

## A NEW SEDIMENT TRANSPORT FORMULA FOR LOCAL SCOUR PREDICTION

Xibing S. Dou (1) and J. Sterling Jones (2)

(1) Ph.D., MASCE, Senior Research Engineer, GKY & Associates, Inc., 5411-E Backlick Road, Springfield, VA 22151; Mailing Address: 6300 Georgetown Pike, McLean, VA 22101; Tel. (202) 493-3058; Fax (202) 493-3442; E-mail: [Xibing.Dou@fhwa.dot.gov](mailto:Xibing.Dou@fhwa.dot.gov)

(2) MASCE, Hydraulic Research Engineer, Federal Highway Administration HRDI-07, 6300 Georgetown Pike, McLean, VA 22101; Tel. (202) 493-3043; Fax (202) 493-3442; E-mail: [Sterling.Jones@fhwa.dot.gov](mailto:Sterling.Jones@fhwa.dot.gov)

### *Abstract*

Numerical models have been developed to adequately simulate three dimensional flow around obstructions, such as bridge piers, in the flow field. Previously, however, sediment transport formulas available for predicting scour were developed for general scour in unobstructed flow. The capability for numerical models to predict local scour around bridge piers was severely restricted by the sediment transport formulas. This paper describes a new effective Sediment Transport Capacity (*STC*) formula which accounts for the dominate factors-diving currents, strong vortices and high turbulence-which characterize the local scour process. The new formula adds a new term to a classic sediment transport formula from the Chinese literature. Coefficients for the added term were derived from physical laboratory experimental data and it becomes zero when there is no flow obstruction. That means that the new formula can be used for both general scour and local scour when incorporated into a reliable numerical flow model. The new formula has been tested against independent physical model results and applied to a full scale study of the proposed Woodrow Wilson replacement bridge. Results from the Woodrow Wilson bridge study are presented in this paper. This paper presents an analogy between the Sediment Transport Capacity and Stream Power and shows that they are directly related. Just as an effective Sediment Transport Capacity is needed to predict local scour with a numerical model, an effective Stream Power is needed to analyze local scour with empirical methods.

### *Introduction*

Local scour is a very complex three-dimensional phenomenon. The basic mechanism of local scour is the formation of vortices (known as the horseshoe vortex) and downflow (FHWA, 1995; Ettema, 1980). Because the difference in scour mechanism, sediment transport involved in local scour process is different from that in general scour. Although numerous equations and formulas have been published for general scour, no sediment transport formulas had been developed for local scour prior to this study. Most of local scour studies were limited in predicting maximum scour depth based on empirical equations. With the development of current computer technology, using numerical models to simulate local scour process, which may provide more detailed and complete information of a scour hole under different flow conditions and scales, becomes feasible. Many numerical models

have been developed to simulate local scour process so far. However, most numerical models adopted sediment transport formulas developed for general scour to study local scour problems and, accordingly, the results are not accurate. A sediment transport function to evaluate the rate of sediment transport for local scour is extremely important to simulate local scour numerically. This paper describes such a function which has tremendous potential for numerical modeling as well as for augmenting other procedures that require stream power or shear stress variation with scour depth.

### ***The Effective Sediment Transport Capacity Formula***

A commonly used sediment transport function is the Sediment Transport Capacity (*STC*). The *STC* represents the maximum amount of sediment a flow can carry under equilibrium conditions. Dou (1974) proposed a *STC* formula for general scour, i.e.,

$$STC = f_0 \frac{U^3}{gH\omega} \quad (1)$$

where  $U$  is the depth-averaged velocity;  $H$  is the flow depth;  $\omega$  is the fall velocity;  $g$  is the gravitational acceleration;  $f_0$  is the coefficient expressed as follows:

$$f_0 = \frac{K_s}{C_0^2} \frac{\gamma\gamma_s}{\gamma_s - \gamma} \quad (2)$$

where  $K_s$  is the constant (= 0.034);  $C_0$  is the dimensionless Chezy's coefficient (=  $C / (g)^{0.5}$ ,  $C$  is the Chezy's coefficient.);  $\gamma$  and  $\gamma_s$  are the weight of water and sand, respectively. Equation (1) can not be directly applied to estimate sediment transport rate for local scour at hydraulic structures because the mechanism of local scour is different from that of general scour. In order to simulate local scour by using a 3-D numerical model, Dou (1997) proposed an effective Sediment Transport Capacity formula, which accounts for downflow, vortices and local turbulence. These are the dominant factors that characterize local scour processes. That effective *STC* formula has been revised and can be written as:

$$STC = f_0 \frac{U^3}{gH\omega} + f_1 \Sigma \cdot \left[ e^{-k(y_s/b)} + f_2 (1 - e^{-k(y_s/b)}) \right] \quad (3)$$

where  $y_s$  is the scour depth;  $b$  is the characteristic length, which is the pier width for local pier scour;  $f_1$ ,  $f_2$  and  $k$  are the coefficients to be determined;  $\Sigma$  is a distribution parameter constructed by a linear average among three normalized parameters which come directly from the magnitude of downflow, the strength of vortices and the turbulence intensity induced by the presence of a structure.  $\Sigma$  has non-zero value only around the hydraulic structures. Figure 1 shows the value of  $\Sigma$  at a cylindrical

pier. In the above proposed effective *STC* formula, the first term, which comes directly from the original *STC* formula, is for general scour while the second term is created only for local scour. Because the second term in the above formula contains the parameter  $\Sigma$  that becomes zero when flow is far away from the structure, the proposed effective *STC* formula automatically becomes the original *STC* formula shown in (1) if the structure is not presented in the flow. The parameter  $\Sigma$ , which ranges from 0 to 1, can be obtained by applying CCHE3D, a 3-D hydrodynamic and sediment transport model developed originally by the Center for Computational Hydroscience and Engineering at the University of Mississippi. Dou (1997) enhanced this model for studying local scour by implementing the Stochastic Turbulence Modeling (Dou, 1982) into this 3-D model. The coefficients  $f_1$  and  $f_2$  in the proposed effective *STC* formula can be determined using experimental laboratory results. The local scour experiments performed in FHWA hydraulic laboratory show that local scour starts at the upstream corner of a pier when the ratio between the approach velocity and critical velocity is about 0.45; Other studies have found that the maximum scour depth for local clear-water scour is about 2.4 times of the pier width when the velocity ratio equals 1, i.e.,

$$\begin{aligned}
 U_{app} / U_{cr} &\approx 0.45 && \text{if } y_s / b = 0 \\
 U_{app} / U_{cr} &\approx 1.0 && \text{if } y_s / b = 2.4
 \end{aligned}
 \tag{4}$$

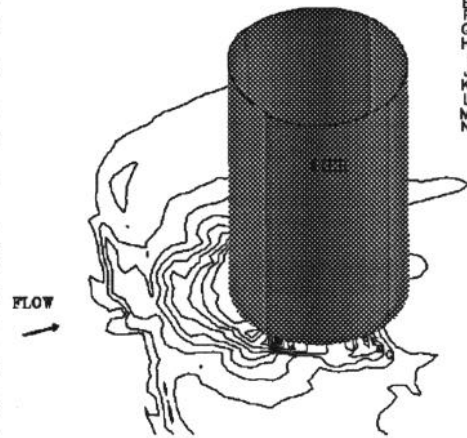
where  $U_{app}$  is the depth-averaged approach velocity;  $U_{cr}$  is the critical velocity. For an approach flow, because parameter  $\Sigma$  is zero, the proposed *STC* formula shown in equation (3) only has the first term, which is proportional to  $U^3$ . Based on the condition listed in equation (4), the relationship between *STC* of an approach flow and the critical *STC* is established as follows:

$$\begin{aligned}
 U_{app}^3 / U_{cr}^3 = STC_{app} / STC_{cr} &= 0.09 && \text{if } y_s / b = 0 \\
 U_{app}^3 / U_{cr}^3 = STC_{app} / STC_{cr} &= 1.0 && \text{if } y_s / b = 2.4
 \end{aligned}
 \tag{5}$$

or,

$$\begin{aligned}
 11 \cdot STC_{app} &= STC_{cr} && \text{if } y_s / b = 0 \\
 STC_{app} &= STC_{cr} && \text{if } y_s / b = 2.4
 \end{aligned}
 \tag{6}$$

Figure 1.  
Parameter  $\Sigma$



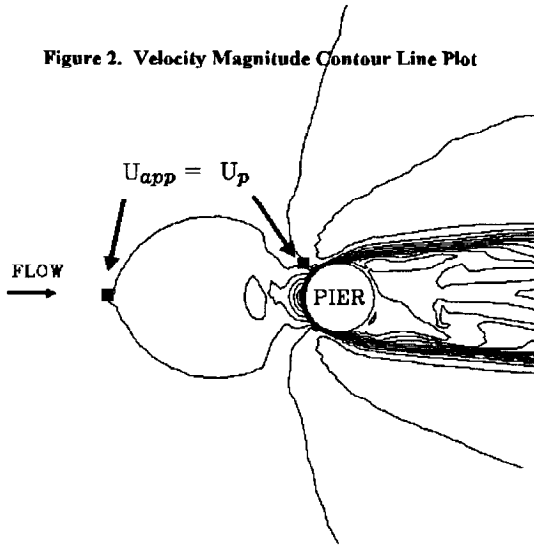
A	0.009
B	0.077
C	0.154
D	0.231
E	0.308
F	0.385
G	0.462
H	0.538
I	0.615
J	0.692
K	0.769
L	0.846
M	0.923
	1.000

where  $STC_{app}$  is the effective Sediment Transport Capacity of approach flow;  $STC_{cr}$  is the critical effective Sediment Transport Capacity. When local scour initiates at a bridge pier, the effective Sediment Transport Capacity at the pier ( $STC_p$ ) equals to  $STC_{cr}$ , and it is expressed as follows:

$$STC_p = STC_{cr} = 11 \cdot STC_{app} = f_0 \frac{U_p^3}{gH\omega} + f_1 \Sigma_{p,flat} \cdot \left[ e^{-k(y_s/b)} + f_2 (1 - e^{-k(y_s/b)}) \right] \quad (7)$$

where  $U_p$  is the depth-averaged velocity at  $p$  near the pier;  $\Sigma_{p,flat}$  is the value of  $\Sigma$  at the location  $p$  where local scour initiates (i.e., the bed is still flat). Laboratory experiments for local pier scour studies show that  $p$  is at the upstream corner of the pier. Because  $y_s$  is zero under the threshold condition of local scour, from equation (7), it gives:

Figure 2. Velocity Magnitude Contour Line Plot



$$11 \cdot STC_{app} = f_0 \frac{U_p^3}{gH\omega} + f_1 \Sigma_{p,flat} \quad (8)$$

Numerous numerical simulations for flow around piers show that  $U_p$  equals  $U_{app}$  if the bed is flat (Figure 2). Thus, coefficient  $f_1$  is determined as follows:

$$f_1 = \frac{10 \cdot STC_{app}}{\Sigma_{p,flat}} \quad (9)$$

On the other hand, for the maximum equilibrium scour depth of local clear-water scour, i.e., when  $y_s/b = 2.4$ , the effective Sediment Transport Capacity at the location  $p$  can be written as:

$$STC_p = STC_{cr} = STC_{app} = f_0 \frac{U_p^3}{gH\omega} + \frac{10 \cdot STC_{app}}{\Sigma_{p,flat}} \Sigma_{p,scoured} \cdot \left[ e^{-k(2.4)} + f_2 (1 - e^{-k(2.4)}) \right] \quad (10)$$

where  $\Sigma_{p,scoured}$  is the value of  $\Sigma$  at  $p$  where the bed has reached a maximum equilibrium scour depth. Numerical studies showed that the distribution parameter at the pier ( $\Sigma_p$ ) has little change with the variation of scour depth. It can be assumed that  $\Sigma_{p,scoured} \approx \Sigma_{p,flat} = \Sigma_p$ . If the coefficient  $k$  in the above equation is taken as 3.0, therefore, equation (10) can be simplified as follows:

$$STC_{app} = f_0 \frac{U_p^3}{gH\omega} + 10 \cdot STC_{app} \cdot \left[ e^{-3.0(2.4)} + f_2 (1 - e^{-3.0(2.4)}) \right] \quad (11)$$

In the above equation  $e^{-3.0(2.4)}$  ( $= 0.000747$ ) and  $U_p^3$  (due to the deep scour hole) are very small numbers. Therefore, the coefficient  $f_2$  is determined as follows:

$$f_2 = \frac{STC_{app}}{10 \cdot STC_{app}} = 0.1 \quad (12)$$

Substituting  $f_1$  and  $f_2$  back to equation (3), the proposed  $STC$  formula for both general scour and local scour is finally obtained as:

$$STC = f_0 \frac{U^3}{gH\omega} + \frac{10 \cdot STC_{app}}{\Sigma_p} \Sigma \cdot \left[ e^{-3.0(y_s/b)} + 0.1 \left( 1 - e^{-3.0(y_s/b)} \right) \right] \quad (13)$$

The conception of  $STC$  is similar to those of stream power ( $SP$ ) and unit stream power ( $USP$ ), which were adopted by Bagnold (1973) and Yang (1973) to predict sediment transport load. Dou (1997) found that the dimensionless  $USP$ , which is the stream power per unit weight, is equated to  $STC$ . Therefore, stream power can be calculated using the following formula:

$$SP = (\rho g H \omega) \cdot STC \quad (14)$$

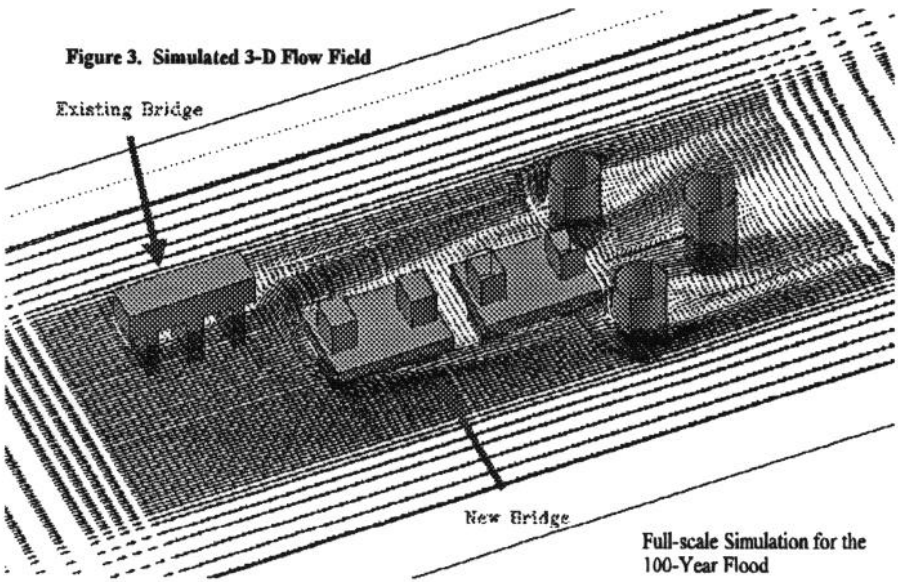
where  $\rho$  is the density of water. According to the definition,  $SP$  is proportional to  $U^3$  while shear stress ( $SS$ ) is proportional to  $U^2$ . Therefore, the ratio between the shear stress of flow near a pier at location  $p$  ( $SS_p$ ) and the shear stress of an approach flow ( $SS_{app}$ ) is expressed as:

$$\frac{SS_p}{SS_{app}} = \left( \frac{SP_p}{SP_{app}} \right)^{2/3} \quad (15)$$

Because they come from the effective  $STC$ , equations (14) and (15) calculate the effective stream power and shear stress. Similar to the effective  $STC$  formula, (14) and (15) are in the general forms that are capable of estimating sediment yield for both general scour and local scour. Both equations are of great importance for augmenting other procedures that require stream power or shear stress variation with scour depth.

### ***The Application of the Effective $STC$ Formula***

The proposed effective  $STC$  formula, as well as the Stochastic Turbulence Modeling (Dou, 1982), has been implemented in CCHE3D, a 3-D hydrodynamic and sediment transport model developed originally by the Center for Computational Hydroscience and Engineering at the University of Mississippi for local scour simulation (Dou, 1997). This 3-D model was used to simulate local scour for the existing and proposed new Woodrow Wilson Bridge over the Potomac River near

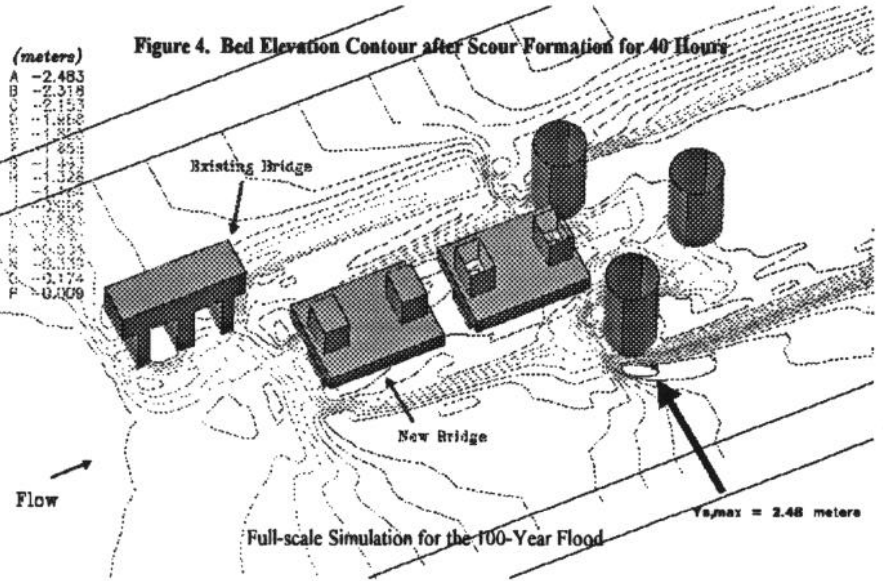


Washington DC. Full-scale local scour simulations were conducted for the 100-year and 500-year floods and for tidal surges with different pier combinations. Figure 3 shows the velocity field and bed elevation after local scour formation at the existing bridge pier E1 and the proposed new bridge pier M1, which will be in the river together during part of the construction period. In order to verify

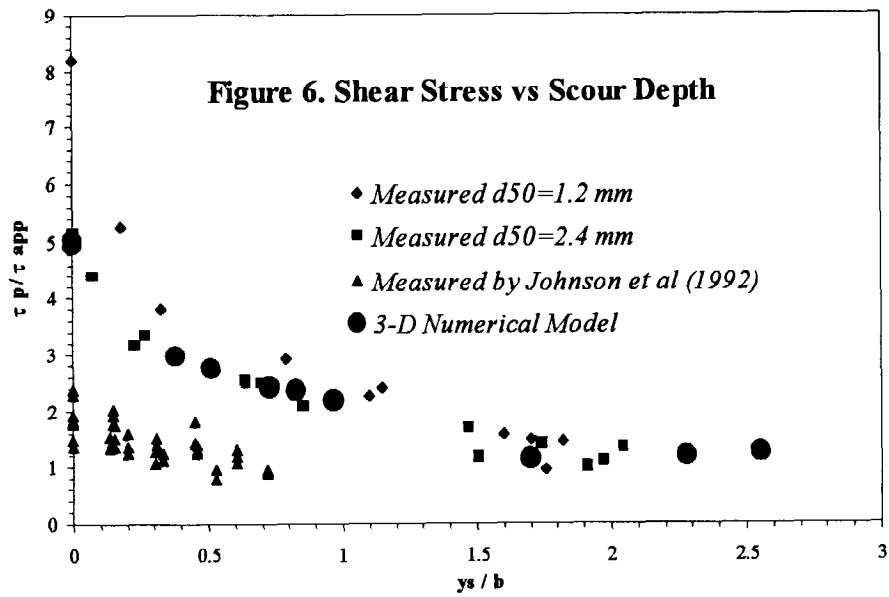
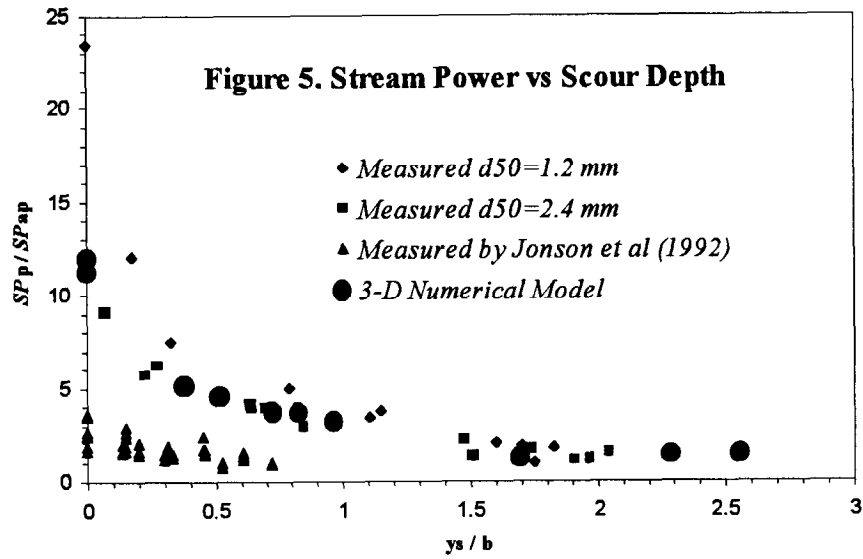
the proposed effective *STC* formula and the 3-D model, numerical simulations of local scour at pier M1 and three 45' dolphins of the proposed Woodrow Wilson Bridge have been performed to repeat the small-scale (1:100 and 1:40) physical experiments for local scour, which were conducted in the laboratory at Turner-Fairbank Highway Research Center, Federal Highway Administration. Figure 4 compares these results. Figure 5 and 6 show that the effective stream power (*SP*) and shear stress (*SS*) decay with the increase of scour depth as calculated using the *STC* function and compared to laboratory measurements that required up to eight days to generate a single point. This figure was developed to support procedures proposed by Annandale (1999) and Smith (1994) to estimate local scour in erodible rock.

**Conclusions**

A sediment transport function or effective *STC* formula described in this paper does represent the basic mechanisms that drive the local scour process. The effective *STC* can be incorporated into numerical models to simulate the local scour process without extensive calibration. The effective *STC* can be directly related to the effective stream power and effective shear stress that might be used by the other procedures for estimating local scour. The 3-D model using the proposed effective *STC* formula can



be used to simulate local scour for different scales. The numerically predicted scour depths precisely match the ones obtained from the laboratory experiments.



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