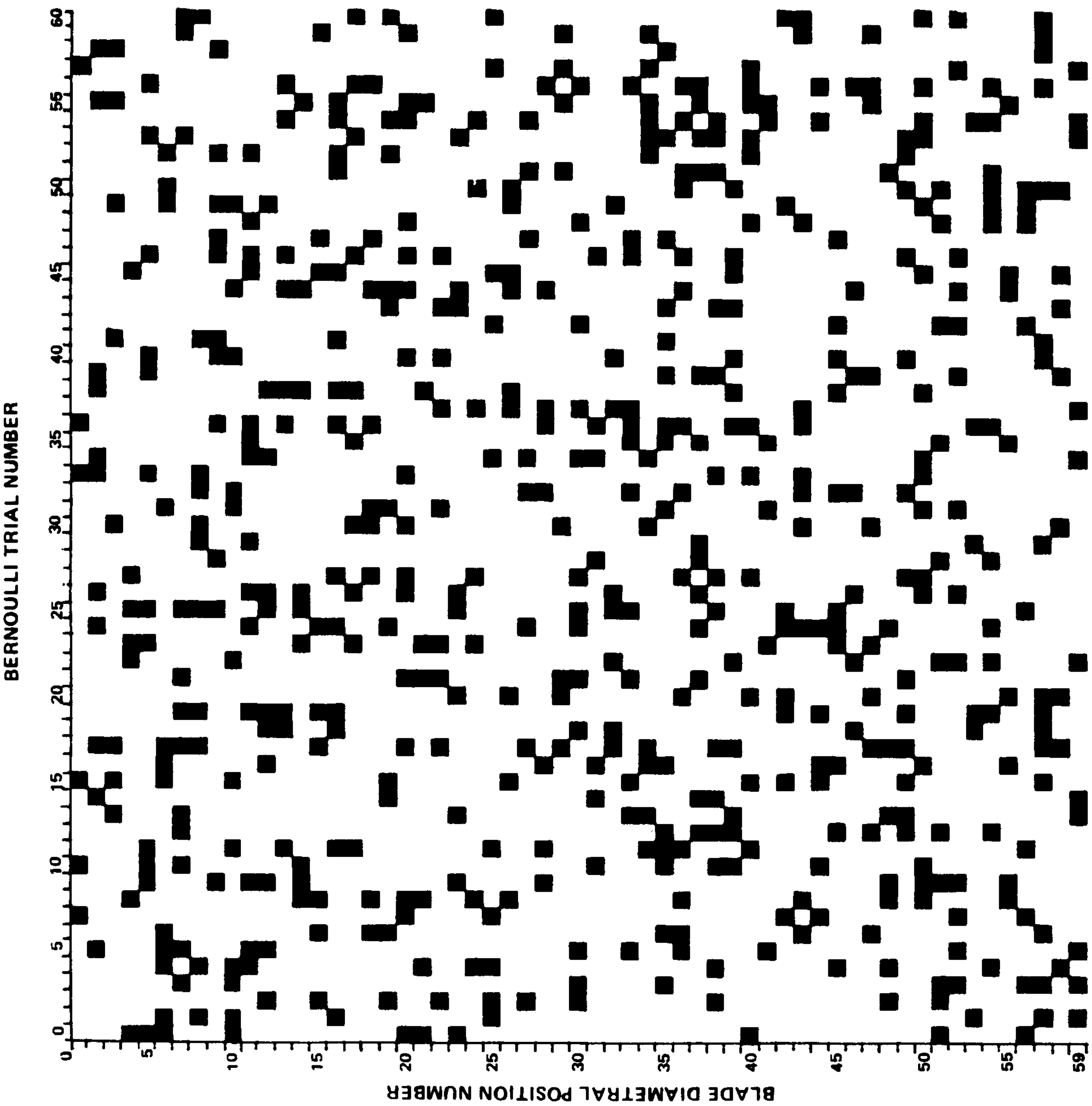


Figure 8. Monte Carlo simulation.

number of trials may indicate that the arrangement may not be random. Using the symbols of cracked and uncracked blades, the sampling distribution of the total number of runs can be approximated closely by a normal distribution with the mean, $\mu_u$, and the standard deviation, $\sigma_u$. The statistic, z, determines the test of the null hypothesis that the arrangement of the sample is random. Randomness of events is based on their outcomes being unable to be predicted. Examination of Table 21 shows that in the first 60 Bernoulli trials, only one trial yields nonrandonmess at the level of significance $\alpha = 0.05$ for the two-tailed test of $\pm z_{0.025} = \pm 1.96$. The values of the z statistic for those 59 trials are obviously not significant, which explains that the cracked and uncracked blades do not tend to cluster or cycle in the Bernoulli trials.

## XIII. CONCLUSIONS

This statistical analysis has not been able to find the cause of cracks or any peculiarities of the cracked blades. They seem to be just average blades, having no distinctive features. The statistical analysis has not discovered in what respect the uncracked and cracked blades differ. They do not differ in frequencies or variances and are not clustered. The histograms show most cracked blades in the neighborhood of sample mean so there are more uncracked blades for five vibrational modes. Overall results seem to indicate that the cracked blades are not different from the uncracked blades. The crack failures are probably random events caused by the fact that the blades are operating at their marginal stress levels. Literature search has not been initiated and generated to support this analysis.

APPROVAL

STATISTICAL ANALYSIS OF 59 INSPECTED
SSME HPFTP TURBINE BLADES
(UNCRACKED AND CRACKED)

By John T. Wheeler

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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