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Exchange Rates, Optimal Debt Composition, and Hedging in Small Open Economies

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Abstract

This paper develops a model of the firm's choice between debt denominated in local currency and that denominated in foreign currency in a small open economy characterized by exchange rate risk and hedging possibilities. The model shows that the currency composition of debt and the level of hedging are endogenously determined as optimal firms' responses to a tradeoff between the lower cost of borrowing in foreign debt and the higher risk of such borrowing due to exchange rate uncertainty. Both the composition of debt and the level of hedging depend on common factors such as foreign exchange rate risk and the probability of financial default, interest rates, the size of firms' net worth, and the costs of managing exchange rate risk. Results of the model are broadly consistent with the lending and hedging behavior of the corporate sector in small open economies that recently experienced currency crises. In particular, unlike the predictions of previous work in the literature on currency crises, the model can explain why the collapse of the fixed exchange rate regime in Brazil, in early 1999, caused no major change in the currency composition of debt of the corporate sector.

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1 Introduction

Recent currency crises in East Asia and Latin America have been characterized by currency mismatches between assets and liabilities, as well as inadequate hedging, in the balance sheets of the corporate sector.[1] Some researchers have argued that the mismatch between foreign currency liabilities and domestic currency assets in firms' balance sheets has been the root cause of the large output collapses following currency depreciations (Krugman, 1999; Aghion, Bacchetta, and Banerjee, 2001; and Schneider and Tornell, 2000). Under a fixed exchange rate regime, firms view fixed exchange rates as a guarantee and thus fail to insure their foreign exposure.[2] A direct implication of this line of reasoning is that once the exchange rate is allowed to float, firms will recognize their exposure, and foreign currency loans will be viewed as more costly so that firms will reduce their currency mismatches by downsizing their foreign currency borrowing. Contrary to this prediction, firm-level evidence from Brazil over the period 1997-2001 suggests that the collapse of the fixed exchange rate regime in 1999 caused no major change in the currency composition of corporate-sector debt. This paper attempts to explain this apparently surprising result.

To study this phenomenon, the paper develops a theoretical framework that examines the firm's choice of local and foreign currency debt in the presence of exchange rate risk and hedging possibilities. The model shows that decisions about the currency composition of debt and the optimal level of hedging are interrelated and depend on common factors such as exchange rate risk and the probability of financial default, interest rates, the size of firms' net worth, and the costs of managing foreign currency risk. The key element driving the model's results is the tradeoff that firms face between the lower cost of borrowing in foreign currency and the higher risk of such borrowing due to exchange rate uncertainty. When affordable, hedging reduces firms' exposure to currency risk and expands their capacity for borrowing in foreign currency. Thus, after a large currency depreciation—say, the collapse of a fixed exchange rate regime—hedged firms are able to

[1] A currency mismatch occurs when a large fraction of a firm's debt is denominated in foreign currency while all of its income and assets are denominated in domestic currency.

[2] This implication is consistent with another strand in the literature, which emphasizes that a moral hazard problem is introduced by the implicit bailout guarantees that the government provides when fixing the exchange rate. These guarantees bias the composition of debt toward foreign currency and eliminate incentives to hedge risk (Burnside, Eichenbaum, and Rebelo, 2001).

maintain or even increase their access to foreign currency borrowing.

In the model, forward-looking firms make their borrowing decisions based on expectations of currency depreciation. Thus, even under fixed exchange rates, as long as such firms perceive that the regime is no longer credible, they will anticipate a future depreciation and will start managing the devaluation risk by engaging in hedging activities. As seen in section 2, this strategy seems to have been pursued by some firms in Brazil, which were hedged with currency derivatives even during the fixed exchange rate regime. Furthermore, when financial markets provide enough tools to mitigate the effects of the devaluation risk, the collapse of the fixed exchange rate may cause only small changes in the currency composition of debt because, given the opportunity to hedge, companies may not reduce their foreign currency debt after the devaluation.

The model's predictions are broadly consistent with the lending and hedging behavior of corporations in small open economies that recently faced currency crises. The theory advanced by the model suggests that, when the economy moves from fixed to floating exchange rates, some firms change their financing policies, and the population of firms exposed to foreign exchange risk is altered. Firms with insufficient net worth and those unable to afford a hedge lose access to capital markets. Firms with high enough net worth and those with an ability to hedge increase their foreign debt. Firms that have intermediate net worth but are unable to hedge borrow less in foreign currency, turn to domestic banks, and are monitored by such banks to maintain their access to foreign capital markets. In a macroeconomic environment characterized by a moderate probability of currency depreciation and inexpensive hedging, these changes in the population of firms can offset each other so that the average currency composition of debt varies little across regimes.

The paper builds on the Holmstrm and Tirole (1997) model of credit extended to the small open economy. The analytical framework most closely related to this paper is that of Martinez and Werner (2002). They also adapt the model of Holmstrm and Tirole to the case of the small open economy and find that before the Mexican crisis of 1994, the decision to borrow in pesos or dollars depended on the exchange rate regime to the extent that the government provided an implicit guarantee by fixing the exchange rate. However, the authors treat the exchange rate as a deterministic variable, and no hedging strategies are discussed. In their model, as in most of the previous literature, large amounts of foreign currency debt implicitly represent higher exposure to exchange rate risk.

As mentioned earlier, previous theoretical models suggest that a large devaluation has a negative effect on companies' foreign currency borrowing and that it reduces currency mismatches. However, empirical findings on the relationship between exchange rate regimes and currency mismatches in corporate-sector balance sheets are mixed and still subject to debate. Some studies find that the adoption of a floating regime reduces currency mismatches by decreasing foreign currency borrowing and increasing levels of hedging, whereas others find that firms borrow even more after a currency crisis.[3] As Cowan, Hansen, and Herrera (2005) point out, many of the previous studies use the ratio of foreign debt to total debt as an exogenous proxy for currency mismatches. Using data from corporations in Chile, the authors argue that the lack of empirical consensus about the importance of balance sheet effects results from the endogeneity of the currency composition of debt and not from the absence of such effects. The model in the current paper accounts for the endogenous determination of both the level of hedging and the currency composition of debt and helps explain the apparent contradiction between theory and empirical evidence.[4]As demonstrated in the model, hedging allows corporations, in the aggregate, to maintain their access to international capital markets and to keep their currency composition of credit almost unchanged after a currency crisis.

The reminder of the paper is organized as follows. Section 2 outlines the motivating empirical facts for the model, using firm-level data from Brazil. Section 3 describes the model of optimal

[3]Empirical findings on currency mismatches and exchange rate exposure during fixed exchange rate regimes are in Burnside et al (2001) in the Asian crisis, Tesar and Dominguez (2001) in 8 non-industrialized and emerging markets and Bonomo et al (2003) a study of the financial data of selected Brazilian companies. In a different study of Brazilian firms, Rossi (2004) finds that the adoption of a floating regime reduces the foreign vulnerability of the corporate sector by negatively affecting firms' foreign borrowing and positively affecting their hedging. However, Martinez and Werner (2002) find that firms in Mexico were able to increase their foreign debt borrowing after the crisis of 1995. Similarly, Allayannis et al (2002) find that nonfinancial firms in Asian countries were able to maintain substantial levels of foreign currency debt even after the currency crisis. But except for only a few studies, such as Rossi (2004) and Allayannis et al (2002), most research in the empirical literature used data only on the currency composition of debt at the firm level, as data on hedging are often not available or hard to collect.

[4]Accounting for endogeneity in the decision to hedge, Berrospide, Purnanandam, and Rajan (2007) show that hedging with currency derivatives increased the foreign debt capacity of firms in Brazil and allowed them to maintain their investment after the currency crisis.

debt allocation and hedging at the firm level. Section 4 offers the main results of the model under fixed and flexible exchange rates. Section 5 concludes the paper.

2 Empirical Evidence on Foreign Currency Exposure and Hedging: Brazil, 1997-2001

This section examines firm-level data on Brazilian firms for the period 1997-2001, which covers the two years before the currency crisis of 1999, the two years after, and the crisis year itself. Like other Latin American countries, Brazil suffered unexpected reversals in capital flows after earlier crises in Mexico (1994), East Asia (1997), and Russia (1998). In Brazil, the macroeconomic adjustment caused by substantial capital outflows at the end of 1998 brought large and persistent swings in the exchange rate that finally led to the collapse of the crawling peg and a sharp devaluation of its currency, the *real*, in January 1999.

Firm-level data consist of financial information for a balanced panel of 145 nonfinancial companies that were publicly traded on the São Paulo Stock Exchange (BOVESPA) over the period 1997-2001.[5] The information was taken directly from companies' annual financial reports. Data contain information on balance sheet variables such as the book value of assets, the currency composition of debt, and shareholders' equity. Data on hedging transactions were hand-collected and consist of the year-end notional value of currency derivatives (forwards, futures, swaps, and options), as reported in the explanatory notes to financial statements available at BOVESPA. Financial and state-owned firms are excluded because of their different motivation for using currency derivatives.

Table 1 illustrates the currency composition of firms' debt and the extent of their hedging activities across exchange rate regimes. As shown in the table, when the Brazilian economy moved from a fixed to a floating exchange rate regime, there was no major change in the currency

[5] The period was chosen because the corresponding data provide valuable information for a comparative analysis of firms' behavior regarding financing and risk-management policies before and after the currency crisis of 1999. Disclosure of information on derivatives was mandatory only after 1995.

composition of debt, measured by the ratio of foreign debt to total debt.[6] Over the sample, the ratio remains stable at around 43 percent. Across firms grouped by size, the foreign debt ratio seems to exhibit a similar pattern except for small firms that, on average, slightly decrease their ratios. In contrast, the extent of hedging operations, approximated by the ratio of the notional value of currency derivatives to foreign debt, increased from 5 percent in 1997 to 25 percent in 2001. Interestingly, the data show that hedging operations increased during the floating exchange rate regime but had already been observed during the fixed exchange rate period, especially for medium and large corporations. Large companies increased their hedging ratio, on average, from 6 percent during the fixed exchange rate regime to 22 percent during the floating regime. The observation of hedging activities under fixed exchange rates indicates that even before the currency crisis, and most likely after crisis episodes in East Asia and Russia, some firms indebted in foreign currency were managing the devaluation risk by hedging their positions with currency derivatives. As will be shown later, this behavior is consistent with the model's predictions concerning the hedging features of financing policies in small open economies subject to currency risk. Table 1 also suggests some evidence of costly hedging, as the increase in the hedging ratio occurred mostly at large and medium-sized companies, whereas small firms appear to have hedged only a small fraction of their foreign debt, even during the floating regime.[7]

An interesting feature regarding the hedging behavior of the corporate sector in Brazil is that most financial hedging transactions involve currency swap contracts. Unlike large U.S. nonfinancial corporations, which report currency forwards and options as the most commonly used tool to manage exchange rate exposure, similar Brazilian firms seem to prefer currency swaps, which effectively convert foreign debt into domestic debt through simultaneous transactions in the spot and forward markets. Broad preference for currency swaps is also consistent with costly hedging, as a swap reduces transaction costs by allowing companies to arrange with only one contract what

[6] Even after correcting for the valuation effect of currency depreciation in the ratio of foreign debt to total debt, there is still no major change in the currency composition of debt.

[7] Chan-Lau (2005) reports that as of 2004 the cost of using currency forwards, measured by the bid-ask spread as a percentage of the forward rate, was relatively high in Brazil (15 basis points) compared with the cost in other emerging-market countries (8 basis points in both Chile and Korea).

may otherwise take several transactions (for example, forward contracts) to replicate. [8]

Table 2 shows the number of firms that held financial debt (in domestic currency, foreign currency, or both), and the number of such firms that used currency derivatives, over the sample period. These data also provide evidence of minor changes in the corporate sector's borrowing behavior. Some firms dropped out of the sample, and the number of firms that borrowed only in domestic currency increased slightly during the floating regime. After the collapse of the exchange rate regime, the number of firms that hedged with currency derivatives increased from 7 in 1997 to 40 in 2001. Interestingly, about 46 percent of the firms with foreign debt remained unhedged in 2001. Overall, the evidence presented in this section seems to support the view that no significant changes occurred in the borrowing patterns of the Brazilian corporate sector during the period 1997-2001—that is, before, during, and after the currency crisis. Admittedly, obtaining a genuine measure of the extent of firm-level hedging is difficult, and the previous observations about hedging refer only to the use of currency derivatives. Firms may use hedging strategies other than derivatives, such as holdings of assets denominated in foreign currency and revenues from exports, which are not examined in the current analysis. That said, one point can be emphasized: The evidence from the sample of Brazilian firms is contrary to the results predicted in previous research. After the large depreciation that followed the collapse of the fixed exchange rate regime, firms still borrowed significantly in foreign currency, and more companies hedged this debt by participating in currency derivatives markets. As shown in the next section, the model's predictions of optimal debt composition and hedging levels are consistent with these empirical facts.

[8] Bonomo, Martins, and Pinto (2003) point out that Brazilian firms prefer currency swaps because they can obtain advantageous swap contracts from local banks. Banks, in turn, can offer these contracts because they are not exposed to exchange rate risk, as they hold government bonds indexed to the dollar in their portfolios. According to these authors, in the end, the hedge appears to be provided by the government through bank intermediation.

3 A Model of the Optimal Currency Composition of Debt and Hedging

Consider a small open economy described by a two-date model ($t = 0, 1$). The economy is populated by a continuum of risk-neutral firms, domestic banks, and foreign banks. Firms are run by wealth-constrained entrepreneurs who need to raise funds to cover their investment outlays. Firms' investment projects can be financed by borrowing in either domestic currency from local banks or in foreign currency from foreign banks. Given the uncertainty about the exchange rate, firms can choose to hedge their foreign exchange risk by signing forward contracts offered by local banks. At $t = 0$, firms sign debt contracts and make investment, borrowing and hedging decisions. At $t = 1$, exchange rate and investment returns are realized and claims are settled. Agents are protected by limited liability so that no party can end up with negative payoffs.

3.1 Investment Projects

Each firm has access to a project requiring an investment of fixed size I, where $I > 0$ at $t = 0$; at $t = 1$ the investment yields a verifiable return in domestic currency, R, in case of success and nothing in case of failure. Firms differ only in their initial capital, A (which is publicly observable). The distribution of firms is described by the cumulative distribution function, $F(A)$. It is assumed that $A < I$, so that firms need external funding to undertake their investments. Investment projects are of three types, as described in Figure 1: good, with a high probability of success, P_G; bad, with a low probability of success, P_B ($P_G > P_B$); or worse, also with a low probability of success. The bad project gives the entrepreneur a low private benefit, b, and the worse project gives the entrepreneur a high private benefit B, where $B > b > 0$. Firms face a moral hazard problem in choosing a project. In the absence of proper incentives or outside monitoring, entrepreneurs can divert resources by deliberately reducing a project's probability of success (from P_G to P_B) to enjoy the private benefit.

Local banks can monitor firms to reduce the moral hazard problem and eliminate the worse

Project	Good	Bad	Worse
Probability of success	P_G	P_B	P_B
Private Benefit	0	b	B

Figure 1: Three Types of Investment Projects
Note: For explanation of variables, refer to text.

project. Monitoring is costly so that local banks face a fixed cost, C, where $C > 0$. Foreign banks are uninformed investors (that is, they are unable to monitor firms) and have access to alternative projects that earn a gross rate of return, r^*, in international markets. Furthermore, it is assumed that given the rate of return on investor capital in domestic currency, r^f, only the good project has a positive expected net present value (NPV), even if the private benefit of the firm is included in the calculation:

$$P_G\ R - r^f\ I > 0 > P_B\ R - r^f\ I + B \tag{1}$$

3.2 Financing decisions and exchange rate risk

In what follows, firms are represented by the index f, domestic banks by the index m, and foreign banks by the index u. At $t = 0$, a representative firm invests all its funds, A, and signs debt contracts to borrow from local banks an amount in domestic currency, I_m, and from local banks an amount in foreign currency, I_u, so that

$$A + I_m + s_L\ I_u \geq I \tag{2}$$

where $s_L > 0$ is the exchange rate at $t = 0$, quoted as s_L units of domestic currency per unit of foreign currency.[9] Note that both the investment return and the exchange rate are uncertain, a departure from the original credit model of Holmstrm and Tirole (1997). In addition to

[9]It is also assumed that the internal rate of return of investment projects exceeds the market rate, r^f, which means that funds invested in the firm earn the external rate of return plus an amount due to the incentive effect. Moreover, there is no other source of funding (such as equity financing) in the model. As in emerging economies in Asia and Latin America, a large part of the funding for investments is provided in the form of bank loans.

incorporating the moral hazard structure of that model, the current setup extends the firm's investment problem not only by having the firm borrow in foreign currency from international capital markets but also by adding a new dimension to the problem—namely, there is the possibility of default due to exchange rate risk, and firms use hedging activities to mitigate that risk. Investment projects, then, are subject to two types of bad events: failure (economic default) or bankruptcy due to currency-led default (financial default). Exchange rate fluctuations, therefore, can turn solvent firms into bankrupt firms even when they undertake successful projects. It is assumed that in the event of any type of default, firms' creditors are left with nothing.[10]

For simplicity, consider only two possible states for the exchange rate: low (L) and high (H). At $t = 0$, the exchange rate is s_L. At $t = 1$, if the economy operates under a fixed exchange rate regime, then $s_1 = s_L$. Under a floating exchange rate regime, the exchange rate can either depreciate to $s_H > s_L$ with probability q, or remain unchanged with probability $1 - q$. To avoid losses due to exchange rate fluctuations, companies may hedge their foreign exchange risk with currency forward contracts.

Let F be the one-period forward exchange rate charged to the firm and defined as units of domestic currency per unit of foreign currency. Given the possibility of economic and financial default, it is assumed that forward markets are efficient so that F is the expected exchange rate at $t = 0$ and is given by the following expression:[11]

$$F = \begin{cases} (1-q)\ s_L + q\ s_H & \text{if financially solvent in both states } L, H \\ s_L & \text{if financially solvent only in state } L \\ s_H & \text{if financially solvent only in state } H \end{cases} \tag{3}$$

Note that $s_L \leq F \leq s_H$. Expression (3) states that the forward rate is adjusted according to the risk of financial default so that overhedging (for example, infinite hedging in case of zero transaction costs) is ruled out, as will be clear in the next paragraphs. Another important feature of the

[10] For example, in case of financial default, the firm is declared bankrupt, and its residual value is completely used to pay bankruptcy costs.

[11] As shown later, the forward rate is related to interest rates according to a covered interest parity condition.

hedging contract is that payments in either direction are contingent on the project's success. In other words, if the project fails and state L occurs, then the firm has nothing, so it pays nothing on the hedging contract; if state H occurs, the firm receives no payment either. This particular feature guarantees that a hedged firm will always be able to obtain a forward contract at the rate $F = (1 - q)\ s_L + q\ s_H$, which occurs because the hedging contract is offered by domestic banks so that payments in both directions are netted out on average.[12]

Let h be the amount of forward contracts, in foreign currency, purchased by a firm, and let ϕ be the transaction costs per unit of forward contract. For simplicity, let ϕ also be expressed in domestic currency per unit of foreign currency. At $t = 1$, if the project succeeds, proceeds from hedging operations in domestic currency are

$$\Pi^H = h\ (s_1 - F) - \phi\ |h| \tag{4}$$

Note that firms in the model are forward-looking agents. They make their debt and hedging decisions based on expectations of currency depreciation. Moreover, q can also be interpreted as the probability of devaluation under fixed exchange rates when the regime is not credible. In that case, q would be the conditional probability that the currency is devalued at $t = 1$ after having been fixed at $t = 0$.

A debt contract must specify the amount borrowed from, and paid to, each lender under all circumstances so that: 1) if the project fails, it pays zero to all parties; 2) at $t = 1$, if the project succeeds and the firm is solvent, the parties divide the total proceeds, which consist of project return R plus the proceeds from hedging operations (if any). Out of the total proceeds, the firm receives $R_f(s_1) \geq 0$, domestic banks receive $R_m \geq 0$, foreign banks receive $s_1 R_u \geq 0$, and forward sellers receive $\phi|h| \geq 0$.

Note that debt payments on the forward contract are made only if the project is successful and the firm is solvent. Note also that debt payments to lenders are independent of the exchange

[12]For simplicity, it is assumed that firms buy forward contracts from domestic banks other than those from which they borrow so that the transaction costs of hedging do not represent revenues to domestic lenders.

rate s_1 provided that the firm is solvent. However, returns to the entrepreneur (the equity-holder) depend on s_1.

Financial default occurs at $t = 1$ when the firm, even if undertaking a successful investment, cannot meet its financial obligations and goes bankrupt. Financial default is then given by the following condition:

$$R - R_f(s_1) - R_m(s_1) - s_1 R_u(s_1) + h(s_1 - F) - \phi|h| < 0 \tag{5}$$

As mentioned above, the forward rate in expression (3) rules out overhedging. For example, consider a firm that wants to buy a significantly large amount of forward contracts to increase its expected payment when s_H occurs; but default when s_L occurs. Because the firm is solvent only in state H, expression (5) becomes positive in state H but negative in state L. By setting $F = s_H$, sellers of forward contracts eliminate incentives to overhedge, as a firm that overhedges receives zero proceeds from the forward contract in the depreciation state.

The preceding analysis implies that the firm hedges its foreign exchange exposure to reduce the possibility of financial default. Furthermore, a firm that is hedged ex ante cannot be insolvent, or solvent in only one state. Because financial default may occur in either state L or H, let 1_L and 1_H be indicator variables such that

$$1_H = \begin{cases} 1 & \text{if solvent in state } H \\ 0 & \text{if default in state } H \end{cases}$$

$$1_L = \begin{cases} 1 & \text{if solvent in state } L \\ 0 & \text{if default in state } L \end{cases} \tag{6}$$

Accordingly, expected cash flows to the entrepreneur with zero proceeds from hedging, when the firm invests in the good project are:

$$R_f(s_H) = \begin{cases} P_G\ [R - R_m - s_H\ R_u] & \text{if solvent in state } H \\ 0 & \text{if default in state } H \end{cases}$$

$$R_f(s_L) = \begin{cases} P_G\ [R - R_m - s_L\ R_u] & \text{if solvent in state } L \\ 0 & \text{if default in state } L \end{cases} \tag{7}$$

3.3 Firm's expected profits

At $t = 0$, a firm with initial capital A and investment I chooses the currency composition of its debt (I_m and I_u), creditors' payments (R_m and R_u), its hedging amount (h) and its expected payments $R_f(s_L)$ and $R_f(s_H)$ to maximize expected total profits:

$$E[\Pi_{TOT}] = P_G[1_L(1-q)[R-R_m-s_LR_u]+1_Hq[R-R_m-s_HR_u]-\phi|h|]+r^f(I-A-I_m-s_LI_u) \tag{8}$$

subject to resource constraints, incentive and participation constraints and non-negativity constraints (for example limited liability).

In setting up the problem in this way, it is assumed that the firm undertakes only good projects with probability of success P_G. This assumption always holds because incentive constraints are imposed and only good projects have $NPV > 0$, which means that if bad projects are undertaken, no borrowing is obtained from creditors. Depending on the exchange rate realization, when the firm is financially solvent, project return, R, and proceeds from hedging operations are distributed among all the parties according to the following resource constraints:

$$R_f(s_L) + R_m + s_L\ R_u = R + h\ (s_L - F) - \phi|h| \tag{9}$$

$$R_f(s_H) + R_m + s_H\ R_u = R + h\ (s_H - F) - \phi|h| \tag{10}$$

When the firm defaults in state H or state L, it pays nothing to the other parties, and the residual value covers bankruptcy costs.

Let 1_m be an indicator variable such that $1_m = 1$ if the firm borrows from the local bank (with monitoring) and $1_m = 0$ if the firm borrows directly from foreign banks and not from local banks (without monitoring). Given the two possible states for the exchange rate, and the solvency condition associated with them, the firm invests in a good project whenever it obtains an expected payment greater than or equal to the expected payment of a bad project including private benefits. In other words, the firm invests in the good project when:

$$P_G\ [1_L\ (1-q)\ R_f(s_L)+1_H\ q\ R_f(s_H)] \geq P_B[1_L\ (1-q)\ R_f(s_L)+1_H\ q\ R_f(s_H)]+1_m b+(1-1_m)B \tag{11}$$

This is the firm's incentive constraint and can also be written as:

$$1_L\ (1-q)\ R_f(s_L) + 1_H\ q\ R_f(s_H) \geq 1_m\ \frac{b}{\Delta p} + (1-1_m)\ \frac{B}{\Delta p} \tag{12}$$

where $\Delta p = P_G - P_B > 0$.

3.4 Domestic bank lending

A local bank monitors the firm and finances the project whenever it receives an expected payment sufficient to cover the fixed monitoring cost, C. The bank's incentive constraint is given by

$$[1_L\ (1-q) + 1_H\ q]\ R_m \geq 1_m\ \frac{C}{\Delta p} \tag{13}$$

The bank is willing to finance the project if it receives a net expected payment (net of monitoring costs) that is, at least, equal to the opportunity cost of its funds. The participation constraint for the bank is given by

$$P_G\ [1_L\ (1-q) + 1_H\ q]\ R_m - 1_m\ C \geq r^f\ I_m \tag{14}$$

Let r be the domestic lending rate that local banks charge on I_m funds lent to the firm, defined as

$$r = \frac{P_G\ [1_L\ (1-q) + 1_H\ q]\ R_m}{I_m} \tag{15}$$

The minimum domestic rate of return, r, that is acceptable for a bank that decides to finance an investment project in domestic currency is determined by the condition

$$\frac{P_G\ C}{\Delta p} - C = r^f\ \frac{P_G\ C}{r \Delta p} \tag{16}$$

Expression (16) is obtained by combining conditions (13) through (15), all of them holding with equality, and assuming that the bank lends to the project so that $1_m = 1$. This condition, in turn, implies that

$$r = r^f \frac{P_G}{P_B} \tag{17}$$

Expression (17) states that the cost of domestic funds incorporates a risk premium relative to the risk-free rate. At the minimum rate of return acceptable for a bank, the risk premium equals the ratio of success probability of the good project to the success probability of the bad project.

Local banks can borrow and lend internationally and are able to replicate a forward contract. Perfect competition and non-arbitrage conditions ensure that banks are indifferent toward two available options: (1) lending in domestic currency at r^f; or (2) converting these funds, at $t = 0$, into foreign currency at the spot rate s_L; lending abroad in foreign currency, where the funds earn r^*; and converting them back to domestic currency at the forward rate F. These transactions imply that

$$r^f = r^* \frac{F}{s_L} \tag{18}$$

This is a covered interest rate parity condition and can also be expressed in terms of the domestic lending rate, r, and the international rate, r^*, by plugging expression (18) into expression (17):[13]

$$r = r^* \frac{F}{s_L} \frac{P_G}{P_B} \tag{19}$$

In this expression, $F \geq s_L$ and $P_G > P_B$, so that the bank lending rate, r, is always greater than the international interest rate, r^*. Thus, foreign debt is always preferred to domestic debt. The result that foreign currency debt is a cheaper source of capital than domestic debt is due to the moral hazard structure associated with investment projects. Intuitively, given that banks face a fixed cost to monitor firms, the lending rate in domestic loans incorporates not only the expected change in the exchange rate, given by the ratio F/s_L, but also a risk premium due to asymmetric information given by the ratio P_G/P_B. As a result, when necessary, firms minimize borrowing from a local bank and obtain the rest of their funds from foreign lenders.

The firm will be able to borrow in foreign currency if foreign banks are promised an expected payment greater than or equal to what they could obtain by investing their funds in international capital markets at the rate r^*. The foreign bank participation constraint, in foreign currency, is then

$$P_G \; [1_L \; (1-q) + 1_H \; q] \; R_u \geq r^* \; I_u \tag{20}$$

[13] Covered interest rate parity and efficient markets hypothesis are certainly very strong and simplifying assumptions. The incorporation of parameter ϕ, denoting the transaction costs involved in setting up a hedging program, is intended to add some friction to the model and make it a more realistic representation of foreign exchange markets. In practice, these transaction costs are expressed as bid-ask spreads in forward and spot rate quotations.

3.5 Firm's profit maximization

The firm's problem is to choose variables h, I_m, I_u, R_m, R_u, $R_f(s_L)$, $R_f(s_H)$, given exogenous parameters such as the firm's initial assets A; its fixed investment, I; the foreign rate of return, r^*; the probability of currency depreciation, q; and the cost of hedging, ϕ, in order to:

maximize $E[\Pi_{TOT}] =$

$$P_G[1_L(1-q)[R - R_m - s_L R_u] + 1_H q[R - R_m - s_H R_u] - \phi|h|] + r^f(I - A - I_m - s_L I_u)$$

subject to

$$A + I_m + s_L\ I_u \geq I \qquad \text{(i)}$$

If the firm is solvent, the following two conditions apply:

$$R_f(s_L) + R_m + s_L\ R_u = R + h\ (s_L - F) - \phi|h| \qquad \text{(ii)}$$

$$R_f(s_H) + R_m + s_H\ R_u = R + h\ (s_H - F) - \phi|h| \qquad \text{(iii)}$$

$$1_L\ (1-q)\ R_f(s_L) + 1_H\ q\ R_f(s_H) \geq 1_m\ \frac{b}{\Delta p} + (1-1_m)\ \frac{B}{\Delta p} \qquad \text{(iv)}$$

$$[1_L\ (1-q) + 1_H\ q]\ R_m \geq 1_m\ \frac{C}{\Delta p} \qquad \text{(v)}$$

$$P_G\ [1_L\ (1-q) + 1_H\ q]\ R_m - 1_m\ C \geq r^f\ I_m \qquad \text{(vi)}$$

$$P_G\ [1_L\ (1-q) + 1_H\ q]\ R_u \geq r^*\ I_u \qquad \text{(vii)}$$

$$R_f(s_H) \geq 0 \qquad \text{(viii)}$$

$$R_f(s_L) \geq 0 \qquad \text{(ix)}$$

Conditions (i) through (iii) are resource constraints, conditions (iv) and (v) are the incentive constraints for the firm and the domestic bank, conditions (vi) and (vii) are the participation constraints for domestic and foreign banks, and conditions (viii) and (ix) are non-negativity constraints for the firm's payments. Only one of the two non-negativity constraints will be binding when the firm defaults in one state of the exchange rate but is solvent in the other. Note that there are nine constraints to solve for seven choice variables, which means that in equilibrium at least seven constraints must be binding.

3.6 Equilibrium

The determination of the equilibrium, as well as the closed-form solution for the currency composition of debt and the optimal level of hedging are explained in detail in appendix 1. As an illustration, consider that both borrowing and hedging decisions are endogenously determined as optimal firms' responses to a tradeoff between the lower cost of borrowing in foreign debt and the higher risk of such borrowing due to exchange rate uncertainty. As explained earlier, foreign currency debt is preferred to domestic debt so that the firm always tries to borrow directly from foreign banks. The smaller the size of its initial assets, A, the more the firm demands from international banks. Because exchange rate uncertainty makes foreign debt risky, foreign banks demand that a firm have a minimum net worth and that it conduct hedging operations through currency forwards to ensure that the firm is solvent enough to repay its debt. If the firm's initial net worth is insufficient to meet the foreign bank's requirement, then the firm must be monitored and must first borrow from domestic banks to be able to borrow from international markets. Depending on the size of its net worth, the monitored firm may also need to hedge to demonstrate financial solvency.

Given that the internal rate of return is greater than the external rate on firm capital, entrepreneurs prefer to invest all their funds in the project. Therefore, constraint (i) in the firm's problem will be binding in most cases. The only time it will not be binding is when a firm must borrow only in domestic currency from local banks. Recall that local banks must cover the fixed monitoring costs when lending in domestic currency. This condition implies that, in equilibrium, local banks lend all firms the same fixed amount in domestic currency. Therefore, when the firm has a net worth insufficient to borrow only in foreign currency but high enough to borrow the fixed amount in domestic currency, and the sources of funds exceed the size of the investment, then constraint (i) is not binding and the firm must invest its excess funds at the market rate r^f. This situation is captured by the last term in the objective function of the firm's maximization problem.

Another key feature of the model is that hedging decisions are perfectly observed by creditors. An immediate implication is that firms have incentives to hedge their exchange rate risk to reduce the probability of financial default. When affordable, hedging increases expected profits by reducing the cost of capital to the firm —that is, by expanding the possibility of borrowing in foreign currency at a lower cost. The equilibrium level of hedging corresponds to any value within an optimal range defined by constraints (ii), (iii), (viii) and (ix) as shown in the appendix. Depending on exogenous parameters (in particular, q and ϕ) the minimum level of hedging in this range can be positive or negative (negative hedging means that the firm sells forward contracts). As shown in what follows, intermediate values of q and a small enough ϕ make the minimum level of hedging positive, and a profit maximizing firm will always choose this minimum level. Let $\overline{q}$ be the probability of depreciation that makes the minimum level of hedging equal to zero, and let $\overline{\phi}$ be the transaction cost in forward markets above which, for any given level of positive q, forward contracts are too costly to provide insurance.[14]These two cutoff values for parameters q and ϕ allow one to identify situations in which the firm optimally chooses not to hedge. When the probability of depreciation is too high—say, when $q > \overline{q}$ (for example, under extreme exchange rate volatility)— firms will find it optimal to choose a zero level of hedging. When hedging is too costly—say, when $\phi > \overline{\phi}$—then hedging, even if available, will not be affordable, and firms will also choose not to hedge.

A representative firm solves its profit maximization problem under two situations: when it hedges enough to avoid financial default and when it does not hedge at all.[15] Given that hedging is necessary but not sufficient for financial solvency, a firm that has sufficiently high net worth can borrow only small amounts of foreign currency debt and be financially solvent without hedging. Therefore, when the firm does not hedge at all, there are two additional possibilities: The firm is solvent even without hedging, or the firm defaults if it is not hedged. Each of these possibilities exists for a monitored company that borrows in both currencies and for a company that borrows only in foreign currency. These multiple cases imply that creditors demand different net worth levels depending on the firm's composition of credit and its hedging strategy.

Appendix 1 illustrates how to find these net worth requirements. For the purpose of analysis,

[14]It can be shown that $\overline{q} = 1 - (\frac{b}{\Delta p}\frac{s_H}{s_H - s_L})/(R - \frac{C}{\Delta p})$ and $\overline{\phi} = s_H - F = (1-q)(s_H - s_L)$ respectively.

[15]Partly hedging is ruled out because, in the case of currency depreciation, this level of hedging is insufficient to avoid default, and so the firm is forced to default as if hedging were zero in the first place. When the costs of hedging are sufficiently small but positive, the firm will prefer zero hedging to partly hedging.

the following paragraphs outline a brief description of all such requirements. Lower-bar net worth levels denote minimum net worth requirements for firms that are being monitored and that must first borrow from domestic banks, and upper-bar net worth levels denote the requirements for firms that borrow only from foreign banks. Accordingly, $\underline{A}_H$ is the minimum net worth for an optimally hedged firm that borrows in both domestic and foreign currency. Similarly, $\overline{A}_H$ is the minimum level of assets required for an optimally hedged firm that wants to borrow only in foreign currency from international banks.

When the firm does not hedge and, as a result, is not solvent in state H, the minimum net worth requirements are $\underline{A}_{NH}$ and $\overline{A}_{NH}$ for firms being monitored by local banks and for firms that want to borrow directly from foreign banks respectively. For a firm to be solvent in state H even if unhedged, the minimum net worth required by creditors is higher: $\underline{A}_{SNH}$ if monitored by local banks and $\overline{A}_{SNH}$ if it wants to borrow only from foreign banks. An unhedged but solvent firm faces the highest net worth requirements, whereas a firm that hedges to be solvent faces the lowest such requirements. In equilibrium, depending on the size of the firm's initial assets and its hedging strategy, financing possibilities vary. To decide on the optimal currency composition of its debt (for example, how much to borrow from each source) and the extent of hedging activities, the firm compares its initial assets, A, with these cutoff levels.

4 Hedging Costs, Exchange Rate Risk, and the Segmentation of Firms in Equilibrium

4.1 Equilibrium with costless hedging ($\phi = 0$)

A costless hedging equilibrium can be thought of as a long-run equilibrium in which currency forward markets are competitive and well developed so that the transaction costs in setting up a hedging program are negligible. The next two results illustrate the optimal financing policies when the economy operates in steady-state equilibrium under fixed exchange rates or under floating exchange rates.

Lemma 1 *In an economy with credible fixed exchange rates—that is, when $q = 0$—the optimal strategy for a firm is to borrow in foreign currency without hedging.*

Proof: Refer to appendix 2

This result explains why, for some firms, a fixed exchange rate biases the currency composition of their debt toward foreign currency and eliminates incentives to operate in forward markets. By fixing the exchange rate, the government provides the corporate sector with free risk management by creating a perception of no foreign exchange risk. [16] Consequently, some firms that otherwise would not be able to borrow at all, or some others that should be monitored to gain access to international capital markets, are now able to obtain foreign currency debt without constraints. This situation creates incentives for entrepreneurs to borrow extensively in foreign currency without hedging and to maintain currency mismatches in their balance sheets. As predicted by the balance sheet approach in the literature on currency crises, in the event of unexpected and large currency depreciation, the corporate sector in this economy could face widespread bankruptcy.

Lemma 2 *In an economy with floating exchange rates—that is, when $0 < q < \overline{q}$—the optimal strategy, in equilibrium, for a firm with net worth A such that $\underline{A}_H < A < \underline{A}_{SNH}$ or $\overline{A}_H < A < \overline{A}_{SNH}$ is to hedge its foreign currency debt enough, through currency forwards, to avoid bankruptcy.*

Proof: Refer to appendix 2

According to this result, during a floating exchange rate regime or a noncredible fixed exchange rate regime, a positive probability of currency depreciation creates incentives for a firm exposed to exchange rate risk to hedge and reduce the possibility of financial default. By reducing the net worth requirement, hedging allows companies to maintain or increase their funding from foreign banks, even in the presence of currency risk. In the absence of transaction costs in forward markets, hedging is always preferred to not hedging, and any firm that is insolvent in state H has incentives to hedge its currency exposure enough to avoid financial default. This is the case when the firm uses a mixture of domestic and foreign debt or when it borrows only in foreign currency. Note also

[16] If hedging is costless and $q = 0$, then net worth ratios are at their lowest values, and those for hedged and unhedged firms are equal, so that the firm is indifferent about how much it hedges because $F = s_L$.

that hedging is beneficial only for firms that are insolvent in state H; firms with net worth above the minimum requirement for solvency in state H do not need to hedge.

The previous two results, in equilibrium, for an economy in which hedging is costless determine a particular ordering of net worth requirements and define an equilibrium segmentation of firms into different categories depending on their demand for bank loans and their hedging strategy, as shown in figure 2. Well-capitalized firms, with net worth $A > \overline{A}_H$, finance their investment directly in foreign currency from international banks. Poorly capitalized firms, with $A < \underline{A}_H$, cannot invest at all since they have no access to any type of finance. Between these segments, somewhat capitalized firms, with $\underline{A}_H < A < \overline{A}_H$, can invest only to the extent that they are monitored and demand domestic bank loans. Firms in this monitoring region $[\underline{A}_H, \overline{A}_H]$ finance their investment with a mixture of domestic and foreign debt. As already mentioned, whether firms hedge their foreign debt or not depends on the size of their initial asset. Well-capitalized firms need not hedge if $A > \overline{A}_{SNH}$ but must hedge if $A \in [\overline{A}_H, \overline{A}_{SNH}]$. Similarly, somewhat capitalized firms need not hedge if $A \in [\underline{A}_{SNH}, \overline{A}_H]$ but must hedge if $A \in [\underline{A}_H, \underline{A}_{SNH}]$.

Although a typical firm in the monitoring region uses a mixture of domestic and foreign currency debt to finance its investment, some firms in the region use only domestic currency debt. As discussed earlier, firms with net worth $A < \overline{A}_H$ but $A + I_m > I$ demand bank loans only in domestic currency and invest their excess funds outside the firm at the market rate. The minimum net worth requirement for these firms is A_B, which is less than $\overline{A}_H$.

As shown in figure 2, firms in the monitoring region have different demands for bank loans and different hedging strategies. Some firms borrow in domestic and foreign currency and must hedge, whereas others borrow only in domestic currency and need not hedge. Note that in the equilibrium segmentation, firms may borrow only in domestic currency after having the chance to borrow in both currencies, but they may not reverse the order of that sequence of events. This feature of the model is explained by the fact that domestic loans are more expensive and are supplied in a fixed amount due to fixed monitoring costs. Therefore, firms in the monitoring region use the fixed amount in domestic currency only after exhausting the possibility of borrowing in foreign currency;

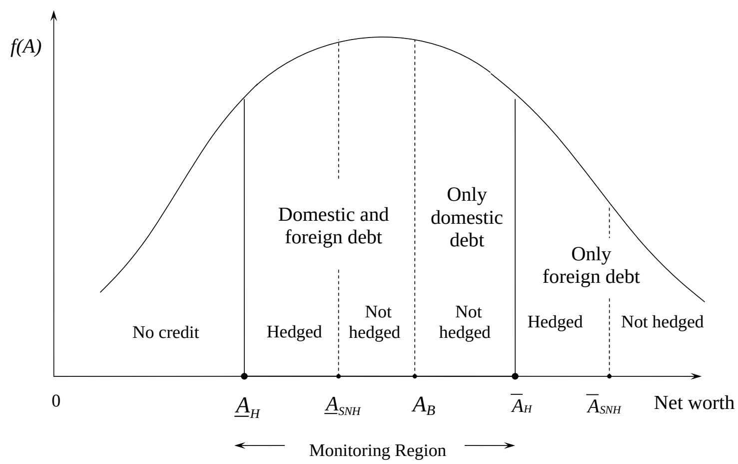

Figure 2: Equilibrium segmentation of firms

Note: The equilibrium segmentation is determined by net worth, currency composition of debt, and hedging strategy. Variables represent minimum net worth requirements for firms and are defined as follows: $\underline{A}_H$, for a firm that must hedge to be solvent and to borrow in domestic and foreign debt; $\underline{A}_{SNH}$, for a firm to be solvent even without hedging and to borrow in foreign debt; $\underline{A}_B$, for a firm to borrow only in domestic currency; $\overline{A}_H$, for a firm that must hedge to be solvent and to borrow directly from foreign banks; $\overline{A}_{SNH}$, for a firm to be solvent even without hedging and to borrow directly from foreign banks.

the extent of such borrowing depends on the size of a firm's net worth. For firms in the monitoring region, unlike firms that borrow only in foreign currency, bank monitoring makes the moral hazard problem less severe.

Lemmas 1 and 2 are consistent with previous predictions in the literature on currency crisis. In particular, they are consistent with the government guarantees approach, which highlights the incentives to borrow extensively in foreign currency, without hedging, when the exchange rate is fixed. When the exchange rate regime is flexible, or when the fixed exchange rate is not credible so that firms anticipate a currency depreciation ($q > 0$), firms will manage the foreign exchange risk by hedging enough to avoid financial default. Interestingly, the mechanism emphasized in the model is that, by lowering the net worth required by creditors for a given size of investment, hedging allows firms to maintain or increase their access to foreign currency debt.

4.2 Equilibrium with costly hedging ($\phi \geq 0$)

In the previous analysis when $\phi = 0$ net worth requirements $\underline{A}_{NH}$ and $\overline{A}_{NH}$ are irrelevant for the hedging decision. But this is not the case when hedging is costly. The next result shows that currency mismatches may arise during floating regimes because hedging opportunities, even if available, can be unaffordable for all firms when transaction costs are sufficiently high.

Proposition 1 Given costly hedging and a positive probability of depreciation, the optimal strategy for a firm in the monitoring region, regarding debt composition and hedging, depends on q and ϕ such that

1. when $q \geq \overline{q}$, the firm borrows the least it can in domestic currency from local banks, borrows the rest in foreign currency from foreign banks, and does not hedge its foreign debt.

2. when $0 < q < \overline{q}$ and $0 < \phi < (1-q)(s_H - s_L)$, the firm finds it optimal to borrow the least it can in domestic currency from local banks and the rest in foreign currency from foreign banks; moreover,

   2.1 if ϕ is small enough then the firm with $\underline{A}_H < A < \underline{A}_{SNH}$ will always hedge its foreign debt;

   2.2 if ϕ is sufficiently high, then only some firms with $\underline{A}_{NH} < A < \underline{A}_{SNH}$ will hedge their foreign debt.

3. when $\phi \geq (1-q)(s_H - s_L)$, the firm finds it optimal to borrow the least it can in domestic currency from local banks and the rest in foreign currency from foreign banks, and does not hedge its foreign debt.

Proof: Refer to appendix 2

Figure 3 illustrates all different hedging possibilities implied by Proposition 1. Notice that the minimum net worth requirement to obtain credit in both domestic and foreign currency varies depending on parameters q and ϕ. In case (1), an extremely high probability of depreciation makes the optimal hedging level zero so that no firm participates in currency forward markets. A similar situation occurs in case (3), when some firms have incentives to hedge but forward contracts are too expensive or nonexistent. Although these two situations seem similar in that firms do not hedge, they are in fact different and have different interpretations. Intuitively, higher values of both q and ϕ increase the net worth requirement, thereby making it harder to borrow in both domestic and foreign currency. In case (1), q is too high and the firm is forced to borrow only small amounts in foreign currency so that it is solvent in state H without hedging. In case (3), however, a firm that is insolvent in state H is still able to borrow large amounts of foreign debt even when it cannot afford to hedge. Such a firm has a currency mismatch in its balance sheet and would default if state H occurred.

The second case in proposition 1 corresponds to intermediate values of q and ϕ, which create incentives for firms to hedge. When transaction costs in hedging markets are small enough, any firm that needs to hedge to be solvent will opt to hedge. This situation -shown in figure 3 as case (2.1)— is similar to the result in Lemma 2. However, as the hedging cost increases, some firms have incentives to borrow in both currencies but find it optimal not to hedge their foreign debt exposure. In that case -shown in the figure as case (2.2)— $\underline{A}_{NH}$ becomes the relevant cutoff level defining the lower limit in the equilibrium segmentation. The proof of proposition 1 (refer to appendix 2) describes the cutoff level A^*, below which firms do not hedge because hedging is too costly. These firms would default in the event of currency depreciation. The situation in case (2.2) may be the result of economies of scale in hedging operations: Companies with small net worth tend to be highly leveraged and demand more foreign currency debt than larger firms; however, they cannot cover this exposure because the level of hedging required turns out to be unaffordable. In contrast, firms with higher net worth demand less foreign currency debt and require a lower level of hedging to manage their foreign exchange risk even if forward contracts are expensive.

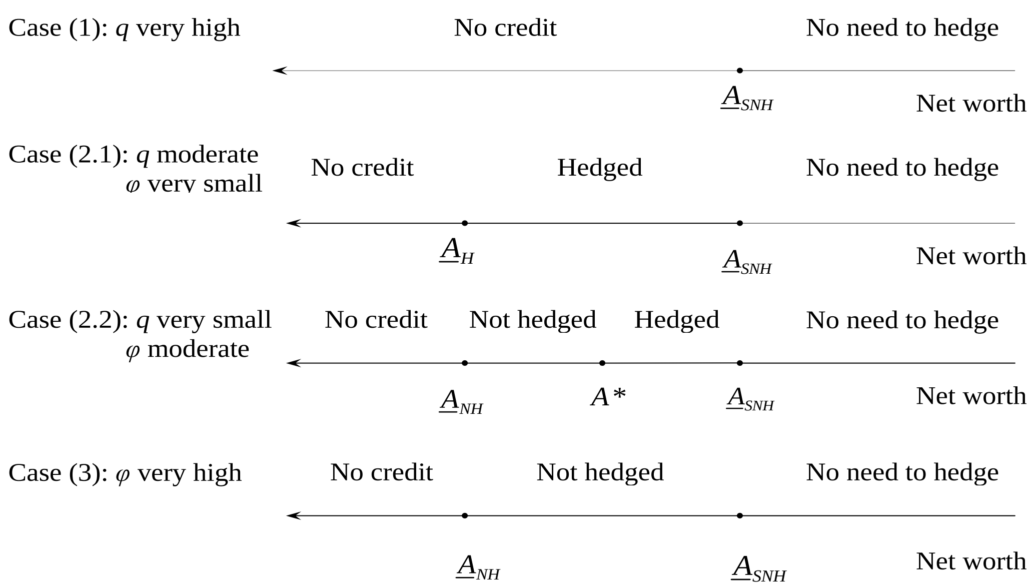

Figure 3: Hedging strategies for firms in the monitoring region.

Note: For explanation of each case, refer to text. Parameter q is the probability of currency depreciation; parameter ϕ is the transaction cost per unit of forward contract. Other variables represent net worth cutoff levels, defining hedging strategies, for firms that use a mixture of foreign and domestic debt. $\underline{A}_H$, is the minimum net worth requirement for a firm that must hedge to be solvent; $\underline{A}_{NH}$, is the minimum net worth requirement when the firm is not hedged and is insolvent in state H; $\underline{A}_{SNH}$, is the minimum net worth requirement for a firm to be solvent even without hedging, and A^* is a specific net worth requirement for firms that must hedge when hedging is moderately costly.

The preceding analysis shows that the equilibrium segmentation is determined by endogenous net worth requirements, which, in turn, depend on parameters describing the macroeconomic environment such as interest rates, exchange rates, the probability of currency depreciation, and hedging costs. All these parameters are the same for every firm. Hence, every investment project has a unique segmentation in equilibrium. Depending on its initial assets, a firm determines both its debt composition and its hedging strategy by locating itself within the segmentation. The next result, described by proposition 2, illustrates how different exchange rate regimes and different stages of development in forward markets affect the allocation of domestic and foreign debt and the selection of a hedging strategy.

Proposition 2 When the economy moves from fixed ($q = 0$) to floating exchange rates and, in the floating regime, $0 < q < \overline{q}$ and $\phi < (1-q)(s_H - s_L)$, all else equal:

1. fewer firms obtain funding for their investment.
2. some firms that borrow only in foreign currency from international banks during the fixed exchange rate increase their demand for domestic loans during the floating regime.
3. hedged firms remain able to borrow in both domestic and foreign currency, and increase their foreign loans after the economy moves to floating exchange rates.

Proof: Refer to appendix 2

To illustrate these results, consider that in equilibrium, domestic currency borrowing, I_m, is a fixed amount, and each firm in the economy demands the same fixed amount from local banks. The aggregate demand for domestic bank loans, D_m, is then

$$D_m(r^*, q) = [F(\overline{A}_H(r^*, q)) - F(\underline{A}_H(r^*, q))]I_m(r^*, q) \tag{21}$$

where the individual demand, I_m, is written as a decreasing function of both r^* and q. The aggregate demand for foreign currency loans is given by

$$D_u(r^*, q) = \int_{\underline{A}_H(r^*,q)}^{\overline{A}_H(r^*,q)} [I - I_m(r^*, q) - A]dF(A) + \int_{\overline{A}_H(r^*,q)}^{\infty} [I - A]dF(A) \tag{22}$$

Given perfect competition in the domestic banking sector, the supply of domestic funds is perfectly elastic at the domestic lending rate, r, and the supply of foreign loans is perfectly elastic at the international interest rate, r^*, which means that D_m and D_u determine the aggregate amounts of domestic and foreign lending in equilibrium.

An increase in q has an ambiguous effect on D_m because both cutoff levels, $\underline{A}_H(r^*, q)$ and $\overline{A}_H(r^*, q)$, increase, and there are two opposite effects. A first group of firms, those with insufficient net worth and those unable to hedge, cannot borrow at all; therefore, they drop out of the market, and D_m decreases. A second group of firms, those with insufficient net worth or an inability to hedge, cannot borrow only in foreign currency; therefore, they turn to domestic banks, and D_m increases.

An increase in q also has an ambiguous effect on D_u. Firms that had previously borrowed in foreign debt and then turned to domestic debt reduce their demand for foreign debt so that D_u drops. However, firms with intermediate net worth and those with the ability to hedge remain in the monitoring region and increase their demand for foreign debt, as I_m decreases for them; as a result, D_u increases. Notice that, by increasing the number of firms with access to foreign currency debt, hedging facilitates the rise in D_u, which compensates for, and may even surpass, the previous decline so that the demand for foreign currency debt may end up increasing on net.

In summary, some firms borrow less in both currencies, and others borrow more in foreign currency. The extent to which these changes in the population of firms affect the aggregate currency composition of debt depends on the distribution function, $F(A)$. Figure 4 illustrates the joint implications of propositions 1 and 2 for changes in the population of firms exposed to currency risk after the collapse of the fixed exchange rate regime. Note that, after the collapse of the regime, all net worth requirements shift to the right except $\overline{A}_{SNH}$ —the requirement for firms that borrow directly from foreign banks and are solvent without hedging. This is the case because $\overline{A}_{SNH}$ does not depend on the probability of depreciation, which means that firms with very large net worth are not concerned about currency risk when they borrow in foreign currency.

The switch to the floating regime shifts the monitoring region, which lies between $\underline{A}_H$ and $\overline{A}_H$ to the right, thereby making it more difficult for firms to borrow in foreign currency. However, firms that hedge are able to reduce their exposure, maintain their access to credit, and increase their foreign currency debt. Changes in hedging behavior can be significant or small depending on the size of q and ϕ. For example, as q increases and approaches $\overline{q}$, the number of firms that hedge

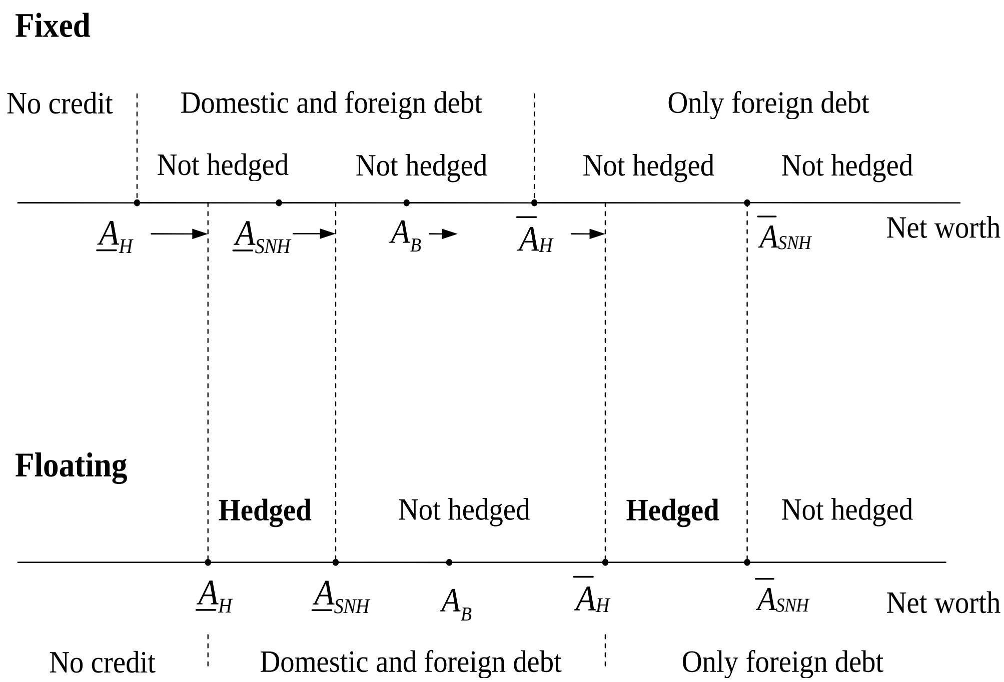

Figure 4: Changes in the equilibrium segmentation of firms from fixed to floating exchange rates
Note: For definition of variables that represent minimum net worth requirements, refer to note in figure 2.

with currency forwards decreases and can become very small if is sufficiently high as to prevent some firms from hedging.

4.3 The Brazilian experience

As a final illustration, consider how the results of the model match up with the currency crisis in Brazil in early 1999. The model's results suggest that the lack of major changes in the borrowing behavior of the corporate sector in Brazil could have reflected a moderate increase in the probability of currency depreciation and the existence of somewhat costless hedging. These outcomes, predicted by the model, are corroborated by the data in tables 1 and 2. After the floating regime was implemented in Brazil, the number of firms borrowing in both currencies declined, some firms dropped out of the sample (8 firms did so in 1999), and the number of firms borrowing only in domestic currency increased. Among the firms that continued borrowing in both currencies (for example, firms in the monitoring region), many were able to increase their debt in foreign currency, and the number of hedged firms increased (from 7 in 1997 to 40 in 2001). The increase in the number of hedged firms was also consistent with a rise in the volume of hedging operations, from 5 percent to 25 percent, within the same period. These changes affecting the demand for domestic and foreign currency debt seem to have offset each other. Even though the collapse of the fixed exchange rate regime increased the risk of borrowing in foreign currency, hedging allowed many firms to mitigate the risk and to maintain or increase their access to foreign debt so that the aggregate currency composition of debt did not change significantly across regimes.

Needless to say, in the real world, various other aspects of the macroeconomic situation, as well as firm-specific characteristics, affect the currency composition of lending and the hedging behavior of firms. Moreover, in contrast to what the model assumes, a currency crisis most likely affects firms' net worth so that the distribution of firms may not be constant across regimes. Furthermore, other parameters, such as the probability of success and the investment payoff, R, are certainly different across firms and are also affected by the collapse of the exchange rate regime. How companies deal with higher foreign exchange risk depends on changes in these variables, which are treated as invariant parameters in the model. For example, depending on specific characteristics such as export status, type of ownership, or the existence of other sources of funding, firms can adopt hedging strategies other than the use of currency forwards. Nevertheless, the model developed in the paper suggests changes in the behavior of a representative firm, and effects on the population of firms, that are broadly consistent with the empirical facts observed in recent currency crises in small open economies and, in particular, in Brazil over the period 1997-2001.

5 Conclusion

This paper seeks to understand the sources of currency mismatches in corporate-sector balance sheets in countries that recently faced currency crises. It does so by introducing hedging decisions in a model of optimal debt composition at the firm level. The model explains why some firms with access to foreign currency debt hedge their currency exposure and others do not, as an optimal response to appropriate incentives given by the macroeconomic environment. Under fixed exchange rates, firms borrow extensively in foreign currency and do not hedge, as they have no incentives to do so given that the government provides a type of free risk management. When the economy adopts a floating exchange rate regime, firms have incentives to use currency forwards to hedge their exposure to exchange rate risk and reduce the probability of financial default. Hedging reduces the net worth required by creditors and thus allows hedged firms to maintain or expand their capacity for borrowing in foreign capital markets.

Despite the obvious limitations of a partial-equilibrium analysis and some simplifying assumptions, the model provides an analytical framework to determine endogenously the currency composition of credit and the optimal degree of hedging at the firm level. In line with the empirical evidence in Brazil during the period 1997-2001, the model predicts that, given a moderate probability of currency depreciation and somewhat costless hedging, the changes in the population of firms after the economy adopts a floating regime can offset each other so that the currency composition of debt does not vary significantly across regimes.

The currency composition of debt remained stable in Brazil after the collapse of the exchange rate regime, and costless hedging operations seemed to provide an effective vehicle to reduce foreign exposure without affecting significantly the aggregate level of borrowing. The model highlights the importance of the development of currency derivatives markets to help the corporate sector manage the increase in foreign exchange risk resulting from macroeconomic shocks.

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Appendix 1: Model Solution

Determination of Equilibrium

There are four possible cases in the profit maximization problem of a representative firm. The maximized objective function is different in each of the four cases:

A. $1_H = 1$ and $1_L = 1$ if the firm is solvent in both states

B. $1_H = 1$ and $1_L = 0$ if the firm is solvent only in state H

C. $1_H = 0$ and $1_L = 1$ if the firm is solvent only in state L

D. $1_H = 0$ and $1_L = 0$ if the firm is insolvent in both states

Case (D) is the uninteresting case and is ruled out, as no lender will lend the firm any amount, thereby preventing the firm from undertaking the project. Moreover, given the features of the hedging contracts, firms that are solvent in only one state of the exchange rate are firms that chose $h = 0$. Therefore, expected profits for the remaining three cases are as follows

A. $E[\Pi_{TOT}] = P_G[R - R_m - F\ R_u - \phi|h|]$

B. $E[\Pi_{TOT}] = P_G[q(R - R_m - s_H\ R_u)]$

C. $E[\Pi_{TOT}] = P_G[(1-q)(R - R_m - s_L\ R_u)]$

Case (B) is also ruled out because it is always dominated by case (A). To understant why this statement is true, note that when $\phi = 0$ for any $0 < q < 1$, then $s_H > F$ and profits are greater in case (A) than in case (B). When $\phi > 0$, profits in case (B) will exceed those in case (A) only if both $q > \overline{q}$ and $\phi > \overline{\phi}$ which, in turn, will make $h = 0$, in which case profits in case (B) will again be smaller. Intuitively, when the firm is solvent in state H only, it is also solvent in both state H and state L even without hedging. Moreover, given positive transaction costs, there is no reason to hedge a positive amount when the firm is solvent in state H.[17] Therefore, the only relevant cases

[17] If solvent in state H, a firm can hedge only if hedging is negative—that is, when the firm sells forward contracts. However, for any positive transaction cost ϕ, negative hedging is also dominated by zero hedging.

to evaluate are (A) and (C), which means that in equilibrium the firm is always solvent in state L ($1_L = 1$) and case (C), insolvency in state H, happens only when the firm is not hedged at all.

Given that the firm is the residual claimant in debt contracts, constraints (v) through (vii) will be binding. This is the case because, to maximize profits, the firm chooses the minimum payments that make its creditors willing to participate financing the project.

A representative firm solves its maximization problem at $t = 0$ using the following algorithm:

1. Suppose that, given the lower cost of foreign debt, the firm decides to borrow directly from foreign banks ($1_m = 0$). Furthermore, suppose the firm judges itself as financially solvent without hedging ($1_H = 1, 1_L = 1$ and $h = 0$). Conditions (v) and (vi) jointly determine $I_m = 0$ and $R_m = 0$. Given A and I, the firm finds I_u using (i), which holds with equality, and then R_u is determined by (vii). Given that expected payments R_m and R_u are already pinned down, constraint (iii) is used to verify whether the firm is solvent in state H. If it is, then the firm can in fact borrow directly from foreign markets without hedging.

2. If the firm is insolvent in state H and decides not to hedge, foreign creditors adjust their expected payment. R_u depends on q and is larger to compensate for the possibility of default. Constraint (viii) is binding so that $R_f(s_H) = 0$ (for example, if state H occurs, the residual value of the firm covers bankruptcy costs). Constraint (ii) gives the firm's own payment, $R_f(s_L)$. If constraint (iv) is met, then borrowing in foreign currency without hedging is feasible. Otherwise, the firm can still borrow directly from foreign markets but must use forward contracts to reduce the possibility of default in state H.

3. If the firm hedges its foreign exchange risk with currency forwards, then constraints (ii) and (iii), combined with constraints (viii) and (ix), determine an optimal range for hedging $[\underline{h}, \overline{h}]$, given by

$$\frac{s_H R_u + R_m - R}{s_H - F - \phi} \leq h \leq \frac{R - R_m - s_L R_u}{F - s_L + \phi}$$

Profit maximization implies that for any positive and small ϕ, the firm chooses the minimum level to avoid default so that optimal hedging is $h = \underline{h}$ with $R_m = 0$. As a result, constraint (viii) is binding and $R_f(s_H) = 0$. Note that when $\phi = 0$, there are multiple solutions for the optimal hedge because the firm is indifferent about choosing any value within the range $[\underline{h}, \overline{h}]$. If constraint (iv) is met, then borrowing in foreign currency and hedging are feasible. The firm determines whether hedging is optimal or not by comparing the profits in this case with those in the no-hedging case. If constraint (iv) is not met, then the firm cannot borrow directly from foreign markets and must turn to domestic banks.

4. Suppose the firm borrows from domestic banks ($1_m = 1$) and believes it is financially solvent without hedging ($1_H = 1, 1_L = 1$ and $h = 0$). As before, constraints (v) and (vi) jointly determine I_m and R_m. These two variables are now,

$$R_m = \frac{C}{\Delta p}, \; I_m = \frac{P_B}{r^*} \frac{s_L}{F} \frac{C}{\Delta p}$$

Note that when the firm is solvent in state H, R_m does not depend on q. Given I and A, and having determined I_m: if $A + I_m > I$, then resource constraint (i) is not binding, the firm borrows only in domestic currency, and invests excess funds $I - A - I_m$ at the market rate, r^f. If $A + I_m \leq I$, then constraint (i) is binding, and the firm uses it to determine I_u so that the currency composition of debt is established. As in steps (2) and (3) of this algorithm, the firm can borrow now in both currencies without hedging if it is solvent in state H, or can it face higher payments if it is insolvent in state H and does not hedge. Whether the firm must hedge or not depends on exogenous parameters (in particular q and ϕ). The firm solves for its optimal hedging decision by comparing the profits in each case. If the firm must hedge, the optimal range for hedging is $[\underline{h}, \overline{h}]$ with $R_m > 0$. As before, the minimum level $h = \underline{h}$ is chosen when $\phi > 0$ or any level within $[\underline{h}, \overline{h}]$ when $\phi = 0$. Finally, if constraint (iv) is not met, even with a positive level of hedging, then the firm is poorly capitalized (it has insufficient net worth) and will not be able to borrow at all.

Determination of the firm's minimum net worth requirements in equilibrium

To find the minimum net worth requirements under different hedging strategies, suppose that instead of using its initial assets as an exogenous variable, the entrepreneur wants to determine the size of A necessary to maximize expected profits and meet all constraints. Net worth A can then be treated as an additional endogenous variable that the entrepreneur finds in equilibrium. The entrepreneur compares A with the actual initial assets, renamed A_0, to determine whether the firm is solvent without hedging or whether it must borrow form local banks and be hedged.

1. The firm borrows a mixture of domestic and foreign currency debt ($1_m = 1$) so that constraint (i) is binding. There are three different cases:

   a. The firm is solvent in both states only if it is hedged ($1_H = 1$ and $1_L = 1$). Constraints (ii) and (iii), combined with (viii) and (ix), define the range $[\underline{h}, \overline{h}]$ for optimal hedging, h, and for any positive and small transaction cost, ϕ, the firm chooses $h = \underline{h}$. This condition is equivalent to making constraint (viii) binding and (ix) nonbinding. Therefore, the firm hedges enough to avoid default, and it receives zero cash flow in the devaluation state. Given that constraints (i) through (viii) hold with equality, in equilibrium, the minimum net worth required by foreign banks is

$$\underline{A}_H(r^*, q, \phi) = I - \frac{P_B}{r^*}\frac{s_L}{F}\frac{C}{\Delta p} - \frac{P_G}{r^*}\frac{s_L}{F+\phi}[R - \frac{b+C}{\Delta p} + \frac{b\phi}{\Delta p(1-q)(s_H - s_L)}]$$

   b. The firm is solvent in both states even if it is not hedged ($1_H = 1, 1_L = 1$ and $h = 0$). This situation requires higher net worth A. The criterion for solvency is that the firm can at least pay its debt even if it must forgo its own expected payment during the depreciation state—that is, $R_f(s_H) = 0$. A firm will be able to invest if, in return, it expects to receive higher payments during the nondepreciation state. Therefore, constraints (iv) and (ix) are nonbinding. In equilibrium, the minimum net worth required by foreign banks is

$$\underline{A}_{SNH}(r^*) = I - \frac{P_B}{r^*}\frac{s_L}{F}\frac{C}{\Delta p} - \frac{P_G}{r^*}\frac{s_L}{s_H}[R - \frac{C}{\Delta p}]$$

As is evident, when $h > 0$, $\underline{A}_H < \underline{A}_{SNH}$, which means that creditors demand higher net worth than in the case when firms must hedge to be solvent.

c. The firm is insolvent if it is not hedged ($1_H = 0, 1_L = 1$ and $h = 0$). Since the firm is insolvent in state H, constraint (iii) is ruled out and constraint (viii) is binding so that $R_f(s_H) = 0$. Moreover, constraint (ix) is nonbinding so that $R_f(s_L) > 0$. The equilibrium solution implies a minimum net worth

$$\underline{A}_{NH}(r^*, q) = I - \frac{P_B}{r^*}\frac{s_L}{F}\frac{C}{\Delta p} - \frac{P_G}{r^*}(1-q)[R - \frac{b+C}{\Delta p(1-q)}]$$

2. The firm borrows only in foreign currency from international banks ($1_m = 0$). This strategy has also three cases:

a. The firm is solvent in both states if it is hedged ($1_H = 1$ and $1_L = 1$). As before, constraints (ii) and (iii), combined with constraints (viii) and (ix), define the range $[\underline{h}, \overline{h}]$ for optimal hedging, h. For any positive and small transaction cost, ϕ, the firm chooses the minimum level to avoid default so that optimal hedging is $h = \underline{h}$. This result implies that constraint (viii) is binding and constraint (ix) is nonbinding. The firm hedges enough to avoid default and it receives zero cash flow in the devaluation state. In equilibrium, the minimum net worth required by foreign banks is

$$\overline{A}_H(r^*, q, \phi) = I - \frac{P_G}{r^*}\frac{s_L}{F+\phi}[R - \frac{B}{\Delta p} + \frac{B\phi}{\Delta p(1-q)(s_H - s_L)}]$$

b. The firm is solvent in both states even if it is not hedged ($1_H = 1, 1_L = 1$ and $h = 0$). Intuitively, again, this firm has high enough net worth (that is, higher than the minimum A when it must hedge to be solvent). As in the case of $1_m = 1$, now constraints (iv) and (ix) are nonbinding. The equilibrium solutions implies a minimum net worth

$$\overline{A}_{SNH}(r^*) = I - \frac{P_G}{r^*}\frac{s_L}{s_H}R$$

Note that when $h > 0$, $\overline{A}_H < \overline{A}_{SNH}$, which means that banks demand higher net worth than in the case when firms must hedged to be solvent.

c. The firm is insolvent if it is not hedged ($1_H = 0, 1_L = 1$ and $h = 0$). Because constraint (ii) is ruled out and constraint (viii) is binding, $R_f(s_H) = 0$. Constraint (ix) is nonbinding so that $R_f(s_L) > 0$. The minimum net worth requirement is,

$$\overline{A}_{NH}(r^*, q) = I - \frac{P_G}{r^*}(1-q)[R - \frac{B}{\Delta p(1-q)}]$$

In all the previous cases, it is assumed that the monitoring technology is socially valuable.[18] As a result, $\underline{A}_H < \overline{A}_H$ and $\underline{A}_{NH} < \overline{A}_{NH}$.

3. The firm borrows only in domestic currency from local banks ($R_u = I_u = 0$). It can be shown that the firm is indifferent between hedging and not hedging if $\phi = 0$. Hence, it can be concluded that in this case the firm is always solvent and does not hedge because it does not need to. Constraints (viii) and (ix) are nonbinding, and constraint (vii) is irrelevant. Profit-maximizing firms pay domestic banks just the least amount banks demand to participate so that constraints (v) and (vi) are binding and the only possible nonbinding constraint is (iv). Therefore, if constraint (i) is made binding, the minimum net worth required by local banks is now

$$A_B(r^*, q) = I - \frac{P_B}{r^*}\frac{s_L}{F}\frac{C}{\Delta p}$$

Note that $A_B > \underline{A}_{SNH}$. Furthermore, it is assumed that monitoring costs are large enough to guarantee that some firms will borrow in domestic debt. This assumption implies that $A_B < \overline{A}_H$. [19]

[18]Monitoring is valuable when $C[P_G - \frac{s_L}{F}P_B] < P_G[B - b]$

[19]Small enough monitoring costs would imply that $A_B > \overline{A}_H$, in which case firms would never borrow only in domestic currency because the net worth required to do so would be high enough to allow them to borrow only in foreign currency at a lower cost.

Appendix 2: Proofs

Proof of Lemma 1

If $q = 0$ and $\phi \geq 0$, then the net worth requirements for an unhedged firm are less than or equal to those for a hedged firm —that is, $\underline{A}_{NH} \leq \underline{A}_H$ and $\overline{A}_{NH} \leq \overline{A}_H$. Therefore, any profit-maximizing firm with net worth $A \geq \underline{A}_{NH}$ borrows in both currencies and is solvent without hedging. Similarly, any profit-maximizing firm with net worth $A \geq \overline{A}_{NH}$ borrows only in foreign currency and is solvent without hedging. Therefore, regardless of ϕ, not hedging is optimal for any firm borrowing in foreign currency. ■

Proof of Lemma 2

When hedging is costless ($\phi = 0$) and $q > 0$, a hedging firm faces a lower net worth requirement than does a firm that decides not to hedge—that is, $\underline{A}_H < \underline{A}_{NH}$ and $\overline{A}_H < \overline{A}_{NH}$. Consider a profit-maximizing firm with net worth A such that $\underline{A}_H < A < \underline{A}_{SNH}$. If the firm's net worth is $\underline{A}_H < A < \underline{A}_{NH}$, then the firm borrows in both domestic and foreign currency and must hedge; otherwise, it will not be able to borrow at all. If the firm's net worth is instead $\underline{A}_{NH} < A < \underline{A}_{SNH}$, then the firm borrows in domestic and foreign currency. If it decides not to hedge, then $R_m = \frac{C}{\Delta p(1-q)}$ and $R_u = \frac{r^* I_u}{P_G(1-q)}$, each of which is greater than $R_m = \frac{C}{\Delta p}$ and $R_u = \frac{r^* I_u}{P_G}$, respectively, if the firm decides to hedge. Therefore, the optimal decision for the firm is to hedge because, for a given amount of debt, hedging allows lower payments to creditors and, thus, greater profits.

Consider a firm with net worth A such that $\overline{A}_H < A < \overline{A}_{SNH}$. By a similar argument, a firm with net worth $\overline{A}_H < A < \overline{A}_{NH}$ borrows in foreign currency only and must hedge; otherwise, it will have to borrow in domestic currency first. A firms with net worth $\overline{A}_{NH} < A < \overline{A}_{SNH}$ will also hedge because, for a given amount of debt, hedging allows lower payments to foreign banks and therefore, greater profits.■

Proof of Proposition 1

This proposition refers to monitored firms with $A < \overline{A}_H$. There are three different cases, depending on parameters q and ϕ, so that each case will be separately proven.

1. When $q \geq \overline{q}$, (regardless of ϕ) $\underline{A}_{SNH} \leq \underline{A}_H < \underline{A}_{NH}$ so that the the minimum net worth in the monitoring region is $\underline{A}_{SNH}$ and any profit-maximizing firm within the monitoring region is solvent in state H without hedging. Therefore, zero hedging is optimal.

2. When $0 < q < \overline{q}$ and $\phi < (1-q)(s_H - s_L)$, $\underline{A}_H < \underline{A}_{NH}$ and $\underline{h} > 0$. Two possible cases arise:

   2.1 If q is sufficiently high such that $\underline{A}_{NH} > \underline{A}_{SNH}$, then any profit-maximizing firm with $A > \underline{A}_{SNH}$ is solvent without hedging. A profit-maximizing firm with net worth A such that $\underline{A}_H < A < \underline{A}_{SNH}$ must hedge to be able to borrow in both currencies and $0 < \phi < (1-q)(s_H - s_L)$ guarantees that the optimal level of hedging is positive and equal to $\underline{h}$.

   2.2 If q is sufficiently small such that both cutoff levels $\underline{A}_{NH}$ and $\underline{A}_H$ are smaller than $\underline{A}_{SNH}$, then the hedging decision depends on whether $\underline{A}_H < \underline{A}_{NH}$ or $\underline{A}_{NH} < \underline{A}_H$. Define $\phi^*(q) = \frac{FRq-(F-s_L)(R-b/\Delta p-c/\Delta p)}{(1-q)R-b/\Delta p-c/\Delta p-(s_L b/\Delta p)/(s_H-F)}$ as the cost of hedging for a given level of q that makes $\underline{A}_H = \underline{A}_{NH}$. Then:

      i. when $\phi < \phi^*(q) < (1-q)(s_H - s_L)$ then $\underline{A}_H < \underline{A}_{NH}$. Any profit-maximizing firm with net worth A such that $\underline{A}_H < A < \underline{A}_{NH}$ must hedge to be able to borrow in domestic and foreign currency. This is similar to case 2.1.

      ii. When $\phi^*(q) < \phi < (1-q)(s_H - s_L)$, $\underline{A}_{NH} < \underline{A}_H$. Given parameters ϕ and q, there exists a net worth A^* with $\underline{A}_H < A^* < \underline{A}_{SNH}$ such that the profit-maximizing firm is indifferent between hedging and not hedging so that:

         * if $\underline{A}_{NH} < A < \underline{A}_H$, then the firm does not hedge because it is solvent without hedging;
         * if $\underline{A}_H < A < A^*$, then not hedging is preferred over hedging.
         * if $A^* < A < \underline{A}_{SNH}$, then hedging is preferred over not hedging.

As is evident, not all firms with $A > \underline{A}_H$ prefer to hedge; only those with $A > A*$ will do so. Therefore, only some firms with net worth A such that $\underline{A}_{NH} < A < \underline{A}_{SNH}$ will be hedged.

3. When $\phi \geq (1-q)(s_H - s_L)$, optimal hedging is zero because, regardless of q, both $\underline{A}_{SNH}$ and $\underline{A}_{NH}$ are lower than $\underline{A}_H$; therefore, a profit-maximizing firm in the monitoring region can borrow in domestic and foreign currency and be solvent without hedging. Depending on q, the minimum net worth defining the lower limit of the monitoring region is $\underline{A}_{NH}$ or $\underline{A}_{SNH}$. ■

Proof of Proposition 2

The results follow directly from the minimum net worth requirements that define the monitoring region, $\underline{A}_H$ and $\overline{A}_H$ or $\underline{A}_{NH}$ and $\overline{A}_{NH}$, depending on the size of the hedging costs. First, higher q increases the cutoff net worth defining the lower limit of the monitoring region so that poorly capitalized firms lose their funding, which implies that aggregate investment drops. Second, higher q also increases the upper limit of the monitoring region so that some firms borrowing only in foreign currency from international banks during the fixed exchange rate regime lie within the monitoring region after q increases. As a result, fewer firms are able to borrow directly from foreign banks, and those moving to the monitoring region demand more domestic debt from local banks. Finally, firms that borrowed in both currencies during the fixed exchange rate regime and hedge after q increases, remain in the monitoring region, borrow less in domestic currency, and more in foreign currency. ■

Table 1: Currency composition of debt for Brazilian firms and the extent of their hedging operations, by size of firm, 1997-2001 (in means)

	Exchange rate regime					Average	
	Fixed		Floating				
Item	1997	1998	1999	2000	2001	Fixed	Floating
Foreign debt/Total debt	43.2	43.4	43.4	43.3	44.0	43.3	43.6
Small	32.2	23.3	24.2	24.6	18.7	27.7	22.5
Medium	40.7	42.2	44.9	44.7	47.3	41.5	45.6
Large	58.5	64.7	61.5	61.9	65.5	61.6	63.0
Currency derivatives/Foreign debt	4.9	4.5	3.3	6.0	24.5	4.7	11.3
Small	0.0	0.0	0.0	0.0	3.0	0.0	1.0
Medium	5.5	4.8	3.5	4.9	18.2	5.2	8.8
Large	6.3	6.3	4.9	12.1	49.7	6.3	22.2

Note: Total debt is the sum of foreign and domestic debt, and both are measured in local currency. Currency derivatives correspond to the notional value of forwards, futures, swaps and options. The sample of firms is divided into quartiles according to total assets. Small firms constitute the first quartile, medium firms the second and third quartiles, and large firms the fourth quartile. Source: Financial annual reports and footnotes to financial statements of companies publicly traded on the São Paulo Stock Exchange.

Table 2: Number of Brazilian firms, by type of financial debt, and number of firms that hedged with currency derivatives, 1997-2001

Item	Exchange rate regime: Fixed 1997	1998	Floating 1999	2000	2001
Total number of firms	145	145	145	145	145
Firms with no financial debt	0	5	8	8	6
Firms with only domestic debt	29	27	30	31	32
Firms with foreign and domestic debt	114	112	105	102	104
Firms hedged using currency derivatives	6	16	14	19	40
Firms not hedged	108	96	91	83	64
Firms with only foreign debt	2	1	2	4	3
Firms hedged using currency derivatives	1	0	0	1	0
Firms not hedged	1	1	2	3	3

Note: Firms with no financial debt are those that dropped out of the sample after 1997.
Source: Financial annual reports and footnotes to financial statements of companies publicly traded on the São Paulo Stock Exchange.