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ON BUCKLING OF LAMINATED PLATES

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EFFECTS OF THICKNESS AND PLY ORIENTATION ON BUCKLING OF LAMINATED PLATES

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Abstract

The buckling loads of laminated plates are predicted using a new theory which takes into account transverse shearing effects. This new theory assumes trigonometric terms through-the-thickness in the displacements to take into account transverse shearing effects in thick plates. Buckling loads predicted by the new theory and by traditional theories are compared for isotropic and laminated plates. The effect of ply orientation on the buckling loads predicted by each theory is demonstrated.

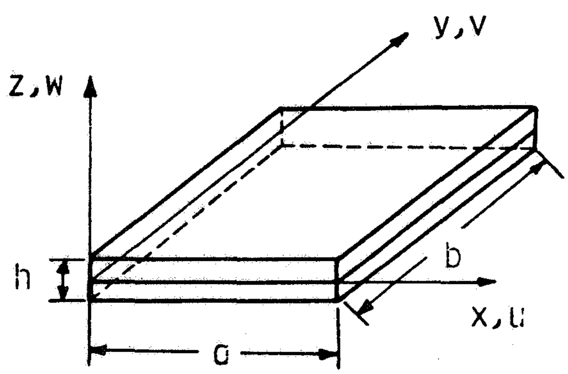
Introduction

Classical plate theory accurately predicts the inplane stresses, displacements and buckling loads of thin plates of homogeneous materials. Conventional transverse shear deformation theory accurately predicts the stresses, displacements and buckling loads of thicker plates of homogeneous materials. However, these theories are not as accurate when applied to laminated plates as they are when applied to isotropic plates because they do not adequately take into account transverse shearing effects. Since transverse shearing can significantly affect the response of laminated plates, a theory which predicts the buckling loads and transverse shearing effects more accurately than traditional theories is essential for predicting the behavior of laminated plates. Such a theory is developed in reference 1.

The purpose of the present paper is to apply the theory developed in reference 1 to simply supported flat plates and to compare the buckling loads for the theory of reference 1, for classical theory and for the conventional transverse shear deformation theory for plates of different thicknesses. In addition, the effect of changes in ply orientation on buckling load for thick laminated plates is discussed.

Analysis Approach

The results presented in this paper are obtained by applying the theory developed in reference 1. In reference 1 the potential energy method is used to obtain equations for the buckling of a flat plate with the coordinate system and plate dimensions shown in the sketch.



The series chosen to represent the plate displacements are shown in equations (1) and have three types of terms.

$$u(x,y,z) = [u^o + u^a (z/h) + u^1 \sin(\pi z/h)] \cos(n\pi x/a) \sin(m\pi y/b)$$

$$v(x,y,z) = [v^o + v^a (z/h) + v^1 \sin(\pi z/h)] \sin(n\pi x/a) \cos(m\pi y/b) \qquad (1)$$

$$w(x,y,z) = [w^o + w^1 \cos(\pi z/h)] \sin(n\pi x/a) \sin(m\pi y/b)$$

The traditional terms from classical plate theory (those independent of position in the z direction), and from conventional transverse shear deformation theory (those linear in z) are represented by the terms with superscripts o and a, respectively. The assumed displacement series also includes trigonometric terms in the through-the-thickness or z direction, which are represented by the terms with superscript 1. By including trigonometric terms in the assumed displacement series, a more accurate solution is obtained because more transverse shear deformation is permitted than in the solutions with fewer assumed displacement terms.

Differential equations for predicting the buckling loads of flat plates are developed in reference 1 by using the potential energy method and the assumed displacement series shown above. The equilibrium equations for the flat plate are obtained from:

$$\int_{-h/2}^{h/2} (\sigma_{x,x} + \tau_{xy,y})\, \delta u^{o} dz = 0$$

$$\int_{-h/2}^{h/2} (\sigma_{y,y} + \tau_{xy,x})\, \delta v^{o} dz = 0$$

$$\int_{-h/2}^{h/2} [(\sigma_x w^{o}_{,x})_{,x} + (\sigma_y w^{o}_{,y})_{,y} + (\tau_{xy} w^{o}_{,x})_{,y} + (\tau_{xy} w^{o}_{,y})_{,x}$$

$$+ \tau_{xz,x} + \tau_{yz,y}]\, \delta w^{o} dz = 0$$

$$\int_{-h/2}^{h/2} [(\sigma_{x,x} + \tau_{xy,y}) \frac{z}{h} - \tau_{xz} \frac{1}{h}]\, \delta u^{a} dz = 0 \qquad (2)$$

$$\int_{-h/2}^{h/2} [(\sigma_{y,y} + \tau_{xy,x}) \frac{z}{h} - \tau_{yz} \frac{1}{h}]\, \delta v^{a} dz = 0$$

$$\int_{-h/2}^{h/2} [(\sigma_{x,x} + \tau_{xy,y}) \sin \frac{\pi z}{h} - \frac{\pi}{h} \tau_{xz} \cos \frac{\pi z}{h}]\, \delta u^{1} dz = 0$$

$$\int_{-h/2}^{h/2} [(\sigma_{y,y} + \tau_{xy,x}) \sin \frac{\pi z}{h} - \frac{\pi}{h} \tau_{yz} \cos \frac{\pi z}{h}]\, \delta v^{1} dz = 0$$

$$\int_{-h/2}^{h/2} [\frac{\pi}{h} \sigma_z \sin \frac{\pi z}{h} + (\tau_{xz,x} + \tau_{yz,y}) \cos \frac{\pi z}{h}] \delta w^1 dz = 0$$

For the case of $\overset{o}{w} = 0$ before buckling, the differential equations of equilibrium can be converted into buckling equations by considering small changes in deformations that occur at buckling. This assumption reduces the third of equations (2) to:

$$\int_{-h/2}^{h/2} [\tau_{xz,x} + \tau_{yz,y}] \, dz + \text{loading terms} = 0 \qquad (3)$$

while the other equations of equations (2) remain unchanged.

The differential equations represented by equations (2) and (3) can be reduced to linear equations containing only the displacement and loading terms as unknowns by using the following stress-strain and strain-displacement relations

$$\begin{aligned}
&\{\sigma_\tau\} = [C_{ij}] \{^{\varepsilon}_{\gamma}\} \\
&\varepsilon_x = u_{,x} + \frac{1}{2} \overset{o}{w}{}_{,x}^{2} \\
&\varepsilon_y = v_{,y} + \frac{1}{2} \overset{o}{w}{}_{,y}^{2} \\
&\varepsilon_z = w_{,z} \\
&\gamma_{xy} = u_{,y} + v_{,x} \\
&\gamma_{xz} = u_{,z} + w_{,x} \\
&\gamma_{yz} = v_{,z} + w_{,y}
\end{aligned} \qquad (4)$$

where $[C_{ij}]$ is the material stiffness matrix.

Inserting equations (4) into equations (2) and (3) and integrating in the thickness direction gives the following linear equations.

$$\alpha h\bar{u}^{o} + kh\bar{v}^{o} = 0$$

$$\beta h\bar{v}^{o} + kh\bar{u}^{o} = 0$$

$$G_{xz}\left(\frac{\pi n}{a}\bar{u}^{a} + \frac{2\pi n}{a}\bar{u}^{1}\right) + G_{yz}\left(\frac{m\pi}{b}\bar{v}^{a} + \frac{2\pi m}{b}\bar{v}^{1}\right) + \rho\left(h\bar{w}^{o} + \frac{2h}{\pi}\bar{w}^{1}\right) + \text{loading terms} = 0$$

$$\alpha\frac{h}{12}\bar{u}^{a} + \kappa\bar{v}^{a}\frac{h}{12} + \alpha\bar{u}^{1}\frac{2h}{\pi^{2}} + \kappa\frac{2h}{\pi^{2}}\bar{v}^{1} + G_{xz}\left(\frac{1}{h}\bar{u}^{a} + \frac{n\pi}{a}\bar{w}^{o} + \frac{2}{h}\bar{u}^{1} + \frac{2n}{a}\bar{w}^{1}\right) = 0$$

$$\beta\frac{h}{12}\bar{v}^{a} + \kappa\frac{h}{12}\bar{u}^{a} + \beta\frac{2h}{\pi^{2}}\bar{v}^{1} + \kappa\frac{2h}{\pi^{2}}\bar{u}^{1} + G_{yz}\left(\frac{1}{h}\bar{v}^{a} + \frac{m\pi}{b}w^{o} + \frac{2}{h}\bar{v}^{1} + \frac{2m}{b}\bar{w}^{1}\right) = 0$$

$$\alpha\frac{2h}{\pi^{2}}\bar{u}^{a} + \kappa\bar{v}^{a}\frac{2h}{\pi^{2}} + \alpha\frac{h}{2}\bar{u}^{1} + \kappa\frac{h}{2}\bar{v}^{1} + G_{xz}\left(\frac{\pi^{2}}{2h}\bar{u}^{1} + \frac{n\pi^{2}}{2a}\bar{w}^{1} + \frac{2}{h}\bar{u}^{a} + \frac{2n\pi}{a}w^{o}\right) = 0$$

$$\beta\frac{2h}{\pi^{2}}\bar{v}^{a} + \kappa\frac{2h}{\pi^{2}}\bar{u}^{a} + \beta\frac{h}{2}\bar{v}^{1} + \kappa\frac{h}{2}\bar{v}^{1} + G_{yz}\left(\frac{\pi}{2h}\bar{v}^{1} + \frac{n\pi^{2}}{2a}\bar{w}^{1} + \frac{2}{h}\bar{v}^{a} + \frac{2n\pi}{a}\bar{w}^{o}\right) = 0 \qquad (5)$$

$$Q_{33}\frac{\pi^{2}}{2h}\bar{w}^{1} + G_{xz}\left(\frac{2n}{a}\bar{u}^{a} + \frac{n\pi^{2}}{2a}\bar{u}^{1}\right) + G_{yz}\left(\frac{2m}{b}\bar{v}^{a} + \frac{m\pi^{2}}{2b}\bar{v}^{1}\right) + \rho\frac{2h}{\pi}\bar{w}^{o} + \rho\bar{w}^{1}\frac{h}{2} = 0$$

and

$$\alpha = \pi^{2}\left(Q_{11}\left(\frac{n}{a}\right)^{2} + G_{xy}\left(\frac{m}{b}\right)^{2}\right)$$

$$\beta = \pi^{2}\left(Q_{22}\left(\frac{m}{b}\right)^{2} + G_{xy}\left(\frac{n}{a}\right)^{2}\right); \quad \rho = \left(\frac{n\pi}{a}\right)^{2}G_{xz} + \left(\frac{m\pi}{b}\right)G_{yz}; \quad \kappa = \frac{\pi^{2}nm}{ab}\left(Q_{12} + G_{xy}\right)$$

The displacements ($\bar{u}^o$, $\bar{u}^a$, $\bar{u}^1$,...) in equations (5) are now the small changes which occur at buckling and n and m are the number of half waves in the x and y directions, respectively.

Loading terms in the form $\{N_x(n\pi/a)^2 + N_y(m\pi/b)^2\}w^o$ for compressive buckling are added to the third of equations (5). Buckling loads can be found from these equations by reducing equations (5) (with the loading terms) to the form of equation (6).

$$[K] \begin{Bmatrix} \bar{u}^o \\ \bar{u}^a \\ \bar{u}^1 \\ \bar{v}^o \\ \bar{v}^a \\ \bar{v}^1 \\ \bar{w}^o \\ \bar{w}^1 \end{Bmatrix} = 0 \qquad (6)$$

The lowest value of N_x or N_y for which the determinant of [K] = 0 is the buckling load. The following section contains numerical results for the compressive buckling of simply supported plates loaded in one direction.

Results and Discussion

A comparison is made between the results from classical plate theory in which no transverse shearing is assumed, from conventional transverse shear deformation theory in which the transverse shear stress is assumed to vary only along the length and width of the plate, and from the theory of reference 1 in which the transverse shear stress varies in all three coordinate directions. The conventional transverse shear deformation theory can be obtained by neglecting the trigonometric terms in the theory of reference 1 and the classical theory can be obtained by assuming the plate to be very thin. These three theories are applied to an isotropic plate, a $[\pm 45]_{4s}$ laminated plate and a $[0]_{16}$ laminated plate. All of the results presented are for square plates. A variety of thicknesses ranging from very thin to one-third the length are examined. The buckling loads and stresses at buckling predicted by each theory are compared. The relationship between ply orientation and buckling load is also studied for $[\pm\theta]_{4s}$ laminates for several thicknesses and for ply orientations ranging from 0 to 90 degrees.

The buckling load parameter, $k = a^2 N_x/(\pi^2\sqrt{D_{11}D_{22}})$, is shown in figure 1 for a square plate with thicknesses varying from 0 to .3 times the length for the three theories. In the case of the thin isotropic plate, the difference between the buckling loads predicted by each theory is less than one percent. The classical theory, which assumes no transverse shearing ($\tau_{xz} = \tau_{yz} = 0$), gives a buckling load parameter which is independent of thickness and therefore only accurate for thin plates. The theory in which the transverse shear stress is assumed to be constant through-the-thickness ($\tau_{xz}(x,y)$; $\tau_{yz}(x,y)$) gives a buckling load which is slightly lower than the buckling load from classical theory for thin plates and a buckling load which is 30 percent lower for very thick plates ($t/a = .3$). The theory of reference 1, in which the transverse shear stress is a function of all three coordinates ($\tau_{xz}(x,y,z)$; $\tau_{yz}(x,y,z)$), gives a buckling load which is 36 percent lower than the buckling load from classical theory for the thickest plate studied.

The buckling load parameter versus the thickness-to-length ratio is shown for a laminated plate of $[\pm 45]_{4s}$ ply orientation in figure 2. Here the same trend occurs as for the isotropic plate. For very thin plates all theories give essentially the same buckling loads. But for a plate which has a thickness-to-length ratio of .3, the conventional transverse shear deformation theory and the theory of reference 1 have predicted buckling loads that are 22 and 36 percent lower, respectively, than results from classical theory.

The buckling stresses versus the thickness-to-length ratio for a $[\pm 45]_{4s}$ laminated plate and for a $[0]_{16}$ plate are shown in figure 3. While the addition of the trigonometric terms decreases the predicted buckling stress in both cases, the more significant decrease is caused by including the trigonometric terms in the $[\pm 45]_{4s}$ laminate.

A comparison of the buckling load parameter for the three theories is shown in figure 4 for ply orientations ranging from 0 to 90 degrees. The plates examined here have thickness-to-length ratios (t/a) of .05 for a very thin plate, .20 for a plate with intermediate thickness, and .30 for a very thick plate. The classical theory gives the same curve as the conventional transverse shearing theory for the thinnest plate. Since no thickness effects are taken into account in the classical theory, all thicknesses give the same curve. There is almost no difference in the buckling loads predicted by each theory for the very thin plate for all ply orientations. The difference between the conventional transverse shearing theory ($\tau_{xz}(x,y)$ and $\tau_{yz}(x,y)$ are constant through-the-thickness) and the theory of reference 1 for the plate with intermediate thickness (t/a=.20) varies from 5 percent for plates with $\theta = 45°$ to 13 percent for $\theta = 30°$. For the thick plate (t/a = .30) the reduction in buckling load is greater than 12 percent for all orientations for the theory of reference 1 compared to the conventional transverse shear theory. The largest difference in buckling load is a reduction of 17 percent for a plate with $\theta = 45°$. In all cases except the very thin plate, the buckling loads predicted by the theory of reference 1 are lower than those predicted by the classical theory and the conventional transverse shear deformation theory. The theory of reference 1 allows more flexibility than the other two theories by including more terms in the assumed displacement series. It is a higher order theory and is, therefore, more accurate.

Concluding Remarks

Results are obtained for a new theory for predicting the buckling load of simply supported laminated plates. The new theory uses trigonometric terms in the assumed displacement series to predict the effects of transverse shearing. Transverse shearing causes significant decreases in the buckling load results of thick laminated plates. For thick plates the new theory predicts buckling loads which are as much as 30 percent lower than those predicted by traditional theories.

A parametric study indicates that the ±45° ply orientation is the orientation most affected by transverse shearing in thick laminates. A comparison of the buckling load results from a theory which does not include the trigonometric terms (conventional transverse shear deformation theory) to the results from the new theory shows a decrease in buckling load results from 12 to 17 percent, depending on the ply orientation.

References

1. Stein, M.: Nonlinear Theory for Laminated and Thick Plates and Shells Including the Effects of Transverse Shearing. Presented at the 26th AIAA/ASME/ASCE/AHS Structures, Structural Dymanics and Materials Conference, Orlando, Florida, April 15-17, 1985. AIAA paper 85-0671-CP, April 1985.

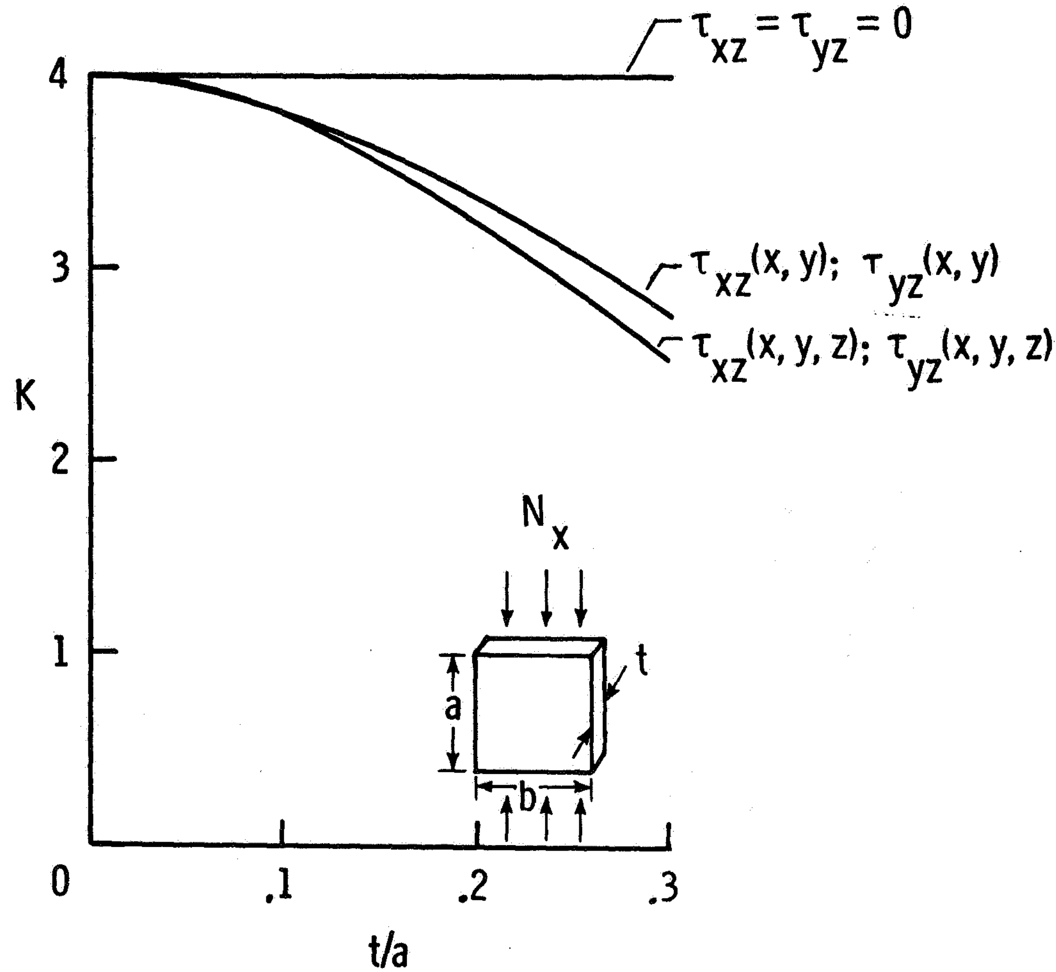


Figure 1. Comparison of buckling load parameter of an isotropic square plate as a function of thickness-to-length ratio obtained by several approximations.

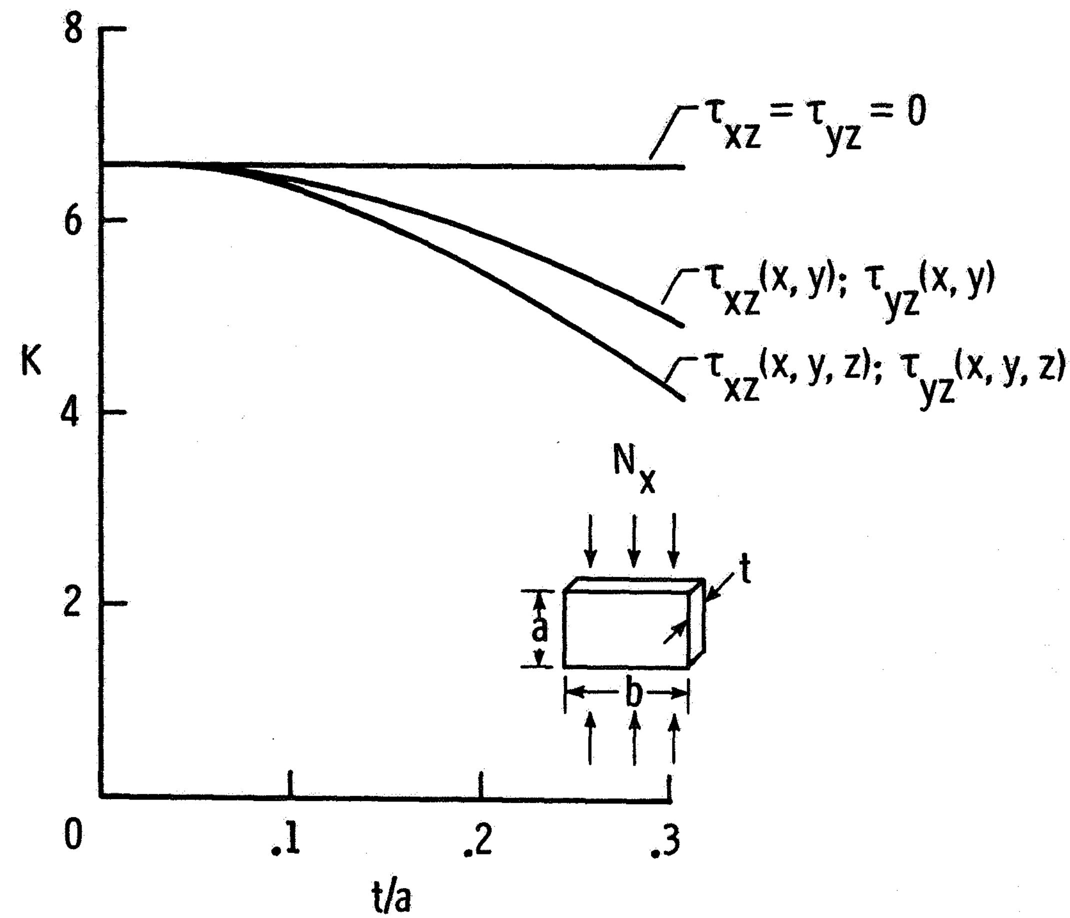
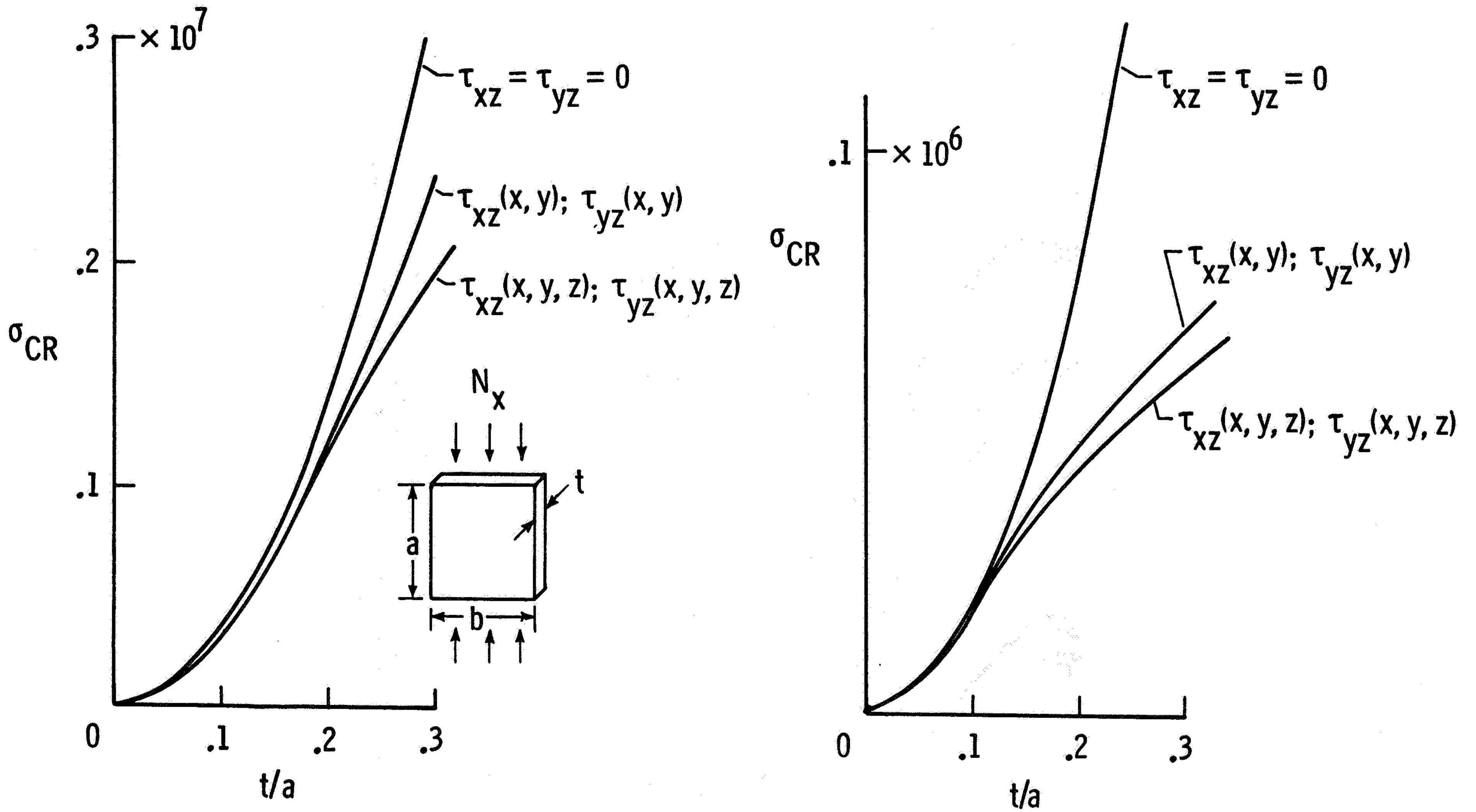


Figure 2. Comparison of buckling load parameter of a laminated graphite-epoxy, $[\pm 45]_{4s}$, square plate as a function of thickness-to-length ratio obtained by several approximations.



(a) $[\pm 45]_{4s}$ (b) $[0]_{16}$

Figure 3. Comparison of average stress at buckling for laminated graphite-epoxy square plates as a function of thickness-to-length ratio obtained by several approximations.

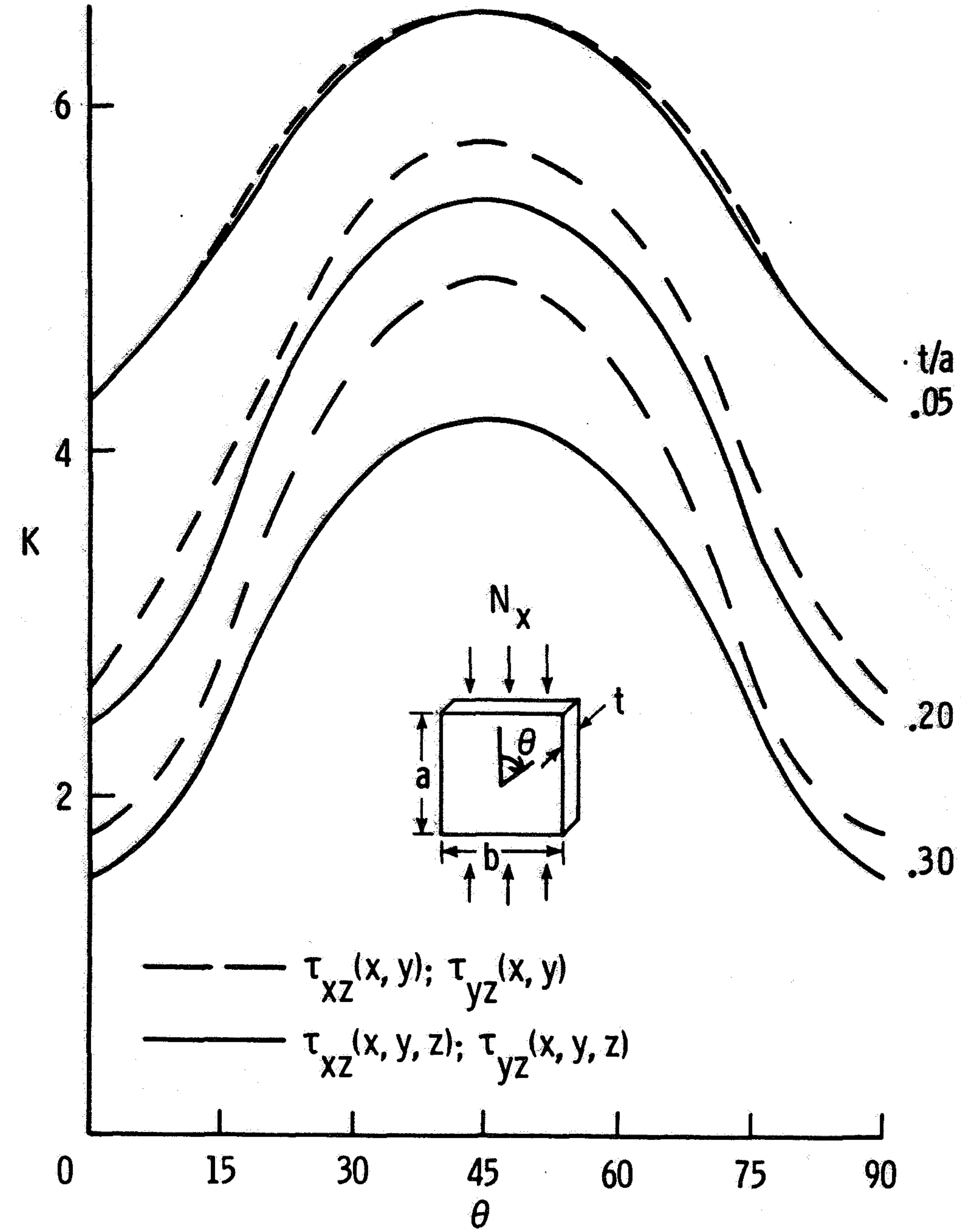


Figure 4. Comparison of buckling load parameter for square $[\pm\theta]_{4s}$ graphite-epoxy laminated plates as a function of ply orientation.

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16. Abstract

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