Errors-In-Variables County Poverty and Income Models

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1. Introduction

At present, the Small Area Income and Poverty Estimates (SAIPE) program at the U.S. Census Bureau estimates state, county, and school district poverty and state and county median household income (MHI). The current county models of child poverty are Empirical Bayes models where the direct survey estimates of poverty and MHI are shrunk with weighted-regression model-based estimates of poverty and MHI, respectively.[1] These weighted regressions assume that direct Annual Social and Economic Supplement (ASEC) estimates of county poverty and MHI are a function of administrative record and survey data.[2] We address a fundamental assumption of regression models that is violated in this application: that the predictors are measured without error. Practical experience, as well as past research (Gee, 2001) suggests that the data for small areas like counties are often measured with error for many reasons. Recent developments in experimental SAIPE county-level poverty models (Fisher, 2003) derive a measure of "true" poverty as a function of several independent measures of poverty, all of which are assumed to possess non-negligible variances; i.e., an "errors-in-variables" model. We adapt the method in Fisher (2003) to estimating the county number of related children ages 5-17 in poverty and MHI. For brevity, we focus the discussion on estimating children ages 5-17 in poverty.

The Hierarchical Bayes (HB) model of children in poverty assumes that as statisticians, we do not observe the object of interest—the county number of related children ages 5-17 in poverty—directly. We only observe measures of that object; all of which possess non-negligible variance conditioned on the object of interest. This leads us to an appealing quality of the HB model: the contribution from a particular source of data to the county estimates is determined by the precision of the data source. Therefore, if decennial census data is conditionally a more precise measure of poverty than administrative records from the U.S. Food and Nutrition Service's Food Stamp Program (FSP), then decennial census data will contribute more to the final estimate of poverty than will FSP data. Note that the relative precision of data sources can change according to population, sample size, and so on, according to our modeling assumptions.

A second advantage to the HB model is the ability to model specific attributes or qualities of a data source as it relates to the object of interest. For instance, the present SAIPE county number of children in poverty model, the model is linear in logarithms. Since the sample size in a single county can be quite small in the ASEC, there are many counties with no sample children in poverty (resulting in a direct survey estimate of zero). Consequently, these counties are treated as having no ASEC sample in the model, potentially biasing the regression estimates (Maiti and Slud, 2002). The proposed model offers a straightforward method of controlling for such censoring.

In this paper, we describe a theoretical model and outline some empirical results. All empirical results are based on income-year 2000 estimates; i.e., estimates that are about income and poverty during 2000. The paper proceeds as follows: First, we discuss the generic form of a theoretical model. Second, the specifics of the children in poverty are covered. Third, we present model fit and general results. Fourth, we

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[1] We estimate the number of children ages 5-17 in families in poverty. For brevity, we often refer to them as simply children.

[2] Poverty estimates from the Current Population Survey (CPS) Annual Social and Economic Supplement (ASEC) are the official measures of poverty. The ASEC was referred to as the CPS March Supplement in the past.

discuss future work and provide concluding remarks.

2. The Generic Model

2.1 The Model

Assume that there are M counties, indexed by i, for which we are interested in estimating μ_i. We do not directly observe μ_i. Instead there are J measures, each of which is observed for some counties, related to μ_i. Denote the j^{th} measure for county i as X_{ij}. Conditioned on the μ_i and parameters a_j and b_j the X_{ij} have a normal distribution:

$$X_{ij} \mid \mu_i, a_j, b_j, \sigma_{ij} \sim \mathrm{N}\left(b_j \mu_i + a_j, \sigma_{ij}^2\right).$$

The θ-parameters describe the conditional bias of measure X_{ij} conditioned on μ_i. We therefore refer to them as the bias parameters for the remainder of the paper. Although we specify a linear conditional relationship between the measure and μ_i in the above equation for convenience, other configurations are possible. The conditional variance parameters, σ_{ij}, are typically modeled as being proportional to some function of sample size or total population.

The object of interest, μ_i, conditioned on parameters η and σ_μ, has a normal distribution given by

$$\mu_i \mid \eta, \sigma_\mu \sim \mathrm{N}\left(f(\eta), \sigma_\mu^2\right).$$

The parameters η and σ_μ describe how the object of interest varies across counties. In our proposed models, the function $f(\cdot)$ is always linear.

We can see the dependencies through a graphical representation of the model displayed in **Figure 1**. Recall that we only observe the $j = 1, 2, \ldots, J$ measures of the object of interest, μ_i, located at the terminal nodes of the diagram. To describe the process in a perhaps overly simple manner, we postulate a relationship between μ_i and the J observed measures and apply Bayes Theorem to derive estimates of μ_i, as well as the other parameters of the model.

2.2 Model Fit

We use two methods to evaluate model fit. In the first, we examine scatterplots of standardized Bayesian residuals based on the mean of the posterior distribution instead of the linear prediction (as in a regression) against various model inputs and total population. In the second, we examine posterior predictive p-values (PPP-values) in a fashion similar to the aforementioned "residuals."

Standardized Bayesian residuals are quite intuitive relative to the ordinary regression diagnostics. Estimates of the mean and standard error are obtained through the parameter estimates of the posterior distribution.[3] A PPP-value is defined as

$$p = \Pr\left(T\left(X_{obs}, \theta_{rep}\right) < T\left(X_{rep}, \theta_{rep}\right) \mid data\right)$$

where $T(\ )$ is some function chosen to evaluate an aspect of the model and θ is a vector containing the model's parameters. More simply, a PPP-value compares characteristics of the replicated data—data drawn from the hypothetical posterior distribution of the model—to observed data. A simple example would be $T(x,\theta) = x$. One would calculate the probability that the replicated value is greater than the observed value. We use two functions to evaluate the model for each measure of μ_i.

$$T_1(x,\theta) = x$$

$$T_2(x,\theta) = \left(x - E[X \mid \theta]\right)^2$$

The straightforward interpretation of these functions allows one to examine T_1 and T_2 to make inferences about the first and second moments of the model, respectively. If one observes a predominate proportion of PPP-values based on the function T_1 that are close to 0 or 1, that implies that the model is consistently underestimating or overestimating the variable in question. If a large majority of PPP-values based on the function T_1 is either above or below ½, then we can infer that the model is biased upward or downward respectively. A similar observation with the function T_2 implies that estimates of the variable's variance are suspect. More generally, we can apply the PPP-value concept to test virtually any aspect of the model.[4]

During the development of the original SAIPE poverty and MHI models, counties were classified according to 1990 Census characteristics. The models' residuals and differences from the 1990 Census were examined for each category such that dependencies or biases relative to the decennial census or other model failure might be detected. The methodology described in this paper is somewhat different, but analogous evaluations can be done by examining PPP-values for the various categories with the expectation that some

[3] The definition of a standardized Bayesian residual is analogous to a standardized residual from regression. See Carlin and Louis (2000) for more details.

[4] For an in-depth discussion of PPP-values, see Gelfand (1996), Gelman and Meng (1996), or Meng (1994).

model failures would be detected in the event that PPP-values are extreme for some variable in some category. Based on those original categories, we developed new categories from the Census 2000. The bases for these county categories are

- Percentage of the population who is Black
- Percentage of the population that is Hispanic
- Whether the county is classified as metropolitan or non-metropolitan
- Total population
- The associated Census Division/Region
- Percentage of the population living at rural addresses
- Percentage of the population residing in Group Quarters
- Percentage of the population in Poverty

For each set of categories, we examined box plots of the PPP-values for the various response variables.

3. Child Poverty Model

3.1 Data

We use measures of poverty that are the same predictors in the current SAIPE county child poverty model. The county measures of the number of children in poverty, prefixed by the abbreviations used in all subscripts, are the following:

1. ***ASEC*—**Direct survey estimates of child poverty from the Annual Social and Economic Supplement (ASEC) of the Current Population Survey (CPS).
2. ***CEN*—**Estimates of the number of children in poverty from the most recent decennial census.
3. ***ACS*—**Direct survey estimates of number of children in poverty from the American Community Survey (ACS). These estimates are based on data collected in a single year.[5]
4. ***TAX*—**Tabulations of Internal Revenue Service (IRS) 1040 tax return child exemptions with income below the poverty threshold.[6]
5. ***NF*—**The number of nonfilers defined as the child population minus the total number of IRS tax return child exemptions.
6. ***FSP*—**The number of the Food Stamp Program (FSP) recipients.

We use demographic resident child population estimates from the Census Bureau's Population Division.[7]

3.2 Model

Past work has shown that the assumption that the conditional expectations of the measures are linear functions of poverty generally fit well. Therefore, all of the measures and the objects of interest are in logarithms. For this section, we denote the object of interest, the logarithm of the number of children ages 5-17 in poverty—as LNP_i for some county ***i***. This leads to an important digression from the generic model in its application to the log number of children ages 5-17 in poverty. A county's sample size in the ASEC and ACS can be quite small, often resulting in no sample children in poverty. A direct estimate of zero children in poverty is problematic due to the logarithmic transformation. In past SAIPE models, we omitted counties with ASEC sample children but with zero sample children in poverty from the regression, thus treating them like counties without ASEC sample. However, as discussed in Maiti and Slud (2002), omitting the counties exposes the model to the well-understood bias associated with censored data. The new model allows us to address the concern directly. For $j \in$ {ASEC, ACS}, the conditional distributions of X_{ij} are

$$f_{X_{ij}}\left(x_{ij} \mid \mu_i, a_j, b_j, \sigma_{ij}\right) = \begin{cases} \phi\left(\dfrac{x_{ij} - \left(b_j \mu_i + a_j\right)}{\sigma_{ij}}\right) & \text{if } x_{ij} > \gamma_{ij} \\ \Phi\left(\dfrac{\gamma_{ij} - \left(b_j \mu_i + a_j\right)}{\sigma_{ij}}\right) & \text{otherwise} \end{cases}$$

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[5] Broadly speaking, the ACS is a rolling survey designed to imitate the decennial long-form survey. ACS questionnaires are mailed to households during the entire year. In that the reference period for the ACS income question is the previous twelve months before completing the survey, using data collected over a twelve-month period implies that there is a "reference-error" associated with the ACS estimate. That is, surveys completed in January are theoretically about income earned during the previous January through December. Surveys completed the following month, however, are about income earned during the previous February through January. Hence, some of the income refers to a period outside the targeted income year.

[6] Henceforth, we refer to the measure as "tax poverty."

[7] See the website http://eire.census.gov/popest/estimates.php for more information on demographic total resident population estimates.

where ϕ denotes the standard normal density function. If the observed number in poverty in the sample is zero, leading to an undefined value of X_{ij}, we assume it was censored so we know that $X_{ij} < \gamma_{ij}$. The censoring threshold γ_{ij} is modeled simply as

$$\gamma_{ij} = \log(pop_i) - \log(k_{ij})$$

where pop_i is the number of 5-17 related children and k_{ij} is the sample size for county i. Poverty status in the ASEC is determined by the family's income relative to its size and composition. Consequently, either an entire family is in or out of poverty. Consider that the smallest observable value of the number of children in poverty is the number of children in an in-sample family. Thus, our threshold model, γ_{ij}, is the average sum of the sample weights for the children in a single family in logarithms.

In the Monte Carlo Markov Chain (MCMC), discussed later, it is straightforward to replicate the survey result conditioned on the event that it fell below the threshold.

$$f_{X_{ij}}(x_{ij} \mid \mu_i, a_j, b_j, \sigma_{ij}, X_{ij} < \gamma_{ij}) = \frac{\dfrac{1}{\sqrt{2\pi}\sigma_{ij}} \exp\left\{-\dfrac{1}{2}\dfrac{(x_{ij} - (b_j\mu_i + a_j))^2}{\sigma_{ij}^2}\right\}}{\Phi\left(\dfrac{\gamma_{ij} - (b_j\mu_i + a_j)}{\sigma_{ij}}\right)},$$

where Φ denotes the standard normal distribution function. In the MCMC, it is straightforward to replicate the unobserved direct estimate from this truncated distribution.

A second difference between the generic and child poverty models is that the relationship between LNP_i and the log number of FSP recipients, $X_{i,FSP}$, is modeled to be quadratic such that

$$X_{i,FSP} \mid LNP_i, \theta_{FSP}, \sigma_{i,FSP} \sim \mathrm{N}\left(c_{FSP}LNP_i^2 + b_{FSP}LNP_i + a_{FSP}, \sigma_{i,FSP}^2\right)$$

Conditioned on the data and all parameters, the posterior distribution of LNP_i is not Normal. We can show that the posterior distribution of LNP_i conditioned on the data and parameters is a N_3 density as discussed by Cobb, Koppstein, and Chen (1983).

The function $f(\ )$ for the poverty models is of the form $f(\eta) = \eta + \log(pop_i)$ where pop_i is the number of related children 5-17. In this form, the parameter η can be interpreted as the log national poverty rate while σ_μ^2 describes the variation of log county child poverty rates around the national average.

Motivated by the fact that the direct survey ASEC estimate is the official measure of poverty, we constrain the log ASEC bias parameters—$\{a_{ASEC}\ b_{ASEC}\}$—to be zero (a_{ASEC} = 0) and one (b_{ASEC} = 1) respectively. Furthermore, the constraint allows us to identify the bias parameters for the other measures of poverty.

The current SAIPE county poverty models distinguish between a random effect—the deviation of "true" child poverty around the regression model—and the sampling error associated with the ASEC. We preserve a similar quality in the proposed model. Notice that we model the observed log ASEC measure, X_{iASEC}, conditioned on the "true value" of LNP_i. Therefore, the estimate of σ_{iASEC} is the measure of ASEC sampling error around true child poverty. We can also obtain the variance of LNP_i through its replicated values; i.e., an estimate of its model-error or random-effect variance. The variances of the other measures of LNP, other than the log decennial census estimate of child poverty, are modeled with a single parameter. In these cases, we need to either assume that there is no sampling error—a reasonable assumption with IRS tax return data—or that the single variance term comprises both sampling and modeling error. Models for the variance terms appear below.

1. The conditional variance of the direct ASEC measure is inversely proportional to the square root of sample size. We arrived at this form through a series of empirical studies.[8]
2. The conditional variance of the decennial census measure has two components. The first component is a coefficient times a function that mimics the decennial census generalized variance function in the logarithmic scale. The second component, motivated by the idea that time has passed since the last decennial census, is a fixed constant.

[8] There are strong reasons to begin with the assumption that the variance is proportional to the inverse of sample/population. However, a long series of SAIPE research beginning with Fisher and Asher (2000) has consistently resulted in a form proportional to the square root of sample/population.

3. The conditional variance of the direct ACS measure is inversely proportional to the square root of sample size.
4. The conditional variance of tax child poverty is inversely proportional to the square root of the population.
5. The conditional variance of the number of nonfilers is inversely proportional to the square root of population.
6. The conditional variance of the number of FSP recipients is a constant.

In **Figure 2**, one can view a graphical representation of the children in poverty model.

We chose the model on the basis of experience with the data, as well as experimental runs with similar county data for earlier income years.

3.3 Priors

The hyperparameters were chosen through a combination of our theoretical relationships between the observed data and empirical trial and error. In formulating our priors, we began with what we considered broad and unrestrictive priors. We fit the models with these priors and basic model assumptions with data from previous income years and, finally, applied them to model and results presented in this paper. This avoids, somewhat, the problem of fitting models to the same data from which estimates are formed. Although we took care while choosing the priors, the abundance of data made the results extremely robust to our choices of priors.[9] This property allowed us to maintain the unrestrictive priors and preserve the cross-sectional quality of the estimates. That is, the estimates from one year are independent of the relationship between the measures of poverty from a previous year, which allows us to avoid having to model how the relationships change over time.

Since all of the measures are directly related to poverty, we give normally distributed priors for the bias parameters centered on each measure being unbiased—i.e., $(a,b) = (0,1)$—and give them a large enough variance such that we feel comfortable labeling the priors as noninformative. As for the variance-parameter-priors, we relied more heavily on initially choosing unrestrictive priors and making changes after viewing the results from earlier years of data. We use noninformative (in our opinion) gamma priors for the variance parameters. Tables listing our priors are available upon request.

[9] In our experimental runs, we would often change the priors with little effect on the posterior estimates of number of children in poverty.

4. Results from the Children in Poverty model

Our initial assessment is that the model fits the data—at least its first two moments—well. Tables with the posterior means and standard deviations are available upon request.

In general, the PPP-values failed to indicate serious failure in the model; for each of the response variables they are close to 0.5, ranging from 0.48 to 0.52. Plots of the PPP-values versus the variables described in the section on MHI model fit failed to show correlation with those variables with the following exceptions. There may be a negative correlation between the T_1 PPP-values for X_{TAX} and percent Hispanic, between the T_1 PPP-values for X_{FSP} and percent Hispanic, between the T_1 PPP-values for X_{FSP} and percent Black, and between the T_1 PPP-values for X_{CEN} and percent Hispanic. Previous research by Gee and Fisher (2003) found evidence of demographic effects—the percent of the county that is Hispanic, non-Hispanic White, and non-Hispanic Black (by omission)—on the Food Stamp Program participation and county poverty within the context of the new model. The aforementioned research suggests that addressing these issues will improve the reliability and precision of the final county-level estimates.

The mean posterior standard deviation of *LNP* under the proposed model for income year 2000 is 0.104. The estimated standard error for the analogous quantity under the current random-effects regression model is 0.145. These two quantities are not strictly comparable, but the interpretations of the numbers are similar. These numbers should not be used to discriminate between the models, since a better estimate of the variances or a better fitting model may produce a more realistic larger posterior SD or standard error.

5. Conclusions

Overall, we feel that the proposed HB model potentially represents a meaningful improvement over the current SAIPE production models. To begin, the new model both characterizes the data well and simulates the way people interpret the data in a natural and intuitive manner. Consider how an ardent baseball fan might make inferences on the major league performance of a rookie player. The fan would observe the player's performance in high school and the minor leagues. He or she might read sportswriters' opinions on the upcoming season's prospects. The fan would have an idea of the attributes and characteristics of an average, all-star, and "bench-warmer" major league baseball player. The fan would also understand that none of these measures are perfect; e.g., performance in the minor leagues is not a perfect predictor of performance in the major leagues. The fan would consider all of these measures of ability, weighting these measures according to their accuracy, compare them to the average player, and then make some prediction on future performance. While an average fan might not go through the process described above explicitly, we would argue that the fan does so implicitly. Now consider how the new county models mimic the baseball fan's implicit model of rookie performance. For the object of interest, county number of children in poverty and median household income, we have several measures of each object of interest. After several years of observing these measures, we have a good understanding of their relationship and realize that the measures possess a non-negligible variance relative to their global relationship. There is a general understanding of the relative accuracies associated with the measures. We possess good estimates of the national child poverty rate. Moreover, we have experience and beliefs regarding the across-county distribution of child poverty. In our county model, however, all of the intuitive machinery that underlies the ardent fan's final prediction of major league potential is made explicit.

In our view of the world, the proposed model accounts for far more attributes and characteristics of the data than does the current SAIPE production model—such as measurement error and censoring—such that they represent a meaningful improvement in of themselves. But the proposed model has several other advantages. One, the proposed model provides bias-parameter estimates that are far more interpretable than the coefficients of the current regression-like model. A major concern with any production process is its associated quality assurances. The proposed model has an intrinsic quality assurance in that radical and unexpected bias-parameter estimates imply that either the relationship between the measures suddenly changed or that an error occurred sometime during the estimation procedure including data collection and processing. Two, ignoring the definitional differences between a standard error from a frequentist and Bayesian model, the proposed model produces estimates with greater precision. Three, the proposed model can handle unbalanced—meaning that a source of data is only available for a subset of all counties—data sources easily. In our experimental runs we demonstrated this ability by using both ASEC and ACS data as measures of child poverty. Theoretically, we could apply this to any data source that ostensibly is related to our objects of interest. Hence, if there is a precise and highly correlated measure of poverty only captured in, say, Western states, we could incorporate it into the model.

There are several extensions of the model that we need to address. Previous work by Fisher and Gee (2003) demonstrated that much of the FSP participation variation around poverty could be explained by other observable county characteristics. We now have evidence that some of the other measures might suffer from a similar bias. Refining these relationships will allow more precise measurements of the underlying county poverty. Further research is needed to isolate and control for these effects. The present program fails to allow for values of zero in the measures of poverty other than ASEC and ACS. This results in approximately 30 counties omitted from the model. Given the small number of counties involved and their relatively small size, we plan on modeling these counties as possessing randomly missing data, since there is little information to be gained. Important future research includes refinements to the variance models. In particular, we can consider models where the variance depends on the replicated mean given the ease that they are implemented in simulated-based models. Furthermore, there may be advantages to modeling the true number of children in poverty as a binomial or Poisson distribution, which leads to a different variance function than the one we use now. More research on the conditional variances of the survey measures could be conducted and perhaps improved. Such research might shed light on the question of why the variances are such surprising functions of the sample sizes. Lastly, there is some research needed with regards to the censoring threshold. One, as opposed to using the average number of children in a household, it would be more accurate to assume a distribution of the number of children in a household. Two, we are looking for methods to test the model fit and effects of the censoring threshold.

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7. Appendix

Figure 1: Graphical Representation of the Generic Model

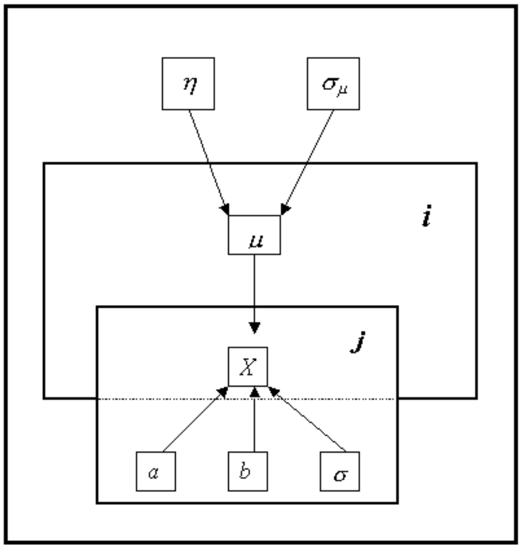

Figure 2: Graphical Representation of Log Number of Children Ages 5-17 in Poverty