APPENDIX C

MTI CHANNEL TRANSFER PROPERTIES

INTRODUCTION

Radars in the 2.7 to 2.9 GHz band generally employ coherent Moving Target Indicator (MTI) phase detector. Coherent MTI radars make use of the phase fluctuations in the target return signal to recognize the doppler component produced by a moving target. In these systems, the amplitude fluctuations are removed by the phase detector. A 30 MHz signal coherent with the transmitted pulse signal is mixed with the 30 MHz target IF output response to produce a signal proportional to the phase difference between the target return and the coherent reference signal. The video signal out of the phase detector is then low pass filtered and fed to the cancellers which suppress stationary targets since their phase difference is the same from pulse-to-pulse.

Both single channel and dual channel (inphase and quadrature) MTI processing is employed by radars in the 2.7 to 2.9 GHz band. The single channel MTI processors are either analog or digital types. The analog MTI processors do not have A-D converters and use delay line cancellers, while digital MTI processors have A-D converters and use shift register cancellers. The digital shift register cancellers have a much higher stability and do not decay in time. Figure C-1 shows a block diagram of a typical single channel digital MTI processor. Figure C-2 shows the block diagram of a dual channel, inphase (I) and quadrature (Q), digital MTI processor.

This appendix discusses the noise, desired signal, and interfering signal transfer properties of the MTI phase detector, low pass filter, and cancellers. The transfer properties of analog and digital MTI cancellers can be treated identically with the exception of the quantization noise due to A-D conversion, roundoff, and truncation inherent in digital processing. For dual MTI channel processing, it is only necessary to analyze one channel since both the I and Q channels are identical with the exception of the COHO signal being shifted 90 degrees and to take into account the transfer properties of the combiner. The signal processing properties of the IF linear-limiting amplifier are discussed in Section 3 and Appendix B.

PHASE DETECTOR TRANSFER PROPERTIES

Figure C-3 shows a simplified block diagram of a coherent MTI radar phase detector. The analysis of the signal transfer properties of the MTI phase detector is very similar to the radar mixer (Appendix A). Assuming linear transfer properties for the phase detector, the noise and signal can be treated separately.

Noise

To calculate the noise power at the input and output of the phase

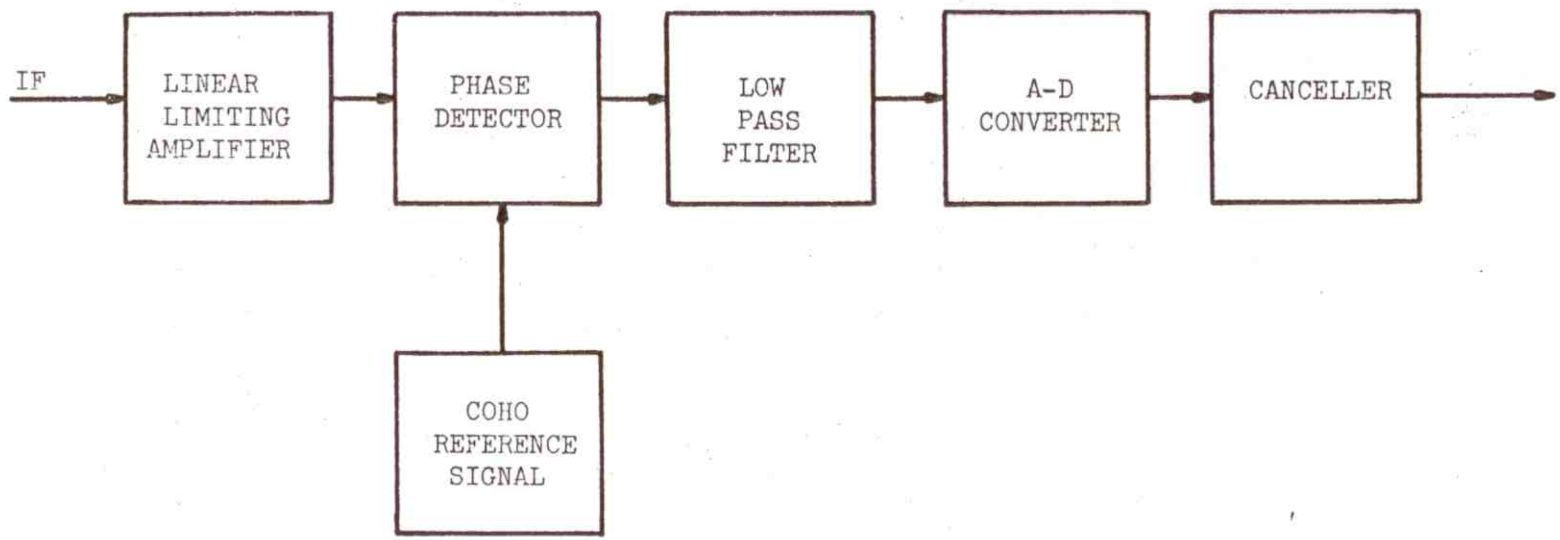


Figure C-1. Digital MTI Channel Block Diagram

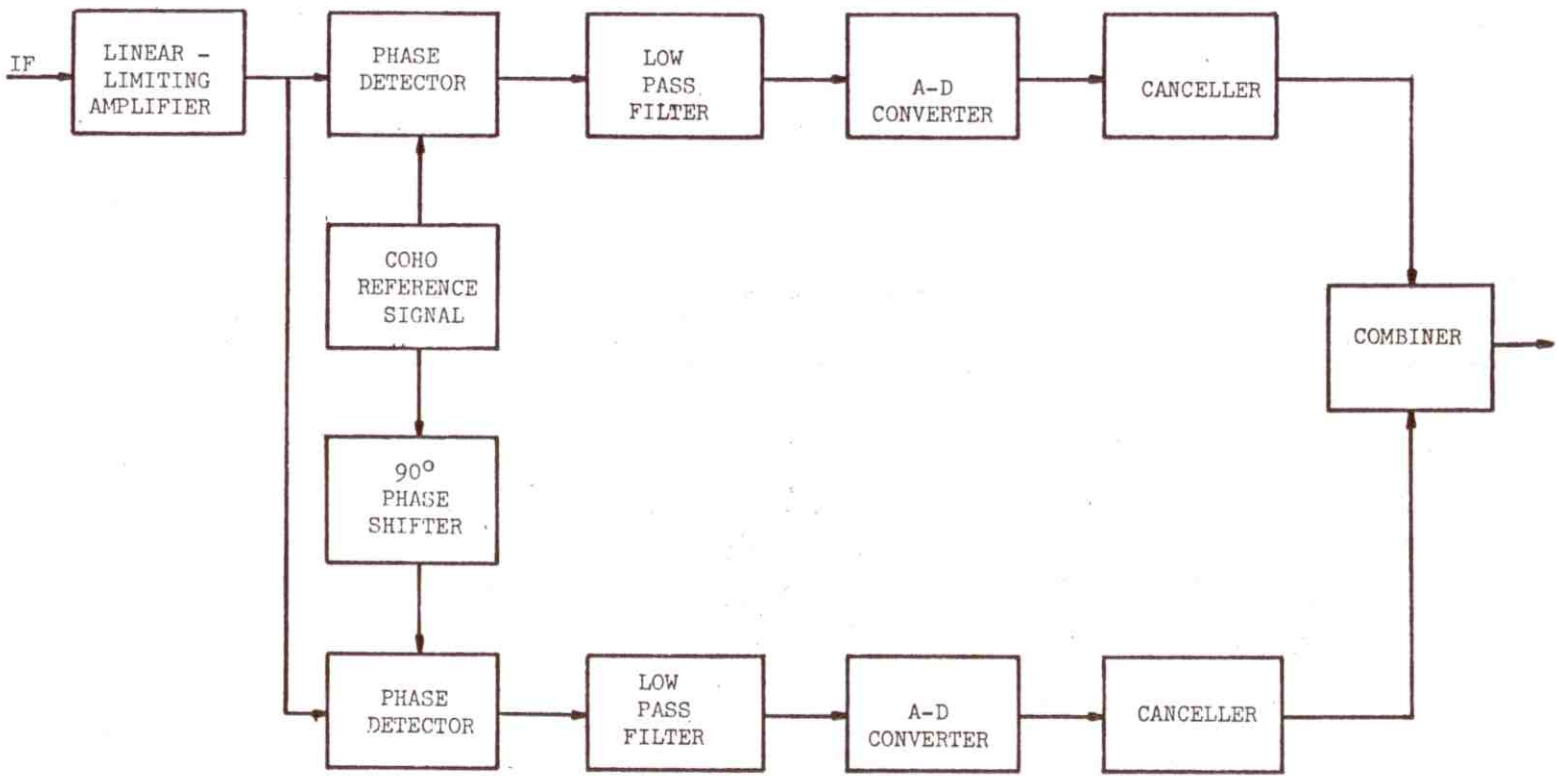


Figure C-2. Inphase and Quadrature Digital MTI Channel Block Diagram

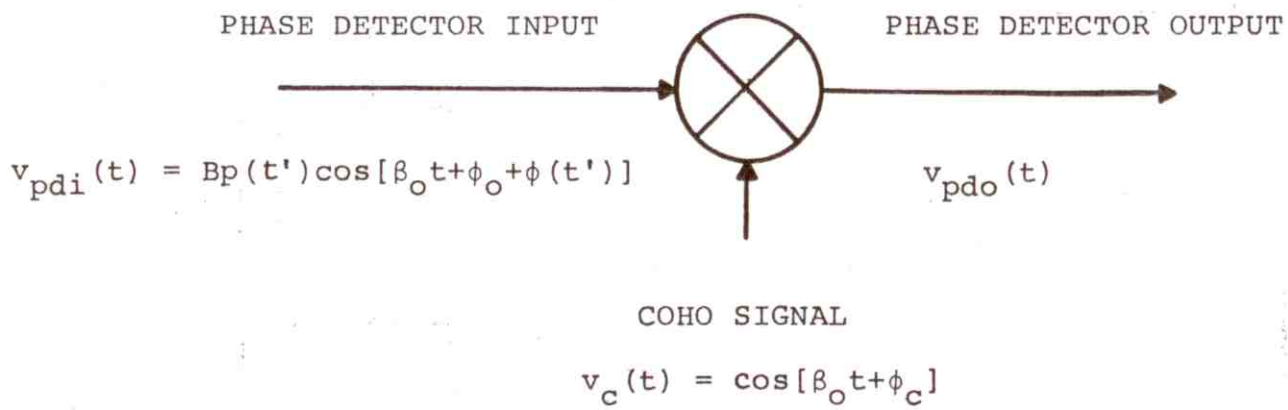


Figure C-3. Radar Coherent MTI Phase Detector

detector (N_{pdo} and N_{pdi}), the bandpass noise model is used, and the input noise signal, $n_{pdi}(t)$, is given by:

$$n_{pdi}(t) = n_c(t) \cos \beta_o t + n_s(t) \sin \beta_o t \qquad \text{(C-1)}$$

Where $n_{pdi}(t)$ is the IF bandpass noise at the input of the MTI phase detector, and the noise power of $N_{pdi}(t)$ is given by:

$$N_{pdi} = \overline{n_{pdi}^2(t)} \qquad \text{(C-2)}$$

Generally, the COHO signal power level is greater than 0 dBm. Therefore, the COHO noise figure (conversion loss) is very small and can be neglected. Also, the COHO signal is usually filtered and the noise suppressed, thus permitting the COHO signal noise level to be neglected. If $n_{pdi}(t)$ is applied at the input of the phase detector (which multiplies the incoming noise signal, $n_{pdi}(t)$, by $v_c(t)$, then $n_{pdo}(t)$, the output noise of the MTI phase detector is given by:

$$n_{pdo}(t) = n_c(t) \cos \beta_o t . \cos (\beta_o t + \phi_c)$$

$$+ n_s(t) \sin \beta_o t . \cos (\beta_o t + \phi_c) \qquad \text{(C-3a)}$$

$$= \frac{n_c(t)}{2} [\cos (2\beta_o t + \phi_c) + \cos (\phi_c)]$$

$$+ \frac{n_s(t)}{2} [\sin (2\beta_o t + \phi_c) - \sin (\phi_c)] \qquad \text{(C-3b)}$$

The terms $\cos (2\beta_o t+\phi_c)$ and $\sin (2\beta_o t+\phi_c)$ represent the spectra of $n_c(t)$ and $n_s(t)$, respectively, shifted at $(2\beta_o t)$ and are filtered out by the low pass video filter at the phase detector output. Hence, $n_{pdo}(t)$ is given by:

$$n_{pdo}(t) = \frac{n_c(t)}{2} \cos (\phi_c) - \frac{n_s(t)}{2} \sin (\phi_c) \qquad \text{(C-4)}$$

and the MTI phase detector output noise power is:

$$N_{pdo} = \overline{n^2_{pdo}(t)} = \overline{\tfrac{1}{4}n^2_{pdi}(t)} \quad \text{(C-5a)}$$

$$= \tfrac{1}{4}N_{pdi} \quad \text{(C-5b)}$$

Since the phase detector is sensitive to both the amplitude and phase of the detector input, the noise amplitude distribution at the phase detector output is Gaussian since the phase detector input noise amplitude distribution is Gaussian.

Desired/Interfering Signal

It is shown in Appendix B that the desired and interfering signal IF output time waveform ($V_{IFo}(t)$, Equation B-33), MTI phase detector input time waveform, can be expressed as:

$$V_{pdi}(t) = Bp(t') \cos [\beta_o t + \phi_o + \phi(t')] \quad \text{(C-6)}$$

Since we are only concerned with the peak power of the signal, p(t') equals 1, and phase detector input signal peak power (the mean square of $v_{pdi}(t)$) is:

$$S_{pdi} = \overline{V^2_{pdi}(t)} = \frac{B^2}{2} \quad \text{(C-7)}$$

The radar COHO signal can be defined as:

$$v_c(t) = \cos [\beta_o t + \phi_c] \quad \text{(C-8)}$$

Where:

β_o = Receiver tuned IF frequency, in radians per second

ϕ_c = Phase of COHO signal, in radians

The signal voltage time waveform at the phase detector output can be found by performing the operation shown in Figure C-3, and is given by:

$$v_{pdo}(t) = v_{pdi}(t) \cdot v_c(t) \quad \text{(C-9a)}$$

$$= Bp(t') \cos[\beta_o t + \phi_o + \phi(t')] \cdot \cos[\beta_o t + \phi_c] \quad \text{(C-9b)}$$

$$= \frac{Bp(t')}{2} [\cos[2\beta_o t + \phi_o + \phi(t') + \phi_c]$$

$$+ \cos[\phi(t') + \phi_o - \phi_c]] \quad \text{(C-9c)}$$

The first term of Equation C-9c is filtered out by the low pass video filter at the phase detector output, resulting in a signal at the phase detector output of:

$$v_{pdo}(t') = \frac{Bp(t')}{2} \cos[\phi(t') + \phi_o - \phi_c] \quad \text{(C-10)}$$

and a signal output peak power (the mean square of $v_{pdo}(t)$) of:

$$S_{pdo} = \overline{[v_{pdo}{}^2(t)]} = \frac{B^2}{8} \quad \text{(C-11a)}$$

$$= \tfrac{1}{4} S_{pdi} \quad \text{(C-11b)}$$

The term $\phi(t')$ in Equation C-5 is the phase modulation produced by the radar IF filter transfer properties when the pulsed interfering signal is off-tuned. $\phi(t')$ consists of a steady state term which is a function of the frequency separation (ΔF) between the interfering signal and victim radar tuned frequency, and a transient term. A detailed discussion of this phase modulation on the interfering signal is given in Appendix B (see Figure B-15). A further discussion of the steady state and transient phase modulation properties of off-tuned pulsed interference will follow in the MTI video low pass filter transfer properties.

In the case where the signal is on-tune, $\Delta F=0$, the term $\phi(t')$ equals zero for the steady state portion of the pulse. For this case the signal at the phase detector output is given by:

$$v_{pdo}(t) = \frac{Bp(t')}{2} \cos (\phi_o - \phi_c) \tag{C-12}$$

and the signal phase detector output peak power is given by:

$$V_{pdo} = \overline{[v_{pdo}^2(t)]} = \frac{B^2}{4} \cos^2 (\phi_o - \phi_c) \tag{C-13}$$

The term $(\phi_o - \phi_c)$ is the phase difference between the signal and the reference COHO signal. Therefore, the signal voltage waveform at the phase detector output will vary between $\pm$ B/2. The signal at the phase detector input can be assumed to have a uniform phase difference distribution between 0 to 360 degrees with the reference COHO signal. Thus, if the signal power at the phase detector output is averaged over all phase angles, the signal power is given by:

$$S_{pdo} = \frac{1}{\pi} \int_o^{\pi} V_{pdo}(\phi) d\phi = \frac{B^2}{8} \tag{C-14a}$$

$$S_{pdo} = \frac{1}{4} S_{pdi} \tag{C-14b}$$

Using Equations C-5b, C-11b, and C-14b, the MTI phase detector SNR and INR transfer properties are given by:

$$SNR_{pdo} = SNR_{pdi} \tag{C-15}$$

Signal-Plus-Noise Distribution

The noise probability density function at the phase detector output is Gaussian. This is due to the fact that the phase detector is amplitude as well as phase sensitive. The noise amplitude Probability Density Function (PDF) at the input to the phase detector is Gaussian distributed, and the noise phase PDF is uniform distributed. Therefore, the noise amplitude PDF at the phase detector output is Gaussian. If one considers a fixed amplitude (non-fluctuating) desired or interfering signal at the phase detector input, the signal-plus-noise at the phase detector output is given by:

$$v_{pdo} (t) = N (t) + A \cos \phi \tag{C-16}$$

where:

$N(t)$ = Noise amplitude which is Gaussian distributed, in volts

A = Desired or interfering signal amplitude, in volts

ϕ = Phase difference between received signal and COHO which is uniform distributed

Papoulis (1965) derives the probability density function for Equation C-16 which can be expressed as:

$$p(v,A) = \frac{1}{\pi\sigma\sqrt{2\pi}} \int_{0}^{\pi} e^{\frac{-(v-A\cos\phi)^2}{2\sigma^2}} d\phi \tag{C-17}$$

where:

σ = rms noise level, in volts

Equation C-17 can not be written in closed form. The MTI channel phase detector output was simulated to investigate trade-offs in suppressing asynchronous interfering signals. The simulation of the radar MTI channel is discussed in Appendix E. Figure C-4 shows the PDF given by Equation C-17 as a function of the signal-to-noise ratio. The PDF was obtained by simulation.

MTI LOW PASS FILTER TRANSFER PROPERTIES

The low-pass filter in the MTI channel is used to reduce the MTI channel video noise level. The low- pass filter bandwidth (B_{lpf}) is approximately equal to $1/\tau$ where τ is the desired signal pulse width. The MTI channel IF bandwidth is always much greater than $1/\tau$ (generally 5.0 MHz) to produce a constant phase characteristic across the desired pulse. Therefore, a narrower low-pass filter bandwidth will improve the video signal-to-noise ratio. The MTI channel low pass filters are usually several stages. The low-pass filter frequency response can be expressed as:

$$A_{LP}(F) = \frac{1}{[1 + j\,(\frac{F}{F_L})\gamma]^n} \tag{C-18}$$

where:

F_L = Low pass filter 3 dB cutoff frequency, in Hz

n = Number of low pass filter stages

$\gamma = \sqrt{2^{1/n}-1}$

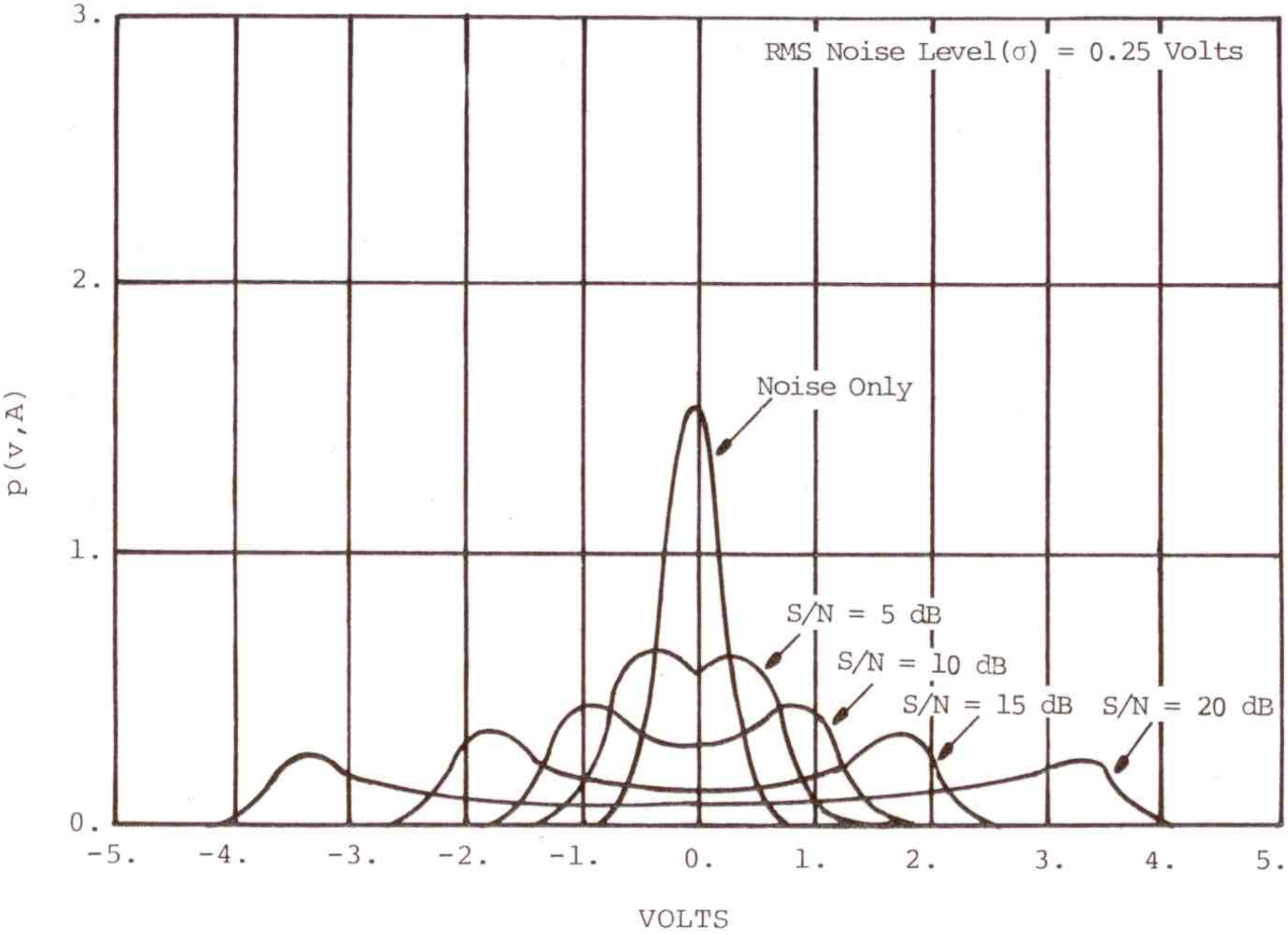


Figure C-4. Probability Density Function for Noise Only and for Signal-Plus-Noise at the MTI Phase Detector Output.

Noise

The noise transfer properties of the MTI low pass filter can be expressed as:

$$N_{lpo} = N_{lpi} + 10 \log \frac{B_{lpf}}{B_{IF}} \tag{C-19}$$

For a radar with a 5.0 MHz IF bandwidth and a 1.2 MHz low pass filter bandwidth, the noise power at the low pass filter output is given by:

$$N_{lpo} = N_{lpi} - 6.2 \text{ dB} \tag{C-20}$$

Therefore, the low pass filter reduces the MTI channel noise level by 6.2 dB.

Desired/Interfering Signal

The desired and interfering signal low pass filter output time response, $V_{lpo}(t)$, can be obtained by multiplying in the frequency domain the low pass filter frequency response with the phase detector output signal frequency spectrum, $V_{pdo}(F)$, and taking the Fourier transform. That is:

$$V_{lpo}(t) = \mathcal{F}^{-1} [V_{pdo}(F) . A_{LP}(F)] \tag{C-21}$$

Measurements made on an ASR-8 showed that as an interfering signal is off-tuned (ΔF), the signal level followed the low pass filter selectivity due to the sinusoidal modulation during the steady state portion of the pulse. However, for ΔF greater than 3 MHz, the off-tuned interfering signal response started to follow the interfering signal RF frequency spectrum, and the low pass filter interfering signal output response was caused by the phase modulation that occurs during the transient part (rabbit ear response occurring at the leading and trailing edge of the IF filter output response, see Figures B-10, B-11, B-13, and B-14) of the interfering signal time waveform. Figure C-5 shows a photograph of the phase detector output time waveform of a 60 μs pulse signal off-tuned 200 KHz. The measurement was made on an ASR-8 radar. The pulse width was set at 60 μs to demonstrate the 200 KHz sinusoidal modulation during the steady state portion of the pulse which is caused by the phase modulation produced by the IF filter transfer

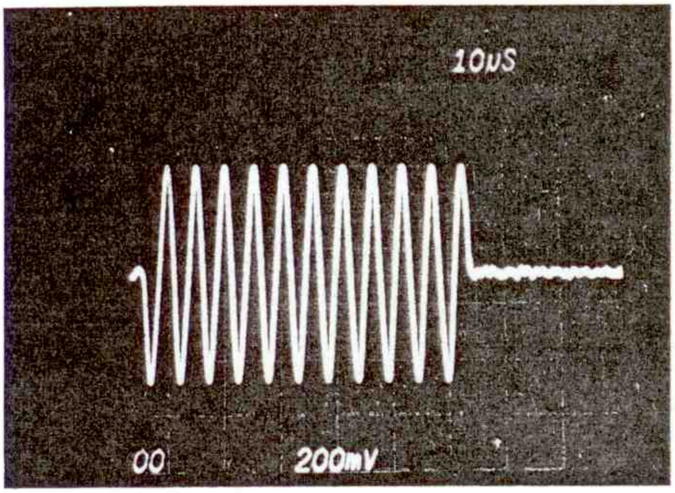


Figure C-5. Measured MTI Low Pass Filter Output Time Waveform

τ = 60 μsec

ΔF = 200 kHz

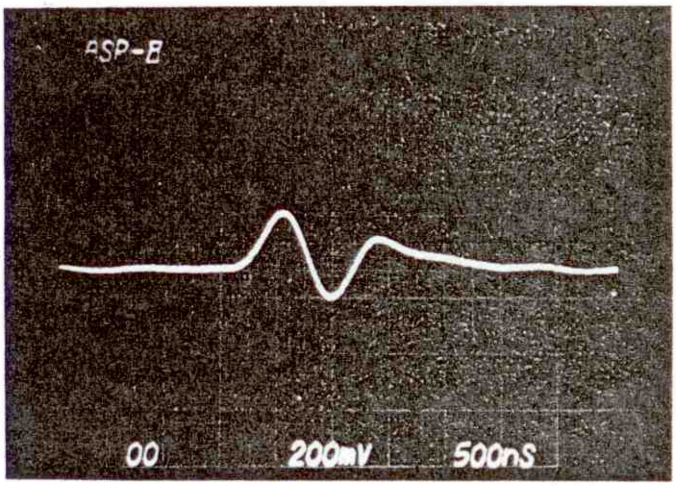


Figure C-6. Measured MTI Low Pass Filter Output Time Waveform

τ = .83 μsec

ΔF = 5.0 MHz

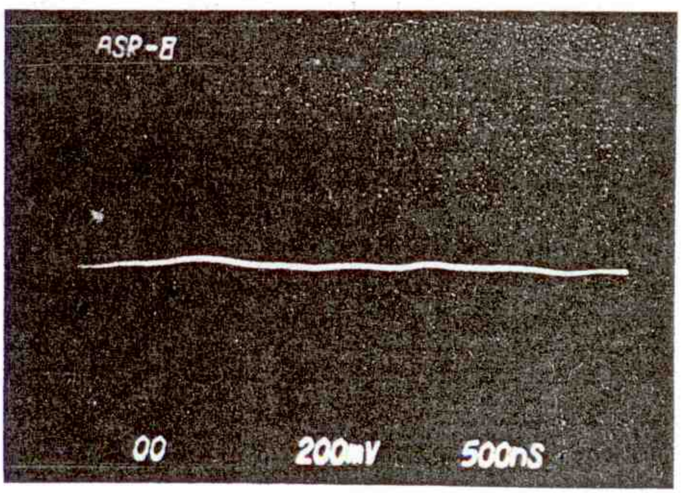


Figure C-7. Measured MTI Low Pass Filter Output Time Waveform

τ = .83 μsec

ΔF = 5.0 MHz

properties, and is equal to the off-tuned frequency (ΔF). Figure C-6 shows a photograph of an ASR-8 low pass filter output of a .83 μs pulse off-tuned 5.0 MHz. Since the low pass filter interfering signal output response for off-tuned interference is due to the phase modulation which occurs during the transient portion of the IF filter response, the low pass filter output interfering signal time response changes from pulse-to-pulse. However, the average pulse amplitude appears to follow the interfering signal RF spectrum envelope. Figure C-7 shows a photograph of the low pass filter output for the same interfering signal parameters as in Figure C-6. However, the phase modulation during the transient portion of the interfering signal did not cause a low pass filter output response.

MTI CANCELLER TRANSFER PROPERTIES

The target return signals after phase detection are processed in the MTI cancellers. The MTI canceller circuits provide for cancelling fixed target signals (clutter), such as buildings, hills, and trees and allowing only moving targets, such as aircraft to be displayed on the PPI. The action of the MTI cancellers is that of a linear filter. In the ideal case, the clutter components will appear at zero frequency and at integral multiples of the radar Pulse Repetition Frequency (PRF), and will be suppressed.

Most radars in the 2.7 to 2.9 GHz band employ both single and double cancellers. Generally, the radars are operated in the double canceller mode which provides broader clutter-rejection nulls than the single cancellers. Some of the radars in the band also have the capability of introducing feedback in the double-canceller mode to improve the velocity response of the MTI filter.

Both analog and digital filtering is used by the radars in the 2.7 to 2.9 MHz band. Radars using analog filters (delay lines) have the capability of continuously variable feedback control, while the digital filter (shift registers) radars generally have several modes of fixed feedback.

Staggered PRF is also generally employed by radars in the 2.7 to 2.9 GHz band to extend the blind speed of the radars. Analog radars typically have two or three staggers, while digital radars with their greater stability may have more than three staggers. The ASR-8 has a four-stagger system, and the ASR-7 has a six-stagger system. A radar operating in the staggered mode affects the range bins in which the interfering pulses may occur. However, the peak interfering signal transfer properties through the MTI canceller are not affected when operating in the staggered mode. Therefore, the staggered PRF modes of the radars can be neglected for analytical simplicity in determining the interfering signal peak power transfer properties of MTI cancellers.

Single Stage Canceller Transfer Properties

Figure C-8 shows a block diagram of a single stage canceller (first-order nonrecursive filter). The canonical form of the single stage

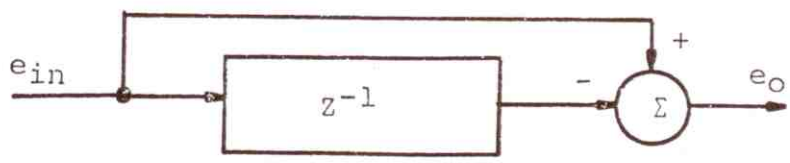


Figure C-8. First-Order Nonrecursive Filter

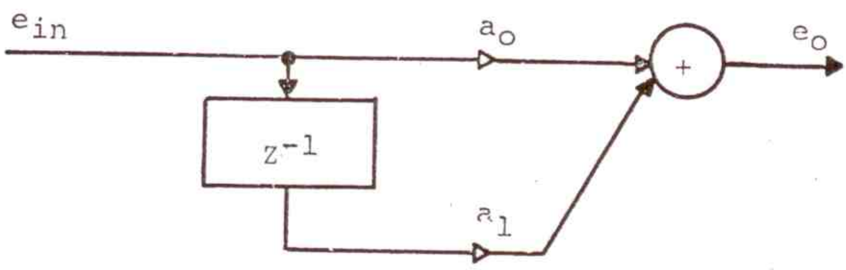


Figure C-9. Canonical Form of First-Order Nonrecursive Filter canceller is shown in Figure C-9. The Z-Transfer function of the first-order nonrecursive filter is given by:

$$H(Z) = \frac{e_o}{e_{in}} = 1 - Z^{-1}$$

$$= a_o + a_i Z^{-1} \qquad (C\text{-}22)$$

where:

Z^{-1} = Unit delay (t - T)

T = Radar pulse period, in seconds

a_i = Binomial weighting factors, $(-1)^i \binom{n}{i}$

n = Filter order, 1

The frequency response magnitude of the first-order nonrecursive filter transfer function is determined by letting $Z^{-1} = e^{-j\omega_d T}$, and is given by:

$$|H(e^{j\omega_d T})| = |1 - (\cos\omega_d T - j \sin \omega_d T)| \qquad (C\text{-}23a)$$

$$= \left|2 \sin \frac{\omega_d T}{2}\right| \qquad (C\text{-}23b)$$

where:

ω_d = Doppler frequency of return signal, in radians per second

Figure C-10 shows the frequency response characteristics for a first-order nonrecursive filter. The improvement factor (I) is 21.3 dB. The improvement factors are defined as the signal-to-clutter ratio at the output of the MTI system compared with that at the input, where the signal is understood as that averaged uniformly over all radial velocities, that is:

$$I = 10 \log \frac{\overline{S_o}/C_o}{S_i/C_i} \qquad (C\text{-}24)$$

Equation C-22 can be expressed in canonical form (Figure C-9) as a n-order difference equation relating the input signal, e_{in}, to the output signal, e_o, by:

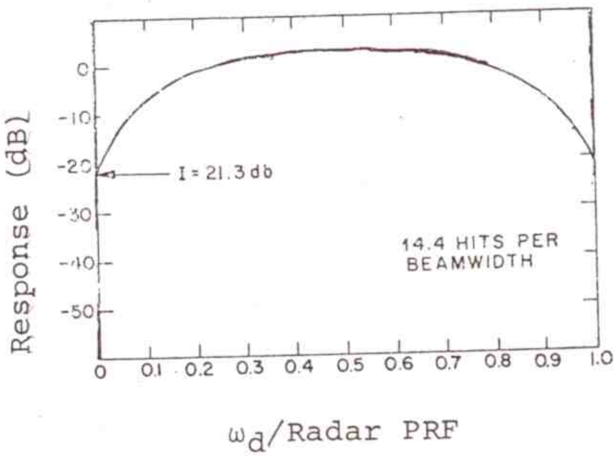
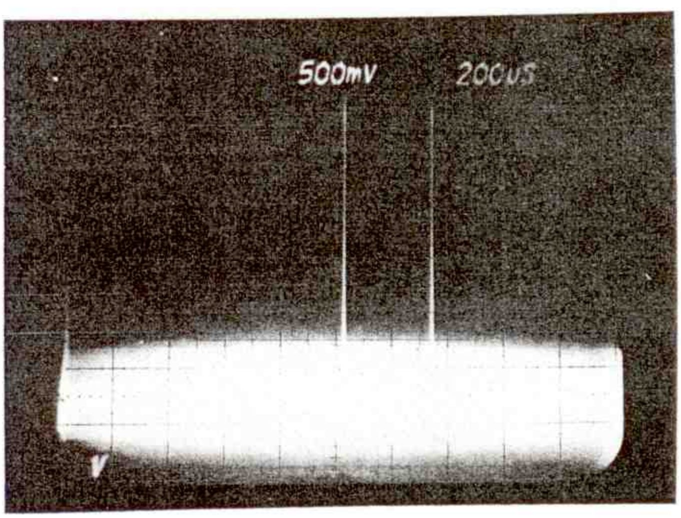


Figure C-10. Frequency Response for a Single Stage MTI Canceller



τ = .83 µsec

PRF_I = 800

Figure C-11 Measured Single Stage MTI Canceller Output Time Waveform