RISK REDUCTION AND SENSOR NETWORK DESIGN

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Abstract

Contamination warning systems (CWS) are a promising approach for detecting contaminants in drinking water systems in time to allow for the effective reduction of public health or economic impacts. In this work, the authors utilize TEVA-SPOT, the Threat Ensemble Vulnerability Assessment and Sensor Placement Optimization Tool, to address one of the most significant outstanding questions about sensor network design: how many sensors are needed in a water distribution system to reduce the risk of contamination incidents? Previous work has assumed that the number of sensors was determined prior to optimization for the best locations, and was typically limited by budget constraints.

In this paper, the authors use network models for several different water utilities that range in size from small to large systems. Sensor network designs are developed that minimize several objectives of concern to water utilities. The number of sensors needed is a question of acceptable risk and is shown to be dependent on the detection goals of the community, utility-specific hydraulics, the sensor technology, and the speed at which an effective emergency response can be achieved. The implications of these results to the widespread applicability and sustainability of CWS will be discussed.

1. INTRODUCTION

Contamination warning systems (CWS) are a promising approach for detecting contaminants in drinking water systems in time to allow for the effective reduction of public health or economic impacts (AWWA, 2005; USEPA, 2005a; ASCE, 2004). A CWS utilizes a variety of monitoring and surveillance methods such as online water quality monitoring and customer complaint surveillance. Optimization tools have been developed by many researchers to support the design of utility-specific CWS (Berry et al., 2005a, 2006b; Ostfeld and Salomons, 2004; Watson, Greenberg and Hart, 2004; Uber et al. 2004). One of these methods has been packaged into a software tool called TEVA-SPOT, the Threat Ensemble Vulnerability Assessment and Sensor Placement Optimization Tool (USEPA, 2008).

Applications of computational optimization methods have shown that this approach is superior to water industry experts in identifying effective sensor locations, achieving up to 80% better performance in reducing public exposure (Ostfeld et al., 2008; Berry et al., 2005b). Applications to several large-scale real water distribution systems have shown that the optimization methods are best utilized as a decision-making tool, with close involvement by policy makers and technical staff (Murray et al., 2008). Utilities are beginning to implement these tools themselves to design their own CWS (Skadsen et. al, 2008).

Previous work assumed that the number of sensors was known prior to optimization for the best locations. In practice, the number of sensors is usually limited by budget constraints, as the capital costs of sensors can be in the thousands of dollars. However, the question of how many sensors a utility needs in order to reliably reduce the risks of contamination incidents has not been answered.

In this work, the authors utilize TEVA-SPOT to address one of the most significant outstanding questions about sensor network design: how many sensors are needed by a community to reduce the risk of contamination incidents? Eight example network models from different water utilities that range in size from small to large systems are used in this analysis. Four of these networks have been introduced in previous research studies. Sensor network designs are provided for these networks in the appendix for different objectives and numbers of sensors. In addition, several criteria for acceptable risk are considered and results are provided for all networks.

2. METHODOLOGY AND ASSUMPTIONS

Modeling, simulation, and optimization tools are used to simulate contamination incidents in a variety of water networks, optimally select locations for sensors, and compare levels of acceptable risk. Several water distribution networks are considered in this analysis, from small test networks to large real-world networks.

EPANET is used to simulate contamination incidents. Each incident is assumed to involve the injection of a 55-gal drum of a biological contaminant into a single location in the distribution system. The biological contaminant is introduced over a 24-hour period beginning at midnight. The characteristics of the biological contaminant are purely hypothetical for the purposes of this analysis: the dose at which 50% of the population is infected is 100,000 organisms; the incubation period is 7 days, and the untreated fatality rate is 30%. The contaminant behaves like a conservative tracer in the water distribution system. In order to incorporate the uncertainty about likely injection locations, incidents are simulated at every node with a user demand in the network model.

TEVA-SPOT is a software tool developed by the U. S. Environmental Protection Agency, Sandia National Laboratories, Argonne National Laboratory, and the University of Cincinnati (USEPA, 2008). TEVA-SPOT incorporates optimization algorithms into a fast and flexible software tool; its capabilities are summarized by Hart et al. (2008) and described in detail by Berry et al. (2005a, 2006b). TEVA-SPOT allows a user to specify a wide range of modeling inputs and performance objectives for CWS design. The development of this tool has focused on ensuring that it can be applied to large drinking water systems on typical desktop computers with 2 gigabytes of RAM memory.

In this paper, three objectives are used when optimizing for sensor placement: the number of persons exposed to a contamination incident (PE), the number of incidents detected (ID), and the extent of contamination (EC) measured in pipe feet. TEVA-SPOT has the ability to handle multi-objective optimization; however, that is not the focus of this paper.

TEVA-SPOT requires users to select modeling parameters that describe sensor performance and utility response time. In this analysis, sensors are assumed to be "perfect" in that they have a zero detection limit and are always accurate (no false positives or false negatives). Utility response to sensor detection is also assumed to be perfect and instantaneous, meaning that following detection, a "Do Not Use" order is issued and made effective immediately, preventing all further consumption. These assumptions are referred to throughout the paper as "perfect sensors and perfect response." It should be noted, however, that these do not translate into perfect performance of a sensor network. For example, even with perfect sensors and perfect response, some incidents may not be detected by a sensor network at all, or may be detected many hours after the incident first began.

The perfect sensors and perfect response assumptions are not a necessary constraint of the modeling tools; rather, they were chosen purposefully to demonstrate the upper bound on sensor network design performance. The results in the rest of the paper, then, reflect the best one could hope for from a sensor network. This is intended to provide readers with a lower bound on the number of sensors needed to meet various risk criteria defined by the authors.

3. DESCRIPTION OF NETWORKS

In this paper, 8 test networks are examined. Two networks are taken from EPANET tutorials (Net 2 and Net 3). Two of the networks (Net1 and Net 4) were used in the Battle of the Water Sensor Networks (BWSN), see Ostfeld et al., 2008. The remaining four networks are models of very large water distribution systems (serving more than 100,000 customers) but are not available due to security restrictions.

Net1. The first test network is Network 1 from BWSN. This network has 126 junctions, 1 reservoir, 2 tanks, 168 pipes, 2 pumps, and 8 valves. Assuming that the average person utilizes 200 gallons per day, the total population served by Net 1 is 6,200 people, and the total length of pipe is 123,000 feet.

Net 2. The second test network is EPANET example network 1 with 9 junctions, 1 reservoir, 1 tank, 12 pipes, and 1 pump. The average residence time for the entire network is 19 hours, with a maximum of 96 hours. The total population served by Net 2 is 7,600. The total length of pipe in the system is 63,530 feet.

Net 3. The third test network is EPANET example network 3 with 92 junctions, 2 reservoirs, 3 tanks, 117 pipes, and 2 pumps. The average residence time for the network is 14 hours, while the maximum is 130 hours. Based on the average base demands, the total population served by Net 3 is 66,394. The total length of pipe in the system is 215,712 feet.

Net 4. The fourth test network was also used in the BWSN. This network has 12,523 junctions, 2 reservoirs, 2 tanks, 14,822 pipes, 4 pumps, and 5 valves. The total population is 142,500 people, and the total number of pipe feet is 5,600,000 feet.

Net 5. The fifth test network has approximately 3,000 junctions, 40 reservoirs, 20 tanks, 4,000 pipes, 100 pumps, and 20 valves. Based on the average base demands, the total population served by Net 5 is 200,000. The total length of pipe in the system is 4,100,000 feet.

Net 6. The sixth test network has approximately 2,200 junctions, 10 reservoirs, no tanks, 6,000 pipes, 1 pump, and 100 valves. Based on the average base demands, the total population served by Net 6 is 450,000. The total length of pipe in the system is 2,700,000 feet.

Net 7. The seventh test network has approximately 8,000 junctions, 2 reservoirs, 20 tanks, 10,000 pipes, 100 pumps, and 200 valves. Based on the average base demands, the total population served by Net 7 is 850,000. The total length of pipe in the system is 9,400,000 feet.

Net 8. The eighth test network has approximately 2,500 junctions, 5 reservoirs, 50 tanks, 4,000 pipes, 100 pumps, and 20 valves. Based on the average base demands, the total population served by Net 8 is 1,200,000. The total length of pipe in the system is 7,500,000 feet.

4. PERFORMANCE OF SENSOR NETWORKS

Even with perfect sensors and perfect response, a sensor network may not detect every event, detect events in a timely manner, prevent all exposure to contaminants, or prevent contamination of pipes. To achieve this perfect performance, in most networks, sensors would need to be placed at almost every junction. This is clearly not feasible in practice; thus, the importance of selecting a small number of sensor locations using optimization software.

Given the available sensor placement tools, like TEVA-SPOT, users can select a single optimization objective that determines the best locations for the sensors. This allows one to develop trade-off curves (see Figure 1) that demonstrate the relationship between the number of sensors (cost) and the benefit provided by the sensor network (calculated for a single objective).

Figure 1 shows the trade-off curve based on the PE objective for Net 1 with 126 junctions. In the absence of a sensor network, over all 79 incidents simulated, an average of 1,145 persons would be exposed to the contaminant, out of a total population of 6,200. A single sensor, optimally located, reduces exposures to 410 people on average. Thus, the first sensor prevents an average of 735 exposures. This can also be stated by saying that the first sensor provides a marginal benefit of 735. Two sensors, optimally located, reduce exposures to 166 people. The second sensor, then, provides a marginal benefit of 244. The third sensor provides a marginal benefit of 31 people. After 10 sensors have been placed, the average number of exposures is reduced to 40 people, and it takes 40 sensors to reduce the average exposure to zero. Each additional sensor yields less and less benefit, reflecting the diminishing marginal returns of sensor placement optimization algorithms.

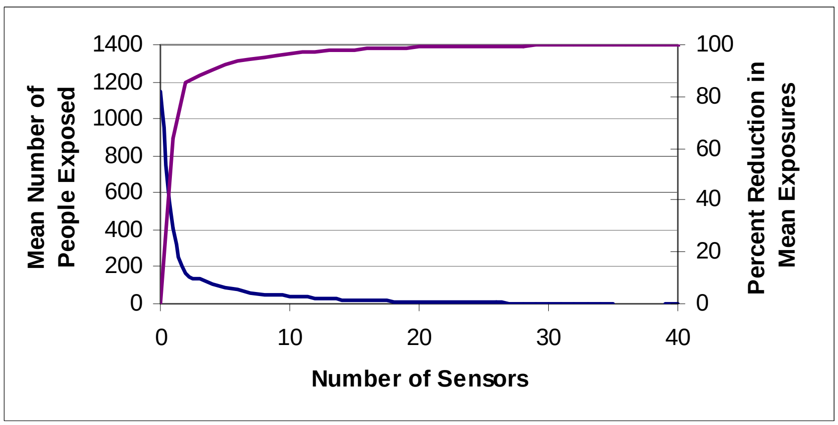
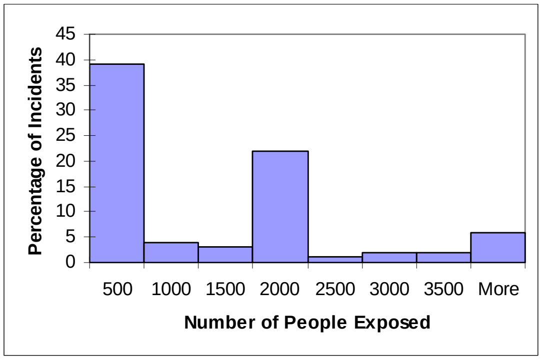


Figure 1: Sensor network design trade-off curve for Example Net 1 based on the number of people exposed (PE) objective. The decreasing line shows the number of people exposes decreases as the number of sensors increases. The increasing line shows the percent reduction in mean exposures.

Decision makers could make reasonable yet conflicting conclusions from Figure 1. For instance, a decision maker could say that given budget concerns, placing two sensors provides substantial public health benefit to the system but that no additional costs can be justified (because the marginal benefits decrease dramatically). Another decision maker could say that designing a system that would expose more than 150 people (or 2.4% of the population) on average is not acceptable in any circumstances.

For the PE objective, the best two locations selected by TEVA-SPOT are Junction-23 and Junction-118 (referenced by the EPANET node ID’s). If the sensor network consists of two sensors, the number of people exposed is reduced by 86% over all the contamination incidents simulated. Figure 2 shows the distribution of PE over the 79 scenarios, first in the absence of sensors and then for the two-sensor design. The mean and maximum number of people exposed is reduced significantly by two sensors; however, there are 4 remaining incidents that impact more than 500 people. For this design, 47% of the incidents are detected with the two sensors (i.e. 53% are not), and an average of 12,000 pipe feet are contaminated. Is this level of risk reduction acceptable? Are there additional criteria that should be considered?



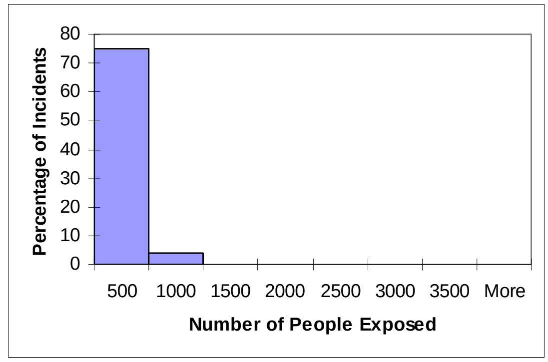


Figure 2: Histograms of the percentage of incidents resulting in a given number of people exposed for the case with no sensors (left) and the case with a two-sensor design (right).

Net 1. Figure 3 shows trade-off curves for three different objectives for Net 1. Nine sensors are needed to detect all incidents in the network; however, it takes 40 sensors to prevent all exposures and 72 sensors to prevent all pipe contamination. Table A1 lists the top twenty sensor locations selected by TEVA-SPOT (the numbers listed are the EPANET node ID's).

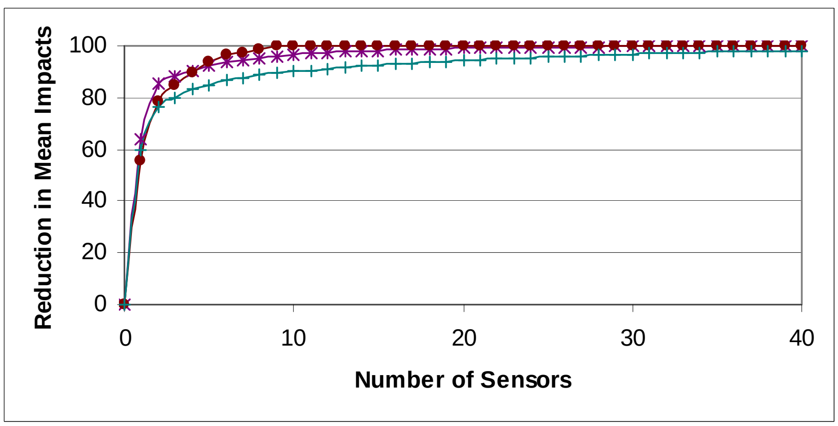


Figure 3: Net 1 trade-off curves for the number of people exposed (stars), the number of incidents not detected (diamonds – equivalent to the percentage of incidents detected), and the number of pipe feet contaminated (plus signs).

Net 2. Figure 4 shows trade-off curves for three different objectives for Net 2. Only two sensors are needed to detect all incidents in the network; however, in the time that it takes for these sensors to detect all incidents, the contaminant may have exposed several people and contaminated pipe walls, leading to substantial impacts. It takes seven sensors to prevent all exposures and prevent all pipe contamination.

Table A2 in the Appendix lists the sensor locations selected by TEVA-SPOT (the numbers listed are the EPANET node ID's). For the EC objective, locations are selected that are near to the source and tank, locations that can influence the entire network resulting in large EC values. On the other hand, the PE objective selects locations in the central part of the network, closer to high population/demand nodes.

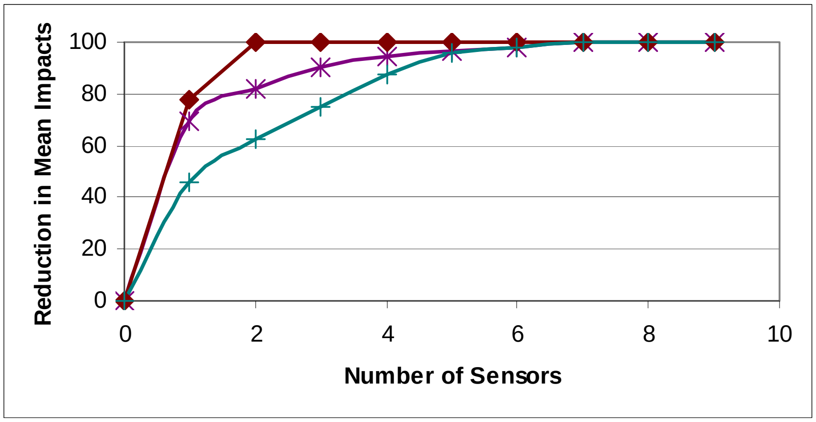


Figure 4: Net 2 trade-off curves for the number of people exposed (stars), the number of incidents not detected (diamonds– equivalent to the percentage of incidents detected), and the number of pipe feet contaminated (plus signs).

Net 3. Figure 5 shows trade-off curves for three different objectives for Net 3. Eleven sensors are needed to detect all incidents in the network; however, it takes 29 sensors to prevent all exposures and 49 sensors to prevent all pipe contamination. Table A3 lists the top twenty sensor locations selected by TEVA-SPOT (the numbers listed are the EPANET node ID's).

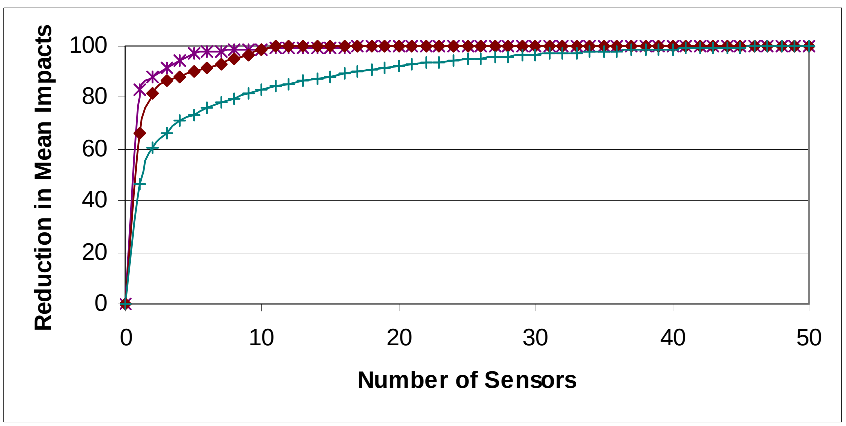


Figure 5: Net 3 trade-off curves for the number of people exposed (stars), the number of incidents not detected (diamonds – equivalent to the percentage of incidents detected), and the number of pipe feet contaminated (plus signs).

Net 4. Figure 6 shows trade-off curves for three different objectives for Net 4. In this large detailed network, nearly 30 sensors are needed to detect 50% of all incidents in the network. This network is much more detailed than some of the others (i.e., less skeletonized) and therefore it is much more difficult to detect a large percentage of events. However, it should be noted that many of the events have very few impacts.

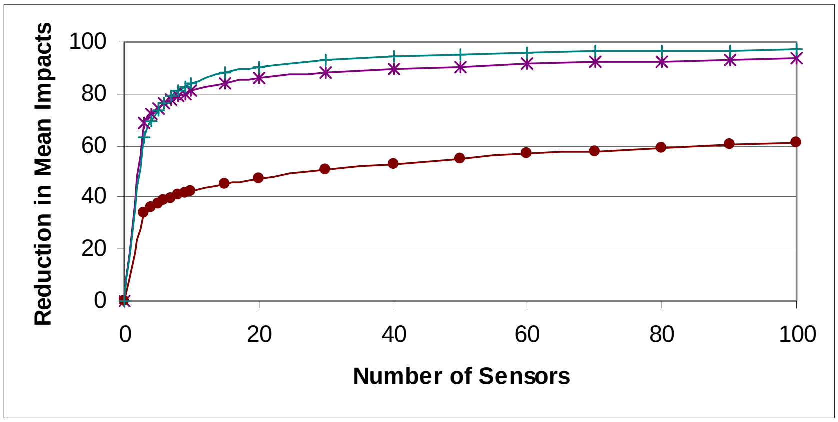


Figure 6: Net 4 trade-off curves for the number of people exposed (stars), the number of incidents not detected (diamonds - equivalent to the percentage of incidents detected), and the number of pipe feet contaminated (plus signs).

Similar curves can be generated for the remaining four networks. These plots are not shown, however the results from these networks are captured in the data tables in the following section.

5. ACCEPTABLE RISK

In this section, multiple metrics for acceptable risk are considered and presented for each of the eight networks. The metrics include specific limits on the number of people exposed (PE), the number of incidents detected (ID), and the number of pipe feet (extent of pipe) contaminated (EC).

Table 1 lists the number of sensors needed to meet each of several public health metrics across the 8 example networks. The public health metrics specify a maximum value of PE, varying between 10 and 10,000 persons. Table 2 lists the number of sensors needed to meet several coverage objectives across the 8 example networks. The coverage metrics measure the percentage of contamination incidents detected by the sensor network, from 40-90% of incidents. Table 3 lists the number of sensors needed to meet several economic objectives for the 8 networks. The economic metric is measured in terms of the number of pipe feet contaminated, from 1-100 miles.

Table 1: Number of sensors needed to achieve public health objective. PE is the number of people exposed to a contaminant. *This metric is beyond the resolution of the utility network model because of skeletonization.

Metric/Network	Net 1	Net 2	Net 3	Net 4	Net 5	Net 6	Net 7	Net 8
Population	6.2K	7.6K	66K	142K	200K	450K	840K	1,200K
Mean PE < 10,000	0	0	1	0	0	0	1	2
Mean PE < 1,000	1	1	3	19	6	10	38	154
Mean PE < 500	1	2	5	85	21	47	125	*
Mean PE < 100	5	5	11	*	*	*	*	*
Mean PE < 10	24	7	24	*	*	*	*	*

Table 2: Number of sensors needed to achieve coverage objective (number of incidents detected (ID)). ID is the percentage of incidents detected. +Note that the sensor placements were only calculated for up to 100 sensors, and these metrics required more than 100 sensors.

Metric/Network	Net 1	Net 2	Net 3	Net 4	Net 5	Net 6	Net 7	Net 8

Incidents	79	9	90	11,000	1,800	2,200	7,000	1,400
Mean ID > 40%	1	1	1	8	2	1	1	1
Mean ID > 50%	2	1	1	30	2	2	3	2
Mean ID > 60%	2	1	1	90	2	4	6	5
Mean ID > 70%	2	1	2	+	4	5	21	11
Mean ID > 80%	3	1	2	+	25	15	75	28
Mean ID > 90%	5	2	6	+	+	+	+	80

Table 3: Number of sensors needed to achieve economic objective. EC is the extent of contaminated pipe measured in feet. *This metric is beyond the resolution of the utility network model given existing pipe lengths.

Metric/Network	Net 1	Net 2	Net 3	Net 4	Net 5	Net 6	Net 7	Net 8
Total pipe feet	123K	64K	216K	5.6M	4.1M	2.7M	9.4M	7.5M
EC < 528,000	0	0	0	0	1	0	1	1
EC < 52,800	0	0	0	7	12	10	25	27
EC< 5,280	7	4	16	*	*	*	*	*

6. DISCUSSION

Typically, water utilities use a combination of budget constraints and sensor network design performance curves in order to determine the appropriate number of sensor stations to install in a distribution network. In Net 1, an analysis of Figure 1 may lead one to determine that 2 sensors is the most appropriate number. However, Tables 1-5 show that with only 2 sensors, a contamination incident would be likely to result in more than 1 mile of contaminated pipe, 30% of incidents not detected, and more than 100 people exposed. Using additional risk criteria may persuade the utility to install additional sensors.

There is no defined “acceptable risk criteria” for CWS. In this paper, the authors consider several measures of risk based on the number of people exposed, the percentage of incidents detected, and the extent of contamination in a distribution system. The results of this paper show the following:

- The number of sensors depends on the risk reduction goals of the water utility.
- The number of sensors predicted for the eight example networks does not increase linearly with population. This is due to at least two factors: (1) the skeletonization of models (which results in overestimation of impacts and underestimation of detection time) and (2) system-specific hydraulics and operations.
- Given that real-world implementation of CWS will not use perfect sensors or involve a perfect response, the number of sensors needed is larger than the numbers given in Tables 1-3.
- Cheaper, more reliable sensor technologies are needed to support high performing contamination warning systems that satisfy the risk reduction goals of water utilities.

The number of sensors needed in a water distribution system is a question of acceptable risk. Acceptable risk must be defined by the water utility, and thus is highly dependent on the detection goals of the community. The risk reduction goals of communities can vary widely from striving to detect only catastrophic incidents, to detecting as many incidents as possible (including accidental cross connections). The utility may have broad goals, such as widespread coverage of the distribution system (for example, sensors in every pressure zone), detection of a large number of contaminants, and specific goals, such as preventing events that would be expected to impact more than 100 people. Using a multi-objective analysis may help to improve the performance of sensor designs across several objectives; however, there will always be a trade-off in performance when balancing performance with costs. In order to design and implement an effective contamination warning system, utilities must comprehend the performance trade-offs of the system they designed.

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APPENDIX: Sensor Locations

Number of Sensors /Performance Objective	Number of people exposed	Number of incidents detected	Number of pipe feet contaminated
1	5	81	9
2	23 118	83 126	20 23
3	23 118 131	83 123 126	20 23 28
4	23 81 118 131	83 100 123 126	17 20 23 28
5	23 81 97 102 118	10 83 100 123 126	17 20 23 28 33
6	17 23 81 97 118 131	10 45 83 100 123 126	17 23 28 33 35 52
7	17 23 81 97 118 126 131	10 45 83 100 123 124 126	17 23 28 33 35 52 77
8	17 28 58 81 97 118 126 131	10 45 83 100 114 123 124 126	17 28 30 33 35 52 53 77
9	17 28 35 58 81 97 118 126 131	11 19 45 83 100 114 123 124 126	17 28 30 33 35 52 53 70 77
10	17 28 35 58 81 93 98 118 126 131	11 19 45 83 100 114 123 124 126 (9 sensors)	17 28 30 33 35 52 53 70 77 102
15	17 28 34 35 45 52 55 68 81 93 98 102 118 123 126	11 19 45 83 100 114 123 124 126 (9 sensors)	4 17 22 23 28 30 33 34 35 52 53 70 77 89 102
20	4 17 28 34 35 39 45 52 55 68 70 76 82 84 93 98 102 118 123 126	11 19 45 83 100 114 123 124 126 (9 sensors)	0 5 20 22 23 25 30 33 34 35 39 44 52 53 70 77 89 97 102 104

Table A1: Optimal sensor locations for Example Net 1.

Number of Sensors /Performance Objective	Number of people exposed	Number of incidents detected	Number of pipe feet contaminated
1	22	23	12
2	21 22	23 32	10 12
3	13 21 32	23 32	10 11 12
4	21 22 23 32	23 32	10 11 12 21
5	13 21 22 23 32	23 32	10 11 12 21 22
6	13 21 22 23 31 32	23 32	10 11 12 21 22 31
7	12 13 21 22 23 31 32	23 32	10 11 12 13 21 22 31

Table A2: Optimal sensor locations for Example Net 2.

Number of Sensors /Performance Objective	Number of people exposed	Number of incidents detected	Number of pipe feet contaminated
1	203	243	189

2	179 203	15 243	111 207
3	179 203 255	15 219 243	111 141 209
4	15 179 203 255	15 131 219 243	111 141 179 209
5	15 35 111 203 247	15 35 131 219 243	111 121 141 179 209
6	15 35 111 184 203 255	15 35 131 219 231 243	111 121 123 141 179 209
7	15 35 111 147 184 203 255	15 35 131 166 219 231 243	111 119 121 123 141 179 209
8	15 35 111 147 184 203 229 255	15 35 131 166 203 219 231 243	101 111 119 121 123 141 179 209
9	15 35 101 111 147 184 203 229 255	15 35 131 166 203 219 225 231 243	101 111 119 121 123 141 179 209 229
10	15 35 105 109 149 191 203 209 229 255	15 35 131 166 203 219 225 231 243 253	101 111 119 121 123 127 145 179 209 229
15	15 35 101 103 109 111 147 185 191 203 209 217 229 247 253	15 35 131 166 167 203 219 225 231 243 253 (11 sensors)	101 109 113 119 121 123 127 145 157 159 161 185 209 229 255
20	15 35 40 101 103 109 113 131 147 191 203 205 211 215 219 229 247 251 253 3	15 35 131 166 167 203 219 225 231 243 253 (11 sensors)	101 107 109 117 119 121 123 127 141 147 157 159 161 163 185 191 201 211 229 255

Table A3: Optimal sensor locations for Example Net 3.