APPENDIX D

INTEGRATOR TRANSFER PROPERTIES

INTRODUCTION

The process of summing the echo pulses from a target is called integration. This appendix discusses the transfer properties of integrators (enhancers) used by radars in the 2.7 to 2.9 GHz band. The discussion includes the integrator transfer properties of noise, desired signal, and asynchronous interfering signals. It should be noted that this appendix does not discuss the cathode ray tube display integration, or the integration properties of the eye and brain of the radar operator, but is limited to the discussion of electronic device integrators only.

Integrators are generally used in the primary radars for two reasons:

1. To enhance weak desired targets for PPI display.

2. To suppress asynchronous pulsed interference.

The principle of the radar video integrator is that radar signal returns from a point target consist of a series of pulses generated as the radar antenna beam scans past the target, all of which fall in the same range bin in successive periods (synchronous with the system). It is this series of synchronous pulses from a target which permits integration of target returns to enhance the weak signals. The number of pulse returns (N) from a target depends upon the radar antenna beamwidth (BW), the rate of antenna rotation (RPM), the radar pulse repetition rate (PRF) and the target characteristics. The equation for the number of pulses from a point target is given by (Skolnik, 1962):

$$N = \frac{PRF \cdot BW}{6 \cdot RPM} \qquad (D-1)$$

Where:

PRF = Radar pulse repetition frequency, in PPS

BW = Antenna 3 dB beamwidth, in degrees

RPM = Antenna scan rate, in rpm

A range value of N for radars in the 2.7 to 2.9 GHz band is 12 to 20. The integrator will suppress asynchronous interference since the interfering pulses will not be separated in time by the radar period, and thus will not occur in the same range bin in successive periods (asynchronous with the system). Therefore, the asynchronous interference will not add-up, and can be suppressed.

All radars in the 2.7 to 2.9 GHz band employ post detection or noncoherent integrators. The types of post detection integrator employed in radars in the 2.7 to 2.9 GHz band can be categorized either as a feedback integrator or a binary integrator. Radars employing feedback integrators may be of analog (delay line) or digital (shift register) type. Only digital binary integrators are used.

The following is a discussion of the transfer properties of the feedback and binary integrators. The noise, desired signal and asynchronous interfering signal transfer properties are discussed along with the tradeoffs of the desired signal transfer properties in suppressing asynchronous interference. The transfer properties were investigated by measurements, analytically and by simulating the noise, desired signal, interfering signals, and the feedback and binary integrator hardware. Appendix E contains a detailed discussion of the methods used to simulate the noise, desired signal, interfering signal, and the actual radar hardware simulated.

FEEDBACK INTEGRATOR

Figure D-1 shows a block diagram of a typical feedback integrator. The radar period delay (1/PRF) for digital radars is achieved by clocking a shift register, and the actual integrator hardware is essentially represented in Figure D-1. Analog radars in the 2.7 to 2.9 GHz band generally use quartz delay lines, thus accomplishing the delay acoustically to reduce the size of the delay line. However, the inherent loss of quartz delay lines requires use of additional hardware, such as, modulators, attenuators, amplifiers, detectors, and balancing circuits (AGC) to achieve integration. If the analog integrator balancing circuitry is aligned properly, the transfer properties of an analog integrator can, for analytical simplicity, be modeled by the operations shown in Figure D-1. Digital integrators will also introduce some roundoff or truncation error. However, the error due to roundoff and truncation is generally very small and can be neglected in most cases.

The feedback integrator depicted in Figure D-1 consists of an input limiter, an adder, and a feedback loop with an output limiter and a delay equal to the time between transmitter pulses (1/PRF). The following is a discussion of the transfer properties of these feedback integrator elements to noise, desired signal and asynchronous interference.

Input Limiter Transfer Properties

The integrator input limiter serves as a video clipping circuit to provide constant level input pulses to the feedback integrator, and is a necessary integrator circuit element to suppress asynchronous interference. The input limiter limit level is always adjustable, and controls the transfer properties of the feedback integrator. The effect of the limiter limit level setting on the desired signal, interfering signal, and noise transfer properties of the integrator are discussed in detail in latter sections of this appendix. The desired signal-to-noise (SNR) transfer properties of the

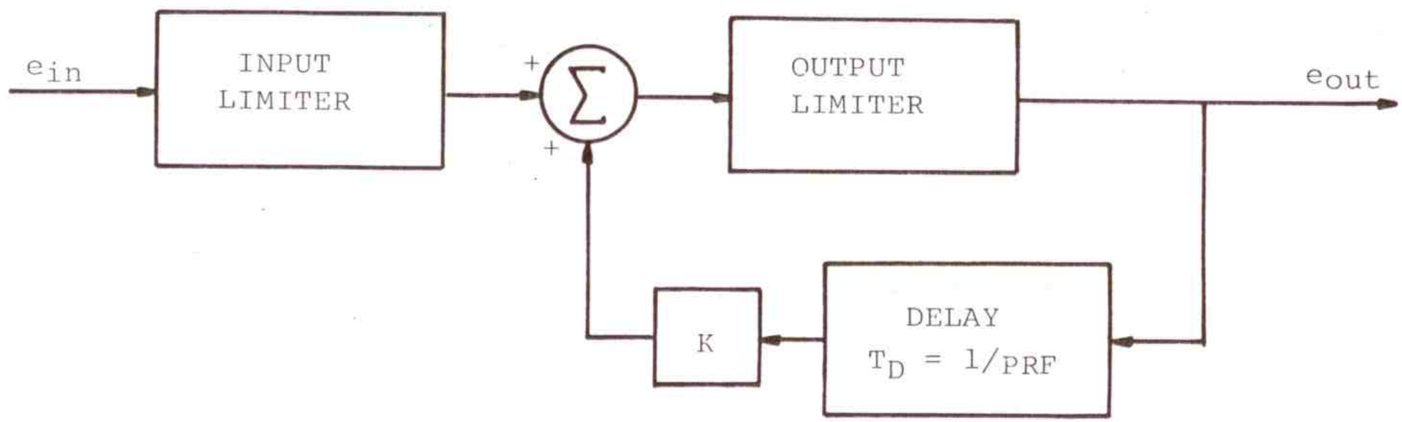

Figure D-1. Feedback Integrator Block Diagram limiter are given by:

$$SNR_o = \begin{cases} SNR_i ;\text{for } SNR_i < LNR \\ LNR ;\text{for } SNR_i \geq LNR \end{cases} \quad \text{(D-2)}$$

Where:

SNR_o = Limiter output peak signal-to-rms noise ratio, in dB

SNR_i = Limiter input peak signal-to-rms noise ratio, in dB

LNR = Limiter limit-to-rms noise ratio, in dB

The interference-to-noise (INR) transfer properties of the input limiter can also be expressed by Equation D-2. Also Equation D-2 represents the transfer properties of the output limiter. Generally the output limiter limit level is set at 4.0 volts.

Feedback Integrator Loop Transfer Properties

The feedback integrator loop consists of an adder, an output limiter, a feedback loop with a delay equal to the time between transmitter pulses (1/PRF) and a loop gain of K. The overall gain, K, of the feedback loop is less than unity to prevent instability.

The signal transfer properties of the integrator feedback loop to noise, desired signal, and asynchronous interference will be treated separately. This is possible since the integrator feedback loop is a linear device. First consider noise.

Noise

The noise voltage amplitude distribution at the feedback integrator input is either Rayleigh distributed (normal channel or inphase and quadrature MTI channel) or a one-sided Gaussian distributed (single channel MTI). Both the Rayleigh and one-sided Gaussian distributions have a non-zero mean. To minimize the noise gain through the feedback integrator, prior to the feedback integrator circuitry shown in Figure D-1, the radar receivers generally have an attenuator, subtractor and bottom clipper. The noise transfer properties of a non-zero mean distribution can be expressed as:

$$n_{Io} = \left[\sum_{j=o}^{\infty} K^{2j} \sigma^2_{nIi} + \left(\sum_{j=o}^{\infty} K^j \mu_{nIi} \right)^2 \right]^{1/2} \quad \text{(D-3)}$$

Where:

$\sigma_{n_{Ii}}$ = The input noise amplitude distribution rms level, in volts

$\mu_{n_{Ii}}$ = The input noise amplitude distribution mean level, in volts

K = Integrator feedback factor.

The sum series in the above equation are a geometric series, and the equation can be rewritten as:

$$n_{Io} = \left[\sigma^2_{n_{Ii}} \left(\frac{1}{1 - K^2} \right) + \mu^2_{n_{Ii}} \left(\frac{1}{1 - K} \right)^2 \right]^{\frac{1}{2}} \qquad \text{(D-4a)}$$

The affect of the attenuator, subtractor, and bottom clipper is to reduce the mean level of the noise amplitude distribution. Alignment procedures of this circuitry generally result in the rms noise level being much greater than the mean noise level. In this case, the second term of Equation D-4a can be neglected for analytical simplicity, and the noise treated as an uncorrelated zero mean amplitude distribution. Making this assumption, Equation D-4a becomes:

$$n_{Io} = \sigma_{n_{Ii}} \left[\frac{1}{1-K^2} \right]^{\frac{1}{2}} \qquad \text{(D-4b)}$$

Therefore, the noise power transfer function of a feedback integrator loop for the radar normal channel is:

$$N_{Io} = N_{Ii} - 10 \log_{10} (1-K^2) \qquad \text{(D-5)}$$

In digital radars such as the ASR-8, the analog-to-digital (A/D) converter causes quantization errors due to truncation. The truncation at low signal levels (less than one volt) has an effect of causing a non-linear feedback constant, K. For low signal levels the feedback signal can vary between K and approximately 1/2. This has an overall affect of reducing the feedback integrator noise gain given by Equation D-4b.

Since the feedback integrator sums or convolves the noise distribution at the input of the feedback loop with itself continuously, the central limit theorem applies which states that the noise distribution at the integrator feedback loop output will be Gaussian distributed even though the input noise distribution was Rayleigh (normal or dual MTI channel).

The noise out of the MTI channel is correlated from range/azimuth cell to range/azimuth cell. Figure D-2 shows how the noise becomes correlated by the transfer properties of a second-order MTI filter. The figure shows the input noise samples (N_1 through N_5) for a specific instant in time (T_1 through T_5). The noise level at the canceller output as a function of time (T_1 through T_5) is also shown. The figure shows that at time T_5, the noise output consists of noise samples N_3, N_4, and N_5. At time T_4, the noise output consists of noise samples N_2, N_3, and N_4. Since noise samples N_3 and N_4 contribute to the noise level at time T_4 and T_5, the noise at the MTI canceller output is correlated. The increase in noise amplitude caused by the integration of the correlated MTI channel noise, is a function of the MTI canceller Type (single stage canceller, double stage canceller, etc.). The MTI channel noise level increase was simulated by Trunk, 1977, and found to be approximately 1 dB for a single stage canceller and 1.8 dB for a double stage canceller. Therefore, the feedback integrator noise transfer properties for the MTI channel can be obtained using Equation D-5 and adding 1 dB for a single stage MTI canceller and 1.8 dB for a double stage MTI canceller.

Desired Signal

The desired signal phases at the input of the feedback loop integrator are coherent and add directly since the feedback loop delay is equal to the desired signal pulse period. For the feedback loop shown in Figure D-1, the general output response (Y_N) at the Nth input pulse (X_N) is given by:

$$Y_N = X_N + Y_{N-1} \qquad \text{(D-6)}$$

Equation D-6 is a first order constant coefficient linear difference equation with a solution

$$Y_N = \left(\frac{1-K^N}{1-K}\right) X_N \qquad \text{(D-7)}$$

Recognizing the transfer function in Equation D-7 as the geometric progression for a series, Equation D-7 becomes:

$$Y_N = [1+K+K^2+\ldots+K^{N-1}] \cdot X_N \qquad \text{(D-8a)}$$

$$Y_N = \sum_{i=1}^{N} K^{(i-1)} \cdot X_N \qquad \text{(D-8b)}$$

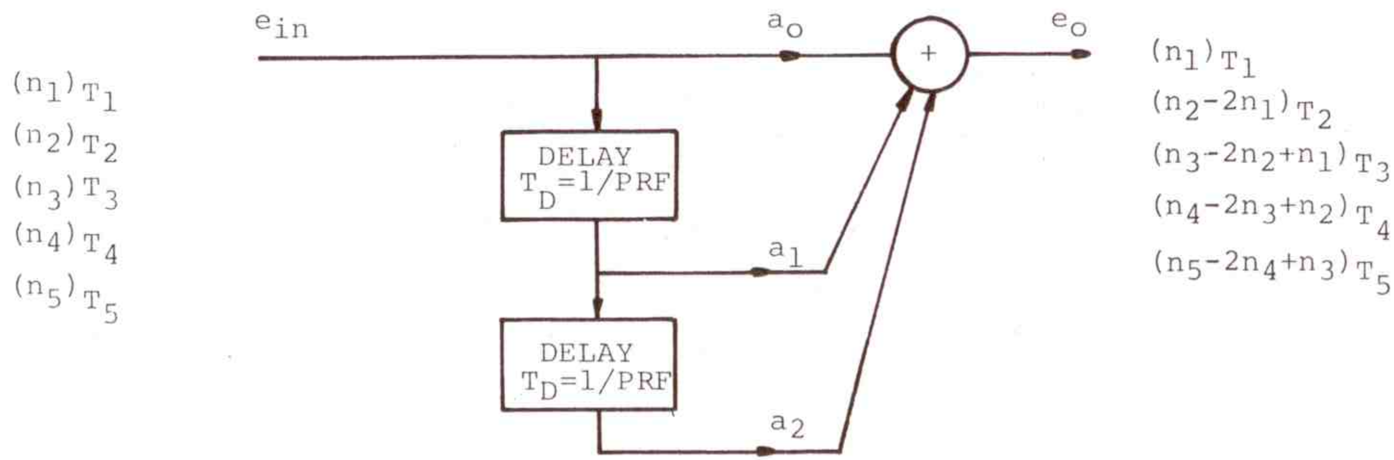

Figure D-2. Canonical Form of Second-Order MTI Canceller Filter Showing Noise Correlation at MTI Channel Output

Thus, for N equal amplitude voltage input pulses (S_{Ii}), the Nth pulse amplitude at the integrator output (S_{Io}) can be expressed as:

$$S_{Io} = S_{Ii} \sum_{i=1}^{N} K^{(i-1)} \qquad \text{(D-9)}$$

The series exprpssed in Equation D-8a can be intuitively obtained using Figure D-1 (feedback loop), and recursively applying a series of N equal voltage amplitude pulses to the input.

The signal-to-noise ratio enhancement (SNR_E) factor for the integrator feedback loop for the normal channel can be obtained from Equations D-4b and D-7, and is given by:

$$SNR_E = 20 \log_{10} \left[\left(\frac{1-K^N}{1-K} \right) \quad (1-K^2)^{1/2} \right] \qquad \text{(D-10)}$$

It should be noted that Equation D-10 is only a first order approximation of the actual signal-to-noise ratio enhancement of a feedback integrator since it assumes a zero mean noise amplitude distribution, and a constant desired signal level. For the MTI channel the signal-to-noise ratio enhancement given by Equation D-10 should be reduced by 1.0 dB for a single stage canceller and 1.8 dB for a double stage MTI canceller due to the correlation of noise by the MTI cancellers.

Equation D-10 can be used to approximate the optimum feedback factor K for the radar design by maximizing the signal-to-noise ratio enhancement (SNR_E). TABLE D-1 shows the SNR_E as a function of the integrator feedback factor (K) for a range of number of pulses integrated (N) between 12 and 20, which is a typical range of N for radionavigation radars in the 2.7 to 2.9 GHz band. The asterisks in each column of the table indicate the maximum SNR_E and optimum integrator feedback factor (K) for that particular value of N. For the range of number of pulses integrated (N) between 12 and 20, the optimum feedback factor (K) ranges from .90 to .94. The table also shows that if an integrator feedback factor of .92 is used for radars with a range of N between 12 and 20, the loss in actual SNR_E and maximum SNR_E is less than .1 dB. Therefore, the design of the integrator feedback factor (K) for a range of number of pulses integrated is not that critical.

Actually, the optimum value of K is difficult to obtain due to fluctuations in target returns (scintillation) and antenna pattern amplitude modulation, and thus requires a complicated Monte Carlo model to determine. Skolnik (1970) gives an empirical value of K as:

$$K = 1 - \frac{1.56}{N} \qquad \text{(D-11)}$$

TABLE D-1

FEEDBACK INTEGRATOR PEAK SIGNAL-TO-NOISE ENHANCEMENT
FOR NORMAL CHANNEL **

Feedback Factor K	Peak Signal-to-Noise Enhancement (SNR_E)				
	N=12	N=14	N-16	N=18	N=20
.70	7.41	7.47	7.50	7.52	7.53
.71	7.56	7.63	7.67	7.69	7.70
.72	7.71	7.80	7.84	7.86	7.87
.73	7.87	7.96	8.01	8.04	8.05
.74	8.02	8.13	8.19	8.22	8.23
.75	8.17	8.29	8.36	8.40	8.42
.76	8.32	8.46	8.54	8.59	8.62
.77	8.48	8.64	8.73	8.78	8.82
.78	8.63	8.81	8.92	8.98	9.02
.79	8.78	8.98	9.10	9.18	9.23
.80	8.92	9.15	9.29	9.38	9.44
.81	9.07	9.32	9.49	9.59	9.66
.82	9.21	9.49	9.68	9.80	9.88
.83	9.34	9.66	9.87	10.01	10.11
.84	9.46	9.82	10.06	10.22	10.34
.85	9.58	9.97	10.24	10.43	10.57
.86	9.68	10.11	10.42	10.64	10.80
.87	9.77	10.25	10.59	10.84	11.03
.88	9.84	10.36	10.75	11.03	11.25
.89	9.89	10.46	10.89	11.21	11.46
.90	9.90*	10.53	11.01	11.37	11.66
.91	9.89	10.57*	11.10	11.51	11.84
.92	9.82	10.56	11.15*	11.61	11.99
.93	9.69	10.50	11.14	11.66*	12.09
.94	9.48	10.36	11.06	11.64	12.12*
.95	9.16	10.10	10.87	11.51	12.05
.96	8.66	9.68	10.52	11.23	11.83
.97	7.89	8.98	9.90	10.68	11.36
.98	6.62	7.79	8.78	9.64	10.39
.99	4.10	5.35	6.43	7.36	8.19

*Maximum signal-to-noise ratio enhancement

**For MTI channel signal-to-noise ratio enhancement reduce the values in the table by 1.0 dB for a single stage canceler and 1.8 dB for a double stage canceler.

For N = 20, the value for K from Equation D-11 is .92, which is close to the value of K obtained using Equation D-10 (see TABLE D-1).

Studies (Trunk, 1970) have been made which take into account pulse amplitude variations due to the antenna beam shape. Trunk concluded that the SNR_E could be reduced by as much as 1.6 dB due to the antenna beam shape pulse amplitude variations. The actual reduction in SNR_E due to the radar antenna pattern is between 0 and 1.6 dB, and is also a function of the integrator input limiter limit level setting.

In congested areas where there is potential for asynchronous interference, adjustments to the integrator input limit level setting are required to suppress the asynchronous interference. However, adjustments of the limiter level setting affect the desired signal pulse train processing characteristics of the feedback integrator. Desired signal pulse train characteristics which are effected include: target azimuth shift, angular resolution and probability of detection. The following is a discussion of the desired signal transfer properties of a feedback integrator loop as a function of the input signal-to-noise ratio (SNR) and input limiter level setting.

Figures D-3 through D-6 show a simulated radar performance of a feedback enhancer for the normal channel as a function of the signal-to-noise ratio (SNR). The enhancer input limiter limit level (V_L) was set at 2.0 volts, and the feedback factor (K) was set at 0.94. The desired target return pulse train consists of 20 pulses. Each figure shows the simulated radar output with the enhancer off (unintegrated) and enhancer on (integrated). Figures D-3 (SNR = 3 dB) and D-4 (SNR = 5 dB) show that with the enhancer off the desired signal is down in the noise. However, when the enhancer is on the signal is pulled out of the noise by the feedback enhancer. Figures D-7 through D-10 show the affect on the desired signal of adjusting the input limiter limit level (V_L) of the feedback enhancer. The SNR for Figures D-7 through D-10 was 15 dB. For comparison with the simulated integrator model (Figures D-7 through D-10), Figures D-11 through D-13 show actual measured ASR-8 normal channel integrated output for input limiter switch settings of 15, 9, and 7, respectively. The receiver input desired signal level was set at -84 dBm.

The target azimuth shift (ω_s) due to integration can be calculated by:

$$\omega_s = \Delta C_p \cdot \phi_s \qquad \text{(D-12)}$$

Where:

ΔC_p = Shift of the center pulse position of an integrated video pulse train relative to an unintegrated pulse train (Figure D-6a).

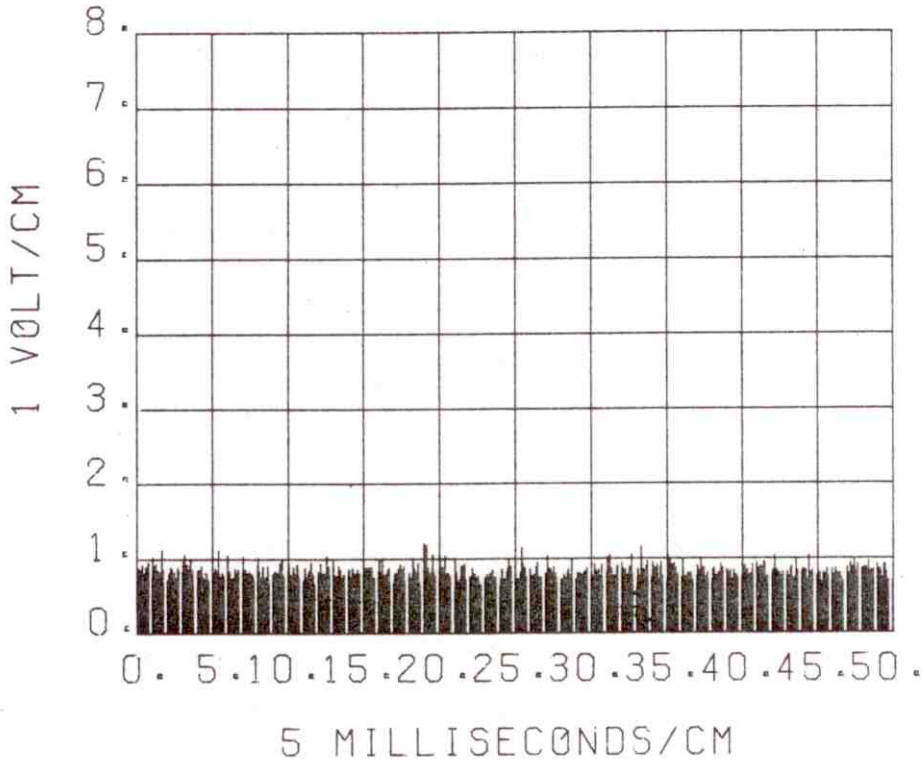

Figure D-3a. Simulated Normal Channel Unintegrated Target Return Pulse Train for a SNR = 3dB

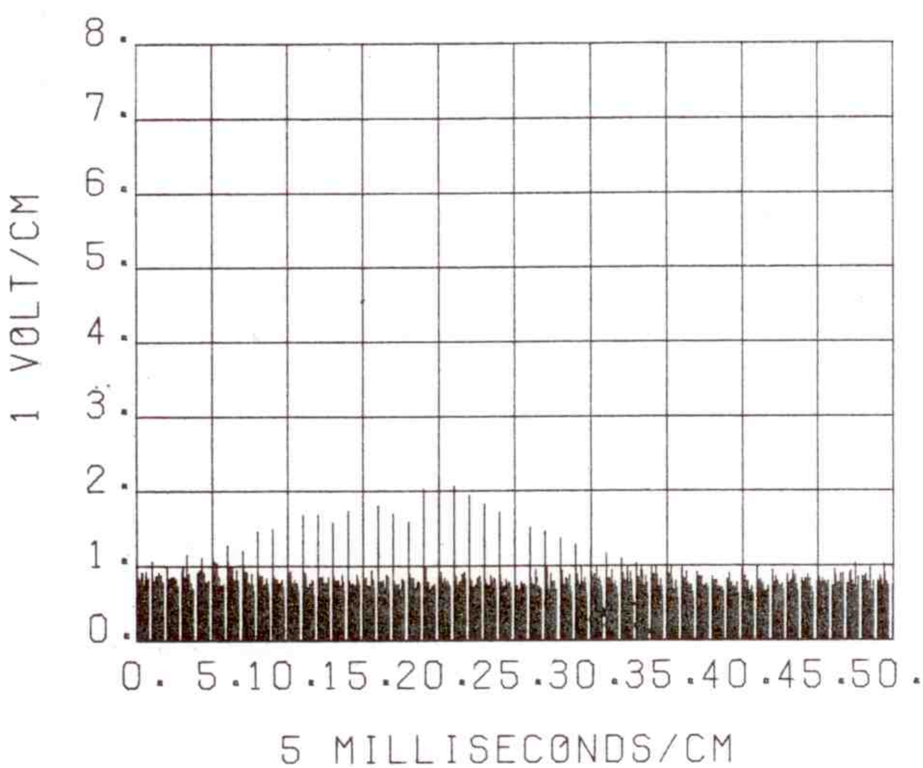

Figure D-3b. Simulated Normal Channel Integrated Target Return Pulse Train for a SNR = 3dB (V_L = 2.0)

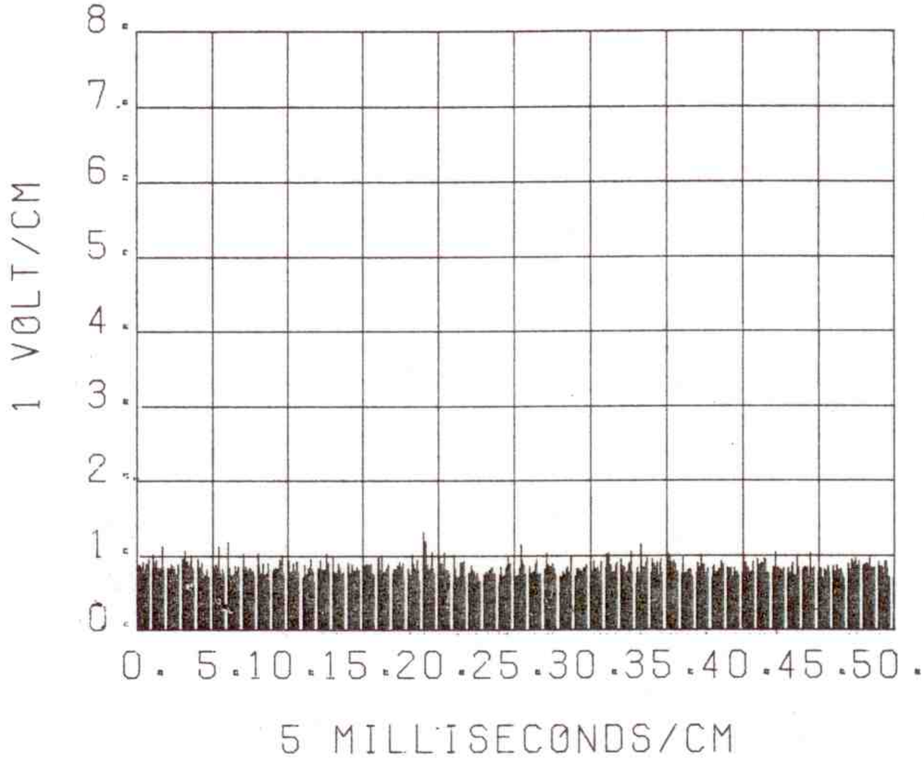

Figure D-4a. Simulated Normal Channel Unintegrated Target Return Pulse Train for a SNR = 5dB

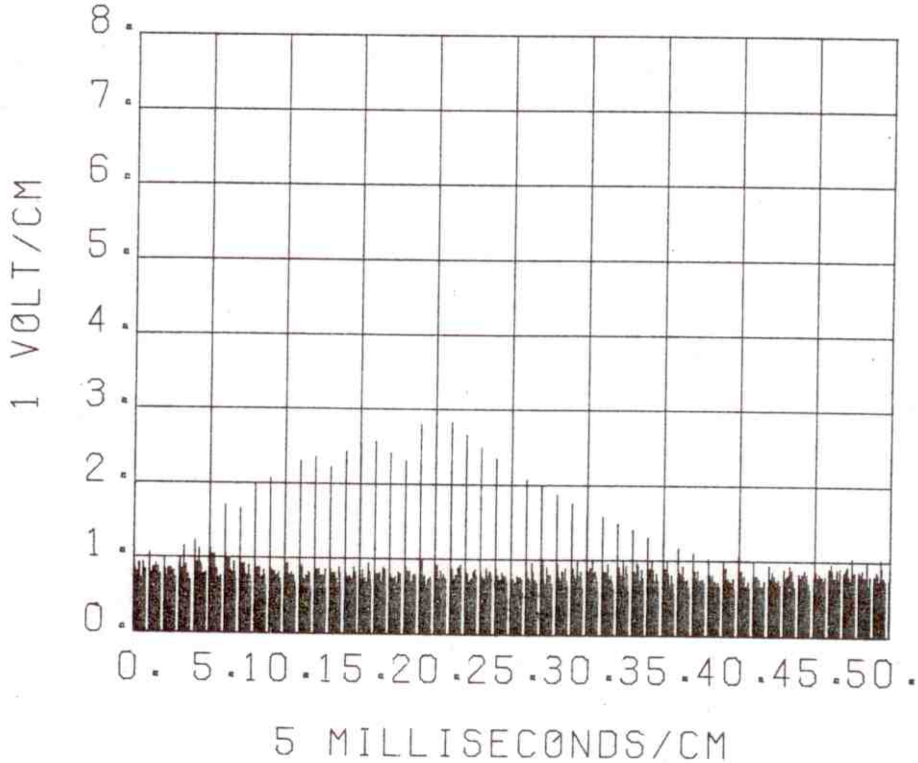

Figure D-4b. Simulated Normal Channel Integrated Target Return Pulse Train for a SNR = 5dB (V_L = 2.0)

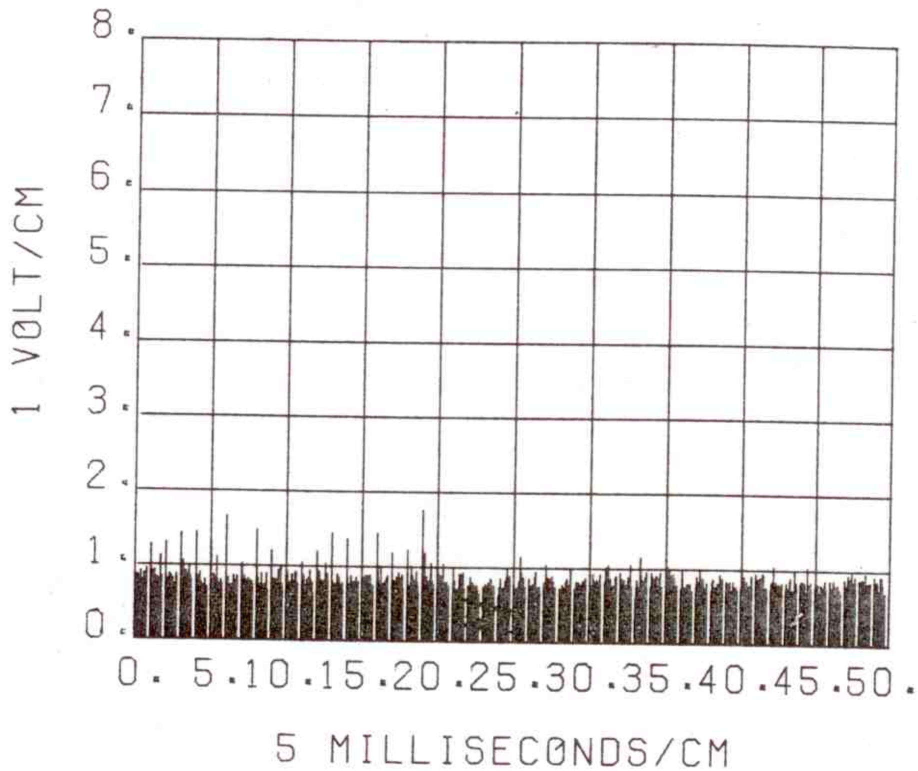

Figure D-5a. Simulated Normal Channel Unintegrated Target Return Pulse Train for a SNR = 10dB

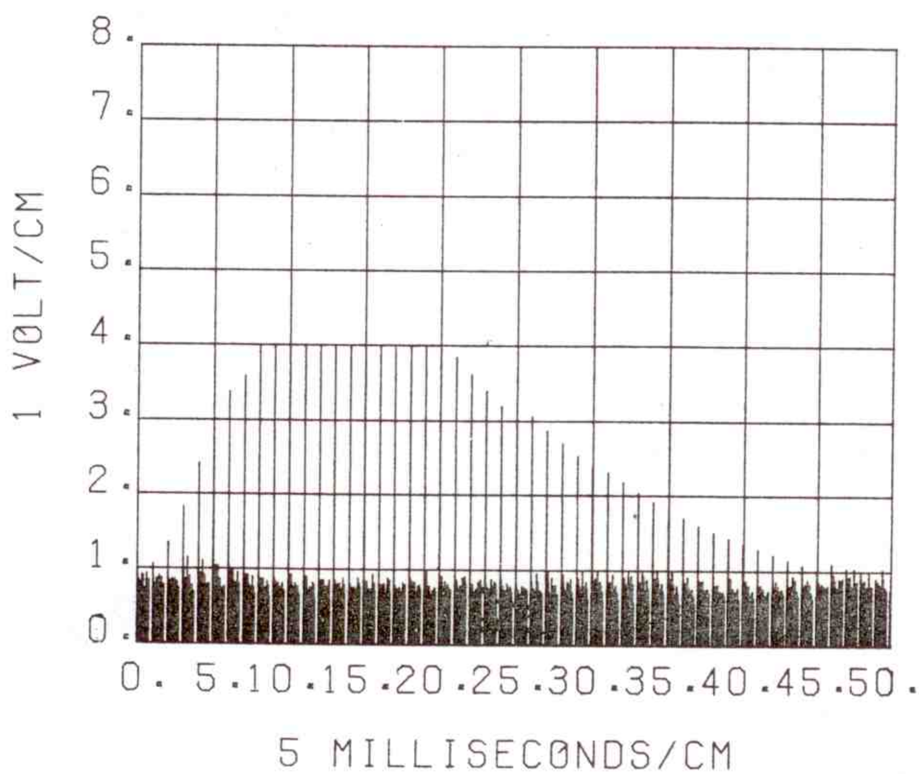

Figure D-5b. Simulated Normal Channel Integrated Target Return Pulse Train for a SNR = 10dB (V_L = 2.0)

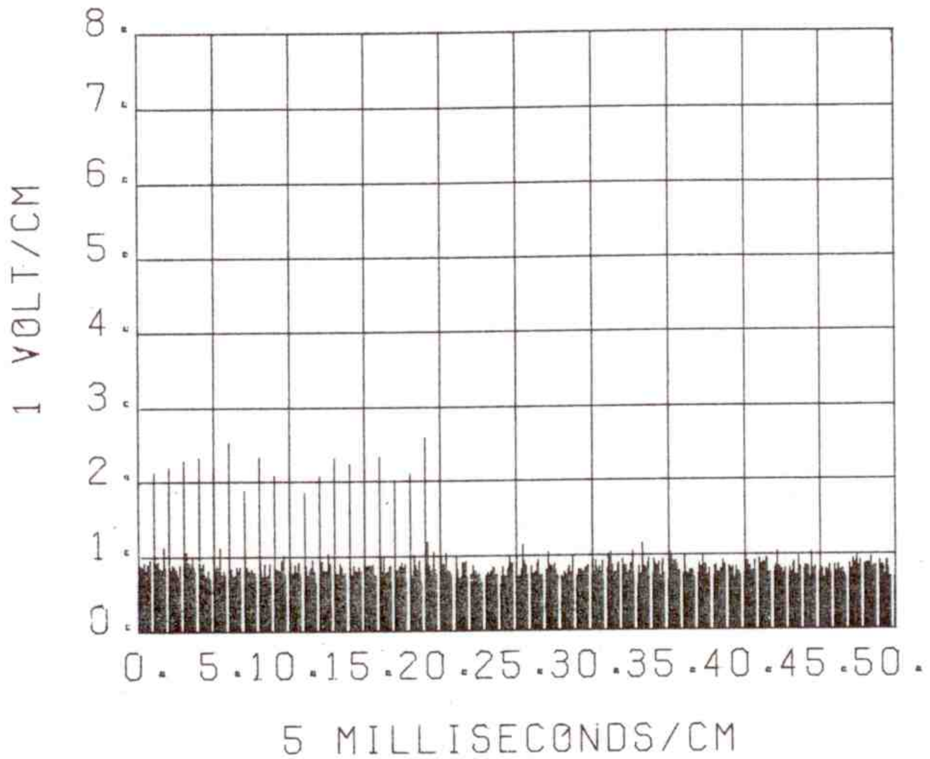

Figure D-6a. Simulated Normal Channel Unintegrated Target Return Pulse Train for a SNR = 15dB

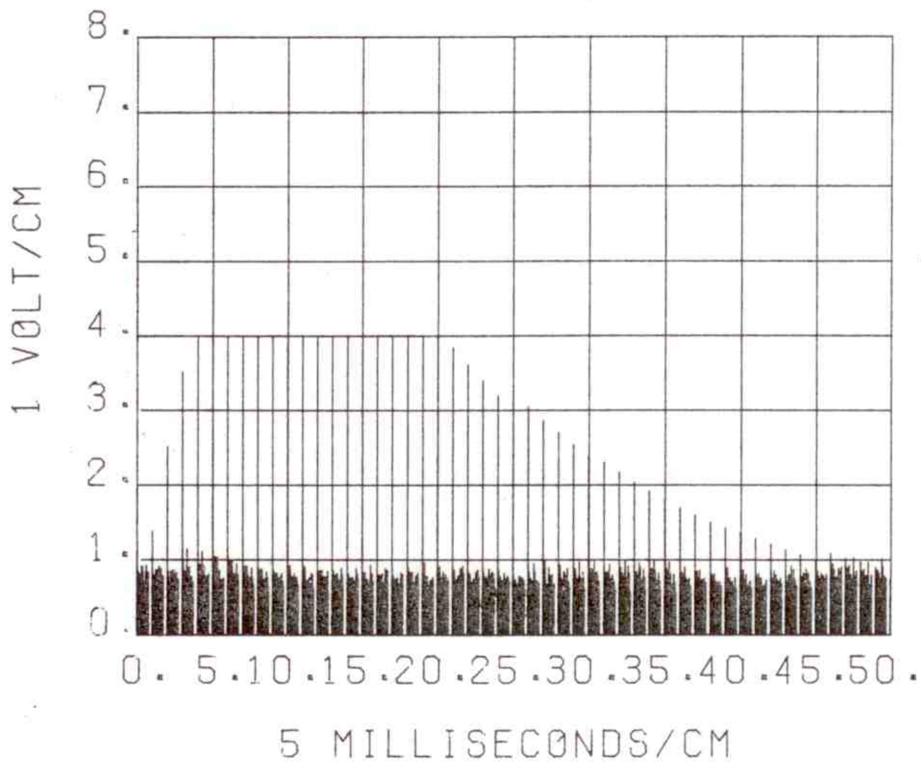

Figure D-6b. Simulated Normal Channel Integrated Target Return Pulse Train for a SNR = 15dB (V_L = 2.0)

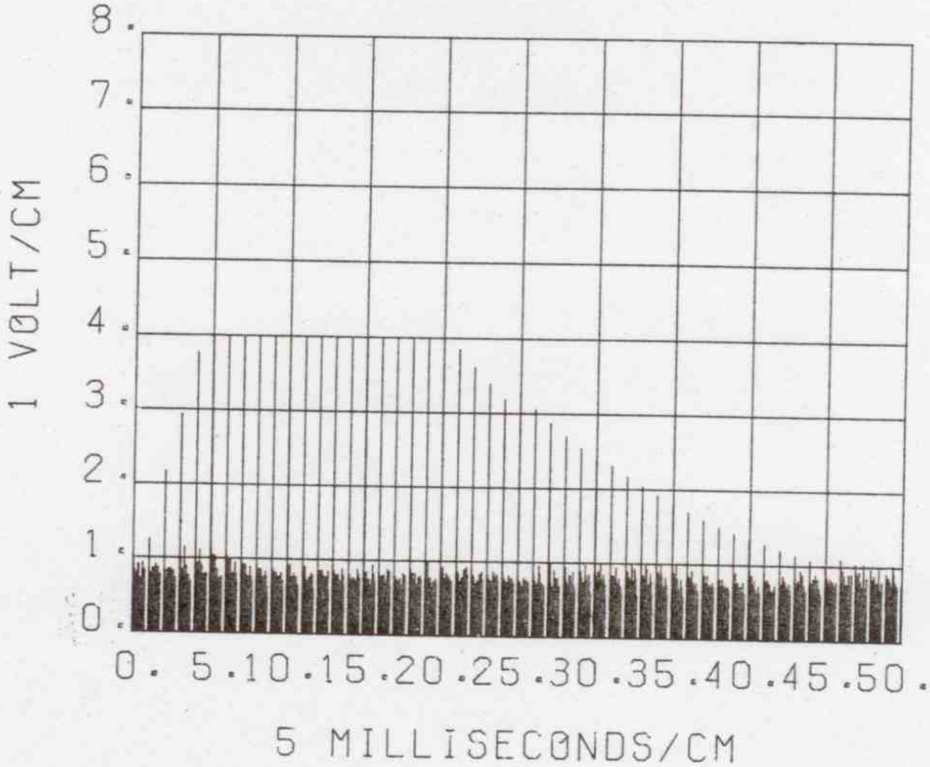

Figure D-7. Simulated Normal Channel Integrated Target Return Pulse Train for a SNR = 15 (V_L = 1.0)

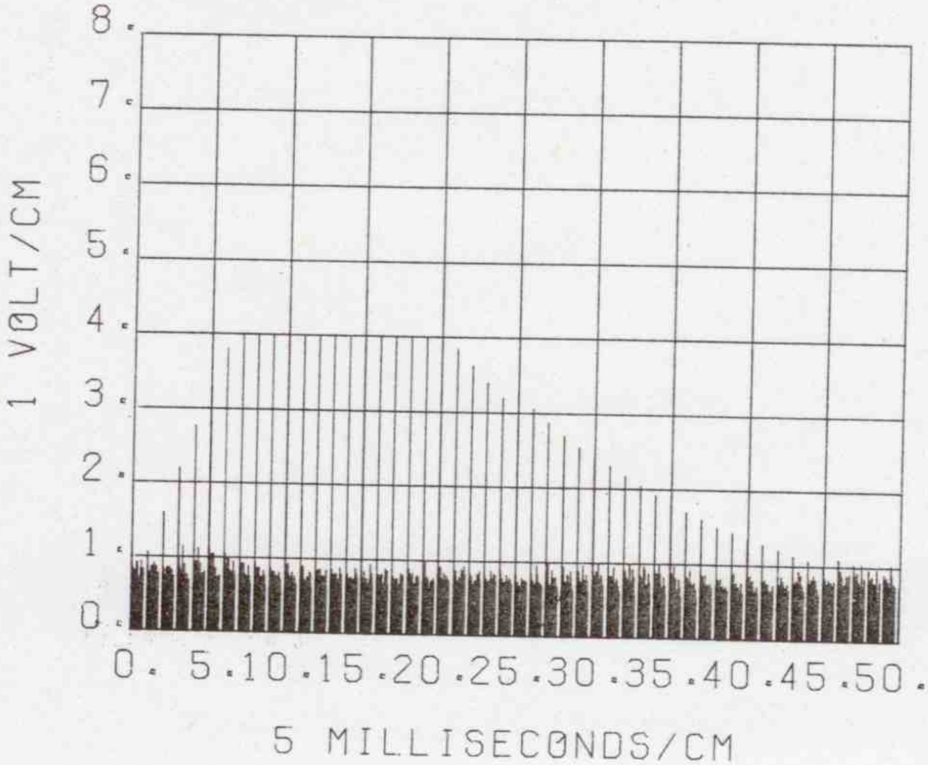

Figure D-8. Simulated Normal Channel Integrated Target Return Train for a SNR = 15 (V_L = 0.7)

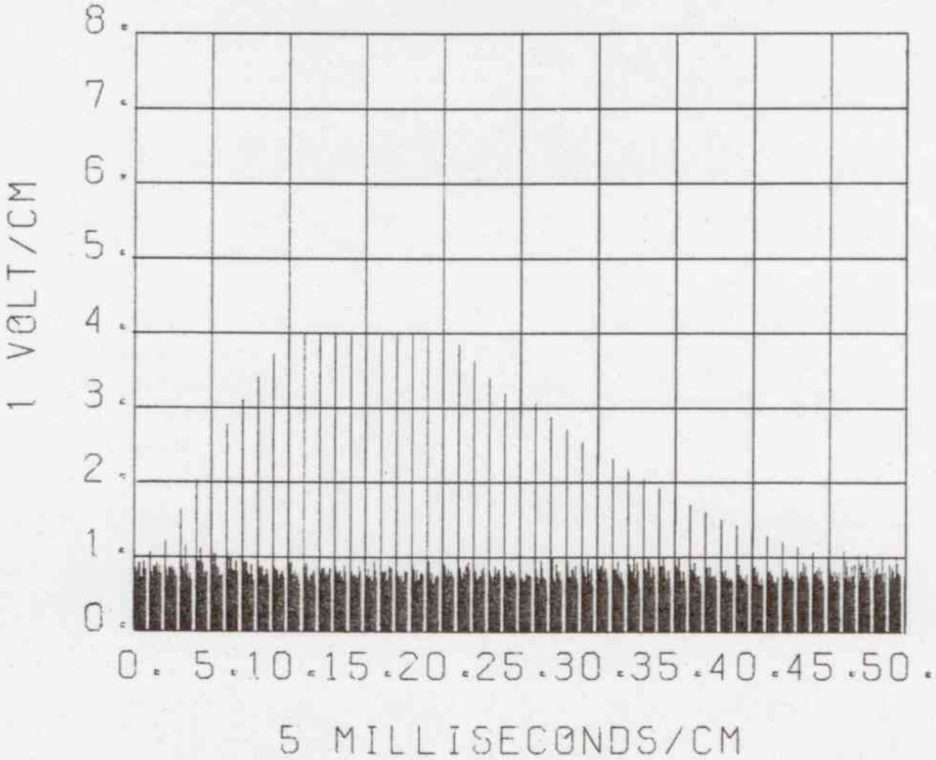

Figure D-9. Simulated Normal Channel Integrated Target Return Pulse Train for a SNR = 15 (V_L = 0.5)

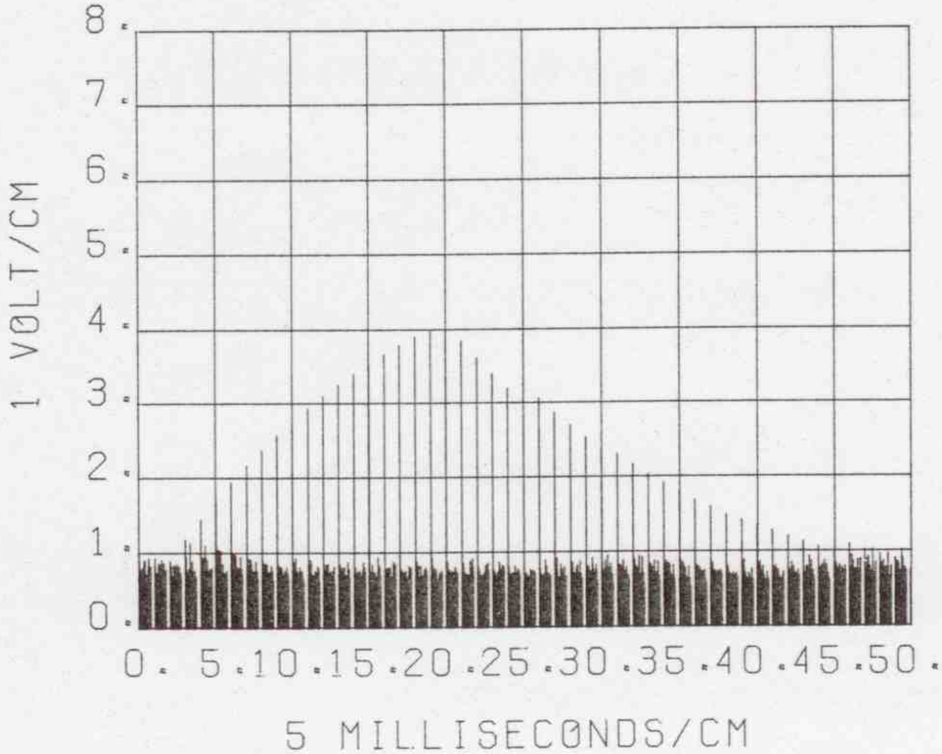

Figure D-10. Simulated Normal Channel Integrated Target Return Pulse Train for a SNR = 15 (V_L = 0.34)

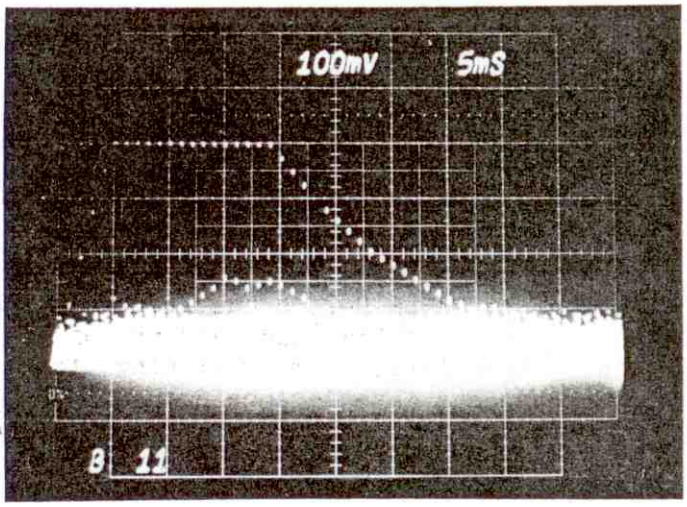

Figure D-11. Measured ASR-8 Normal Channel Integrated Output

Limit Adjust = 15
S = -84 dBm

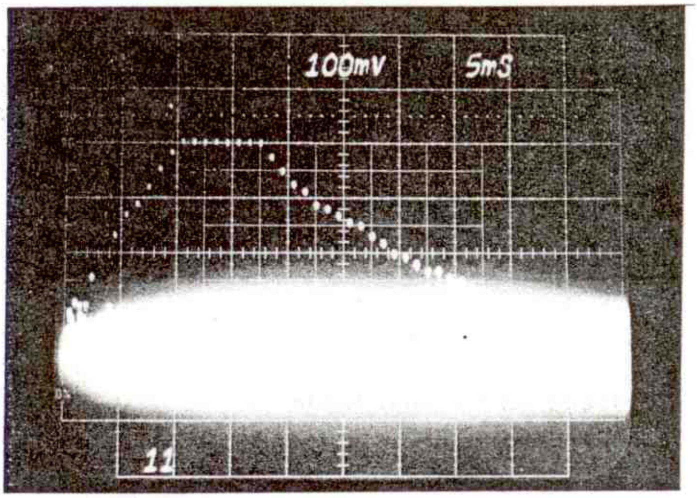

Figure D-12. Measured ASR-8 Normal Channel Integrated Output

Limit Adjust = 9
S = -84 dBm

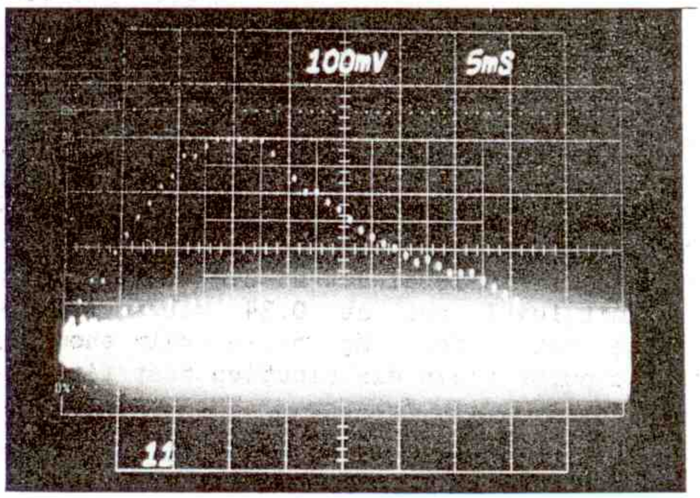

Figure D-13. Measured ASR-8 Normal Channel Integrated Output

Limit Adjust = 7
S = -84 dBm

ϕ_s = Degrees of antenna rotation per pulse

$$\phi_s = \frac{6 \cdot RPM}{PRF}$$

For an ASR-8 with an antenna RPM of 12.5 and PRF equal to 1000, ϕs equals 0.075 degrees per pulse. TABLE D-2 shows the target azimuth shift (Equation D-12) due to target integration for various SNRs and feedback integrator input limiter limit level settings. The table shows a target azimuth shift due to integration of approximately .900 degrees relative to an unintegrated target return pulse train. Also shown in TABLE D-2 is the fact that the target azimuth shift due to integration is essentially independent of the integrator input limit level setting. Therefore, hard limiting in the integrator input limiter to suppress asynchronous interference does not cause a significant increase in target azimuth shift. However, the feedback integrator causes a target azimuth shift of .900 degrees over an unintegrated target return pulse train.

The property of the radar to distinguish between targets is called resolution. A comparison of Figure D-6a (unintegrated target return pulse train) with Figures D-6b through D-10 (integrated target return pulse train) shows a decrease in target angular resolution when the feedback enhancer is used. The decrease in target resolution is determined by:

$$\Delta RES = \Delta P \cdot \phi_S \qquad (D-13)$$

Where:

ΔP = Difference in number of pulses above the noise level at the integrator input and output.

Figures D-6b through D-10 show a decrease in target angular resolution due to use of the feedback integrator of 1.2 to 1.5 degrees. For aircraft at a range of 60 nautical miles, this would result in a decrease in angular resolution of 1.5 nautical miles.

In order to determine the effect of the integrator input limit level settings on the probability of target detection, the integrator output pulse train distribution must be best fitted to a Chi-square distribution by determining the degrees of freedom of the Chi-square distribution which best fit the integrator output pulse train. The probability of target detection was determined for the integrator input limit level set at 2.0 volts (Figure D-6b), and for the integrator input limit level set at 0.34 volts (Figure D-10). The mean peak signal-to-noise ratio for the pulse train shown in Figure D-6b was 20.35 dB, and the pulse train distribution best fitted a Chi-square distribution with four degrees of freedom. The mean peak signal-to-noise ratio for the pulse train shown in Figure D-10 was 18.48 dB, and the pulse train distribution best fitted a Chi-square distribution with

TABLE D-2

TARGET AZIMUTH SHIFT CAUSED BY FEEDBACK INTEGRATION

Figure Reference	Signal-to-Noise Ratio (dB)	Input Limiter Limit (Volts)	Center Pulse Position Relative to Unintegrated Pulse (ΔCp)	Target Azimuth Shift (Degrees)
Figure D-3b	3	2.0	8	0.600
Figure D-4b	5	2.0	10	0.750
Figure D-5b	10	2.0	12	0.900
Figure D-6b	15	2.0	13	0.975
Figure D-7	15	1.00	13	0.975
Figure D-8	15	0.7	13	0.975
Figure D-9	15	.50	12	0.900
Figure D-10	15	.34	12	0.900

two degrees of freedom. For a Swerling Case III target and probability of false alarm of 10^{-6}, the probability of target detection for the integrator output pulse trains shown in Figures D-6b and D-10 was determined using probability of detection curves given by Meyer and Mayher (1973) and making appropriate adjustments for the different degrees of freedom. The probability of target detection was determined to be .9998 and .9995 for the integrator output pulse trains shown in Figures D-6b and D-10, respectively. The high probability of detection for the two target return pulse trains shown in Figures D-6b and D-10 is due to the high mean signal-to-noise ratio of the pulse trains, and the number of pulses at the integrator output (approximately 35 pulses above 1.25 volts). Therefore, the probability of target detection does not change significantly by varying the integrator input limit level from 2.0 volts to .34 volts.

Also, measurements were made on the ASR-8 radar using observers to determine the PPI Minimum Discernible Signal (MDS) level as a function of integrator input limit level settings. The measurements showed the desired target PPI MDS level was independent of the integrator (enhancer) limit level adjust switch settings greater than five. This was due to the fact that the integrated (enhanced) target PPI MDS level was -104 dBm (peak signal-to-noise approximately 5.0 dB) which produced a SNR approximately equal to the integrator input limit level setting five. With the integrator limit level adjust switch set at five, the integrator suppressed non-synchronous interference for both the normal and MTI channel. Therefore, the PPI MDS measurements made on the ASR-8 indicate that asynchronous interference can be suppressed by the feedback integrator without a significant change in the radar PPI MDS level.

Interference

The following is a discussion of the feedback integrator signal processing properties to asynchronous normal and MTI channel interference. To analyze the signal processing properties of a feedback integrator, a careful investigation of the Pulse Repetition Frequency (PRF) relationship of the interfering and desired signal must be made. If the interfering signal PRF is different from the desired signal, the recirculated interference pulses will not arrive back at the input to the integrator coincidentally with the next pulse in the interfering train, and integration of successive pulses will not occur. However, for certain PRF's, partial integration can occur.

If the relationship in the following equation holds, partial integration will occur:

$$r\,(PRF_{INT}) = s\,(PRF_{DSR}) \qquad (D-14)$$

Where: