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Estimation of Infiltration Rate in the Vadose Zone: Compilation of Simple Mathematical Models

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ESTIMATION OF INFILTRATION RATE IN THE VADOSE ZONE: COMPILATION OF SIMPLE MATHEMATICAL MODELS

Volume I

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NOTICE

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FOREWORD

The U.S. Environmental Protection Agency is charged by Congress with protecting the Nation's land, air, and water resources. Under a mandate of national environmental laws, the Agency strives to formulate and implement actions leading to a compatible balance between human activities and the ability of natural systems to support and nurture life. To meet these mandates, EPA's research program is providing data and technical support for solving environmental problems today and building a science knowledge base necessary to manage our ecological resources wisely, understand how pollutants affect our health, and prevent or reduce environmental risks in the future.

The National Risk Management Research Laboratory is the Agency's center for investigation of technological and management approaches for reducing risks from threats to human health and the environment. The focus of the laboratory's research program is on methods for the prevention and control of pollution to air, land, water, and subsurface resources; protection of water quality in public water systems; remediation of contaminated sites and ground water, and prevention and control of indoor air pollution. The goal of this research effort is to catalyze development and implementation of innovative, cost-effective environmental technologies; develop scientific and engineering information needed by EPA to support regulatory and policy decisions; and provide technical support and information transfer to ensure effective implementation of environmental regulations and strategies.

Infiltration estimation methods are an integral part of the assessment of contaminant transport and fate. This document provides an extensive compilation of methods, in addition to a categorization of those methods. Appendices are included for methods based on the Green-Ampt model and the Richards Equation, and for the estimation of soil hydraulic properties.

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ABSTRACT

The unsaturated or vadose zone provides a complex system for the simulation of water movement and contaminant transport and fate. Numerous models are available for performing simulations related to the movement of water. There exists extensive documentation of these models. However, the practical application of these infiltration models has not been adequately addressed in the literature. In recent years, the use of vadose zone models has increased for the purpose of estimating contaminant levels in soils related to different types of remedial decision-making. The rate of infiltration of water is generally the most important parameter required in such models. Often these models use an over-simplified estimate of the infiltration rate, which has little basis in reality and actual field conditions. This document presents a compilation of simple infiltration models for quantifying the rate of water movement. A great majority of the compiled models are based on widely-accepted concepts of soil physics. These models, represented by simple mathematical expressions, can be readily implemented in a spreadsheet environment. Proper use of these models should provide a rational and scientific basis for remedial decision-making related to soil contaminant levels.

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1.0 Introduction

The vadose zone is an integral component of the hydrological cycle, directly influencing infiltration, storm runoff, evapotranspiration, interflow, and aquifer recharge. Understanding the nature of water movement in the vadose zone and its quantification is essential to solving a variety of problems. Examples of such problems are: prediction of runoff from given precipitation events for the purposes of erosion control; sediment transport and flood control; estimation of water availability for plant growth; estimation of water recharge to the underlying aquifer; and assessment of the potential for aquifer contamination due to migration of water-soluble chemicals present in the vadose zone. Consequently, the study of soil-water movement has interested scientists from diverse disciplines such as soil science, hydrology, agriculture, civil engineering, and the environmental sciences for several years.

The use of vadose zone models for the determination of contaminant cleanup levels, preliminary remediation goals (PRG), soil screening levels (SSL), and other related terms has proliferated in recent years. In providing technical support to various consultants, EPA Regional Offices, and the state agencies, it has become apparent that there exists a lack of guidance in the estimation of model parameters. In an attempt to address this problem, an EPA Issue Paper was prepared by Breckenridge *et al.* (1991), which was later published as a journal article (Breckenridge *et al.*, 1994). This paper discussed the techniques for characterizing soils for their chemical, physical, and hydraulic properties. It also provided a list of available field and laboratory measurement techniques and look-up methods for these parameters. In addition to the identification of the parameters, the document provided a table of common unsaturated zone models and their parameter requirements. An extensive survey of unsaturated zone models was provided by van der Hiejde (1994).

A related effort is the release of the Soil Screening Guidance (U.S.EPA, 1996). This Guidance provides the public with a tool for determining risk-based, site-specific, soil screening levels (SSLs) for the evaluation of the need for further investigation at NPL (National Priority List) sites. The release of this document with an accompanying technical background document has resulted in a growth in emphasis by several states on the development of state soil screening levels, and remediation goals and thus, a renewed interest in the prediction of contaminant movement in the unsaturated zone, in most cases with simple analytical models for the estimation of water movement. Often these models utilize an over-simplified estimate of the infiltration rate, which may have very little basis in physics and actual site characteristics.

This document presents a compilation of simple mathematical models for quantifying the rate of soil-water movement due to infiltration. The rate of infiltration is usually a required input parameter for simpler screening level models which describe contaminant migration in the vadose zone. There are numerous methods available for the measurement/estimation of infiltration rate. In an activity and report supported by the American Petroleum Institute, Daniel B. Stephens & Associates provided a good review of various field and laboratory methods, as well as numerical models (API,1996). The distinguishing feature and primary motivation for the present study is the compilation of relatively simple mathematical expressions (models), implementable in a spreadsheet or symbolic math software environment. A great majority of the compiled models are based on widely-accepted concepts of soil physics. Proper use of these models should provide a rational and scientific basis for the determination of site-specific PRGs and SSLs. The primary intended audience for this document would be applied hydrologists, soil scientists, and modelers working on field problems. Soil physicists and hydrologists in the research community with a need to locate infiltration models and mathematical solutions may also find this document useful.

The mathematical models compiled here have several advantages over other estimation techniques:

(a) They are relatively easy to use.
(b) They are based on widely-accepted concepts of soil physics (except for the empirical models).
(c) Required hydraulic parameters can be easily obtained from the published literature as well as electronic databases.
(d) Site-specific measurement of all the parameters, although recommended, is not necessary for obtaining preliminary estimates of water flux to be used in screening level contaminant transport models.
(e) The models presented here are ideal for simulating the infiltration component in the watershed models since they incorporate all the essential physics of the soil-water flow phenomena without being complicated.
(f) Spatial variability of the soil parameters can be more easily incorporated into the analytical models either deterministically or statistically (*e.g.*, a Monte-Carlo type approach).
(g) The mathematical solutions compiled here may also be useful as benchmarks for establishing the validity and accuracy of the finite-difference or finite-element discretization schemes.

2.0 Estimation Methods for Infiltration Rate

There are numerous techniques available for the estimation of water infiltration rate through the vadose zone. API (1996) provides a good review of such techniques identified as: 1) soil-water balance, 2) lysimeter measurements, 3) Darcy flux method, 4) plane of zero flux method, 5) soil temperature methods, 6) electromagnetic methods, 7) ground-water basin outflow method, 8) water-level fluctuations, 9) stream gauging, 10) chemical tracers and isotopes, 11) chloride mass balance, 12) water balance models, and 13) numerical models based on the Richards equation. However, none of these methods offers a quick and easy way to obtain an estimate of infiltration rate for the purposes of preliminary analysis and decision-making. Therefore, the objective of this study is to compile estimation methods which are easy-to-use and yield scientifically-based estimates using soil-hydraulic and climatic parameters representative of the prevailing site conditions. The compiled methods can be divided into three broad categories of: a) empirical models, b) Green-Ampt models, and c) Richards equation models. Numerous formulations of these models have then been developed for utilization with specific site conceptualizations, or boundary conditions. Before discussing these models, a brief introduction to the processes involved in soil-water movement will be presented. For more detailed discussion of these processes the reader is referred to several excellent soil physics and hydrology textbooks such as Hillel (1980), Hanks and Ashcroft (1980), Guymon (1994), Stephens (1996), Dragun (1988), and Fetter (1988).

Water movement in the vadose zone is generally conceptualized as occurring in the three stages of infiltration, redistribution, and drainage or deep percolation as illustrated in Figure 1. For this conceptualization, infiltration is defined as the initial process of water entering the soil resulting from application at the soil surface. Capillary forces, or matric potentials, are dominant during this phase. Redistribution occurs as the next stage where the infiltrated water is redistributed within the soil profile after the cessation of water application to the soil surface. During redistribution, both capillary and gravitational effects are important. Simultaneous drainage and wetting takes place during this stage, and the impact of hysteresis may be important. Hysteresis is the phenomenon illustrated in Figure 2, which implies that the wetting and drying curves of a specific soil will not be identical. Evapotranspiration takes place concurrently during the redistribution stage, and will impact the amount of water available for deeper penetration within the soil profile. The final stage of water movement is termed deep percolation or recharge, which occurs when the wetting front reaches the water table. For the purposes of this report, the term "infiltration" will lump all the three stages of water movement through vadose zone. Also the terms, "water flux," "infiltration rate," and "rate of water movement" will be used interchangeably.

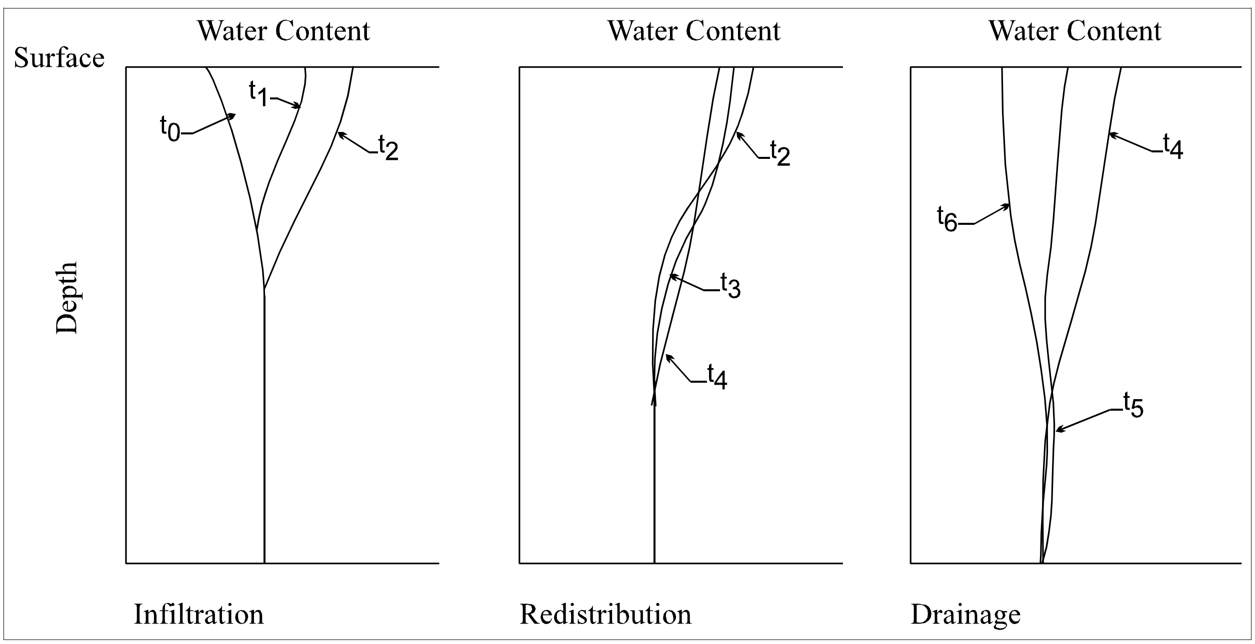

Figure 1. Conceptualization of water content profiles during infiltration, redistribution, and drainage (deep percolation).

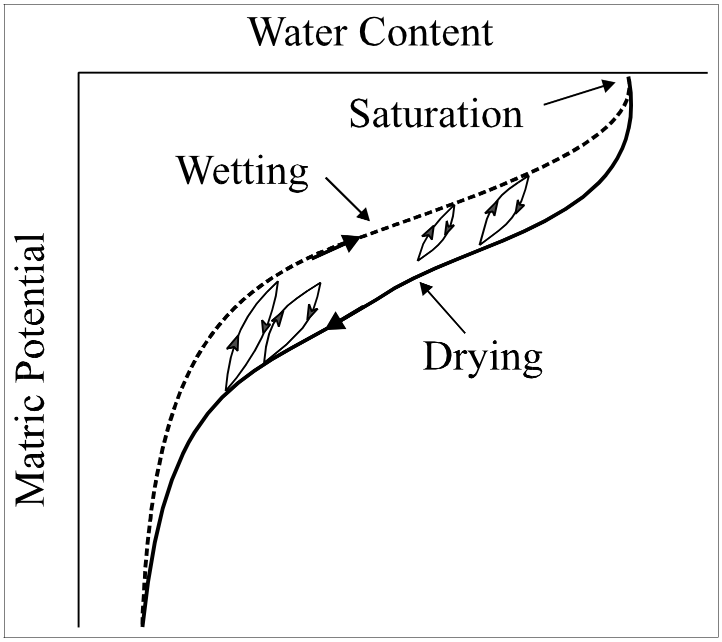

Figure 2. Illustration of the impact of hysteresis on wetting and drying curves for water content (Hillel, 1980).

3.0. Empirical Models

Empirical methods are usually in the form of simple equations, the parameters of which are derived by means of curve-fitting the equation to actual measurements of cumulative water infiltration. These equations only provide estimates of cumulative infiltration and infiltration rates, and do not provide information regarding water content distribution. Most are derived on the basis of a constant water content being available at the surface. A few of the commonly used infiltration equations, which have no apparent physical basis, are presented below. Detailed discussions of these empirical models are provided by Philip (1957), Swartzendruber and Hillel (1973), Dunin (1976), and Parlange and Haverkamp (1989).

3.1 Kostiakov's Equation

Kostiakov (1932) proposed the following equation for estimating infiltration,

$$i(t) = \alpha t^{-\beta} \tag{1}$$

where i is the infiltration rate at time t, and α $(\alpha>0)$ and β $(0<\beta<1)$ are empirical constants. Upon integration from 0 to t, Eq. 1 yields Eq. 2, which is the expression for cumulative infiltration, $I(t)$.

$$I(t) = \frac{\alpha}{1-\beta} t^{(1-\beta)} \tag{2}$$

The constants α and β can be determined by curve-fitting Eq. 2 to experimental data for cumulative infiltration, $I(t)$. Since infiltration rate, i, becomes zero as $t \to \infty$, rather than approach a constant non-zero value, Kostiakov proposed that the Eqs. 1 and 2 be used only for $t < t_{max}$, where t_{max} is equal to $(\alpha/K_S)^{(1/\beta)}$, and K_S is the saturated hydraulic conductivity of the soil. Kostiakov's equation describes the infiltration quite well at smaller times, but becomes less accurate at larger times (Philip, 1957; Parlange and Haverkamp, 1989).

3.2 Horton's Equation

Horton (1940) proposed to estimate infiltration in the following manner,

$$i(t) = i_f + (i_0 - i_f)\, e^{-\gamma t} \tag{3}$$

and

$$I(t) = i_f t + \frac{1}{\gamma}(i_0 - i_f)(1 - e^{-\gamma t}) \tag{4}$$

where i_0 and i_f are the presumed initial and final infiltration rates, and γ is an empirical constant. It is readily seen that $i(t)$ is non-zero as t approaches infinity, unlike Kostiakov's equation. It does not, however, adequately represent the rapid decrease of i from very high values at small t (Philip, 1957). It also requires an additional parameter over the Kostiakov equation. Parlange and Haverkamp (1989), in their comparison study of various empirical infiltration equations, found the performance of Horton's equation to be inferior to that of Kostiakov's equation.

3.3 Mezencev's Equation

In order to overcome the limitations of Kostiakov's equation for large times, Mezencev (Philip, 1957) proposed the following as modifications to Eqs. 1 and 2. Mezencev proposed infiltration estimated by

$$i(t) = i_f + \alpha t^{-\beta} \tag{5}$$

and

$$I(t) = i_f t + \frac{\alpha}{1-\beta} t^{(1-\beta)} \tag{6}$$

where i_f is the final infiltration rate at steady state.

3.4 SCS Equation

The USDA Soil Conservation Service (1957) developed an equation for rainfall-runoff relationship based on daily rainfall data as input:

$$R = \frac{(P - 0.2F_w)^2}{P + 0.8F_w} \tag{7}$$

where P is the daily rainfall, R is the runoff, and F_w is a statistically derived parameter bearing some resemblance to the initial soil moisture deficit. Infiltration is calculated as the excess of rainfall over runoff:

$$I = P - R \tag{8}$$

3.5 Holtan's Equation

The empirical infiltration equation devised by Holtan (1961) is explicitly dependent on soil water conditions in the form of available pore space for moisture storage:

$$i(t) = i_f + a\,b\,(\omega - I)^{1.4} \tag{9}$$

where *a* is a constant related to the surface conditions varying between *0.25* and *0.8*, *b* is a scaling factor, ω is the initial moisture deficit or the pore space per unit area of cross section initially available for water storage (cm), and *I* is the cumulative infiltration (cm) at *t*. This equation has been found to be suitable for inclusion in catchment models because of soil water dependence, and satisfactory progress has been reported for runoff predictions (Dunin, 1976).

3.6 Boughton's Equation

Boughton (1966) modified the rainfall-runoff relationship of USDA-SCS (1957) given by Eq.(7) as follows:

$$R = P - F_r \tanh\left(\frac{P}{F_r}\right) \tag{10}$$

where F_r is an empirical parameter; however, some success has been reported when interpreted as the initial soil moisture deficit (Dunin, 1976). Infiltration is calculated using Eq. (8).

4.0. Green-Ampt Models

4.1 Basic Concepts

Green and Ampt (1911) derived the first physically based equation describing the infiltration of water into a soil. The Green-Ampt model has been the subject of considerable developments in applied soil physics and hydrology owing to its simplicity and satisfactory performance for a great variety of hydrological problems. For many hydrological problems the use of more sophisticated approaches (*e.g.*, the models based on the nonlinear Richards equation,) is both impractical and inefficient due to more information on soil hydraulic parameters (*e.g.* water retention and hydraulic conductivity functions) being required. In these methods, the entire soil moisture-pressure profile is generally evaluated, even though the main quantity of interest is the flux at one or both of the boundaries. Therefore, not surprisingly, the Green-Ampt equation has been the choice model of infiltration estimation in many physically-based hydrologic models (Freyberg *et al*., 1980). Also, the USDA's Agricultural Research Service (ARS) has done extensive work to develop empirical relations for obtaining the Green-Ampt model parameters (Brakensiek and Onstad, 1977; McCuen *et al*., 1981; Rawls and Brakensiek, 1982; Springer and Cundy, 1987), thus providing additional impetus for inclusion in many watershed models (Goldman, 1989).

Green and Ampt assumed a piston-type water content profile (Figure 3) with a well-defined wetting front. The piston-type profile assumes the soil is saturated at a volumetric water content of θ_s (except for entrapped air) down to the wetting front. At the wetting front, the water content drops abruptly to an antecedent value of θ_0, which is the initial water content. The soil-water pressure head at the wetting front is assumed to be h_f (negative). Soil-water pressure at the surface, h_s, is assumed to be equal to the depth of the ponded water.

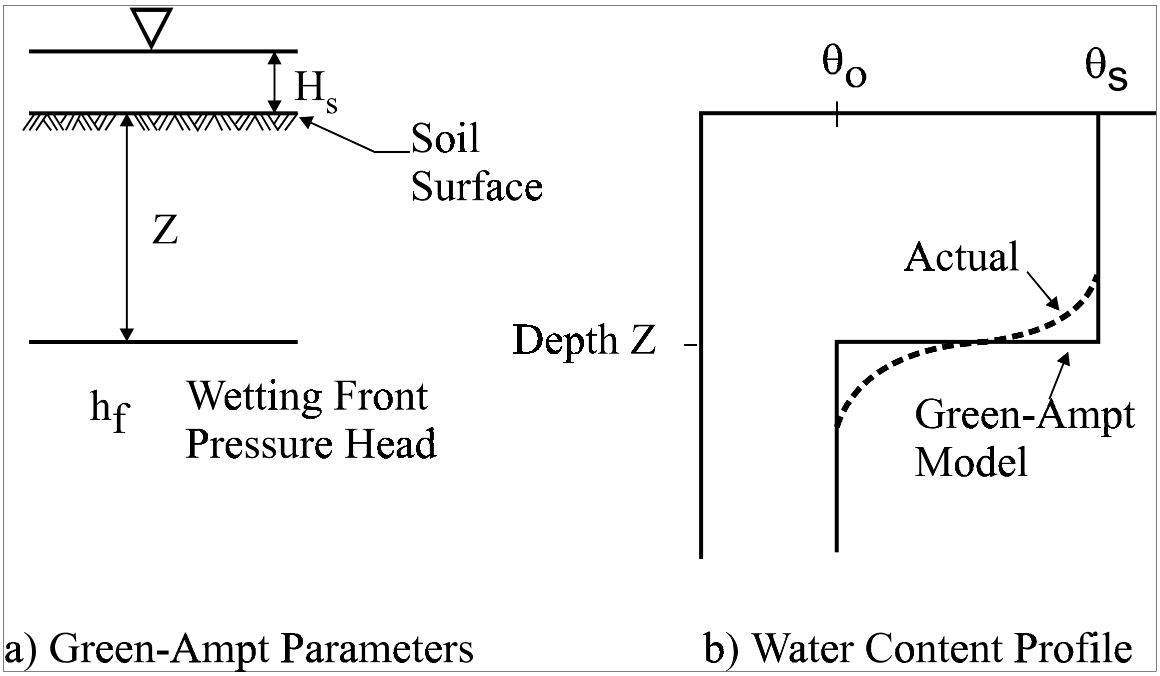

Figure 3. Illustration of Green-Ampt parameters and the conceptualized water content profile, which demonstrates the sharp wetting front.

At any time, *t*, the penetration of the infiltrating wetting front will be *Z*. Darcy's law can be stated as follows:

$$q = \frac{dI}{dt} = -K_s\left(\frac{h_f - (h_s + Z)}{Z}\right) \tag{11}$$

where K_s is the hydraulic conductivity corresponding to the surface water content, and *I(t)* is the cumulative infiltration at time *t*, and is equal to $Z(\theta_s - \theta_0)$. Using this relation for *I(t)* to eliminate *Z* from Eq. 11, and performing the integration yields,

$$I = K_s t - (h_f - h_s)(\theta_s - \theta_0)\log_e\left[1 - \frac{I}{(h_f - h_s)(\theta_s - \theta_0)}\right] \tag{12}$$

Equation 12 is precisely the statement of the Green-Ampt model. Philip (1957) demonstrated that the Green-Ampt equation can also be obtained as an exact solution of the Richards equation when the diffusivity function is assumed to be a Dirac Delta-type function with a non-zero value only at the saturated water content. Philip called the Green-Ampt model the "delta-function" model.

4.2 Compilation of Green-Ampt Models

Even though originally developed for idealized conditions (*i.e.*, homogeneous soil and constant surface ponding depth), the Green-Ampt model has been extended to take into account more-realistic features. Refer to Table 1 for a list of methods to estimate water flux based on the Green-Ampt model. Also, Appendix A is provided as an annotated bibliography pertaining to the Green-Ampt approach. The primary utility of the Green-Ampt approach lies in the estimation of the water flux, and it must be emphasized that the actual water content distribution with depth, $\theta(z)$, cannot be simulated, since the model formulation assumes a sharp wetting front.

Table 1. Models of Soil Water Movement Based on the Green-Ampt Concept[1]

Model Author(s)	Important Features / Limitations
Green and Ampt (1911)	Equation 9 for cumulative infiltration, implicit in time, is the Green-Ampt model; sharp wetting front; constant ponding depth; homogeneous soil; uniform antecedent water content.
Bouwer (1969)	Layered soils; non-uniform antecedent water content; constant ponding depth.
Childs and Bybordi (1969)	Implicit equation (Eq. 5) for cumulative infiltration in layered soils; constant ponding depth; uniform antecedent water content.
Mein and Larson (1973)	Pre-ponding (Eq. 6) and ponded infiltration (Eq. 8); Eq. 8 has to be integrated after replacing f_p with dF/dt and the lower limits of integration (t_s,F_s); constant rainfall rate greater than K_{sat}; homogeneous soil; uniform antecedent water content.
Swartzendruber (1974)	Constant surface water flux greater than saturated hydraulic conductivity; pre-ponding and ponding infiltration; Eqs. 4 and 5 yield time to ponding and cumulative infiltration prior to ponding, respectively; Eq. 8 or 12 gives cumulative infiltration, implicitly in time, after ponding; two approximate explicit equations (Eqs. 23 and 30) for cumulative infiltration as a function of time; homogeneous soil; uniform antecedent water content.
Morel-Seytoux and Khanji (1974)	Infiltration under the two-phase flow of air and water (Eq. 17 for total velocity); a rigorous derivation of expressions (Eqs. 18 and 26) for the wetting front suction, h_f, and a viscous correction factor, β, in terms of initial water content; homogeneous soil; uniform antecedent water content.
James and Larson (1976)	Intermittent (piecewise-constant) rainfall; infiltration (Eqs. 2 and 3) and redistribution (Eq. 1); homogeneous soil; uniform antecedent water content.

[1] Equation numbers referenced in this table correspond to those in the cited article. They *do not* correspond to the equation numbers in this report.

Model Author(s)	Important Features / Limitations
Li et al. (1976)	Explicit approximations for calculating the Green-Ampt model cumulative infiltration and infiltration rate as functions of time (Eqs. 11, 25, and 6); homogeneous soil; constant head ponding at the surface; uniform antecedent water content.
Smith and Parlange (1978)	Two 2-parameter models for ponding time (Eqs. 6 and 9) and infiltration rate (Eqs. 20 and 26); arbitrary transient rainfall; homogeneous soil; uniform antecedent water content.
Chu (1978)	Transient rainfall; pre-ponding (Eqs. 18, 19, and 23-26) and ponded infiltration (Eqs. 18, 19 and 27); homogeneous soil; uniform antecedent water content.
Flerchinger *et al*. (1988)	An explicit equation for cumulative infiltration for layered soils; extension of Li et al. (1976) approach for layered soils; constant head at the surface.
Philip (1992)	A solution for falling head ponded infiltration (Eq. 4). Solution form is the same as that for constant head infiltration; only the values of the constants change.
Philip (1993)	Variable-head ponded infiltration (Eq. 11) due to constant or arbitrarily transient rainfall; Eq. 26 for constant rainfall, and Eq. 35 for piecewise constant rainfall; homogeneous soil; uniform antecedent water content.
Salvucci and Entekhabi (1994)	Four-term expression explicit approximations for Green-Ampt infiltration rate and cumulative infiltration (Eqs. 9 and 10) as a function of t; homogeneous soil; uniform antecedent water content.

4.3 Parameter Estimation for the Green-Ampt Models

The popularity of the Green-Ampt models is primarily due to simplicity, adaptability to varying scenarios, and the availability of characteristic parameter values for various soil textures and conditions. Extensive studies conducted by the USDA's Agricultural Research Service (ARS) have resulted in the development of empirical relations for the model parameters in terms of easily-measurable variables. This has provided an additional impetus for the inclusion of Green-Ampt models in many watershed models (Goldman, 1989).

Bouwer (1966) demonstrated that K_s in Eq. 12 is not equal to the saturated hydraulic conductivity of the soil (K_{sat}), but can be approximated as $0.5*K_{sat}$. He also suggested that h_f can be treated as air-exit suction head. Neuman (1976) derived expressions for h_f, valid for small, intermediate, and large times. Useful empirical expressions and various statistical correlations are available for K_s and h_f (Brakensiek and Onstad, 1977; McCuen et al., 1981; Rawls and Brakensiek, 1982). Appendix C is provided to include additional references pertaining to the estimation of soil hydraulic parameters.

5.0. Richards Equation Models

5.1 Basic Concepts

The Darcy-Buckingham law (Eq. 13), which is the vadose zone analog of Darcy's law, for soil water flux is given as follows:

$$q = -K(\theta)\nabla\psi(\theta) \tag{13}$$

where q is the water flux (cm/s), θ is the volumetric water content as a function of location and time t, K is the unsaturated hydraulic conductivity of soil (cm/s) as a function of volumetric water content, and ψ is the total soil-water head (cm) as a function of the volumetric water content, and z is the vertical coordinate. For this formulation, z is positive in the direction of gravity with $z=0$ being the top surface. The primary difference between the Darcy-Buckingham law and its counterpart in the saturated flow is the dependence of hydraulic conductivity and total head on the volumetric water content. The total head ψ is the sum of capillary head, *h(which is dependent on moisture content θ)*, and the elevation head, z. The Darcy-Buckingham law can be combined with the continuity (*i.e.*, the differential water balance) equation to obtain a general form of the Richards equation:

$$\frac{\partial\theta}{\partial t} = \nabla\left(K(\theta)\nabla h(\theta)\right) - \frac{\partial K(\theta)}{\partial z} \tag{14}$$

Sometimes, specifically when solving multi-dimensional problems, it is helpful to make a change of dependent variable known as the Kirchoff transformation:

$$U = \int_{h_0}^{h} K(\alpha)\,d\alpha \tag{15}$$

where the lower limit can be chosen as arbitrarily as is convenient. U is known as the matric flux potential. With this transformation Eq. 14 reduces to:

$$\frac{\partial U}{\partial t} = D\nabla^2 U - D\frac{\partial K}{\partial z} \tag{16}$$

where $D = K\,dh/d\theta$ is the diffusivity of the soil. This is the U-based formulation of the Richards equation.

A vast majority of the analyses of Richards equation has considered only the vertical soil water movement, treating the problem as essentially one-dimensional. In which case, Eq. 14 can be rewritten as

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left(K(\theta)\frac{\partial h(\theta)}{\partial z}\right) - \frac{\partial K(\theta)}{\partial z} \tag{17}$$

Eq. 17 involves the two dependent variables h and θ, and can be rewritten either in terms of θ or with h as the dependent variable. The θ-based formulation of the one-dimensional Richards model is:

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left(D(\theta)\frac{\partial\theta}{\partial z}\right) - \frac{dK}{d\theta}\frac{\partial\theta}{\partial z} \tag{18}$$

Similarly, an *h*-based formulation can be written as follows:

$$C\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}\left(K(h)\frac{\partial h}{\partial z}\right) - \frac{dK}{dh}\frac{\partial h}{\partial z} \tag{19}$$

where $C(h) = d\theta(h)/dh$.

Equations 18 and 19 are not completely equivalent (Philip, 1969). As an example, Eq. 19 may still apply when *h* exceeds the air-entry value (*i.e.*, the value at which air enters an initially saturated soil), h_b, or when positive, as may be the case when a depth of free water is ponded over the soil. Eq. 18 cannot be applied to these scenarios. This results from the water diffusivity, $D(\theta)$, being infinite due to $dh/d\theta$ becoming infinite for all $h > h_b$. Under such saturated conditions Eq. 19 reduces to the Laplace equation, since $C(h) = 0$ and $K(h) = constant$. Also, Eq. 18 cannot apply to layered soils due to the existence of discontinuities in water content at the layer interfaces.

In the formulation of Eqs. 18 and 19 the assumption is tacitly made that $\theta(h)$ and $K(h)$ are single-valued functions such that the inverse functions of $h(\theta)$ and $K(\theta)$ exist, and are single-valued. For soils exhibiting hysteresis these functions are two-valued, and special care should be exercised in the solution of the Richards equation. Soil hysteresis is usually understood in terms of the curve depicting the equilibrium relationship between water content and capillary pressure. This equilibrium relationship can be obtained in two ways: (1) in drying, by taking an initially saturated soil sample and applying increasing capillary pressure to dry the soil while recording equilibrium water content at each applied suction; and (2) in wetting, by gradually wetting up an initially dry soil sample while reducing capillary pressure. Each of these methods yields a continuous curve, but the two curves generally will not be identical (refer to Figure 2). The equilibrium soil water content at an applied suction is greater in drying than in wetting. This dependence of the equilibrium water content upon the direction of the process is referred to as hysteresis (Hillel, 1980).

Several limitations exist to the general applicability of the Richards model as represented by Eqs. 18 and 19. Important limitations are (refer to Philip (1969) for a detailed discussion of these) as follows:

- The specification of a representative elementary volume or a Darcy scale may not be possible (*e.g.*, preferential pathways or macropores may be present).
- Colloidal swelling and shrinking of soils may demand that the water movement be considered relative to the movement of the soil particles; this phenomenon may also cause significant changes in soil permeability.
- Two-phase flow involving air movement may be important when air pressures differ significantly from atmospheric pressure.
- Thermal effects may be important, especially for evaporation during the redistribution of infiltrated water, in which case the simultaneous transfer of both

heat and moisture needs to be considered.

- Soil hysteresis may be very significant after infiltration ceases and redistribution begins; wetting and drying occur simultaneously; different points in the soil follow different scanning (h versus θ) curves; and there is no definite relationship between gradients of h and gradients of θ.
- Sources and sinks (*e.g.*, root extraction) are neglected, but may be readily included.
- Flow is one-dimensional; this is reasonable for rainfall or irrigation infiltration over a large area.

The following system inputs are needed to obtain soil water flux by solving the Richards equation, Eqs. 18 or 19:

- A boundary condition (BC) at the water-supply surface, $z = 0$; either a concentration-type, $\theta = \theta_s(t)$ or $h = H(t)$, or a flux-type, $K - D\, \partial\theta/\partial z = R(t)$ or $K(1 - \partial h/\partial z) = R(t)$.
- An initial condition for all z, $\theta(z,t=0) = \theta_0(z)$ or $h(z,t=0) = h_0(z)$, and $R(t)$ is the rate of water entry at the surface as a function of time.
- Soil hydraulic parameters, $K(\theta)$ and $h(\theta)$.

The boundary and initial conditions are formulated according to the nature of the problem to be solved. Different conditions are required for infiltration, redistribution, evaporation, and drainage problems (Refer to page 83 of Hanks and Ashcroft, 1980). Except for drainage problems where the soil water profile drains to the ground water, a second boundary condition is not explicitly formulated because the soil domain is generally assumed to be semi-infinite. Where the water depth over the supply surface (or the hydrostatic head under which water is supplied) is not negligibly small, and also where the air-entry value of the soil is (arithmetically) large, the h-based formulation of the Richards equation, Eq. 19, along with appropriate boundary conditions in terms of h, is the correct model. In all other cases, the θ-based formulation is more convenient to solve.

The mathematical analysis of the Richards equation has almost exclusively dealt with the absorption (moisture movement without gravity) and infiltration problems. Relatively little, if any, attention has been paid to the redistribution and drainage problems. Philip (1991) gives three reasons for this relative paucity of mathematical-physical studies of redistribution. First, the initial conditions for the redistribution process tend to be complicated. Second, various mathematical techniques useful in the infiltration context do not, generally, carry over to redistribution problems. Third, redistribution processes involve the very considerable complication of capillary hysteresis. Therefore, the analyses of redistribution and drainage processes using the Richards model have almost totally relied upon numerical techniques.

Depending on the simplicity (or complexity) of these input parameters, the Richards equation can be solved exactly or numerically. Therefore, the models presented here will be separated according to such a classification as simple infiltration equations, analytical or quasi-analytical models and numerical models. The infiltration equations only relate the cumulative infiltration to the time; they do not provide information on the moisture profile or water flux distribution. There are numerous analytical/quasi-analytical and numerical solutions to Richards equation. These provide estimation of moisture and flux distribution as well as infiltration rates. Only some of the more significant approaches will be presented here; however, all are subject to the limitations previously discussed, unless stated otherwise.

5.2 Simplistic Models of Infiltration

There are numerous simple infiltration equations which are solutions to the Richards equation under highly ideal conditions. They are quite restrictive since they only describe infiltration from a water-ponded surface into a semi-infinite, homogeneous soil with a uniform antecedent water content. Swartzendruber and Clague (1989) list several such infiltration equations, some of which are presented here. Each of these equations are given in terms of the dimensionless time *(T)*, and dimensionless cumulative infiltration *(Y)*, which are defined as follows:

$$T = K_s^2 t / S^2 \tag{20}$$

and

$$Y = K_s I / S^2 \tag{21}$$

where K_s is the saturated hydraulic conductivity and S is the sorptivity defined by Philip (1957).

Philip (1957)

$$Y = T^{1/2} + \lambda T \tag{22}$$

where λ is a constant between *0* and *1*.

Philip (1969)

$$Y = \frac{1}{4}\left[2T^{1/2}\exp(-T/\pi) + (2T+\pi)\,erf[(T/\pi)^{1/2}] + 2T\right] \tag{23}$$

Knight (1973)

$$Y = \frac{\pi}{4}\ln\left[1 + erf(4T/\pi)^{1/2}\right] + T \tag{24}$$

Parlange (1975)

$$2Y - \left[1 - \exp(-2T^{1/2})\right] = 2T \tag{25}$$

Brutsaert (1977)

$$Y = T + \left[\frac{T^{1/2}}{(1+\alpha T^{1/2})}\right] \tag{26}$$

where α is either *2/3* or *1*.

Collis-George (1977)

$$Y = T + \frac{1}{N}\left[\tanh(N^2 T)\right]^{1/2} \tag{27}$$

where N is a dimensionless constant that varies between *1* and *4*.

Swartzendruber and Clague (1989)

$$Y = T + \frac{1}{\alpha}\left[1 - \exp(-\alpha T^{1/2})\right] \tag{28}$$

where α is a constant related to soil hydraulic parameters. It is seen that Eq. 25 is a special form of Eq. 28 with $\alpha=2$.

5.3 More Realistic Models of Infiltration

The solutions discussed in Table 2 are more general than the simple infiltration equations presented in Section 5.2, and the Green-Ampt models in Section 4.2. Whereas the Green-Ampt models and the simple infiltration equations are based on idealizations such as sharp wetting front, delta-function or constant diffusivity, linear $K(\theta)$ function, ponded surface, homogeneous soil, and uniform initial moisture distribution, the analytical solutions presented in Table 2 relax one or many of these restrictions. For example, some of the solutions are valid for constant or transient rainfall infiltration; some are valid for heterogeneous soils; some are valid for non-uniform antecedent moisture distribution; and some are valid for realistic nonlinear soil hydraulic functions, $K(\theta)$ and $h(\theta)$. Appendix B provides a comprehensive annotated bibliography of various analytical and quasi-analytical approaches to the Richards equation.

Table 2. Analytical Solutions of the Richards Equation[2]

Model Author(s)	Important Features / Limitations
Philip (1969)	Boltzmann similarity transformation used to obtain a power-series solution in $t^{1/2}$ (Eqs. 83,87,88,91 and 92); the coefficients, which are functions of θ, are obtained (numerically) as the solutions of a set of linear, ordinary integro-differential equations; homogeneous soil; constant water content at the supply surface ; uniform initial water content; there is a practical time limit beyond which the series may not converge; an asymptotic solution valid for large times is also provided; a computer code, INFIL, based on this solution is available (El Kadi, 1983); it can accept various $K(\theta)$ and $h(\theta)$ functions as input.
Philip (1969)	A steady-state (large t) solution (Eq. 117) to Kirchoff-transformed infiltration equation (a "quasilinear" equat-ion); solution (Eq. 122) is also provided for infiltration from a point source into a three-dimensionally infinite region; homogeneous soil; $K(h)$ is equal to $K_s\ exp(\alpha h)$.
Philip (1969)	Solutions (Eqs. 146,132 and 145) to a linearized form of the Richards equation; homogeneous soil; constant water content at the supply surface ; uniform initial water content; $D(\theta)=D_*$ and $dK/d\theta=k$ are constants; D_* is obtained as a function of sorptivity, S, by matching the linear and nonlinear one-dimensional absorption solutions; k is the velocity of the "profile at infinity."
Philip (1969)	Solutions to linearized Richards equation describing multi-dimensional infiltration from the surfaces of cylindrical (Eqs. 156 and 155), and spherical (Eqs. 160 and 155) cavities; source radius is small; homogeneous soil; constant water content at the supply surface; uniform initial water content; linearized solutions are accurate at large t when applied to multi-dimensional problems.
Philip (1972)	Solutions for quasilinear, steady infiltration in heterogeneous soils; Eq. 2 describes $K(h,z)$; solutions for buried, surface, and perched point sources, Eqs. 29, 34, and 38, respectively; solutions for buried, surface, and perched line sources, Eqs. 41, 43, and 46, respectively; source strengths are known.
Raats (1972)	A solution (Eq. 8 or 9) for matric flux potential, $U(h)$, for steady infiltration from a single point source (of known strength) at arbitrary depth; exponential $K(h)$; homogeneous soil; also, Eq. 6 (Philip, 1971) relates the solution for surface source (point, line, or areal) to that for a source buried in an infinite soil mass which is easily obtainable; Philip's (1971) superposition theorem for surface sources (Eq. 6) is generalized to an arbitrary distribution of sources at arbitrary depths (Eq. 12); Eq. 14 provides the inverse transformation for obtaining the pressure head $h(U)$.

[2] Equation numbers referenced in this table correspond to those in the cited article. They *do not* correspond to the equation numbers of this report.

Model Author(s)	Important Features / Limitations
Philip (1974)	Exact solutions to the Burger's equation which is a minimally nonlinear form of Richards equation; these were developed by Knight (1973); a constant $D(\theta)$ and quadratic $K(\theta)$ are assumed; Eqs. 24 and 25 provide the moisture profile and infiltration rate for constant moisture content BC; Eq. 29 provides the moisture profile for constant flux BC for pre-ponding when the flux is greater than K_{sat}.
Warrick (1974a)	An analytical solution (Eq. 11) for steady, quasilinear, one-dimensional infiltration for an arbitrary plant-water uptake function; $K(h)$ is exponential; semi-infinite and finite soil domains; constant surface flux; homogeneous soil.
Lomen and Warrick (1974)	Analytical solutions (Eqs. 12 and 17) for linearized, unsteady, two-dimensional infiltration from buried and surface line sources; $D(\theta)$ is constant and $K(h)$ is exponential; uniform initial water content; zero-flux at the surface except at the source; homogeneous soil.
Warrick (1974b)	Analytical solutions (Eqs. 13 and 16) for linearized, unsteady, three-dimensional infiltration from buried and surface point sources; $D(\theta)$ is constant and $K(h)$ is exponential; uniform initial water content; zero-flux at the surface except at the source; homogeneous soil.
Warrick (1975)	An analytical solution (Eqs. 13-15) for one-dimensional, linearized Richards equation; time-varying surface flux or water content and arbitrary initial conditions; $D(\theta)$ is constant and $K(h)$ is exponential.
Warrick and Lomen (1976)	Analytical solutions for linearized, unsteady, infiltration from strip and disc sources; Eqs. 10 and 14 for buried and surface strip sources, respectively; Eqs. 20 and 25 for buried and surface disc sources, respectively; $D(\theta)$ is constant and $K(h)$ is exponential; uniform initial water content; zero-flux at the surface except at the source; homogeneous soil.
Eagleson (1978)	Equations for soil-water flux during dry and wet periods; Eq. 59 for flux due to capillary rise from the water table during dry periods; Eq. 60 for infiltration during a storm; Eq. 61 for exfiltration or upward movement due to evapotranspiration and capillary rise from the water table; Eq. 62 for deep percolation to water table; Eqs. 60 and 61 are based on Philip's (1969) series solution; moisture profile is assumed to be uniform before storm and during interstorm periods at an appropriate average value; homogeneous soil.
Lomen and Warrick (1978)	An analytical solution (Eqs. 21, 17 and 19) for the linearized one-dimensional Richards equation with plant-water uptake; $D(\theta) = constant$ and $K(h)$ is exponential; the sink (water uptake) function is a sequence (for successive time intervals) of depth-dependent functions; surface flux can be time-dependent; homogeneous soil.
Sisson *et al*. (1980)	Equations for redistribution and drainage based on kinematic wave theory; only gravity effects (unit total potential gradient) are considered; a first-order, hyperbolic PDE results; solution is based on the method of characteristics; Table 1 provides the procedure and actual examples for obtaining moisture profiles, $\theta(z,t)$, for realistic $K(\theta)$ functions; uniform initial moisture profile before redistribution; homogeneous soil.

Model Author(s)	Important Features / Limitations
Parlange *et al*. (1982)	A three-parameter equation (Eq. 13) for estimating cumulative infiltration; constant surface water content (non-ponding); uniform initial water content; homogeneous soil.
Dagan and Bresler (1983)	A sharp front or a rectangular profile model for one-dimensional infiltration and redistribution; Eq. 36 for constant water content BC; Eqs. (47) or (49), and (53) for time-varying or constant flux infiltration; for redistri-bution Eqs. (68) and (53) are valid; ponding time is given by Eq. (55); uniform initial water content.
Smith (1983)	Similar to Sisson's (1980) approach, but a little more general; does not neglect capillary gradients; assumes $q = K(\theta) - D(\theta)\, \partial\theta/\partial z$ to be only a function of θ; $z/t = dq/d\theta$; either moisture content or flux specified at the surface; homogeneous soil.
Philip (1984a)	A solution (Eq. 28) for quasilinear (Kirchoff-transformed), steady infiltration from circular cylindrical cavities; non-zero source radius; water content or suction specified at the source boundary; $K(h)$ is exponential; homogeneous soil.
Philip (1984b)	A solution (Eq. 20) for quasilinear, steady infiltration from spherical cavities; finite source radius; water content or suction specified at the source boundary; $K(h)$ is exponential; homogeneous soil.
Morel-Seytoux (1984)	A rectangular profile model for redistribution based on Brooks-Corey functions for $h(\theta)$ and $K(\theta)$; soil is at residual saturation prior to a wetting event; water content within the actual profile is equal to that within the rectangular profile; cumulative infiltration prior to redistribution is known; Eqs. 16, 17 and 18 provide the moisture profile, front velocity (in the Darcy sense) and front location, respectively, during redistribution; evaporation is neglected; homogeneous soil.
Charbeneau (1984)	Eq. 7 yields the moisture profile based on kinematic wave theory; surface moisture content is specified as $g(t)$; capillary gradients are neglected; an inconsistency in Smith's (1983) formulation relating to the non-neglect of capillary gradients is shown; homogeneous soil.
Parlange *et al*. (1985)	An equation, Eq. 23, along with Eqs. 1 and 19b, for estimating cumulative infiltration, based on Parlange *et al.* (1982); ponded conditions at the surface; uniform initial water content; homogeneous soil.
Sander *et al*. (1988)	An exact, parametric solution (Eqs. 10a-10d), with parameter ξ, for unsteady, one-dimensional, constant flux infiltration; both $D(\theta)$ and $K(\theta)$ are nonlinear, but of special forms (Eqs. 4 and 6); Eq. 8 provides the $h(\theta)$ relation; uniform initial water content; homogeneous soil.
Broadbridge and White (1988)	An exact, parametric solution (Eqs.41-44) with parameter ζ, for unsteady, one-dimensional, constant flux infiltration; almost identical to that of Sander *et al*. (1988); $D(\theta)$ and $K(\theta)$ are of the form given by Eqs. 4 and 5, respectively, enabling the Richards equation to be reduced to the minimally nonlinear Fokker-Planck equation (Philip, 1974); the actual expressions of $D(\theta)$ and $K(\theta)$ are, respectively, Eqs. 21 and 11; Eq. 22 provides the $h(\theta)$ relation; uniform initial water content; homogeneous soil.

Model Author(s)	Important Features / Limitations
Broadbridge *et al*. (1988)	The results of Broadbridge and White (1988) for semi-infinite domain are extended to a finite soil column with zero-flux BC at the bottom; Eqs. (A27-A30) provide the parametric solutions; there appears to be a slight discrepancy between Eqs. 16 & 17 and Eqs. A28 & A30.
Swartzendruber and Clague (1989)	A listing of various, simple infiltration equations (Eqs. 5-24) in terms of two dimensionless variables which are defined by Eqs. 3 and 4; constant water content at the surface; uniform initial water content; homogeneous soil.
Warrick (1990)	An analytical solution (Eqs. 26-28) for one-dimensional soil-water movement based on Broadbridge and White (1988); arbitrary initial water content; constant surface flux, R, but it can be positive, zero, or negative; when R is negative, Eqs. 36 and 39 may have to be used instead of Eq. 27; homogeneous, semi-infinite soil.
Barry and Sander (1991)	A parametric, analytical solution (Eqs. 13a,13b, 11a, and 11b), with parameter ζ, for one-dimensional soil-water movement; arbitrarily time-varying flux at the surface; arbitrary initial water content; $D(\theta)$ and $K(\theta)$ given by Eqs. 2b and 2c; homogeneous, semi-infinite soil.
Warrick *et al*. (1991)	An analytical solution (Eqs. 26-30) for one-dimensional, time-varying infiltration; based on Broadbridge and White (1988); $K(\theta)$ and $D(\theta)$ functions are the same as in Broadbridge and White (1988); arbitrary initial water content; surface flux is treated as a series of constant rates stepped over arbitrary time intervals; homogeneous, semi-infinite soil.
Sander *et al*. (1988)	An exact solution to nonlinear, nonhysteretic redistribution of water in a vertical soil column of finite depth; both $D(\theta)$ and $K(\theta)$ are nonlinear, but of special forms (Eqs. 3 and 4); no flow at the surface; arbitrary initial water content distribution; homogeneous soil.
Protopapas and Bras (1991)	Solutions to the linearized, one- (Eq. 8) and two- (Eq. 15) dimensional, unsteady unsaturated water flow in terms of matric flux potential; $K(h)$ is exponential and $D(\theta)$ is constant; K_{sat} is assumed to vary exponentially in space; uniform initial water content; constant matric flux potential BC at the surface; soil is infinite laterally and semi-infinite vertically.
Hills and Warrick (1993)	An exact solution to Burger's equation for soil water flow in a finite length domain (Eqs. 37, 32, and 21); D is constant and $K(\theta)$ is a quadratic function (Eq. 7); constant flux at the top surface; constant water content at the bottom; homogeneous soil.

5.4 Parameter Estimation for the Richards Equation Models

The solution of Richards equation requires, among other things, the specification of soil characteristic functions, $K(\theta)$ and $h(\theta)$. Based on experiments, these functions can be estimated in a tabulated form; however, algebraic forms are preferred in order to facilitate analytical or numerical solutions of the Richards equation. There are several empirical functional forms of the water retention function, $h(\theta)$, and the hydraulic conductivity function, $K(\theta)$ (El-Kadi, 1985).

The coefficients of water retention function can be determined from experimental data of *h* versus θ, using nonlinear regression techniques (van Genuchten *et al*., 1991). Even though $K(\theta)$ can be estimated in a similar manner, the measurement of unsaturated hydraulic conductivity for a fixed water content is very difficult. Therefore, $h(\theta)$ is customarily used in conjunction with theoretical models such as those of Burdine (1953) and Mualem (1976) to obtain $K(\theta)$. For each of these models an additional parameter, the saturated hydraulic conductivity (K_{sat}) is required.

The most widely used $h(\theta)$ functions are those developed by Brooks and Corey (1964) and van Genuchten (1980). Numerous sources in the soil science literature provide estimates of the model parameters to calculate $h(\theta)$ and $K(\theta)$. The three most comprehensive sources for these model parameters are Brakensiek *et al*. (1981), Panian (1987), and Carsel and Parrish (1988). These papers provide estimates of saturated hydraulic conductivity and water retention parameters for soils of various textures. Johnson and Ravi (1993) evaluated the basis of these databases and discussed their potential limitations and possible uncertainties with respect to use in simulating water movement in the vadose zone. van Genuchten *et al*. (1992) presented various indirect methods for estimating the hydraulic properties of unsaturated soils. An electronic database called UNSODA (Leij et al., 1996) is another good source for soil hydraulic parameters. Also refer to Appendix C for an annotated bibliography on soil hydraulic parameter estimation.

6.0 Summary and Recommendations

A compilation of simple mathematical models has been presented for quantifying the rate of soil-water movement due to infiltration. The rate of infiltration is usually required as an input parameter by the simpler, screening level models which describe contaminant migration in the vadose zone. Even though numerous techniques are available for estimating infiltration rate (API, 1996), none offers a quick and easy way to obtain an estimate for the purposes of preliminary analysis and decision-making. The estimation methods presented here are easy-to-use, and yield scientifically-based estimates using soil-hydraulic and climatic parameters representative of the prevailing site conditions. The compiled methods were divided into three types: a) empirical models, b) Green-Ampt models, and c) Richards equation models. These methods (except the empirical models) are based on widely-accepted concepts of soil physics. Proper use of them should provide a rational and scientific basis for the determination of site-specific PRGs and SSLs.

Evaluation of selected models is presented in an accompanying document (Ouyang *et al.*, 1998). These models should be adequate for quantifying infiltration rates under conditions which are commonly-encountered at many hazardous waste sites. The document includes examples illustrating the proper application of these models under different scenarios. Also, sources of information for the model parameters are identified.

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van Genuchten, M. Th., F.J. Leij, and L.J. Lund. 1992. Indirect Methods for Estimating the Hydraulic Properties of Unsaturated Soils. University of California, Riverside, CA.

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APPENDIX A

Green-Ampt Models

Annotated Bibliography

GREEN-AMPT BASED METHODS

1. Aggelides, S., E.G. Youngs. The Dependence of the Parameters in the Green and Ampt Infiltration Equation on the Initial Water Content in Draining and Wetting States. *Water Resources Research*, 14:857-862 (1978).

    The parameters in the Green and Ampt infiltration equation were determined from infiltration experiments in a sand column at various uniform initial water contents in both draining and wetting states and were compared with various estimates obtained from the soil water properties measured on the same experimental column. The estimates of the soil water pressure head at the wetting front were generally more negative than the values obtained from the directly measured parameters. It was found that the calculated cumulative infiltration as a function of time was fitted better by using Bouwer's crude 'water entry' estimate than values deduced by approximating Richards' flow equation.

2. Ahuja, L.R. Applicability of the Green-Ampt Approach to Water Infiltration through Surface Crust. *Soil Science*, 118:283-288 (1974).

    Applicability of the Green-Ampt approach to describe water infiltration through surface crust after the rapid initial stage is evaluated by testing the equations of Hillel and Gardner (1970) and two modifications of these against numerical-solution data for Yolo soil. The modifications were designed to approximately account for the transient nature of soil-water content at the crust-soil boundary. One of the modifications involved piece-wise application of Hillel and Gardner's (1970) equations, and the other treated the boundary water content as a continuous variable. The results of tests showed that the unmodified equations of Hillel and Gardner (1970) were fairly good for a smaller crust resistance but involved considerable error for a larger resistance. Piecewise application of these equations with as small as four subranges gave results in close agreement with the numerical solution for both resistances. The other modifications involving continuously variable boundary water content showed very good agreement for both resistances in early stages. In the intermediate stages, it showed errors of large relative magnitude, which decreased with increase in resistance.

3. Barry, D.A., J.-Y. Parlange, and R. Haverkamp. Short Communication, Falling-Head Ponded Infiltration with Evaporation--Comment. *Journal of Hydrology*, 162:211-213 (1994).

4. Bouwer, H. Rapid Field Measurement of Air Entry Value and Hydraulic Conductivity of Soil as Significant Parameters in Flow System Analysis. *Water Resources Research*, 2:729-738 (1966).

    Field measurements of air entry value and hydraulic conductivity of soil are obtained with a covered cylinder infiltrometer equipped with standpipe and vacuum gage. Tests are normally completed in approximately 30 minutes. The resulting data are used to construct step functions relating hydraulic conductivity to (negative) soil water pressure for sorption and desorption. These functions may be used as simplified hydraulic conductivity characteristics to include negative pressure flow in the analysis of subsurface water movement. Several field and laboratory studies demonstrate the validity of the concepts and the technique.

5. Brakensiek, D.L., C.A. Onstad. Parameter Estimation of the Green and Ampt Infiltration Equation. *Water Resources Research* , 13:1009-1012 (1977).

    Infiltrometer data are utilized to test a fitting procedure for the Green and Ampt infiltration equation parameters. The spatial variation of the estimated parameters is averaged to give lumped parameter values for watershed modeling. An appropriate scale for parameter averaging is discussed. which transforms parameter values to normal deviates. A sensitivity analysis for the equation parameters is performed for both infiltration estimates and runoff hydrograph volume and peak rates. The capillary pressure parameter shows least sensitivity; however, variation in the fillable porosity and effective conductivity parameters has a major influence on infiltration and runoff amounts and rates.

6. Brakensiek, D.L. Estimating the Effective Capillary Pressure in the Green and Ampt Infiltration Equation. *Water Resources Research*, 13:680-682 (1977).

A considerable amount of research indicates that the Green and Ampt infiltration equation can be used in hydrologic modeling with confidence. The parameters, however still require considerable field and or laboratory effort to determine estimated values. Reported here are alternatives for determining the average wetting front capillary pressure parameter. Only a moisture characteristic would be required.

7. Charbeneau, R.J. and R.G. Asgian. Simulation of the Transient Soil Water Content Profile for a Homogeneous Bare Soil. *Water Resources Research*, 27(6):1271-1279 (1991).

Characterization of the fate and transport of solutes through the vadose zone requires estimation of average water contents and travel times through the unsaturated profiles. This paper reviews a model for long-term simulation of the soil water content profile for a homogeneous bare soil using physically based parameters. For an arbitrary rainfall record, the model calculates the cumulative infiltration, runoff, evaporation, and recharge components, as well as the time average reduced saturation and its variance as a function of depth. The model is computationally efficient and easily allows long-term simulation for parameter sensitivity investigations.

8. Childs, E.C., M. Bybordi. The Vertical Movement of Water in Stratified Porous Material 1. Infiltration. *Water Resources Research*, 5:446-459 (1969).

The infiltration law of Green and Ampt has long been established as an adequate approximation in uniform soil profiles. It is here extended to include heterogeneous profiles. Formulas are developed to express three relationships. Firstly, the infiltration law is derived for a profile consisting of a succession of n different layers with conductivity decreasing from the surface. Secondly, an expression is derived for the conductivity profile that will give a specified infiltration law, and in particular a linear infiltration law with specified parameters. Thirdly, this expression serves to determine the conductivity when the infiltration law is observed. Experiments are described to measure the infiltration rates into layered columns or porous material, the results of which confirm the theory.

9. Chu, S.T. Infiltration During an Unsteady Rain. *Water Resources Research*, 14(3):461-466 (1978).

Infiltration during a rainfall event can be divided into two distinct stages: a stage with surface ponding and a stage without surface ponding. Few of the infiltration models in current use are suitable to describe infiltration for both stages. In this paper the Green and Ampt equation was applied to determine the time that separates these two stages so that infiltration for the different stages can be treated separately. To obtain an integrated form of the Green and Ampt equation, it is traditionally assumed that the cumulative infiltration is zero at the time when surface ponding starts. But in a rainfall event the cumulative infiltration equals the water infiltrated into the soil profile prior to the ponding time, which is usually not zero. Therefore, a modification in the traditional Green and Ampt equation is needed to describe infiltration during a rainfall event. It is shown in this paper that this modification is equivalent to a shift of the time scale by an amount which is referred to as the pseudotime by the author. The modified version of the Green and Ampt equation was applied to determine rainfall excess and to predict total runoff for three major storms recorded by the Agricultural Research Service from 1957 to 1959 on a watershed near Oxford, Mississippi. A comparison of the prediction and the measured total runoff appeared to be promising.

10. Freyberg, D.L., J.W. Reeder, J.B. Franzini, I. Remson. Application of the Green-Ampt Model to Infiltration Under Time-Dependent Surface Water Depths. *Water Resources Research*, 16:517-528 (1980).

The performance of the Green-Ampt model for infiltration problems where the depth of water above the ground surface is varying with time is investigated. In order to yield infiltration rates that agree with those predicted by the Richards equation for flow in a homogeneous, nondeforming, and nonhysteretic soil the effective suction head parameter in the Green-Ampt model must be considered a function of time, surface water depth, initial moisture content, and soil type. However, when a constant value of the effective suction head is assumed, the response of the Green-Ampt model to variations in surface water depth is qualitatively equivalent to the response of the Richards model. The effectiveness of a number of definitions of the suction head parameter that have been proposed in the literature is investigated. While the differences In predicted infiltration rates and cumulative infiltration obtained by using the different definitions are, in general, of marginal significance, the best choice of the value of the effective suction head depends upon the particular problem to which the model is being applied.

11. Green, W.H. and C.A. Ampt. <u>Studies on Soil Physics I. The Flow of Air and Water through Soils</u>. *J. Agr. Sci.*, IV (Part I):1-24 (1911).

12. Hillel, D., W.R. Gardner. <u>Transient Infiltration into Crust-Topped Profiles</u>. *Soil Science*, 109:69-76 (1970).

13. Mein, R.G. and C.L. Larson. <u>Modeling Infiltration during a Steady Rain</u>. *Water Resources Research*, 9(2):384-394 (1973).

    Few of the infiltration models in current use are suitable for the situation in which the rainfall intensity is initially less than the infiltration capacity of the soil. In this paper a simple two-stage model is developed for infiltration under a constant intensity rainfall into a homogeneous soil with uniform initial moisture content. The first stage predicts the volume of infiltration to the moment at which surface ponding begins. The second stage, which is the Green-Ampt model modified for the infiltration prior to surface saturation, describes the subsequent infiltration behavior. A method for estimating the mean suction of the wetting front is given. Comparison of the model predictions with experimental data and numerical solutions of the Richards equation for several soil types shows excellent agreement.

14. Morel-Seytoux, H.J. and J. Khanji. <u>Derivation of an Equation of Infiltration</u>. *Water Resources Research*, 10(4):795-800 (1974).

    Since the publication of the article of Green and Ampt (1911) the two constants appearing in their ‘equation of infiltration’ have been considered to be empirical constants. In fact, they can be deduced simply from the soil characteristics, and they have a very precise physical meaning. However, the assumption of an abrupt front separating a saturated zone from the zone at the initial water content can lead to prediction errors of the order of 10-70%. Derived formulas display explicitly the functional dependence of the effective (or flowing) capillary head and of the viscous correction factor as a function of the initial water content. These formulas are both simple and accurate and of practical value to the hydrologist.

15. Neuman, S.P. <u>Wetting Front Pressure Head in the Infiltration Model of Green and Ampt</u>. *Water Resources Research*, 12:564-566 (1976).

    A theoretical expression relating the wetting front pressure head in the infiltration model of Green and Ampt (1911) to soil characteristics is derived. This expression is identical to that previously suggested by Bouwer (1964) on the basis of an analogy with horizontal flow. It differs from a more complete expression recently proposed by Morel-Seytoux and Khanji (1974) in that the effect of air mobility is neglected, the need for determining the functional relationship between the relatively permeability of air and water saturation thus being avoided.

16. Philip, J.R. <u>Variable-Head Ponded Infiltration Under Constant or Variable Rainfall</u>. *Water Resources Research*, 29(7):2155-2165 (1993).

    This study of the effect of excess rainfall, ponded without runoff, on the dynamics of infiltration into a deep homogeneous soil uses the Green-Ampt, or delta function, approximation. Exact solutions are given for constant and for piecewise constant rainfall. An inverse method yields solutions for arbitrary hyetographs. Ponding can increase markedly both the total duration of the infiltration event and the total amount infiltrated. The analysis suggests that capillary hysteresis emerges during the

ponding recession phase toward the end of a storm. In long-duration heavy rainstorms infiltration rate may increase with time, contrary to the classical picture. The asymptotic large time infiltration rate under continuous constant rainfall at rate R, with $R \geq K$ (the conductivity parameter in the Green-Ampt formulation) is an increasing function of both K and R. (With ponding neglected, it is simply K). An appendix gives some exact results on the influence of hyetograph shape on time to ponding.

17. Philip, J.R. Short Communication, Falling-Head Ponded Infiltration with Evaporation--Reply. *Journal of Hydrology*, 162:215-221 (1994).

18. Salvucci, G.D. and D. Entekhabi. Explicit Expressions for Green-Ampt (Delta Function Diffusivity) Infiltration Rate and Cumulative Storage. *Water Resources Research*, 30(9):2661-2663 (1994).

The sharp wetting front model of infiltration (Green and Ampt, 1911; Philip, 1954) yields through simple integration an exact solution relating the infiltration rate (i), cumulative infiltration (I), and time (t). The relation, however, is implicit for i or I; i.e., it is of the form $t = A[1 - B \ln(1 + I/B)]$. Numerical iteration is required to find the infiltration rate, and furthermore, analytic manipulations are limited using this traditional formulation of the Green-Ampt infiltration. In this note we present an accurate expression for the infiltration rate in the form of a rapidly converging series in the variable $\tau = t/(t + \chi)$. Truncating the series at four terms yields a useful expression for $i(t)$. The proposed four-term expression gives less than 2% error at all times and is readily integrated to yield the cumulative infiltration $I(t)$. In conjunction with the exact expression for time (t) given i or I, the proposed expression is useful in infiltration/runoff calculations that necessitate the time compression approximation (TCA).

19. Smith, R.E. and J.-Y. Parlange. A Parameter-Efficient Hydrologic Infiltration Model. *Water Resources Research*, 14(3):533-538 (1978).

By adopting two extreme assumptions concerning the behavior of unsaturated soil hydraulic conductivity K near saturation, we derived a two-branched model for ponding time and infiltration rate decay for arbitrary rainfall rates. One assumption was that K varies slowly near saturation and leads to an expression for ponding time and infiltration decay. For initially ponded conditions, ponding time is zero, and with rainfall rate r 6 4, the familiar Green and Ampt (1911) expression results. The other, rather opposite assumption was that K varies rapidly, e.g., exponentially, near saturation. This model also holds for both rainfall and ponded surface conditions, and for ponded conditions the expression is identical to that of Parlange (1971). Each model uses only two parameters, saturated soil conductivity K_s and a parameter that is roughly related to sorptivity and responds nearly linearly to variations in initial saturation. Both parameters are physically related to measurable soil properties. Methods are presented to estimate parameters of either model from infiltrometer tests. The two models are compared with a precise numerical solution of the unsaturated soil water diffusion equations for three soils that represent a range of soil behaviors near saturation. Our results show that either assumption would be an excellent model for most hydrologic purposes, and the relative goodness of fit of each model is generally consistent with the appropriate behavior of K(θ 6 θ_s).

20. Swartzendruber, D. Infiltration of Constant-Flux Rainfall into Soil as Analyzed by the Approach of Green and Ampt. *Soil Science*, 117:272-281 (1974).

Infiltration of water into soil from an application of constant water flux r is analyzed theoretically by treating the resulting soil-water profiles as step functions, as assumed in the approach of Green and Ampt (1911). Important features of the infiltration process then follow in fairly straightforward mathematical form. As r decreases toward the near-saturated soil hydraulic conductivity K_b, the time to of onset of surface-water excess is sufficiently increased so that the curves of infiltration flux i versus cumulative time t are shifted toward larger times, progressively away from the curve of i versus *t* for continuous ponding. But *i* is not a simple function of *t*, primarily because the cumulative water infiltration *I* is an implicit function of *t*. Two explicit-form equations are then proposed for *i* versus *t*, both equations being integrable into explicit forms of *I* versus *t*. The one explicit equation is matched with the implicit function within ±11 percent, over the whole range from $r = K_b$ to r .

APPENDIX B

Richards Equation Models

Annotated Bibliography

RICHARDS' EQUATION (ANALYTICAL / QUASI-ANALYTICAL)

1. Babu, D.K. Infiltration Analysis and Perturbation Methods 1. Absorption with Exponential Diffusivity. *Water Resources Research*, 12(1):89-93 (1976).

   Simple perturbation methods are employed to analyze the horizontal absorption of moisture in unsaturated soils. The special case treated assumes the diffusivity to be an exponential function of the concentration and the concentration at the boundaries to be constant. The solution emerges as an explicitly determined power series in the Boltzmann variable.

   The resulting profiles are compared with some others found in the existing literature. A discussion about the relevance and advantages of this type of analysis forms the concluding part of the paper.

2. Babu, D.K. Infiltration Analysis and Perturbation Methods 2. Horizontal Absorption. *Water Resources Research*, 12(5):1013-1018 (1976).

   Perturbation methods are employed to analyze the horizontal absorption of moisture in unsaturated soils. The present study applies to absorption under the general diffusivity function, subject to constant boundary concentration. The concept of the wetting front moving with a finite velocity plays a fundamental role in this set of papers. The solution emerges as a series of terms that can be explicitly calculated in terms of the integrals involving diffusivity functions. The examples of exponential and constant diffusivity functions, as well as the case of Yolo light clay, are computed to one or two terms to exhibit the advantages over, and the good agreement with, other published data of Parlange, Philip, etc. The method possesses great potential, and in a subsequent paper the study of vertical infiltration will be presented by utilizing the perturbation mechanism.

3. Babu, D.K. Infiltration Analysis and Perturbation Methods 3. Vertical Infiltration. *Water Resources Research*, 12(5):1019-1024 (1976).

   Simple perturbation methods continue to yield good results in the solution of the problem of vertical infiltration of water into unsaturated soils. The initial approximation is valid for all times and gives useful information about important flow characteristics. The require computation are always direct and simple. Some numerical work pertaining to Yolo light clay is presented to exhibit the advantages, relevance, and simplicity of the present analysis compared to other exiting works on this topic.

4. Barry, D.A., *et al.* A Class of Exact Solutions for Richards' Equation. *Journal of Hydrology*, 142:29-46 (1993).

   A new solution satisfying Richards' equation is derived. The solution, which may be applied for infiltration or capillary rise, is valid for the condition of an arbitrary moisture tension imposed at the soil surface. When written in terms of the moisture tension, the new result is very simple, being derived m terms of a similarity variable. The solution applies when the form of the soil moisture characteristic curve is a particular weighted integral of the gradient of the unsaturated hydraulic conductivity. Thus, if the soil moisture characteristic curve is selected a priori, then this condition determines the hydraulic conductivity. The converse of this statement also applies. The cumulative infiltration derived from the solution is of the form of the Green-Ampt infiltration equation; however, there is no need to assume a steep wetting front as Green and Ampt did. Finally, using the correspondence between Richards' equation and the convection-dispersion equation with a non-linear solute adsorption isotherm, a new exact solution for adsorptive solute transport is derived.

5. Barry, D.A., and G.C. Sander. Exact Solutions for Water Infiltration with an Arbitrary Surface Flux or Nonlinear Solute Adsorption. *Water Resources Research*, 27:2667-2680 (1991).

Several authors have now used the Backlund transformation to linearize Richards' equation and so obtain analytical solutions for infiltration and redistribution processes for the linearizable class of diffusivities and hydraulic conductivities. After transformation a convection-diffusion equation is obtained. To date, all the analytical results making use of the Backlund transform have been based on a specified, constant flux that is assumed to exist at the surface boundary. We generalize the previous work by removing this restriction and obtain a general solution for an arbitrary temporal variation in the surface flux. The solution contains a function that is specified by a Volterra integral equation. In practice, the Volterra equation must be evaluated numerically. This task is accomplished using a relatively straightforward algorithm. The analytical results for infiltration can be applied to the process of solute transport subject to a nonlinear adsorption isotherm since these two phenomena are equivalent under another mapping. Our new analytical results, therefore, also apply to this class of solute transport processes. The general form of the adsorption isotherm for which exact results can be obtained is derived. This isotherm appears capable of representing a relatively large family of physically relevant cases, although this topic remains open for future investigation.

6. Batu, V. Time-Dependent, Linearized Two-Dimensional Infiltration and Evaporation from Nonuniform and Nonperiodic Strip Sources. *Water Resources Research*, 18:1725-1733 (1982).

Using a linearized equation describing unsaturated homogeneous and isotropic porous media flow, a general, two-dimensional, time-dependent mathematical model is presented for infiltration and/or infiltration-evaporation from nonuniform and nonperiodic strip sources located at the soil surface. The analysis is based on an exponential relationship between the unsaturated hydraulic conductivity and the soil water pressure head and also assumes a constant value for the derivative of unsaturated hydraulic conductivity with respect to water content. In the mathematical analysis, Laplace transform and Fourier analysis techniques are used simultaneously, and a general equation in an integral form for the distribution of matric flux potential has been obtained. The result of Warrick and Lomen (1976) for a single strip source is shown to be a special case of this general model. The solution for infiltration from two strip sources is presented as another special case. Also, a third special case is presented for infiltration from two strip sources and evaporation from another strip located in the middle part. The horizontal and vertical flux components for these cases are presented. All results are expressed in integral forms and calculated with a computer using a numerical integration method. The solutions predict the matric flux potential and flux components as functions of space and time. Examples that illustrate the applicability of some of the solutions are presented. These solutions are of interest in the design of both sprinkler and furrow irrigation systems. The results may also be used for different purposes in civil and agricultural engineering applications.

7. Ben-Asher, J., D.O. Lomen, and A.W. Warrick. Linear and Nonlinear Models of Infiltration from a Point Source. *Soil Sci. Soc. Am. J.*, 42:3-6 (1978).

Numerical and analytical solutions for water flow from a point source are compared. The numerical solution was for the nonlinear moisture flow equation and the analytical solution for the corresponding linearized form. For cyclic conditions, results were approximately the same with regard to range in values between the wettest and driest values. However, the numerical results show a faster response both for wetting and drying. Computational times required of the analytical solution were of the order of 1/20 to 1/200th of that required for the finite difference solutions.

8. Bolt, G.H. Discussion on "Absorption and Infiltration in Two- and Three Dimensional Systems." In: *Water in the Unsaturated Zone*, IASH/AIHS - Unesco (1969).

9. Boulier, J.F., J. Touma, and M. Vauclin. Flux-Concentration Relation-Based Solutions of Constant-Flux Infiltration Equation: I. Infiltration into Nonuniform Initial Moisture Profiles. *Soil Sci. Soc. Am. J.*, 48:245-251 (1984).

   The quasi-analytical solution of the infiltration equation based on the flux-concentration relation for constant flux condition and initially uniform water content profile is extended here to nonuniform moisture profiles for fluxes either smaller or greater than the saturated hydraulic conductivity. In the latter case, the solution is developed for the postponding stage. It is shown that the measured flux-concentration relation is well-approximated by $F(\theta^*) = \theta^*$ and that no significant time dependence can be observed. The quasi-analytical solution is then successfully compared with both laboratory experiments performed on a sandy soil column and numerical solution of the Richards equation for the nonponding, preponding, and postponding stages of infiltration.

10. Broadbridge, P., J.H. Knight, and C. Rogers. Constant Rate Rainfall Infiltration in a Bounded Profile: Solutions of a Nonlinear Model. *Soil Sci. Soc. Am. J.*, 52:1526-1533 (1988).

    We present new exact solutions to a versatile analytic nonlinear model of single phase vertical unsaturated flow during constant rate rainfall infiltration in a bounded soil profile with an impermeable base. The nonlinear flow equation in a fixed finite region is transformed to a linear diffusion problem with boundary conditions on a shrinking domain. The linear problem is treated by King's method of Laplace-transform boosts. The analytic solutions illustrate the theoretical differences in the basement moisture build-up when the water content dependence of the soil hydraulic properties varies from strong to weak. When the rainfall rate exceeds a critical value which is estimated here, surface ponding precedes basement saturation. In this case, time to ponding is approximated well by the expression for the infinite column. When the rainfall rate is less than the critical value (which is greater than the conductivity at saturation), basement saturation precedes surface ponding. In this case, the time to basement saturation is close to the time taken for the rainfall to fill the available pore space.

11. Broadbridge, P. and I. White. Constant Rate Rainfall Infiltration: A Versatile Nonlinear Model 1. Analytic Solution. *Water Resources Research*, 24(1):145-154 (1988).

    Analytic solutions are presented for a nonlinear diffusion-convection model describing constant rate rainfall infiltration in uniform soils and other porous materials. The model is based on the Darcy-Buckingham approach to unsaturated water flow and assumes simple functional forms for the soil water diffusivity $D(\theta)$ and hydraulic conductivity $K(\theta)$ which depend on a single free parameter C and readily measured soil hydraulic properties. These $D(\theta)$ and $K(\theta)$ yield physically reasonable analytic moisture characteristics. The relation between this model and other models which give analytic solutions is explored. As C -> , the model reduces to the weakly nonlinear Burgers' equation, which has been applied in certain field situations. At the other end of the range as C ->1, the model approaches a Green-Ampt-like model. A wide range of realistic soil hydraulic properties is encompassed by varying the C parameter. The general features of the analytic solutions are illustrated for selected C values. Gradual and steep wetting profiles develop during rainfall, aspects seen in the laboratory and field. In addition, the time-dependent surface water content and surface water pressure potential are presented explicitly. A simple traveling wave approximation is given which agrees closely with the exact solution at comparatively early infiltration times.

12. Broadbridge, P. and I. White. Time to Ponding: Comparison of Analytic, Quasi-Analytic, and Approximate Predictions. *Water Resources Research*, 23(12):2302-2310 (1987).

    An analytic expression for time to ponding is introduced using the nonlinear model of Broadbridge and White (1987). The hydraulic properties of this model can encompass properties ranging from those of a highly nonlinear Green-Ampt-like soil to those satisfying the weakly nonlinear Burgers' equation. Because of its versatility, this analytic solution is used as a benchmark against which extant analytic, quasi-analytic, and approximate expressions are compared. Time to ponding is parameterized here in terms of the readily measured field properties, sorptivity and hydraulic conductivity. In the limit of Green-Ampt-like properties the analytic solution reduces exactly to the Parlange and Smith (1976) approximation. A similar functional dependence of time to ponding on rainfall rate is found from quasi-analytic approximations. Based on this, a modified approximation is suggested which should give time to ponding for most soils to within ±10%. Some existing approximations are found to have unacceptable deviations from the analytic solution, and their continued use appears unwarranted. Finally, we address the field problem of predicting time to ponding at any antecedent water content, given sorptivity measured at only one initial water content.

13. Brutsaert, W. More on an Approximate Solution for Nonlinear Diffusion. *Water Resources Research*, 10:1251-1252 (1974).

A method of solution developed earlier (1970) for a diffusivity, that is, a power function of concentration, is shown to be more generally applicable. One of the conditions of its applicability is the existence of a sharp diffusing front. As an illustration the solution is worked out herein for a diffusivity that has been useful in the analysis of soil water problems.

14. Brutsaert, W., and R.N. Weisman. Comparison of Solutions of a Nonlinear Diffusion Equation. *Water Resources Research*, 6:642-644 (1970).

Four solutions of a general nonlinear diffusion equation are given. A numerical analysis serves as reference for one series solution and for two more simple approximate solutions. The accuracy of the methods is discussed and presented graphically; the results of the numerical analysis are given in tabular form. It was found that the usefulness of the two approximate solutions lies in their mathematical simplicity and the accuracy with which they describe infiltration.

15. Charbeneau, R.J. Kinematic Models for Soil Moisture and Solute Transport. *Water Resources Research*, 20(6):699-706 (1984).

The kinematic theory of soil moisture and solute transport in the vertical direction for unsaturated groundwater recharge is considered. The general theory of kinematic models is reviewed and applied for an isolated wetting event wherein the soil starts at and eventually drains to field capacity. Analytical expressions are developed for the water content and moisture flux as a function of depth and time. The approach is extended for an arbitrary sequence of surface flux or water content boundary conditions. The transport of a solute with a general nonlinear sorption isotherm is also considered. It is shown that for the general isotherm the vertical displacement of a solute isochore during an arbitrary wetting sequence depends only on the total depth of water infiltrated and not on how the infiltration rate varies with time.

16. Cisler, J. Note on the Parlange Method for the Numerical Solution of Horizontal Infiltration of Water into Soil. *Soil Science*, 117:70-73 (1974).

Parlange's approximate analytical solution for the horizontal infiltration of water into soil is numerically tested for the case of two-step diffusivity function and also for the Yolo Light Clay diffusivity of Philip (1957). It is shown that Parlange's second approximation for the Boltzman variable $\phi = x \bullet t^{-1/2}$ as a function of moisture content gives in both cases too high values, as compared to the analytical solution for two-step diffusivity, or Philip's solution for the soil-water case. The error decreases as the diffusivity function approaches more closely to the delta-function type of diffusivity. The improved second approximation of Parlange is, however, very precise, even for the diffusivities, which are not of delta-function type. The increase of labor for this improved approximation is very small. A recurrence formula for the successive ϕ's is also examined, but it is found, however, that this formula does not give convergent values and the computational effort is larger than the improved second approximation.

17. Clothier, B.E., J.H. Knight, and I. White. Burgers' Equation: Application to Field Constant-Flux Infiltration. *Soil Science*, 132(4) (1981).

    We present the analytical constant-flux solution to Burgers' equation. Burgers' equation is a minimally nonlinear Fokker-Planck diffusion equation, applicable to infiltration into soils with a constant diffusivity and quadratic conductivity-water content relationship. Field Bungendore fine sand has a near-constant diffusivity-water content relationship, and analytical solutions of Burgers' equation are in good agreement with field profiles of water content obtained in situ with a rainfall simulator. The solutions and experimental data all relate to nonponding infiltration with fluxes less than the saturated hydraulic conductivity; however, a wide range of elapsed times and flux rates are covered. Predicted wet-front penetration is in good agreement with the field experimental data. 'The complete analytical solution can be evaluated using a programmable hand calculator. The simpler "profile-at-infinity" solution is shown to be surprisingly accurate over a wide range of times, giving useful results easily. An expression for the time to ponding is also presented.

18. Corradini, C., F. Melone, and R.E. Smith. Modeling Infiltration During Complex Rainfall Sequences. *Water Resources Research*, 30(10):2777-2784 (1994).

    An extension of the conceptual model earlier developed by Smith et al. (1993) is presented. Their basic model considered the problem of point infiltration during a storm consisting of two parts separated by a rainfall hiatus, with surface saturation and runoff occurring in each part. The model is here extended toward further generality, including the representation of a sequence of infiltration-redistribution cycles with situations not leading to soil surface saturation, and rainfall periods of intensity less than the soil infiltration capacity. The model employs at most a two-part profile for simulating the actual one. When the surface flux is not at capacity, it uses a slightly modified version of the Parlange et al. (1985) model for description of increases in the surface water content and the Smith et al. (1993) redistribution equation for decreases. Criteria for the development of compound profiles and for their reduction to single profiles are also incorporated. The extended model is tested by comparison with numerical solutions of Richards' equation, carried out for a variety of experiments upon two contrasting soils. The model applications yield very accurate results and support its use as part of a watershed hydrologic model.

19. Drake, R.L., *et al.* Similarity Approximation for the Radial Subsurface Flow Problem. *Water Resources Research*, 5(3):673-684 (1969).

    A useful mathematical technique in the study of radial flow of underground water is the application of similarity transformations, such as Boltzmann's transformation. The transformations lead to approximations that are shown to be valid if certain physical parameters satisfy given limiting conditions.

    A nonlinear partial differential equation is formulated describing the radial flow. of soil moisture. A similarity approximation is used to transform the partial differential equation into a nonlinear ordinary differential equation. Criteria by which to judge the applicability of the similarity approximation are derived.

    Several monotonicity properties of the transformed moisture as a function of the similarity variable are derived. The properties are used to develop a numeric procedure for the solution of the transformed equation. The procedure is used to study the radial flow of soil moisture to a cylindric sink for diffusivities which vary linearly, quadratically, and exponentially with soil moisture. The solutions show that the moisture front steepens with increasing nonlinearity.

20. Drake, R.L. and C.P. Peterson. Application of a Local Similarity Concept in Solving the Vertical Subsurface Flow. *Water Resources Research*, 7(5):1241-1255 (1971).

    A technique is described for solving various problems in continuum mechanics through the application of similarity transformations when all the requirements for the application of such transformations are not satisfied. The transformed equations usually involve a stray time variable that is interpreted as a system parameter. When this parameter is properly interpreted, the solutions of the transformed systems are very good numerical approximations for the solutions of the original systems. The advantages of considering the transformed systems are usually two-fold: numerical solutions can be obtained with less computer time, and qualitative analysis of the transformed systems is usually easier than that of the original systems. The problems discussed in this paper include certain previously published results and some new results obtained for the nonlinear Fokker-Planck equation in which the vertical movement of soil moisture under the influence of gravity is described.

21. Gardner, W.R. Some Steady-State Solutions of the Unsaturated Moisture Flow Equation with Application to Evaporation from a Water Table. *Soil Science*, 4(85):228-232 (1957).

22. Ghosh, R.K. Modeling Infiltration. *Soil Science*, 130(6):297-302 (1980).

    The equation $I = at^b + K_st$ is proposed as a model of infiltration. This is a combination of the Lewis-Kostiakov equation and Philip's two-term infiltration equation, but it has the advantage of eliminating the lapses of both those equations. The formulas by which a and b can be predicted in advance are also presented.

23. Ghosh, R.K. A Note on the Infiltration Equation. *Soil Science*, 136(6):333-338 (1983).

    Philip's two-term infiltration equation $I = St^{1/2} + At$, sometimes fails to describe field results precisely. Though physically less appropriate, $I = mt^n$, the Lewis-Kostiakov equation, is advantageous in that it can accommodate varieties of field results. For the case where n is greater than l/2, the term t^n can be expanded suitably, and if only two terms of this expanded time series are considered, the equation $I = m_ot^{1/2} + m_1t^{3/2}$ is formed. This equation describes the field results more precisely than Philip's two-term equation. Two sets of published data (Lewis 1937) and six sets of experimentally observed data for various time ranges are examined here. The agreement between the observed and generated data (using the present model) is found to be very good.

24. Gilding, B.H. Qualitative Mathematical Analysis of the Richards Equation. *Transport in Porous Media*, (5):651-666 (1991).

    The Richards equation is widely used as a model for the flow of water in unsaturated soils. For modelling one-dimensional flow in a homogeneous soil, this equation can be cast in the form of a specific nonlinear partial differential equation with a time derivative and one spatial derivative. This paper is a survey of recent progress in the pure mathematical analysis of this last equation. The emphasis is on the interpretation of the results of the analysis. These are explained in terms of the qualitative behavior of the flow of water in an unsaturated soil which is described by the Richards equation.

25. Haverkamp, R., *et al.* Infiltration Under Ponded Conditions: 3. A Predictive Equation Based on Physical Parameters. *Soil Science*, 149(5):292-300 (1990).

    We derived a new infiltration equation that takes into account the possibility of an infinite diffusivity near saturation. Using the example of two soils (clay and coarse sand), we showed that this new infiltration equation has a sound physical basis. In particular, all parameters used are true soil properties that are constant with time and independent of the water depth imposed as a surface boundary condition. Compared with analytical, numerical, and experimental results, the equation shows a great precision ($\sigma^2 < 5.10^{-3}$ cm^2) at all times.

The present law introduces a significant improvement over the law obtained in part 1 of this series dealing with ponded infiltration by introducing the physical effect of an infinite diffusivity at saturation.

26. Haverkamp, R., *et al.* Three-Dimensional Analysis of Infiltration from the Disc Infiltrometer 2. Physically Based Infiltration Equation. *Water Resources Research*, 30(11):2931-2935 (1994).

In situ measurement of soil hydraulic properties may be achieved by analyzing the unconfined efflux from disc tension infiltrometers, once consistent infiltration equations can be derived. In this paper an analytical, three-dimensional infiltration equation is developed, based on the use of parameters with sound physical meaning and adjustable for varying initial and boundary conditions. The equation is valid over the entire time range. For practical purposes, a simplified solution is also derived. The full and simplified equations give excellent agreement with published experimental results and are particularly useful for determining soil hydraulic properties through application of inverse procedures.

27. Irmay, S. A Linearization Technique for the Study of Infiltration. In: *Water in the Unsaturated Zone*, IASH/AIHS - Unesco (1969).

The partial differential equation (PDE) of unsaturated flow of liquids in porous media, especially of water in stable soils, is discussed: steady and unsteady flows, with or without gravity, in one-, two-, and three-dimensional cases, in cartesian and cylindrical coordinates. The initial and boundary conditions are described, and the hodograph sphere (circle) method of saturated flow is extended to unsaturated media. The inverse PDE are also developed.

Various auxiliary functions are used: effective concentration C, hydraulic conductivity K, capillary potential ψ, and diffusivity D, but the equations become much simplified when expressed in terms of the diffusivity potential F, when they often become linear in steady flow. It is shown that $K^{1/3}$ is linear in C, and ψ is linear in log C in a wide range.

A number of steady flow solutions is studied analytically and graphically, either transforming the PDE into an ordinary differential equation (ODE), finding a first integral or the general solution. The cases treated include: horizontal parallel, radial and two-dimensional flows; vertical flows and flows in a vertical plane and axi-symmetric. In downflow two cases are possible: from higher to lower C, and conversely.

In unsteady flows the PDE are transformed into an ODE by the Boltzmann transformation, by a linear wave-like transformation, by similarity transformations, by separation of variables and by other methods. Among the new solutions the effect of a rising or falling aquifer is described, and unsteady linear inclined flow, evaporation and infiltration effects.

The method outlined above can be used in solving similar diffusion equations in other fields.

28. Knight, J.H. and J.R. Philip. On Solving the Unsaturated Flow Equation: 2. Critique of Parlange's Method. *Soil Science*, 116(6):407-415 (1974).

The physical content of Parlange's method of solving the flow equation is explored. His first approximation, which was developed by Macey (1969), satisfies continuity in the integral sense, but the second and higher approximations do not. Approximations beyond the first lack any constraining link between the separate steps of 'satisfying continuity' and 'satisfying Darcy's law', which make up each iteration. There is, consequently, nothing in the procedure to ensure convergence.

A detailed investigation establishes the nonconvergence of the method when applied to one-dimensional sorption. It is found that the first approximation is best, and that the higher approximations make oscillations of increasing magnitude about the exact solution. Two illustrative examples are given. There is no reason to expect the procedure to be any more useful in other cases.

The utility of Parlange's method is thus simply the utility of the first approximation: the dependence of this on the shape of the diffusivity function and on the flux-concentration relation is discussed.

Contrary to Parlange's claim, the method cannot be applied to two- and three-dimensional systems other than radially symmetrical ones.

29. Kobayashi, H. The Mathematics of Unsaturated Flow, A Theoretical Analysis and Numerical Solutions of Unsaturated Flow in Soil. In: *Water in the Unsaturated Zone*, Chapter 7, IASH/AIHS - Unesco (1969).

Vertical flow is investigated by Forward type difference equation, using S-function, because the error of numerical calculation can be reduced in this way.

In the case where the fundamental equation can be transformed to an ordinary differential equation, the author shows the necessary conditions that the auxiliary function has to satisfy and indicates that there are two kinds of auxiliary functions, i.e., the product from and the linear form. Furthermore a generalized Boltzmann's transformation of the product form has been shown in three dimensional form

$$\Phi = \{(ax + by + cz)^2 \bullet t^{-1}\}^1$$

By using the generalized Boltzmann's transformation, the author solved the problem for one dimensional horizontal flow by the iterative method and compared the solution with the solution of the fundamental solution by the difference method by Hanks et al. Next, the author derived the Forward difference equation of vertical flow, investigated the variation of computed results by the mesh size Δx of the difference method and compared the solution with the solution by the above iterative method.

These results were computed at relatively small values of t, but they seem to be reasonable with the influence of the gravity term.

30. Koussis, A.D. Nonlinear Sorption of Water in Soil. *Soil Science*, 132(4):262-266 (1981).

We derive analytical solutions, based on integral approximations of the governing equation, for sorption of water in power law soil. The parameters of the solution are functions of the soil-characterizing exponent and are determined by considering the extreme medium behavior of "linear" and "delta" type. A particular parameter choice reproduces an earlier solution due to Brutsaert. We compare several solution configurations, including that of Brutsaert, to other approximate methods and to exact numerical integration.

31. Lomen, D.O. and A.W. Warrick. Time-Dependent Linearized Infiltration: II. Line Sources. *Soil Sci. Soc. Amer. Proc.*, 38:568-572 (1974).

Water flow from line sources is analyzed using a linearized form of the moisture flow equation. Both single and parallel line sources are considered. Results are particularly relevant for high-frequency irrigation, such as by trickle sources, for which the soil moisture at any particular point varies over a relatively small range. Numerical calculations include lines of constant matric flux potential (or equal moisture content) as a function of time and the time-dependent response to a cyclic input.

Although the results are developed for surface sources, the analysis may easily be extended to buried sources.

32. Lomen, D.O. and A.W. Warrick. Time-Dependent Solutions to the One-Dimensional Linearized Moisture Flow Equation with Water Extraction. *Journal of Hydrology*, 39:59-67 (1978).

An exact, analytical solution is developed for one-dimensional soil water flow. Assumptions are that the unsaturated conductivity is exponential with pressure head and that the soil-moisture diffusivity is constant. The surface boundary condition is taken as a time-dependent specified-intake velocity. A sink function for plant-water uptake is described by a sequence of depth-dependent functions which change at specified times. Numerical examples include drainage, infiltration, steady uptake and cyclic surface flux and uptake patterns. The linearizing assumptions are the most realistic for short cycles, but in any case the exact nature of the answers make them ideal for checking numerical approximations, such as finite differencing.

33. Molz, F.J., *et al.* Soil Moisture Availability for Transpiration. *Water Resources Research*, 4(6):1161-1169 (1968).

Potential transpiration is a measure of the rate at which water can be transmitted from evaporation sites in plant leaves to the atmosphere. Potential soil-moisture availability is defined as a measure of the capacity of a soil to transmit water to a root site. A differential equation is presented describing radial flow of soil moisture to a single vertical sink (root) in an infinite soil mass which is initially at a uniform moisture content. The relationship between moisture content and diffusivity for the soils studied may be represented by an exponential function. A numerical solution of the differential equation is used to determine the soil-moisture flux. The results show that for scientific soils the decrease in soil moisture with time occurs mainly in the immediate vicinity of the sink. Moisture flux increases with initial moisture content but is essentially time independent. In natural systems the flux would probably decrease with time because of multiple root interference. At large soil moisture contents, actual transpiration is limited by and equivalent to potential transpiration. At small soil-moisture contents, actual transpiration is limited by and equivalent to potential soil-moisture availability.

34. Morel-Seytoux, H.J. Analytical Results for Prediction of Variable Rainfall Infiltration. *Journal of Hydrology*, 59:209-230 (1982).

The basic equations that govern water movement in unsaturated soils are presented for a boundary condition of variable rainfall rate at the soil surface. In particular, a differential equation for the water content at the soil surface is derived. Solutions are then obtained for the time evolution of water content at the soil surface assuming a power law form for the relative permeability to water as a function of normalized water content. Ponding time formulae are obtained and compared with other previously published relations. Water content profiles are also obtained and their shapes are displayed graphically for the case of an exponent of 2 in the power law form of the relative permeability to water for a constant rainfall rate. Formulae are also derived for the situation of a variable rainfall pattern. A methodology for the coupling of the analytical procedures with an implicit numerical solution of the water content at the soil surface is suggested as a cost-effective alternative to the strict numerical solution of the partial differential equation that governs the water content profile evolution. Postponding infiltration formulae and water content profile equations are also provided.

35. Morel-Seytoux, H.J. Drainage Rates from a Vertical Column. *Hydrological Sciences Bulletin*, 20(2):249-255 (1975).

When both water and air movements are included in the formulation of infiltration or drainage problems, the problems of infiltration and drainage exhibit a natural symmetry. Methods of solution for one problem are applicable to the other type of problem. an equation of drainage was derived and it had precisely the same functional form as the equation derived for infiltration in a previous paper.

36. Morel-Seytoux, H.J. From Effective Infiltration to Aquifer Recharge: A Derivation Based on the Fluid Mechanics of Unsaturated Porous Media Flow. Colorado State University, Interim Report for FY 83-84, CER83-84HJM6 (1983).

An approximate unit hydrograph of aquifer recharge due to effective percolation is derived from the fluid mechanics of unsaturated flow. The parameters appearing in the expression of the unit hydrograph have clear physical meaning. Comparison of theory with field observations indicates that the theory works well. A new tool for estimation of the temporal variation of aquifer recharge from that of effective infiltration is now available. The tool is practical (simple), physical and apparently works well as evidenced by comparison with field observations.

37. Morel-Seytoux, H.J. From Excess Infiltration to Aquifer Recharge: A Derivation Based on the Theory of Flow of Water in Unsaturated Soils. *Water Resources Research*, 20(9):1230-1240 (1984).

An approximate unit of hydrograph of aquifer recharge due to effective percolation is derived from a theory describing flow of water in unsaturated soils. The parameters appearing in the expression of the unit hydrograph have physical meaning. Comparison of the theory with field observations indicates that the theory works well. A new tool for estimation of the temporal variation of aquifer recharge from that of excess infiltration is now available. The tool is practical (simple) and physical and apparently works well, as evidenced by comparisons with field observations.

38. Morel-Seytoux, H.J. Two-Phase Flows in Porous Media. In: *Advances in Hydroscience,* V.T. Chow, Ed., Academic Press, New York, NY. pp. 119-202 (1973).

With the proliferation of scientific and technological journals it is becoming more and more difficult to read the complete literature in one's own field. Treatises that synthesize the current state of the art are welcome additions since they extract the essential information from hundreds of articles, permit the reader to bypass the task of actually reading most of these articles, and direct him to the really fundamental papers in his field of interest. Two more such treatises in the field of flow through porous media (Bear, 1972) and soil and water (Hillel, 1971) have recently appeared.

When the books cover a broad field they run two risks: one of being at times rather shallow and the other of being partly obsolete by the time of publication. It is hoped that the present article will avoid these two hurdles because it is both narrow in scope and limited in length.

This article was conceived with three purposes in mind. The first purpose was to present a review of what had already been done in the field of two-phase flow of interest to the soil scientist and the hydrologist. The second purpose was to review the "petroleum industry know-how" in the area of two-phase flow in porous media in a form which would appeal to the hydrologist or soil scientist. Finally, the third purpose was to show how these particular techniques and their extensions can be used fruitfully for a better understanding and description of the movement of water and air in soils.

In summary, the basic philosophy of this article gravitates around the following central themes: (1) water movement in the soil is (at least) a two-phase phenomenon and may not be at times adequately described or understood by the prevailing current unsaturated (one-phase) approach, and (2) much of the petroleum literature know-how can be used effectively to solve two-phase flow problems.

39. Morel-Seytoux, H.J. and C. Miracapillo. Groundwater Flow in Unsaturated/Saturated Zones. In: *Appropriate Methodologies for Development and Management of Groundwater Resources in Developing*

Countries. Volume III. Proceedings of an International Workshop held February 23-March 4; 1989, Rotterdam (1989).

The unsaturated zone plays a crucial role in the hydrologic cycle as the link between surface and groundwater. Though there are many aspects to this interaction, examination of a single aspect, that of aquifer recharge, will illustrate nevertheless most essential elements in the linkage. Typically intermittent streams (wadis) and (artificial) infiltration basins are not in permanent hydraulic connection with the underlying water table aquifer. As water becomes available in the wadi or in the basin, infiltration proceeds. Due to the frequent presence of a clogged layer the water flow below the wadi (or basin) bed is unsaturated. Recharge occurs only after the unsaturated wetting front reaches the water table. On the other hand, long after infiltration has ceased at the soil surface, recharge continues from the unsaturated storage in excess of field capacity, which has accumulated above the water table during the infiltration phase. The three-dimensional character of the flow is rendered by a technique that consists of matching a flow solution in a vertical cross-section with a flow pattern in a horizontal plane. Practical examples illustrate the influence of the parameters on the water table level below the infiltrating area and on the lateral recharge rate into the part of the aquifer which is not overlain by the river or basin bed. Comparison with field observations shows the technique to be accurate, inexpensive, and easy to apply.

40. Morel-Seytoux, H.J. and C. Miracapillo. Prediction of Water Table Mound Development and Aquifer Recharge from an Infiltrating Area. In: *Unsaturated Flow in Hydrologic Modeling, Theory and Practice,* H.J. Morel-Seytoux, Ed., Kluwer Academic Publishers, Boston, MA. pp. 241-272 (1989).

Typically intermittent streams (wadis) and zones of infiltration during and shortly after a rain or a flood are not in permanent hydraulic connection with the underlying water-table aquifer. Due to the frequent presence of a clogged layer the water flow below the infiltration zone is often unsaturated. Recharge occurs only after the unsaturated wetting front reaches the water-table. On the other hand, long after infiltration has ceased at the soil surface, recharge continues from the unsaturated storage which has accumulated above the water table during the infiltration phase. This distinct phase of recharge after infiltration has stopped is called the drainage phase. The problem is two or three-dimensional in nature. To simplify the predictive tool and make it very practical, the multi-dimensional character of the flow problem is rendered, approximately but effectively, by a special technique that consists of matching two unidimensional flows, a vertical and a horizontal one. Practical examples illustrate the influence of the parameters on the water table level below the infiltrating area and on the lateral recharge rate into the part of the aquifer which is not overlain by the river or infiltration basin bed. Comparison with field observations shows the technique to be accurate, inexpensive and easy to apply.

41. Novak, M.D. Quasi-Analytical Solutions of the Soil Water Flow Equation for Problems of Evaporation. *Soil Sci. Soc. Am. J.,* 52:916-924 (1988).

Quasi-analytical solutions of the one-dimensional soil water flow equations applied to problems of evaporation are derived and compared with exact numerical solutions available in the literature. Constant-concentration and constant-flux surface boundary conditions and semi-infinite and finite soil columns are considered. For the semi-infinite soils the quasi-analytical technique of Philip and Knight (1974) is very successful in predicting the water content profiles for the constant-concentration condition, and only moderately so for the constant-flux condition. Their iterative procedure for the flux concentration function converges rapidly in the constant-concentration case but an analogous procedure diverges in the constant-flux case.

For finite soils the simple assumption that the rate of drying of the column is independent of depth in the flow equation leads to accurate predictions of the water content profiles for the constant-concentration cases and for the constant-flux case if the potential evaporation is low, the soil is

shallow, and/or the initial hydraulic diffusivity is high. For conditions other than these roughly accounting for the higher rate of drying that occurs near the surface in this case greatly improves the agreement.

42. Parlange, J.-Y. Convergence and Validity of Time Expansion Solutions: A Comparison to Exact and Approximate Solutions. *Soil Sci. Soc. Am. Proc.*, 39:3-6 (1975).

The convergence of series solutions for the diffusion equation by time expansion is discussed quantitatively, on the basis of the linear and delta function solutions for a spherical cavity. It is shown that convergence alone is a poor criterion to justify the validity of the series solutions. A counter example, diffusion in the presence of an impervious wall, shows that the series may converge for all times but be entirely erroneous. By comparison an approximate integral technique yields a solution which agrees very well with the exact result.

43. Parlange, J.-Y. Theory of Water-Movement in Soils: 1. One-Dimensional Absorption. *Soil Science*, 111(2):134-137 (1971).

44. Parlange, J.-Y. Theory of Water-Movement in Soils: 2. One-Dimensional Infiltration. *Soil Science*, 111(3):170-174 (1971).

45. Parlange, J.-Y. Theory of Water-Movement in Soils: 3. Two and Three-Dimensional Absorption. *Soil Science*, 112(5):313-317 (1971).

46. Parlange, J.-Y. Theory of Water-Movement in Soils: 4. Two and Three-Dimensional Steady Infiltration. *Soil Science*, 113(2):96-101 (1972).

47 . Parlange, J.-Y. Theory of Water-Movement in Soils: 5. Unsteady Infiltration from Spherical Cavities. *Soil Science*, 113(3):156-161 (1972).

48. Parlange, J.-Y. Theory of Water-Movement in Soils: 6. Effect of Water Depth Over Soil. *Soil Science*, 113(5):308-312 (1972).

49. Parlange, J.-Y. Theory of Water-Movement in Soils: 7. Multidimensional Cavities Under Pressure. *Soil Science*, 113(6):379-382 (1972).

50. Parlange, J.-Y. Theory of Water-Movement in Soils: 8. Multidimensional Cavities Under Pressure. *Soil Science*, 114(1):1-4 (1972).

51. Parlange, J.-Y. Theory of Water-Movement in Soils: 9. The Dynamics of Capillary Rise. *Soil Science*, 114(2):79-81 (1972).

52. Parlange, J.-Y. Theory of Water Movement in Soils: 10. Cavities with Constant Flux. *Soil Science*, 116:1-7 (1973).

Analytical expressions are derived for the absorption of water from cylindrical and spherical cavities, when the flux of water at the cavity is imposed. The solution reduces to the one-dimensional case in the short time limit. After a certain time the soil around the cavity may become saturated and the growth of the saturated region with time is investigated. Correction terms due to gravity are also derived in the case of a spherical cavity.

53. Parlange, J.-Y., R.D. Braddock, and B. T. Chu. First Integrals of the Diffusion Equation; An Extension of the Fujita Solutions. *Soil Sci. Soc. Am. J.*, 44:908-911 (1980).

In general, solutions of the nonlinear diffusion equation have to be obtained by numerical or analytical iterative integration. It is shown here that if the diffusivity has a dependence on the water content which obeys a power law, then iterations can be avoided, and the solution obtained at once. This unique case results from the existence of a first integral of the diffusion equation. Previously known analytical results, i.e., the constant diffusivity solution, the delta-function solution, and the Fujita solutions belong to that general class.

For this general class of solutions it is also possible to show that the sorptivity has a dependence on the surface water content which obeys a power law. This represents the only known case when such an analytical relationship exists. This relationship is used to discuss the representation of the square of the sorptivity as an integral of the diffusivity, when the latter has an arbitrary dependence on the water content.

54. Parlange, J.-Y., R. Haverkamp, and J. Touma. Infiltration Under Ponded Conditions: 1. Optimal Analytical Solution and Comparison with Experimental Observations. *Soil Science*, 139:305-311 (1985).

Under ponded infiltration the cumulative infiltration is a function of soil properties, initial conditions, and water layer thickness above the soil surface. For arbitrary soil properties and arbitrary dependence of water layer thickness on time but uniform initial water content, an optimization technique predicts the cumulative infiltration from the integration of an ordinary differential equation. If the thickness of the water layer is constant with time, a fully analytical result is obtained. Careful experimental observations carried out in the laboratory illustrate the validity and accuracy of the result.

55. Parlange, J.-Y., I. Lisle, R.D. Braddock, and R.E. Smith. The Three-Parameter Infiltration Equation. *Soil Science*, 133:337-341 (1982).

A new infiltration equation is obtained by introducing a new parameter in addition to the usual sorptivity and conductivity at saturation. This third parameter can be related to the conductivity of the soil. In particular, if this parameter is equal to zero, then the infiltration equation reduces to the well-known Green and Ampt equation. Comparison with accurate simulations carried out earlier for a sand and clay indicates that sorptivity, conductivity at saturation, and this additional parameter are sufficient to describe cumulative infiltration accurately. Finally, the two examples discussed in detail suggest that this parameter is not very sensitive to soil structure, i.e., it has about the same value for the clay and the sand. Hence, in practice, use of the new law may require a sorptivity and conductivity determination only, i.e., two rather than three parameters. Finally, this new law has about the same precision as existing laws that require knowledge of up to six parameters.

56. Parlange, M.B., *et al.* Optimal Solutions of the Bruce and Klute Equation. *Soil Science*, 155(1):1-7 (1993).

A simple analytical expression is derived to describe soil water profiles satisfying the Bruce and Klute equation. Two parameters have to be determined from integral conditions. The simplicity and accuracy of the technique is checked when the soil-water diffusivity obeys an exponential or a power law function. For those two examples no numerical work is necessary as the results are expressed in terms of tabulated functions only. The method extends to the profile determination the optimal technique used earlier for the determination of the sorptivity, with the same precision.

57. Perroux, K.M., D.E. Smiles, and I. White. Water Movement in Uniform Soils During Constant-Flux Infiltration. *Soil Sci. Soc. Am. J.*, 45:237-240 (1981).

An analysis is presented for constant-flux infiltration of water in soil based on the flux-concentration relation. The analysis is compared with laboratory experiments on constant-flux infiltration into columns of fine sand and silty clay loam. It is shown that the effect of gravity is small for the early

stage and that during this stage sufficiently accurate predictions of moisture profile development can be made by using the simpler absorption analysis of I. White, D. E. Smiles, and K. M. Perroux.

58. Philip, J.R. Absorption and Infiltration in Two- and Three-Dimensional Systems. In: *Water in the Unsaturated Zone*, IASH/AIHS - Unesco (1969).

The theory of the dynamics of one-dimensional infiltration is well developed, but many practical problems involve two- and three-dimensional systems. This study is directed towards the two-dimensional problem of infiltration from a semi-circular furrow and the three-dimensional problem of infiltration from a hemispherical cavity. The latter problem conserves the principal elements of the problem of infiltration from a shallow ring infiltrometer.

The paper begins by examining the problems of absorption in two- and three dimensional radial systems from furrows or cavities of finite radius. 'Exact' series solutions of these non-linear problems are found which are appropriate for small times. For the three-dimensional case a steady large-time solution exists, and is evaluated. The corresponding linear and 'delta function' solutions are also found; they agree well with the exact solutions. These various results support the author's argument that previous attempts to apply the 'Boltzmann transformation' to this group of problems have been erroneous.

The corresponding infiltration problems are formulated and the linear solutions developed. The results are of considerable practical interest. It is shown that: 1) the wetting region for both two- and three-dimensions is finite no matter how great the time (unlike for one dimension); and 2) the final steady state infiltration rate depends not only on the hydraulic conductivity, but also on the capillary properties of the soil. The influence of gravity in distorting the pattern of wetting decreases as the radius of the cavity or furrow decreases; and it is very much less in three dimensions than in two.

59. Philip, J.R. Comments on the Paper by R. Singh and J.B. Franzini, 'Unsteady Flow in Unsaturated Soil-Water Movement from a Cylindrical Source.' *Journal of Geophysical Research*, 73:3968-3970 (1968).

60. Philip, J.R. The Dynamics of Capillary Rise. In: *Water in the Unsaturated Zone,* IASH/AIHS - Unesco (1969).

The problem of the dynamics of capillary rise associated with the sudden immersion the bottom of a soil column in free water (and of the corresponding sub-irrigation problem) examined. An 'exact' method developed for analysing the dynamics of one-dimensional infiltration is appropriate at small times, but problems of convergence set limits to its applicability. The equilibrium situation approached at large times is, of course, readily found; but the 'exact bridging of the gap between the 'small-time' solution and the final equilibrium appears to require the use of high-speed computers.

61. Philip, J.R. The Function Inverfc θ. *Australian Journal of Physics*, 3:13-20 (1960).

The function inverfc θ arises in certain diffusion problems when concentration is taken as an independent variable. It enters into a general method of exact solution of the concentration-dependent diffusion equation. An account is given of the properties of this function, and of its derivatives and integrals. The function

$$B(\theta)=(2/\pi^{1/2}) \exp [—(\text{inverfc } \theta)^2]$$

is intimately connected with the first integral of inverfc θ and with its derivatives. Tables of inverfc θ and B(θ) are given.

62. Philip, J.R. General Method of Exact Solution of the Concentration-Dependent Diffusion Equation. *Australian Journal of Physics*, 13:1-12 (1960).

Only three forms of $D(\theta)$ have previously been known to yield exact solutions of the equation

$$\partial\theta/\partial t = \partial/\partial x \,(D(\theta)\, \partial\theta/\partial t),$$

subject to the conditions $\theta = 0, x > 0, t = 0; \theta = 1, x = 0, t > 0$. The present paper reports a general method of establishing a very large class of $D(\theta)$ functions which yield exact solutions. A similar method enables exact solutions of the same equation subject to the conditions $\theta = 0, x > 0$, and $\theta = 1$, $x < 0, t = 0; \int_0^1 x d\theta = 0, t > 0$. In this case also a very large class of $D(\theta)$ functions yield exact solutions. Examples are given for both cases.

Many of the exact solutions which are most readily found tend to lead to zero or infinite values of D at one or two points of the θ-range. Means of avoiding this difficulty are devised. Practical use of the method is discussed.

63. Philip, J.R. Horizontal Redistribution with Capillary Hysteresis. *Water Resources Research*, 27(7):1459-1469 (1991).

Exact and relatively simple analysis is possible for one hysteretic redistribution process that in long horizontal columns with the two parts, $x < 0$ and $x > 0$, at different uniform moisture contents θ_1 and θ_2 ($\theta_1 > \theta_2$) at time $t = 0$. We formulate and solve this problem. The key to its simplicity is its similarity character, with moisture profile $x(\theta)$ proportional to $t^{1/2}$. At the interface $x = 0$ there is, for $t > 0$, a permanent and constant moisture content jump θ_3 to θ_4 ($\theta_3 > \theta_4$). The operative moisture potential is the primary drying potential for $x < 0$, $\theta_1 \geq \theta \geq \theta_3$ and the primary wetting potential for $x > 0$, $\theta_4 \geq \theta \geq \theta_2$. The operative moisture diffusivity is the primary drying diffusivity for $x < 0$ and the primary wetting diffusivity for $x > 0$. The drying diffusivity is greater than the wetting diffusivity near $\theta = \theta_1$ and less near $\theta = \theta_2$ Solutions are found for illustrative examples corresponding to four values of H, the relative magnitude of the hysteresis loop. The resorptivity R, the total wet-to-dry exchange of water across $x = 0$ in reduced form, is analogous to the sorptivity S in absorption and infiltration. For the calculated examples, R is about one third of S for the corresponding absorption process and varies only mildly with H. Nonhysteretic calculations may thus give reasonable estimates of R for horizontal hysteretic redistribution, though they cannot give details such as the moisture jump at $x = 0$. Desorption moisture profiles for $x < 0$ are very gradual, with a large depth of penetration. Absorption profiles for $x > 0$ are relatively steep, and the penetration depth small. It follows that in experiments the initially wet section of the column should be made some 5-10 times longer than the initially dry sector.

64. Philip, J.R. A Linearization Technique for the Study of Infiltration. In: *Water in the Unsaturated Zone*, IASH/AIHS - Unesco (1969).

The paper introduces a linearization technique for the solution of non-linear problems in infiltration and other problems of water movement in unsaturated soils. The method consists, essentially, in matching exactly linear and non-linear solutions at small times, and in matching the solutions in some integral sense at large times. The method is worked out for one-dimensional infiltration. It is shown to yield results agreeing closely with the "exact" non-linear solution and to provide algebraic infiltration equations equally valid for small and large times.

The parallel use of similarity (or better, "delta function") solutions provides a good method of approximate analysis, since the linear and the "delta function" solutions represent extremes of soil characteristics and, by implication, of hydrological behaviour. We may, therefore, obtain insight into the general character of the phenomenon and the extent to which it may vary, and we may obtain, at the same time, an indication of the accuracy of the results. For one-dimensional infiltration, the similarity (or "delta function") solution agrees well with the linear solution.

65. Philip, J.R. On Solving the Unsaturated Flow Equation: 1. The Flux-Concentration Relation. *Soil Science*, 116(5):328-335 (1973).

The flux-concentration relation [F(**(-)**)] expresses in reduced form the dependence of flux density on moisture content during unsteady flow phenomena in unsaturated soils. In this first paper of a series, the general properties of F(**(-)**) are established for various phenomena: one-dimensional sorption and infiltration with constant concentration conditions and one-dimensional sorption and infiltration with constant concentration conditions and one-dimensional sorption (for the 'linear' soil only) with constant and variable flux and variable concentration conditions. These studies provide a guide to the properties of F(**(-)**) in more complicated phenomena in one-, two-, and three-dimensional systems.

F(**(-)**) is independent of time for some phenomena and it varies only mildly with time for some others; and for some phenomena, the dependence of F(**(-)**) on the 'shape' of the moisture diffusivity function lies within fairly narrow bounds, which are fixed by reference to the extreme cases of the 'delta function' and the 'linear' soil. This relative stability of F(**(-)**) makes it a useful aid to the study of methods of solving the unsaturated flow equation. It is employed in later papers in a critique of Parlange's method and in the development of a new quasi-analytical technique.

An appendix demonstrates that the approximation of step function moisture profiles implies a delta function moisture diffusivity function.

66. Philip, J.R. Recent Progress in the Solution of Nonlinear Diffusion Equations. *Soil Science*, 117:257-264 (1974).

Important contributions of the late Dr. E. C. Childs included early recognition of the nonlinear diffusion form of the nonhysteretic flow equation for unsaturated nonswelling soils, and of the importance of a predictive system based on the flow equation. The full equation is a nonlinear Fokker-Planck equation, and reduces to a diffusion equation when gravity may be neglected. Equations of the same form describe water movement and volume change in swelling soils. This paper reviews recent work on quasianalytical and analytical methods of solving these equations.

Quasilinear solutions of the steady equation in two and three dimensions have received much attention. The extension to heterogeneous soils can be taken further to include sloping stratification. Theorems enable solutions for surface sources, and sources at arbitrary finite depth, to be deduced from mathematically simpler solutions in a region extending infinitely in all directions. A generalization to the case of sloping soil surface is given.

In many real-world problems where similarity methods do not apply, integral methods offer the same possibility of effectively reducing the number of independent variables. Green and Ampt introduced a simple integral method; and more sophisticated methods have entered soil-water studies through the work of Parlange. Recent studies reveal the importance of preserving integral continuity in integral methods involving iteration.

For diffusivity $D = a(b—\theta)^{-2}$, all problems of one-dimensional nonlinear diffusion subject to arbitrary initial conditions plus a flux boundary condition can be linearized. Exact solutions are therefore available for various problems, some of them in explicit form.

Burgers' equation

$$\partial\theta/\partial t = D\, \partial^2 q/\partial z^2 - (Aq + B)\, \partial q/\partial z$$

is of interest as an especially simple nonlinear Fokker-Planck equation. The nonlinearity of the term in $\partial q/\partial z$ suffices to give a 'profile at infinity' solution for one-dimensional infiltration. The linear Fokker-Planck equation, on the other hand, has no such feature. Knight has found the exact Burgers solutions corresponding to constant-concentration infiltration and to constant-flux infiltration. Both solutions yield the same 'profile at infinity' behavior at large times, as one expects on physical grounds. The Burgers model is more informative and more accurate than simple linearization.

67. Philip, J.R. Steady Infiltration from Buried, Surface, and Perched Point and Line Sources in Heterogeneous Soils: I. Analysis. *Soil Sci. Soc. Amer. Proc.*, 36:268-273 (1972).

The quasilinearized steady infiltration equation is generalized to apply to heterogeneous soils with conductivity depending exponentially on both moisture potential and depth. Mathematical developments, including a theorem connecting surface and buried source solutions, follow closely those established previously for homogeneous soils. Solutions are found for buried, surface, and perched point and line sources. Physically relevant solutions are limited to the following ranges of β, the dimensionless coefficient of dependence of conductivity on depth: for buried and surface sources, $\beta \geq 0$; for perched point sources, $\beta \leq -1$; for perched line sources, $\beta < -1$. It is of interest that the homogeneous medium ($\beta = 0$) is an extreme case for existence of buried and surface source solutions; and that perched source solutions (relevant to subirrigation) exist only for soils with conductivity increasing rapidly enough with height above the impermeable base. Detailed mapping of flows and general discussion is left to Part 2.

68. Philip, J.R. Steady Infiltration from Circular Cylindrical Cavities. *Soil Sci. Soc. Am. J.*, 48:270-278 (1984).

The problem of quasilinearized steady infiltration from circular cylindrical- cavities, with the moisture potential fixed at the cavity surface, is solved exactly. Solutions are presented numerically and graphically for values of the dimensionless cavity radius, R_o, ranging from 0.01 to 10. The dependence on R_o of the average infiltration rate around the cavity surface, and of the distributions of moisture content and potential, are examined. As R_o increases, the effects of gravity on the phenomenon increasingly dominate those of capillarity. Gravity very strongly distorts the moisture distribution from the symmetry produced by capillarity alone: for R_o as small as 0.01, the depth of the effectively wetted region exceeds 250 times its horizontal width on the cavity center line; and the ratio increases rapidly with R_o, exceeding 50 000 for $R_o = 5$. An earlier small R_o approximation proves useful only for $R_o < 0.2$. The cavity flow, evaluated by the present analysis, may be combined with the line source solution to give approximate results useful for the region deeper than 50 radii below the cavity.

69. Philip, J.R. Steady Infiltration from Spherical Cavities. *Soil Sci. Soc. Am J.*, 48:724-729 (1984).

The problem of quasilinearized steady infiltration from spherical cavities, with the moisture potential fixed at the cavity surface, is solved exactly. Solutions are presented numerically and graphically for values of the dimensionless cavity radius R_o in the range 0 to 10. The dependence on R_o of total cavity flow, of the variation of infiltration rate around the cavity surface, and of the distributions of moisture content and potential, are examined. As R_o increases, gravity increasingly distorts the moisture distribution from the symmetry produced by capillarity alone. This distortion, although marked, is less than one-hundredth that for infiltration from circular cylindrical cavities: a vivid illustration of the stronger dominance of gravity over capillarity in two-dimensional systems than in three-dimensional ones. An earlier approximate solution of this problem proves accurate only for $R_o \leq 0.27$. The cavity flow, evaluated by the present analysis, may be combined with the point source solution to give approximate results useful for the region deeper than 20 radii below the cavity.

70. Philip, J.R. Theory of Infiltration. Academic Press, New York, NY, Vol. 9, pp. 215-295 (1969)

Introduction--A very large fraction of the water falling as rain on the land surfaces of the earth moves through unsaturated soil during the subsequent processes of infiltration, drainage, evaporation, and the absorption of soil-water by plant roots. Hydrologists, and their textbooks and handbooks, have tended, nevertheless, to pay relatively little attention to the phenomenon of water movement in unsaturated soils. Most research on this topic has been done by soil physicists, concerned ultimately with agronomic or ecological aspects of hydrology; but their colleagues in engineering hydrology have exhibited an increasing interest in this field in recent years.

The present article deals with the theory of infiltration which is one important outcome of the mathematical-physical approach to the study of water movement in unsaturated soils which has been developed over the past 15 years or so, principally in the U.S., England, and Australia. We shall consider briefly the physical basis for the general formalism of this approach and the limits of its applicability; but we shall be concerned principally with developing the general flow equation (a nonlinear Fokker-Planck equation), with describing methods for its solution, and with presenting the solutions and discussing their physical significance

We define infiltration as the process of the entry into the soil of water made available (under appropriately defined conditions at its surface. This "surface" may be the natural, more or less horizontal upper surface of the soil; or it may be the bed of a natural or artificial furrow or stream, or the walls of a natural or artificial tunnel or cavity. As we shall see, a fundamental point of departure for the study of infiltration is the study of absorption, the particular case of infiltration when the effect of gravity may be neglected, as in horizontal systems, in the early stages of infiltration, and in fine-textured soils in which the influence of moisture gradients dominates that of gravity.

71. Philip, J.R. The Theory of Infiltration: 1. The Infiltration Equation and Its Solution. *Soil Science*, 83:345-357 (1957).

72. Philip, J.R. The Theory of Infiltration: 2. The Profile of Infinity. *Soil Science*, 83:435-448 (1957).

73. Philip, J.R. The Theory of Infiltration: 3. Moisture Profiles and Relation to Experiment. *Soil Science*, 84:163-178 (1957).

74. Philip, J.R. The Theory of Infiltration: 4. Sorptivity and Algebraic Infiltration Equations. *Soil Science*, 84:257-264 (1957).

Many situations in applied hydrology require that the dynamics of infiltration be characterized by a small number of parameters. These parameters are most appropriately the coefficients of an algebraic equation representing the variation of i or v_0 with t.

Parts 1 and 2 of this series provided a detailed analysis of infiltration and part 3 a general discussion of the physical significance of the analytical results. In the present paper we use the preceding work as the basis for a study of the available (generally empirical) algebraic infiltration equations.

This study is facilitated if we first give some attention to a new physical property of porous media which enters the subsequent developments here and which we shall have occasion to use also in later papers of this series.

75. Philip, J.R. The Theory of Infiltration: 5. The Influence of the Initial Moisture Content. *Soil Science*, 84:329-339 (1957).

76. Philip, J.R. The Theory of Infiltration: 6. Effect of Water Depth Over Soil. *Soil Science*, 85:278-286 (1957).

77. Philip, J.R. The Theory of Infiltration: 7. *Soil Science*, 85:333-337 (1958).

78. Philip, J.R. and J.H. Knight. On Solving the Unsaturated Flow Equation: 3. New Quasi-Analytical Technique. *Soil Science*, 117(1):1-13 (1974).

We present a new quasi-analytical technique for solving the flow equation. It has affinities with Parlange's method, but offers the following advantages: freedom to choose an initial assumed flux-concentration relation, F_1, greatly improves the possible accuracy of the first approximation, and the higher approximations preserve integral continuity and therefore behave more stably. The first of these advantages is of practical importance, but the second is more basic. This paper treats only solutions subject to concentration conditions; the related technique for solutions subject to flux conditions will be developed in a later paper.

The technique is studied analytically and numerically for one-dimensional sorption subject to constant concentration conditions. It is found to be convergent for a wide range of shapes of the diffusivity function. For the unfavorable ease of the 'linear' soil, the mean error is 3 percent after two iterations and 1 percent after three. For absorption in Yolo light clay the corresponding figures are 0.57 percent and 0.07 percent.

The general iterative scheme for one-dimensional infiltration subject to constant concentration conditions is presented. Three choices of F_1 should yield useful first approximations: (A) F_{lA} - $\lim_{t\text{->}0}$ F (t is time) = F_{abs}, the F for the analogous absorption process; (B) F_{1B} = limt-> F = θ; and (C) F_{1C}, an interpolation function which is exact in the limits as t->0 and t-> . F_{1A} should lead to a good lower bound for the infiltration rate function q(t), F_{1B} an upper bound, and F_{1C} close upper bound for all except very large t, and the quality of the estimates of moisture profiles should be comparable. Detailed calculations for Yolo light clay bear out these expectations; the three estimates are wholly consistent with the power series solution of Philip (1957b). The error of the approximation based on F_{1C} increases from 0 percent at t = 0 to about 1 percent at t =10^6 sec. This first approximation is accurate enough to render iteration unnecessary for most purposes. Parallel calculations confirm the nonconvergence of Parlange's method when applied to infiltration.

General iterative schemes are given also for two- and three-dimensional sorption subject to constant concentration conditions.

79. Poulovassilis, A., *et al.* An Investigation of the Relationship Between Ponded and Constant Flux Rainfall Infiltration. *Water Resources Research*, 27(7):1403-1409 (1991).

The physics of the rainfall infiltration under a constant rainfall flux greater than the hydraulic conductivity at saturation are investigated and compared with those pertaining to the infiltration process under flooding. From the arguments developed theoretically the following conclusions are reached: The moisture profile prevailing at the time the soil surface becomes saturated during the rainfall process $(t = T)$ is identical with the profile which has been developed during flooding at an earlier time at which the infiltration rate is equal to the rainfall flux $(t - t_c < T)$. The rainfall profile development after T is the same as that of the profile under flooding after t_c the two profiles having a constant time lag equal to $(T - t_c)$. The Green and Ampt analysis as has been applied to the constant flux rainfall infiltration presupposes the identity of the profiles mentioned above. In addition to the theoretical arguments, the flow equation was solved numerically for four different soils under the conditions that define the two infiltration processes. The numerical results obtained conform with the theory.

80. Prasad, S.N. and M.J.M. Romkens. An Approximate Integral Solution of Vertical Infiltration Under Changing Boundary Conditions. *Water Resources Research*, 18(4):1022-1028 (1982).

An approximate solution of the Richards equation was obtained with a spectral series solution for rain infiltration under changing boundary conditions. Prior to surface ponding a flux boundary condition was employed, which was followed by a concentration boundary condition. During this latter stage a flux matching boundary condition was used to describe the movement of the interface between the saturated and unsaturated zones. The solutions yield a general expression for ponding time, wetting front advance, and soil water profile. Infiltration rates beyond ponding include the effect of time dependent ponding height and saturation depth.

81. Protopapas, A.L. and R.L. Bras. Analytical Solutions for Unsteady Multidimensional Infiltration in Heterogeneous Soils. *Water Resources Research*, 27(6):1029-1034 (1991).

The problem of unsteady multidimensional infiltration in heterogeneous soils is approached analytically. By assuming that the hydraulic conductivity is an exponential function of the soil matric potential and a linear function of the soil moisture. the governing flow equation is linearized for a special variation of the saturated hydraulic conductivity in space. By using integral transform methods, analytical solutions to the one- and two-dimensional flow problem are derived and shown to be generalized versions of known results for homogeneous soils. The development aims at obtaining quantitative criteria for the validity of the one-dimensional approximation, which is commonly used in hydrologic applications.

82. Protopapas, A.L. and R.L. Bras. The One-Dimensional Approximation for Infiltration in Heterogeneous Soils. *Water Resources Research*, 27(6):1019-1027 (1991).

By using analytical solutions to the problem of one- and two-dimensional linearized unsteady infiltration from a strip source in a soil with saturated hydraulic conductivity varying exponentially in space, explicit criteria for the validity of the one-dimensional approximation are derived, in terms of parameters describing the soil type, the uniformity of the infiltrating source at the surface, and the soil heterogeneity. Numerical solutions for the linearized infiltration problem are developed for any variation of hydraulic conductivity in space. Extensive simulations suggest that for a sizable strip source and moderate variation of hydraulic conductivity, a set of independent neighboring soil columns represents well the multidimensional infiltration problem. The possibility of defining sufficient parameters controlling the accuracy of the one-dimensional approximation is discussed.

83. Pullan, A.J. The Quasilinear Approximation for Unsaturated Porous Media Flow. *Water Resources Research*, 26:1219-1234 (1990).

This paper gives a review of the quasilinear approximation, an exponential relationship between hydraulic conductivity and moisture potential, and the uses this has been put to since the time of its creation. One of the major attractions of this approximation is that under steady regimes the highly nonlinear Richards equation governing unsaturated flow can be linearized. This simplification has enabled analytic and semianalytic solutions to be found for many unsaturated flow problems. These are reviewed, and the current research areas, in which the quasilinear approximation is being used, are discussed.

84. Raats, P.A.C. Steady Infiltration from Sources at Arbitrary Depth. *Soil Sci. Soc. Amer. Proc.*, 36:399-401 (1972).

This discussion of steady infiltration is based on the assumption that the hydraulic conductivity is an exponential function of the pressure head. The solution for infiltration from a single point source at arbitrary depth is presented. On the basis of this solution, J. R. Philip's superposition theorem for surface sources is generalized to an arbitrary distribution of sources at arbitrary depths. General expressions for the pressure head, the total head, and the components of the flux are also given.

85. Remson, I., *et al.* Vertical Drainage of an Unsaturated Soil. *Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers*, 91(HY1):55-74 (1965).

The general equation describing vertical unsaturated flow of soil water is solved by numerical approximation for several cases. The computer programs are described. The results show: (1) The intermediate belt of a well-drained unsaturated soil approximates field capacity during times of small evapotranspiration and intermittent penetrating rainfall; (2) infiltrated rainfall moves as a pulse with a steep wetting front; (3) the belt of soil water drains from saturation to approximately field capacity within two days for zero evapotranspiration.

86. Salvucci, G.D. An Approximate Solution for Steady Vertical Flux of Moisture Through an Unsaturated Homogeneous Soil. *Water Resources Research*, 29(11):3749-3753 (1993).

An approximate solution is found to the differential equation governing the steady state vertical flux of moisture through an unsaturated homogeneous soil for which the dependence of hydraulic conductivity on tension head is given in the form $K(\psi) = A(B + \psi^n)^{-1}$. The solution expresses the depth from the water table as a function of the capillary tension for a given rate of percolation or capillary rise. It is easily inverted to obtain explicit expressions both for the capillary tension as a function of depth and flow rate. and for the steady flow rate in terms of the capillary tension at a given depth. The latter is shown to compare favorably both to existing limiting case solutions of percolation to a water table at infinite depth, and to capillary rise from a shallow water table to an infinitely dry surface. More importantly, this solution applies to the intermediate condition where the magnitude and direction of flow are sensitive to both the near-surface capillary tension and the water table depth.

87. Sander, G.C., *et al.* Exact Nonlinear Solution for Constant Flux Infiltration. *Journal of Hydrology*, 97:341-346 (1988).

Recently, an analytical nonlinear solution to the problem of two phase oil and water infiltration under a constant flux boundary condition was derived. We show that this solution also applies to the problem of constant infiltration of water by introducing a very simple change in the independent variables of space and time.

88. Sander, G.C., J.-Y. Parlange, V. Kuhnel, W.L. Hogarth, and J.P.J. O'Kane. Comment on "Constant Rate Rainfall Infiltration: A Versatile Nonlinear Model 1. Analytic Solution" by P. Broadbridge and I. White. *Water Resources Research*, 24:2107-2108 (1988).

89. Sisson, J.B., A.H. Ferguson, and M.T. van Genuchten. Simple Method for Predicting Drainage from Field Plots. *Soil Sci. Soc. Am. J.*, 44:1147-1152 (1980).

When the one-dimensional moisture flow equation is simplified by applying the unit gradient approximation, a first-order partial differential equation results. The first-order equation is hyperbolic and easily solved by the method of P. D. Lax. Three published $K(\theta)$ relationships were used to generate three analytical solutions for the drainage phase following infiltration. All three solutions produced straight lines or nearly straight lines when log of total water above a depth was plotted versus log of time. Several suggestions for obtaining the required parameters are presented and two example problems are included to demonstrate the accuracy and applicability of the method.

90. Smettem, K.R.J., *et al.* Three-Dimensional Analysis of Infiltration from the Disc Infiltrometer 1. A Capillary-Based Theory. *Water Resources Research*, 30(11):2925-2929 (1994).

The hydraulic properties of an unsaturated homogenous and isotropic soil can be obtained from the unconfined flux out of a disc infiltrometer into the soil over the depth of wetting. The disc infiltrometer is becoming increasingly popular, but methods of analysis have generally relied on the restrictive assumptions of one-dimensional flow at early times or quasi-steady state flow at large times. We provide an approximate analytical expression for three-dimensional unsteady, unconfined flow out of a disc infiltrometer, and this includes the geometric effect of the circular source but ignores gravity. This physically based solution is tested against data obtained from laboratory

experiments on repacked material. The results illustrate that the difference between three-dimensional and one-dimensional flow is linear with time.

91. Smith, R.E. Approximate Soil Water Movement by Kinematic Characteristics. *Soil Sci. Soc. Am. J.*, 47:3-8 (1983).

Using the Richard's equation in the Fokker-Planck nonlinear diffusion form, unsaturated soil water flow may be treated as a diffusion-convection wave process. If $\alpha\theta/\alpha z$ is assumed a function of θ alone, the unsaturated flow equation may be solved by the method of characteristics, and when $\alpha\theta/\alpha z$ becomes sufficiently small, the Peclet number is assumed large enough to treat unsaturated flow kinematically. Changes in θ with depth in the soil profile are treated as waves, moving downward. Advancing and receding “waves” are treated differently in the approximate analytical technique described here, with advancing wetting fronts described by kinematic “shocks.” The method is compared to the complete solution to Richard's equation for a complex rain pattern and found to predict well the location of deeper moving fronts and also general θ patterns. The kinematic method is also shown to apply to root water extraction zones and to layered soil situations.

92. Smith, R.E., C. Corradini, and F. Melone. Modeling Infiltration for Multistorm Runoff Events. *Water Resources Research*, 29(1):133-144 (1993).

We present a relatively simple analytical/conceptual model for rainfall infiltration during complex storms. It is an approximate but physically based model which can treat intervals of either no rain, low rain, or evaporation. The infiltration model is based on the very general three-parameter analytic model of Parlange et al. (1982), extended to treat soils with very high initial water content. The redistribution model is based on profile extension with shape similarity. A wide range of soil types can be simulated. The model is tested by comparison with numerical solutions of Richards's equation carried out for a variety of events upon four selected soils. The model simulates the solution to Richards's equation quite accurately, provided basic soil retention relations are parametrically represented. It simulates redistribution particularly well for redistribution intervals up to 20 hours. The model usefulness in comparison with the common and simple approach which disregards soil water redistribution is also shown.

93. Sposito, G. and J.V. Giraldez. On the Theory of Infiltration in Swelling Soils. pp. 107-118.

The generalization of the one-dimensional Richards Equation to the case of swelling soil is presented. Although the generalized infiltration equation has the same mathematical form as the classical Richards Equation, it differs physically from the latter in containing material coordinates and in having an additional component in the total soil water potential that is due to applied pressure (e.g., overburden pressure). A new material coordinate transformation is introduced to cast the solution of the generalized infiltration equation into a coordinate frame that is fixed with respect of the experimenter. This transformation can be determined empirically through dual-energy gamma-ray measurements of the bulk density of the swelling soil. Lastly, a model expression for the shrinkage curve is presented that may be employed, for a wide variety of swelling soils, to describe mathematically the contribution of overburden pressure to the gradient of the total soil water potential.

94. Swartzendruber, D. A Quasi-Solution of Richards' Equation for the Downward Infiltration of Water into Soil. *Water Resources Research*, 23:809-817 (1987).

A new equation is presented that expresses depth Z of soil water content θ at time t as an explicit function of θ and t and holds for all $t \geq 0$. The new $Z(\theta, t)$ includes the classical J. R. Philip (1957b, c) solution for asymptotically large times. At moderate times, the new $Z(\theta, t)$ is matched to the classical Philip solution expressed as $\phi t^{1/2} + \chi^t + \psi t^{3/2} + \varpi t^2 + \ldots$, where $\phi, \chi, \psi, \varpi, \ldots$ are different

functions of θ alone. The matching process enables the characterizing functions (or parameters) within $Z(\theta, t)$ to be evaluated in straightforward fashion from the moderate-time Philip functions (ϕ, χ, ψ, $\tilde{\omega}$, . . .). When evaluated for either Philip's data or the Green and Ampt step-function solution, the new $Z(\theta, t)$ in each case describes the water content profiles with excellent accuracy for all times. Also, for all $t \geq 0$, the new $Z(\theta, t)$ can be directly integrated to provide a new equation for cumulative quantity of water infiltrated or flux, both as explicit functions of time.

95. Swartzendruber, D., and F.R. Clague. An Inclusive Infiltration Equation for Downward Water Entry into Soil. *Water Resources Research*, 25:619-626 (1989).

A new infiltration equation, derived exactly from a recent quasi-solution of the governing differential flow equation, is capable of expressing eleven previously published infiltration equations. On the basis of least squares fitting, the new equation in three-parameter dimensionless form expressed each of the eleven previous equations within ±0.31% over all times. If only one parameter was retained and fitted in the dimensionless form, nine of the eleven equations were still expressed within ±2.5%. This one-parameter dimensionless form implies the dimensional infiltration equation $I = (S/A_o)[1 - \exp(-A_o t^{1/2})] + Kt$, which is therefore proposed as a realistic blend of rigor and utility for the complete range of time t ($0 \leq t \leq x$), where I is the cumulative quantity of water infiltrated, S is the sorptivity, A_o is a constant arising from the quasi-solution, and K is the sated (satiated, near-saturated) hydraulic conductivity. Because of its broad matching capability, this equation is deemed to hold considerable promise for describing and fitting experimental infiltration data, whether from the laboratory or the field.

96. Swartzendruber, D. and W.L. Hogarth. Water Infiltration into Soil in Response to Ponded-Water Head. *Soil Sci. Soc. of Am. J.*, 55(6):1511-1515 (1991).

The pressure head of water ponded on the soil surface can increase the infiltration of water into soil, but the effect has often been complicated to describe mathematically. This study was conducted to devise a simpler mathematical description without undue sacrifice of accuracy. A new three-parameter infiltration equation was examined for its capability in describing the effect of soil-surface-ponded water head, h, on the cumulative quantity of water infiltrated into the soil with time. The infiltration equation, as reduced to two-parameter dimensionless form, was fitted by nonlinear least squares to dimensionless data generated from mathematical descriptions of infiltration that included the effect of h in somewhat complicated parametric form. The fitted two-parameter equation gave an excellent description of all the generated data, in terms of both goodness of fit and in recovery of the dimensionless ponded head p used as an input into the generated data. From an overall perspective, recovery of h was achieved within a relative error of ±1.4% across the complete range of the generated data, thus validating the new and relatively simple equation in its description of the general effect of ponded head on the cumulative infiltration process.

97. Swartzendruber, D., and E.G. Youngs. A Comparison of Physically-Based Infiltration Equations (Note). *Soil Science*, 117:165-167 (1974).

Three infiltration equations, each derived in some manner from a consideration of the physics of flow through porous media, are compared quantitatively over the whole time range from zero to infinity. Compared with the Green and Ampt equation, the two-term Philip equation is never more than 15.1 percent higher, while the equation arising out of Philip's linearization analysis is never less than 9.2 percent lower. For practical purposes, these discrepancies may frequently be ignored. Because of its mathematical simplicity and adaptability, the two-term Philip equation, however, would be the preferred choice.

98. Wallach, R. Soilwater Distribution in a Nonuniformly Irrigated Field with Root Extraction. *Journal of Hydrology*, 119:137-150 (1990).

A steady-state solution is developed for two-dimensional distributions of irrigation water flux at the soil surface and water extraction by plant roots. The solution of the linear equation is based on the small perturbations method, in which a small parameter is introduced into the governing differential equation. For the sprinkler irrigation method, this parameter is determined by scaling the soil characteristic length, α, by the distance between the sprinklers. Linearization is attained by assuming that the unsaturated hydraulic conductivity is exponentially related to the pressure head. Vertical distribution of the two-dimensional plant-root uptake is based on the one-dimensional model, which meets the approximation that the extraction pattern of plant roots is 40, 30, 20, and 10% of the total transpiration requirements in each successively deeper quarter of the root zone. Water extraction by plant roots increases lateral unsaturated flow beyond that which occurs as a result of irrigation nonuniformity. However, for a homogeneous soil profile, this lateral flow component increases water availability to those parts of the field where water shortages occur because of irrigation nonuniformity. Indeed, actual irrigation uniformity increases as measured by the decrease in the amplitude of the vertical flux around the mean. Younger plants suffer more from spatial fluctuations of irrigation water than do older plants, which have deeper roots. Actual irrigation uniformity, as measured by water availability to roots, is also affected by the distance between sprinklers and by soil type.

99. Warrick, A.W. Additional Solutions for Steady-State Evaporation from a Shallow Water Table. *Soil Science*, 146:63-66 (1988).

In this paper, I consider evaporation from a shallow water table and present analytical solutions. An unsaturated conductivity function K of the form $K = a/(S^n + b)$ is used, with S the matric suction and a, b, and n constants. The results of Gardner and Fireman (1958) are generalized to all $n > 1$, including fractional values and b = O, for which K reduces to the form commonly referred to as that of Brooks and Corey. A table is presented from which evaporation rates follow for a wide range of depths and hydraulic conductivity functions of the above form.

100. Warrick, A.W. An Analytical Solution to Richards' Equation for a Draining Soil Profile. *Water Resources Research*, 26(2):253-258 (1990).

Analytical solutions are developed for the Richards' equation following the analysis of Broadbridge and White. Included here is the solution for drainage and redistribution of a partially or deeply wetted profile. Additionally, infiltration for various initial conditions is examined as well as evaporation at the upper boundary. In all cases the surface flux is constant, whether it be zero for drainage, positive for infiltration, or negative for evaporation. The solutions assume specific forms for the soil water diffusivity and hydraulic conductivity functions: $a(b-\theta)^{-2}$ and $\beta + \gamma(b-\theta) + \lambda/[2(b-\theta)]$, respectively. Here θ is the water content and $a, b, \beta, \gamma,$ *and* λ are constants.

101. Warrick, A.W. Analytical Solutions to the One-Dimensional Linearized Moisture Flow Equation for Arbitrary Input. *Soil Science*, 120:79-84 (1975).

Analytical solutions to the one-dimensional linearized moisture flow equation are presented. The surface boundary condition is taken either as a time-varying flux, time-varying water content, or with the flux as a linear function of the water content. A semi-infinite vertical flow medium is considered with arbitrary initial conditions. Calculations for a cyclic flux input are compared to the associated nonlinear finite difference solution for five simulations using data from two soils. The results are encouraging in that the same general profiles are obtained. The solution is believed to have particular merit to describe moisture regimes resulting from a high frequency irrigation schedule.

102. Warrick, A.W. Inverse Estimations of Soil Hydraulic Properties with Scaling: One-Dimensional Infiltration. *Soil Sci. Soc. Am. J.*, 57(3):631-636 (1993).

Inverse estimations of hydraulic properties are of immense importance to effectively provide input for water flow modeling and descriptions of soil systems. Here inverse estimations were made using scaled forms of Richards' equation and infiltration measurements in two steps. In Step 1, transient experimental data of cumulative infiltration rate, or wetting front position were "best-fit" using Philip's quasi-analytical, algebraic forms. This was demonstrated with linear regression although a maximum likelihood method could have been used. The first step does not depend on the form of the hydraulic functions and is valid for all uniform soils with constant initial and boundary conditions. Step 2 related the algebraic coefficients to the soil properties and was dependent on the model assumed for the hydraulic functions. Computations were generally algebraic and extremely easy relative to alternative methods that require a numerical simulation for each combination of properties considered. Clearly, the number of estimated parameters cannot exceed the significant number of terms fit to the algebraic forms. For small times or "noisy" data, perhaps only one parameter can be estimated that, for the examples presented, leads to an estimate of the ratio of saturated conductivity to an inverse characteristic length. For this situation, further measurements are needed to provide additional information if more parameters are to be found. Three commonly used forms of hydraulic functions were used, but the methodology is not limited to these specific forms. Scaling could also he used for more complex geometrics and moisture regimes; however, the convenience of the algebraic forms will not generally be available in those cases.

103. Warrick, A.W. Solution to the One-Dimensional Linear Moisture Flow Equation with Water Extraction. *Soil Sci. Soc. Amer. Proc.*, 38:573-576 (1974).

The one-dimensional, steady-state moisture flow equation is solved for arbitrary plant water withdrawal functions using the matric flux potential of earlier investigators to obtain a linearized form. A semi-infinite flow medium and a finite-depth medium overlying a shallow water table are considered with the surface boundary condition taken as a flux. Tables are presented giving the matric flux potential (from which the pressure head is easily determined) for several withdrawal functions. Numerical examples show the effects of different surface fluxes and rooting depths on the matric flux potential and pressure-head profiles. The results are particularly relevant for high-frequency irrigation.

104. Warrick, A.W. Time-Dependent Linearized Infiltration. I. Point Sources. *Soil Sci. Soc. Amer. Proc.*, 38:383-386 (1974).

Water flow from a point source is analyzed using a linearized form of the moisture flow equation. Time-dependence is assumed with the results simplifying to those of previous investigators for steady-state conditions. Discrete time-distributed inputs such as might occur for trickle or high frequency irrigation is amenable to the solution. Numerical simulations include (i) the advance of a wetting front during infiltration, (ii) moisture variation resulting from a cyclic input as during irrigation, and (iii) the matric flux potential field for a two-source problem.

105. Warrick, A.W. Traveling Front Approximations for Infiltration into Stratified Soils. *Journal of Hydrology*, 128:213-222 (1991).

Two approximations are developed for infiltration into a stratified soil profile. Both satisfy the initial conditions, give the correct water content at a given depth for large time, obey mass conservation and give the correct rate of advance for large time. The first is referred to as the “square front” solution and is analogous to Green-Ampt theory in that the profile is either wet or dry without a transition zone. The second solution is based on a homogeneous analog for which Philip’s solution for a profile at large time is invoked. Examples are given for surface boundary conditions both as a specified head and as a flux density.

106. Warrick, A.W. and A.A. Hussen. Scaling of Richards' Equation for Infiltration and Drainage. *Soil Sci. Soc. Am. J.*, 57:15-18 (1993).

Scaling techniques allow a single solution to Richards' equation to suffice for numerous specific cases of water flow in unsaturated soils. Here, such solutions are presented for the well-known hydraulic functions of Brooks and Corey. Infiltration for a constant water content or a constant flux at the soil surface is considered as well as drainage for a semi-infinite profile. Although limited to a single form of hydraulic properties, the results are otherwise more general than those previously available. Of particular interest is that the hydraulic conductivity and air-entry values are embedded within the reduced variables. This is an advantage for computational efficiency, such as when using parameter optimization techniques. Three examples are presented. In the first, a single solution is presented,valid for four combinations of saturated hydraulic conductivity and surface water content. A second example relates to drainage, also with one solution to describe four combinations of physical parameters. The final example is for constant flux infiltration.

107. Warrick, A.W., A. Islas, and D.O. Lomen. An Analytical Solution to Richards' Equation for Time-Varying Infiltration. *Water Resources Research*, 27:763-766 (1991).

An analytical solution is developed for Richards equation for time-varying infiltration. The infiltration rates must be specified as piecewise constants with time. The answer is expressed as a sum of two terms, the first is a function of the instantaneous infiltration and the second an integral which accounts for the water distribution within the profile for the previous infiltration history. The solutions assume specific forms for the soil water diffusivity and hydraulic functions used earlier by Broadbridge and White (1988).

108. Warrick, A.W., D.O. Lomen, and A. Ammozegar-Fard. Linearized Moisture Flow with Root Extraction for Three-Dimensional, Steady Conditions. *Soil Sci. Soc. Am. J.*, 44:911-914 (1980).

A mathematical model is presented to simulate plant water uptake assuming steady state conditions and an unsaturated hydraulic conductivity as an exponential function of pressure head. By choosing the conductivity to be of this form, the moisture flow equation is linear, allowing analytical solutions to be developed in a closed form. Geometries considered possess radial symmetry and are appropriate to describe trickle irrigation. These include matrix potentials for points, discs, and cylinders which can be taken either as sources or sinks. These provide the basic building blocks to simulate varied steady-state withdrawal patterns. The solution for the cylinder is newly derived; the other solutions were already available. Nonsymmetric cases can result from superpositions. Examples include pressure head distributions for four sink-source combinations.

109. Warrick, A.W., and D.O. Lomen. Time-Dependent Linearized Infiltration: III. Strip and Disc Sources. *Soil Sci. Soc. Amer. J.*, 40:639-643 (1976).

Water flow from strip and disc surface sources is analyzed using an approach similar to previous investigations with line and point sources. A line of closely spaced trickle irrigation emitters often wets a surface strip of finite width; similarly, for a single emitter the surface wetted pattern is a disc. Lines of equal matric flux potential are wider and more shallow for these sources than wetting patterns for the line and point. The moisture regime is independent of the source shape and depends only on the total flow rate for regions beyond approximately two times the strip width or disc radius.

110. Warrick, A.W., D.O. Lomen, and S.R. Yates. A Generalized Solution to Infiltration. *Soil Science Society of America Journal*, 49:34-38 (1985).

A generalized solution the moisture flow (Richards') equation is developed for infiltration. The equation is expressed in terms of dimensionless time, depth, and water content. The reduced form

applies for the hydraulic functions of van Genuchten or of Brooks and Corey as well as to other scaled forms and the solution utilizes the procedure of Philip. Answers are presented in concise tables which, in principle, allow for finding moisture profiles, wetting front, intake rate and cumulative intake for a variety of soils and for varying initial water contents. For application, specifically measured hydraulic conductivities can be best-fitted to the forms used here or generalized empirical values can be used from the literature. Thus, either specific or generic parameters may be used, depending on availability for a given problem.

111. White, I. and P. Broadbridge. Constant Rate Rainfall Infiltration: A Versatile Nonlinear Model 2. Applications of Solutions. *Water Resources Research*, 24(1):155-162 (1988).

In paper 1 (Broadbridge and White, this issue) an analytical nonlinear model for constant rate rainfall infiltration was proposed which promised considerable versatility. In it a wide range of soil hydraulic properties are generated through the variation of a single free parameter C. Here three techniques are advanced for determining this parameter. The first, a one-dimensional technique, involves simultaneous determination of sorptivity and wetting front position. The second uses measured values of surface water content at long infiltration times for rainfall rates less than the saturated conductivity. In the third, three- and one-dimensional flow rates are measured on the same soil sample. All are suitable for field applications. The practical range of the C parameter, for a variety of repacked and in situ soils, is found to be restricted to between 1 and 2. The hydraulic conductivities and diffusivities of the model are in general agreement with independent measurements. Its moisture characteristics $\psi(\theta)$, which are not matched in any way to measured characteristics, follow closely those observed. Also, the model permits predictions of the dependence of sorptivity on antecedent water content given a single measurement of sorptivity. The analytic solutions for constant flux infiltration given in paper 1 (Broadbridge and White, this issue) describe satisfactorily the evolution of water content profiles and surface water pressure potential in the laboratory and field without a posteriori adjustments. The mathematically simple, traveling wave approximation agrees well with observations at comparatively short infiltration times. Finally, field and laboratory measured times to ponding are predicted satisfactorily by the model's analytic expression.

112. White, I., D.E. Smiles, and K.M. Perroux. Absorption of Water by Soil: The Constant Flux Boundary Condition. *Soil Sci. Soc. Am. J.*, 43:659-664 (1979).

Absorption of water into soil as the result of a constant flux condition at the soil surface is examined.

Experiments for a fine sand show that the surface water content, movement of the wetting front, and the water content profiles may be predicted from the soil water diffusivity function using the notion of the flux-concentration relation of Philip (1973).

Reduced space and time variables $X = V_o x$ and $T = V_o^2 t$ are introduced. Use of these variables greatly simplifies treatment of the system and reveals that the surface water content and the reduced position of the wetting front are uniquely defined by T while the water content profiles at any value of T are unique in terms of X.

113. Whisler, F.D. and A. Klute. Numerical Analysis of Ponded Rainfall Infiltration. In: *Water in the Unsaturated Zone*, IASH/AIHS - Unesco (1969).

A numerical solution of the flow equation for water in soil was obtained for a flow system consisting of a vertical column of stratified soil materials which had been drained from saturation to equilibrium with a water table. Water was assumed to be applied at the top of the column as either rainfall or ponded water. Infiltration into such a system involves hysteresis; i e., the soil at each point wets along a different wetting scanning curve. This part of the hysteresis phenomena was taken into account in the solution of the problem. The solution of the equation depicts the time and depth distributions of water content and pressure head during the resulting infiltration as well as infiltration rate and accumulated water content.

The properties of the soil that influence the flow are the water capacity and hydraulic conductivity. Tables of χ, a dimensionless conductivity, and γ, a dimensionless water capacity, were calculated from tables of Θ, a dimensionless water content, and φ, a dimensionless pressure head, for a set of moisture characteristic curves. These tables were fed into a digital computer which solved the problem.

Several hypothetical cases were examined. These were coarse-textured soils overlying fine-textured soils, fine-textured lenses in coarse-textured soils, and coarse-textured lenses in fine-textured soils.

Comparisons were made between these cases and also nonstratified cases on the basis of pressure head-time profiles, moisture content-time profiles, infiltration rate changes, and accumulated water content changes The numerical solution predicts a hold up effect on the wetting front as it goes from a fine-textured layer into a coarse-textured one. As one would expect, the rate of advance of the fonts is faster for ponded infiltration than for rainfall infiltration.

114. Youngs, E.G. and A. Poulovassilis. The Different Forms of Moisture Profile Development During the Redistribution of Soil Water After Infiltration. *Water Resources Research*, 12(5):1007-1012 (1976).

Two different forms of moisture profile development may occur during the redistribution of soil water after infiltration. In one of the profile shape continues to be similar to that during the infiltration process, with a fairly uniform water content region near the soil surface above a steep wetting front, and the rate of redistribution is inversely related to the infiltration depth. In the other, water drains from near the surface to appear as a step in the moisture profile below the wetting front at the cessation of infiltration, and the rate of redistribution is directly related to infiltration depth. Classical theory of soil water movement is shown to account for both forms of moisture profile development, which depend on the moisture profile shape at the cessation of infiltration and on the soil water properties of the porous material. Experimental results of moisture profile development in a sand column illustrate how different profile forms develop as the depth of infiltration is increased and, by inclining the experimental column, how they depend on the relative importance of the soil water pressure head gradients compared with that of gravity.

115. Zimmerman, R.W., G.S. Bodvarsson, and E.M. Kwicklis. Absorption of Water into Porous Blocks of Various Shapes and Sizes. *Water Resources Research*, 26(11):2797-2806 (1990).

Approximate solutions are presented for absorption of water into porous spherical, cylindrical, and slablike blocks whose characteristic curves are of the van Genuchten-Mualem type. The solutions are compared to numerical simulations of absorption into blocks of the Topopah Spring member (Paintbrush tuff) from the site of the proposed nuclear waste repository at Yucca Mountain, Nevada. Guided by these results, a scaling law, based on the ratio of surface area to volume, is then proposed for predicting the rate of absorption into irregularly shaped blocks. This scaling law is tested against a numerical simulation of absorption into an irregularly shaped, 2-dimensional polygonal block and is shown to be a good approximation.

APPENDIX C

Soil Hydraulic Property Estimation

Annotated Bibliography

ESTIMATION OF SOIL HYDRAULIC PARAMETERS

1. Block, L.O., M.J. Hendry, and G.W. Chan. MRCON: Computer Program for Estimating Soil Moisture Retention and Unsaturated Hydraulic Conductivity. *Ground Water*, 26(2):218-221 (1988).

MRCON is an interactive computer program designed to approximate soil moisture retention (θ-ψ) and/or unsaturated hydraulic conductivity (Kθ) for soils. MRCON, which is written in BASIC, uses empirical methods obtained from the literature to calculate K-θ and a modified method to calculate θ-ψ. Input data for the program consists of a saturated moisture content and a minimum of three values of θ-ψ. A measured value of saturated hydraulic conductivity can be used as input to better approximate the K-θ curves to field conditions.

2. Bohne, K., *et al*. Rapid Method for Estimating the Unsaturated Hydraulic Conductivity from Infiltration Measurements. *Soil Science*, 155(4):237-244 (1993).

A method is proposed for estimating the unsaturated hydraulic conductivity from observed infiltration data. Infiltration measurements are generally easier to obtain than experimental data required for in situ determination of the hydraulic conductivity. The problem was formulated in terms of a nonlinear least-squares parameter optimization method which combines Philip's two-term infiltration equation with an analytical description of the unsaturated soil hydraulic properties according to Mualem and van Genuchten. Reliable estimates for the hydraulic parameters could be obtained with an inverse procedure when independently measured water retention data were included. The results indicate that soil water content measurements at very low values of the soil water pressure head are especially important to ensure parameter uniqueness. The method provides rapid and cost-effective estimates of the hydraulic properties of field soils.

3. Brutsaert, W. Some Methods of Calculating Unsaturated Permeability. In: *Winter Meeting of the American Society of Agricultural Engineers*, Chicago, IL (1965).

4. Clapp, R.B. and G.M. Hornberger. Empirical Equations for Some Soil Hydraulic Properties. *Water Resources Research*, 14(4):601-604 (1978).

The soil moisture characteristic may be modeled as a power curve combined with a short parabolic section near saturation to represent gradual air entry. This two-part function—together with a power function relating soil moisture and hydraulic conductivity—is used to derive a formula for the wetting front suction required by the Green-Ampt equation. Representative parameters for the moisture characteristic, the wetting front suction, and the sorptivity, a parameter in the infiltration equation derived by Philip (1957), are computed by using the desorption data of Holtan et al. (1968). Average values of the parameters, and associated standard deviations, are calculated for 11 soil textural classes. The results of this study indicate that the exponent of the moisture characteristic power curve can be predicted reasonably well from soil texture and that gradual air entry may have a considerable effect on a soil's wetting front suction.

5. Clothier, B.E. and I. White. Measurement of Sorptivity and Soil Water Diffusivity in the Field. *Soil Sci. Soc. Am. J.*, 45:241-245 (1981).

An analysis for scaling the exponential soil water diffusivity function, D(θ), from concurrent measurements of sorptivity and wet front advance is presented. This analysis provides a field

method for measuring β, the slope of the exponential D(θ). For Bungendore fine sand in the field it was found that β = 3. This is contrary to absorption experiments on a variety of repacked soils that have consistently found β = 8 and led to the suggestion that β = 8 may be the "universal" value applicable to all soils. Although the exponential function with β = 3 provided a usable approximation over the entire range, D(θ) data from undisturbed cores showed that at high water contents D was effectively constant, and for lower water contents the undisturbed data were well described by an exponential function with β = 8. Measurements of sorptivity, wet front advance, and hydraulic conductivity were made in the field using a device that supplied water to the soil surface at the small tension of 4 cm. This gives the physical properties of the soil matrix, since larger channels and voids (>0.75 mm in diam) do not affect the measurements made at a small tension. Field heterogeneity due to such macropores is consequently eliminated. This sorptivity of Bungendore fine sand is about 0.4 that from ponded infiltration affected by the macropores.

6. Cook, F.J. and A. Broeren. Six Methods for Determining Sorptivity and Hydraulic Conductivity with Disc Permeameters. *Soil Science*, 157(1):2-11 (1994).

The disc permeameter has become a popular apparatus for measuring in situ the sorptivity, S, and hydraulic conductivity, K, of the soil at some prescribed potential. A number of different methods have been proposed for calculating S and K using the flow rate, Q(t), from the disc. Measurements of Q(t) on a Kokotau silt loam soil were made using discs with radii of 60 and 102 mm. Measurements were made at potentials of -20, -40, and -100 mm with the 102-mm disc and at potentials of -20 and -40 mm with the 60 mm disc.

S and K were calculated using six different methods. Three of the methods use Wooding's equation of flow from a disc:

$$Q_4 / \pi r^2 = \Delta K \, [1 + 4\lambda_c / \pi r]$$

where Q_4 is the steady-state flow rate; r is the radius of the disc source; $\Delta K = K(\psi_o) - K(\psi_n)$ (where $+\psi_o$ and ψ_n are, respectively, the potential of the source and the initial potential of the soil); and λ_c is the macroscopic capillary length scale. These three methods and a method based on a linear diffusion model gave similar values of S and/or K. The method of Youngs, which uses the early- to medium-time infiltration data to calculate K, gave very low and inconsistent values of K.

Calculations of K were also made from values of S at two different potentials. Here S as S_0 was first calculated by using the early-time values of $Q(t^{1/2})$. These values of K were similar to those calculated using the methods based on Wooding's equation. This suggests that measuring S at two or more different potentials, using the disc permeameter, would provide a very rapid method for characterizing the hydraulic properties of soils, provided the soil is fine textured and S can be measured either from the early-time behavior or using the method of Warrick.

7. El-Kadi, A.I. On Estimating the Hydraulics Properties of Soil, Part 1. Comparison Between Forms to Estimate the Soil-Water Characteristic Function. *Advances in Water Resources*, May 1985 (1985).

The hydraulic properties of soil include the soil-water characteristic function [h(θ) in which θ is water content and h is pressure head (suction)] and the hydraulic conductivity function [K(θ) or K(h)]. These functions are essential to the solution of unsaturated groundwater flow problems. A number of empirical and semi-empirical forms have been proposed in the literature to estimate these functions. The present paper employs a nonlinear least-square analysis for comparison

between some of the available forms, using a large number of experimental measurements of h(θ) for different classes of soil. Suitability of the forms for predicting the hydraulic conductivity function is examined. In the absence of accurate measurements, the paper facilitates modeling providing estimates for the parameters of the soil-water characteristic function.

8. Ghosh, R.K. Estimation of Soil-Moisture Characteristics from Mechanical Properties of Soils. *Soil Science*, 130(2):60-63 (1980).

   Soil-moisture characteristics can be established directly from the physical properties of soils and a single measurement of water potential, ψ, in bars, at some moisture content, θ, on a volume basis. The method is applicable for those soils for which the ψ - θ relations can be expressed as $\psi = \psi_e (\theta/\theta_0)$-β, where ψ_e is the air-entry water potential in bars, θ_0 is the saturated water content in volume per volume, and β is an empirically determined constant. The procedure is useful when a reliable soil moisture characteristic curve is not available. We noted considerable agreement between estimated values using this procedure and those experimentally determined and published for eight soils.

9. Gupta, S.C. and W.E. Larson. Estimating Soil Water Retention Characteristics from Particle Size Distribution, Organic Matter Percent, and Bulk Density. *Water Resources Research*, 15(6):1633-1635 (1979).

   Regression models are presented for estimating soil-water-retention curves from particle-size distribution, percentage of organic matter, and bulk density. Models were developed from the measured soil-water-retention curves of artificially packed cores (7.6 X 7.6 cm) of 43 soil materials. These soil materials included 13 agricultural soils. Curves predicted with these models approximated reasonably well the measured water retention of 61 Missouri soils. Because conventional methods of obtaining retention curves are expensive and time consuming, these equations will be valuable for modeling salt and water flow in soils and for estimating available water capacities.

10. Hills, R.G., *et al.* Modeling One-Dimensional Infiltration into Very Dry Soils 2. Estimation of the Soil Water Parameters and Model Predictions. *Water Resources Research*, 25(6):1271-1282 (1989).

    Using a water content based one-dimensional finite difference algorithm, we model infiltration into a 6-m-lysimeter column containing alternating layers of air dried clay loam and sand. We use van Genuchten's equation to model the h-θ relationship and both Campbell's and Mualem's equations to model the K-θ relationship. Several sets of model parameters are estimated using data generated from various combinations of laboratory drainage experiments, laboratory measurements of saturated hydraulic conductivity, lysimeter observations of initial conditions, and lysimeter observations of water redistribution. We find that predictions of infiltration based on field redistribution data give the best agreement with the observed infiltration into the lysimeter. We also find that the use of Campbell's K-θ relationship results in closer agreement between the infiltration model predictions and the lysimeter observations than does the use of Mualem's K-θ equation. The results of this study show that infiltration model predictions for a carefully controlled field scale lysimeter are very sensitive to the field and laboratory techniques used to estimate the soil water parameters.

11. Hussen, A.A. and A.W. Warrick. Alternative Analyses of Hydraulic Data From Disc Tension Infiltrometers. *Water Resources Research*, 29(12):4103-4108 (1993).

Hydraulic conductivity values were compared based on alternative analyses of data from disc tension infiltrometers. The first method was based on a single disc and tension and depends on the estimate of sorptivity and steady state flow. A second method used steady state flow measurements for two different disc radii, 52 and 118 mm. A third method used a single disc with multiple tensions from which steady state flow was obtained at three or more tensions which could be at the same location. A fourth method used a single disc with two tensions from which steady state flow was obtained at two tensions. Finally, values based on soil cores were compared. The results show reasonable agreement between methods for the hydraulic conductivity with the largest differences for data collected for zero tension. For the most part, there were no significant differences in hydraulic conductivity due to the disc infiltrometer radius. A single-disc method with multiple tensions (more than 3 points) and large disc radius gave results which were the most stable, accurate, and repeatable.

12. Klute, A. The Determination of the Hydraulic Conductivity and Diffusivity of Unsaturated Soils. *Soil Science*, 113(4):264-276 (1972).

Quantitative application of the theory of unsaturated flow to field or laboratory flow systems requires knowledge of the hydraulic conductivity and water retention characteristics of the soils involved. The purpose of this paper is to survey the various methods available for the measurement of hydraulic conductivity and diffusivity and to identify the principles, advantages, and disadvantages of each. Space will not permit a complete description of each method. The reader is advised to consult the original references for more information on any given method of interest.

13. Kool, J.B., J.C. Parker, and M.T. van Genuchten. Parameter Estimation for Unsaturated Flow and Transport Models—A Review. *Journal of Hydrology*, 91:255-293 (1987).

This paper reviews the current status of parameter estimation techniques and their utility for determining key parameters affecting water flow and solute transport in the unsaturated (vadose) zone. Historically, hydraulic and transport properties of the unsaturated zone have been determined by imposing rather restrictive initial and boundary conditions so that the governing flow and transport equations can be inverted by analytical or semi-analytical methods. Contrary to these direct methods, parameter estimation techniques do not impose any constraints on the model, on the stipulation of initial and boundary conditions, on the constitutive relationships, or on the treatment of inhomogeneities via deterministic or stochastic representations. While parameter estimation analyses of subsurface saturated flow are increasingly common, their application to unsaturated flow and transport processes is a relatively new endeavor. Nevertheless, a number of laboratory and field applications currently exist that show the great potential of parameter estimation techniques for improved designs and analyses of vadose zone flow and transport experiments. Several practical examples for determining unsaturated soil hydraulic functions and various transport parameters are presented, and advantages and limitations of the estimation process are discussed. Specific research areas in need of future investigation are outlined.

14. Malik, R.S., C. Laroussi, and L.W. De Backer. Physical Components of the Diffusivity Coefficient. *Soil Sci. Soc. Am. J.*, 43(4):633-637 (1979).

Isothermal horizontal infiltration experiments were conducted in 53-63 µm, 74-88 µm, or 105-125 µm glass bead fractions with water, 6% ethyl alcohol water solution, 20% ethyl alcohol water solution, ethyl alcohol, and methyl alcohol. From these experiments, the diffusivity coefficient was determined using the Bruce and Klute (1956) procedure and D(θ) was established for the fluid content, θ, from 0% to saturation.

The D(θ) variation in the fluid content range between 3 and 30% was investigated. The functional relationships between D(θ) and the properties of the fluid, the geometry and the solid matrix were investigated. The physical components of D(θ) were determined using:

$$D(\theta) = (r^{*2} / R^{*2} \bullet \sigma \cos \alpha / \mu - 4.10^{-4} \Delta H_v + 5) \bullet 10^{-2} \theta$$

where r* and R* are the mean radius of pores and beads, respectively, σ, μ and ΔH_v. are the surface tension, viscosity and molar heat of vaporization of the fluid, respectively, and α is the fluid-solid contact angle.

The physical significance of the diffusivity coefficient given by Laroussi and De Backer (1975) defining this coefficient as the ease with which the fluid particles spread from a wetting front, is clarified and confirmed.

15. Marion, J.M., *et al.* Evaluation of Methods for Determining Soil-Water Retentivity and Unsaturated Hydraulic Conductivity. *Soil Science*, 158(1):1-13 (1994).

The transport of dissolved contaminants through the vadose zone is a major source of soil and groundwater contamination. Soil hydraulic properties must be determined to accurately describe water and contaminant transport and potential environmental impacts. Comparisons were made of three field and three laboratory methods for estimating soil-water retention, θ(ψ) and unsaturated hydraulic conductivity functions K(θ). Instrumentation was installed in 36 field plots and two redistribution cycles were conducted. Field data obtained from each cycle were utilized in three outflow-based field methods: (i) instantaneous profile method (ii) Libardi's method and (iii) a nonlinear least squares approach. Undisturbed soil cores were extracted from 24 field plots at six depths and used in laboratory tests. Techniques consisted of (i) a multi-step outflow approach (a) coupled with (a) "inverse methodology" for transient conditions and (b) a least-squares approach for equilibrium conditions and (ii) a particle size distribution model. Parametric models were coupled with the modeling efforts. The results obtained by the in situ instantaneous profile method for both soil hydraulic functions were considered to hold the greatest validity. However the multi-step outflow methods produced feasible θ(ψ) curves and the inverse methodology was time efficient. Libardi's method for determining K(θ) relationships was accurate at deep profile depths but failed at shallow ones.

16. McCuen, R.H., W.J. Rawls, and D.L. Brakensiek. Statistical Analysis of the Brooks-Corey and the Green-Ampt Parameters Across Soil Textures. *Water Resources Research*, 17(4):1005-1013 (1981).

Infiltration is a major component of the hydrologic cycle for most watersheds. Therefore, it is important to have a method that can provide infiltration estimates for soils within ungaged watersheds. Both the analysis of variance and the multivariate analysis were used to examine whether estimated Brooks-Corey and Green-Ampt parameters differ singularly or collectively across soil texture classes. The analysis indicated that the parameters for the two models examined varied collectively across soil texture classes. Mean parameter values and standard errors for soil textures are presented for possible use on ungaged watersheds.

17. Mishra, S. and J.C. Parker. Methods of Estimating Hydraulic and Transport Parameters for the Unsaturated Zone. In: *Conference on New Field Techniques for Quantifying the Physical and Chemical Properties of Heterogeneous Aquifers*, Dallas, TX, National Water Well Association (1989).

Methods of estimating hydraulic and transport parameters for the unsaturated zone are presented. Two approaches to parameter estimation are discussed. The first approach is based on inversion of the governing initial-boundary value problem for unsaturated flow and/or transport using data from transient experiments. The second approach utilizes particle size distribution data to obtain estimates of soil hydraulic properties. Example applications of the two different approaches are presented and their advantages and limitations are discussed. Procedures for identifying and quantifying sources of uncertainty in parameter estimates are examined. Ongoing research on the scale-up of parameters for applications in large-scale numerical simulations is reviewed.

18. Mishra, S. and J.C. Parker. Effects of Parameter Uncertainty on Predictions of Unsaturated Flow. *Journal of Hydrology*, 108:19-33 (1989).

This study uses first-order first and second moment analysis to evaluate the error in predictions of unsaturated flow models caused by parameter uncertainty, expressed in terms of a mean value for each parameter and an error covariance matrix. These are related to the mean and variance in the model predictions by a first-order Taylor expansion. Two applications of the methodology are presented. The first involves the estimation of parameters and their error covariance matrix from particle size distribution data for a layered field soil. These are used to predict water contents and associated errors for a static retention test and a dynamic drainage experiment. The second case involves parameter estimation using a numerical inversion method from a transient flow experiment for a hypothetical homogeneous soil. Predictions of water contents and their errors for a rainfall-redistribution event using the estimated parameters compare well with predictions corresponding to actual parameters. Uncertainty predictions with the first-order error analysis procedure also agree well with results of a Monte Carlo simulation study.

19. Mishra, S., J.C. Parker, and N. Singhal. Estimation of Soil Hydraulic Properties and Their Uncertainty from Particle Size Distribution Data. *Journal of Hydrology*, 108:1-18 (1989).

A unified approach to the estimation of soil hydraulic properties and their uncertainty from particle size distribution data is presented. Soil hydraulic properties are represented by the parametric models of Van Genuchten and/or Brooks and Corey Particle size distribution data are used to generate theoretical soil-water retention data using a modified form of the model proposed by Arya and Paris, which was calibrated in this study using a data set of 250 soil samples. Parameters in the van Genuchten model are fitted to the predicted water content—capillary pressure data by nonlinear regression methods and may be optionally converted to equivalent Brooks-Worey retention parameters using an empirical procedure. Saturated conductivity is estimated from particle size data using a modified Kozeny-Carman equation which was developed from the data set of 250 soil samples. Uncertainty in parameter estimates is evaluated using first-order error analysis methods. Application of the proposed methodology to three soils which were not in the calibration set indicated water content-capillary pressure relations can be predicted with reasonable accuracy and precision. Uncertainty in predicted saturated hydraulic conductivity will be rather large making direct measurement of this variable highly desirable.

20. Mishra, S. and J.C. Parker. On the Relation Between Saturated Conductivity and Capillary Retention Characteristics. *Ground Water*, 28(5):775-777 (1990).

A simple closed-form expression relating saturated hydraulic conductivity to the van Genuchten capillary retention model parameters is derived. Application of this equation to an experimental data set shows reasonable agreement between measured and predicted saturated conductivity values. The proposed equation provides a consistent theoretical basis for estimating both saturated and unsaturated hydraulic conductivity from statistical pore structure models.

21. Moldrup, P., *et al.* Estimation of the Soil-Water Sorptivity from Infiltration in Vertical Soil Columns. *Soil Science*, 157(1):12-18 (1994).

This paper presents a simple method for estimating the soil-water sorptivity (S) from vertical infiltration experiments at ponding conditions. The proposed estimation method is based on the Haverkamp et al. (R. Haverkamp, J.-Y. Parlange, J. L. Starr, G. Schmitz, and C. Fuentes. 1990. Infiltration under ponded conditions. 3. A predictive equation based on physical parameters. Soil Sci. 149:292-300) predictive infiltration equation. This equation is simplified, so that a simple analytical solution is obtained for the sorptivity. The new S estimation method requires measurement of cumulative infiltration versus time and independent measurement of the saturated hydraulic conductivity on the same soil cores as used for the infiltration experiment. Criteria for avoiding numerical instability when using the new estimation method are given. Use of the proposed method is illustrated on infiltration data measured on undisturbed sand cores. The method gave good and robust estimates of the soil-water sorptivity (coefficient of variation <25% for all S estimates). Horizontal and vertical infiltration tests for homogeneous packed soil columns showed good agreement between the new S estimation method and the classical Philip method for horizontal infiltration at nonponding conditions.

22. Mous, S.L.J. Identification of the Movement of Water in Unsaturated Soils: The Problem of Identifiability of the Model. *Journal of Hydrology*, 143:153-167 (1993).

The estimation of model parameters using non-linear regression techniques is one of the aspects of inverse modelling and is known as the identification problem. The solution of which may be non-unique. The main causes of this non-uniqueness are the structure of the model and the design of the input signal. It will be shown that the parameters can be estimated only if a model with different parameter values yields different output signals. A model that has this feature is called identifiable. As an example, the identification of a model for the movement of water in unsaturated soils is used. This model appears to be non-identifiable, which results in non-unique solutions.

23. Paige, G.B. and D. Hillel. Comparison of Three Methods for Assessing Soil Hydraulic Properties. *Soil Science*, 155(3):175-189 (1993).

Three methods for assessing soil hydraulic properties were conducted and their results compared for two soils in Western Massachusetts. The methods compared are: the Instantaneous Profile Method, the Guelph Permeameter, and laboratory determination using intact soil cores. The saturated hydraulic conductivity and unsaturated conductivity function, as well as the moisture retention relationship when possible, were determined and the results compared with respect to their ranges of applicability and the respective limitations of each method. We found close agreement for the moisture retention relationships determined by the instantaneous profile method and the soil cores for the ranges of pressures and moisture contents they have in common. In addition, there was also close agreement between the $K(\psi)$ relationship measured using the instantaneous profile method and that predicted using the van Genuchten and Mualem models. The field saturated conductivity results determined using the Guelph Permeameter were one to three orders of magnitude less than the saturated conductivity results determined from soil cores and those determined by the instantaneous profile method. The unsaturated $K(\psi)$ relationship using Gardner's definition of matric potential and the results from the Guelph permeameter predicted hydraulic conductivity values three to four orders of magnitude less then the other two methods at 200 cm of pressure.

24. Ragab, R., J. Feyen, and D. Hillel. Comparative Study of Numerical and Laboratory Methods for Determining the Hydraulic Conductivity Function of a Sand. *Soil Science*, 131(6):375-388 (1981).

We compared experimentally measured hydraulic conductivity-water content relationships with those predicted by three models. Sand was the medium used for this comparative study. The laboratory methods used included: instantaneous profile-internal drainage; infiltration through crust; hot air drying; pressure plate outflow; and unit gradient drainage. For the numerical approach we used the Irmay, the Jackson and the Mualem models. The moisture desorption curve and the saturated hydraulic conductivity were used as inputs. Additional parameters were estimated from the literature. We found close agreement between the $K(\theta)$ relationship measured according to the instantaneous profile method and estimates based on Irmay's equation; Jackson 's prediction method with a pore interaction exponent of 0.5; and Mualem' s model, with an effective saturation term exponent of 0.5. In addition, the predicted $K(\theta)$ relationship with the Mualem model using 0.75 as exponent of the effective saturation term, and the predicted values with the Millington Jackson 's equation (being Jackson 's model with p = 4/3) corresponded with the experimental relationship obtained by the gypsum-sand crust technique. The predicted $K(\theta)$ relationships obtained by the above models also matched the $K(\theta)$ values obtained by the hot air method in the wet range.

25. Ragab, R. and J.D. Cooper. Variability of Unsaturated Zone Water Transport Parameters: Implications for Hydrological Modelling. 2. Predicted vs. In Situ Methods and Evaluation of Methods. *Journal of Hydrology*, 148:133-147 (1993).

Two methods to estimate the saturated and unsaturated hydraulic conductivity of soil were compared in a catchment in south-west England. The catchment consists of three land use domains--arable, permanent grass and woodland. The methods used were in situ measurements using a Guelph permeameter and the predictive model of Rawls and Brakensiek, which uses percentage of sand, percentage of clay and porosity as input. In grassland and woodland no significant differences were found between the mean saturated and unsaturated hydraulic conductivity estimated by the two methods. It is suggested that a significant difference found for the arable land is due to the effect of farming operations on soil permeability. On a catchment scale there were no significant differences between the methods the results obtained using the predictive model are encouraging and worth further investigation. The advantages and disadvantages of the two methods are evaluated.

26. Ragab, R. and J.D. Cooper. Variability of Unsaturated Zone Water Transport Parameters: Implications for Hydrological Modelling. 1. In Situ Measurements. *Journal of Hydrology*, 148:109-131 (1993).

Physically based hydrological models require values of saturated and unsaturated hydraulic conductivity to solve the equations used to describe subsurface flow. Variability of these parameters within and between various land uses and soil types is an important consideration. If hydrological behaviour is to be described realistically.

A catchment in south-west England has been chosen to validate the European Hydrological System Systeme Hydrologique Europeen SHE). The catchment includes three land use domains--arable including temporary grass), permanent grassland and woodland. For the unsaturated zone component of the SHE both saturated hydraulic conductivity, K_s, and unsaturated hydraulic conductivity as a function of matric suction, $K(\psi)$, are required. The aim of this investigation

was to measure the parameters for input to the model to assess the variability of these parameters and to determine whether or not parameters should be used for each domain independently. A Guelph permeameter was used to obtain K_s and α, which describes the slope of the logarithmic unsaturated hydraulic conductivity function ($K(\psi) = K_s \exp(\alpha\psi)$) . Measurements at four depths were carried out. Domains were chosen according to land use arable land (including temporary grass), permanent grassland and woodland. Both K_s and a were found to be lognormally distributed. The K_s values were highest in the grassland and lowest in arable land. The difference between means was significant for K_s but not for α in the three domains. High values of K_s were attributed to preferential flow along slate faces and root-induced channels in the grassland and woodland, respectively, as indicated by dye tests. Finally it was concluded that, as K_s and $K(\psi)$ differ significantly between domains, different hydrological behaviour would be expected. It is therefore recommended that different parameter sets be used for modelling the individual domains.

27. Rawls, W.J., D.L. Brakensiek, and K.L. Saxton. Soil Water Characteristics. In: *1981 Winter Meeting of American Society of Agricultural Engineers, Paper No. 81-2510*, Chicago, IL, ASAE (1981).

Relationships of soil vater tension and conductivity with soil water content are needed to quantify plant available water and to model the movement of water and solutes in and through soils. Field and laboratory measurement of these hydraulic soil properties ia very difficult, laborious, and costly. To provide the best estimates possible from previous analyses, a comprehensive search of the literature and data sources for hydraulic conductivity and related soil-water data was made in 1978. From this search, data for 1,323 soils with about 5,350 horizons from 32 states were summarized. Reported here are summaries of the soil profile descriptions, soil textures, particle size distributions, organic carbon contents, bulk densities, selected chemical data, hydraulic conductivities, soil water retention data, sample location, and the specific reference or data source. The Brooks and Corey equation parameters, soil water retention volumes at 0.33 bar and 15 bar, and saturated conductivities for the major soil textures classes are reported. Relationships for predicting water retention volumes for particular tensions and saturated hydraulic conductivities based on soil properties are also presented.

28. Rawls, W.J. and D.L. Brakensiek. Estimating Soil Water Retention from Soil Properties. *Journal of Irrigation and Drainage, Proceedings of the American Society of Civil Engineers*, 108(IR2):166-171 (1982).

29. Rawls, W.J. and D.L. Brakensiek. Prediction of Soil Water Properties for Hydrologic Modeling. In: *Symposium on Watershed Management*, ASCE (1985).

Water flow in soils can be characterized for many boundary and initial conditions by solving governing differential equations. Thus, a physically consistent means of quantifying water flow in soils in terms of the soil properties governing the movement of water and air exists. There are several reasons why this state-of-the-art technology is not yet fully utilized. One reason may be complexity and expense of computer-based numerical solutions. However, a more important reason is the difficulty of obtaining the required inputs which are the relationships between matric potential and hydraulic conductivity as a function of soil water content. Approximate water flow models based on physical principles or empirical results may simplify the computational requirements, however, inputs are still required. Therefore, it is the objective of this paper to present procedures to (1) estimate input parameters for the Brooks-Corey, Campbell and Van Genuchten soil water retention and hydraulic conductivity functions based on readily available soil properties, and (2) estimate infiltration parameters.

30. Reynolds, W.D. and D.E. Elrick. <u>A Method for Simultaneous In Situ Measurement in the Vadose Zone of Field-Saturated Hydraulic Conductivity, Sorptivity and the Conductivity-Pressure Head Relationship</u>. *Ground Water Monitoring Review GWMRDU*, 6(1):84-95 (1986).

The Guelph Permeameter (GP) method for simultaneous, in situ measurement in the vadose zone of field-saturated hydraulic conductivity (K sub fs), sorptivity (S) and the conductivity-pressure head relationship (K(theta)) is described and discussed. The method involves measuring the steady-state liquid recharge, Q, necessary to maintain a constant depth of water, H, in an uncased, cylindrical well of radius, a, finished above the water table. An 'in-hole' Mariotte bottle device is used to maintain H and to measure Q. Step-by-step procedures with example calculations are given for obtaining K sub fs, S and K(theta). Techniques for assessing the results are also given. The GP method will (theoretically) yield simultaneous, in situ measurements of K sub fs and K(theta) for infiltration of any wet liquid, and yields a 'point' measurement requiring replication. The GP apparatus is inexpensive, simple and easily operated by one person. The method can be used to obtain vertical profiles of K sub fs, S and alpha, requiring less time per measurement than most other methods.

31. Ross, P.J. and K. Smettem R.J. <u>Describing Soil Hydraulic Properties with Sums of Simple Functions</u>. *Soil Sci. Soc. Am. J.*, 57:26-29 (1993).

Simple functions do not adequately describe the hydraulic properties of many field soils, particularly those with substantial macroporosity. By considering the soil pore-size distribution $f(\psi) = \Sigma^N_{\iota=1} \phi_i f_i(\psi)$ corresponding to the effective saturation $S(\psi) = \Sigma^N_{\iota=1} \phi_i S_i(\psi)$, where ψ is matric pressure head, the ϕ_i are fractions of effective porosity, the $S_i(\psi)$ are simple water retention functions in common use, and $f_i(\psi) = S_i(\psi)$, we show that the relative hydraulic conductivity according to the Mualem model is $K_r(\psi) = S^p[\Sigma^N_{\iota=1} \phi_i g_i(\psi) / \Sigma^N_{\iota=1} \phi_i g_i(O)]^2$, where $g_i(\psi) = \int^{\psi}_{-x} \psi^{-1} f_i(\psi)\, d\psi$ and p is a pore interaction index. If the pores of the distributions do not interact, the appropriate relation is $K(\psi) = \Sigma^N_{\iota=1} K_{si} K_{ri}(\psi)$ where K_{si} is the saturated conductivity of distribution i and $K_{ri} = S^p [g_i(\psi)/g_i(0)]^2$. We note that the van Genuchten function $S(\psi) = [1 + (-\alpha\psi)^n]^{-m}$ with the restriction $m = 1 - 1/n$ leads to an infinite slope $K'(\psi)$ at $\psi = O$ unless $n \geq 2$, which is unrealistic for field soils if a wide range of matric pressure heads is considered. Hydraulic conductivity near saturation is often expressed as $K'(\psi) = K_s \exp(a\psi)$. We introduce the function $S(\psi) = (1 - \alpha\psi) \exp(a\psi)$, which gives, according to Mualem's model, a conductivity $K(\psi) = K_s(l - a\psi)p \exp[(p + 2)\alpha\psi]$ that approximates $K_s \exp(a\psi)$ near saturation if $a = 2\alpha$ and is exactly equal if $p = O$. As an example, a function using this model for one pore-size distribution and the van Genuchten model for the other was compared with a function using two van Genuchten distributions. The latter gave a slightly improved fit to water content and conductivity data for an aggregated soil.

32. Ross, P.J. and J.-Y. Parlange. <u>Comparing Exact and Numerical Solutions of Richards' Equation for One-Dimensional Infiltration and Drainage</u>. *Soil Science*, 157(6):341-344 (1994).

Analytical solutions to Richards' equation are valuable in testing numerical models of water movement in soils, but few such solutions are available, especially for general soil hydraulic properties. We present such a solution, valid for restricted boundary conditions during infiltration and drainage, and illustrate its use by comparing numerical and analytic solutions. In the case of infiltration the solution is based on an earlier result which is extended here to describe drainage from soil with a uniform initial water content. Accurate numerical solutions could be obtained with a coarse spatial grid of eleven points.

33. Ross, P.J. and J.-Y. Parlange. <u>Investigation of a Method for Deriving Unsaturated Soil Hydraulic Properties from Water Content Profiles</u>. *Soil Science*, 157(6):335-340 (1994).

Hydraulic properties of unsaturated soil profiles are needed for quantitative analysis of water movement. A recently proposed method of estimating some of these properties from water content profiles after drainage was examined for a uniform soil satisfying certain constraints that allowed a new exact analytical solution to be obtained. A new analytical and numerical examination of the method, assuming correct property functional forms and boundary conditions, established that it can give good estimates of unsaturated hydraulic conductivities. If a matric potential value is available at one depth in the profile, matric potentials at other depths can also be estimated accurately. The method seemed to work reasonably well even when assumed property functions and boundary conditions had an incorrect form, although hydraulic conductivities were then in error up to 25 percent. This seemingly large error reflects the difficulty in measuring a highly variable dynamic property, like conductivity, in soils.

34. Satterwhite, M.B. Evaluating Soil Moisture and Textural Relationships Using Regression Analysis. U.S. Army Corps of Engineers, ETL-0226 (1980).

Soil moisture and textural conditions are described for 179 soil samples from an arid to semiarid climate. Stepwise multiple regression analysis of these data produced four regression equations that related (1) the percent sand and clay and (2) the percent fines, with the percent soil water held at 0.33 bar (FC) and the 15 bar (WP) potentials. Evaluation of these equations showed no differences between the estimates at the 0.33 bar potential using either the percent sand and clay or the percent fines. Better estimates for the WP were obtained when the percent sand and clay were used instead of the percent fines. The differences between the estimated soil moisture at FC or WP varied less than 30 percent from the measured soil moisture values for 161 (90 percent) of the 179 soil samples. The differences between the estimated and the measured soil moisture values were not significant at the 95 percent level of confidence.

The regression equations provide a method by which the potential percent soil water held at the FC or WP can be estimated from soil textural data. The accuracy and precision of the results of applying these equations to soils of other areas has not been determined. It would seem, however, that they would be applicable in those instances where only general working estimates are needed.

35. Shamsai, A. and N. Sitar. Method for Determination of Hydraulic Conductivity in Unsaturated Porous Media. *Journal of Irrigation and Drainage Engineering*, 117(1):64-78 (1991).

A new procedure for in situ determination of saturated hydraulic conductivity of an initially unsaturated soil is proposed. The procedure is based on the measurement of the rise of a groundwater mound in an initially unsaturated zone above an impervious layer receiving uniform recharge from an infiltrometer. In this paper analytical solutions of equations describing the growth of a groundwater mound above an impervious layer are compared with laboratory and field experiments to demonstrate the applicability of the method to the determination of saturated hydraulic conductivity of soils in shallow subsurface.

36. Springer, E.P. and T.W. Cundy. Field-Scale Evaluation of Infiltration Parameters from Soil Texture for Hydrologic Analysis. *Water Resources Research*, 23(2):325-334 (1987).

Recent interest in predicting soil hydraulic properties from simple physical properties such as texture has major implications in the parameterization of physically based models of surface runoff. This study was undertaken to (1) compare, on a field scale, soil hydraulic parameters predicted from texture to those derived from field measurements and (2) compare simulated

overland flow response using these two parameter sets. The parameters for the Green-Ampt infiltration equation were obtained from field measurements and using texture-based predictors for two agricultural fields, which were mapped as single soil units. Results of the analyses were that (1) the mean and variance of the field-based parameters were not preserved by the texture-based estimates, (2) spatial and cross correlations between parameters were induced by the texture-based estimation procedures, (3) the overland flow simulations using texture-based parameters were significantly different than those from field-based parameters, and (4) simulations using field-measured hydraulic conductivities and texture-based storage parameters were very close to simulations using only field-based parameters.

37. Stolte, J., *et al.* Comparison of Six Methods to Determine Unsaturated Soil Hydraulic Conductivity. *Soil Sci. Soc. Am. J.,* 58:1596-1603 (1995).

Knowledge of soil hydraulic properties is required for soil-water flow models. Although many studies of individual methods exist, comparisons of methods are uncommon. Therefore, we compared application ranges and results for six laboratory methods for determining hydraulic conductivity or diffusivity on eolian sand, eolian silt loam, marine sandy loam, and fluviatile silt loam. The methods, hot air, sorptivity, crust, drip infiltrometer, Wind's evaporation, and one-step outflow, fall into three groups: (i) those that only yield a conductivity curve; (ii) those that yield a simultaneous estimate of conductivity, diffusivity, water content, and pressure head; and (iii) those that yield a diffusivity curve. Diffusivities were converted to conductivities with a water retention curve. One main difference between the methods was the pressure head-water content range. Despite the large differences between the methods, the results for the first two groups tended to be similar. The results of the third group did not match well with those of the first two. It proved difficult to compare these methods correctly due to hysteresis.

38. Tseng, P.-H. and W.A. Jury. Simulation of Field Measurement of Hydraulic Conductivity in Unsaturated Heterogeneous Soil. *Water Resources Research,* 29(7):2087-2099 (1993).

In this study the procedures used in common methods for field measurement of unsaturated hydraulic conductivity in heterogeneous soil were simulated through numerical experiments. It was assumed that the heterogeneity of the field soil can be described by a scaling factor δ which is treated as a lognormally distributed second-order stationary random variable in space. After the hypothetical random field was generated using property variability and correlation length scales typical of those observed in field measurements it was used to test various field measurement methods currently in practice. One detailed and three simplified unsteady drainage flux methods were used to estimate the hydraulic conductivity under plausible field measurement conditions. In addition the soil water retention data were used to predict hydraulic conductivity from a pore-size distribution model. It was found that even under the assumption that there were no reading or instrument errors other sources of errors can still be created due to the limitations of the observation techniques. These error sources include limited number of readings in space and time separation distances between the readings of water content and pressure head and volume-averaged readings rather than point estimates with common instrumentation. Under the conditions studied in the simulations the detailed method of estimation produced reasonable estimates of hydraulic conductivity, but only when very dense measurements of water content and matric potential were used in the estimation. None of the approximate methods yielded accurate measurements of the true hydraulic conductivity of the field region measured.

39. Valiantzas, J.D. Analysis of Outflow Experiments Subject to Significant Plate Impedance. *Water Resources Research,* 26(12):2921-2929 (1990).

A new method is developed for the determination of the diffusivity-water content relationship from pressure-outflow experiments when the impedance of the supporting porous plate has to be taken into account. The classic one-step outflow procedure for the determination of soil water diffusivity does not use the early linear part of the outflow versus the square root of time, t, curve, where the effect of plate impedance is not negligible. The present method can be applied also to the linear part of the outflow curve which is considerably extended when the effect of the plate impedance is significant. The suggested procedure requires additional information, namely, the retention curve and the plate impedance value. The method was tested by simulating outflow experiments with significant plate impedance and using data obtained to calculate diffusivities, as well as by analyzing experimental outflow data for a real soil.

40. van Genuchten, M.T., F.J. Leij, and S.R. Yates. The RETC Code for Quantifying the Hydraulic Functions of Unsaturated Soils. U.S. EPA, Robert S. Kerr Environmental Research Laboratory, EPA/600/S2-91/065 (1991).

This summary describes the RETC computer code for analyzing the soil water retention and hydraulic conductivity functions of unsaturated soils. These hydraulic properties are key parameters in any quantitative description of water flow into and through the unsaturated zone of soils. The program uses the parametric models of Brooks-Corey and van Genuchten to represent the soil water retention curve, and the theoretical pore-size distribution models of Mualem and Burdine to predict the unsaturated hydraulic conductivity function from observed soil water retention data. Some of the analytical expressions used for quantifying the soil water retention and hydraulic conductivity functions are presented. A brief review is also given of the nonlinear least-squares parameter optimization method used for estimating the unknown coefficients in the hydraulic models. The program may be used to predict the hydraulic conductivity from observed soil water retention data assuming that an estimate of the saturated hydraulic conductivity is available. The program also permits one to fit analytical functions simultaneously to observed water retention and hydraulic conductivity data, or to predict the hydraulic curves from specified model parameters if no retention and conductivity data are available. The actual report, summarized here, serves as both a user manual and reference document.

This Project Summary was developed by EPA's Robert S. Kerr Environmental Research Laboratory, Ada, OK, to announce key findings of the research project that is fully documented in a separate report of the same title.

41. Vereecken, H., J. Maes, and J. Feyen. Estimating Unsaturated Hydraulic Conductivity from Easily Measured Soil Properties. *Soil Science*, 149(1):1-12 (1990).

We measured hydraulic conductivity (saturated and unsaturated) on 127 soil cores, which were taken in different horizons of a wide variety of Belgian soil series. The hot air method (Arya et al. 1975) and the crust method (Bouma et al. 1983) were combined to obtain the complete range of hydraulic conductivity from saturation to air-dry.

The textural composition in nine fractions, the organic carbon content, and the dry bulk density were determined for each of the sample horizons as well. Four different empirical models were evaluated on their performance in describing the measured hydraulic conductivity curves. The model parameters were estimated by linear and nonlinear regression techniques. It is concluded that the Gardner equation (1958) with three parameters best described the hydraulic conductivity for the given soils.

Regression equations for estimating the Gardner parameters were established from simple soil properties, such as soil texture, carbon content, bulk density, and saturated hydraulic conductivity. We found that the three parameters can reasonably well be estimated from the textural composition and the saturated hydraulic conductivity. A one-dimensional sensitivity analysis indicates that the n parameter, representing the slope of the hydraulic conductivity pressure head relation in log-log scale, is most sensitive.

42. Warrick, A.W. Soil Water Diffusivity Estimates from One-Dimensional Absorption Experiments. *Soil Sci. Soc. Am. J.*, 58:72-77 (1992).

A method to estimate soil water diffusivity from experimental absorption data was explored. The purpose was to directly match measured sorption curves (water content vs. distance or the Boltzmann transform variable) to scaled forms of the solutions and thus provide consistent hydraulic properties. Evaluation of such parameters is critical to prediction of water and solute transport within the vadose zone. The method is relevant to several, if not most, of the commonly used hydraulic functional relationships. Generally, the results gave two independent relationships of the parameters. An example was considered and calculations performed for three alternative models. The best choice of each model was easily found and the evaluated parameters were shown to be consistent. The soil water diffusivities were compared for the estimated parametric relationships. Individual models provide alternative shapes for the diffusivity function, but tend to converge for the wetter range of water contents.

43. Weaver, J.W., *et al.* Evaluating Parameter Estimation Techniques Applied in Vadose Zone Modeling. In: *First International Ground Water Modeling Conference*, Golden, CO (1993).

The paper discusses vadose zone parameter estimation techniques that may be used in lieu of measured parameter values. In particular, two different data bases that provide generic parameter estimates are reviewed. The effects of parameter variability on vadose zone NAPL flow are illustrated through usage of the Kinematic Oily Pollutant Transport (KOPT) model. The model simulated a laboratory experiment where gasoline was released into a one-meter long chromatography column. The model input parameters were measured, and the position of the leading edge of the gasoline was recorded as a function of time from the experiment. Simulations that use measured or tabulated parameter sets were compared with the experimental data, which represents the correct response of the system to the gasoline input. To assess the impact of using tabulated parameters Monte Carlo simulations were performed using correlated parameter sets generated from the tabulations. These results show that wide distributions of the parameters are required to capture likely variability in input parameter values.

44. Zhang, R. and M.T. van Genuchten. New Models for Unsaturated Soil Hydraulic Properties. *Soil Science*, 158(2):77-85 (1994).

Two relatively simple models are proposed for describing the soil water retention curve. The expressions define sigmoidal or bimodal type retention functions with four or five parameters, respectively. The sigmoidal retention model may be combined with predictive pore-size distribution theories to yield closed-form equations for the unsaturated hydraulic conductivity. Parameters in the proposed hydraulic functions were estimated from observed retention data using a nonlinear least-squares optimization process. The models were tested on hydraulic data for more than 20 soils. Good agreement between predicted values and measured retention and conductivity data was found for most of the soils. The soil hydraulic models can be effectively utilized as inputs for numerical models of water flow and solute transport.