

A Balanced System of Industry Accounts for the U.S. and Structural Distribution of Statistical Discrepancy*

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Abstract

This paper describes and illustrates a generalized least squares (GLS) reconciliation method that can efficiently incorporate all available information on initial data in reconciling a large system of disaggregated accounts and can accurately estimate industry distribution of statistical discrepancy. The GLS reconciliation method is applied to reconciling the 1997 GDP-by-industry accounts and the Input-output accounts. The former measure GDP by industry using industry gross income, and the latter measure GDP by industry as the residual between gross output and intermediate inputs. The GLS method produced balanced estimates and estimated the industry distribution of the statistical discrepancy. The results show that using reliability to reconcile different accounts produces statistically meaningful balanced estimates. The study demonstrates that reconciling a large system of disaggregated accounts is empirically feasible and computationally efficient.

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I. Introduction

The Bureau of Economic Analysis (BEA) estimates GDP based on production, final expenditure, and income data. The National Income and Product Accounts (NIPA) estimate GDP based on final expenditures and gross income. The industry input-output accounts estimate GDP based on by-industry estimates of value-added measured as the residual between industry gross output and intermediate inputs. The GDP-by-industry accounts estimate GDP based on by-industry estimates of value-added measured as the sum of all items of gross business income.

BEA publishes GDP based on final expenditures and Gross Domestic Income (GDI). Although the two estimates are conceptually equivalent, the actual estimates are inconsistent. The residual between estimated GDP and GDI is the aggregate statistical discrepancy, and it is recorded as an income component that reconciles the income side with the expenditure side of the accounts.

The presence of inconsistency between different measures of GDP is due to different sources of errors in the initial data used in different accounts. The main errors include errors induced by inconsistency between initial data from different sources, sampling and non-sampling errors. Over the years, compilers of national and industry accounts at BEA have made consistent efforts to reduce errors in initial data in order to reduce inconsistency between different measures of GDP. However, currently there are no estimates of statistical discrepancy by industry or by expenditure category. Lack of such information hinders a good understanding of the sources of aggregate inconsistency and makes it difficult to identify improvements in source data and estimation methods needed to minimize the statistical discrepancy.

Traditionally, in the GDP-by-industry accounts the aggregate statistical discrepancy is treated as a separate industry, such that estimates of nominal value-added by industry

sum up to the aggregate nominal GDP. During the 2003 comprehensive revision, BEA decided to distribute the aggregate statistical discrepancy to each industry as part of reconciliation between the 1997 benchmark input-output accounts and the 1997 GDP-by-industry accounts (Lawson et al., 2004). The reconciliation proceeded in four steps: 1) Some adjustments were separately made in the estimates of gross operating surplus by industry in the input-output and GDP-by-industry accounts based on GDP estimates from the 2003 benchmark revision; 2) the aggregate statistical discrepancy was distributed to each industry in the GDP-by-industry accounts according to industry shares of gross operating surplus; 3) the inconsistency between estimates of value-added from the two accounts was reconciled by computing the final estimates as a weighted average of the two estimates; and 4) the final estimates of some industries were readjusted manually in order to satisfy industry accounting constraints.

There are several concerns over this reconciliation method. First of all, the method used to allocate the statistical discrepancy suggests that reliability of initial data plays no role in the allocation, and it implies the assumption that industries that have large shares of total gross operating surplus contribute more to the aggregate statistical discrepancy. However, data provide little empirical support to this assumption. Secondly, the weights used to compute the final estimates were based on an assumed distribution and on subjectively assigned standard deviations of the initial estimates of gross operating surplus. Thirdly, the multi-step reconciliation procedure is complicated and difficult to replicate. Therefore, there is strong interest in developing an alternative method to achieve a statistically meaningful reconciliation of the accounts.

The objective of this study is to propose a generalized least squares (GLS) method that can efficiently incorporate all available information on initial data in reconciling different

sets of accounts, and that can correctly estimate the industry distribution of the aggregate statistical discrepancy according to reliabilities of the initial estimates in the input-output and GDP-by-industry accounts.

The GLS method proposed here has two empirical advantages. The first advantage is that it has a firm Bayesian foundation. It allows information on relative reliabilities of the initial data to be used efficiently in the reconciliation process. Using this method, reconciliation is achieved by trading off relative degrees of uncertainty of all data items in the system in order to adjust the initial estimates to satisfy the accounting constraints. This is essentially what is usually done during the final stages of compiling national and industry accounts when major discrepancies exist between data from different sources that need to be reconciled within the accounting framework. The difference is that here it is done in a framework that allows reliabilities of the initial estimates to be used systematically in a procedure to remove inconsistencies between the accounts. This technique is an enhancement of the knowledge about the initial data. It allows producing fully balanced accounts with adjustments that reflect the quality of the initial data.

The second advantage is that it provides great flexibility to the balancing process. For example, reconciliation can be conducted in a hierarchical manner (Dagum and Cholette, 2005). In a first round of reconciliation, the initial estimates at a relatively aggregated level are reconciled (e.g. 2-digit industry classification). In a second round of reconciliation, the initial estimates at a more disaggregated level (e.g. 4-digits) are reconciled, and these reconciled estimates add up to the previously reconciled aggregates. This method also allows additional constraints or restrictions to be easily imposed. For example, upper and lower bounds can be placed on unknown elements, inequality constraints can be added, or a penalty function can be incorporated to restrict solutions to be in the feasible set. Such flexibility is important in order to improve

the information content of the balanced estimates. Moreover, this method allows unobserved or unallocated initial estimates to be estimated (Barker et al., 1984).

The idea of incorporating data reliability in data reconciliation dates back to Stone (1942) when he developed the procedures for compiling national income accounts. Byron (1978) introduced a more efficient alternative procedure based on a conjugate gradient algorithm, and, thus, made it empirically feasible to implement the GLS method to reconciling large accounting systems. Since then, the GLS method has been further developed (Stone, 1982; van der Ploeg, 1982a, b; and Weale, 1982). The GLS method has been applied to balancing small consolidated and large disaggregated systems of accounts (Stone, 1982; Barker et al, 1984), and to developing an accounting matrix for transactions of the world economy (Weale, 1984).

However, despite of these developments, there were two obstacles to the implementation of the GLS method in national accounts. The first one was the limited computer capacity, software capability, and the large computer memory required for reconciling a large disaggregated accounting system. With the advances in computer technology and software, this obstacle has been removed. The second obstacle was, and still is, the availability of objective information on the reliability of initial data. In fact, in the previous applications of balancing national accounts for a given year, reliabilities of the initial data are assigned subjectively. Because initial estimates are adjusted according to their relative reliabilities, subjectively assigned reliabilities may lead to incorrectly reconciled accounts and improperly estimated industry distribution of statistical discrepancy. Tremendous effort has been made in this research to obtain all available information on sampling and non-sampling errors in the initial estimates in order to properly estimate their reliabilities.

In this study the GLS method is applied to reconciling the 1997 U.S. input-output and GDP-by-industry accounts with the

expenditure-based GDP estimates at the details of 65 industries, 69 commodity groups, 3 value-added components, 11 final demand categories, exports and imports. The initial estimates on gross output and intermediate inputs are from the benchmark Input-output accounts, and the initial estimates of value-added are from the GDP-by-industry accounts prior to the allocation of the aggregate statistical discrepancy. The initial estimates of final expenditures are from the 2003 comprehensive revision. Data on sampling errors are provided by the Census Bureau and Statistics of Income (SOI) of the IRS.

The plan for the remainder of the paper is as follows. Section II discusses the major data problems identified in the initial data used in the 1997 accounts. Section III describes the GLS method and the accounting system to be balanced. Section IV discusses reliabilities of the initial estimates. Section V presents the balanced estimates and the industry distribution of the statistical discrepancy. Section VI discusses future research and concludes the paper.

II. Major Data Problems Identified in the 1997 Accounts

Data used in the national and industry accounts are subject to various sources of errors. There are four types of major data problems identified in the initial estimates of the 1997 input-output and GDP-by-industry accounts.

1) Inconsistency between Initial Data from Different Sources.

The primary source data for the 1997 benchmark input-output accounts were from the 1997 Economic Census collected by the Census Bureau. For manufacturing industries, except for selected purchased services, and for the Mining industry, data were compiled directly from the Economic Census data. The selected purchased services for manufacturing industries were estimated using data collected through a sampling survey. For most

services, retail and wholesale trade industries, estimates were compiled using data from the Business Expense Survey (BES). Data for some transportation industries, communication, and construction industries were compiled, respectively, from information collected in the 1997 Transportation Survey, Annual Communication Survey, and Construction Survey. These surveys were related to Economic Census programs.

However, the Census Bureau did not provide production data for all industries. Data for the farm industry were provided by the Department of Agriculture (USDA); data for the utility industry were from the Department of Energy; rail transportation data were from Amtrak and the American Rail Road Association; air transportation data were from the Department of Transportation (DOT) and some trade companies; data for financial industries were partially based on information collected by the Federal Reserve Bank, investment for pension plans, and SOI of the IRS; and data for the insurance industry were provided by a trade company.

The GDP-by-industry accounts contain estimates of value-added by industry based on gross business income. There are three aggregate components in the GDP-by-industry accounts: compensation, taxes and subsidies, and gross operating surplus. Primary source data on wages and salaries, which accounted for 80% of compensation, were largely based on state unemployment insurance reports (UI) tabulated by the Bureau of Labor Statistics (BLS). Data on taxes and subsidies on production and imports were compiled using data from BEA, the Census Bureau, Department of Transportation, IRS, Department of Treasury, and other state government agencies. A major portion of the initial data on gross operating surplus was from SOI provided by the IRS. In addition, data from the Federal Reserve Bank, Census Bureau, some regulatory agencies and some trade companies were also used.

2) Sampling and Non-Sampling Errors in the Source Data.

Production data from BES and from various annual surveys used to construct the input-output accounts were estimated from sampling surveys, and SOI data used to construct the GDP-by-industry accounts were estimates from samples of business income tax returns. Thus, there were sampling errors in these estimates. The Census Bureau and SOI of the IRS provided data on the coefficient of variation (CV) for all their published estimates. In addition, the IRS also provided correlation coefficients for SOI estimates.

In addition to sampling errors, source data for the input-output and GDP-by-industry accounts also suffer from non-sampling errors such as double counting, misallocation, misreporting, misspecification, omission, or simple mistakes. Double counting was encountered in both product and income side of the accounts. For some industries, double counting was a serious source of errors in the source data. For example, gross output of the rental and leasing industry (NAICS 532RL) was primarily based on royalty income data from SOI. However, SOI royalty income overlaps with receipts from the Economic Census for some industries, where these receipts were classified as miscellaneous receipts or receipts for non-employer establishments, and they were not classified as secondary products for those industries. Consequently, this resulted in double counting of royalty income, creating a large gap between initial estimates of industry gross output and total inputs.

Historically, under-reporting and misreporting on income tax returns have consistently been serious sources of non-sampling errors in the national and industry accounts. Non-filers and non-employer establishments contribute additional errors to the income and product side of the accounts. Misallocation of source data is another example of non-sampling error which occurs in both sides of the accounts. For instance, for some industries whose production data were not provided by

the Census Bureau, the only source data available were total receipts. To construct the use table analysts had to allocate total receipts to expense items subjectively. The allocated expenses were inevitably subject to misallocation error.

3) Errors in the Adjustments to the Source Data

Various adjustments were made to source data in both accounts in order to correct non-sampling errors. The adjustments could be categorized into five different types: i) *definitional adjustments* to reconcile differences between the definitions of income and product used in the source data and used in the national and industry accounts; ii) *misreporting adjustments* to adjust for under-reporting and misreporting on business income tax returns, and for non-filers' income or receipts; iii) *double counting adjustments* to correct double counting according to NIPA concepts; iv) *depreciation adjustments* to account for definitional differences in depreciation that exist between source data and national accounts; and v) imputation adjustment to account for portions of income, intermediate inputs and gross output that can not be directly measured using source data.

However, adjustment data themselves introduce additional uncertainty in initial estimates. Some adjustments were based on studies conducted many years ago or based on ad hoc methods. For example, the 1997 misreporting adjustments were primarily based on TCMP Audit Adjustment and Information Return Program Adjustment (IRP) provided by the IRS. Apart from the fact that TCMP and IRP programs were based on a 1986 study using data from 1976, misreporting adjustments were not available at industry level. The industry allocation of misreporting adjustments was conducted in an ad hoc manner. Consequently, misallocation of misreporting adjustments occurred in both product and income side of the accounts. Moreover, some adjustments were based purely on analysts' subjective judgments, inevitably introducing additional

errors in the initial estimates. In sum, adjustments intended to correct non-sampling errors in the source data are also subject to errors, and some errors could be quite significant.

4. The Official Residual Errors.

The official errors between income and expenditure measures of GDP, i.e., the aggregate statistical discrepancy, were a major inconsistency to be removed. The aggregate statistical discrepancy was recorded as a separate item in the GDP-by-industry accounts.

III. A GLS Method of Accounts Reconciliation

The objective here is to reconcile the 1997 input-output and GDP-by-industry accounts with the final expenditure-based GDP. Because the expenditure-based GDP estimate was from the 2003 comprehensive revision, it was considered the most accurate measure of GDP. Thus, initial estimates of final expenditures, exports and imports were considered final and were not to be adjusted¹. The mathematical problem is then to minimize the reliability weighted sum of squares of adjustments of all components of initial estimates in gross output, intermediate inputs and value-added of all industries and all commodities, subject to accounting constraints and restrictions.

Let x , z and v denote initial estimates of gross output, intermediate inputs, and value-added. Let w_x , w_z and w_v denote reliabilities of corresponding initial estimates measured by the variances of the initial estimates. Let y , e and m denote final demand by expenditure category, exports and imports. Let Y^E and Y^I denote aggregate GDP and GDI. Let subscripts i , k , f and d indicate indexes for industry, commodity group, value-added

¹ BEA decided not to adjust expenditure-based GDP in the reconciliation of the 1997 accounts, because recent studies have shown that expenditure-based GDP estimates are very reliable (small revisions) over time (Fixler and Grimm, 2005). However, the mathematical model can be easily modified to allow initial estimates of all elements in all accounts to be adjusted. See the appendix A.

component and final expenditure category, and let superscript "o" indicate the initial estimates. Formally, the reconciliation problem is to minimize

$$(3.1) \text{ Min } S_{\{x,z,v\}} = \sum_{i=1}^{65} \sum_{k=1}^{69} \frac{(x_{ik} - x_{ik}^0)^2}{wx_{ik}} + \sum_{i=1}^{65} \sum_{k=1}^{69} \frac{(z_{ik} - z_{ik}^0)^2}{wz_{ik}} + \sum_{i=1}^{65} \sum_{f=1}^3 \frac{(v_{if} - v_{if}^0)}{wv_{if}},$$

subject to

$$(3.2) \quad \sum_{k=1}^{69} x_{ik} - \sum_{k=1}^{69} z_{ik} - \sum_{f=1}^3 v_{if} = 0,$$

for $i = 1, \dots, 65,$

$$(3.3) \quad \sum_{i=1}^{65} x_{ki} - \sum_{i=1}^{65} z_{ki} - \sum_{d=1}^{11} y_{kd}^o - e_k^o + m_k^o = 0,$$

for $k = 1, \dots, 69,$

$$(3.4) \quad \sum_{i=1}^{65} \sum_{f=1}^3 v_{if} - \sum_{k=1}^{69} \left[\sum_{d=1}^{11} y_{kd}^o - e_k^o + m_k^o \right] = 0,$$

with the initial conditions that satisfy

$$(3.5) \quad \sum_{k=1}^{69} \left[\sum_{d=1}^{11} y_{kd}^o - e_k^o + m_k^o \right] = Y^{E0},$$

$$(3.6) \quad \sum_{i=1}^{65} \sum_{f=1}^3 v_{if}^o = Y^{I0}.$$

The industry constraint (3.2) says that for each industry, final estimates of intermediate inputs and value-added must sum up to final estimate of industry gross output. The commodity

constraint (3.3) states that for each commodity, final estimates of commodities used as intermediate inputs and of commodities sold as final demand must sum up to final estimate of commodity output. Aggregation constraint (3.4) says that value-added estimates of all industries must sum up to total GDP estimate, removing the aggregate statistical discrepancy. Equations (3.5) and (3.6) state the initial conditions that initial estimate of total GDP differs from the initial estimate of total GDI, and the difference between the two initial estimates, $Y^{E0} - Y^{I0}$, is the aggregate statistical discrepancy.

The GLS reconciliation model described above has a unique solution. Proof of the solution's uniqueness can be found in Byron (1978). Van der Ploeg (1982b) discusses the treatment of account items with zero variance.

The system of accounts described here consists of 10062 variables to be solved for and 135 accounting constraints to be satisfied. The reconciliation model is solved using the CPLEX solver of the optimization software package GAMS, a powerful tool for handling large linear or quadratic constrained programming problems. Using this software, the system of accounts described above can be successfully reconciled in less than one second.

IV. Reliability of the Data

This section discusses how reliabilities of the initial estimates were estimated. As pointed out earlier, various types of adjustments were made at national and industry accounts to correct non-sampling errors in the source data. Therefore, initial estimates of gross output, intermediate inputs, and value-added can be decomposed into two components: source data value and adjustment value. Specifically, an item of initial estimate of gross output, intermediate inputs, and value-added in the accounts can be expressed as

$$x_{ik} = x_{ik}^S + x_{ik}^A, \quad z_{ik} = z_{ik}^S + z_{ik}^A, \quad v_{if} = v_{if}^S + v_{if}^A,$$

where superscripts "S" and "A" indicate source and adjustment component of the initial estimate.

Reliabilities of source data were measured by their estimated variances. In the input-output accounts, for data from BES and other annual surveys, the Census Bureau provided coefficients of variation (CV) of all published estimates. For data compiled from the Economic Census, such as gross output, CV = 0 because there were no sampling errors. Thus, variances of source data items used to construct the input-output accounts were estimated using the published estimates and their corresponding CVs.

In the GDP-by-industry accounts, source data on wages and salaries were from the state UI reports compiled from quarterly Census data. Data on taxes and subsidies were provided by federal, state and local governments. Thus, source data on wages and salaries and on taxes and subsidies were treated in the same fashion as data from the Economic Census that had no sampling errors. For the SOI portion of the initial estimates of gross operating surplus (GOS), IRS provided correlation coefficients in addition to CVs of all components of GOS. Therefore, variances of the SOI portion of the GOS estimates were estimated using published SOI estimates, their corresponding CVs and estimated correlation coefficients.

However, estimating reliabilities of adjustment data was less straightforward, because there was little information available about the degrees of uncertainty in the adjustment data. Based on how they were obtained, adjustment data are divided into three categories and are ranked in a decreasing order of reliability: 1) adjustments estimated using data from major source data agencies, such as the Census Bureau, IRS and other regulatory agencies; 2) adjustments estimated using established procedures or fairly reliable sources; and 3)

adjustments estimated using incomplete data or using methods that have serious known problems.

An example of adjustments in category 1 is inventory change in the input-output accounts using data from the Census Bureau. An example of adjustments in category 2 is the depreciation adjustment of both the income and product sides of the accounts estimated using a procedure developed by the National Accounts. One example of adjustment in category 3 is misreporting adjustments based on TCMP and IRP and allocated to each industry using an ad hoc procedure. Another such example is the adjustments base purely on analysts' subjective judgments.

Adjustment data in percentage of total initial estimates and the composition of different categories of adjustments vary largely across industries (see Figure 1 for some details). For a few industries, more than 50% of the initial estimates of gross output or intermediate inputs were from estimated adjustments. Data items from SOI were estimated from company-based business income tax returns, whereas data from the Economic Census were on an establishment basis. To achieve consistency between accounts, company-based SOI estimates were converted into establishment-based estimates. However, the method used for conversion was based on some very strong assumptions about the behavior of the estimates across industries. Consequently, conversion introduced additional uncertainty. Figure 1 shows that for some industries, the converted estimates were hugely different from the pre-converted values.

Since inconsistencies are removed according to relative reliabilities of initial estimates, different degrees of uncertainty in adjustments across industries should be taken into account. However, because there is little information about how most of the adjustments were estimated, objective measures of uncertainty in the adjustments were impossible to obtain. Thus, reliability of the adjustments was assessed subjectively based on the reliability rankings of the adjustment data. Let $\theta = (1, 2,$

3) be the reliability rankings of the adjustment data in categories 1, 2, and 3; let A_θ be an item of adjustment data in an account; and let c be the minimum CV of adjustment data assessed by experienced analysts. The CV of an adjustment data item in each category is assumed to be a linear function of the reliability ranking and the minimum CV² is

$$(4.1) \quad CV(A_\theta) = f(c, \theta) = \theta c.$$

In this study, the minimum CV is set to 10%. Thus, the CVs of adjustment data in categories 1, 2, 3 are 10%, 20%, and 30%. The variance of estimated adjustments in each category is computed as the product of θc and the estimated adjustments. Correlations between different categories of adjustment are ignored due to lack of information.

Total reliability of each initial estimate in gross output, intermediate inputs, and value-added is thus measured by the variance of the sum of the source data and adjustment data. Correlations between source data and adjustments are ignored, because no information is available on how these two components are correlated. For example, the variance of a gross output item in the input-output account is computed as

$$(4.2) \quad \begin{aligned} \text{var}(x_{ik}^S + x_{ik}^A) &= \text{var}(x_{ik}^S) + \text{var}(x_{ik}^A) \\ &= (\text{cv}(x_{ik}) x_{ik}^S)^2 + \sum_{\theta=1}^3 (\theta c x_{ik}^A)^2. \end{aligned}$$

² The CV of adjustment data are assigned subjectively because of insufficient information about the actual uncertainty in the data. The number of categories of adjustments should depend on the analysts' knowledge about the details of the relative reliability of the adjustments according to the sources and methods used to obtain them. Functional forms other than linear could be used if more information is available about the relative degrees of uncertainty in the adjustments in different categories.

Alternatively, we may contrast the reliability measure with a neutral variant defined as the absolute value of initial estimates. Neutral variants of gross output, intermediate inputs, and value-added are estimated as

$$wx_{ik} = \text{abs}(x_{ik}), \quad wz_{ik} = \text{abs}(z_{ik}), \quad wv_{if} = \text{abs}(v_{if}).$$

Reconciliation according to neutral variants does not take into account reliabilities of initial estimates. Using neutral variants is equivalent to assigning identical reliability to each initial estimate, and it implies the assumption that inconsistency between accounts should be removed according to relative sizes of initial estimates. In other words, larger industries should account for larger shares of the aggregate statistical discrepancy. Such neutral variants have been used in previous studies to derive weights for accounts reconciliation (Beaulieu and Bartelsman, 2004). For the purpose of a comparative study, both relative reliability and neutral variant weights were used in this study to reconcile the input-output and GDP-by-industry accounts with the benchmark expenditure-based GDP.

V. Balanced System of Industry Accounts

We now present and discuss the two sets of final estimates. The first set is estimated using weights derived from reliabilities measured by variances of initial estimates and the second set is estimated using weights derived from neutral variants of initial estimates. The main results are: 1) GLS reconciliation model produced two sets of balanced estimates and removed aggregate statistical discrepancies; 2) the sizes of initial gaps between estimates from the input-output and GDP-by-industry accounts affect the sizes of adjustments in initial estimates; and 3) balanced estimates based on relative reliabilities of the initial estimates can be substantially

different from balanced estimates based on neutral variants of the initial estimates. Moreover, using reliability weights to reconcile the accounts, relative reliabilities of initial estimates determine adjustments in the initial estimates and in the industry distribution of the statistical discrepancy. On the contrary, using neutral variants to reconcile the accounts, the relative sizes of the initial estimates and the shares of industry value-added of GDP determine the outcome.

Next, we shall discuss the results in details. Note that the word "adjustment" used in this section refers to the difference between balanced and initial estimate. It is customary to use "adjustment" in the reconciliation literature. All tables and figures of the results are at the end of the paper. A table that contains NAICS codes and the description of the 65 industries is also included at the end of the paper.

1) Balanced Estimates for the 65 Industries and 69 Commodities.

Table 1 contains initial estimates and two sets of balanced estimates for the 65 industries. The left panel shows the initial estimates of gross output, x , intermediate inputs, z , and value-added, v , by industry. It also shows the percentage gap by industry between gross output and total inputs (sum of z and v). Initial gaps reflect degrees of violation of industry constraints across industries. The middle panel displays balanced estimates according to relative reliabilities of the initial estimates. The right panel displays balanced estimates according to neutral variants of the initial estimates. Zeros in the last column in the middle and right panel indicate that 65 industry constraints stated in equation (3.2) are all satisfied.

There are three features to note from the balanced estimates in Table 1. First, if the gap between initial gross output and total inputs is large for an industry, the adjustments will be large, at least for some components, in order to satisfy the industry accounting constraint. Second, the two sets of

balanced estimates for a given industry can be substantially different. Third, percentage adjustments in different components in the account can be quite disproportional if relative reliabilities are used to remove inconsistencies in an industry or a commodity account, whereas percentage adjustments in different components in an account tend to be more proportional if neutral variants are used to remove inconsistencies. This feature shows the potential importance of incorporating information about degrees of uncertainty in the initial data. Third, the aggregate statistical discrepancy is removed. The last row of the column labeled "value-added" in the middle and right panel in Table 1 shows that the sum of value-added of the 65 industries equals GDP, indicating that the aggregation constraint stated in equation (3.4) is satisfied.

In order to see these features and have a flavor of the details, consider the Paper (NAICS 322) and the Electric Equipment industries (NAICS 335) as examples. The initial gap between gross output and total inputs is .32% for industry 322 and 33.18% for industry 335. The balanced estimates for industry 322 show very small changes from the initial estimates in all three components, whereas for industry 335, the adjustments needed to remove the inconsistency are much larger, especially in some components. The balanced estimates based on relative reliabilities can be very different from balanced estimates based on neutral variants. For industry 335, the differences between the two sets of balanced estimates for gross output, intermediate inputs and value-added are, respectively, -8.984, 10.908 and -19.892 billion dollars.

Furthermore, adjustments in different components can be quite disproportionate when relative reliabilities are used to remove inconsistencies. For industry 335, the initial estimates of gross output, intermediate inputs and value-added are adjusted by .17%, -2.08% and -44.39% respectively. This is not surprising because reliability of each component for a given industry can be quite different. For industry 335, variances of initial gross

output, intermediate inputs and value-added are, respectively, 6.15×10^5 , 4.81×10^6 and 1.89×10^8 . In contrast, adjustments in different components tend to be more proportional when neutral variants are used to remove inconsistency in initial estimates. Again for industry 335, percentage adjustments in the initial estimates of gross output, intermediate inputs and value-added are, respectively, 2.89%, -18.27% and -18.9%. The adjustments are more proportional when neutral variants are used because different components in the account for an industry tend to be more proportional to the overall size of the industry.

Table 2 shows the initial and balanced estimates for the 69 commodities. The initial estimates of final uses from the input-output accounts are much closer to the final expenditures from the benchmark revised GDP, and, thus, the initial gaps between gross commodity output, x_k , and total uses of commodities (sum of z_k , y_k , e_k and m_k) are in general smaller.

Balanced commodity estimates in Table 2 show similar features as those observed from balanced industry estimates in Table 1. To see these features, consider the Wholesale (NAICS 42) and the Air transportation industries (NAICS 481) as examples. The sizes of initial commodity gaps affect the sizes of adjustments in the initial estimates. The initial gap is .03% for industry 42 and 8.65% for industry 481. Consequently, the adjustments needed to close the gap are larger for industry 481. Balanced estimates based on relative reliabilities are quite different from balanced estimates based on neutral variants, and adjustments in different components could be more disproportionate when relative reliabilities are used to remove inconsistencies. Recall that the initial estimates of final uses are considered fixed and are adjusted. For industry 481, differences between the two balanced estimates for gross commodity output and intermediate inputs uses are both -6.432 billion dollars. When adjustments in the initial estimates are based on their relative reliabilities, the percentage adjustments

in the initial estimates of gross commodity output and intermediate inputs uses are -7.92% and -15.3%.

The last row of the column labeled "final uses" shows that the sum of final uses from 69 commodities equals GDP. Because commodity uses as final demand are not adjusted, the identical values in the columns labeled "final uses" indicate that the restriction on final expenditures is respected.

2) Industry Distribution of Adjustments in Initial Estimates.

Balanced estimates in Table 1 and 2 reflect very different adjustments in the initial estimates of gross output, intermediate inputs and value-added. Histograms in Figure 2 and 3 along with summary statistics in Table 3 and 4 provide some insights on the adjustments in the initial estimates across industries.

Figure 2 and Table 3 are here

Histograms in Figure 2 are empirical frequency distributions of percentage adjustments in the initial estimates across industries when relative reliabilities are used to reconcile the accounts. In this case, the adjustments are centered on 1.39% for gross output, .63% for intermediate inputs, and 8.25% for value-added. Summary statistics in Table 3 show that if reconciliation is based on relative reliabilities of initial estimates, the mean and median in absolute values and the standard deviation of the percentage adjustments in gross output and intermediate inputs are much smaller than those in value-added. This is expected because for most industries the initial estimates of gross output were directly compiled from the Economic Census data. The initial source data on intermediate inputs were compiled from Economic Census data and related survey estimates which had fairly small sampling errors. Moreover,

except for a few industries, adjustments made to correct non-sampling errors in the source data were a small fraction of the total initial estimates. Therefore, initial estimates of gross output and intermediate inputs had fairly high reliabilities. On the other hand, the initial estimates of value-added were a combination of the SOI estimates and a variety of adjustments made to the source data. Some SOI estimates had fairly large sampling errors (large CVs), and for a large number of industries adjustments made to source data to correct non-sampling errors were a significant portion of the total estimates. As a result, the initial estimates of value-added were less reliable and varied more across industries.

Figure 3 and Table 4 are here

In contrast, Figure 3 and Table 4 show that if reconciliation is based on neutral variants of the initial estimates, percentage adjustments in different components are more proportionate. This is clearly illustrated by the frequency distributions of the percentage adjustments in intermediate inputs and value-added shown in Figure 3. Adjustments in intermediate inputs and value-added are centered on 7.2% and 7.35% respectively. Compared with the summary statistics in Table 3, the mean percentage adjustments in absolute values are larger for gross output and intermediate inputs and smaller for value-added when neutral variants are used in reconciliation. Also compared with Table 3, the standard deviations of percentage adjustments of gross output and intermediate inputs in Table 4 are more than double, but value-added is less than 50%. As a result, the differences in standard deviations of the percentage adjustments in the three components are substantially smaller, even though the reliabilities of the three components are very different.

3) Industry Distribution of the Statistical Discrepancy.

The balanced estimates for the 65 industries show that the aggregate statistical discrepancy is removed. The difference between the balanced and initial estimates of value-added (or the adjustment in the initial estimates of value-added) for each industry measures the estimated statistical discrepancy distributed to that industry. Table 5 shows how the aggregate statistical discrepancy is distributed among the 65 industries, based on the relative reliability or the neutral variant of the initial estimates.

Table 5 is here

Column 2 in Table 5 shows the initial gap between value-added estimates from input-output and GDP-by-industry accounts for each industry, measured in millions of dollars, and column 3 shows the initial gaps in percentage terms. The middle panel contains the distributional results based on the relative reliability of initial estimates. Columns 4 and 5 tabulate the estimated statistical discrepancy by industry measured in millions of dollars and in percentages. Column 6 shows relative variances of initial estimates of value-added by industry from the GDP-by-industry account to that from the input-output account measured as the residual between gross output and intermediate inputs. Column 7 contains the shares of final estimates of value-added to GDP by industry.

Distributional results based on neutral variants of the initial estimates are tabulated in the right panel. Columns 8, 9 and 11 correspond to columns 4, 5, and 7 in the middle panel. Column 10 shows value-added from the GDP-by-industry accounts relative to that from the input-output accounts. In both cases, the gaps in the initial estimates of value-added indicate the total adjustments needed to remove the inconsistency.

However, the distribution of estimated statistical discrepancy is determined differently in each case. Results shown in the middle panel suggest that for an industry if the initial estimate of value-added from the GDP-by-industry account is much less reliable than value-added estimate from the input-output accounts, then, the absolute statistical discrepancy allocated to the GDP-by-industry account of that industry is large. Consider, for example, the Petroleum and coal (NAICS 324) and Textile industries (NAICS 313TT). For Petroleum and coal industry, the initial gap in value-added is -43.164 billion dollars or -64.69%, and the relative variance of value-added is 76.66. Consequently, the adjustment needed to remove the inconsistency goes largely to value-added in the GDP-by-industry account. For Textile industry, the initial gap is -2.175 billion dollars or -7.86%, and relative variance of value-added is .55. The adjustment needed to remove the inconsistency goes largely to gross output and to intermediate inputs in the input-output account.

However, the right panel shows that the relative values of value-added and the industry shares of value-added to GDP jointly determine the industry distribution of the statistical discrepancy. Consider, for example, Real estate (NAICS 531) and Rental and leasing (NAICS 532RL) industries. The industry's share of value-added to GDP is 11.07% for Real estate industry, which is the largest share among all industries. The initial value-added estimate from GDP-by-industry account is 94% of that from the input-output account. Because the share of value-added for industry 531 is substantially larger than for any other industry, the statistical discrepancy allocated to that industry is 36.44 billion dollars, the largest amount among all industries. For Rental and leasing industry, the share of value-added to total GDP is merely 1.2% and the initial estimate of value-added from the GDP-by-industry account is 51% of that from the input-output accounts. The statistical discrepancy allocated

to that industry is 25.93 million dollars or 36% of the initial gap.

Figure 4 and Table 6 are here

Histograms in Figures 4a and 4b, along with summary statistics in Table 6, provide some insight on how statistical discrepancy is allocated across industries in the two cases. Figure 4a shows that if reconciliation is based on relative reliabilities of the initial estimates, the industry's statistical discrepancy centers on 3.94 billion dollars, whereas Figure 4b shows that if reconciliation is based on neutral variants of the initial estimates, the industry's statistical discrepancy centers on 2.62 billion dollars. Summary statistics in Table 6 show that variation in the industry allocation of the statistical discrepancy is much larger if relative reliabilities are used to remove inconsistencies. This is expected because reliability of the initial estimates varies greatly across industries in the input-output and GDP-by-industry accounts. The difference in the industry distribution of the statistical discrepancy observed here reiterates the potential value of incorporating information about the relative degrees of uncertainty in the initial data in reconciling different sets of accounts.

VI. Conclusion

In this study the GLS method has been used to successfully remove inconsistencies between different 1997 accounts and to reconcile the input-output and GDP-by-industry accounts with the benchmark revised GDP. The contributions of this study are: 1) it has shown that using relative reliabilities to remove inconsistencies produces statistically meaningful balanced estimates; 2) the reconciliation process has helped identify some

problems in the source data and in the estimation methods, especially those used to estimate adjustments intended to correct non-sampling errors in the source data; and 3) it has demonstrated that using the GLS method to reconcile disaggregated accounts is empirically feasible and computationally efficient.

As for future research, we should continue to improve reliability measures, especially reliability measures of the adjustments made to correct non-sampling errors in the source data. Expanded coverage of industries and data items in future economic censuses by primary source data agencies, reducing inconsistencies between initial data from different sources through data sharing among federal statistical agencies, and improving the methods used to estimate adjustments to source data are a few ways to improve reliabilities of initial data.

This study should be considered the first step toward a full integration between national and industry accounts. In the current study, expenditure-based GDP is considered final and is not adjusted. However, there is little evidence that there is no uncertainty in the initial data used to estimate final expenditures. A full reconciliation of national and industry accounts could produce balanced estimates based on reliabilities of all data items in national and industry accounts and could estimate the statistical discrepancy by industry and by expenditure categories. The theoretical framework is fully developed and large memory computer capacity and software are available to handle a full reconciliation of a large disaggregated system of accounts. The challenge lies in the effort to obtain estimates of the reliability of the final expenditures.

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Appendix A

If the objective is to reconcile the GDP-by-expenditure, the input-output and the GDP-by-industry accounts, the reconciliation model described in Section III can be easily modified. To generalize the problem, let I , K , F and D denote the total number of industries, the total number of commodities, the total number of value-added categories, and total number of final expense categories.

The mathematical problem is then to minimize the reliability-weighted sum of squares of adjustments of initial estimates in all components of value-added, intermediate inputs, and gross output data, and in all final expenditure categories, over all industries and commodities, subject to accounting constraints,

(A1)

$$\begin{aligned} \text{Min } S = & \sum_{i=1}^I \sum_{k=1}^K \frac{(z_{ik} - z_{ik}^0)^2}{wz_{ik}} + \sum_{i=1}^I \sum_{k=1}^K \frac{(x_{ik} - x_{ik}^0)^2}{wx_{ik}} + \sum_{i=1}^I \sum_{f=1}^F \frac{(v_{if} - v_{if}^0)^2}{wv_i} \\ & + \sum_{k=1}^K \sum_{d=1}^D \frac{(y_{kd} - y_{kd}^0)^2}{wy_{kd}} + \sum_{k=1}^K \frac{(e_k - e_k^0)^2}{we_k} + \sum_{k=1}^K \frac{(m_k - m_k^0)^2}{wm_k}, \end{aligned}$$

subject to

$$(A2) \quad \sum_{k=1}^K x_{ik} - \sum_{k=1}^K z_{ik} - \sum_{f=1}^F v_{if} = 0,$$

for $i = 1, \dots, I$,

$$(A3) \quad \sum_{i=1}^I x_{ki} - \sum_{i=1}^I z_{ki} - \sum_{d=1}^D y_{kd} - e_k + m_k = 0,$$

for $k = 1, \dots, K,$

$$(A4) \quad \sum_{i=1}^I \sum_{f=1}^F v_{if} - \sum_{k=1}^K \left(\sum_{d=1}^D y_{kd} - e_k + m_k \right) = 0,$$

and with initial conditions which satisfy

$$(A5) \quad \sum_{i=1}^I \sum_{f=1}^F v_{if}^0 = Y^{I0},$$

$$(A6) \quad \sum_{k=1}^K \left(\sum_{d=1}^D y_{kd}^0 + e_k^0 - m_k^0 \right) = Y^{E0}.$$

Balanced estimates generate the final estimate of GDP.

Appendix B

Account reconciliation can also be done in a hierarchical manner. In the first stage of reconciliation, initial estimates at a relatively aggregated level are reconciled. In the second stage, the initial estimates at a more disaggregated level are reconciled, and these reconciled estimates add up to the previously reconciled aggregates.

Let x_i^* , z_i^* and v_i^* , $i = 1, \dots, I$, denote the balanced estimates of industry gross output, intermediate inputs, and value-added from the first stage reconciliation. Let x_k^* , z_k^* and y_k^* , $k = 1, \dots, K$, be the corresponding balanced estimates of commodity gross output, intermediate inputs, and final uses. Let $n = 1, \dots, N$ and $m = 1, \dots, M$ denote the indexes for industries and commodities at more disaggregated levels. Let n_i be the number of disaggregated industries in industry i where the total number

of disaggregated industries is $\sum_{i=1}^I n_i = N$. Let m_k be the number of disaggregated commodities in commodity group k where the total number of disaggregated commodities is $\sum_{k=1}^K m_k = M$. Let $f = 1, \dots, F$ and $d = 1, \dots, D$ be the index for value-add component and final use categories. Then the second stage reconciliation model is

$$\begin{aligned}
 \text{(B1) Min } S_{\{x, z, v\}} &= \sum_{n=1}^N \sum_{m=1}^M \frac{(x_{nm} - x_{nm}^0)^2}{wz_{nm}} + \sum_{n=1}^N \sum_{m=1}^M \frac{(z_{nm} - z_{nm}^0)^2}{wz_{nm}} \\
 &+ \sum_{n=1}^N \sum_{f=1}^F \frac{(v_{nf} - v_{nf}^0)^2}{wv_{nf}} + \sum_{m=1}^M \sum_{d=1}^D \frac{(y_{kd} - y_{kd}^0)^2}{wy_{kd}} \\
 &+ \sum_{m=1}^M \frac{(e_m - e_m^0)^2}{we_m} + \sum_{m=1}^M \frac{(m_m - m_m^0)^2}{wm_m},
 \end{aligned}$$

Subject to

$$\text{(B2) } \sum_{m=1}^M x_{nm} - \sum_{m=1}^M z_{nm} - \sum_{f=1}^F v_{nf} = 0,$$

for $n = 1, \dots, N$,

$$\text{(B3) } \sum_{n=1}^N x_{mn} - \sum_{n=1}^N z_{mn} - \sum_{d=1}^D y_{md} - e_m + m_m = 0,$$

for $m = 1, \dots, M$,

$$\text{(B4) } \sum_{n=1}^N \sum_{f=1}^F v_{nf} - \sum_{m=1}^M \left(\sum_{d=1}^D y_{md} - e_m + m_m \right) = 0,$$

$$(B5) \quad \sum_{n=n_{i-1}+1}^{n_i} \sum_{m=1}^M x_{nm} = x_i^*,$$

$$(B6) \quad \sum_{n=n_{i-1}+1}^{n_i} \sum_{m=1}^M z_{nm} = z_i^*,$$

$$(B7) \quad \sum_{n=n_{i-1}+1}^{n_i} \sum_{f=1}^F v_{nf} = v_i^*,$$

$$(B8) \quad \sum_{m=m_{k-1}+1}^{m_k} \sum_{n=1}^N x_{mn} = x_k^*,$$

$$(B9) \quad \sum_{m=m_{k-1}+1}^{m_k} \sum_{n=1}^N z_{mn} = z_k^*,$$

$$(B10) \quad \sum_{m=m_{k-1}+1}^{m_k} \sum_{d=1}^D z_{md} = y_k^*,$$

for $i = 1, \dots, I$, $k = 1, \dots, K$, and $n_0 = m_0 = 0$,

with initial conditions which satisfy

$$(B11) \quad \sum_{i=1}^I \sum_{f=1}^F v_{if}^0 = Y^{I0},$$

$$(B12) \quad \sum_{k=1}^K \left(\sum_{d=1}^D y_{kd}^0 + e_k^0 - m_k^0 \right) = Y^{E0}.$$

Constraints (B5)-(B10) ensure that the final balanced estimates in the more disaggregated accounts add up to the balanced estimates obtained in the first stage.

Figure 1: Percentage Adjustments in Gross Output, Intermediate Inputs and Components of Value-added in Correction of Non-Sampling Errors in the 1997 Source Data

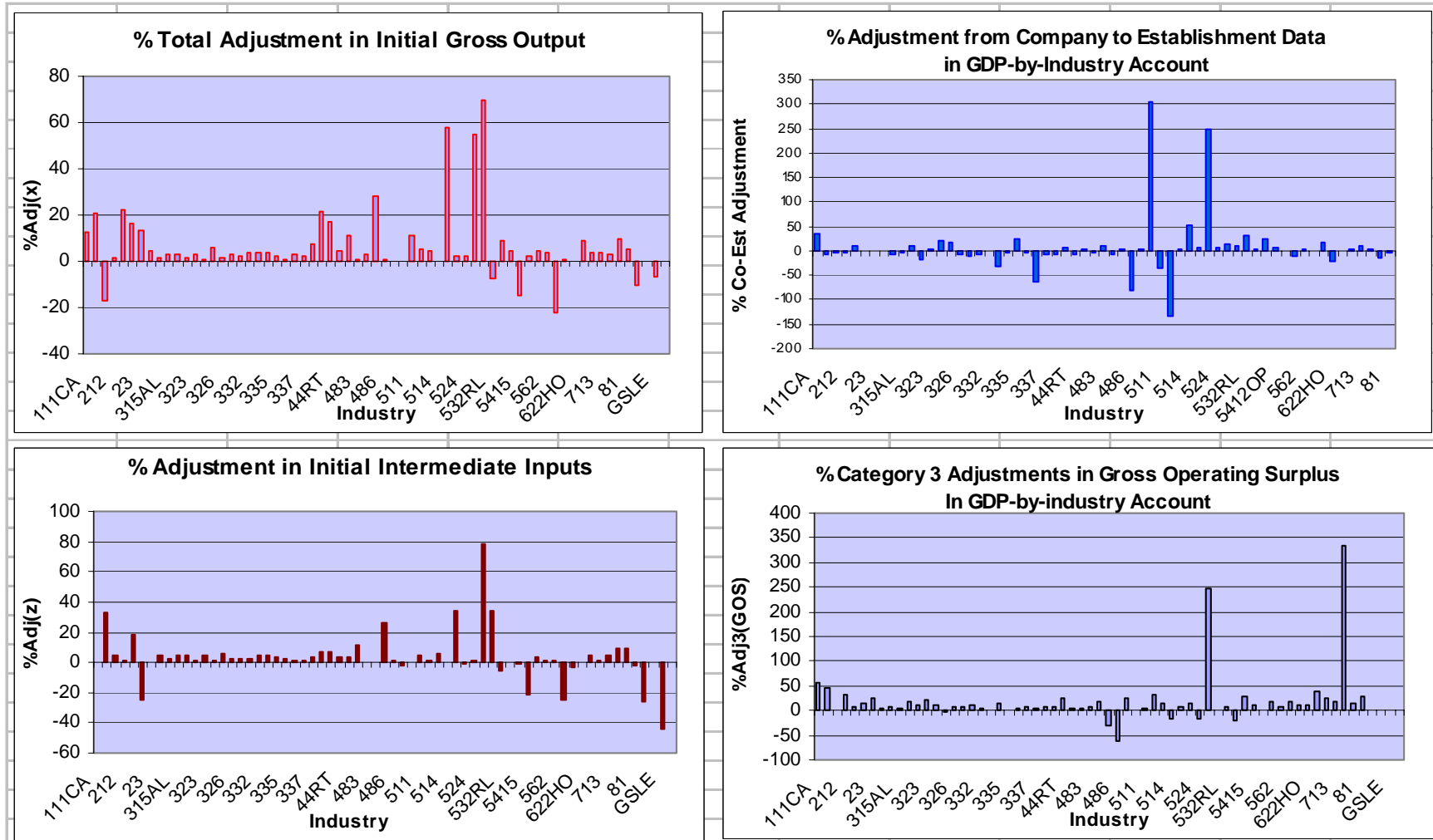


Table 1: Initial and Balanced Estimates for 65 Industries
(in millions of dollars)

1	2	3	4	5	6	7	8	9	10	11	12	13
	Initial Estimates				Balanced Estimates (Relative Reliability)				Balanced Estimates (Neutral Variant)			
	Gross Output	Intermediate Inputs	Value-added	Initial Gap	Gross Output	Intermediate Inputs	Value-added	Industry Constraint	Gross Output	Intermediate Inputs	Value-added	Industry Constraint
Pubcode	x_i^0	z_i^0	v_i^0	$x_i^0 - (z_i^0 + v_i^0) \%$	x_i^*	z_i^*	v_i^*	$x_i^* - z_i^* - v_i^*$	x_i'	z_i'	v_i'	$x_i' - z_i' - v_i'$
111CA	241952	153810	88142	0.00	241952	153810	88142	0	244496	154618	89878	0
113FF	48627	27550	23546	-5.08	48302	27268	21035	0	49419	26816	22603	0
211	91610	50096	58424	-18.46	94177	49802	44375	0	91695	42118	49577	0
212	51919	26212	27608	-3.66	51920	26133	25787	0	52065	25483	26582	0
213	25200	13217	18250	-24.87	25214	12386	12828	0	25447	10682	14765	0
22	289375	126557	178491	-5.42	292619	123986	168633	0	296658	127042	169616	0
23	679314	374546	343859	-5.75	683369	354748	328620	0	687476	358334	329142	0
311FT	519348	365401	129125	4.78	519182	370996	148186	0	515517	378152	137364	0
313TT	89388	63893	27670	-2.43	89410	62395	27014	0	90385	62957	27429	0
315AL	77228	48697	26093	3.16	77295	49203	28092	0	76860	49806	27054	0
321	88476	61742	30430	-4.18	88593	60307	28285	0	89579	60230	29350	0
322	149062	97640	50943	0.32	149169	96748	52421	0	149786	98193	51593	0
323	97586	53623	47035	-3.15	97963	53433	44530	0	100310	52992	47317	0
324	174942	151379	66727	-24.67	175144	150724	24420	0	189264	136312	52953	0
325	408567	260843	148991	-0.31	408670	259906	148764	0	407348	259060	148288	0
326	157721	95088	49546	8.30	157205	100640	56565	0	153285	100588	52697	0
327	85827	44737	37498	4.18	85632	44966	40666	0	84229	45742	38487	0
331	168318	124548	50851	-4.21	168348	121747	46601	0	170290	121307	48983	0
332	239045	124731	101959	5.17	238258	127104	111154	0	235036	129287	105749	0
333	260246	155665	88334	6.24	257697	160148	97549	0	255916	163073	92843	0
334	433139	257160	143401	7.52	432611	266488	166123	0	423578	268570	155008	0
335	109172	67362	78029	-33.18	109352	65963	43389	0	118336	55055	63281	0
3361MV	417807	324381	116195	-5.45	417783	320175	97608	0	420387	310443	109944	0
3364OT	150342	94898	52284	2.10	151049	95340	55709	0	150616	96350	54267	0
337	63529	35470	25433	4.13	63391	36328	27063	0	62828	36503	26325	0
339	102268	54242	47424	0.59	102202	54312	47890	0	102286	54303	47983	0
42	754944	266354	528905	-5.34	762009	263619	498390	0	768585	253795	514790	0
44RT	830070	313587	585081	-8.26	835741	301413	534328	0	844738	290864	553874	0
481	120232	75185	54541	-7.90	110042	64545	45497	0	117143	67873	49270	0
482	42357	18746	22438	2.77	41765	18262	23503	0	41721	18698	23023	0
483	24598	18839	6215	-1.85	24619	18223	6396	0	24951	18741	6210	0
484	169107	86150	75832	4.21	169072	86143	82928	0	167624	89106	78518	0
485	24717	8542	12094	16.51	24017	9040	14977	0	23326	9664	13662	0
486	27527	18617	8044	3.15	27527	19089	8437	0	26723	18837	7886	0

Table 1: Initial and Balanced Estimates for 65 Industries (Continue)
(in millions of dollars)

	Initial Estimates				Balanced Estimates (Relative Reliability)				Balanced Estimates (Neutral Variant)			
	Gross Output	Intermediate Inputs	Value-added	Initial Gap	Gross Output	Intermediate Inputs	Value-added	Industry Constraint	Gross Output	Intermediate Inputs	Value-added	Industry Constraint
Pubcode	x_i^0	z_i^0	v_i^0	$x_i^0 - (z_i^0 + v_i^0)\%$	x_i^*	z_i^*	v_i^*	$x_i^* - z_i^* - v_i^*$	x_i'	z_i'	v_i'	$x_i' - z_i' - v_i'$
487OS	84777	34680	59189	-10.72	84777	34330	50448	0	88934	32605	56329	0
493	27211	8317	19902	-3.71	27501	8303	19199	0	27530	8036	19494	0
511	183378	71328	65295	25.50	176819	77240	99578	0	165737	84881	80856	0
512	61496	35872	22783	4.62	61650	35828	25822	0	60835	36627	24208	0
513	377161	178927	209913	-3.10	378116	178528	199588	0	381986	174054	207932	0
514	47220	16953	18587	24.74	47219	19564	27655	0	42507	19636	22870	0
521CI	418041	146014	249138	5.48	420582	151684	268898	0	416623	152049	264574	0
523	199457	88051	130180	-9.41	199499	80634	118866	0	204458	82319	122139	0
524	350988	168429	215462	-9.37	351079	158167	192912	0	358608	160764	197844	0
525	53059	42899	9822	0.64	53179	43361	9819	0	53140	43450	9690	0
531	1260014	318624	883180	4.62	1256714	338400	918314	0	1248366	328743	919623	0
532RL	176438	31943	73375	40.31	164017	32017	132000	0	141976	42670	99306	0
5411	152096	41414	118401	-5.08	152459	41332	111126	0	154159	39159	115000	0
5412OP	489099	148023	306413	7.09	487683	148005	339678	0	478806	153175	325630	0
5415	101417	31646	86899	-16.89	102621	31203	71418	0	107868	27936	79932	0
55	241960	94294	145738	0.80	241786	93113	148673	0	240088	92621	147467	0
561	305939	78047	197254	10.02	304907	78431	226476	0	295485	82814	212671	0
562	41959	19204	20194	6.10	41956	19533	22423	0	41153	20042	21111	0
61	111493	48250	61209	1.83	111222	48285	62937	0	110705	48061	62644	0
621	381139	116383	260386	1.15	381154	116277	264877	0	380766	115859	264907	0
622HO	368320	164002	199284	1.37	368314	163737	204577	0	367392	164170	203222	0
624	66930	28502	42972	-6.79	66944	28468	38476	0	67223	26373	40850	0
711AS	55432	25987	34440	-9.01	55494	25851	29643	0	56837	24289	32548	0
713	68571	23735	37443	10.78	68408	23811	44597	0	66356	25367	40989	0
721	109988	34159	70517	4.83	109980	34143	75837	0	108985	34932	74053	0
722	299834	150156	132759	5.64	297860	150816	147044	0	295287	154337	140951	0
81	347541	142648	184592	5.84	345855	142591	203264	0	338940	146481	192459	0
GFE	74622	16864	58335	-0.77	74924	16798	58126	0	75134	16908	58225	0
GFG	466570	176186	290870	-0.10	466570	175700	290870	0	466470	174086	292384	0
GSLE	131232	71583	58053	1.22	131329	71809	59520	0	130992	72528	58464	0
GS LG	950643	304840	645781	0.00	950643	304862	645781	0	953069	303411	649658	0
Sum	15217582	6917468	8257803		15202554.3	6898210	8304344	0	15184319	6879975	8304344	0

Note: A table that contains NAICS code and description of the industries is at the end of the papers

Table 2: Initial and Balanced Estimates for 69 Commodities
(In millions of dollars)

1	2	3	4	5	6	7	8	9	10	11	12	13
	Initial Estimates				Balanced Estimates (Relative Reliability)				Balanced Estimates (Neutral Variant)			
	Gross Output	Intermediate Inputs Uses	Final Uses	Initial Gap	Gross Output	Intermediate Inputs Uses	Final Uses	Comm Constraint	Gross Output	Intermediate Inputs Uses	Final Uses	Comm Constraint
Com Code	x_k^0	z_k^0	y_k^0	$(x_k - z_k - y_k)\%$	x_k^*	z_k^*	y_k^0	$x_k^* - z_k^* - y_k^0$	x_k^1	z_k^0	y_k^1	$x_k^1 - z_k^1 - y_k^0$
111CA	235666	193581	42084	0.20	235685	193601	42084	0	238388	196304	42084	0
113FF	55668	55573	95	0.71	55750	55655	95	0	56131	56036	95	0
211	84179	147515	-63336	0.33	86149	149485	-63336	0	83972	147308	-63336	0
212	50908	46183	4726	0.32	50908	46182	4726	0	50993	46267	4726	0
213	25220	2681	22539	-0.06	25235	2696	22539	0	25398	2858	22539	0
22	335214	182958	152256	0.01	338234	185978	152256	0	340426	188170	152256	0
23	763229	95129	668100	-0.01	765704	97605	668100	0	767622	99523	668100	0
311FT	524786	206343	318443	-0.08	524237	205794	318443	0	522698	204255	318443	0
313TT	86513	66187	20326	-0.38	86531	66205	20326	0	87455	67129	20326	0
315AL	76142	17602	58541	-0.15	76203	17662	58541	0	75964	17424	58541	0
321	88337	84760	3577	-0.08	88349	84772	3577	0	89332	85755	3577	0
322	146453	132462	13991	0.07	146566	132575	13991	0	147165	133174	13991	0
323	71052	67562	3490	0.07	71373	67883	3490	0	72470	68981	3490	0
324	173626	117449	56177	-0.37	173745	117568	56177	0	185043	128865	56177	0
325	414505	311243	103262	0.68	414089	310827	103262	0	415757	312495	103262	0
326	155614	141008	14606	-0.26	155437	140832	14606	0	151616	137010	14606	0
327	84489	84183	306	-0.09	84414	84108	306	0	83049	82743	306	0
331	169789	189514	-19725	0.32	169785	189510	-19725	0	171619	191343	-19725	0
332	232993	215503	17490	-0.38	232311	214821	17490	0	229577	212087	17490	0
333	257474	88839	168635	0.06	256437	87802	168635	0	254101	85467	168635	0
334	420874	242384	178490	0.40	420001	241511	178490	0	413539	235049	178490	0
335	107526	75460	32066	-0.13	107695	75629	32066	0	115069	83003	32066	0
3361MV	411995	178164	233831	0.14	412154	178323	233831	0	414016	180185	233831	0
3364OT	148435	63250	85185	-1.53	149094	63909	85185	0	148853	63668	85185	0
337	62588	13706	48882	0.44	62549	13667	48882	0	61954	13071	48882	0
339	98545	39938	58607	0.08	98481	39874	58607	0	98567	39960	58607	0
42	736429	356396	380034	-0.03	744859	364826	380034	0	743952	363919	380034	0
44RT	750417	82944	667473	0.07	754616	87144	667473	0	753242	85770	667473	0
481	124418	64416	60002	8.65	114560	54558	60002	0	120992	60990	60002	0
482	38949	27836	11113	-0.49	38442	27330	11113	0	38566	27453	11113	0
483	24634	6599	18034	-2.30	24655	6621	18034	0	25005	6970	18034	0
484	171443	116817	54626	-0.61	170407	115781	54626	0	170495	115868	54626	0
485	32076	15341	16734	0.20	31935	15201	16734	0	31231	14497	16734	0
486	27284	26281	1003	0.22	27284	26281	1003	0	26488	25485	1003	0

Table 2: Initial and Balanced Estimates for 69 Commodities (Continue)
(In millions of dollars)

Com Code	Initial Estimates				Balanced Estimates (Relative Reliability)				Balanced Estimates (Neutral Variant)							
	Gross Output	Intermediate Inputs	Uses	Final Uses	Initial Gap	Gross Output	Intermediate Inputs	Uses	Final Uses	Comm Constraint	Gross Output	Intermediate Inputs	Uses	Final Uses	Comm Constraint	
	x_k^0	z_k^0	y_k^0	$(x_k^0 - z_k^0 - y_k^0)\%$	x_k^*	z_k^*	y_k^0	$x_k^* - z_k^* - y_k^0$	x_k^0	z_k^0	y_k^0	$x_k^0 - z_k^0 - y_k^0$	x_k^0	z_k^0	y_k^0	$x_k^0 - z_k^0 - y_k^0$
487OS	83861	67676	16186	-6.65	83875	67689	16186	0	88136	71950	16186	0	88136	71950	16186	0
493	29295	28388	906	0.03	29270	28363	906	0	29505	28599	906	0	29505	28599	906	0
511	122672	32465	90207	0.33	118673	28465	90207	0	118508	28301	90207	0	118508	28301	90207	0
512	64848	40441	24407	-0.57	65006	40599	24407	0	64439	40032	24407	0	64439	40032	24407	0
513	319608	172178	147431	-0.33	320588	173157	147431	0	321639	174208	147431	0	321639	174208	147431	0
514	53954	47923	6032	-0.35	53954	47922	6032	0	50064	44032	6032	0	50064	44032	6032	0
521CI	354596	204255	150341	0.17	352845	202504	150341	0	349556	199215	150341	0	349556	199215	150341	0
523	203385	135307	68077	-0.06	203367	135290	68077	0	207210	139133	68077	0	207210	139133	68077	0
524	351128	185752	165376	-0.98	351216	185840	165376	0	358249	192872	165376	0	358249	192872	165376	0
525	59378	2608	56771	-0.26	59496	2725	56771	0	59346	2575	56771	0	59346	2575	56771	0
531	1227089	384429	842660	0.00	1225686	383026	842660	0	1219163	376503	842660	0	1219163	376503	842660	0
532RL	246249	154850	91399	-0.02	239468	148069	91399	0	221342	129943	91399	0	221342	129943	91399	0
5411	152745	96634	56111	-0.05	154038	97927	56111	0	154855	98744	56111	0	154855	98744	56111	0
5412OP	607192	542196	64996	-0.86	603575	538579	64996	0	591865	526869	64996	0	591865	526869	64996	0
5415	141476	43519	97957	-0.52	142253	44295	97957	0	143504	45547	97957	0	143504	45547	97957	0
55	237689	215272	22417	0.00	237551	215134	22417	0	235686	213269	22417	0	235686	213269	22417	0
561	309843	286064	23778	-0.07	307641	283863	23778	0	300323	276544	23778	0	300323	276544	23778	0
562	46927	38189	8738	0.00	46925	38187	8738	0	46478	37740	8738	0	46478	37740	8738	0
61	142495	21866	120629	0.15	142229	21599	120629	0	141957	21328	120629	0	141957	21328	120629	0
621	396741	7106	389634	0.00	396755	7121	389634	0	396752	7117	389634	0	396752	7117	389634	0
622HO	435235	4121	431114	0.00	435229	4116	431114	0	435182	4068	431114	0	435182	4068	431114	0
624	67868	3253	64615	-0.02	67883	3268	64615	0	68050	3435	64615	0	68050	3435	64615	0
711AS	52629	30072	22557	-0.58	52702	30145	22557	0	53712	31155	22557	0	53712	31155	22557	0
713	84857	5411	79447	0.07	84796	5349	79447	0	84479	5032	79447	0	84479	5032	79447	0
721	82707	31275	51433	0.10	82661	31229	51433	0	81915	30483	51433	0	81915	30483	51433	0
722	356604	65285	291318	0.07	353329	62011	291318	0	353661	62343	291318	0	353661	62343	291318	0
81	448282	154314	293968	-0.07	445169	151201	293968	0	446536	152568	293968	0	446536	152568	293968	0
GFE	60927	52267	8660	-0.57	61229	52569	8660	0	61621	52961	8660	0	61621	52961	8660	0
GFG	459378	0	459378	0.00	459378	0	459378	0	459378	0	459378	0	459378	0	459378	0
GSLE	43059	10464	32595	0.01	43055	10460	32595	0	43163	10568	32595	0	43163	10568	32595	0
GSLG	760065	0	760065	0.00	760065	0	760065	0	760065	0	760065	0	760065	0	760065	0
S002	7622	82833	-75211	-2.12	0	75211	-75211	0	0	75211	-75211	0	0	75211	-75211	0
S003	0	11036	-11036	0.00	7622	18658	-11036	0	7513	18549	-11036	0	7513	18549	-11036	0
S004	19711	-14	19725	-0.07	19725	0	19725	0	19725	0	19725	0	19725	0	19725	0
																0
Total	15217582	6913238	15221812		15201130	6896787	8304344		15184319	6879975	8304344		15184319	6879975	8304344	0

Note: y_k represents final expenditures, exports, and imports.

Figure2: Histograms of % Adjustments (Reliability Weights)

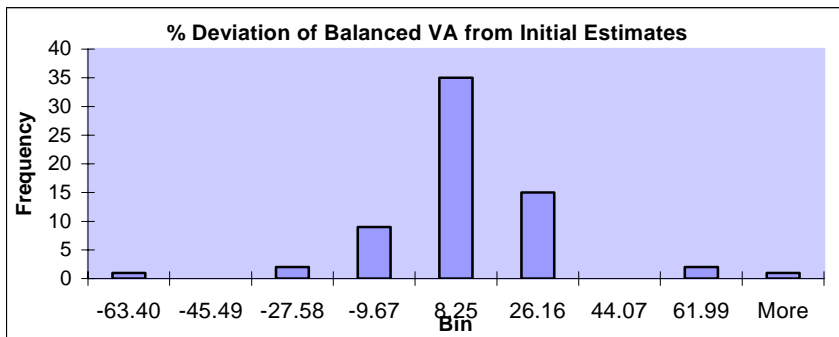
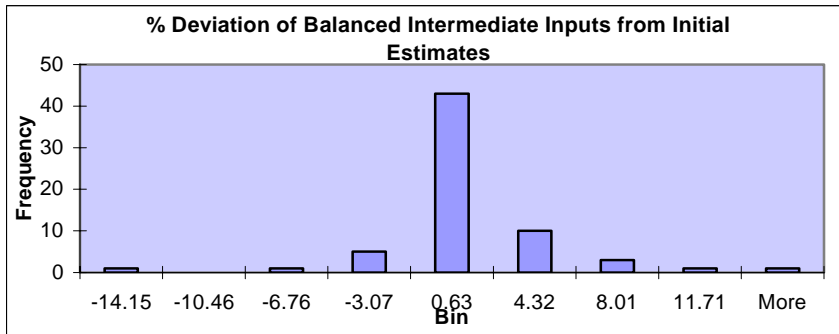
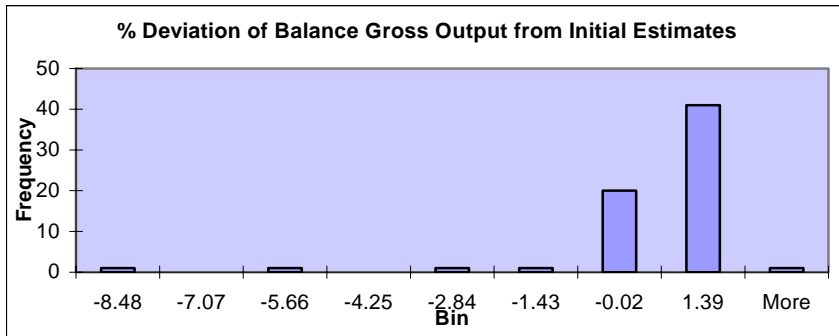


Table 3: Statistics of % Adjustment (Reliability Weights)

% Adjustments in Gross Output

Mean	-0.26
Max	2.80
Min	-8.48
Median	0.00
St Dev	1.57

% Adjustments in Intermediate Inputs

Mean	-0.10
Max	15.40
Min	-14.15
Median	-0.12
St Dev	3.78

% Adjustments in Value Added Estimates

Mean	1.46
Max	79.90
Min	-63.40
Median	1.73
St Dev	19.41

Figure 3: Histograms of % Adjustments (Neutral Variants)

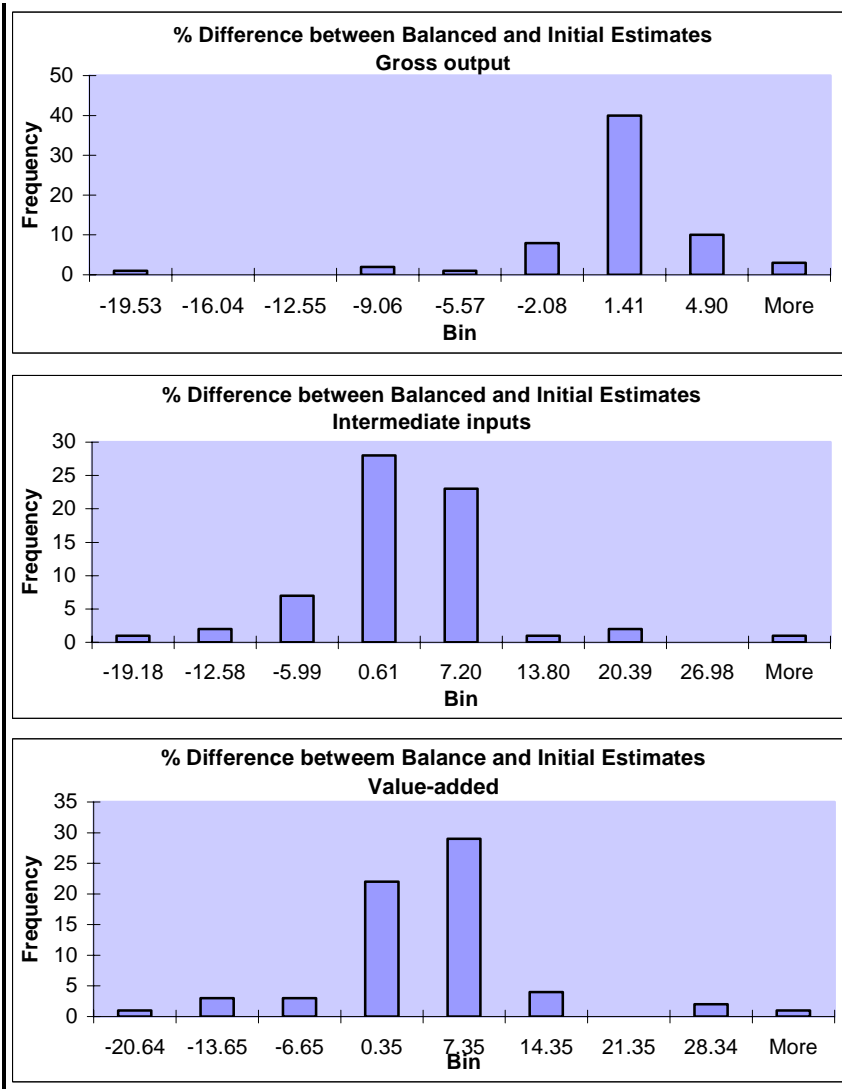


Table 4: Statistics of % Adjustment (Neutral Variants)

% Adjustments in Gross Output	
Mean	-0.38
Max	8.39
Min	-19.53
Median	-0.02
St Dev	3.83
% Adjustments in Intermediate Inputs	
Mean	-0.11
Max	33.58
Min	-19.18
Median	0.10
St Dev	7.81
% Adjustments in Value Added Estimates	
Mean	0.76
Max	35.34
Min	-20.64
Median	0.71
St Dev	8.88

Table 5: Estimated Industry Statistical Discrepancy Based on Reliability and Neutral Variant
 (Initial gap and estimates of statistical discrepancy are in millions of dollars)

1	2	3	4	5	6	7	8	9	10	11
			Estimates based on Relative Reliability				Estimates based on Neutral Variant			
	Initial Gap	Initial Gap%	Industry Stat. discrepancy	% Statistical discrepancy	Relative variance	Value-added Share of GDP	Industry Stat. discrepancy	% Statistical discrepancy	Relative value added	Value-added Share of GDP
Indcode	$v^0 - (x^0 - z^0)$	$[v^0 - (x^0 - z^0)]\%$	$v_i^* - v_i^0$	$(v_i^* - v_i^0)/v^0$	$\frac{\text{var}(v^*)}{\text{var}(x^0 - z^0)}$	V_i^*/GDP	$v_i^1 - v_i^0$	$(v_i^1 - v_i^0)/v^0$	$v_i^0/(x_i^0 - z_i^0)$	v_i^1/GDP
111CA	0	0.00	0	0.00	0.00	1.06	1737	1.97	1.00	1.08
113FF	-2469	-10.49	-2511	-10.67	0.36	0.25	-943	-4.00	1.12	0.27
211	-16910	-28.94	-14049	-24.05	3.17	0.53	-8847	-15.14	1.41	0.6
212	-1901	-6.89	-1821	-6.60	35.20	0.31	-1026	-3.72	1.07	0.32
213	-6268	-34.34	-5422	-29.71	2.66	0.15	-3486	-19.10	1.52	0.18
22	-15673	-8.78	-9859	-5.52	0.36	2.03	-8876	-4.97	1.10	2.04
23	-39091	-11.37	-15239	-4.43	0.20	3.96	-14717	-4.28	1.13	3.96
311FT	24823	19.22	19062	14.76	1.81	1.78	8240	6.38	0.84	1.65
313TT	-2175	-7.86	-656	-2.37	0.55	0.33	-241	-0.87	1.09	0.33
315AL	2438	9.34	1998	7.66	1.96	0.34	960	3.68	0.91	0.33
321	-3696	-12.15	-2145	-7.05	0.95	0.34	-1081	-3.55	1.14	0.35
322	480	0.94	1478	2.90	2.00	0.63	650	1.28	0.99	0.62
323	-3072	-6.53	-2505	-5.33	10.54	0.54	283	0.60	1.07	0.57
324	-43164	-64.69	-42307	-63.40	76.66	0.29	-13775	-20.64	2.83	0.64
325	-1267	-0.85	-227	-0.15	1.97	1.79	-703	-0.47	1.01	1.79
326	13087	26.41	7019	14.17	1.14	0.68	3151	6.36	0.79	0.63
327	3591	9.58	3168	8.45	8.04	0.49	989	2.64	0.91	0.46
331	-7081	-13.92	-4251	-8.36	0.64	0.56	-1868	-3.67	1.16	0.59
332	12356	12.12	9196	9.02	4.16	1.34	3791	3.72	0.89	1.27
333	16247	18.39	9215	10.43	1.12	1.17	4509	5.11	0.84	1.12
334	32578	22.72	22722	15.85	0.74	2.00	11607	8.09	0.81	1.87
335	-36219	-46.42	-34640	-44.39	19.88	0.52	-14747	-18.90	1.87	0.76
3361MV	-22770	-19.60	-18587	-16.00	10.69	1.18	-6251	-5.38	1.24	1.32
3364OT	3160	6.04	3425	6.55	2.09	0.67	1983	3.79	0.94	0.65
337	2626	10.33	1630	6.41	1.70	0.33	892	3.51	0.91	0.32
339	603	1.27	466	0.98	5.38	0.58	560	1.18	0.99	0.58
42	-40316	-7.62	-30515	-5.77	1.09	6.00	-14116	-2.67	1.08	6.2
44RT	-68598	-11.72	-50754	-8.68	2.02	6.43	-31208	-5.33	1.13	6.67
481	-9494	-17.41	-9044	-16.58	0.18	0.55	-5271	-9.66	1.21	0.59
482	1172	5.23	1065	4.75	0.19	0.28	585	2.61	0.95	0.28
483	-456	-7.33	181	2.92	0.13	0.08	-5	-0.08	1.08	0.07
484	7126	9.40	7097	9.36	3.28	1.00	2686	3.54	0.91	0.95
485	4080	33.74	2882	23.83	0.24	0.18	1568	12.96	0.75	0.16
486	866	10.77	393	4.88	0.96	0.10	-158	-1.97	0.90	0.09

**Table 5: Estimated Industry Statistical Discrepancy Based on Reliability and Neutral Variant
(Continue)**

	Initial Gap	Initial Gap%	Industry Stat. discrepancy	% Statistical discrepancy	Relative variance	Value-added Share of GDP	Industry Stat. discrepancy	% Statistical discrepancy	Relative value added	Value-added Share of GDP
Indcode	$v^0-(x^0-z^0)$	$[v^0-(x^0-z^0)]\%$	$v_i^*-v_i^0$	$(v_i^*-v_i^0)/v^0$	$\text{var}(v_i^*)/\text{var}(x_i^0-z_i^0)$	v_i^*/GDP	$v_i^1-v_i^0$	$(v_i^1-v_i^0)/v^0$	$v_i^0/(x_i^0-z_i^0)$	v_i^1/GDP
487OS	-9092	-15.36	-8742	-14.77	37.60	0.61	-2860	-4.83	1.18	0.68
493	-1008	-5.07	-703	-3.53	1.65	0.23	-408	-2.05	1.05	0.23
511	46755	71.61	34283	52.51	0.84	1.20	15561	23.83	0.58	0.97
512	2841	12.47	3039	13.34	4.57	0.31	1424	6.25	0.89	0.29
513	-11679	-5.56	-10325	-4.92	457.31	2.40	-1982	-0.94	1.06	2.5
514	11681	62.85	9069	48.79	3.30	0.33	4284	23.05	0.61	0.28
521CI	22889	9.19	19760	7.93	1.31	3.24	15436	6.20	0.92	3.19
523	-18774	-14.42	-11315	-8.69	1.34	1.43	-8041	-6.18	1.17	1.47
524	-32903	-15.27	-22550	-10.47	0.35	2.32	-17617	-8.18	1.18	2.38
525	338	3.44	-3	-0.03	0.00	0.12	-132	-1.34	0.97	0.12
531	58210	6.59	35134	3.98	3.35	11.06	36443	4.13	0.94	11.07
532RL	71120	96.93	58626	79.90	0.03	1.59	25932	35.34	0.51	1.2
5411	-7719	-6.52	-7275	-6.14	32.75	1.34	-3401	-2.87	1.07	1.38
5412OP	34663	11.31	33265	10.86	34.18	4.09	19217	6.27	0.90	3.92
5415	-17128	-19.71	-15482	-17.82	2.25	0.86	-6967	-8.02	1.25	0.96
55	1928	1.32	2935	2.01	5.14	1.79	1729	1.19	0.99	1.78
561	30638	15.53	29223	14.82	14.18	2.73	15418	7.82	0.87	2.56
562	2561	12.68	2229	11.04	0.69	0.27	918	4.55	0.89	0.25
61	2034	3.32	1728	2.82	0.33	0.76	1435	2.35	0.97	0.75
621	4370	1.68	4491	1.73	339.05	3.19	4521	1.74	0.98	3.19
622HO	5034	2.53	5293	2.66	90.25	2.46	3938	1.98	0.98	2.45
624	-4545	-10.58	-4496	-10.46	7.59	0.46	-2122	-4.94	1.12	0.49
711AS	-4995	-14.50	-4798	-13.93	725.66	0.36	-1892	-5.49	1.17	0.39
713	7393	19.74	7154	19.11	5.46	0.54	3546	9.47	0.84	0.49
721	5312	7.53	5320	7.54	22.35	0.91	3537	5.02	0.93	0.89
722	16919	12.74	14284	10.76	0.69	1.77	8191	6.17	0.89	1.7
81	20301	11.00	18672	10.12	13.58	2.45	7867	4.26	0.90	2.32
GFE	-577	-0.99	-209	-0.36	2.74	0.70	-110	-0.19	1.01	0.7
GFG	-486	0.00	0	0.00	0.00	3.50	1514	0.52	1.00	3.52
GSL E	1596	2.75	1467	2.53	2.37	0.72	411	0.71	0.97	0.7
GSLG	22	0.00	0	0.00	0.00	7.78	3877	0.60	1.00	7.82
Sum			46541				46541			

Figure 4: Histogram of SD by Industry

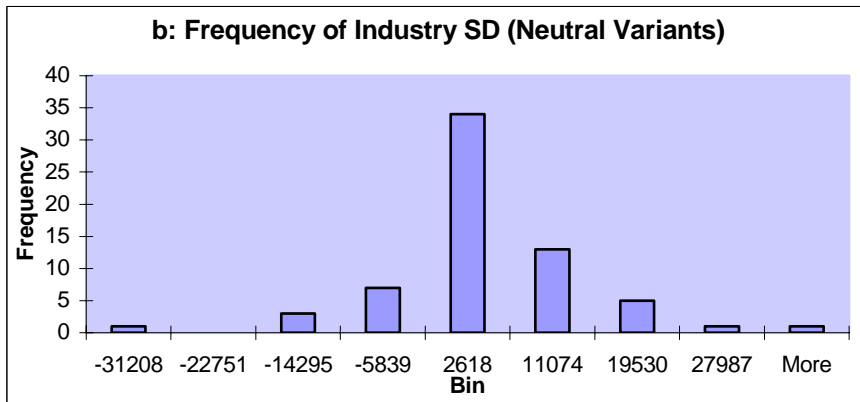
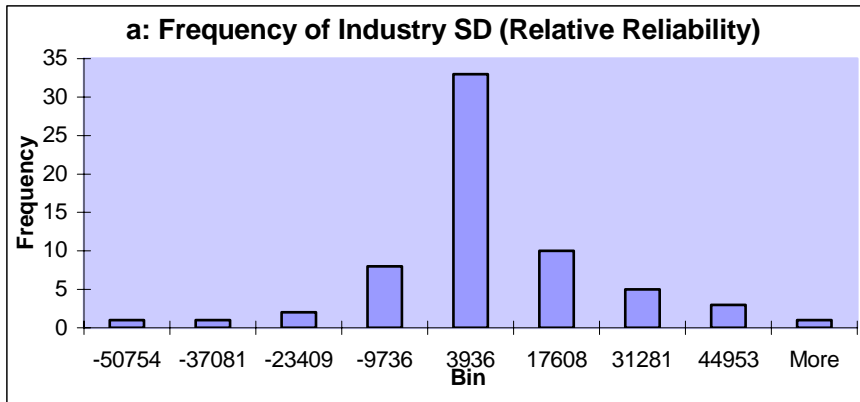


Table 6: Summary Statistics of Estimated Industry Statistical Discrepancy

Summary Statistics (Relative Reliability)	
Mean	716
Max	59868
Min	-50754
Median	393
St Dev	17148
Summary Statistics (Neutral Variants)	
Mean	716
Max	36443
Min	-31208
Median	585
St Dev	9724

NAICS Industry Codes and Industry Description			
Indcode	Industry description	Indcode	Industry description
111CA	Farms	487OS	Other transportation and support activities
113FF	Forestry, fishing, and related activities	493	Warehousing and storage
211	Oil and gas extraction	511	Publishing industries (includes software)
212	Mining, except oil and gas	512	Motion picture and sound recording industries
213	Support activities for mining	513	Broadcasting and telecommunications
22	Utilities	514	Information and data processing services
23	Construction	521CI	Federal Reserve banks, credit intermediation, and related activities
311FT	Food and beverage and tobacco products	523	Securities, commodity contracts, and investments
313TT	Textile mills and textile product mills	524	Insurance carriers and related activities
315AL	Apparel and leather and allied products	525	Funds, trusts, and other financial vehicles
321	Wood products	531	Real estate
322	Paper products	532RL	Rental and leasing services and lessors of intangible assets
323	Printing and related support activities	5411	Legal services
324	Petroleum and coal products	5412OP	Miscellaneous professional, scientific and technical services
325	Chemical products	5415	Computer systems design and related services
326	Plastics and rubber products	55	Management of companies and enterprises
327	Nonmetallic mineral products	561	Administrative and support services
331	Primary metals	562	Waste management and remediation services
332	Fabricated metal products	61	Educational services
333	Machinery	621	Ambulatory health care services
334	Computer and electronic products	622HO	Hospitals and nursing and residential care facilities
335	Electrical equipment, appliances, and components	624	Social assistance
3361MV	Motor vehicles, bodies and trailers, and parts	711AS	Performing arts, spectator sports, museums, and related activities
3364OT	Other transportation equipment	713	Amusements, gambling, and recreation industries
337	Furniture and related products	721	Accommodation
339	Miscellaneous manufacturing	722	Food services and drinking places
42	Wholesale trade	81	Other services, except government
44RT	Retail trade	GFE	Federal government enterprises
481	Air transportation	GFG	
482	Rail transportation	GSLE	State and local government enterprises
483	Water transportation	GSLG	
484	Truck transportation		
485	Transit and ground passenger transportation		
486	Pipeline transportation		