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Topological Defects in Extended Inflation

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Abstract

We consider the production of topological defects, especially cosmic strings, in extended inflation models. In extended inflation, the Universe passes through a first-order phase transition via bubble percolation, which naturally allows defects to form at the end of inflation. The correlation length, which determines the number density of the defects, is related to the mean size of bubbles when they collide. This mechanism allows a natural combination of inflation and large-scale structure via cosmic strings.

There has been a recent renewed interest in the possibility of inflation via a first-order phase transition in the early Universe, in the manner of Guth's "old" inflation scenario.[1] In Guth's model, the Universe was trapped in a false-vacuum state and the energy density of the Universe became dominated by the vacuum energy density. This resulted in an exponential increase in the scale factor of the Universe, solving several cosmological puzzles. The phase transition proceeded by quantum tunnelling to the true vacuum state,[2] which lead to the nucleation of bubbles of true vacuum. Having appeared with a characteristic size determined by microphysics (assuming that gravitational effects upon bubble nucleation are not too large),[3] the bubbles then grew at the speed of light. Unfortunately, because the regions of the Universe in the false-vacuum state were expanding exponentially, the expansion was more effective than the decay to the true vacuum state, bubbles of true vacuum never percolated, and the energy density in the expanding walls of the true vacuum bubbles could not be converted to radiation through bubble wall collisions. This problem became known as the "graceful exit" problem of old inflation.[4]

La and Steinhardt[5] have recently revived the idea of a first-order phase transition in the context of theories in which the gravitational constant may vary, such as Jordan–Brans–Dicke theories, and term their model "extended" inflation. In such theories the expansion of the scale factor of the Universe is a power law rather than exponential during inflation (for example, in a Brans–Dicke theory with parameter ω, the scale factor increases as $t^{\omega+1/2}$ at late times). In such cases the exponential bubble nucleation rate will eventually overcome the power-law expansion rate, and the Universe will exit from the inflationary era. Such a model may relieve many of the fine-tuning problems associated with new and chaotic inflation.

As emphasised by Weinberg[6] and by La, Steinhardt and Bertschinger,[7] the exit from extended inflation based on the Jordan–Brans–Dicke theory is not without prob-

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lems due to the existence of large vacuum bubbles at the end of inflation (the spectrum for bubble formation is nearly scale independent). Large bubbles will not have time to thermalize before recombination, and hence will lead to distortions in the microwave background; arranging for these to be sufficiently small requires the Brans–Dicke parameter ω certainly to be less than 50 and possibly much smaller, which is in conflict with the lower bound from time-delay experiments of $\omega > 500$.

To resolve this conflict, several more complicated (but perhaps more realistic) models have been proposed, for instance allowing ω to vary with time,[8] or by incorporating the scenario into either an induced-gravity model[9] or a Kaluza–Klein model,[10] or by modifying the gravitational couplings of the inflaton.[11] All these models suppress large bubbles (those formed early in the inflationary epoch) and hence allow sufficient thermalization of the bubbles before decoupling to evade microwave background constraints. It has been suggested that the bubbly nature of space after extended inflation may be reflected in observations of frothy large-scale structure.[7]

However there might very well be a different mechanism for the formation of structure after extended inflation, namely the formation of topological defects in the inflaton field formed as it passes through the phase transition. Calculations of the false-vacuum decay rate made so far consider the evolution from a false-vacuum state to a *unique* true-vacuum state. However, the inflaton is far more likely to have degenerate minima, especially if it is part of a grand-unified Higgs sector. For example, if the inflaton is a real scalar field ϕ, then it would be common for it to have two discrete minima at $\phi = +\phi_0$ and $\phi = -\phi_0$, in which case there is the possibility of forming domain walls. More interestingly, if the scalar field is complex, with a potential of the form such that the manifold of possible ground states form a circle of degenerate minima, e.g. $V(\phi) = \lambda(|\phi|^2 - \phi_0^2)^2$, there will be the possibility of cosmic string solutions. Yet more complex models would allow monopole solutions, or hybrid

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creatures such as monopoles connected by strings or walls bounded by strings. The types of defects formed in the phase transition of any given model will depend on the properties of the manifold of degenerate vacuum states.

Recall the standard picture of cosmic string formation in a smooth second-order phase transition.[12] At early times the universe was very hot and the fields describing interactions were in a highly symmetric phase. However as the universe expanded and cooled, symmetry breaking processes spontaneously occurred, which may have left behind remnants of the old symmetric phase, possibly in the form of strings, domain walls or monopoles. Here, we concentrate on strings, which occur when the potential has a circle of degenerate minima parametrized by a phase angle θ. As the system cooled below the critical temperature, ($T_C \sim \phi_0$), the ϕ field began to fall to the minima of its potential. Due to thermal fluctuations of the ϕ field, domains formed of size $\xi_2 \sim (\sqrt{\lambda}\phi_0)^{-1}$ (the length over which the ϕ field is spatially correlated). In separate domains ϕ pointed in arbitary directions in the vacuum manifold, but matched smoothly at the boundary, with θ varying so as to cause defects to form on the edges common to certain domains. This is easily seen. Consider one such edge where θ varies by 2π in encircling the edge, i.e., all around the edge we continuously encircle the minimum of the potential. Such regions line up to minimize the spatial gradient energy, forming a defect line or cosmic string. This corresponds to a thin tube of false-vacuum energy, and these strings are either in the form of closed loops or are infinitely long.

Strings appropriate to galaxy formation are required to have a line density of $G\mu \sim 10^{-6}$, where $\mu \sim \phi_0^2$, corresponding to a breaking scale of 10^{-3} Planck masses. Unfortunately, generic new and chaotic inflationary scenarios occur at or below this energy scale, and hence the strings form before or early in the inflationary epoch and are rapidly inflated away. It has been demonstrated that the universe cannot be made

to reheat after inflation to sufficiently high temperatures as to restore the symmetry of the string-forming field and allow a new phase of string formation after inflation.[13,14] This leads to the incompatibility of cosmic strings with new or chaotic inflation. These arguments apply whether the inflaton and the cosmic string fields are the same field or different ones (in chaotic inflation the inflaton field can never be identified with the cosmic string field as the symmetry is broken even initially). In the case where the cosmic string field is distinct from the inflaton field, models have been proposed which resolve the conflict. The model of Vishniac, Olive, and Seckel[14] couples the inflaton and the string field in a particular way, but the only motivation for doing this is to solve the strings-inflation problem, so their solution appears unnatural. More recently, Yokoyama[15] has suggested that a non-minimal coupling to gravity of the string field can hold it in its symmetric phase during inflation, and allow strings to form at the end of inflation.

Now consider the picture of string formation in extended inflation, where the fact that the transition is first order has crucial consequences. As the Universe cools from high temperatures, a complex scalar field is trapped in a false-vacuum state and the Universe enters a phase of rapid power-law expansion. Bubbles of true vacuum then begin to nucleate and grow at the speed of light. Due to the presence of event horizons in the inflating Universe they grow to a constant comoving volume which depends on their time of formation. The important ingredient to our scenario is that each bubble forms independently of the rest, and so there is no correlation between the choice of true vacuum made in each bubble from the selection of degenerate true vacua. Eventually the bubbles grow and collide, finally percolating the Universe and bringing the inflationary era to an end.

At the end of inflation, the collision of bubble walls (in which all the energy is held) produces particles and causes thermalization of the energy. However, because

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the scalar field is only correlated on the scale of a bubble, we can expect topological defects to be present. The usual arguments state that there is typically of order one cosmic string per correlation volume of the scalar field, and hence we expect roughly one string per mean bubble size at the end of inflation.

All attempts to solve the big-bubble problem of the original extended inflation model have the common feature that large bubbles (bubbles formed early in inflation) are suppressed. Therefore we expect that in any successful extended inflation model large bubbles should be very rare, and there should be a rapid "turn on" of bubble nucleation at the end of inflation. Therefore to a first approximation we will assume that at the end of inflation nearly all of the volume of the Universe is taken up by bubbles nucleated near the end of inflation, with only very few rare large bubbles. The distribution of large bubbles clearly depends upon the model for extended inflation, while the fact that most of the volume of the Universe must be in bubbles nucleated near the end of inflation should be true in any successful extended inflation model.

It is possible to estimate the effective correlation length for the string network formed at the end of extended inflation. In the thin-wall picture a bubble is nucleated with physical size $R_C = 3(\Delta V)^{-1} \int d\phi \sqrt{2V(\phi)}$, where the integral is evaluated between the true and false vacuum values of ϕ, and ΔV is the potential difference between the false and true vacua driving inflation.[2] For typical polynomial potentials, $\Delta V \sim \epsilon \lambda \phi_0^4$, where λ is an overall coupling constant for the potential, ϕ_0 sets the scale for the vacuum expectation value, and ϵ describes the difference in potential energy between the false and true vacua. To get a lower limit on the correlation length, we assume that there has been little growth in the coordinate radius of the mean bubble from the time it was nucleated, $\tilde{t}$ until the end of inflation, t_{end} (bubbles nucleated toward the end of inflation have little growth in their coordinate radius). Then the physical size of the mean bubble at the end of inflation (i.e., the effective correlation

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length) is $\zeta_{\text{eff}} \sim R_C[a(t_{\text{end}})/a(\bar{t})]$.

This leads to a correlation length greater than that typical of the Kibble mechanism where cosmic strings form via a second-order thermal phase transition, and hence a more dilute network will be formed in our model. For example, consider a potential of the form $V(\phi) = \lambda(\phi^2 - \phi_0^2)^2 - \epsilon\lambda\phi^3\phi_0$. In the limit of small ϵ, $R_C \sim \sqrt{\lambda}\,\phi_0^3/\Delta V \sim (\sqrt{\lambda}\,\epsilon\phi_0)^{-1}$. Therefore $\zeta_{\text{eff}} \sim (\sqrt{\lambda}\,\epsilon\phi_0)^{-1}[a(t_{\text{end}})/a(\bar{t})]$. As previously mentioned, a second-order phase transition in such a model leads to a correlation length of $\zeta_2 \sim (\sqrt{\lambda}\,\phi_0)^{-1}$, so the comparison of the effective correlation length in our model to the usual correlation length in a second-order transition is $\zeta_{\text{eff}}/\zeta_2 = \epsilon^{-1}[a(t_{\text{end}})/a(\bar{t})]$. Note that $\epsilon \ll 1$ is required for sufficient inflation. On the other hand, we expect that in successful extended inflation scenarios $[a(t_{\text{end}})/a(\bar{t})]$ is not too much larger than unity.

This model for the formation of strings allows for the existence of large voids, which would be a consequence of the rare large bubbles. Although the typical string separation at the end of inflation is ζ_{eff}, extended inflation allows for the possibility of rare large bubbles, formed by quantum tunnelling early in inflation. The true vacuum formed inside bubbles contains no matter (any matter originally in that volume is assumed to be inflated away while the scalar field dominates the energy density). All the energy of the Universe after inflation is contained in the walls of the expanding bubbles which collide to form matter and to cause thermalization of the energy density. After collisions, matter will flow back into the void, though as it cannot travel faster than light, we can calculate the minimum time the bubble will require to thermalize. A large bubble will have a coherent scalar field vacuum and hence no strings will be formed within it—we can thus expect the interior of the bubble to evolve into a large region void of strings. If cosmic strings are to provide the seeds for galaxy formation, then we can expect to see few or no galaxies within

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the void. The presence of voids is an additional property of this model which may help explain observed large-scale structure.

In fact, at the time of percolation the bubbles may have a range of sizes, which can lead to the formation of an initial string network differing from the usual one. As the correlation length is essentially just the bubble size, and because there would appear to be no *a priori* reason why bubbles everywhere should be *exactly* the same size (at small sizes the assumption of a scale-invariant bubble size distribution would seem more reasonable), the strings will be formed with a randomly spatially varying correlation length. This will presumably lead to higher densities of strings in some regions than others, which again may have implications for structure formation, depending on how much the effects of the initial string distribution might be wiped out by the future evolution and decay of strings. One desirable effect of a more dilute string network would be to avoid the uncomfortable bounds from gravitational wave production, from small string loops.[16] The fact that the correlation length will generically be greater (and in some models perhaps much greater) than that of the Kibble mechanism may also have important implications, though perhaps not as great as one might naïvely suppose if the small strings rapidly disappear from the network once string evolution commences.

These formation arguments can be equally well applied to the cases of domain walls and monopoles, again giving rise to an estimate of order one defect per bubble at the time of bubble collision. In the case of domain walls this will give rise to an excessive number, and will be disallowed on cosmological grounds. Hence, any extended inflation model featuring a potential with domain wall solutions (i.e., a disconnected vacuum manifold) can be ruled out. The situation is less clear for monopoles, because the correlation length may well be substantially greater than that of the Kibble mechanism and hence proportionally fewer monopoles are expected.

However, standard estimates of the cosmological monopole abundance[17] give values of perhaps twenty orders of magnitude in excess of the Parker limit,[18] so the correlation length would have to be increased by seven or eight orders of magnitude before being within experimental limits—such an increase seems very unlikely.

If we consider the unification to be part of a grand-unified theory, the problem of monopole overproduction must be addressed,[5] as any breaking to the symmetry of the standard model must produce monopoles at some stage. The simplest method is to arrange for monopoles to be formed in a partial symmetry breaking and then later inflated away in a second transition. La, Steinhardt and Bertschinger[7] considered this solution though their emphasis was on monopoles produced in a field other then the inflaton.

We also point out that if the string network produced in extended inflation is relevant for formation of large-scale structure, the line density must be about 10^{-6} in Planck units, corresponding to a symmetry breaking scale of 10^{16}GeV. Thus observational limits on the string line density translate into bounds on the mass of the inflaton.

In conclusion, we have shown that extended inflation and cosmic strings are quite compatible. The cosmic string network formed in an extended inflation phase transition will in many ways be identical to the network formed in non-inflationary phase transitions. Possible exceptions to this statement are 1) a larger effective correlation length, which will lower the density of small string loops, lessening the constraints from gravitational wave production, and 2) perhaps rare large volumes void of strings, which may be of importance in formation of large-scale structure. Extended inflation does not solve the monopole or domain wall problem, at least in regard to monopoles and domain walls produced by the inflaton.

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