# Analytical Versus Numerical Estimates of Water-Level Declines Caused by Pumping, and a Case Study of the Iao Aquifer, Maui, Hawaii

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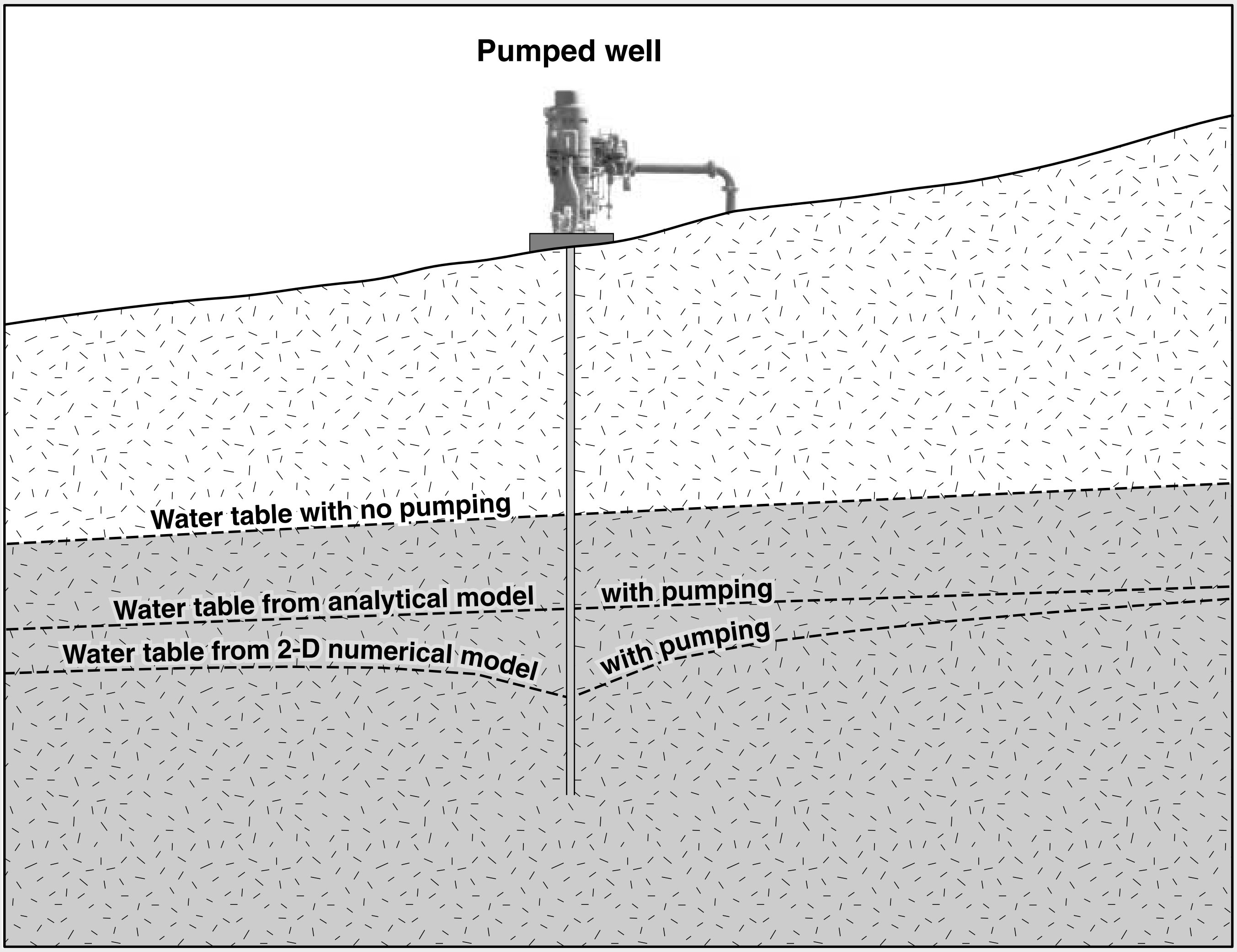

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*By* Delwyn S. Oki and William Meyer

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BRUCE BABBITT, Secretary

U.S. GEOLOGICAL SURVEY
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## Conversion Factors

| Multiply | By | To obtain |
|---|---|---|
| foot (ft) | 0.3048 | meter |
| foot per day (ft/d) | 0.3048 | meter per day |
| million gallons per day (Mgal/d) | 0.04381 | cubic meter per second |

# Analytical Versus Numerical Estimates of Water-Level Declines Caused by Pumping, and a Case Study of the Iao Aquifer, Maui, Hawaii

By Delwyn S. Oki and William Meyer

## Abstract

Comparisons were made between model-calculated water levels from a one-dimensional analytical model referred to as RAM (Robust Analytical Model) and those from numerical ground-water flow models using a sharp-interface model code. RAM incorporates the horizontal-flow assumption and the Ghyben-Herzberg relation to represent flow in a one-dimensional unconfined aquifer that contains a body of freshwater floating on denser saltwater. RAM does not account for the presence of a low-permeability coastal confining unit (caprock), which impedes the discharge of fresh ground water from the aquifer to the ocean, nor for the spatial distribution of ground-water withdrawals from wells, which is significant because water-level declines are greatest in the vicinity of withdrawal wells. Numerical ground-water flow models can readily account for discharge through a coastal confining unit and for the spatial distribution of ground-water withdrawals from wells.

For a given aquifer hydraulic-conductivity value, recharge rate, and withdrawal rate, model-calculated steady-state water-level declines from RAM can be significantly less than those from numerical ground-water flow models. The differences between model-calculated water-level declines from RAM and those from numerical models are partly dependent on the hydraulic properties of the aquifer system and the spatial distribution of ground-water withdrawals from wells. RAM invariably predicts the greatest water-level declines at the inland extent of the aquifer where the freshwater body is thickest and the potential for saltwater intrusion is lowest. For cases in which a low-permeability confining unit overlies the aquifer near the coast, however, water-level declines calculated from numerical models may exceed those from RAM even at the inland extent of the aquifer.

Since 1990, RAM has been used by the State of Hawaii Commission on Water Resource Management for establishing sustainable-yield values for the State's aquifers. Data from the Iao aquifer, which lies on the northeastern flank of the West Maui Volcano and which is confined near the coast by caprock, are now available to evaluate the predictive capability of RAM for this system. In 1995 and 1996, withdrawal from the Iao aquifer reached the 20 million gallon per day sustainable-yield value derived using RAM. However, even before 1996, water levels in the aquifer had declined significantly below those predicted by RAM, and continued to decline in 1997. To halt the decline of water levels and to preclude the intrusion of saltwater into the four major well fields in the aquifer, it was necessary to reduce withdrawal from the aquifer system below the sustainable-yield value derived using RAM.

In the Iao aquifer, the decline of measured water levels below those predicted by RAM is consistent with the results of the numerical model analysis. Relative to model-calculated water-level declines from numerical ground-water flow models, (1) RAM underestimates water-level declines

in areas where a low-permeability confining unit exists, and (2) RAM underestimates water-level declines in the vicinity of withdrawal wells.

## INTRODUCTION

A one-dimensional analytical model of ground-water flow, known as the Robust Analytical Model (RAM) (Mink, 1980), is a commonly used tool for estimating sustainable-yield values for aquifer systems in Hawaii. Sustainable yield, as defined by the State of Hawaii, refers to "the maximum rate at which water may be withdrawn from a water source without impairing the utility or quality of the water source..." (State of Hawaii, 1987). The definition "unequivocally incorporates infinite time as a fundamental condition" of the sustainable-yield estimate (State of Hawaii, 1992, p. 98). In Hawaii, the most common limitation on the rate of withdrawal from an aquifer is the upward movement (into wells) of the brackish-water transition zone between freshwater and saltwater. To preclude saltwater intrusion at a given location, it is necessary to maintain a sufficient water level at that location. Estimates of sustainable yield, therefore, require accurate estimates of the water levels in a ground-water system for a given distribution and rate of ground-water withdrawal.

To estimate the amount of water available from a ground-water system on a long-term basis, water-level declines and the changes in the magnitude and distribution of recharge or discharge within the system caused by withdrawals need to be estimated. These factors are, in turn, dependent on: (1) the hydraulic properties of the system, (2) boundary conditions (hydrogeologic features at the physical limits of the system), and (3) the positioning of development (wells) within the system (Bredehoeft and others, 1982).

RAM does not account for aquifer boundary conditions that commonly exist in Hawaii, nor for the spatial distribution of ground-water withdrawals from wells (RAM is one dimensional). Implicit in the use of RAM are the assumptions that (1) sustainable yield can be estimated without accounting for aquifer boundary conditions, aquifer geometry, and the spatial distribution of hydraulic properties of the system, and (2) sustainable yield is an intrinsic property of an aquifer independent of the locations of wells and rates of withdrawal from wells.

One of the more important boundary conditions that RAM cannot represent is a low-permeability confining unit that exists over the volcanic-rock aquifers near and beyond the shoreline in many areas of the State (see, for example, Hunt, 1996; Meyer and Presley, 2000). Among the volcanic-rock aquifers that are overlain by a low-permeability confining unit are the two most important aquifers in the State, the Pearl Harbor aquifer on Oahu and the Iao aquifer on Maui. [For the purposes of this report, the Iao aquifer system, as delineated by State Commission on Water Resource Management (CWRM), is referred to as the Iao aquifer although it is recognized that the Iao aquifer system is part of a regional ground-water flow system.] The confining unit is formed by a wedge-shaped layer of terrestrial or marine sediments of relatively low permeability and is referred to as caprock in Hawaii. A caprock impedes the discharge of freshwater from the aquifer to the ocean and is an important control on the ultimate water-level decline caused by ground-water withdrawals from the aquifer.

In 1995 and 1996, withdrawal from the Iao aquifer reached the sustainable-yield value derived using RAM. However, even before 1996, water levels in the aquifer had declined significantly below those predicted by RAM, and were still declining in 1997. As a result, withdrawal from the aquifer was reduced below the sustainable-yield value derived using RAM to halt the decline of water levels and preclude the intrusion of saltwater into the four major well fields in the aquifer.

***Purpose and scope*.--**The purpose of this report is to describe (1) comparisons between model-calculated water levels from RAM and those from numerical ground-water flow models that account for appropriate aquifer boundary conditions and spatially distributed withdrawals, and (2) a case study of the Iao aquifer, Maui, where water levels have declined below altitudes predicted by RAM. A site-specific numerical ground-water flow model of the Iao aquifer was not developed for this study. Rather, generic one- and two-dimensional numerical ground-water flow models were used to simulate water-level declines for highly permeable aquifers overlain by caprock near the coast. All numerical models used a sharp-interface code (Essaid, 1990) that simulates flow in ground-water systems containing freshwater and saltwater.

## GEOHYDROLOGIC SETTING OF THE HAWAIIAN ISLANDS

The main islands of Hawaii consist of one or more volcanoes that were formed by submarine and subaerial eruptions. During the principal stage of volcano building, called the shield stage, thousands of lava flows emanate from a central caldera and from two to three rift zones that extend outward from the caldera. Magma may cool and solidify beneath the surface of the volcano and form thin, dense, massive, nearly vertical sheets of intrusive rock known as dikes. Within and near the caldera and rift zones, lava flows are intruded by numerous dikes. Outside the zone containing dikes, lava flows extend to the ocean without intrusions. These latter flows are commonly referred to as flank flows in Hawaii. In many coastal areas of the State, lava flows are overlain by sedimentary deposits that form a confining unit, called caprock, above the volcanic-rock aquifer.

In qualitative terms, permeability describes the ease with which fluid can move through a porous rock (see, for example, Domenico and Schwartz, 1990). Permeability of dike-free volcanic rocks in Hawaii is highly variable, depending to some degree on the thickness of individual lava flows and the extent of weathering that individual flows have undergone. Hydraulic conductivity is a quantitative measure of the capacity of a rock to transmit water. The horizontal hydraulic conductivity of the dike-free volcanic rocks of central Oahu and western Hawaii generally is high (on the order of 1,000 ft/d or more) (Hunt, 1996; Oki, 1999), whereas the horizontal hydraulic conductivity of the volcanic rocks of eastern Kauai and northeastern Maui generally is low (on the order of 1 ft/d or less) (Izuka and Gingerich, 1998; Gingerich, 1998; Gingerich, 1999). The low hydraulic conductivity of volcanic rocks of eastern Kauai and northeastern Maui may partly be caused by the presence of dikes.

Ground-water recharge rates in Hawaii vary greatly and are dependent on factors such as soil properties, land cover, and rates of rainfall, evaporation, and runoff. In southern Oahu, recharge has been estimated to range from about 16 to 21 Mgal/d per mile of aquifer width (measured parallel to the coast), depending on land-use conditions (Giambelluca, 1986). In drier areas, such as the western part of the island of Hawaii (Oki and others, 1999), recharge may be as low as 3 Mgal/d per mile of aquifer width.

Fresh ground water in the Hawaiian islands is found mainly as: (1) a freshwater-lens system (with water levels commonly less than a few tens of feet above sea level) consisting of a lens-shaped body of freshwater floating on and displacing saltwater within dike-free volcanic rocks, (2) dike-impounded water (with water levels that are tens to thousands of feet above sea level) where overall permeability is low due to the presence of dikes, and (3) as perched water. The principal source of fresh ground water for domestic use in the Hawaiian islands is from freshwater-lens systems within the highly permeable dike-free parts of volcanic-rock aquifers, such as the Pearl Harbor aquifer on Oahu and the Iao aquifer on Maui.

Where the permeability of dike-free volcanic rocks is relatively high (hydraulic-conductivity values greater than about 1,000 ft/d), predevelopment water levels in the freshwater-lens system generally are less than 50 ft above sea level. Where the permeability of the dike-free volcanic rocks is relatively low (hydraulic-conductivity values less than about 1 ft/d), predevelopment water levels can range from several hundred to several thousands of feet above sea level, forming a vertically extensive freshwater-lens system.

The general movement of fresh ground water is from mountainous interior areas to coastal discharge areas (fig. 1). Ground water discharges into the ocean or streams or by evapotranspiration near the shoreline. Near coastal discharge areas, movement of fresh ground water in a freshwater-lens system is predominantly upward and across the layered sequence of lava flows and the caprock, where it exists.

## HYDROLOGIC EFFECTS OF WITHDRAWAL FROM A GROUND-WATER SYSTEM

The effects of withdrawal on water levels and discharge can be understood most readily by considering a simple, finite ground-water flow system in which the only source of recharge is from precipitation and all discharge is to the ocean. If the rate of recharge to this ground-water system remains unchanged over time, and if there are no ground-water withdrawals, a predevelopment equilibrium or steady-state condition will eventually be reached in which ground-water levels do not vary with time and the rate of discharge from the system is equal to the rate of recharge.

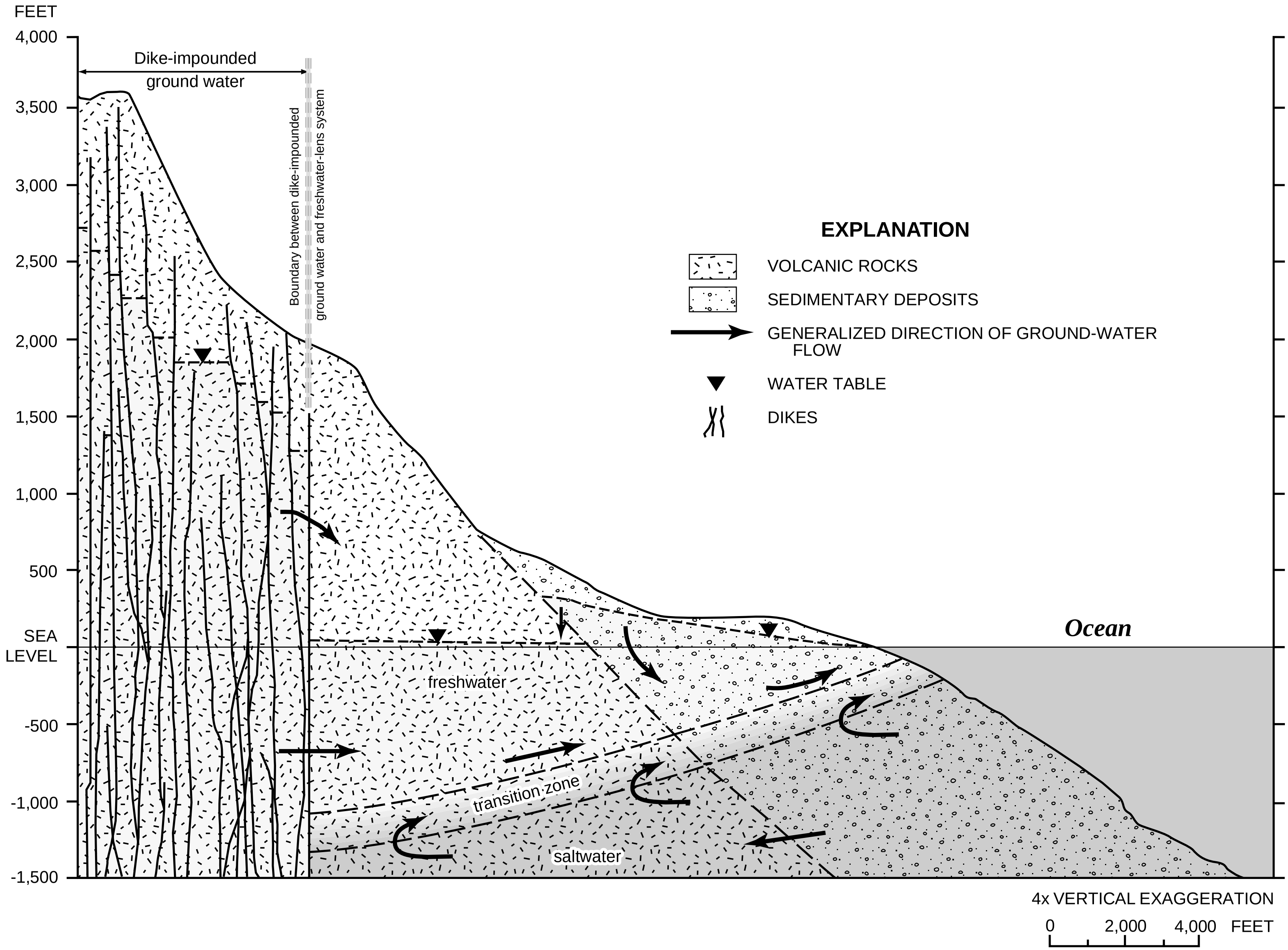


**Figure 1.** Schematic cross section of the regional ground-water flow system of the Iao aquifer, Maui, Hawaii.

When withdrawal from a well begins, water is initially removed from aquifer storage in the vicinity of the well, and water levels in the vicinity of the well begin to decline. If withdrawal from the well continues at a constant rate, the zone over which water levels decline expands outward from the well as additional water is removed from storage. Water-level decline is greatest at the withdrawal site and decreases outward from the well forming what is known as a cone of depression. The cone of depression eventually reaches an area where water is discharging to the ocean. As water levels decline near the discharge area, the rate of discharge to the ocean decreases. If and when the reduction of discharge rate to the ocean is equal to the rate of withdrawal, a new steady-state condition is reached and water levels cease to decline further. The magnitude of the ultimate water-level decline caused by withdrawal is affected by factors including (1) the rate of withdrawal, (2) the hydraulic properties of the aquifer system, and (3) the location of the withdrawal site relative to the discharge boundary of the system. These factors are briefly described in the following paragraphs.

***Rate of withdrawal.***--All other factors being equal, higher rates of withdrawal cause greater water-level declines than lower rates of withdrawal. This is intuitively clear considering that for a withdrawal rate of zero water levels will not decline, and that for a small but positive withdrawal rate water levels will decline to some extent.

***Hydraulic properties of the aquifer system.***--The hydraulic properties at the discharge boundary of the system have an effect on the magnitude of the ultimate water-level decline caused by withdrawal. The lower the permeability of the coastal confining unit, the greater is the water-level decline at the discharge boundary necessary to reduce an equal amount of discharge from the system. This is explained by first considering the case of injecting rather than withdrawing water from a system. Assuming that water is injected at a steady rate for a period sufficiently long to reach steady-state conditions, water levels at the discharge boundary increase to a greater extent (relative to the pre-injection, steady-state condition) the lower the permeability of the confining unit at the discharge boundary because greater hydraulic head is required to force an equal amount of discharge through a low-permeability confining unit than a high-permeability unit. (Hydraulic head at a given point is commonly measured by water levels in wells that are open to the aquifer only at that point.) Thus, to return back to the original, pre-injection, steady-state condition following the cessation of injection, the lower the permeability of the confining unit the greater is the water-level decline at the discharge boundary necessary to reduce an equal amount of discharge from the system.

***Location of the withdrawal site.***--The location of the withdrawal site relative to the discharge boundary has an effect on the magnitude of the water-level decline at the withdrawal site. Consider the case of a one-dimensional, finite aquifer system that is in a steady-state condition prior to any withdrawal. Steady withdrawal from a well at the inland extent of the discharge boundary will cause water levels to decline to a new steady-state level at which the reduction of discharge rate is equal to the withdrawal rate. Because the cone of depression caused by withdrawal from a well is deepest at the well, water-level declines decrease inland from the well. Consider next the case of a well withdrawing at the same rate as in the previous case but located at the inland extent of an identical one-dimensional aquifer system. All other factors being equal, withdrawal from a well at the inland extent of the aquifer will cause water levels to decline at the discharge boundary, and at the inland extent of the discharge boundary, to the same level as in the previous case because steady-state discharge to the ocean is the same in both cases. As in the previous case, water-level declines are greatest at the withdrawal site and, therefore, water-level declines increase from the inland extent of the discharge boundary toward the well. Thus, the water-level decline at an inland withdrawal site is greater than the water-level decline at a withdrawal site near the discharge boundary, all other factors being equal.

In most situations, the source of water derived from wells is from decreased ground-water storage and decreased ground-water discharge. In the above discussion, ground-water discharge was limited to the ocean, which is sometimes the case in Hawaii. However, in some ground-water systems (including those in Hawaii), discharge may be to streams and surface-water bodies other than the ocean, or by evapotranspiration from plants that have roots extending to ground water. Thus, withdrawal from a well may cause a reduction of discharge to streams and other surface-water bodies, or decreased evapotranspiration by plants if the water table is lowered below the level of the roots. In addition, the source of water derived from wells may be from increased recharge. For example, reduction of ground-water levels by withdrawal may induce flow from a stream into the ground-water system or may increase recharge by capturing water that was originally runoff when water levels were at or near the surface.

The hydrologic analysis of a ground-water system generally requires construction of a numerical ground-water flow model. If appropriately constructed, a numerical model can represent the complex relations among the inflows, outflows, changes in storage, movement of water in the system, and other important features.

## CALCULATION OF SUSTAINABLE YIELD USING THE ROBUST ANALYTICAL MODEL (RAM)

The one-dimensional RAM used by CWRM to estimate sustainable yield in Hawaii incorporates the horizontal-flow assumption (see, for example, Bear, 1972) and the Ghyben-Herzberg relation and is described in detail in the appendix. By the assumptions used to derive RAM (Mink, 1980), for any location in the aquifer, the ratio of the hydraulic head squared to the total flow rate through the aquifer is constant. Thus, the following relation is assumed to be true:

$$h_0^2/Q_0 = h_e^2/Q_e, \qquad (1)$$

where,
- $h_0$ = hydraulic head [L], relative to mean sea level, at location $x$ for flow rate $Q_0$,
- $Q_0$ = steady-state rate of flow through aquifer for predevelopment conditions [$L^3/T$],
- $h_e$ = hydraulic head [L], relative to mean sea level, at location $x$ for flow rate $Q_e$,
- $Q_e$ = steady-state rate of flow (less withdrawals from wells or shafts) through aquifer for development conditions [$L^3/T$], and
- $x$ = Cartesian coordinate [L].

Calculation of sustainable yield using RAM involves pre-selection of the steady-state water level that will occur if ground water is withdrawn at a rate equal to the sustainable yield. This water level is referred to as the equilibrium head ($h_e$). For the desired equilibrium head, $h_e$, CWRM defines the sustainable yield, $D$, as the difference between the predevelopment rate of flow through the aquifer minus the reduced rate of flow through the aquifer following development:

$$D = Q_0 - Q_e. \qquad (2)$$

Combining equations (1) and (2), and defining $I$ to be equal to $Q_0$ yields:

$$D/I = 1 - (h_e/h_0)^2. \qquad (3)$$

Equation (3) represents the model (RAM) commonly used to set sustainable yield in Hawaii. To apply this equation, predevelopment values for $h_0$ and $I$ must be known or estimated, and some desired minimum equilibrium head, $h_e$, must be established. In many areas, values for $h_0$ are poorly known and must therefore be estimated. The value for $I$ is generally equated to the recharge from a water budget of predevelopment conditions. The value for $h_e$ is selected to preserve the quality of water produced at steady-state conditions (State of Hawaii, 1992, p. B3).

In Hawaii, RAM is used for all freshwater-lens systems and in areas where dike-impounded water is dominant or extends to the coast (State of Hawaii, 1992, p. 120). According to the State Water Resources Protection Plan, where the initial head, $h_0$, in the aquifer was low, the ratio $h_e$:$h_0$ must be large and the ratio $D$:$I$ must be small (State of Hawaii, 1992, p. B3). Also according to the State Water Resources Protection Plan, the ratio $h_e$:$h_0$ "used to obtain sustainable yield is based on experience with known aquifers, such as those of Honolulu and southern Oahu" (State of Hawaii, 1992, p. B4). Values of $h_e$:$h_0$ and $D$:$I$ used by CWRM for given values of $h_0$ are shown in table 1.

***Limitations of RAM***.--One of the major limitations of RAM for use in estimating sustainable yield in Hawaii is the inability of the model to account for the caprock, which creates resistance to vertical discharge of ground water from the aquifer to the ocean. The overall vertical hydraulic conductivity of dike-free volcanic rocks (including weathered zones) and the caprock is generally one to four orders of magnitude less than the horizontal hydraulic conductivity of the dike-free volcanic rocks. Thus, the resistance to vertical discharge of ground water to the ocean is much greater per unit area than the resistance to horizontal ground-water flow in the aquifer. The rate of vertical discharge is proportional to the overall vertical hydraulic conductivity of the volcanic rocks and caprock divided by the thickness

**Table 1.** Ratios of sustainable yield to recharge used by the State of Hawaii Commission on Water Resource Management for aquifers in Hawaii (State of Hawaii, 1990)

| Initial head, $h_0$, in feet above sea level | Ratio of equilibrium head to initial head, $h_e$:$h_0$ | Ratio of sustainable yield to recharge, $D$:$I$ |
|---|---|---|
| 4–10 | 0.75 | 0.44 |
| 11–15 | 0.70 | 0.51 |
| 16–20 | 0.65 | 0.58 |
| 21–25 | 0.60 | 0.64 |
| >26 | 0.50 | 0.75 |

of these two rock units. The ratio of vertical hydraulic conductivity to thickness is known as leakance:

$$L = K_v / B \tag{4}$$

where, $L$ = leakance [1/T],
$K_v$ = overall vertical hydraulic conductivity of the rocks where vertical discharge occurs [L/T], and
$B$ = overall rock thickness over which vertical discharge occurs [L].

RAM does not account for the concept of leakance although leakance is "all important" in controlling the response of ground-water systems to stresses (Bredehoeft and Hall, 1995). Leakance is important because for withdrawal to be sustained in most areas of Hawaii, natural discharge into the ocean must be reduced by an amount equal or nearly equal to withdrawal. The smaller the value of leakance (or the greater the resistance to the diversion of water to wells), the greater is the water-level decline necessary to reduce an equal amount of natural discharge, and the greater the water-level decline in the well or wells. Because RAM does not account for the presence of a caprock and the concept of leakance, RAM cannot accurately predict water-level declines associated with withdrawals in many Hawaiian ground-water systems due to this limitation alone.

In addition to its inability to represent a caprock, RAM cannot account for spatially distributed withdrawals from wells and the spatial distribution of water-level declines, which are greatest in the vicinity of withdrawal wells. As will be shown in the following sections, the one-dimensional RAM invariably predicts the greatest water-level declines at the inland extent of the aquifer where the freshwater lens is thickest and the potential for saltwater intrusion is lowest.

Because RAM is a one-dimensional model, it cannot accurately account for the spatial distribution of recharge. RAM assumes that all recharge enters the ground-water flow system at the inland extent of the system. Furthermore, because RAM is a one-dimensional model, it cannot adequately account for the geometry of the ground-water flow system. RAM also cannot account for the spatial variability of aquifer hydraulic properties, which affects the distribution of water-level declines caused by withdrawals.

In the following sections of this report, model-calculated water-level declines from RAM are compared with model-calculated water-level declines from one- and two-dimensional numerical ground-water flow models. One-dimensional numerical models are used to demonstrate the importance of the caprock on the hydrologic response of the ground-water system to withdrawals, and two-dimensional (areal) numerical models are used to demonstrate the importance of representing the spatial distribution of ground-water withdrawals from wells. (By addressing the spatial distribution of withdrawals, the two-dimensional models also indirectly address the importance of properly representing the spatial distribution of recharge.)

## COMPARISONS BETWEEN RAM AND ONE-DIMENSIONAL NUMERICAL MODELS

A simple one-dimensional ground-water flow system was used to compare model-calculated water levels from RAM with steady-state water levels from sharp-interface numerical ground-water flow models. The numerical code used was SHARP (Essaid, 1990), which simulates flow in ground-water systems containing freshwater and saltwater and treats freshwater and saltwater as immiscible fluids separated by a sharp interface. The ground-water flow system was assumed to consist of an aquifer that is unconfined inland and that is confined by a caprock near the shore and offshore. The numerical model grid used to represent the flow system consists of 44 cells; each cell is 2,000 ft long and extends to a depth of 6,000 ft below sea level (fig. 2).

Recharge to the system was assumed to be a constant value of 20 Mgal/d per mile of aquifer width and enter the system at the inland extent of the aquifer. The restriction that recharge enter the system at the inland extent of the aquifer is necessary because RAM cannot represent spatially varying recharge.

The horizontal hydraulic conductivity of the aquifer was assumed to be a constant value of 1,500 ft/d, corresponding to a highly permeable volcanic-rock aquifer. The analysis was restricted to highly permeable volcanic-rock aquifers because vertical head gradients are expected to be small in magnitude relative to vertical head gradients in poorly permeable aquifers. Both RAM and the numerical models used in this study assume that flow is horizontal, a condition which is less likely to occur in poorly permeable aquifers.

The confining unit that overlies the aquifer near the coast is represented in the numerical models as a seaward-thickening wedge of coastal sedimentary deposits that is 40 ft thick at the inland extent of the confining unit and 1,000 ft thick at the shore (fig. 2). Offshore, the caprock is assumed to have a constant thickness of 1,000 ft. Discharge through the caprock is assumed to be in the vertical direction. Three different values of caprock vertical hydraulic conductivity were tested with numerical models: 15, 0.15, and 0.075 ft/d. The vertical hydraulic-conductivity value of 0.15 ft/d is representative of the Pearl Harbor aquifer of southern Oahu (Souza and Voss, 1987). The range of leakance values represented in the one-dimensional numerical models is about 0.000075 (=0.075/1,000) to 0.375 (=15/40) feet per day per foot. The range of leakance values tested is consistent with the range of values estimated for Hawaiian ground-water flow systems (table 2).

Discharge from the aquifer to the ocean was modeled as a head-dependent discharge boundary condition. The rate of freshwater discharge is assumed to be linearly related to the leakance and head in the aquifer according to the equation:

$$Q = LA_c(h - h') \tag{5}$$

where, $Q$ = rate of discharge from the aquifer [$L^3$/T],
$L$ = confining unit leakance, [1/T],
$A_c$ = plan area of confining unit [$L^2$],
$h$ = hydraulic head in the aquifer [L], relative to mean sea level, at the discharge boundary, and
$h'$ = hydraulic head above the confining unit [L], relative to mean sea level.

For onshore areas, $h'$ was assumed to be equal to zero. For offshore areas, $h'$ was assigned a value corresponding to the freshwater-equivalent head of the saltwater column overlying the ocean floor within the cell. The freshwater-equivalent head, measured relative to a mean sea level datum, was computed from the equation:

$$h' = -Z/40, \tag{6}$$

where $Z$ is the altitude of the ocean floor.

## Zero Ground-Water Withdrawals

For zero ground-water withdrawals, model-calculated steady-state water levels from the numerical models were 6.3, 30.6, and 52.5 ft above sea level at the seaward extent of the unconfined part of the system for caprock vertical hydraulic-conductivity values of 15, 0.15, and 0.075 ft/d, respectively (figs. 3 and 4). Lower vertical hydraulic-conductivity values for the caprock result in a greater resistance to discharge and higher water levels.

In the absence of ground-water withdrawals, an analytical equation (see equation a4 in the appendix) that forms the basis of RAM can be used to compute the steady-state water-table profile in a one-dimensional aquifer if the water level is known at the seaward extent of the unconfined part of the aquifer. To allow for direct comparisons between the analytical equation and numerical models, the water level at the seaward extent of the unconfined part of the aquifer for the analytical equation was assigned the same value as the corresponding water level from the numerical model. Thus, in the analytical equation, the water level at the seaward extent of the unconfined part of the aquifer was assigned values of 6.3, 30.6, and 52.5 ft above sea level for the three different cases, corresponding to the three caprock vertical hydraulic-conductivity values tested with the numerical models. For zero ground-water withdrawals, the model-calculated water-table profiles from the numerical models are in close agreement with the

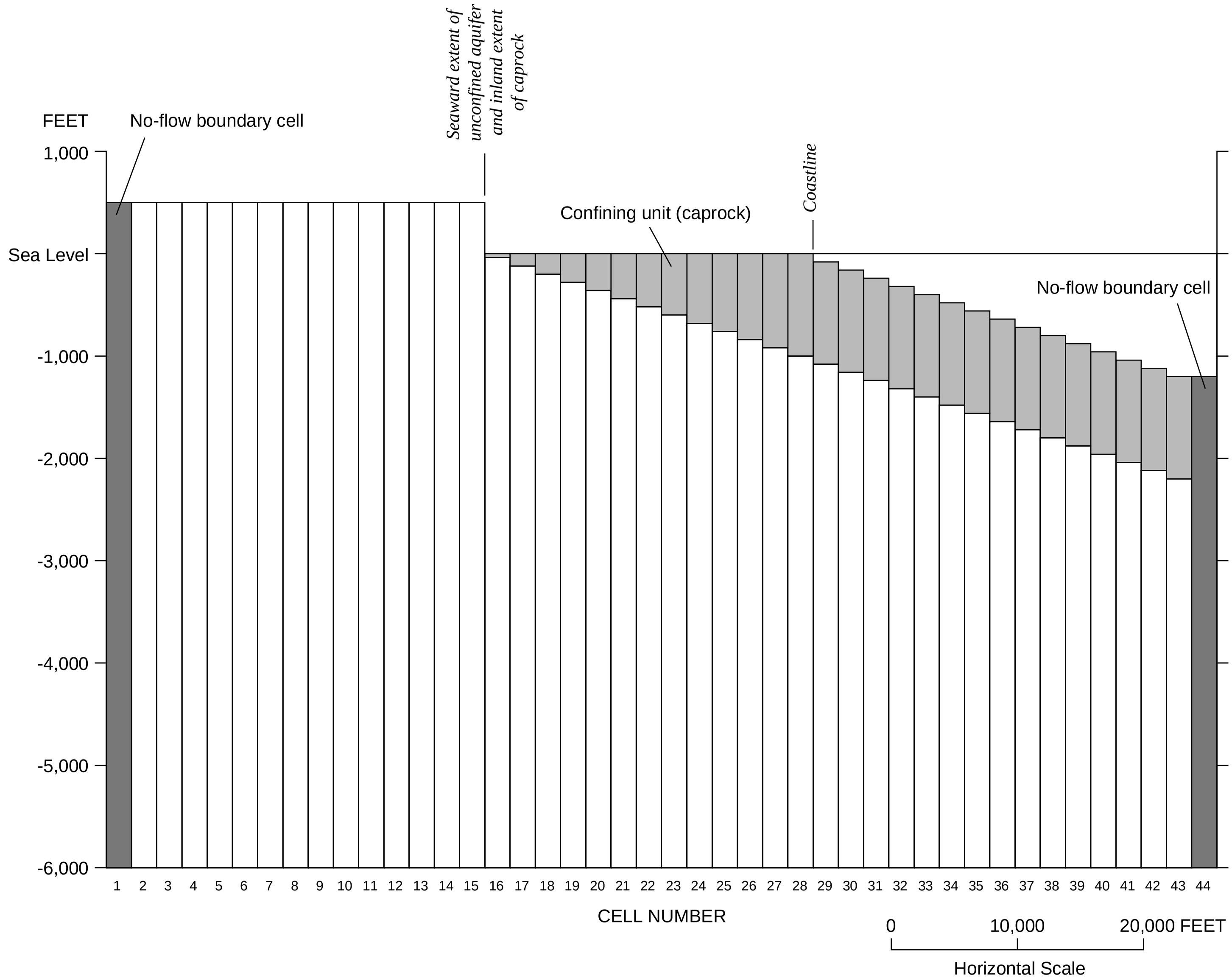


**Figure 2.** Vertical cross section of the one- and two-dimensional numerical-model grids. Model cells 2 to 15 of the one-dimensional model grid (corresponding to rows 2 to 15 of the two-dimensional model grid) are unconfined, water-table cells. Model cells 16 to 43 of the one-dimensional model grid (corresponding to rows 16 to 43 of the two-dimensional model grid) are head-dependent discharge cells. All model cells are 2,000 feet long. Recharge is introduced into model cell 2 of the one-dimensional model grid (row 2 for the two-dimensional model grid). For the one-dimensional model, pumping is simulated from cells 2, 9, or 15, corresponding to pumping from near the inland extent, middle, or seaward extent, respectively, of the unconfined part of the aquifer.

**Table 2.** Values of leakance for coastal discharge areas in Hawaii

| Area | Leakance (feet per day per foot) | Reference |
|---|---|---|
| Oahu, northern | 0.00007–1[a] | Oki, 1998 |
| Oahu, southern | 0.00001–0.03[a] | Oki, 1998 |
| Oahu, southeastern | 0.0004–0.03[a] | Eyre and others, 1986 |
| Molokai, northern | 0.1 | Oki, 1997 |
| Molokai, southern | 0.001–0.3[a] | Oki, 1997 |
| Hawaii, northwestern | 0.01–0.1 | Underwood and others, 1995 |
| Hawaii, western | 0.05 | Oki, 1999 |

[a]Leakance is dependent on the thickness of the confining unit and is therefore spatially variable.

water-table profiles from the analytical equation (figs. 3 and 4).

## Ground-Water Withdrawals

For ground-water systems in Hawaii with a low-permeability coastal confining unit, predevelopment water levels generally ranged from about 10 to 40 ft above sea level. For these systems, CWRM assumes that at least 50 percent of the total ground-water recharge to the aquifer can be withdrawn (table 1). For systems with predevelopment water levels greater than 26 ft above sea level, CWRM assumes that as much as 75 percent of the total recharge to the aquifer can be withdrawn (table 1). Thus, the one-dimensional numerical models were used to simulate steady-state water levels that result from withdrawing 50 percent (fig. 3) or 75 percent (fig. 4) of the recharge to the aquifer.

Water-table profiles were simulated for each of three caprock vertical hydraulic-conductivity values (0.075, 0.15, and 15 ft/d) and for each of three different locations of withdrawal (at the inland extent of the unconfined part of the aquifer, near the middle of the unconfined part of the aquifer, and near the seaward extent of the unconfined part of the aquifer). The seaward extent of the unconfined part of the aquifer is the same as the inland extent of the caprock discharge boundary. In a one-dimensional model, withdrawal is implicitly assumed to occur uniformly along the entire width of the aquifer. In the numerical model, the simulated withdrawal was restricted to the freshwater part of the system; that is, no saltwater was withdrawn. Results from this study are consistent with results from published numerical models, which have shown that leakance is one of the major factors controlling the response of ground-water systems to natural or imposed stresses in Hawaii (Underwood and others, 1995; Oki, 1997; Oki, 1998).

The model-calculated water-table profiles from the numerical models (figs. 3 and 4) indicate that, for a given withdrawal rate and location, lower values of caprock vertical hydraulic conductivity cause greater water-level declines relative to predevelopment (zero withdrawal) conditions. As described previously, the lower the value of caprock vertical hydraulic conductivity (or leakance), the greater is the steady-state water-level decline needed to reduce an equal amount of natural discharge (see the section “Hydrologic Effects of Withdrawal from a Ground-Water System”).

The model-calculated water-table profiles (figs. 3 and 4) from the numerical models also indicate that for a given value of caprock vertical hydraulic conductivity and withdrawal rate (1) the water-level declines at the inland extent of the discharge boundary (caprock) are the same regardless of where the withdrawal site is located inland from the caprock, and (2) water-level declines at withdrawal sites are greater for inland withdrawal sites than for withdrawal sites near the caprock. These results are consistent with the expected response of a ground-water system to withdrawal (see the section “Hydrologic Effects of Withdrawal from a Ground-Water System”).

RAM also was used to compute the water-table profiles that would result if either 50 or 75 percent of the total 20 Mgal/d per mile recharge was withdrawn (figs. 3 and 4). By the assumptions of RAM, all ground-water withdrawals are assumed to occur at the inland extent of the aquifer because withdrawals are represented as a reduction in recharge. RAM predicts that if 50 percent of the recharge is withdrawn, then the resulting steady-state water levels are uniformly 0.707 (equal to the square root of 0.5) multiplied by the predevelopment

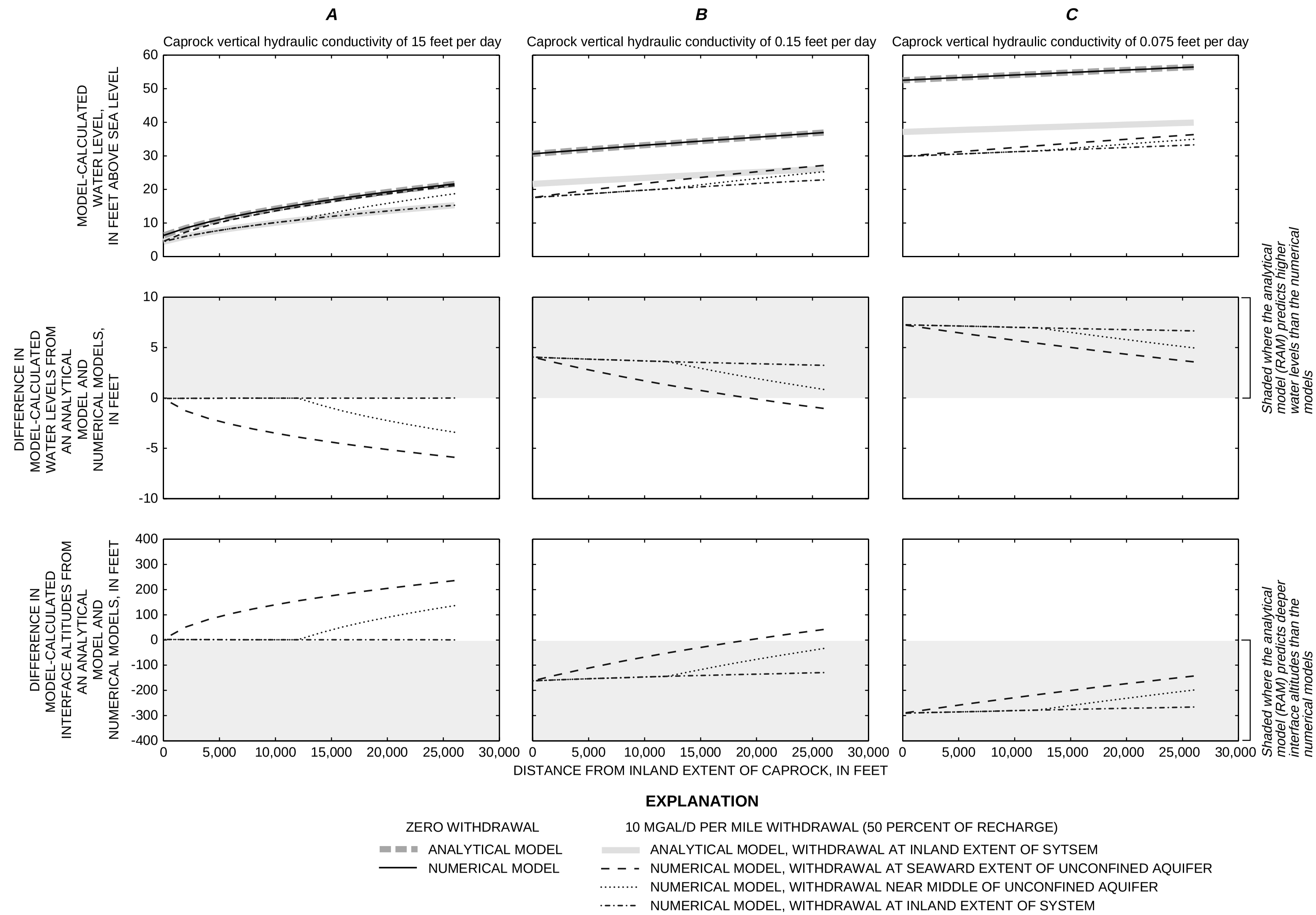


**Figure 3.** Model-calculated steady-state water levels from an analytical model (RAM) and from one-dimensional numerical models for conditions of zero withdrawal and withdrawal of 50 percent of the total recharge to the aquifer for selected caprock confining unit vertical hydraulic-conductivity values: (A) 15 feet per day; (B) 0.15 feet per day; (C) 0.075 feet per day. Also shown are the differences between the model-calculated water levels (and interface altitudes) from the analytical model and one-dimensional numerical models for withdrawal conditions. Aquifer hydraulic conductivity is 1,500 feet per day, and ground-water recharge is 20 million gallons per day per mile of aquifer width. All recharge enters the system at the inland extent of the aquifer.

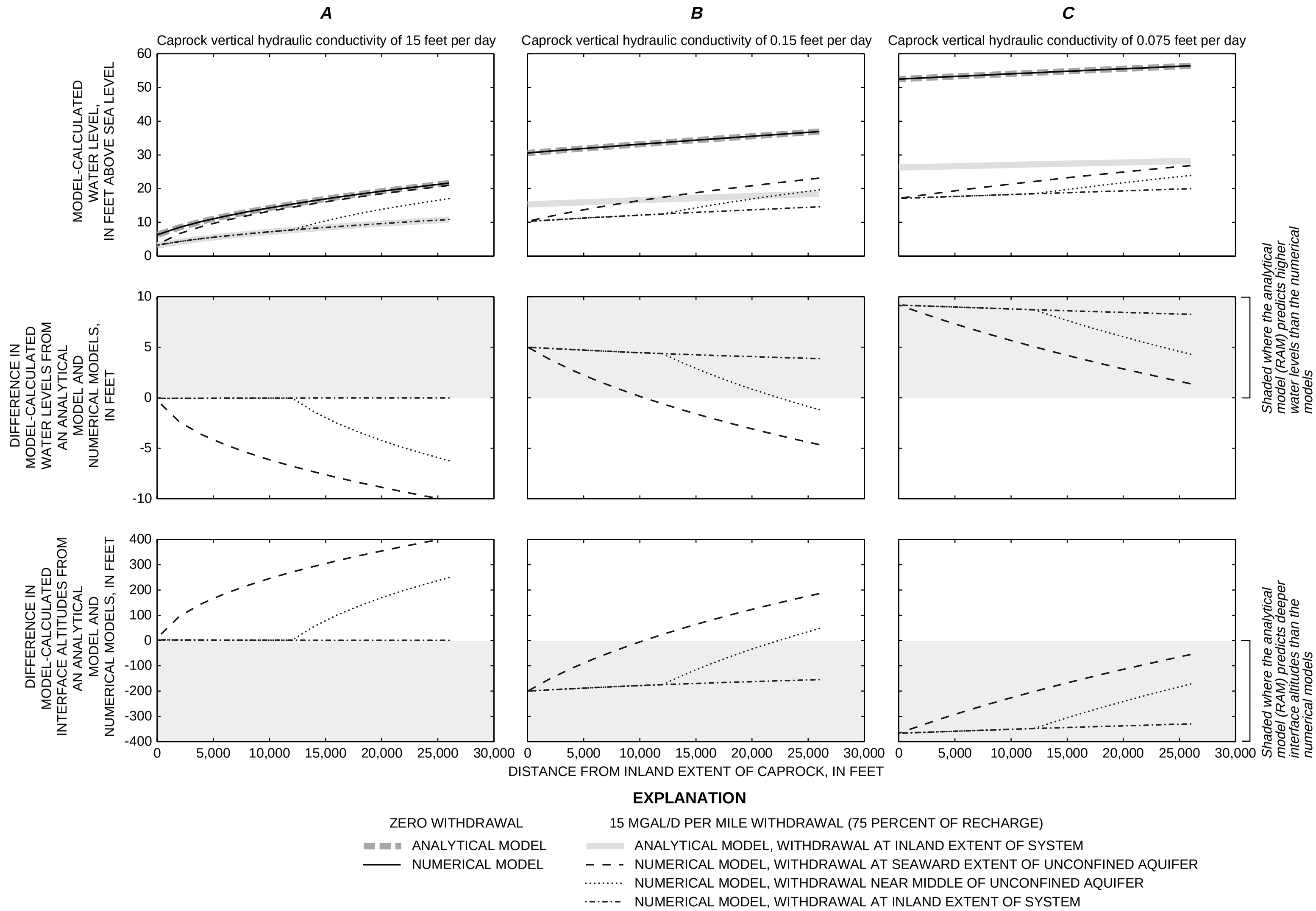


**Figure 4.** Model-calculated steady-state water levels from an analytical model (RAM) and from one-dimensional numerical models for conditions of zero withdrawal and withdrawal of 75 percent of the total recharge to the aquifer for selected caprock confining unit vertical hydraulic-conductivity values: (A) 15 feet per day; (B) 0.15 feet per day; (C) 0.075 feet per day. Also shown are the differences between the model-calculated water levels (and interface altitudes) from the analytical model and one-dimensional numerical models for withdrawal conditions. Aquifer hydraulic conductivity is 1,500 feet per day, and ground-water recharge is 20 million gallons per day per mile of aquifer width. All recharge enters the system at the inland extent of the aquifer.

steady-state water levels (see equation 3). Similarly, RAM predicts that if 75 percent of the recharge is withdrawn, then the resulting steady-state water levels are uniformly 0.5 (equal to the square root of 0.25) multiplied by the predevelopment steady-state water levels (see equation 3).

Model results indicate that for the case of an aquifer overlain by a coastal caprock with a high vertical hydraulic conductivity (15 ft/d), (1) the model-calculated water-table profile from RAM is almost identical to the model-calculated water-table profile from a one-dimensional numerical model if withdrawal in the numerical model is represented at the inland extent of the aquifer, and (2) model-calculated water levels from RAM are generally lower than or at the same altitude as model-calculated water levels from a one-dimensional numerical model if withdrawal in the numerical model is from sites other than at the inland extent of the aquifer (figs. 3A and 4A; table 3). As described previously, withdrawal sites closer to the discharge boundary are expected to cause smaller water-level declines than sites farther from the discharge boundary, all other factors being equal (see the section "Hydrologic Effects of Withdrawal from a Ground-Water System").

For lower values of caprock vertical hydraulic conductivity (0.075 and 0.15 ft/d), the model-calculated water levels from RAM are higher than those from one-dimensional numerical models at the site of withdrawal represented in the numerical models (figs. 3B and C, and 4B and C). For the case of withdrawing 50 percent of the recharge from an aquifer overlain by a caprock with a vertical hydraulic conductivity of 0.15 ft/d, model-calculated water levels from RAM are higher than those from the numerical models at the withdrawal site by 3.2 ft (withdrawal at inland extent of aquifer) to 4.1 ft (withdrawal at inland extent of caprock) (fig. 3B and C; table 3). For the case of withdrawing 75 percent of the recharge from an aquifer overlain by a caprock with a vertical hydraulic conductivity of 0.15 ft/d, model-calculated water levels from RAM are higher than those from the numerical models at the withdrawal site by 3.9 ft (withdrawal at inland extent of aquifer) to 5.0 ft (withdrawal at inland extent of caprock) (fig. 4B and C; table 3). At the site of withdrawal in an aquifer overlain by a low-permeability coastal caprock, the difference in model-calculated water levels from RAM and the numerical models increases with increasing rate of withdrawal. This result indicates that properly accounting for the hydrologic effects of a low-permeability caprock on water levels at the withdrawal site becomes increasingly important as the withdrawal rate increases.

For caprock vertical hydraulic-conductivity values of 0.075 and 0.15 ft/d, the maximum difference between model-calculated water levels from RAM and the numerical models is at the inland extent of the caprock (figs. 3B and C, and 4B and C). For the case of withdrawing 50 percent of the recharge, model-calculated water levels from RAM are higher than those from the numerical models at the inland extent of the caprock by 4.1 and 7.3 ft for caprock vertical hydraulic conductivity values of 0.15 and 0.075 ft/d, respectively (table 3).

The differences in model-calculated water levels from RAM and the numerical models result from the inability of RAM to adequately account for the hydrologic effects of a coastal confining unit. Because RAM assumes that discharge from the system is not impeded by a coastal confining unit, RAM tends to underestimate steady-state water-level declines caused by withdrawals for cases in which a low-permeability confining unit is present.

The Ghyben-Herzberg relation (see appendix) indicates that for every foot of water-level decline, the position of the freshwater-saltwater interface will rise by 40 ft. Except for cases in which water is withdrawn from the inland extent of an aquifer without a low-permeability coastal caprock, model-calculated water-level declines (and interface rises) from RAM and from numerical models generally differ (figs. 3 and 4). RAM predicts that the interface beneath sites of withdrawal will rise to a lesser extent than indicated by one-dimensional numerical models representing aquifers that are confined by a low-permeability caprock (figs. 3B and C, and 4B and C). For the case of withdrawing 50 percent of the recharge from an aquifer overlain by a caprock with a vertical hydraulic conductivity value of 0.15 ft/d, model-calculated water levels from RAM are higher than those from the numerical models at the withdrawal site by 3.2 to 4.1 ft. Thus, at the withdrawal site, the corresponding freshwater-saltwater interface position predicted by RAM is 128 to 164 ft deeper than indicated by the numerical models (fig. 3B). In Hawaii, management practices have generally assumed that it is desirable, where possible, to maintain about a 100 ft zone of freshwater between the bottom of a withdrawal well and the top of the brackish-water transition zone

**Table 3.** Differences between model-calculated water levels from RAM and the numerical models at selected sites

| Numerical model | Location of withdrawal well in the unconfined part of the aquifer | Difference between water levels predicted by RAM and the numerical model at different locations in the unconfined part of the aquifer, in feet[a]: Seaward extent | Middle | Inland extent |
|---|---|---|---|---|
| **50 percent of recharge withdrawn** | | | | |
| **One-dimensional model, $K_v$ = 0.075 feet per day**[b] | seaward extent | 7.3 | 5.4 | 3.6 |
| | middle | 7.3 | 7.0 | 5.0 |
| | inland extent | 7.3 | 7.0 | 6.7 |
| **One-dimensional model, $K_v$ = 0.15 feet per day**[b] | seaward extent | 4.1 | 1.3 | -1.0 |
| | middle | 4.1 | 3.6 | 0.8 |
| | inland extent | 4.1 | 3.6 | 3.2 |
| **One-dimensional model, $K_v$ = 15 feet per day**[b] | seaward extent | -0.1 | -3.9 | -5.9 |
| | middle | 0.0 | 0.0 | -3.4 |
| | inland extent | 0.0 | 0.0 | 0.0 |
| **Two-dimensional model, $K_v$ = 0.15 feet per day**[b] | seaward extent | 11.5 | 1.7 | -1.0 |
| | middle | 4.6 | 9.3 | 1.3 |
| | inland extent | 4.2 | 4.2 | 11.1 |
| **75 percent of recharge withdrawn** | | | | |
| **One-dimensional model, $K_v$ = 0.075 feet per day**[b] | seaward extent | 9.2 | 5.1 | 1.4 |
| | middle | 9.2 | 8.7 | 4.3 |
| | inland extent | 9.2 | 8.7 | 8.2 |
| **One-dimensional model, $K_v$ = 0.15 feet per day**[b] | seaward extent | 5.0 | -0.6 | -4.6 |
| | middle | 5.0 | 4.4 | -1.2 |
| | inland extent | 5.0 | 4.4 | 3.9 |
| **One-dimensional model, $K_v$ = 15 feet per day**[b] | seaward extent | -0.3 | -6.8 | -10.2 |
| | middle | -0.1 | 0.0 | -6.2 |
| | inland extent | -0.1 | 0.0 | 0.0 |

[a]Positive differences indicate that the water level predicted by RAM is greater than the water level predicted by the numerical model. For the two-dimensional numerical model, differences were computed along a line through the well, and perpendicular to the coast.

[b]$K_v$ is the vertical hydraulic conductivity of the caprock confining unit.

(Mink and others, 1988). Therefore, underestimating water-level declines by a few feet is significant and could lead to saltwater intrusion into some wells.

## COMPARISONS BETWEEN RAM AND TWO-DIMENSIONAL NUMERICAL MODELS

Although RAM is a one-dimensional model, steady-state water levels from two-dimensional (areal), sharp-interface numerical ground-water flow models also were compared with water levels from RAM. The numerical code SHARP (Essaid, 1990) also was used for the two-dimensional models. The geometry of the two-dimensional ground-water flow system was the same as the one-dimensional system described previously, except that the two-dimensional system was discretized perpendicular to the coastline. The numerical model grid used to represent the flow system consists of 1,188 square cells, each 2,000 ft long by 2,000 ft wide, arranged in a rectangular array with 44 rows and 27 columns. As with the one-dimensional system, the horizontal hydraulic conductivity of the aquifer was assumed to be a constant value of 1,500 ft/d, and the recharge to the system was assumed to be a constant value of 20 Mgal/d per mile of aquifer width and uniformly enter the system at the inland extent of the aquifer. The coastal confining unit in the two-dimensional system was represented using the same geometry as in the one-dimensional system. The caprock vertical hydraulic-conductivity value tested with the two-dimensional system was 0.15 ft/d, which is a reasonable value for low-permeability coastal sedimentary deposits.

### Zero Ground-Water Withdrawals

For zero ground-water withdrawals, the model-calculated water level at the seaward extent of the unconfined part of the system was 30.6 ft above sea level for the numerical model. The water level at the seaward extent of the unconfined part of the aquifer for the analytical equation (equation a4) was assigned the same value of 30.6 ft above sea level. The water-table profiles from the analytical equation and numerical model were in close agreement. Although the numerical model is discretized in two dimensions (areally), the flow field for this case is one-dimensional because recharge is introduced uniformly along the width of the aquifer at the inland extent of the system. Thus, the water-table profile is identical to the profile from the one-dimensional numerical model without withdrawals (fig. 3B).

### Ground-Water Withdrawals

Unlike in a one-dimensional model in which withdrawal is implicitly assumed to occur uniformly along the entire width of the aquifer, in a two-dimensional (areal) model withdrawal can be nonuniformly distributed at individual sites in the aquifer. Water-table profiles were simulated for each of three different sites of withdrawal: at the inland extent of the unconfined part of the aquifer (fig. 5), near the middle of the unconfined part of the aquifer (fig. 6), and near the seaward extent of the unconfined part of the aquifer (fig. 7). Each of the individual withdrawal sites represented in the two-dimensional system was placed along the centerline (perpendicular to the coast) of the aquifer. The numerical models were used to simulate steady-state water levels that result from withdrawing 50 percent of the total ground-water recharge to the aquifer.

Although RAM predicts that the ratio of development heads to predevelopment heads ($h_e$:$h_0$) is 0.707 at all locations if 50 percent of the recharge is withdrawn, results from the two-dimensional numerical models indicate that for a caprock vertical hydraulic-conductivity value of 0.15 ft/d, the ratio $h_e$:$h_0$ is (1) spatially variable, (2) less than 0.5 near the sites of withdrawal, where maintaining higher water levels is generally most important, (3) less than 0.707 for all locations if water is withdrawn at a site that is inland from the middle of the unconfined part of the aquifer, and (4) equal to 0.707 only along a single line in the aquifer (in plan view) if water is withdrawn near the seaward extent of the unconfined part of the aquifer (fig. 7).

Model results indicate that water levels from RAM are as much as 11.5 ft higher than water levels from a two-dimensional numerical model at the site of withdrawal (table 3). Thus, on the basis of the Ghyben-Herzberg relation, RAM predicts that the position of the freshwater-saltwater interface is as much as 460 ft deeper than indicated by the two-dimensional numerical model. Spatially, the differences between model-calculated water levels from RAM and model-calculated water levels from the numerical models are greatest in the most critical areas, which are near the sites of withdrawal. The inability of RAM to adequately account for the spatial distribution of withdrawals (or recharge) is a major limitation of RAM.

It should be noted that the simulated water-level decline in a numerical-model cell may be much less than the actual decline at the withdrawal well because the simulated water-level decline represents the average decline over an entire cell rather than the maximum at a given point. In addition, the actual water-level decline in the immediate vicinity of partially penetrating withdrawal wells may be much greater than simulated with the numerical model because a single-layer numerical model cannot account for vertical head gradients in the aquifer. On the other hand, because a single-layer numerical model cannot account for vertical flow, the numerical model may overestimate the rise in position of the freshwater-saltwater interface caused by withdrawal from a partially penetrating well, especially for highly anisotropic aquifers in which the vertical hydraulic conductivity is several orders of magnitude less than the horizontal hydraulic conductivity.

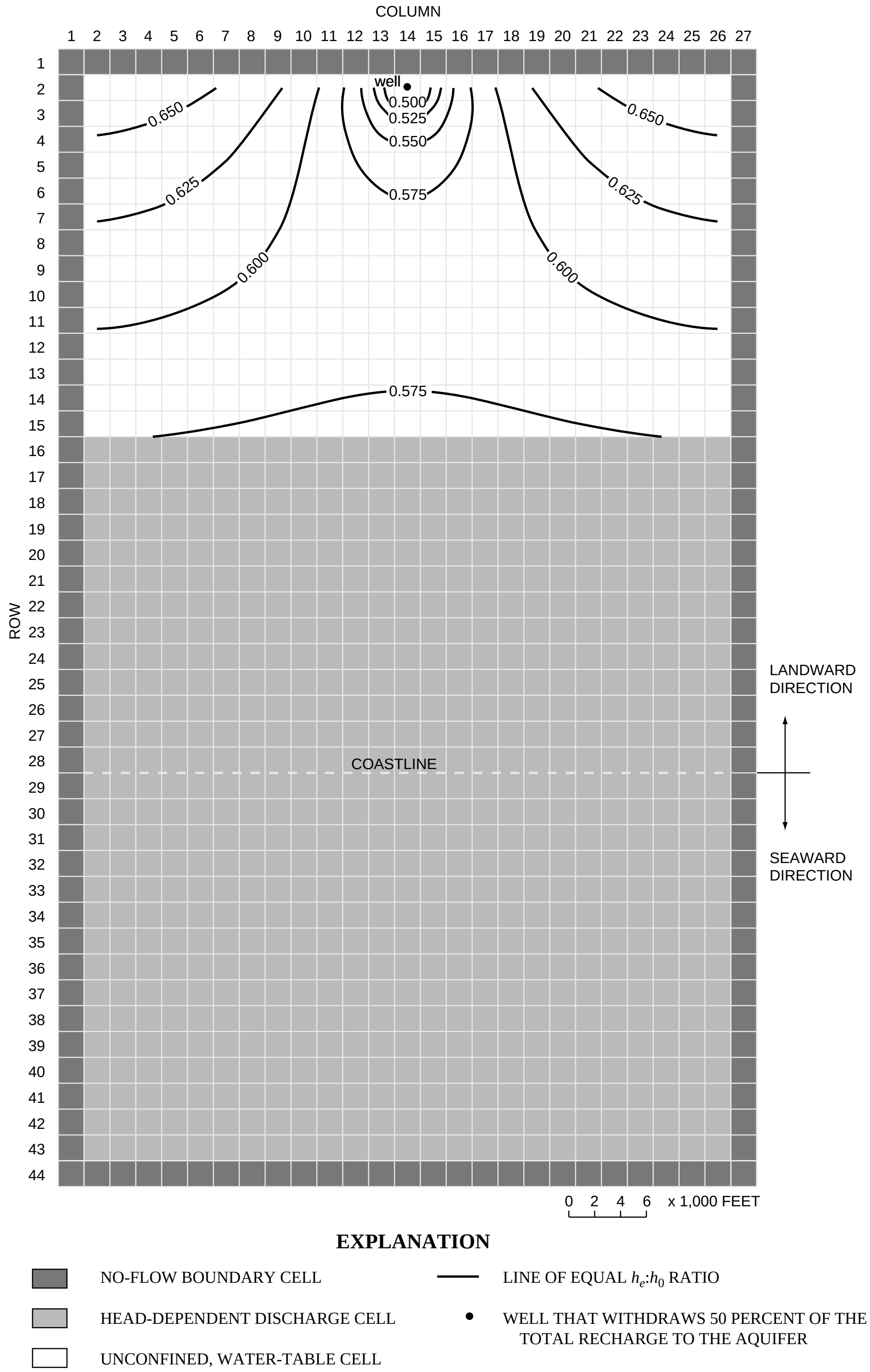


**Figure 5.** Model-calculated ratios (from a two-dimensional numerical ground-water flow model) of steady-state water levels for withdrawal conditions ($h_e$) to steady-state predevelopment water levels ($h_0$) in the unconfined part of the aquifer for the case of withdrawing, from a well near the inland extent of the aquifer, 50 percent of the total ground-water recharge. Recharge enters the system uniformly at the inland extent of the aquifer at the rate of 20 million gallons per day per mile of width. The horizontal hydraulic conductivity of the aquifer is 1,500 feet per day, and the vertical hydraulic conductivity of the caprock confining unit is 0.15 feet per day.

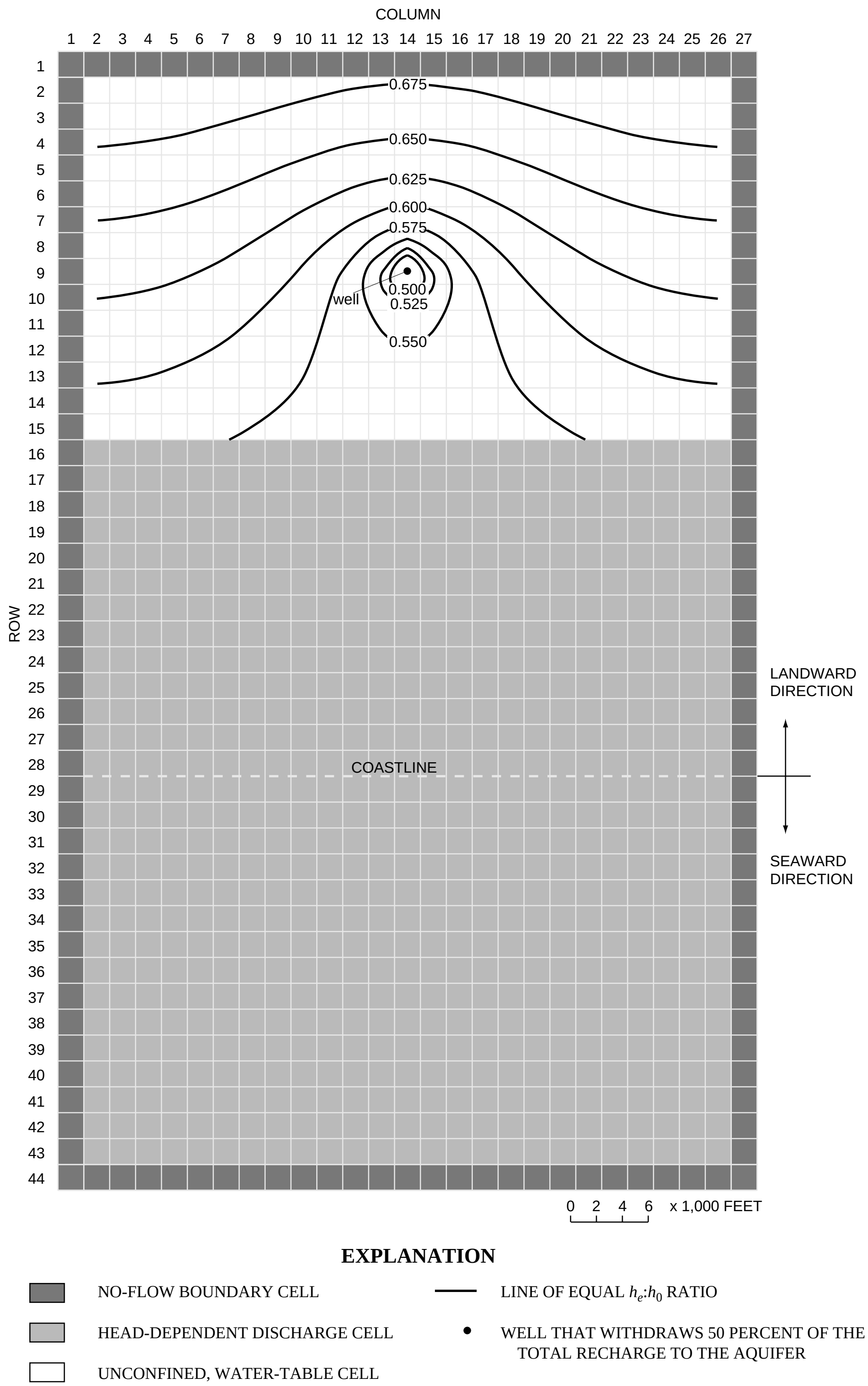


**Figure 6.** Model-calculated ratios (from a two-dimensional numerical ground-water flow model) of steady-state water levels for withdrawal conditions ($h_e$) to steady-state predevelopment water levels ($h_0$) in the unconfined part of the aquifer for the case of withdrawing, from a well near the middle of the unconfined part of the aquifer, 50 percent of the total ground-water recharge. Recharge enters the system uniformly at the inland extent of the aquifer at the rate of 20 million gallons per day per mile of width. The horizontal hydraulic conductivity of the aquifer is 1,500 feet per day, and the vertical hydraulic conductivity of the caprock confining unit is 0.15 feet per day.

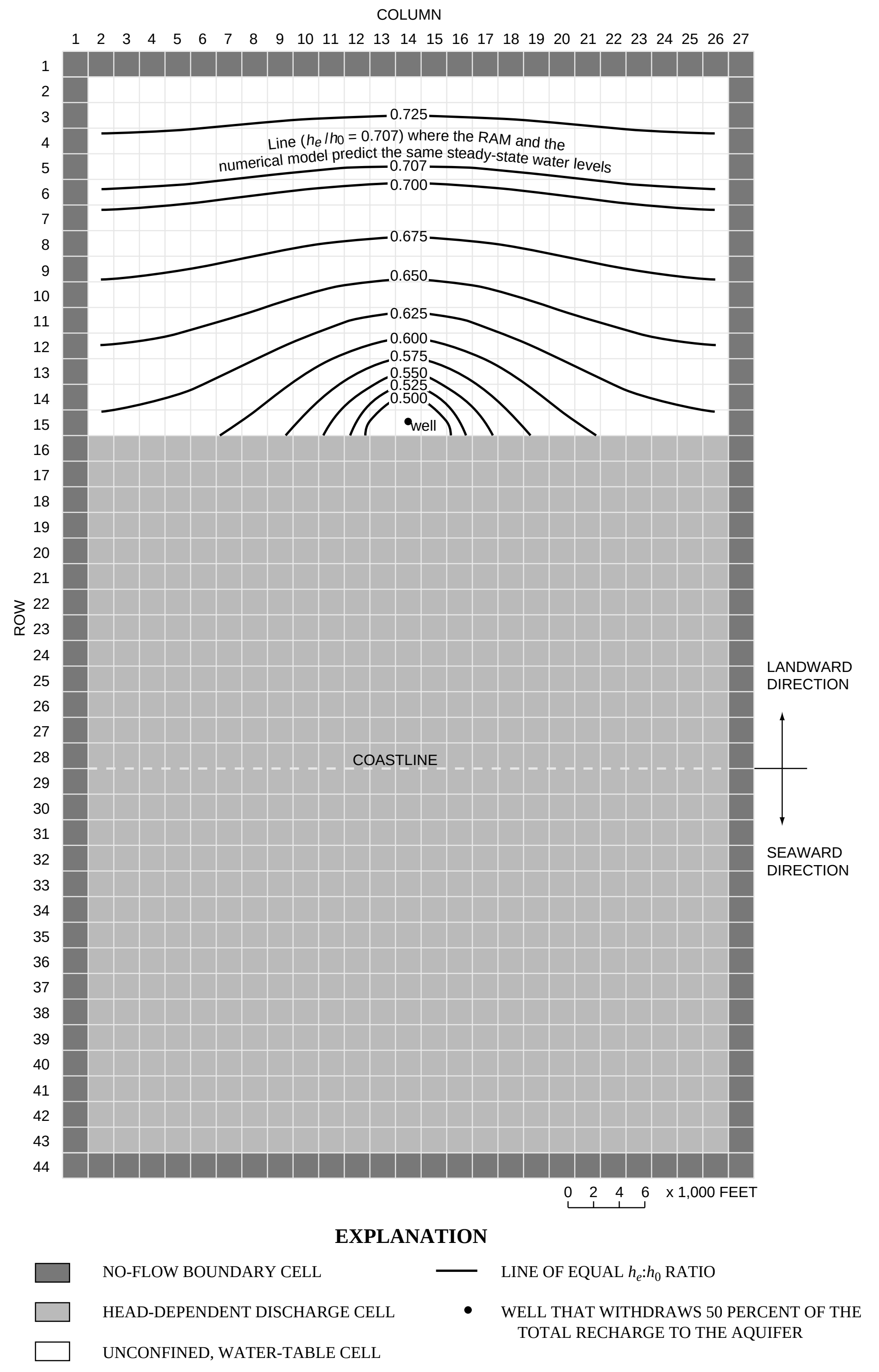


**Figure 7.** Model-calculated ratios (from a two-dimensional numerical ground-water flow model) of steady-state water levels for withdrawal conditions ($h_e$) to steady-state predevelopment water levels ($h_0$) in the unconfined part of the aquifer for the case of withdrawing, from a well near the seaward extent of the unconfined part of the aquifer, 50 percent of the total ground-water recharge. Recharge enters the system uniformly at the inland extent of the aquifer at the rate of 20 million gallons per day per mile of width. The horizontal hydraulic conductivity of the aquifer is 1,500 feet per day, and the vertical hydraulic conductivity of the caprock confining unit is 0.15 feet per day.

## CASE STUDY OF THE IAO AQUIFER, MAUI

The Iao aquifer lies on the northeastern flank of the West Maui Volcano. As delineated by CWRM, the aquifer system extends from the mountainous crest of the volcano to the ocean (fig. 8). The aquifer system is the main source of domestic water for Maui, accounting for about 76 percent of the water supplied by the Maui County Department of Water Supply (DWS) on the island in 1998 (Meyer and Presley, 2000).

### Geohydrologic Setting

This section describes the major features of the geohydrologic setting of the Iao aquifer area. Meyer and Presley (2000) provide a more detailed description.

The West Maui Volcano has a central caldera and two main rift zones that trend in northwestern and southeastern directions from the caldera (fig. 9). Thousands of dikes exist within the rift zones of West Maui Volcano, with the number of dikes increasing toward the caldera and with depth. Additional dikes exist outside the general trends of the rift zones, creating a radial pattern of dikes emanating from the caldera (Macdonald and others, 1983). Thousands of lava flows emanated from vents in and near the caldera and rift zones. Volcanic rocks in the Iao aquifer consist mainly of the shield-stage Wailuku Basalt, which is overlain in places by the Honolua Volcanics (Stearns and Macdonald, 1942; Langenheim and Clague, 1987). The dike-free flank flows of the Wailuku Basalt are generally thin-bedded and highly permeable and extend to depths far below sea level. Volcanic rocks in the Iao aquifer are overlain by sedimentary deposits near the coast (fig. 9).

The general movement of ground water in the Iao aquifer is from the dike-impounded ground-water system near the mountainous interior toward the ocean (fig. 1). Dike-impounded ground water occurs at levels as high as 2,000 ft above sea level. A freshwater-lens system exists within the dike-free volcanic rocks that extend beyond the dike-impounded system. Water levels measured in wells in the freshwater-lens system of the dike-free volcanic rocks have been as high as 37 ft above sea level (Meyer and Presley, 2000). In the Iao aquifer, the less-permeable sedimentary deposits that overlie the Wailuku Basalt near the shoreline form a confining unit that impedes the discharge of water from the volcanic-rock aquifer into the ocean.

### Ground-Water Withdrawals

Four major well fields (shaft 33, Mokuhau, Waiehu Heights, and Waihee) are in the part of the Iao aquifer containing a freshwater lens, and the Kepaniwai well field is in the upgradient area containing dike-impounded water (fig. 8; table 4). The freshwater lens is the main source of water from the Iao aquifer. Major withdrawal of ground water from the part of the aquifer containing a freshwater lens began in 1948 at shaft 33 (fig. 8). Water from shaft 33 was originally used for agricultural purposes. The Mokuhau well field was constructed in 1953 for domestic supply. Two additional well fields were constructed in the late 1970’s (Waiehu Heights in 1977 and Waihee in 1979) and the remaining well field, Kepaniwai, was first used in 1977. Water from all of these well fields is presently (1999) used for domestic supply. Nearly all of the water presently withdrawn from the aquifer is from these five well fields operated by the Maui DWS.

In 1990, the sustainable yield of the Iao aquifer was set at 20 Mgal/d by CWRM (State of Hawaii, 1990). The sustainable-yield value ($D$) was derived assuming a predevelopment recharge rate ($I$) of 31.57 Mgal/d and a ratio of $D$:$I$ equal to 0.64 (Mink, 1995). The $D$:$I$ ratio is obtained from table 1 with a predevelopment head ($h_0$) of 25 ft. Average annual withdrawal in 1990 was 17.31 Mgal/d, a value that was approached once before in 1975 (table 5). Average withdrawal rates increased steadily between 1985 and 1990, however, and this increase continued through 1995 when withdrawal peaked at 20.50 Mgal/d. The average rate of increase from 1985 through 1995 was 0.86 Mgal/d per year. Average 1996 withdrawal was 20.35 Mgal/d, about equal to the 1995 rate. Withdrawal was reduced to 19.10 Mgal/d in 1997 and to 17.90 Mgal/d in 1998 (table 5).

### Measured Water Levels and Comparisons with RAM-Predicted Equilibrium Heads

The use of RAM by CWRM for estimating sustainable-yield values for the State’s aquifers is relatively recent (1990). Data from the Iao aquifer are now available that allow an evaluation of the model’s predictive capability for the aquifer.

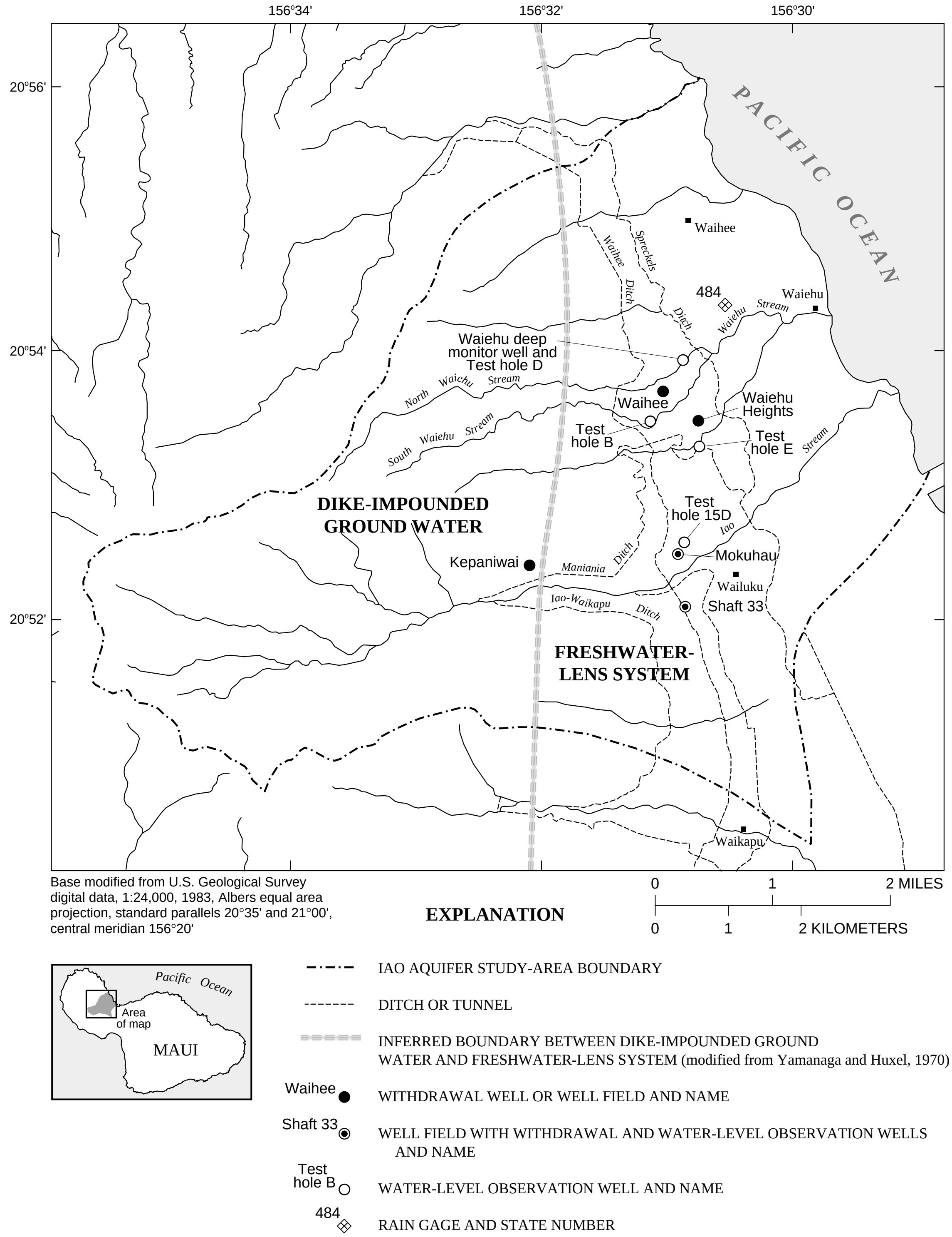


**Figure 8.** Selected wells in the Iao aquifer, Maui, Hawaii.

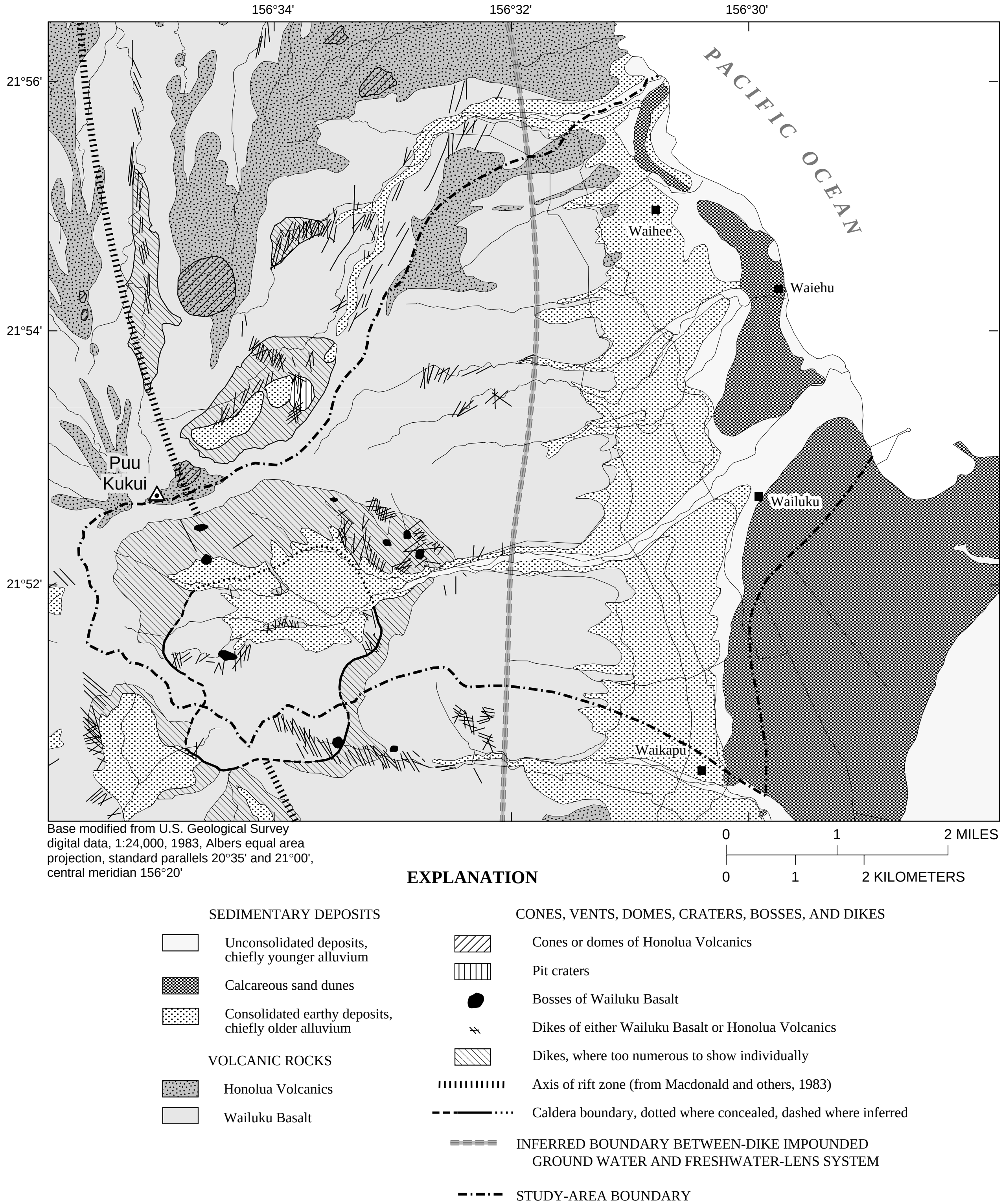


**Figure 9.** Surficial geology of the Iao aquifer area, Maui, Hawaii (modified from Stearns and Macdonald, 1942 and Langenheim and Clague, 1987).

**Table 4.** State numbers and names of selected wells in the Iao aquifer, Maui, Hawaii

| State well number | Well name | Use of well |
|---|---|---|
| 5330-05 | Shaft 33 | withdrawal and water-level observation |
| 5330-07 | Test hole 15D | water-level observation |
| 5330-09 to -11 | Mokuhau | withdrawal and water-level observation |
| 5332-05 | Kepaniwai | withdrawal |
| 5430-01, -02 | Waiehu Heights | withdrawal |
| 5430-03 | Test hole E | water-level observation |
| 5430-04 | Test hole D | water-level observation |
| 5430-05 | Waiehu deep monitor | water-level observation[a] |
| 5431-01 | Test hole B | water-level observation |
| 5431-02 to -04 | Waihee | withdrawal |

[a]Well also used for vertical salinity-profile information.

Water-level data of most interest for this discussion are from the 1990's, when total withdrawal increased and reached (in 1995 and 1996) the sustainable-yield value used by CWRM. Water levels near shaft 33 were measured intermittently from 1940 through 1970. After 1970, water levels were not measured until 1996 when measurements were made in one of the unused wells at shaft 33 (fig. 10). Water levels at and near Mokuhau well field were measured from 1951 to 1979. After 1979, water levels were not measured until 1998 when measurements were made in one of the unused wells in the Mokuhau well field (fig. 11) (Meyer and Presley, 2000). Water levels near Waihee and Waiehu Heights well fields can be inferred from water-level measurements made at the nearby Waiehu deep monitor well and at test holes B, D, and E (fig. 12) (Meyer and Presley, 2000). Given their locations (fig. 8), water levels in the Waiehu deep monitor well and test holes B and D can be considered representative of water levels at Waihee well field, and water levels at test holes B and E can be considered representative of those at Waiehu Heights well field.

Changes in water levels in observation wells in the area of Waiehu Heights and Waihee well fields indicate that water levels in the Iao aquifer respond to changes in withdrawals, rainfall, and recharge from irrigation. Between April 1977 and April 1997, water levels declined by about 6 ft near these wells (from about 15 to 16 ft above sea level in April 1977 to about 9 to 10 ft above sea level in April 1997) (fig. 13). During this period, withdrawals from the Waiehu Heights and Waihee well fields increased and recharge from irrigation decreased. Withdrawals at the Waiehu Heights and Waihee well fields started in 1977 and 1979, respectively. In addition, because of changes in irrigation practices, changes in types of crops grown, and reduction in agricultural acreage, estimated recharge from irrigation decreased from 17 Mgal/d during 1926–79, to 6 Mgal/d during 1980–85, to 2 Mgal/d during 1986–95 (Shade, 1997). The water-level declines (fig. 12) were not continuous, however, indicating that water levels are influenced by a factor or factors in addition to increased withdrawals from the Waiehu Heights and Waihee well fields and decreased recharge from irrigation. During the 1980's, water levels rose above 1977 and 1979 water levels because of reduced withdrawals from the Mokuhau well field and shaft 33 and variations in rainfall. Between 1977 and 1991, trends in water levels in the vicinity of the two well fields correlate closely to the 12-month departure of rainfall from mean rainfall (Meyer and Presley, 2000) (fig. 12). From 1992 onward, however, there is little correlation between rainfall and water levels. Average water levels declined from 1990 through 1996, in apparent response to increased withdrawals from the aquifer between 1990 and 1996. Although withdrawal was reduced in 1997, water levels continued to decline through 1997. Further reduction of withdrawal in 1998 resulted in a slight increase in water levels. Changes in rainfall would be expected to affect water levels, but the effect of withdrawal on water levels is more significant than the effect of recent (1990–98) changes in rainfall. Average 1998 water levels are about 10 ft above sea level at all of the well fields.

The RAM-predicted equilibrium heads, $h_e$, associated with the 20 Mgal/d value of sustainable yield used by CWRM can be determined in two ways: (1) by using

**Table 5.** Annual ground-water withdrawal from the Iao aquifer, Maui, Hawaii
[Values in million gallons per day, or percentage where noted; --, not applicable; data from Maui Department of Water Supply and unpublished data from Wailuku Sugar Co. in U.S. Geological Survey well files, Honolulu]

| Year | Well field | | | | | Total | Domestic | Domestic (percentage of total) |
|---|---|---|---|---|---|---|---|---|
| | Waiehu Heights | Waihee | Kepaniwai | Mokuhau | Shaft 33 | | | |
| 1948 | -- | -- | -- | -- | 2.00 | 2.00 | 0.00 | 0.0 |
| 1949 | -- | -- | -- | -- | 1.99 | 1.99 | 0.00 | 0.0 |
| 1950 | -- | -- | -- | -- | 1.77 | 1.77 | 0.00 | 0.0 |
| 1951 | -- | -- | -- | -- | 3.70 | 3.70 | 0.00 | 0.0 |
| 1952 | -- | -- | -- | -- | 3.05 | 3.05 | 0.00 | 0.0 |
| 1953 | -- | -- | -- | -- | 9.77 | 9.77 | 0.00 | 0.0 |
| 1954 | -- | -- | -- | -- | 6.11 | 6.11 | 0.00 | 0.0 |
| 1955 | -- | -- | -- | 1.10 | 1.31 | 2.41 | 1.10 | 45.6 |
| 1956 | -- | -- | -- | 0.66 | 0.83 | 1.49 | 0.66 | 44.3 |
| 1957 | -- | -- | -- | 1.19 | 6.12 | 7.31 | 1.19 | 16.3 |
| 1958 | -- | -- | -- | 1.22 | 0.67 | 1.89 | 1.22 | 64.6 |
| 1959 | -- | -- | -- | 1.43 | 4.15 | 5.58 | 1.43 | 25.6 |
| 1960 | -- | -- | -- | 1.59 | 5.65 | 7.24 | 1.59 | 22.0 |
| 1961 | -- | -- | -- | 2.25 | 5.64 | 7.89 | 2.25 | 28.5 |
| 1962 | -- | -- | -- | 2.04 | 7.97 | 10.01 | 2.04 | 20.4 |
| 1963 | -- | -- | -- | 2.06 | 0.85 | 2.91 | 2.06 | 70.8 |
| 1964 | -- | -- | -- | 2.79 | 6.00 | 8.79 | 2.79 | 31.7 |
| 1965 | -- | -- | -- | 2.67 | 4.68 | 7.35 | 2.67 | 36.3 |
| 1966 | -- | -- | -- | 3.12 | 4.69 | 7.81 | 3.12 | 39.9 |
| 1967 | -- | -- | -- | 2.91 | 3.08 | 5.99 | 2.91 | 48.6 |
| 1968 | -- | -- | -- | 2.88 | 6.28 | 9.16 | 2.88 | 31.4 |
| 1969 | -- | -- | -- | 3.61 | 1.18 | 4.79 | 3.61 | 75.4 |
| 1970 | -- | -- | -- | 3.98 | 5.08 | 9.06 | 3.98 | 43.9 |
| 1971 | -- | -- | -- | 4.34 | 11.65 | 15.99 | 4.34 | 27.1 |
| 1972 | -- | -- | -- | 4.66 | 9.45 | 14.11 | 4.66 | 33.0 |
| 1973 | -- | -- | -- | 5.16 | 8.11 | 13.27 | 5.16 | 38.9 |
| 1974 | -- | -- | -- | 5.44 | 9.11 | 14.55 | 5.44 | 37.4 |
| 1975 | -- | -- | -- | 6.40 | 10.56 | 16.96 | 6.40 | 37.7 |
| 1976 | -- | -- | -- | 6.69 | 6.37 | 13.06 | 6.69 | 51.2 |
| 1977 | 0.38 | -- | 0.04 | 8.10 | 6.39 | 14.91 | 8.52 | 57.1 |
| 1978 | 0.46 | -- | 0.01 | 8.29 | 3.14 | 11.90 | 8.76 | 73.6 |
| 1979 | 0.48 | 1.37 | 0.02 | 6.51 | 3.29 | 11.67 | 8.38 | 71.8 |
| 1980 | 0.48 | 6.15 | 0.02 | 3.05 | 0.00 | 9.70 | 9.70 | 100.0 |
| 1981 | 0.59 | 4.87 | 0.03 | 5.80 | 1.18 | 12.47 | 11.29 | 90.5 |
| 1982 | 0.49 | 5.20 | 0.007 | 4.60 | 0.00 | 10.30 | 10.30 | 100.0 |
| 1983 | 0.34 | 5.39 | 0.14 | 5.82 | 0.00 | 11.69 | 11.69 | 100.0 |
| 1984 | 0.29 | 5.41 | 0.11 | 6.39 | 0.37 | 12.57 | 12.20 | 97.1 |
| 1985 | 0.18 | 5.12 | 0.03 | 6.52 | 0.00 | 11.85 | 11.85 | 100.0 |
| 1986 | 0.27 | 6.63 | 0.003 | 6.42 | 0.00 | 13.32 | 13.32 | 100.0 |
| 1987 | 0.34 | 8.53 | 0.003 | 5.11 | 0.00 | 13.98 | 13.98 | 100.0 |
| 1988 | 0.35 | 8.06 | 0.00 | 6.71 | 0.00 | 15.12 | 15.12 | 100.0 |
| 1989 | 0.51 | 7.34 | 0.00 | 7.49 | 0.00 | 15.34 | 15.34 | 100.0 |
| 1990 | 0.92 | 8.66 | 0.07 | 7.66 | 0.00 | 17.31 | 17.31 | 100.0 |
| 1991 | 1.96 | 8.22 | 1.03 | 5.72 | 1.90 | 18.83 | 18.83 | 100.0 |
| 1992 | 1.08 | 7.96 | 0.82 | 3.17 | 5.10 | 18.13 | 18.13 | 100.0 |
| 1993 | 1.51 | 7.24 | 0.49 | 3.60 | 5.56 | 18.40 | 18.40 | 100.0 |
| 1994 | 1.20 | 8.15 | 0.45 | 6.49 | 2.91 | 19.20 | 19.20 | 100.0 |
| 1995 | 1.71 | 7.92 | 0.49 | 4.92 | 5.46 | 20.50 | 20.50 | 100.0 |
| 1996 | 1.56 | 8.22 | 0.28 | 5.13 | 5.16 | 20.35 | 20.35 | 100.0 |
| 1997 | 1.23 | 8.94 | 0.80 | 6.29 | 1.84 | 19.10 | 19.10 | 100.0 |
| 1998 | 0.23 | 9.11 | 0.51 | 3.21 | 4.84 | 17.90 | 17.90 | 100.0 |

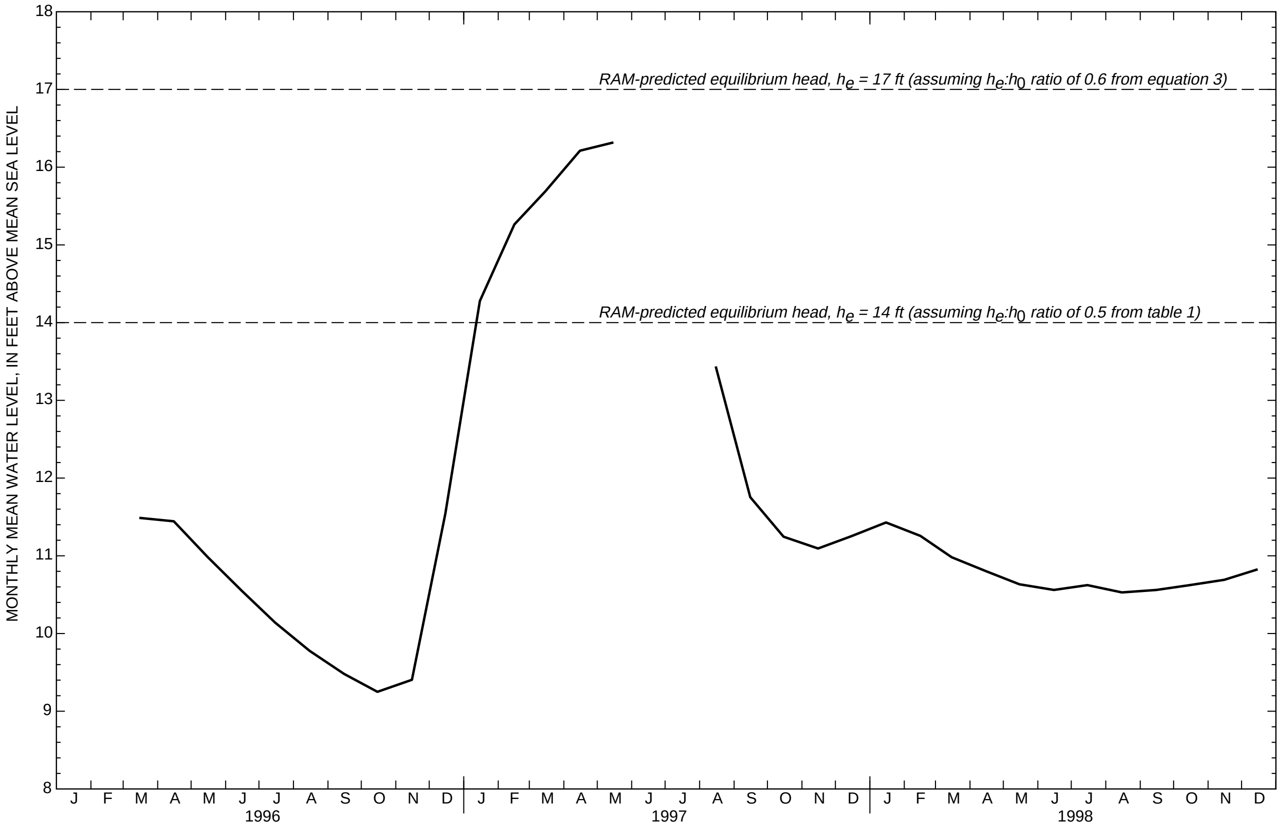


**Figure 10.** Water levels at shaft 33 during 1996–98, Iao aquifer, Maui, Hawaii.

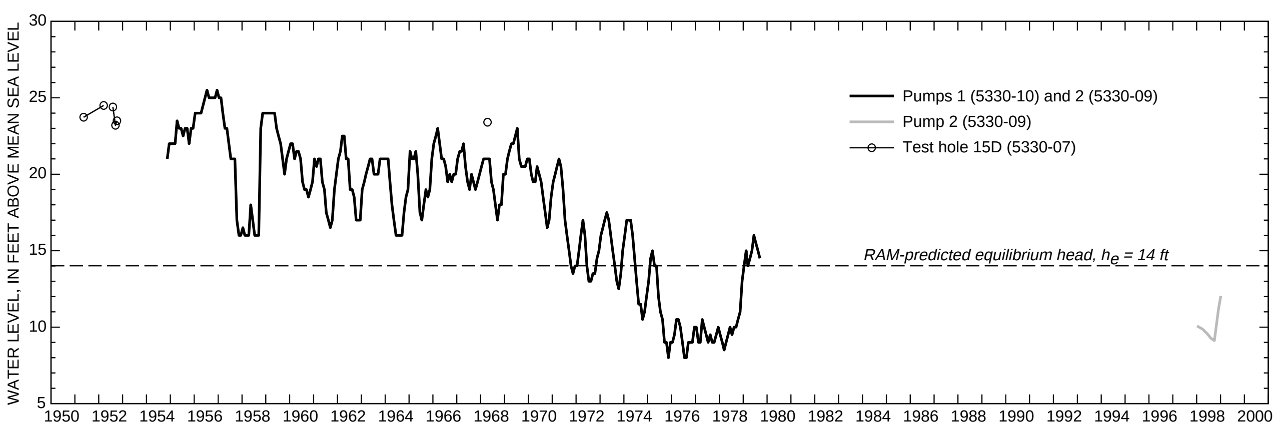


**Figure 11.** Water levels at Mokuhau well field, Iao aquifer, Maui, Hawaii.

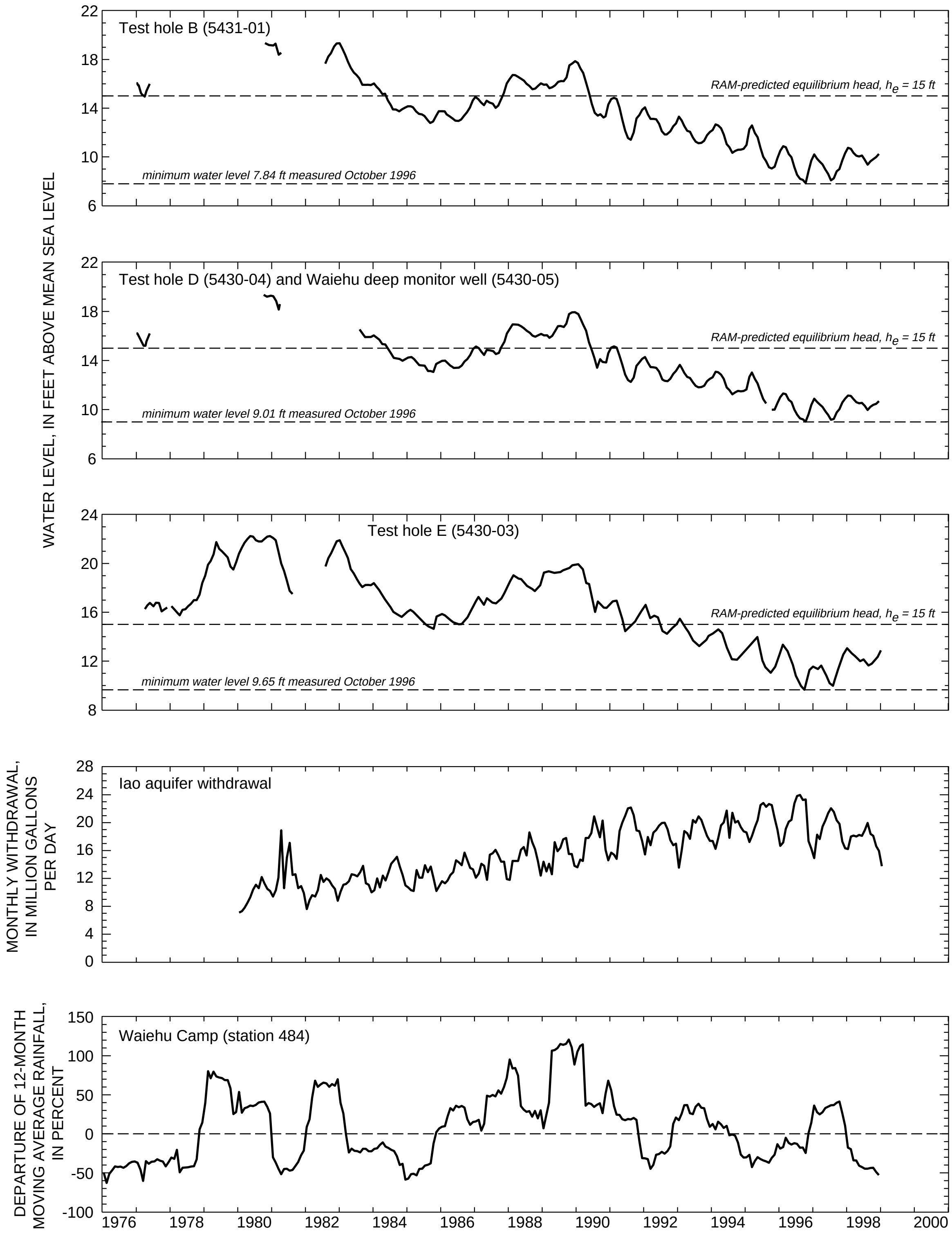


**Figure 12.** Water levels at Waiehu deep monitor well and test holes B and E, departure of backward-looking 12-month moving average rainfall from the long-term average rainfall for Waiehu Camp rain gage, and monthly mean total withdrawal from the Iao aquifer prior to 1999, Maui, Hawaii. (Unpublished rainfall data from Commission on Water Resource Management.)

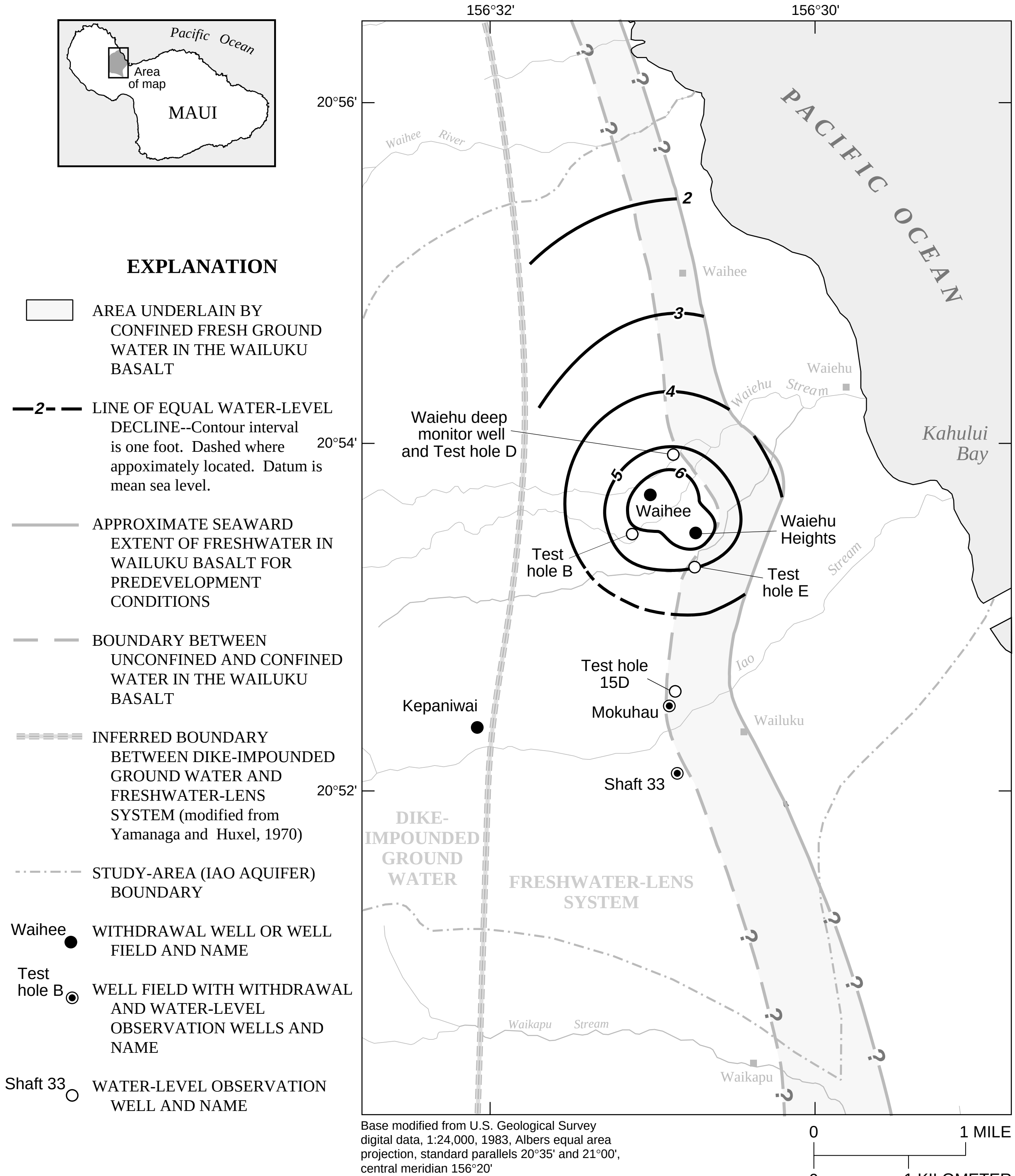


**Figure 13.** Map showing water-level declines in the flank flows of Wailuku Basalt between April 1977 and April 1997 for the northern part of the Iao aquifer, Maui, Hawaii.

equation 3, or (2) by using ratios of $h_e$:$h_0$ from table 1. Using equation 3 and assuming *D* and *I* values of 20 Mgal/d and 31.57 Mgal/d (Mink, 1995), respectively, the ratio $h_e$:$h_0$ is computed to be 0.6 for the Iao aquifer. Predevelopment heads averaged 28.5 ft above sea level near shaft 33 and 24 ft above sea level near the Mokuhau well field. Thus, using a ratio for $h_e$:$h_0$ of 0.6, the RAM-predicted equilibrium heads are about 17 ft and 14 ft for the shaft 33 and Mokuhau well fields, respectively (table 6). No information exists for predevelopment heads at the Waiehu Heights and Waihee well fields, although Yamanaga and Huxel (1970) suggest that they might have been about 25 ft above sea level. For a predevelopment head of 25 ft above sea level, and a ratio of $h_e$:$h_0$ of 0.6, the RAM-predicted equilibrium head for the Waiehu Heights and Waihee well fields is 15 ft above sea level.

For a predevelopment head of 28.5 ft above sea level at shaft 33, the ratio of $h_e$:$h_0$ from table 1 is 0.5. Thus, on the basis of ratios of $h_e$:$h_0$ from table 1, the equilibrium head for shaft 33 is estimated to be about 14 ft, which is lower than the 17-ft value previously calculated using equation 3. At the Mokuhau, Waiehu Heights, and Waihee well fields, predevelopment heads ranged from about 24 to 25 ft above sea level. Thus, the ratio of $h_e$:$h_0$ from table 1 for these well sites is 0.6, which is the same value calculated using equation 3.

In 1995 and 1996, withdrawal from the Iao aquifer reached the sustainable-yield value derived using RAM. However, by 1996 or earlier, water levels at shaft 33 and Waiehu Heights (represented with water levels from test holes B and E) were below RAM-predicted equilibrium heads. Water levels at Waihee well field (represented with water levels from the Waiehu deep monitor well and test hole B) also were below the RAM-predicted equilibrium head and were at an altitude that potentially could have resulted in saltwater intrusion. No water-level data were available for Mokuhau well field in 1996. CWRM held hearings on the status of the aquifer and in April 1997 concluded that “current pumpage rates in current locations cannot be sustained in the long term” (State of Hawaii, 1997, p. 4).

Although the decline in water levels ceased prior to 1999, average 1998 water levels were still below RAM-predicted equilibrium heads for shaft 33, Mokuhau, Waihee, and Waiehu Heights well fields. At their lowest altitudes prior to 1999, water levels were 5 to 7 ft below RAM-predicted equilibrium heads for Waihee and Waiehu Heights well fields, and about 5 ft below the 14-ft RAM-predicted equilibrium head for shaft 33. Water-level data for Mokuhau well field were not available until 1998 when the average water level was about 4 ft below the RAM-predicted equilibrium head for this well field (table 6).

Water levels in the Iao aquifer continued to decline when withdrawal was at the sustainable-yield value determined from RAM. The ultimate decline that would have occurred if withdrawal was permitted to remain at 20 Mgal/d cannot be estimated from available data. The rise in the position of the brackish-water transition zone generally will not occur immediately following a decline in water level. Rather, the change in position of the transition zone will generally lag behind the change in water level (see, for example, Essaid, 1986). Thus, the ultimate rise in the transition zone that would have occurred if withdrawal was permitted to remain at 20 Mgal/d also cannot be estimated from available data. However, even with average 1998 withdrawal at about 18 Mgal/d, water levels were probably still below acceptable long-term values to preclude saltwater intrusion at some of the well fields (Meyer and Presley, 2000).

Although uncertainty associated with the recharge estimate and predevelopment water levels contributes to uncertainty in the equilibrium heads predicted by RAM, the decline of water levels below those predicted by RAM in the vicinity of the well fields in the Iao aquifer is consistent with the results of the preceding numerical model analysis that demonstrate (1) the effect of a caprock on water-level declines caused by withdrawals, and (2) the importance of representing the distribution of withdrawals in an aquifer. The field setting for the Iao aquifer is similar to that shown in figure 7, and although the model does not simulate the Iao aquifer, model results provide insight to why water levels in the Iao aquifer fell below equilibrium heads predicted by RAM. Water availability in the Iao aquifer can be best understood by constructing a numerical model of the ground-water flow system. The data needs, required expertise, and development time are much greater for constructing a numerical model than for using RAM. Although construction of a numerical model is more costly than simply using RAM, a numerical model will lead to an improved understanding of the ground-water system and better management decisions.

**Table 6.** Measured water levels and RAM-predicted equilibrium head ($h_e$) at selected wells, Iao aquifer, Maui, Hawaii

| Well | RAM-predicted equilibrium head, $h_e$ (feet) | | Average measured water level (feet) | Year | Total withdrawal from Iao aquifer (Mgal/d) |
|---|---|---|---|---|---|
| | $h_e$ estimated from $h_e$:$h_0$ ratios in table 1 | $h_e$ estimated from equation 3 | | | |
| Waiehu monitor | 15 | 15 | 13 | 1992 | 18.12 |
| | | | 12 | 1994 | 19.2 |
| | | | 10 | 1997 | 19.12 |
| | | | 10.5 | 1998 | 17.89 |
| Test hole B | 15 | 15 | 13 | 1992 | 18.12 |
| | | | 12 | 1993 | 18.41 |
| | | | 9 | 1997 | 19.12 |
| | | | 10 | 1998 | 17.89 |
| Test hole E | 15 | 15 | 13 | 1994 | 19.2 |
| | | | 12 | 1996 | 20.4 |
| | | | 10.5 | 1997 | 19.12 |
| | | | 12.5 | 1998 | 17.89 |
| Shaft 33 | 14 | 17 | 10 | 1996 | 20.4 |
| | | | 10.5 | 1998 | 17.89 |
| Mokuhau | 14 | 14 | 10.5 | 1998 | 17.89 |

## SUMMARY

Sustainable yield, as defined by the State of Hawaii, refers to “the maximum rate at which water may be withdrawn from a water source without impairing the utility or quality of the water source...” (State of Hawaii, 1987). In Hawaii, sustainable-yield values for a given aquifer system are commonly determined from a one-dimensional analytical model of ground-water flow known as the Robust Analytical Model (RAM). The analytical model incorporates the horizontal-flow assumption and the Ghyben-Herzberg relation to represent flow in an unconfined aquifer that contains a body of freshwater floating on saltwater.

RAM does not account for aquifer-system boundary conditions that commonly exist in Hawaii, nor for the spatial distribution of ground-water withdrawals from wells (RAM is one dimensional). Therefore RAM cannot accurately predict water-level declines associated with withdrawals except under the most restrictive situations.

Two of the State’s most important aquifers, the Pearl Harbor aquifer on Oahu and the Iao aquifer on Maui, are overlain by coastal sedimentary deposits known as caprock. A caprock impedes the discharge of freshwater from the aquifer to the ocean and is an important control on the ultimate water-level decline caused by ground-water withdrawals from the aquifer. For areas where a caprock exists, water-level declines predicted by RAM generally are less than those indicated by numerical ground-water flow models that incorporate this boundary condition. This, in turn, indicates that management of these ground-water systems using a sustainable-yield value determined from RAM could result in some existing or future well fields ultimately being intruded by saltwater.

In addition to its inability to represent a caprock, RAM cannot account for spatially distributed withdrawals from wells, which is significant because water-level declines are greatest in the vicinity of withdrawal wells. The one-dimensional RAM invariably predicts the greatest water-level declines at the inland extent of the aquifer where the freshwater lens is thickest and the potential for saltwater intrusion is lowest.

The use of RAM by the Hawaii Commission on Water Resource Management (CWRM) for establishing the sustainable yield of the State’s aquifers is relatively recent. Data from the Iao aquifer, which lies on the northeastern flank of the West Maui Volcano and which is confined near the coast by a caprock, are now available to evaluate the predictive capability of RAM for this aquifer. In 1995 and 1996, withdrawal reached the sustainable-yield value derived using RAM (State of Hawaii, 1990). However, even before 1996, water levels in the aquifer had declined significantly below those predicted by RAM and were still declining in 1997. As a result, it was necessary to reduce withdrawal from the aquifer below the sustainable-yield value derived using RAM in order to halt the continuing decline of water levels and to preclude the ultimate intrusion of saltwater into the four major well fields in the aquifer.

Although uncertainty associated with the recharge estimate and predevelopment water levels contributes to uncertainty in the equilibrium heads predicted by RAM, the decline of water levels below those predicted by RAM in the Iao aquifer is consistent with the results of the numerical model analysis that demonstrate (1) the effect of a caprock on water-level declines caused by withdrawals, and (2) the importance of representing the distribution of withdrawals in an aquifer. Water availability in the Iao aquifer can be best understood by constructing a numerical model of the ground-water flow system.

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# APPENDIX: DESCRIPTION OF RAM

The one-dimensional Robust Analytical Model (RAM) (Mink, 1980) used by CWRM to estimate sustainable yield incorporates the Dupuit assumption of horizontal flow (see, for example, Bear, 1972) and the Ghyben-Herzberg relation. The steady-state model is described and derived in this appendix.

## Dupuit Assumption

For ground-water flow systems with small water-table slopes, an approximate solution for flow in an unconfined aquifer can be derived. The assumption of small water-table slope is equivalent to assuming that equipotential surfaces are vertical and flow is essentially horizontal, or to assuming that a hydrostatic pressure distribution exists (Bear, 1972, p. 361). This is known as the Dupuit assumption. With the Dupuit assumption, variations in velocity and pressure in the vertical direction are neglected and, thus, two-dimensional flow in a vertical cross section can be approximated by one-dimensional flow in the horizontal direction.

In general, ground-water flow in an unconfined aquifer is three dimensional. If ground-water flow is first assumed to be adequately represented by flow in a two-dimensional vertical cross section, and if the horizontal-flow assumption is then used, the original three-dimensional flow system can be approximated by a one-dimensional system.

## Ghyben-Herzberg Relation

In Hawaii, fresh ground water commonly occurs as a lens-shaped body of freshwater floating on denser, underlying saltwater derived from the ocean. A brackish-water transition zone of varying thickness exists between the freshwater and underlying saltwater. The transition zone is created by mixing of saltwater with seaward flowing freshwater.

For areas where the brackish-water transition zone is thin, the Ghyben-Herzberg relation can be used to estimate the thickness of the freshwater lens. If the specific gravities of freshwater and saltwater are assumed to be 1.000 and 1.025, respectively, then the Ghyben-Herzberg relation predicts that every foot of freshwater above sea level must be balanced by 40 ft of freshwater below sea level. The Ghyben-Herzberg relation is valid for hydrostatic conditions. For dynamic conditions, the Ghyben-Herzberg relation tends to underestimate freshwater-lens thickness near the discharge zone and overestimate lens thickness near the recharge zone.

### One-Dimensional Analytical Equation

The flow rate in a porous medium is proportional to the cross-sectional area of flow and the hydraulic gradient, and can be described by Darcy's law. For one-dimensional steady-state flow, Darcy's law can be written as:

$$Q = -K\, i\, A, \qquad \text{(a1)}$$

where, $Q$ = rate of flow [$L^3/T$],
$K$ = hydraulic conductivity [L/T],
$i$ = hydraulic gradient [L/L], and
$A$ = cross-sectional area of flow [$L^2$].

The constant of proportionality in Darcy's law is the hydraulic conductivity, $K$, which is related to the properties of the porous medium and the fluid. The hydraulic conductivity is a quantitative measure of the capacity of a rock to transmit water. For one-dimensional flow, the hydraulic gradient is given by:

$$i = dh/dx,$$

where, $h$ = hydraulic head [L] measured relative to mean sea level,
$x$ = Cartesian coordinate [L], and
$dh/dx$ = derivative of $h$ with respect to $x$.

Flow is in the direction of decreasing hydraulic head, which accounts for the negative sign in equation a1. The cross-sectional area of flow at any section is given by:

$$A = 41hw,$$

where, $w$ = width of section [L], and
$41h$ = height of section from the Ghyben-Herzberg relation [L].

Thus, Darcy's law can be expressed as:

$$Q = -K\,(dh/dx)\,(41hw). \qquad \text{(a2)}$$

Rearranging terms yields:

$$-[Q/(41Kw)]\,dx = h\,dh. \qquad \text{(a3)}$$

Integration of equation (a3) yields:

$$-[Q/(41Kw)]\,(x_2 - x_1) = 0.5(h_2^2 - h_1^2). \qquad \text{(a4)}$$

If $h_1 = 0$ at $x_1 = 0$, then from equation (a4), $h$ as a function of $x$ can be written as:

$$h^2 = -[2Q/(41Kw)]\,x. \qquad \text{(a5)}$$

Equation (a5) forms the basis of RAM. Equation (a5) can be rearranged as:

$$h^2/Q = -2x/(41Kw). \qquad \text{(a6)}$$

For any given location, $x$, the right hand side of equation (a6) is a constant and, thus, the ratio of $h^2/Q$ is a constant:

$$h_0^2/Q_0 = h_e^2/Q_e, \qquad \text{(a7)}$$

where, $h_0$ = hydraulic head [L] at location $x$ for flow rate $Q_0$,
$Q_0$ = steady-state rate of flow through aquifer for predevelopment conditions [$L^3/T$],
$h_e$ = hydraulic head [L] at location $x$ for flow rate $Q_e$, and
$Q_e$ = steady-state rate of flow (less withdrawals from wells or shafts) through aquifer for development conditions [$L^3/T$].

For some desired equilibrium head, $h_e$, CWRM defines the sustainable yield, $D$, as the difference between the predevelopment rate of flow through the aquifer minus the reduced rate of flow through the aquifer following development:

$$D = Q_0 - Q_e. \qquad \text{(a8)}$$

Combining equations (a7) and (a8), and letting $Q_0 = I$ yields:

$$D/I = 1 - (h_e/h_0)^2. \qquad \text{(a9)}$$

Equation (a9) represents the model (RAM) used by CWRM to set sustainable yield in Hawaii. To apply this equation, predevelopment values for $h_0$ and $I$ must be known. After establishing some desired minimum equilibrium head, $h_e$, equation (a9) is used by CWRM to estimate the sustainable yield of an aquifer. For example, if the desired equilibrium head is 60 percent of the predevelopment head ($h_e/h_0 = 0.6$), then $D/I = 1 - (0.6)^2$, or $D/I = 0.64$. Thus, RAM estimates that the sustainable yield, $D$, is 64 percent of the predevelopment flow rate, $I$, in the aquifer for this case.