

Calibrating a Tensor Magnetic Gradiometer Using Spin Data

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By Robert E. Bracken, David V. Smith, and Philip J. Brown

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Abstract

Scalar magnetic data are often acquired to discern characteristics of geologic source materials and buried objects. It is evident that a great deal can be done with scalar data, but there are significant advantages to direct measurement of the magnetic gradient tensor in applications with nearby sources, such as unexploded ordnance (UXO). To explore these advantages, we adapted a prototype tensor magnetic gradiometer system (TMGS) and successfully implemented a data-reduction procedure.

One of several critical reduction issues is the precise determination of a large group of calibration coefficients for the sensors and sensor array. To resolve these coefficients, we devised a *spin calibration* method, after similar methods of calibrating space-based magnetometers (Snare, 2001). The spin calibration procedure consists of three parts: (1) collecting data by slowly revolving the sensor array in the Earth's magnetic field, (2) deriving a comprehensive set of coefficients from the spin data, and (3) applying the coefficients to the survey data.

To show that the TMGS functions as a tensor gradiometer, we conducted an experimental survey that verified that the reduction procedure was effective (Bracken and Brown, in press). Therefore, because it was an integral part of the reduction, it can be concluded that the spin calibration was correctly formulated with acceptably small errors.

Introduction

A magnetic gradient is measured by finding the difference between two magnetometer readings and then normalizing the difference with respect to the sensor separation. This forms a gradient in the direction of the line between the two sensors. There are three gradients for a given direction and three linearly independent directions, making nine gradients. These gradients can be arranged into a matrix called a magnetic gradient tensor, mathematically expressed as:

$$\mathbf{G} = \begin{bmatrix} \partial b_x / \partial x & \partial b_y / \partial x & \partial b_z / \partial x \\ \partial b_x / \partial y & \partial b_y / \partial y & \partial b_z / \partial y \\ \partial b_x / \partial z & \partial b_y / \partial z & \partial b_z / \partial z \end{bmatrix}$$

Because both the divergence and curl of a magneto-static field must equal zero in a sourceless region, this tensor is traceless and symmetric, having only five independent components. The magnetic gradient tensor and three field components comprise a complete description of the magnetic field to the first order. Therefore, a practical tensor magnetic gradiometer will use a spatial array of vector magnetic sensors for differencing and ultimate derivation of the tensor.

For a tensor gradiometer, calibration is particularly important because the ambient field is much larger than the anomalous source fields, and the gradiometer is a differencing device. For example, if two sensors give different readings in a common uniform field, that error will be included in the measured gradient and will be indistinguishable from a true gradient. Furthermore, if the common field is large, the error will be large and will likely overwhelm the gradient being sought. Therefore, the vector sensors must be calibrated to extraordinary levels of precision, which—if obtained by direct-calibration methods—requires costly, specialized facilities and tedious measurements (Graven and Kenny, 1996).

To avoid these difficulties, we derived a method that uses the Earth's field to advantage, similar to the spin methods used on space vehicles (Snare, 2001); hence, the name spin calibration. By slowly turning the sensor array on a carousel in the ambient field, the full range of each sensor axis can be exercised with thousands of data points. This suggests the use of regressions for coefficient derivations. We use two nonlinear regressions: one combines the spin data with scalar base-station magnetometer data to calibrate each of four vector magnetometers individually; the other uses only spin data to rotate the four sensors precisely into a common reference frame.

Description of the Instrumentation

The TMGS consists of two basic units. Figure 1 shows the sensor array called TESSA (tetrahedral em/mag sensor suspension apparatus), which is a 1-m hollow tetrahedron with four triaxial magnetic sensors at its vertices. The sensors are numbered and located inside the protective cases shown in the figure. The magnetometers are ring-core triaxial fluxgates manufactured by Narod Geophysics (Narod, 1987) for use in magnetic observatories by the U.S. Geological Survey (USGS). The second unit (not pictured) is an enclo-

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sure that provides a thermally controlled environment for the magnetometer interface electronics. It contains circuitry that measures the temperature of each sensor and samples the magnetometer output. The raw data are sent, via an RS-232 link, to a laptop computer for storage. The TMGS was originally designed for detection of volcano magnetic effects from a stationary observation point (Bracken and others, 1998). Therefore, its size and sensor geometry have not been optimized for UXO applications. However, the spin-calibration procedure developed for this instrument can be applied to any tensor gradiometer system.

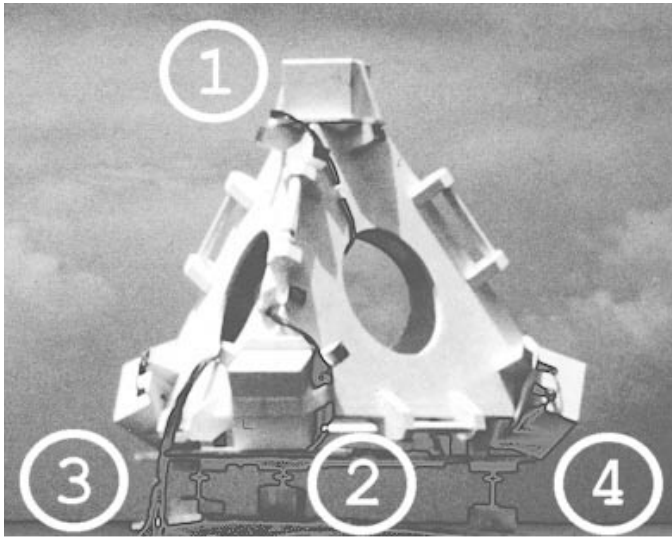


Figure 1. TMGS sensor array (TESSA) with sensors numbered.

Description of the Survey

On March 12, 2003, the USGS used the TMGS to perform an experimental survey over a known UXO target at the Standardized UXO Test Site in Yuma Proving Ground, Arizona (YPG). The target was a 60-mm mortar shell buried 0.25 m deep. We collected the data as 10-s stationary observations at five samples per second over a 3-m-square grid centered on the target and having 0.25-m grid cells. Smith and Bracken (2004) describe the survey in detail. Three categories of data were collected on-site: (a) primary measurements including magnetics, position, and attitude; (b) spin calibration measurements; and (c) thermal baseline measurements. Some calibration data used in the reduction procedure were collected in the laboratory.

Method

Acquiring Spin Data

We erected a nonmagnetic turntable calibration apparatus (fig. 2) in an area of known low magnetic gradients near

a reference base magnetometer. Plastic trestles were used to elevate the sensor array about 1.5 m above the ground in order to reduce noise from near-surface sources. The turntable was driven by a small, high-torque DC electric motor through a 1:50 reduction gear. The motor and gearbox assembly were about 3 m away from the turntable, where a magnetic interference test confirmed the absence of static and dynamic magnetic noise. The turntable completed one revolution (360 degrees) in 48.5 minutes (8.09 seconds per degree). We chose a low spin rate because the TMGS was not designed for high angular velocities. Four data sets were acquired. In each, TESSA was resting on a different face and completed one revolution.

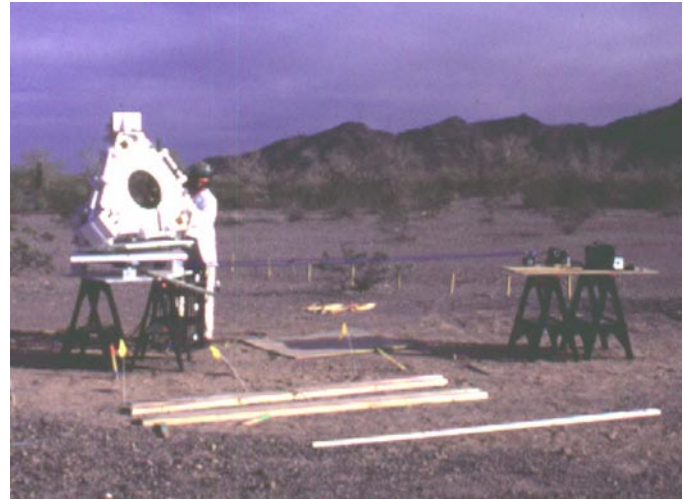


Figure 2. TESSA on spin-calibration turntable with drive assembly on right connected with nonmagnetic drive belt.

Coefficient Definitions

The magnetometers and the sensor array are calibrated using equations that contain the subject coefficients of the spin-calibration regressions. Therefore, we first describe each of three forward calibration equations that are generally applicable to any tensor gradiometer system.

The output voltage of each axis of each magnetometer is converted into magnetic field with a third-order polynomial (axis polynomial). Figure 3 is an exaggerated view showing how the axis polynomial transforms a magnetometer output voltage into a calibrated field component value. The result, F_{jk} , is the calibrated field component value of the j th axis of the k th sensor; V_{jk} is the output voltage for the subscripted axis; and a_{jk} , b_{jk} , c_{jk} , and d_{jk} are the coefficients to be calculated from the spin data (for calibrating linearity, scale factor, and bias):

$$F_{jk} = a_{jk} V_{jk}^3 + b_{jk} V_{jk}^2 + c_{jk} V_{jk} + d_{jk}$$

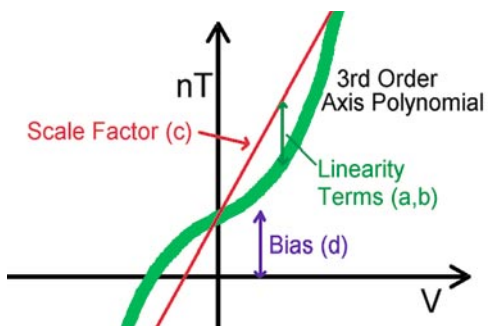


Figure 3. Coefficients in the axis polynomial.

The three calibrated axes of a given magnetometer, which are nominally orthogonal, are aligned into a true orthogonal coordinate system by an orthogonality correction. The result, H_{jk} , is the aligned j th axis of the k th sensor. The aligning angles, α_k , β_k , and γ_k (fig. 4), are to be calculated from the spin data. (Note that the orthogonality matrix in the following equation is inverted.)

$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix}_k = \begin{bmatrix} 1 & 0 & 0 \\ \cos\alpha & \sin\alpha & 0 \\ \cos\gamma & \cos\beta & \sqrt{(1 - \cos^2\beta - \cos^2\gamma)} \end{bmatrix}_k^{-1} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}_k$$

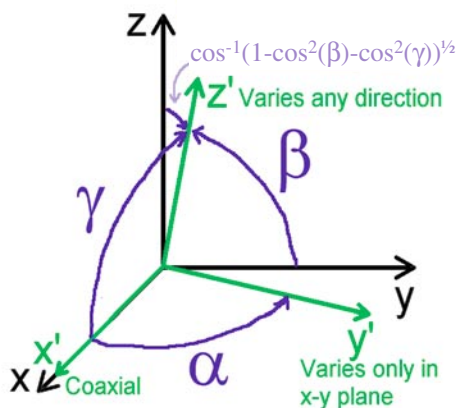


Figure 4. Alignment angles used in the orthogonality correction.

A sensor attitude correction rotates each of the sensors numbered 2, 3, and 4 (fig. 1) into the sensor-1 frame (coordinate system). The sensor-1 frame is nominally the same as the sensor array's frame. The result, B_{jk} , is the field component value of the j th axis of the k th sensor rotated into the sensor array frame. The attitude correction angles, δ_k , ϵ_k , and ζ_k (fig. 5), are rotations about the k th sensor's Z , Y' , and Z'' axes, respectively, to be calculated from the spin data:

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix}_k = \begin{bmatrix} \cos\zeta & \sin\zeta & 0 \\ -\sin\zeta & \cos\zeta & 0 \\ 0 & 0 & 1 \end{bmatrix}_k \begin{bmatrix} \cos\epsilon & 0 & -\sin\epsilon \\ 0 & 1 & 0 \\ \sin\epsilon & 0 & \cos\epsilon \end{bmatrix}_k \begin{bmatrix} \cos\delta & \sin\delta & 0 \\ -\sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{bmatrix}_k \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix}_k$$

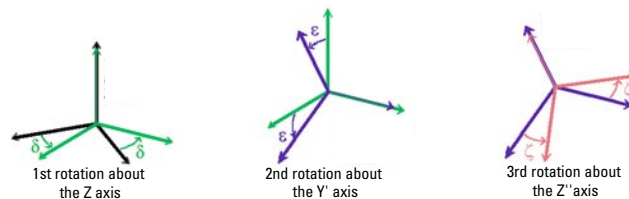


Figure 5. Diagram of attitude correction rotations.

Gradients can then be found by differencing the B_{jk} . For example, the gradient of the y -component along the line from the 1st sensor to the 4th sensor would be expressed as $(B_{24} - B_{21})/E$, where E is the distance between the sensors. Although gradients calculated in this way can be used to derive the final tensor, it is more easily done analytically by fitting a 3-dimensional function to B_{jk} .

Deriving Coefficients from the Spin Data

For a given sensor, the coefficients of its axis polynomial and orthogonality correction are derived simultaneously using a nonlinear regression; each sensor is treated independently. At a given sample point, n , the magnitude of the sensor is derived from the output voltages of its three axes using the axis polynomial and orthogonality equations. The regression finds the coefficients (a_{jk} , b_{jk} , c_{jk} , d_{jk} and α_k , β_k , γ_k) by fitting these derived signals to measured responses (I_n), recorded by the base magnetometer, according to this equation:

$$H_{1kn}^2 + H_{2kn}^2 + H_{3kn}^2 = I_n^2$$

A second regression is then performed to obtain the attitude correction coefficients (δ_k , ϵ_k , ζ_k). The H_{jkn} are first calculated using the coefficients found in the first regression. At a given sample point, n , sensor 2, 3, or 4 is rotated into the sensor-1 frame using the attitude correction equation. The resulting unit field vector (\mathbf{B}_{kn}/B_{kn}) is then dotted with the unrotated unit field vector of sensor 1 (\mathbf{H}_{1n}/H_{1n}). The regression finds the coefficients by fitting the dot product to 1 (minimizing the angle between unit vectors), mathematically collinearizing the two coordinate systems, according to this equation:

$$(\mathbf{H}_{1n} / H_{1n}) \cdot (\mathbf{B}_{kn} / B_{kn}) = 1$$

Before starting either regression, all coefficients are given initial values equal to their nominal values. For the axis polynomial, the nominal values are all zero except for the scale factor, c_{jk} , which should be set to the nominal scale factor of the magnetometer being used—in our case 100 nT/V. The orthogonality aligning angles are nominally 90°. The attitude-correction angles depend on the geometry of the sensor array.

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Table 1. Calibration-coefficient statistics.

Coefficient	Number of samples	Units	Nominal value	Mean	Standard deviation	Gradient effect (nT/m)
Bias	12	nT	0	-12.6	29.8	29.8
Scale factor	12	nT/V	100	99.9	0.681	197
Quadratic linearity	12	nT/V ²	0	1.07e-5	6.61e-5	5.52
Cubic linearity	12	nT/V ³	0	-1.72e-8	16.2e-8	3.91
Orthogonality	12	degrees	90	90.0	0.537	271
Attitude	9	degrees	0	0.220	0.640	322

Examination of figure 5 may assist in deriving values for a particular system.

Results and Conclusions

After performing the spin calibration and running the data through the regression, the derived coefficients showed a reasonably small amount of scatter about the nominal values. In table 1, statistics are given for the 12 samples (attitude has only 9 samples) in each category of coefficients. The nominal

values are close to the mean values, which is to be expected. The standard deviations also appear reasonable and are consistent with the engineering quality of the system. The gradient effect shows the magnitude of a false gradient that would be superimposed on a typical gradient measurement if the coefficient for one of two differenced sensors had an error equal to the standard deviation and the coefficient for the other sensor had no error. The gradient effect is reported in nT/m with a separation of 1 m, assuming a 50,000-nT ambient field. The impact of the gradient effect is fully realized when the sensor platform undergoes large attitude variation.

The gradient effect shows that there is a wide range in

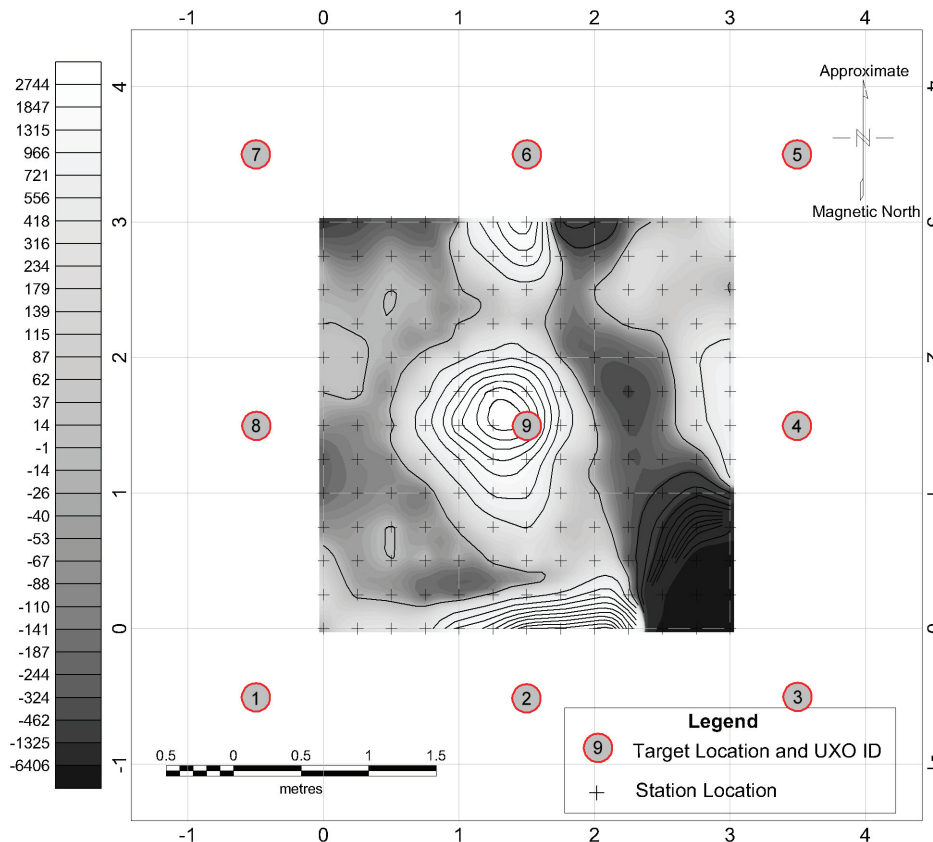


Figure 6. Determinant map of reduced tensor data from experimental survey. UXO #9 produces a distinct anomaly in center of map with peak magnitude near 3,000 nT³/m³. Contour interval is 500 nT³/m³.

significance of the various coefficients, with the linearity terms being least and the attitude correction greatest. However, all of the coefficients are significant by comparison to the experimental survey's central anomaly shown in figure 6. The data reduction (Bracken and Brown, in press), which included this spin calibration, yielded a 14-nT/m target anomaly (derived from the 3,000 nT³/m³ peak in fig. 6). Consequently, if the linearity terms had been neglected, the anomaly could have been severely degraded. Had any other coefficients been neglected in the spin calibration, the anomaly could have been obliterated. Furthermore, even a small error in some of the coefficients could have destroyed the anomaly. For example, a 0.02-degree attitude error would translate into a 20 nT/m false gradient and overwhelm the target anomaly. Therefore, because the anomaly in figure 6 was derived by a reduction that included the spin calibration as an essential sequential processing step, it can be concluded that the spin-calibration procedure works and produces reasonably accurate results.

Future Work

We expect to perform a rigorous analysis of the spin-calibration results to demonstrate attainable accuracy levels. Anticipated improvements to the TMGS will allow spin calibration data to be acquired more rapidly; this will also be investigated. Finally, it is well known that fluxgate magnetometers have significant thermal dependencies. We therefore expect many of the spin-calibration-derivable coefficients to be functions of temperature. These functions may be estimated by performing spin calibrations at a variety of temperatures.

Acknowledgments

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