Lessons learned from analyses of two ‘benchmark’ unconfined aquifer tests

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Acknowledgment- Collaborators on initial portions of this study were Steve Garabedian and Denis LeBlanc of the MA / RI Water Science Center

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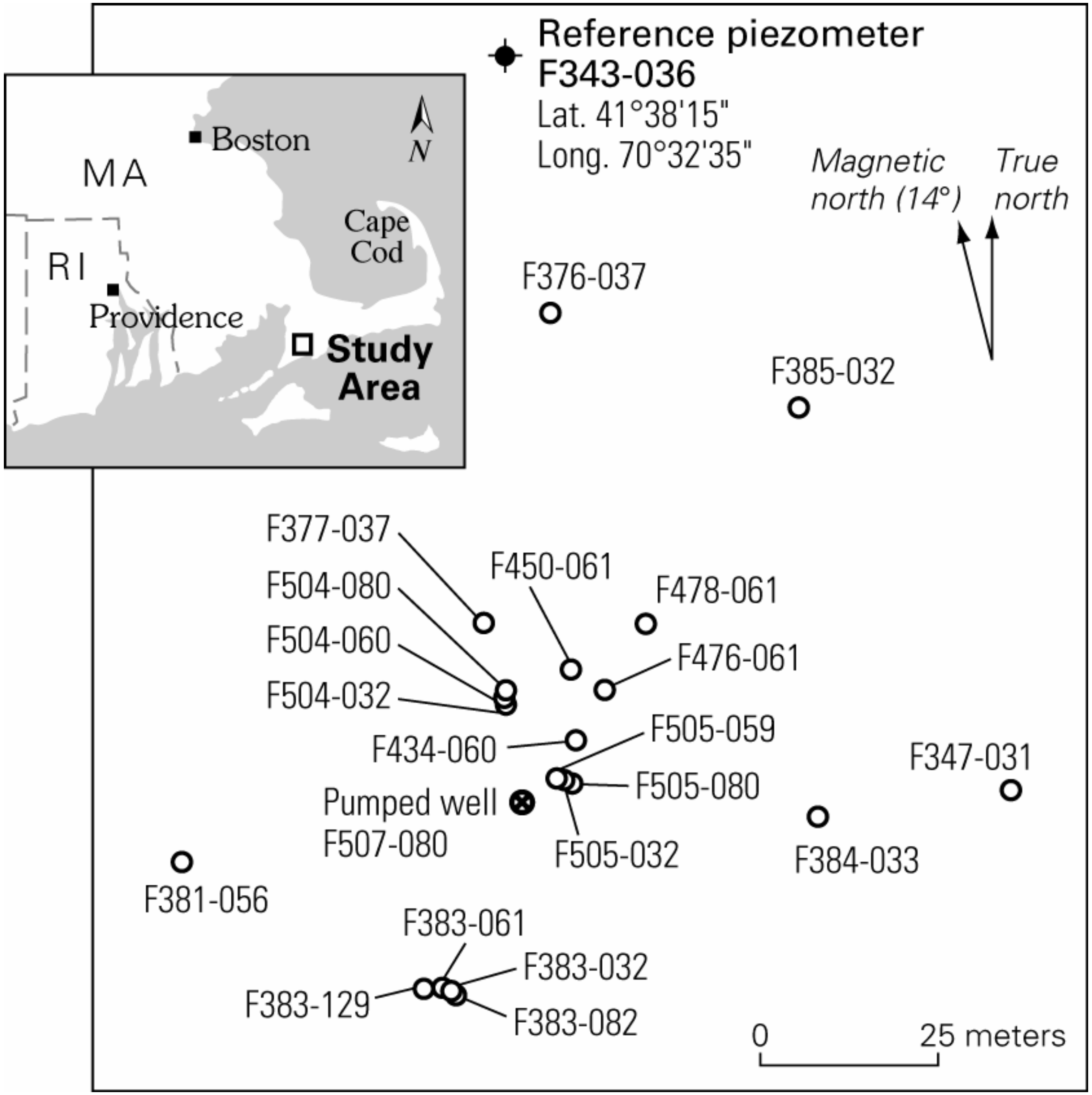
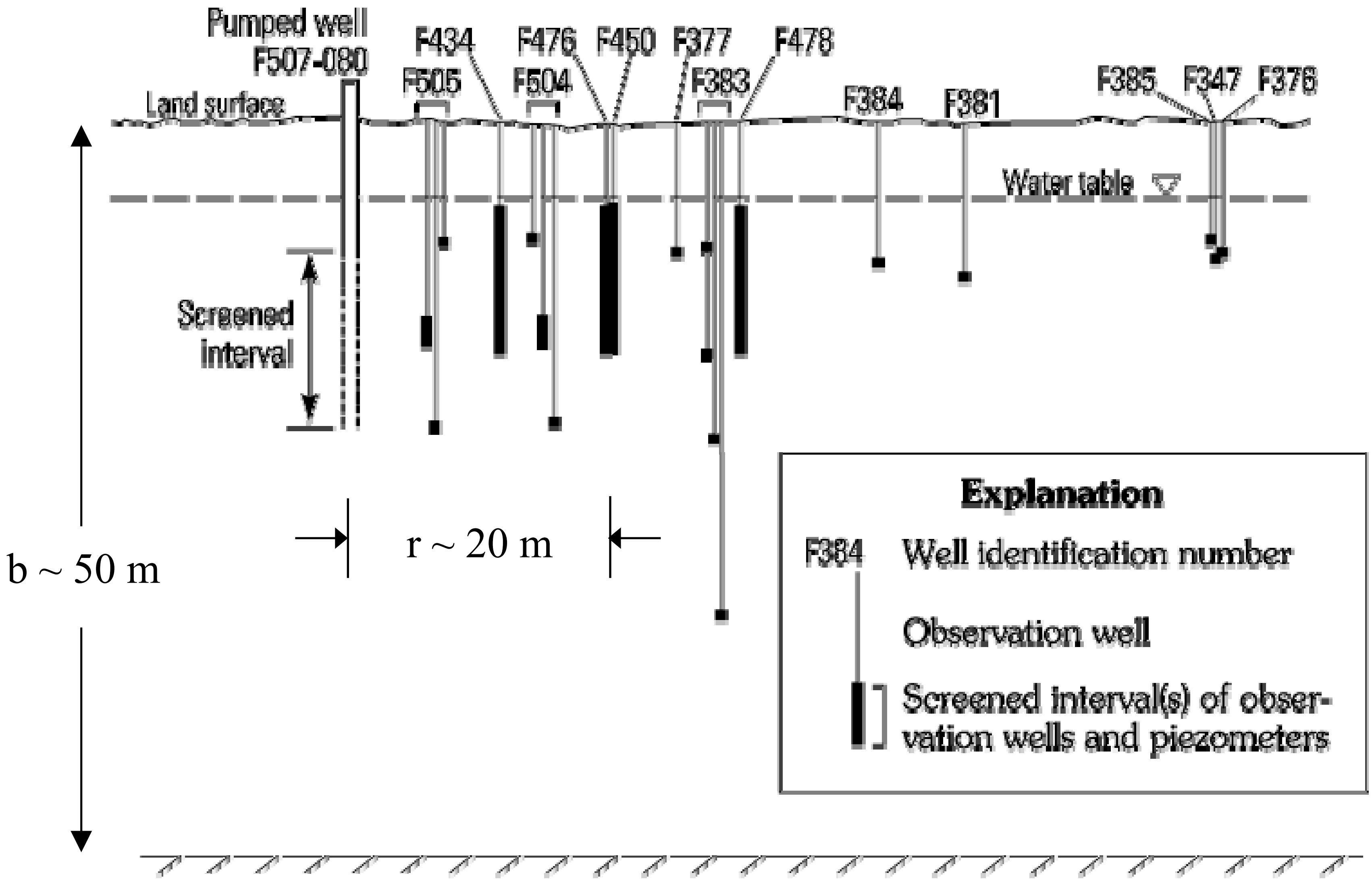
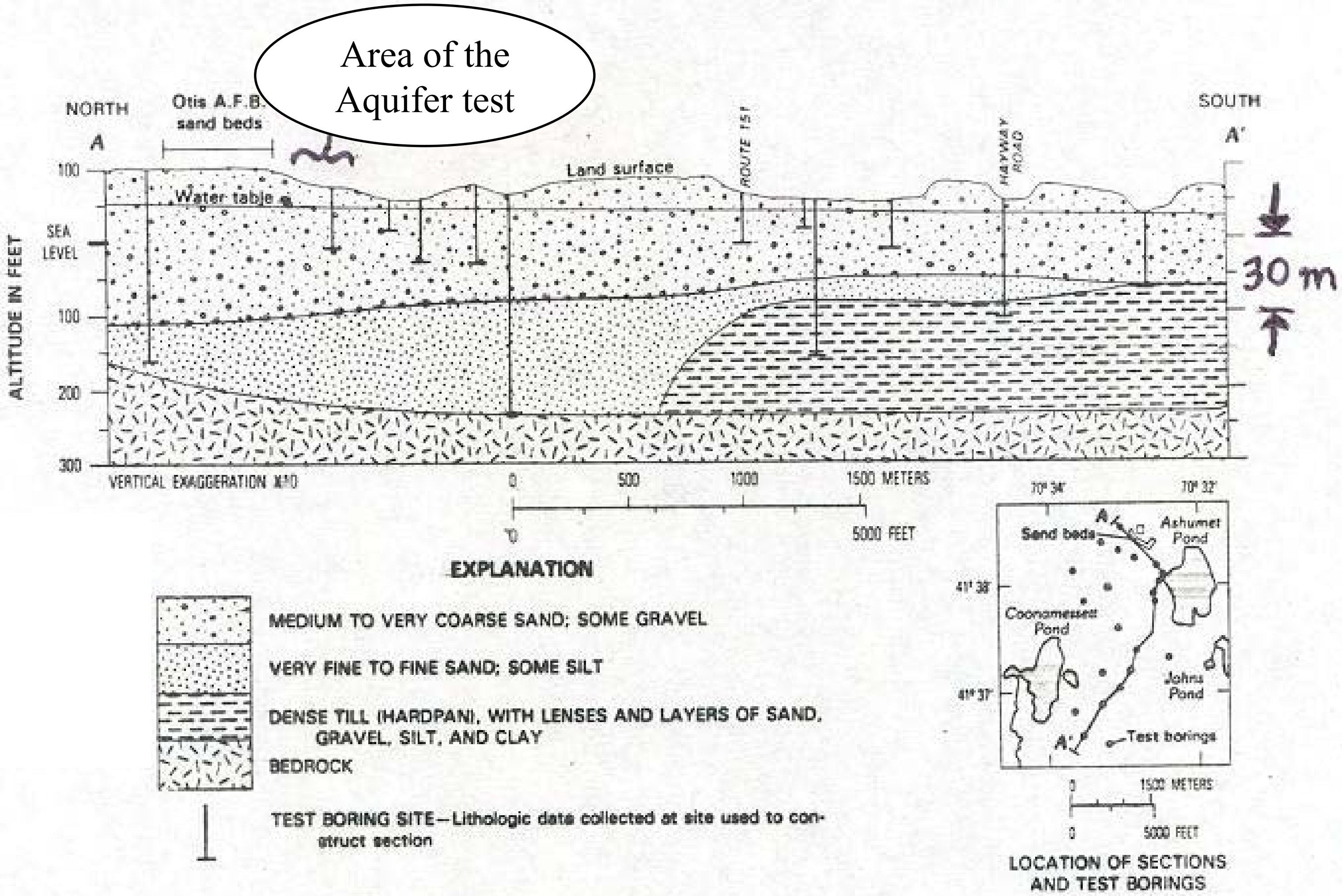
Regional location and local plan view



Vertical cross section drawn approximately to scale



Stratigraphic Section



Test Details:

72-hour, constant-rate test
Q = 1210 liters/minute

20 observation piezometers

Depth to water table ~ 6 m

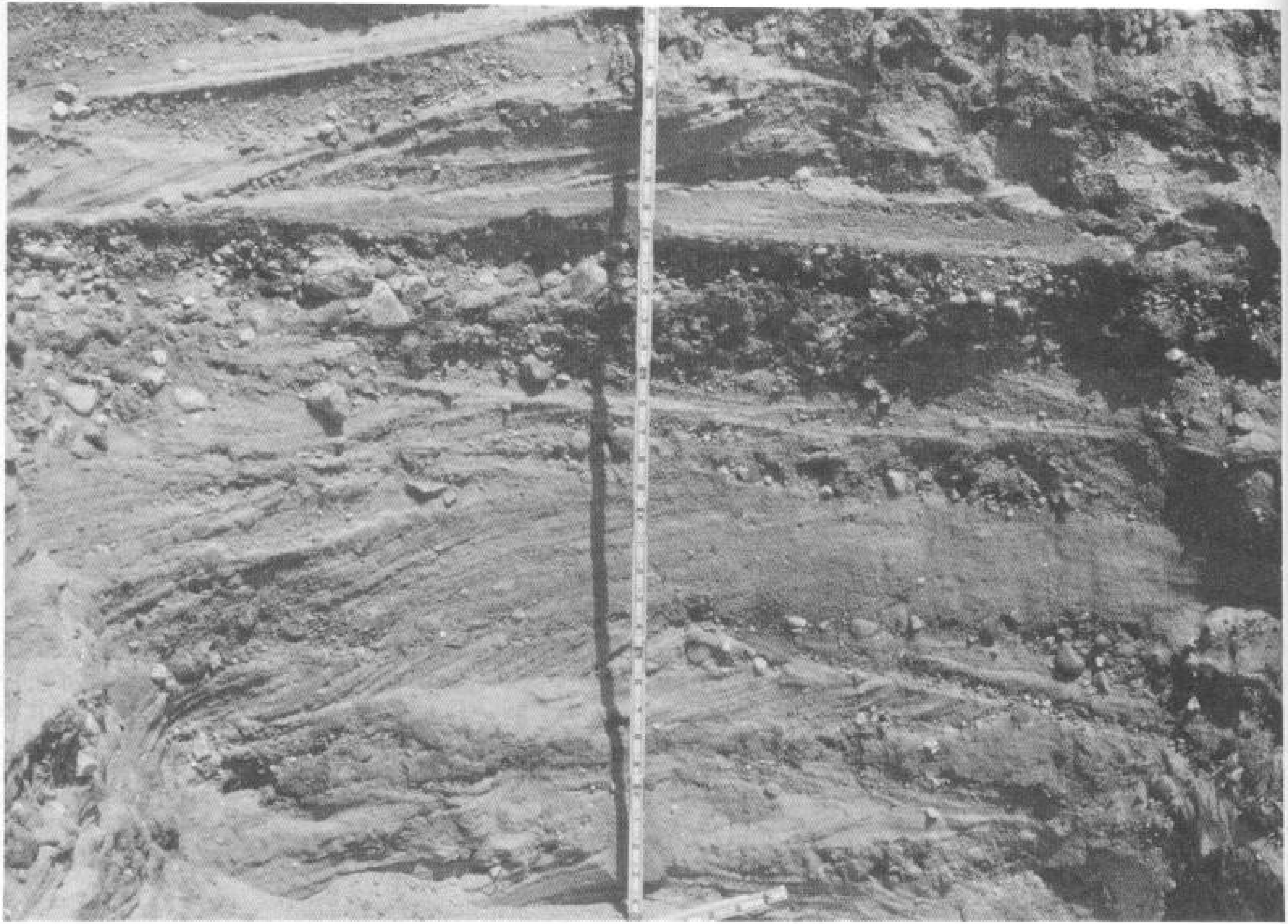


Fig. 3. Fresh exposure of sand and gravel outwash in test pit at tracer test site. Exposure in unsaturated zone about 5 m above water table. Height of section about 1 m. Location of test pit shown in Figure 4.

From: LeBlanc *et al.*, (Water Resour. Res., 1991, p. 895-910)

Hydrogeologic Characteristics

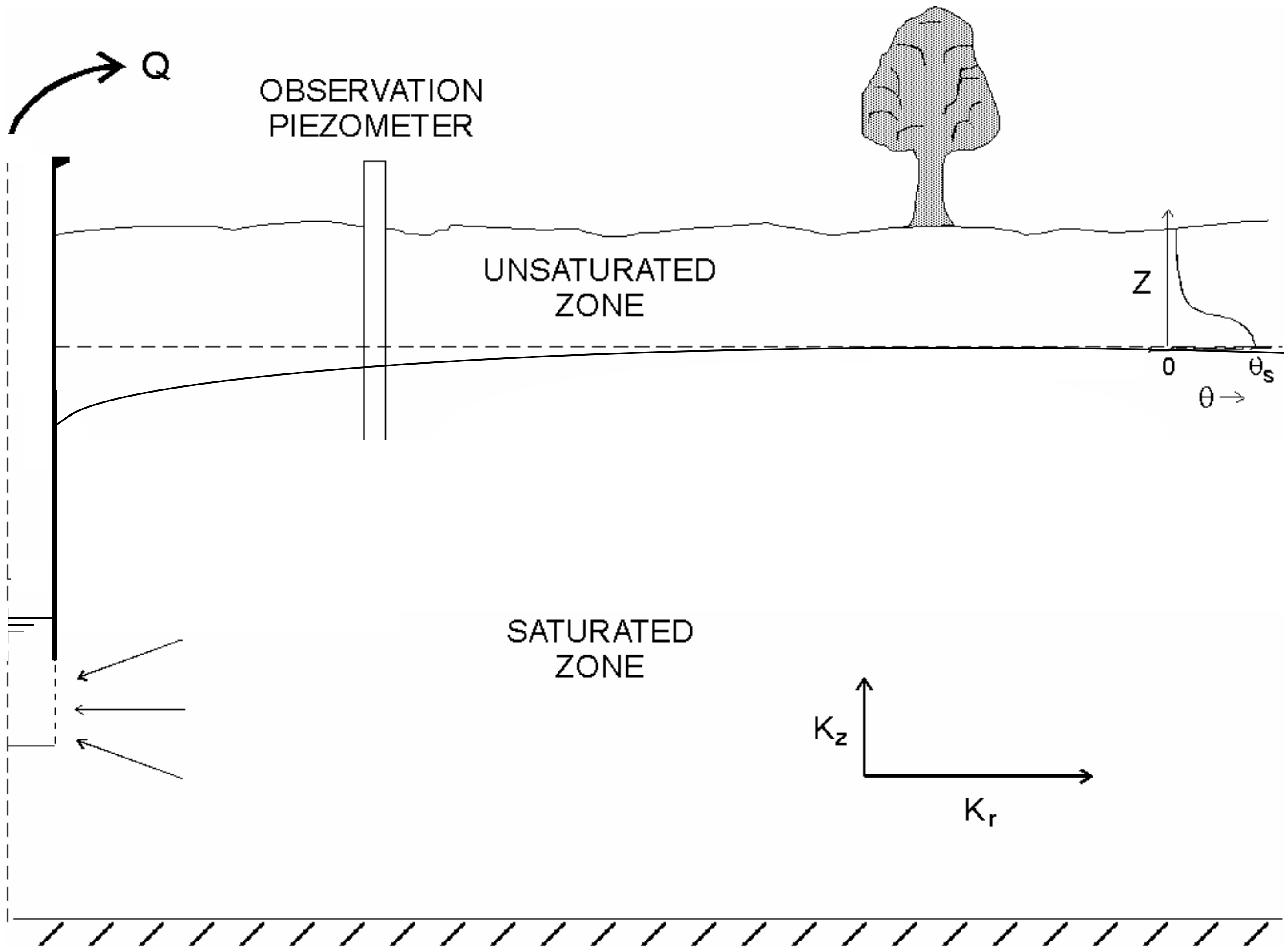
- Coarse-grained, unconsolidated, glacial-outwash deposits composed of *interbedded lenses* of sand and gravel

- *Mildly heterogeneous* (horizontal correlation scale ~ 3-8 m, vertical correlation scale ~ 0.2-0.4 m) [1]

- *Slightly anisotropic* ($K_r/K_z \sim 1.24$, where $K_r = 1.22 \times 10^{-3}$ m/s) [1]

- Underlain by very fine-grained material at ~50 m below w.t.

Schematic diagram of pumped well in a homogeneous aquifer



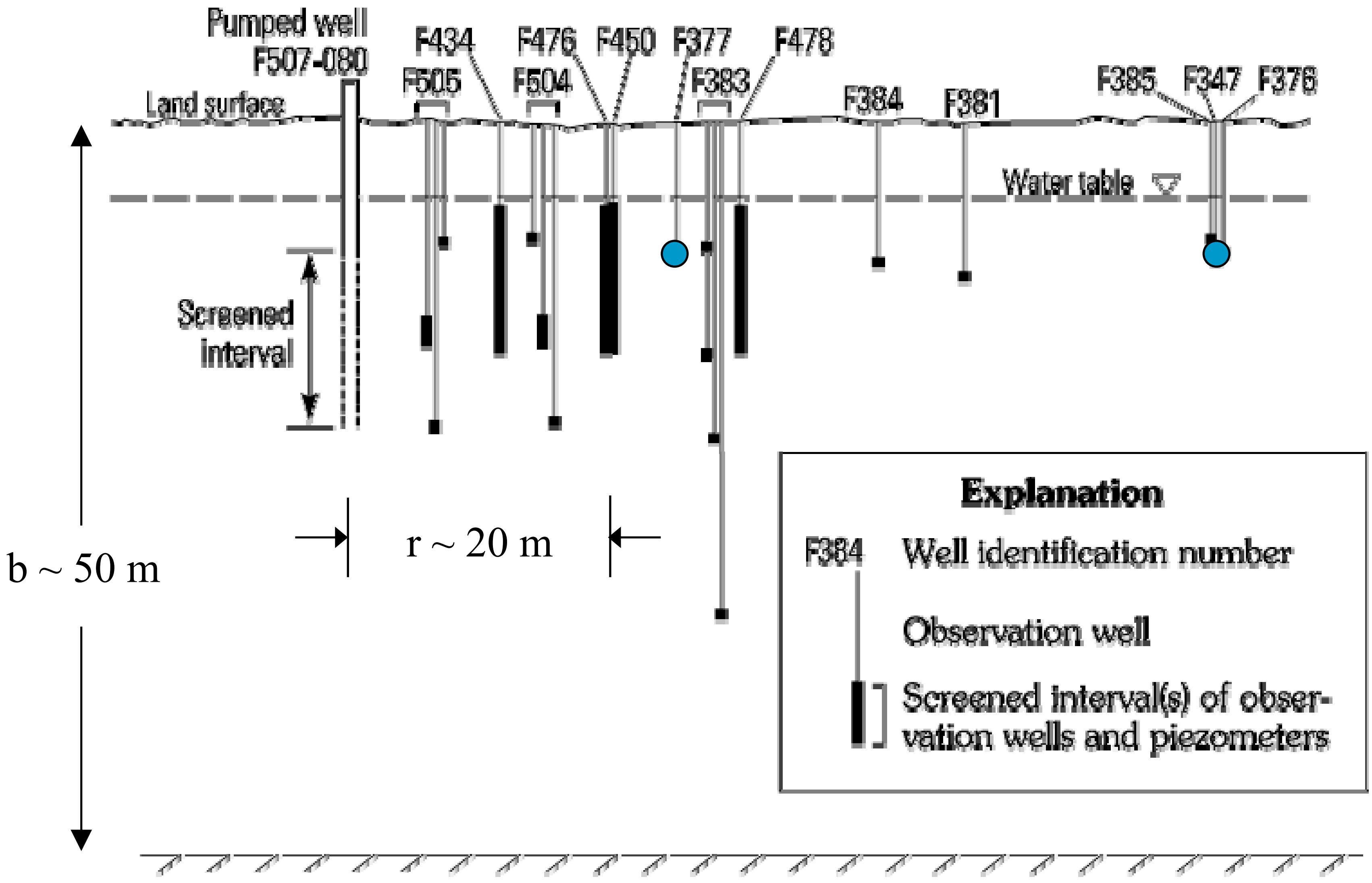
Traditional model assumptions for unconfined aquifer:

- Homogeneous aquifer
- Infinite radial extent
- Impermeable base

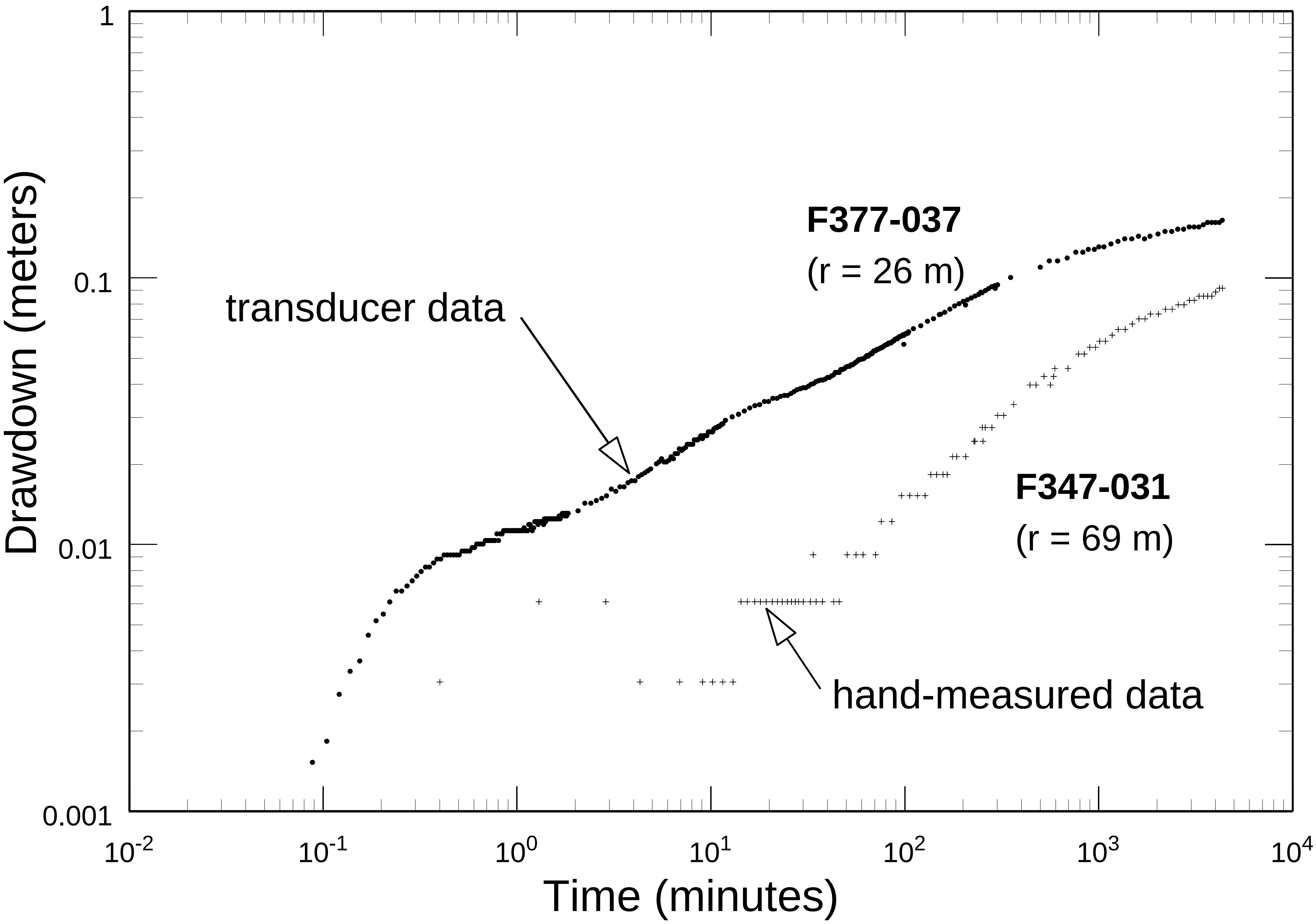
- Constant saturated thickness
- Constant initial condition
- No extraneous interference

- Instantaneous drainage from the vadose zone

Locations of piezometers with transducers



Typical plot of measured drawdown



Mathematical Model

Differential equation:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} + \frac{K_z}{K_r}\frac{\partial^2 h}{\partial z^2} = \frac{S_s}{K_r}\frac{\partial h}{\partial t}$$

Initial condition:

$$h_i - h(r, z, 0) = 0$$

Inner boundary condition, $r = r_w$:

$$2\pi\, r_w(\ell - d)K_r \frac{\partial h}{\partial r}\bigg|_{r=r_w} = Q + C\frac{\partial h_w}{\partial t}$$ wellbore balance

and $$K_s \frac{(h^* - h_w)}{d_s} = K_r \frac{\partial h}{\partial r}\bigg|_{r=r_w}$$ wellbore skin

where $C = \pi r_w^2$ for an open well

Outer boundary condition, $r = \infty$:

$$h_i - h(\infty, z, t) = 0$$

Lower boundary condition, $z = 0$:

$$\frac{\partial h}{\partial z}(r, 0, t) = 0$$

Instantaneous Drainage Assumption

To simplify the analytical solution, the assumption of **instantaneous drainage** from the zone above the water table has been commonly used.

$$K_z \frac{\partial h}{\partial z}(r, b, t) = -S_y \frac{\partial h}{\partial t}$$

Unconfined-aquifer parameters to be estimated with the analytical model:

K_r	horizontal hydraulic conductivity [m/s]
K_z	vertical hydraulic conductivity [m/s]
b	saturated thickness [m]
S_y	specific yield
S_s	specific storage [m^{-1}]

Parameter estimation algorithms
(*e.g.*, PEST or UCODE)

Objective:
To obtain aquifer parameters by minimizing the square of differences between model-generated drawdowns and corresponding measured values.

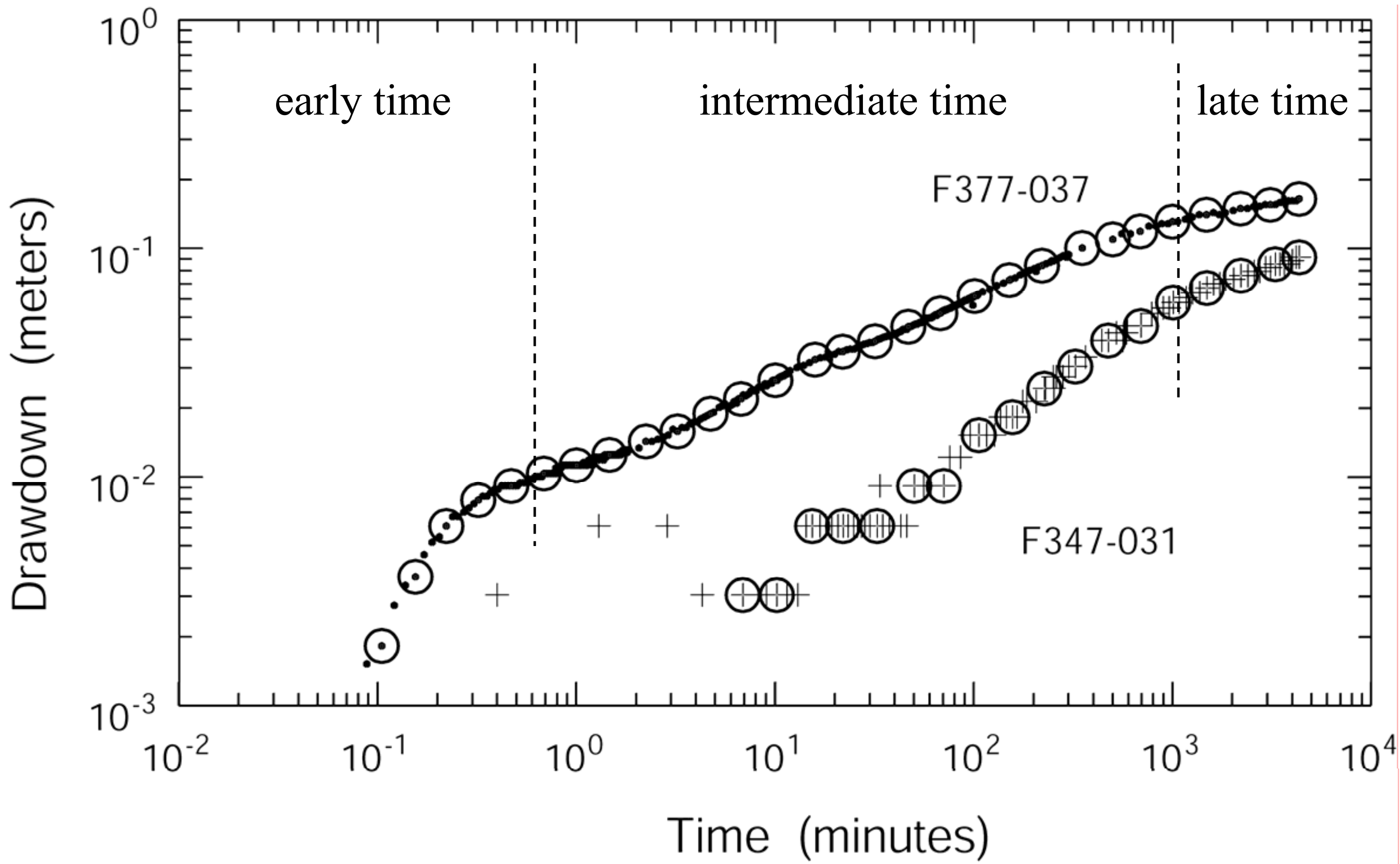
Results are influenced by model error and measurement error.

PART I

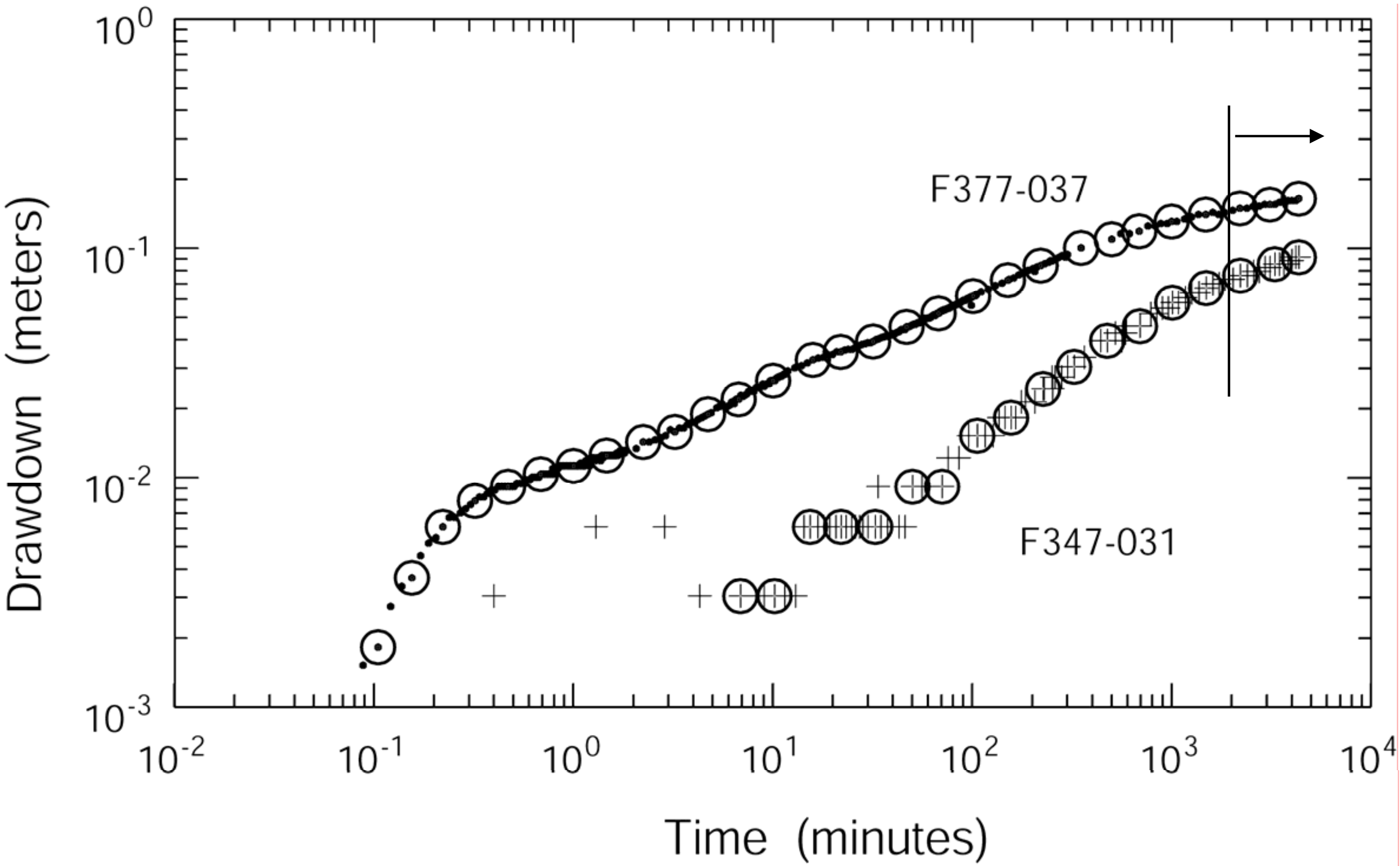
Analysis by analytical model WTAQ*

* Barlow and Moench, 1999, USGS WRI 99-4225

For parameter estimation using PEST about 6 data values (open circles) per log cycle are used



Using only late-time data (*i.e.*, t>2000 min)



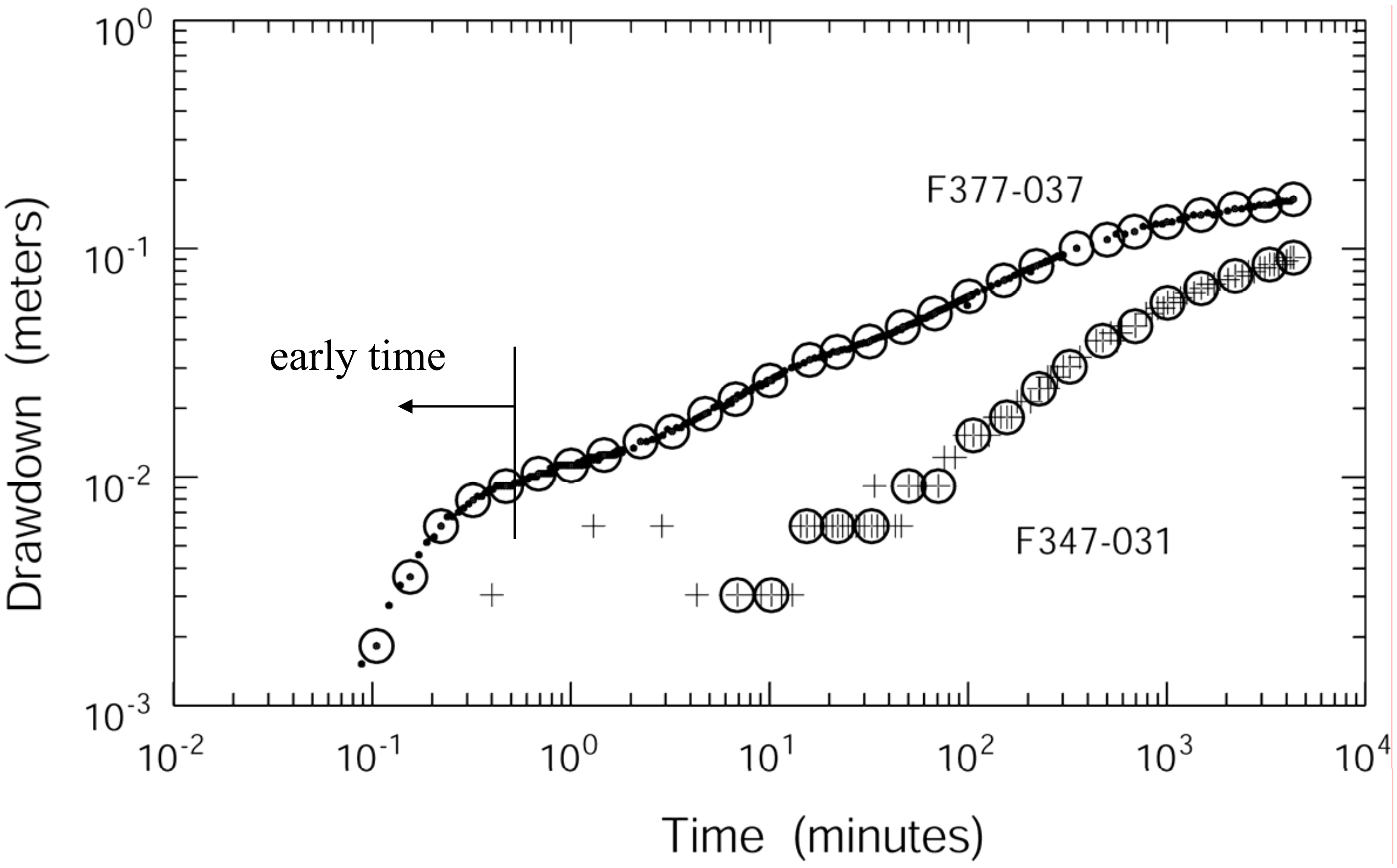
Three hydraulic parameters and *saturated thickness* (b) can be accurately estimated using only late-time data

Parameter	Estimated value	95 percent confidence limits		Initial value
		lower limit	upper limit	
S_y	0.2536	0.2356	0.2730	0.1
b (m)	52.2	50.4	54.1	30.
K_r (m/s)	1.16×10^{-3}	1.15×10^{-3}	1.18×10^{-3}	5×10^{-5}
K_z (m/s)	6.96×10^{-4}	6.69×10^{-4}	7.24×10^{-4}	5×10^{-5}

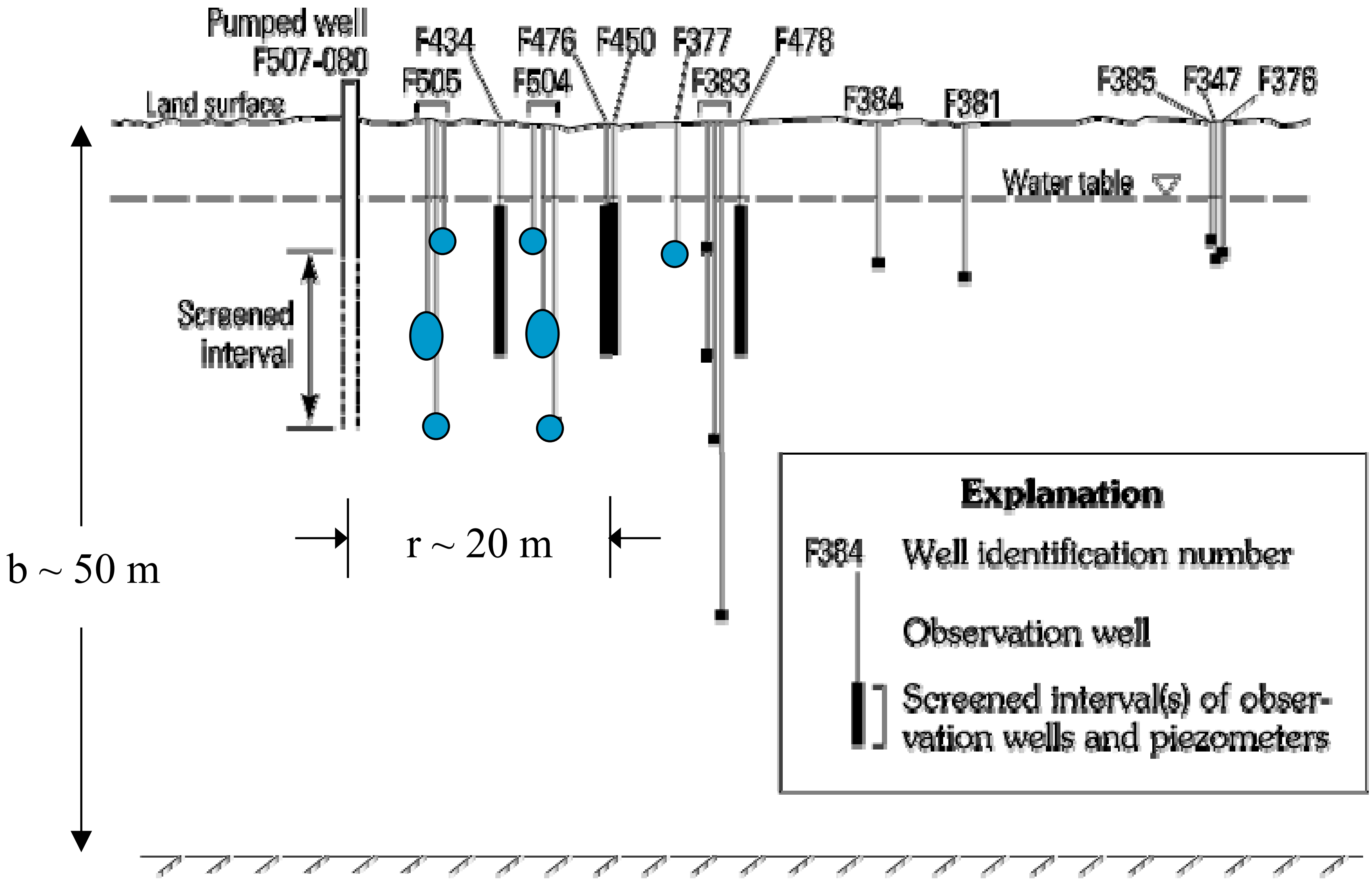
Accurate estimates of S_s require:

- Very early-time data (transducers required)
- Well bore storage
- Well bore skin
  (pumped-well data needed for this)
- Delayed piezometer response

Consider only early-time transducer data (t<0.5 min):



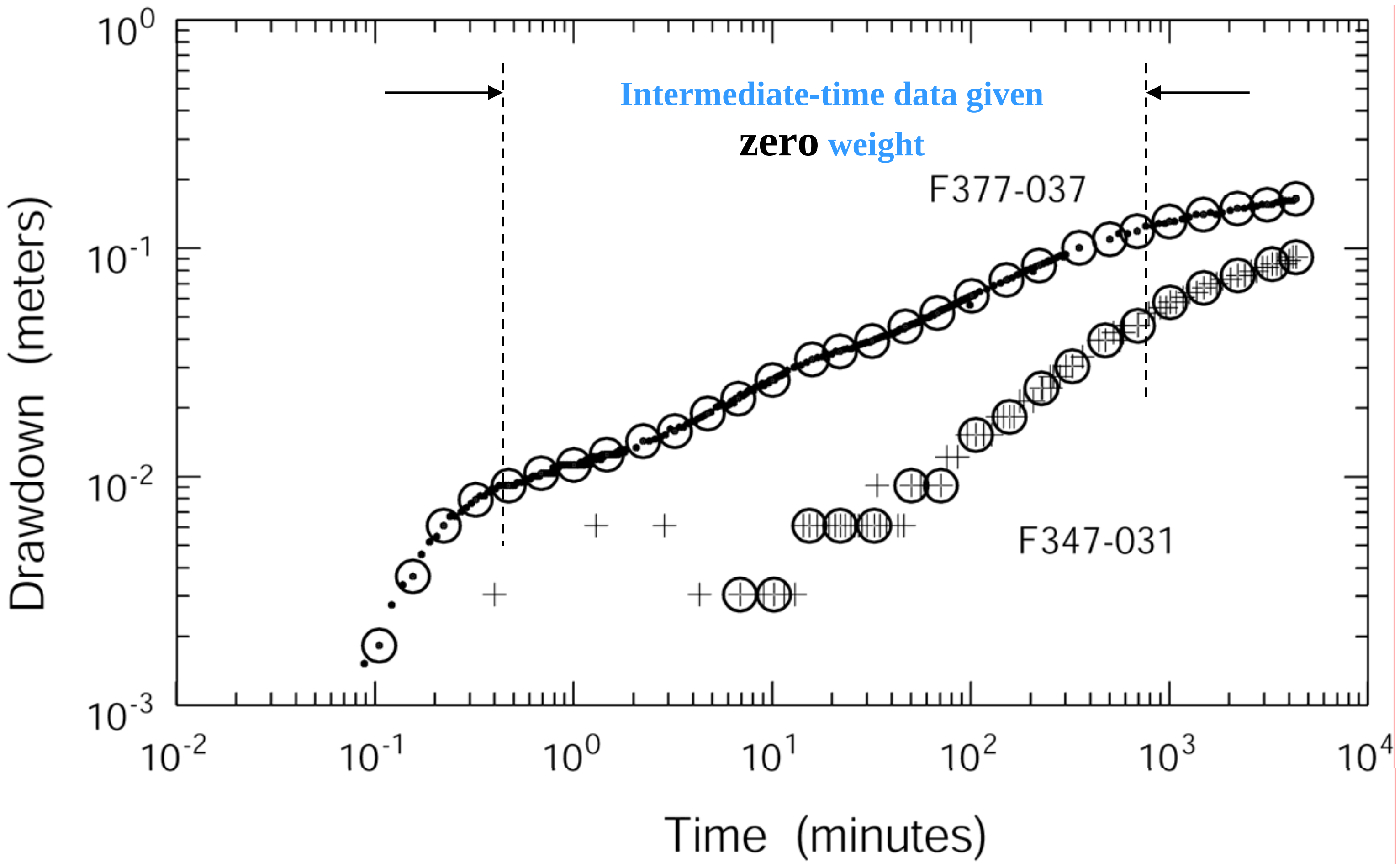
Locations of piezometers with transducers



Specific storage estimated using the seven piezometers that have transducers:

Parameter	Estimated Value	95 Percent Confidence Limits		Initial Value
		Lower limit	Upper limit	
S_s (m^{-1})	4.26E-05	4.07E-05	4.47E-05	3. E-06

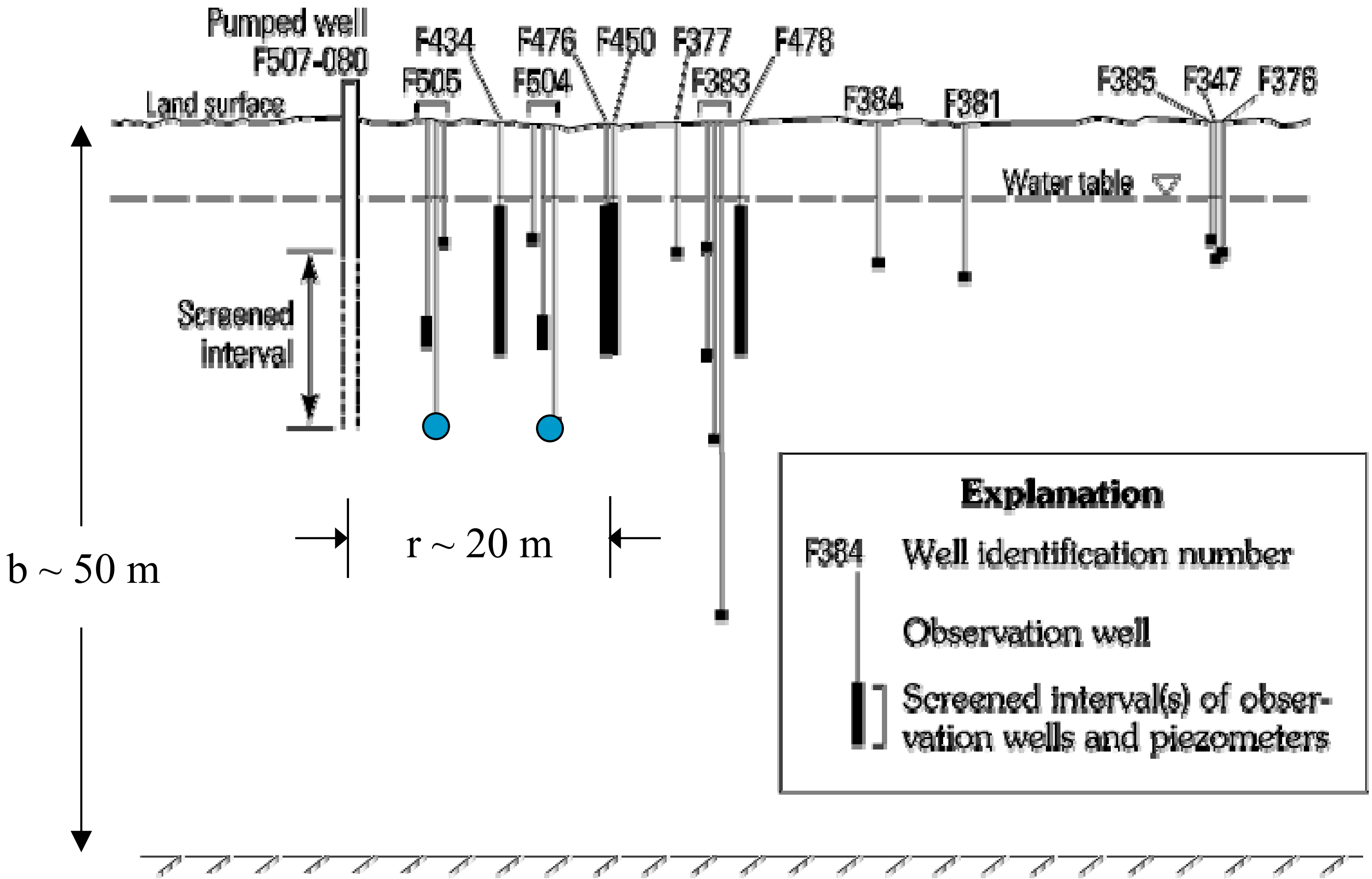
Simultaneous analysis of early and late-time data only:

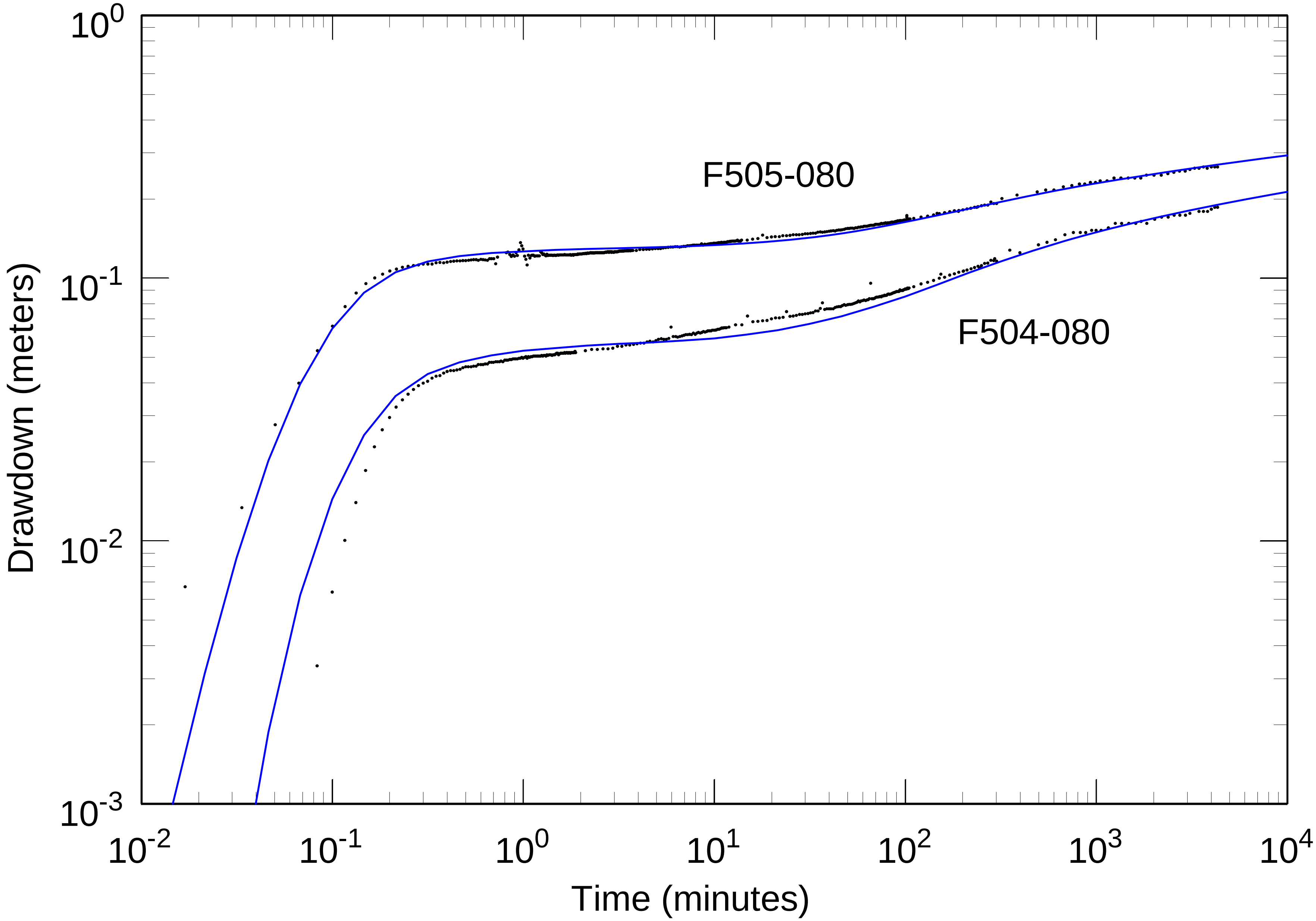


Parameters estimated using zero weight on intermediate-time data

Parameter	Estimated Value	95 percent confidence limits		Initial Value
		lower limit	upper limit	
S_s (m^{-1})	4.26E-05	4.07E-05	4.47E-05	3. E-06
S_y	0.265	0.240	0.291	0.1
b (m)	51.3	49.0	53.7	30.
K_r (m/s)	1.17E-03	1.15E-03	1.19E-03	5.E-05
K_z (m/s)	6.58E-04	6.27E-04	6.89E-04	5.E-05

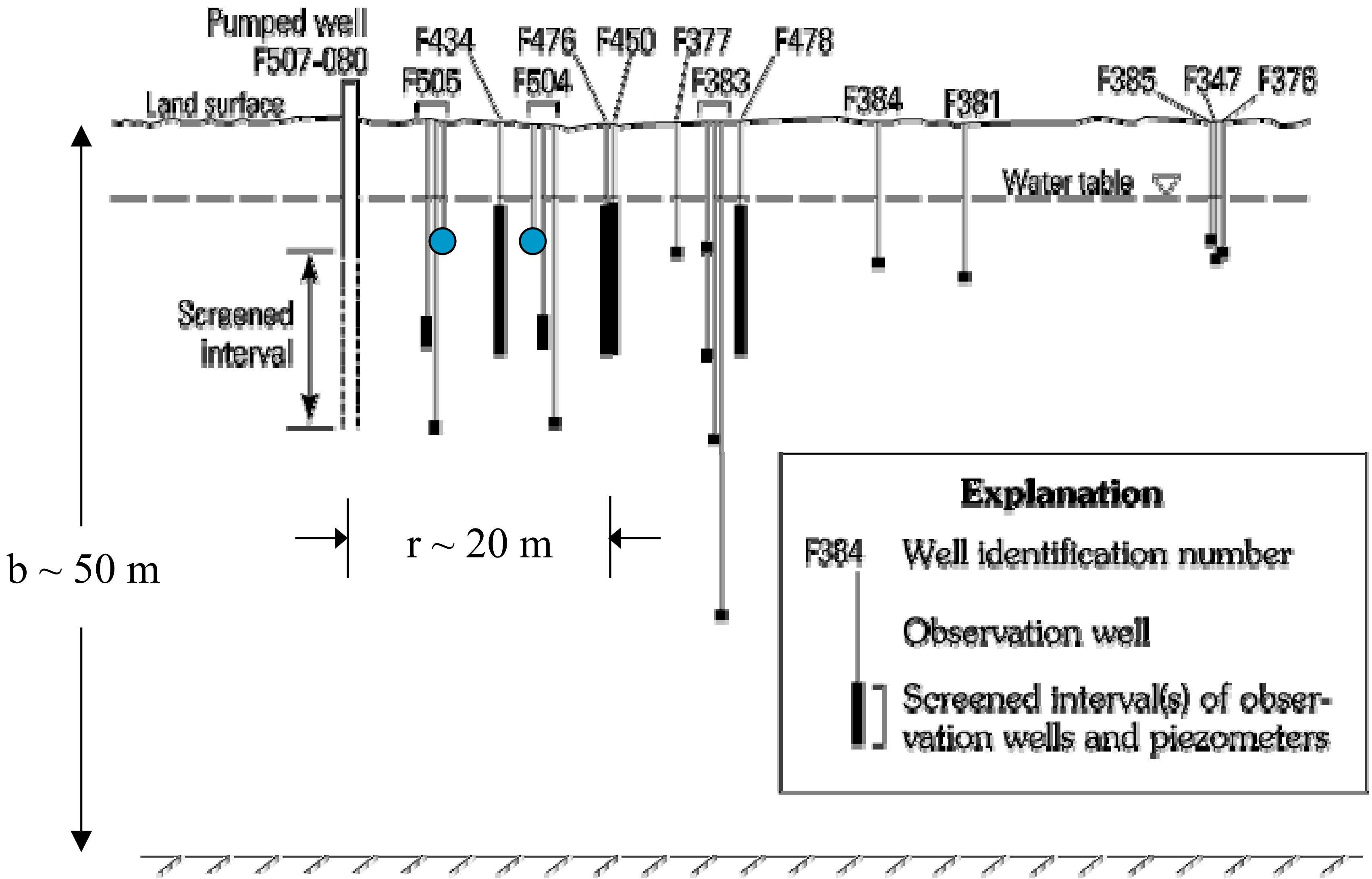
Location of two of the deep-seated piezometers

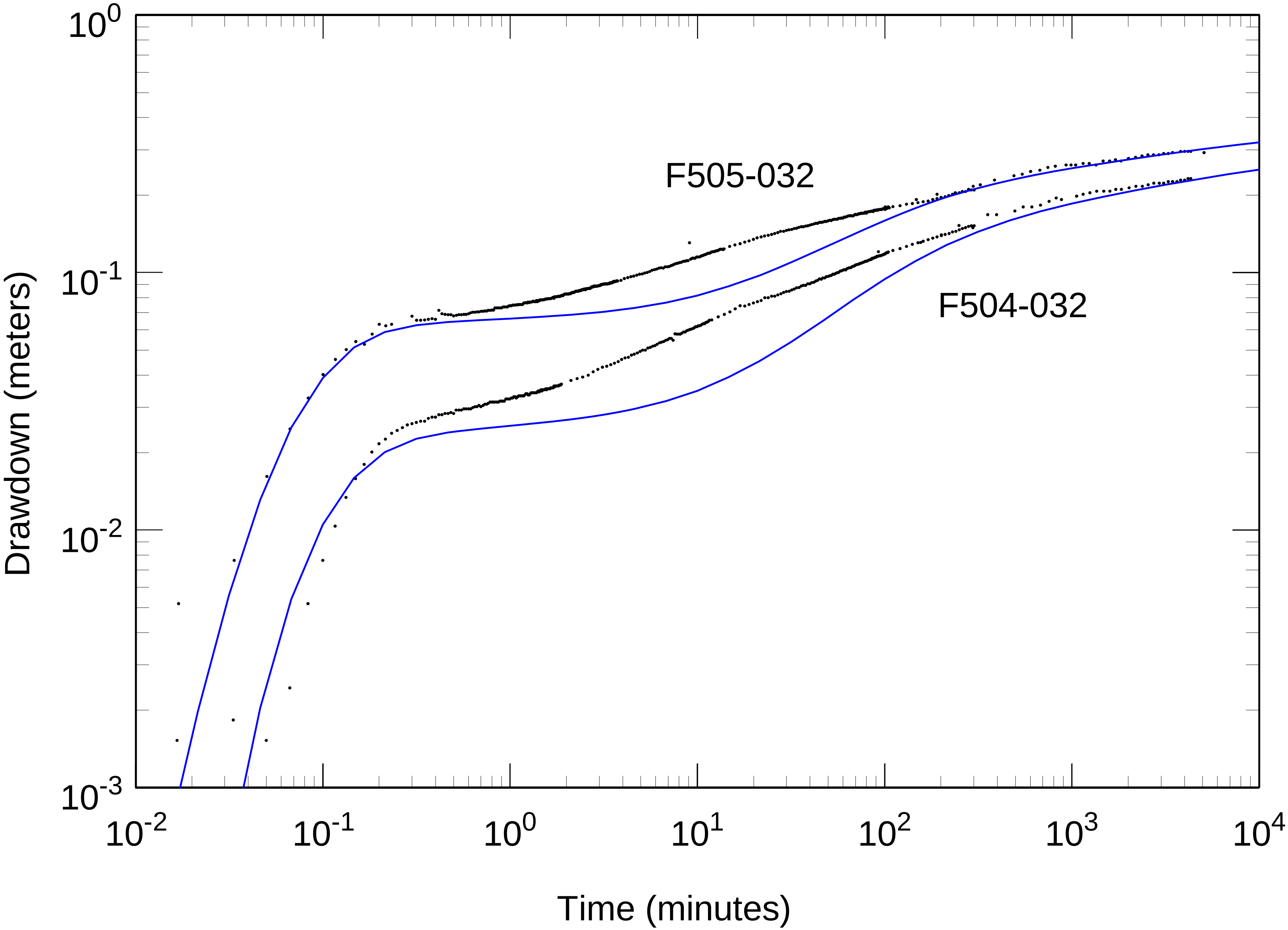




F505-080
F504-080
Drawdown (meters)
Time (minutes)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
10^2
10^3
10^4

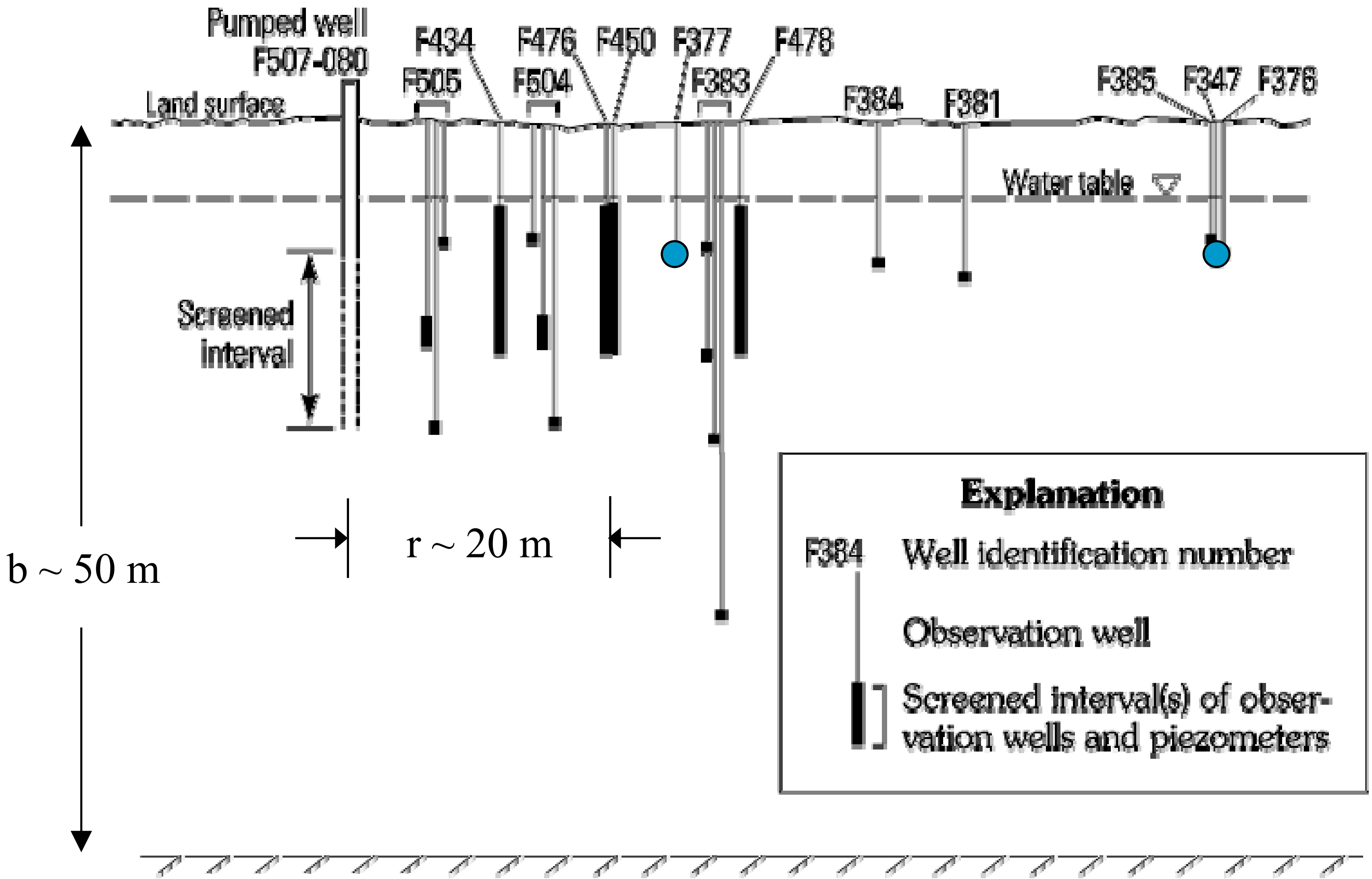
Location of two of the shallow piezometers

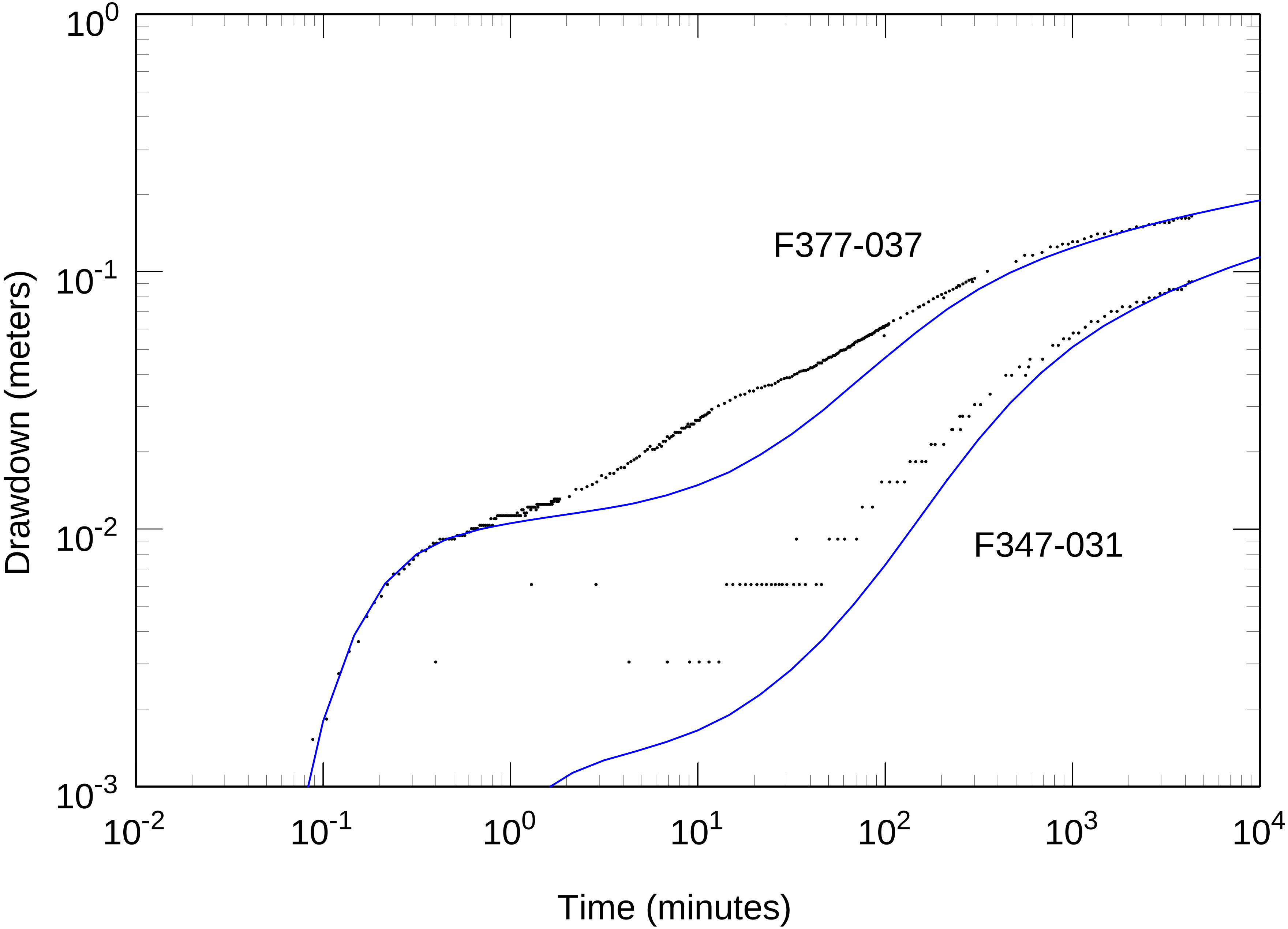




F505-032
F504-032
Drawdown (meters)
Time (minutes)
10^0
10^-1
10^-2
10^-3
10^-2
10^-1
10^0
10^1
10^2
10^3
10^4

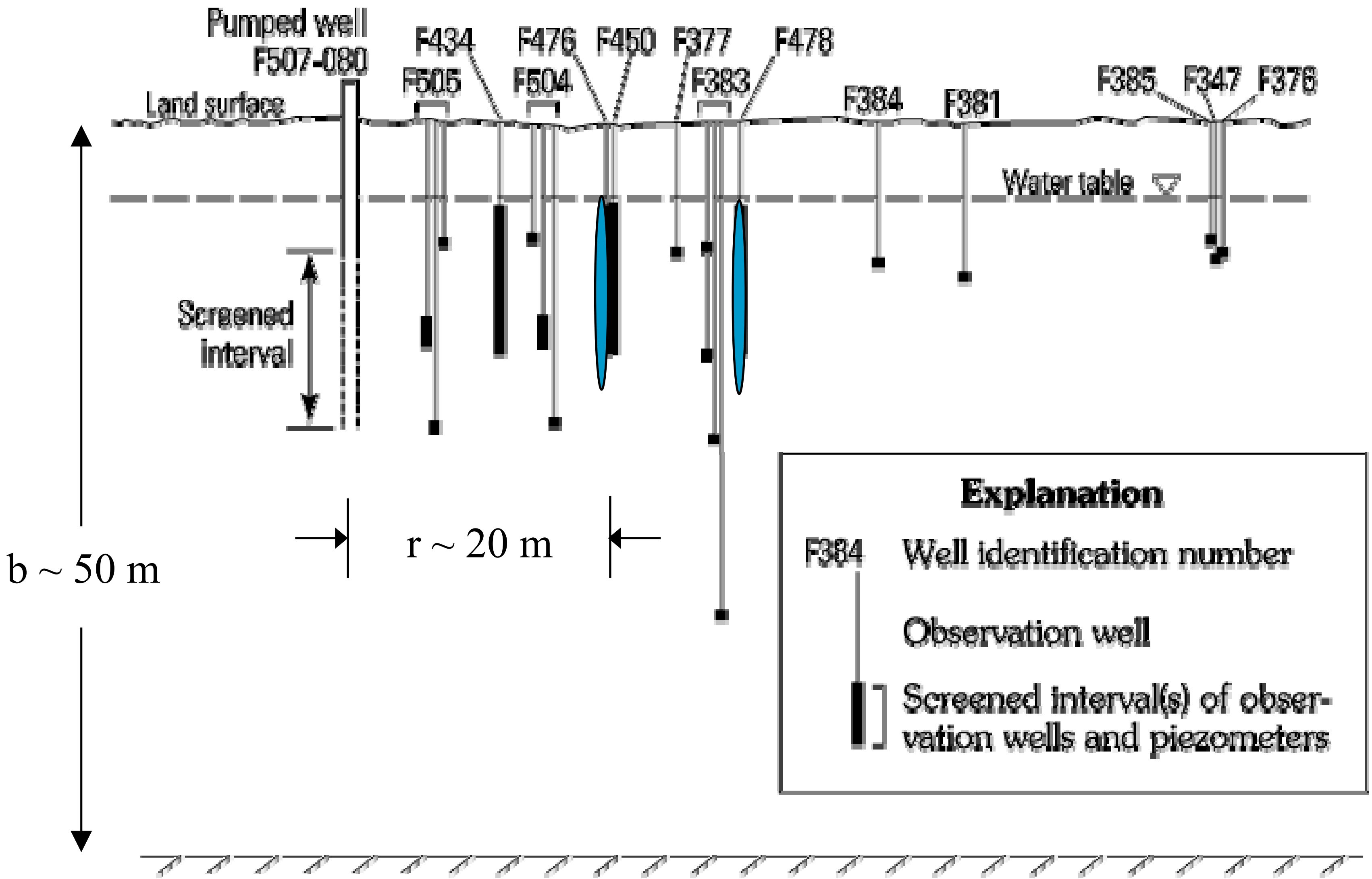
Location of two of the (more distant) shallow piezometers

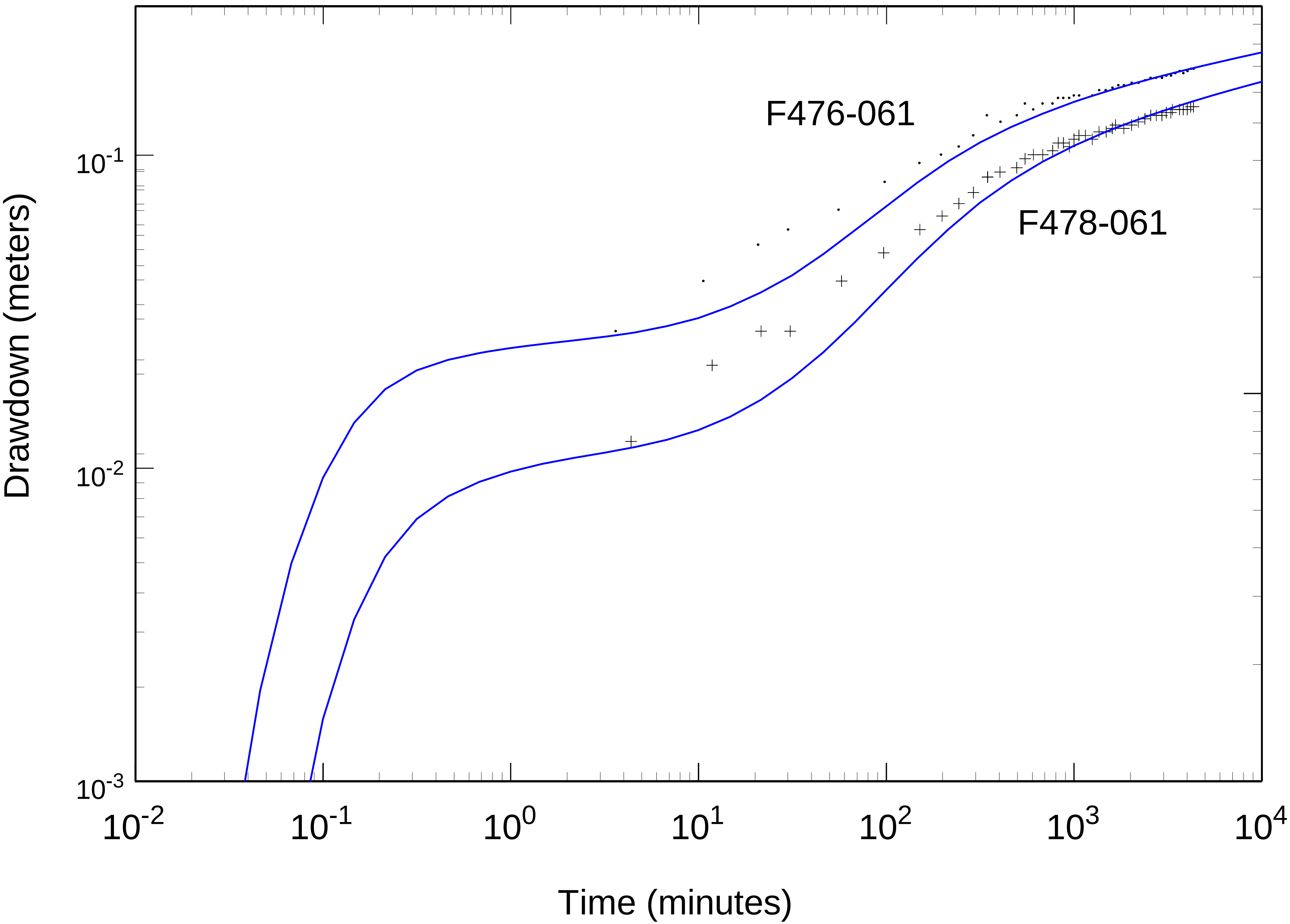




F377-037
F347-031
Drawdown (meters)
Time (minutes)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}

Location of two of the long-screened piezometers





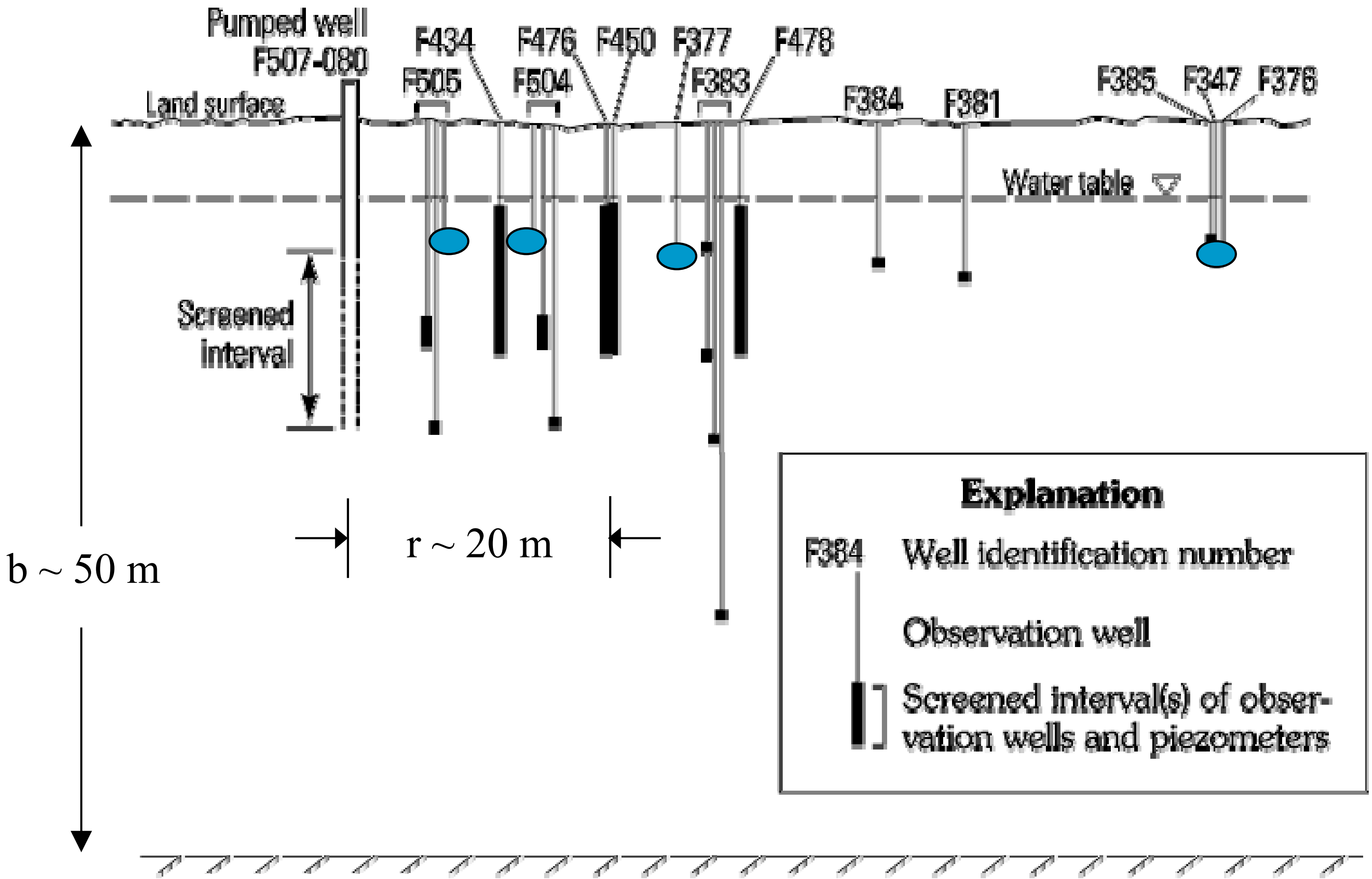
F476-061
F478-061
Drawdown (meters)
10-1
10-2
10-3
10-2
10-1
100
101
102
103
104
Time (minutes)

The “gaps” between measured data and simulated responses are due to *model error* (*i.e.*, the instantaneous drainage assumption).

Note:

These ‘gaps’ occur even though the piezometers are located about three meters below the water table and the aquifer is highly permeable.

What happens under the instantaneous drainage assumption if all measurements are given equal weight?

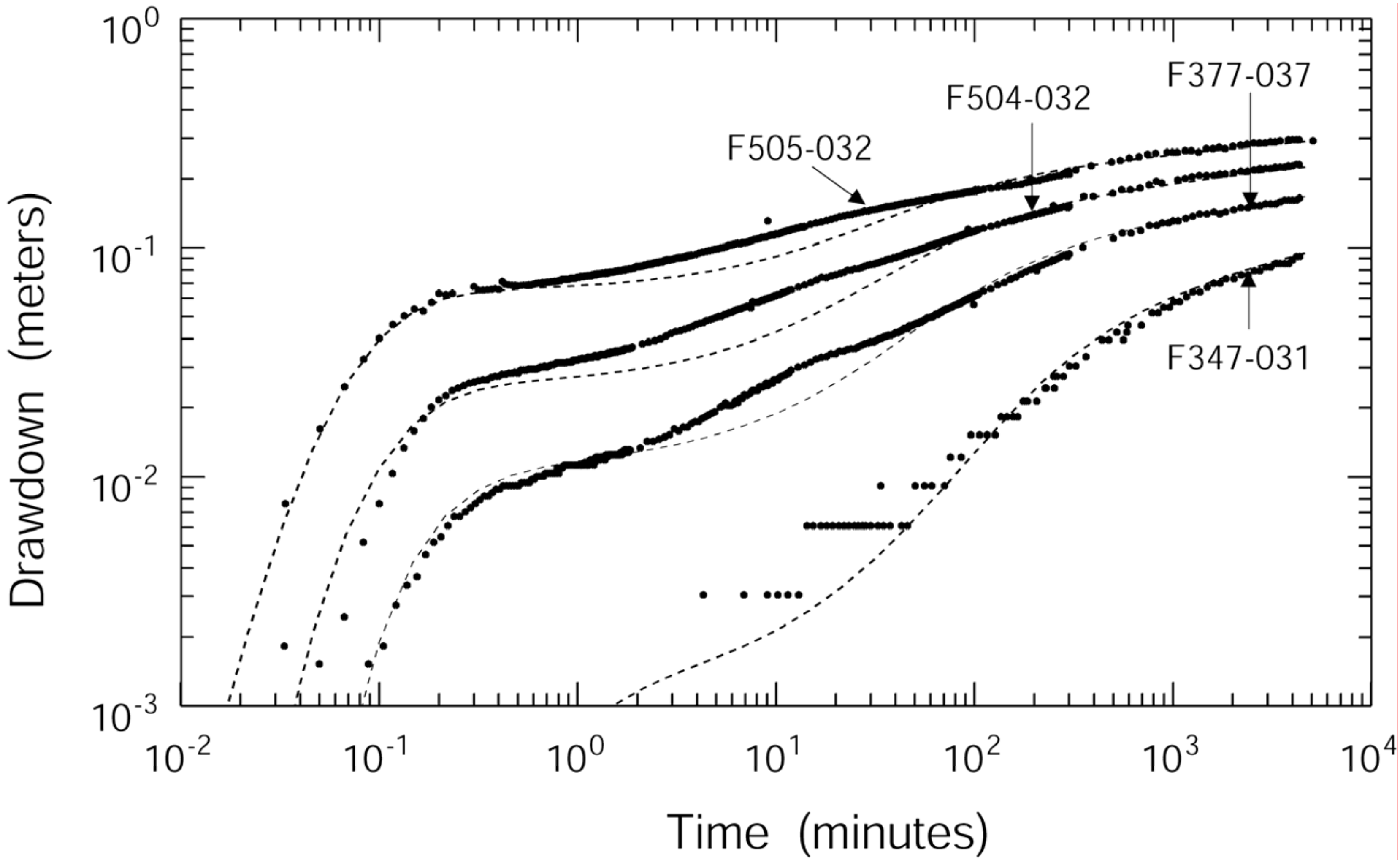


Pumped well
F507-080
F434
F476
F450
F377
F478
F505
F504
F383
F384
F381
F385
F347
F376
Land surface
Water table
Screened interval
b ~ 50 m
r ~ 20 m
Explanation
F384 Well identification number
Observation well
Screened interval(s) of observation wells and piezometers

Parameter estimates obtained using 4 shallow piezometers with all data weighted equally:

Parameter	Estimated Value	95 percent confidence limits		Initial Value
		lower limit	upper limit	
S_s (m^{-1})	4.58E-05	4.12E-05	5.08E-05	3. E-06
S_y	0.145	0.138	0.151	0.1
b (m)	60.2	58.0	62.5	30.
K_r (m/s)	1.24E-03	1.22E-03	1.26E-03	5.E-05
K_z (m/s)	5.96E-04	5.73E-04	6.19E-04	5.E-05

Comparisons of measured and simulated drawdowns using estimated parameters from the previous table:



Upper boundary condition for the analytical solution must be modified to account for *gradual drainage* from the unsaturated zone.

The proposed modified condition is:

$$K_z \frac{\partial h}{\partial z}(r,b,t)$$

$$= -S_y \int_0^t \frac{\partial h}{\partial t'}(r,b,t) \sum_{m=1}^{M} \frac{\alpha_m}{M} \exp[-\alpha_m (t-t')]\, dt'$$

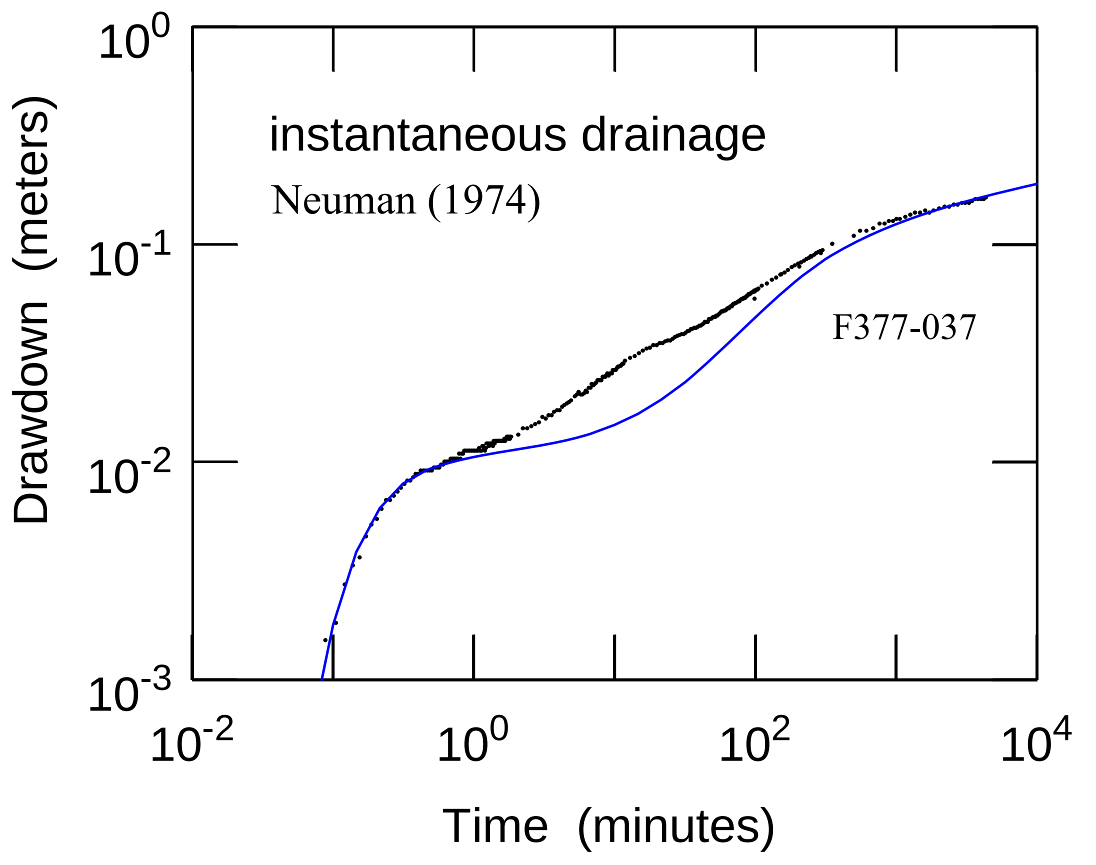
which becomes:

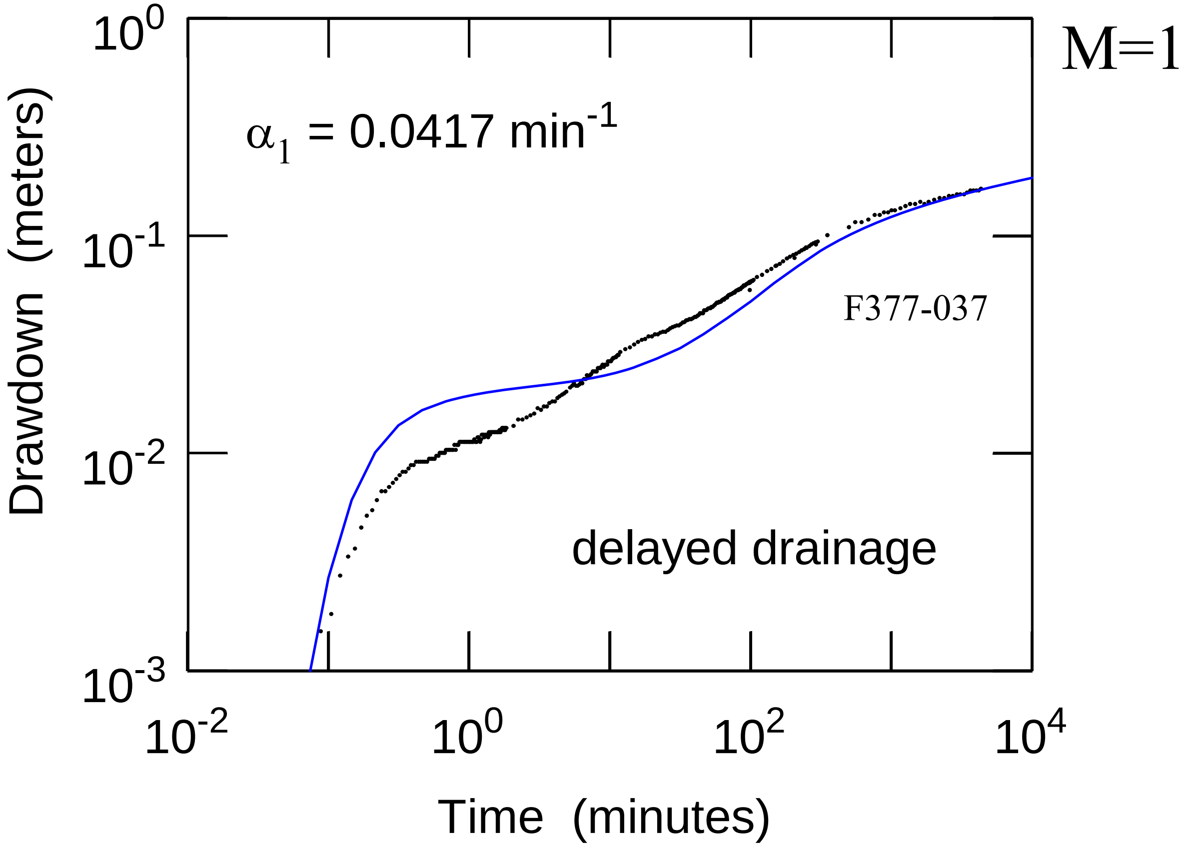
1. the instantaneous drainage condition:

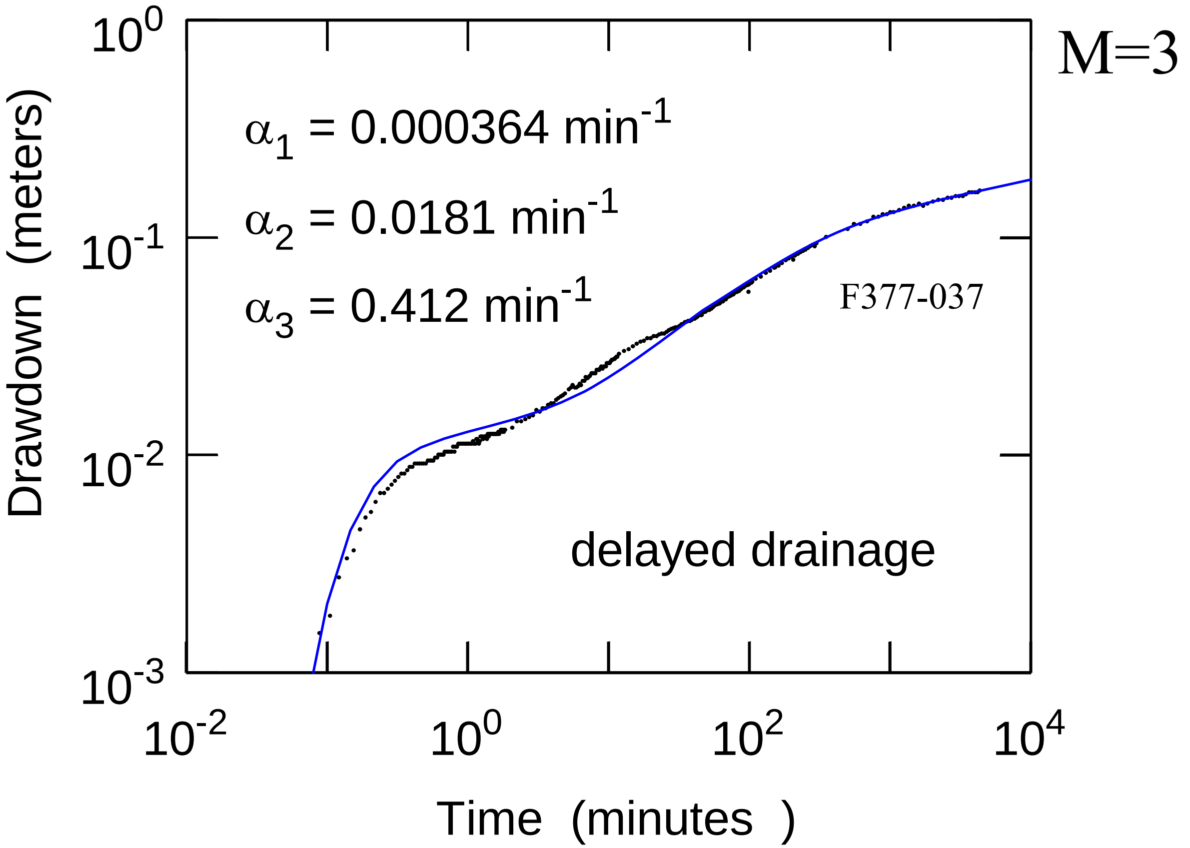
$$K_z \frac{\partial h}{\partial z}(r,b,t) = -S_y \frac{\partial h}{\partial t}$$ if α becomes infinite

2. the condition for a confined aquifer:

$$K_z \frac{\partial h}{\partial z}(r,b,t) = 0$$ if α becomes zero


instantaneous drainage
Neuman (1974)
F377-037
Drawdown (meters)
Time (minutes)
10^0
10^-1
10^-2
10^-3
10^-2
10^0
10^2
10^4


M=1
α_1 = 0.0417 min^{-1}
F377-037
delayed drainage
Drawdown (meters)
Time (minutes)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^0
10^2
10^4

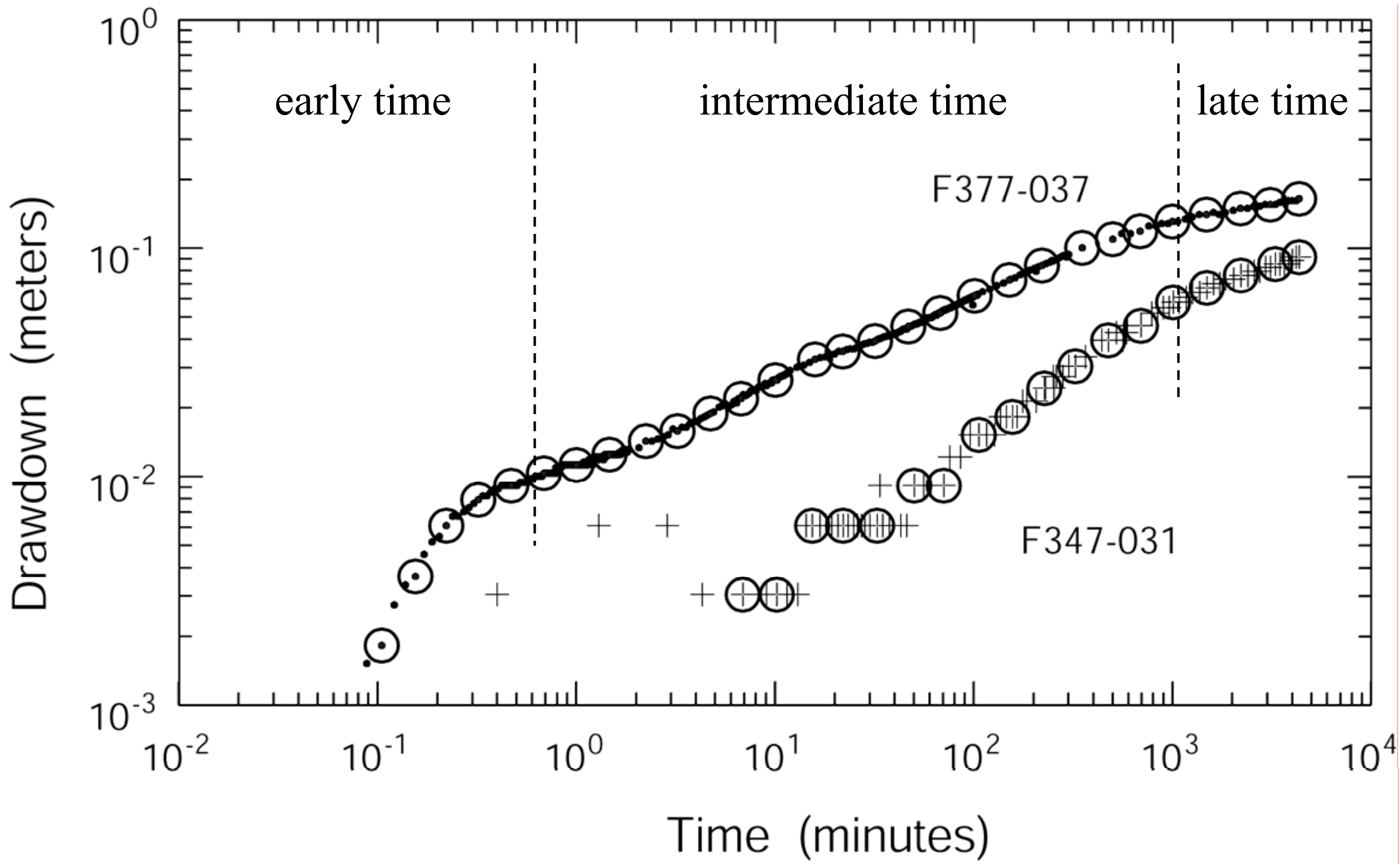

M=3
α_1 = 0.000364 min^{-1}
α_2 = 0.0181 min^{-1}
α_3 = 0.412 min^{-1}
F377-037
delayed drainage
Drawdown (meters)
Time (minutes)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-2}
10^0
10^2
10^4

Regarding the number of empirical constants:

It has been found that M=3 gives satisfactory results for the Cape Cod aquifer test.

The use of additional empirical constants was not justified due to small-scale aquifer heterogeneity.

Now all drawdown data can be given equal weight



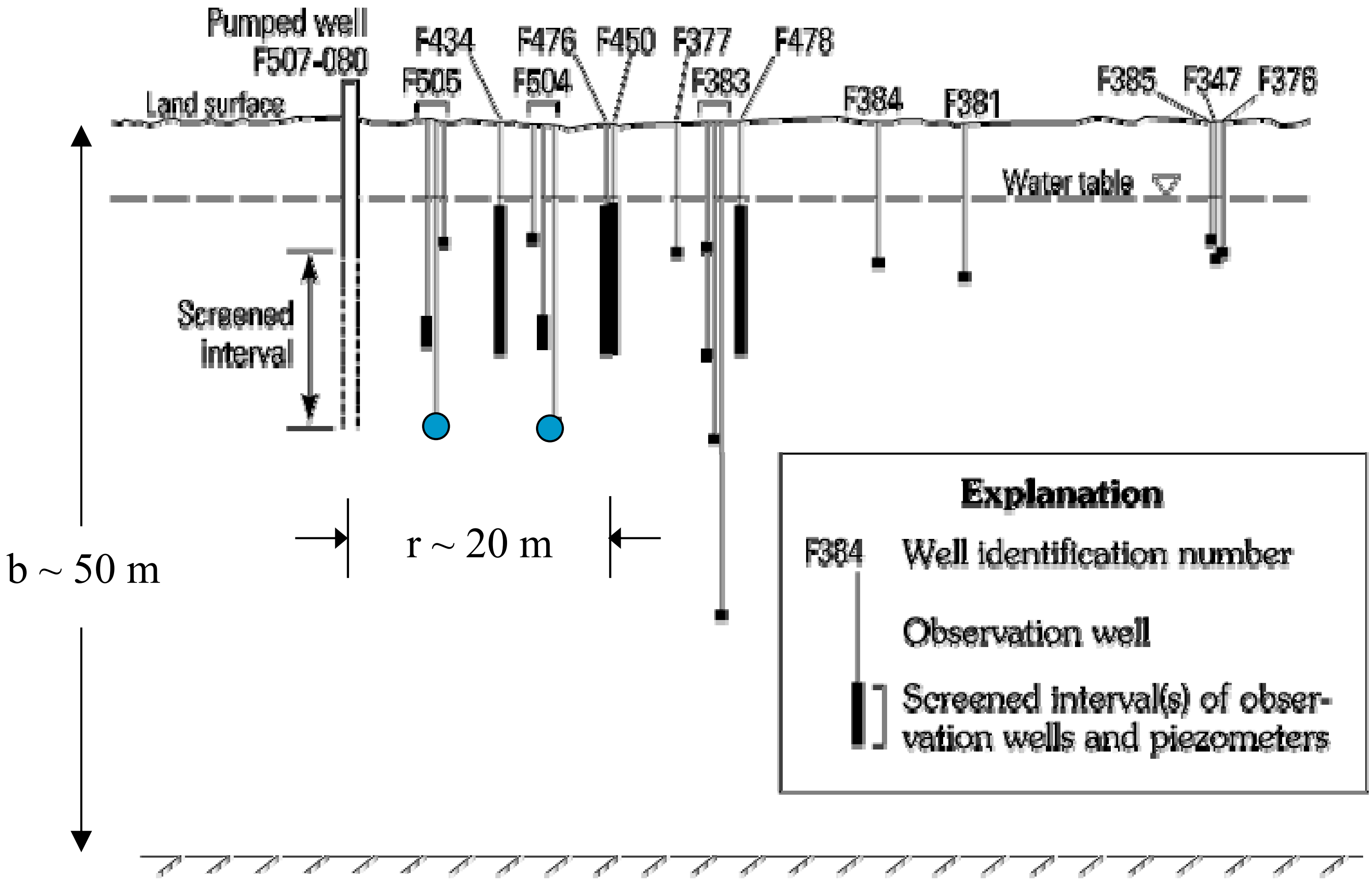
Parameters estimated with the **complete** data set (424 measured values of drawdown)

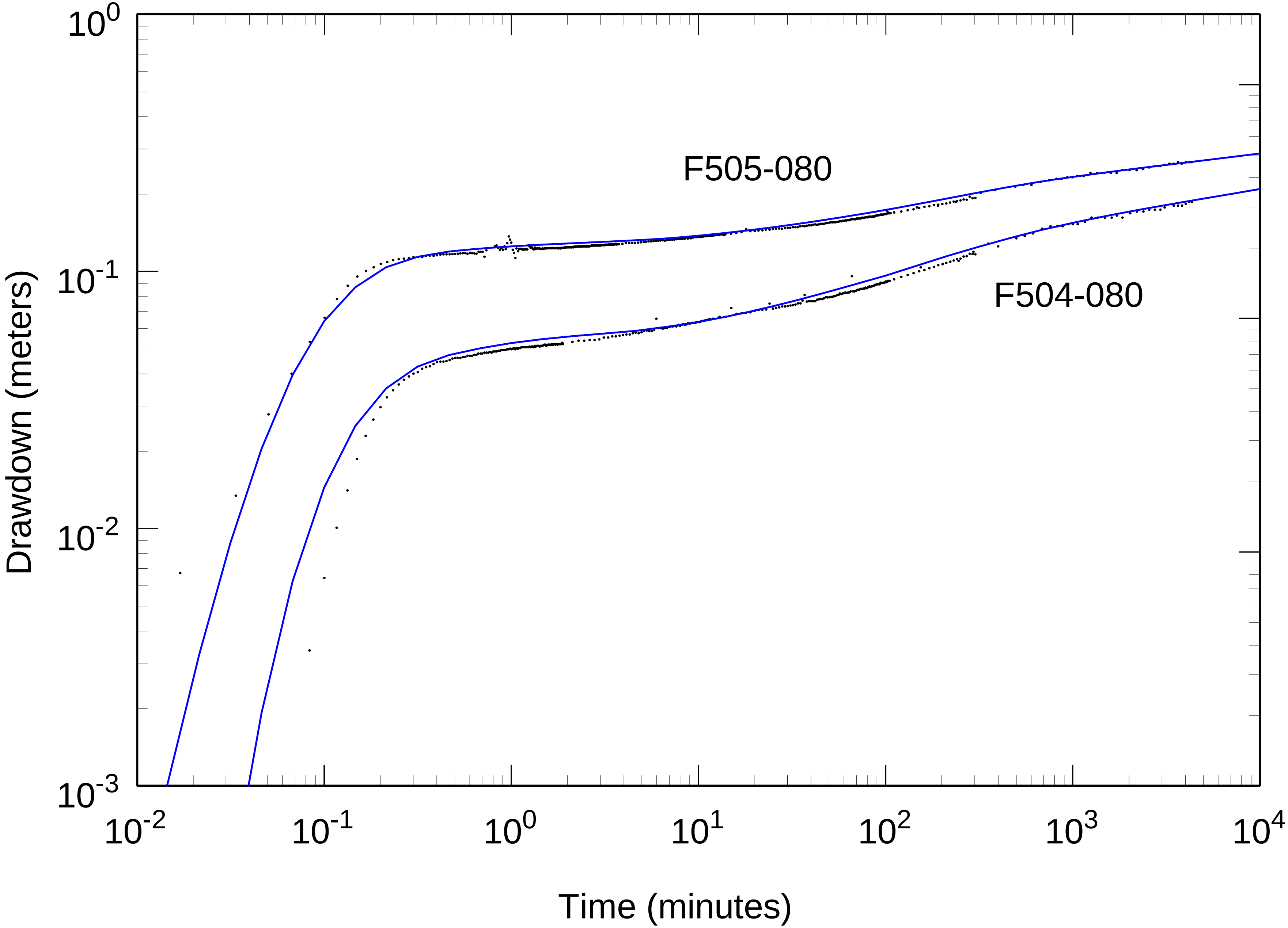
Parameter	Estimated Value	95% Confidence Limits		Initial Value
		Lower limit	Upper limit	
S_s (m^{-1})	4.28E-05	3.95E-05	4.64E-05	3.E-06
S_y	0.266	0.253	0.280	0.1
b (m)	51.5	49.5	53.5	60.
K_r (m/s)	1.18E-03	1.17E-03	1.20E-03	5.E-05
K_z (m/s)	7.20E-04	6.93E-04	7.49E-04	5.E-05
α_1 (min^{-1})	2.78E-04	1.50E-04	5.14E-04	1.E-03
α_2 (min^{-1})	1.68E-02	1.27E-02	2.22E-02	1.E-02
α_3 (min^{-1})	0.416	0.318	0.545	1.E-01

The following slides show

comparisons of measured drawdown with theoretical responses using the estimated values of S_s, S_y, K_r, K_z, b, and the set $\{\alpha_1, \alpha_2, \alpha_3\}$:

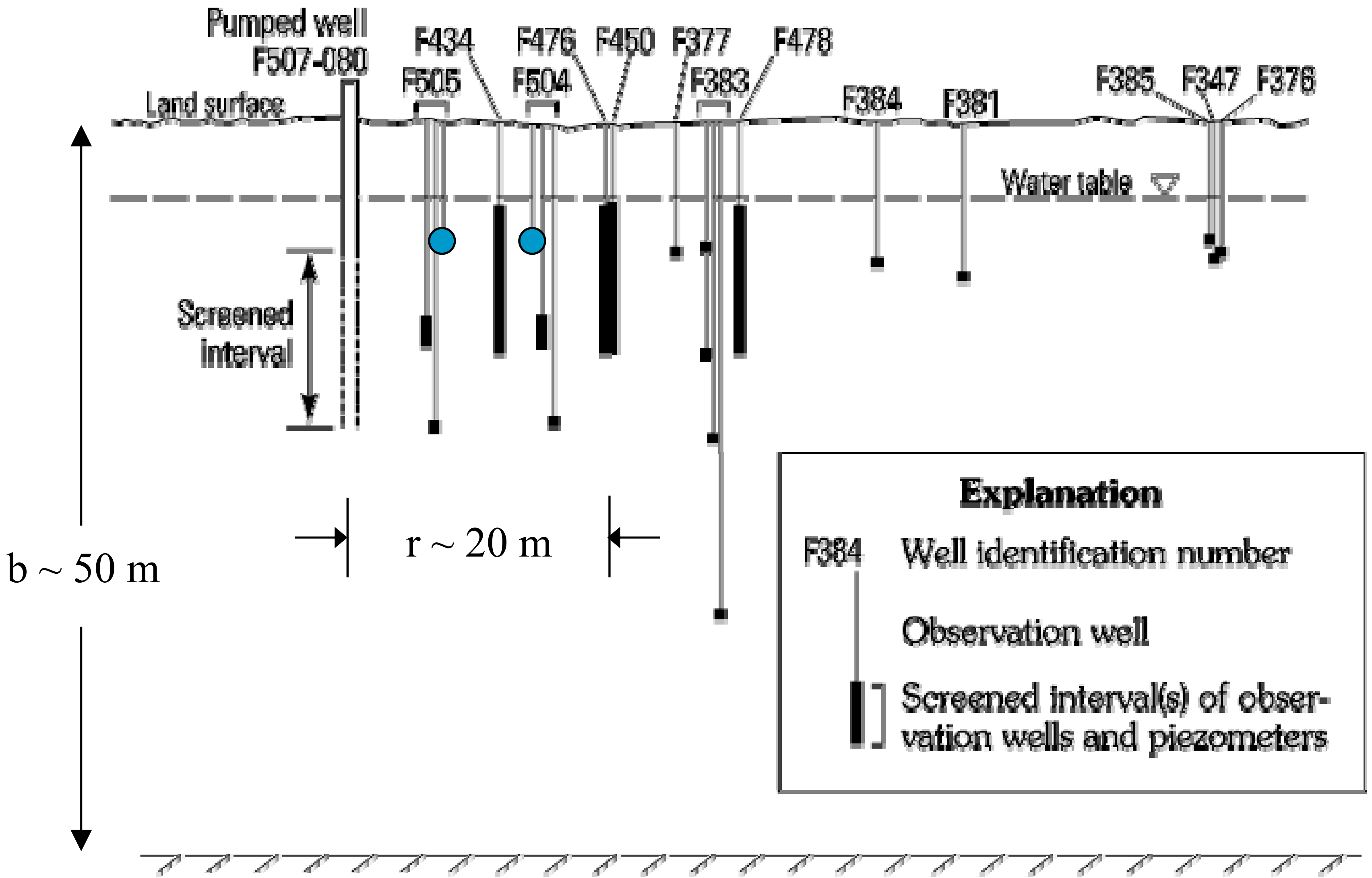
Location of two of the deep-seated piezometers

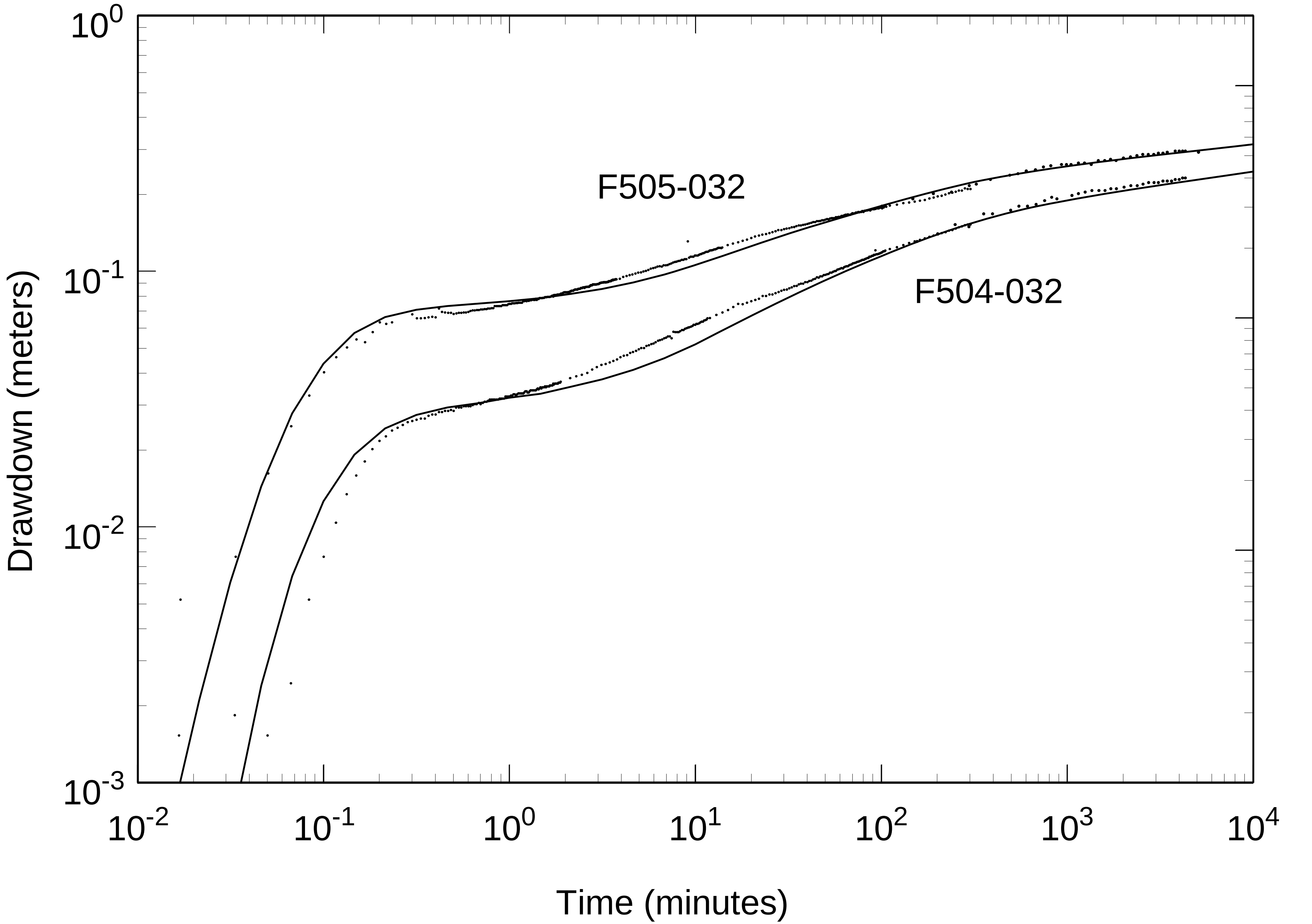




F505-080
F504-080
Drawdown (meters)
Time (minutes)
10^0
10^-1
10^-2
10^-3
10^-2
10^-1
10^0
10^1
10^2
10^3
10^4

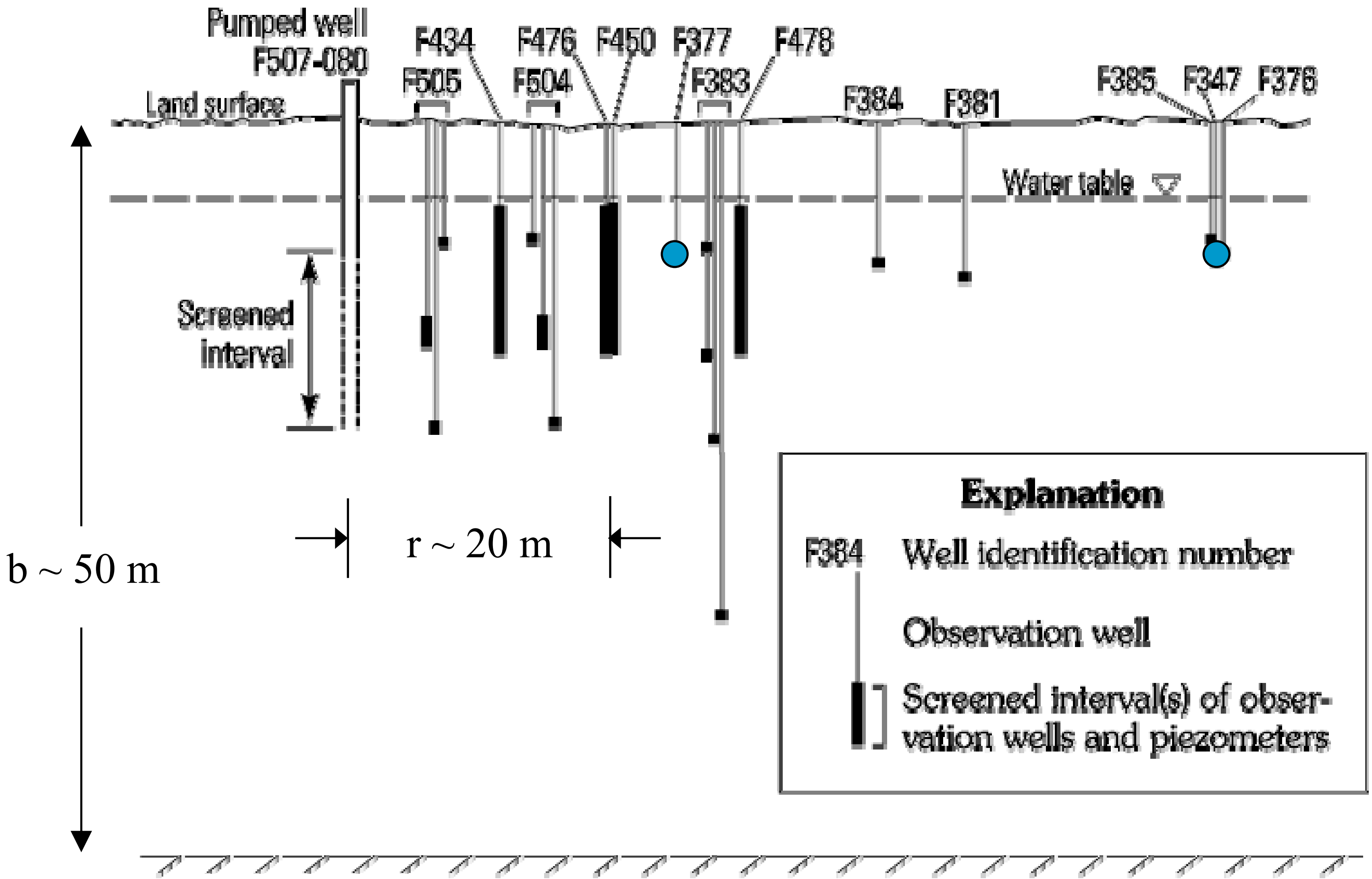
Location of two of the (close-in) shallow piezometers

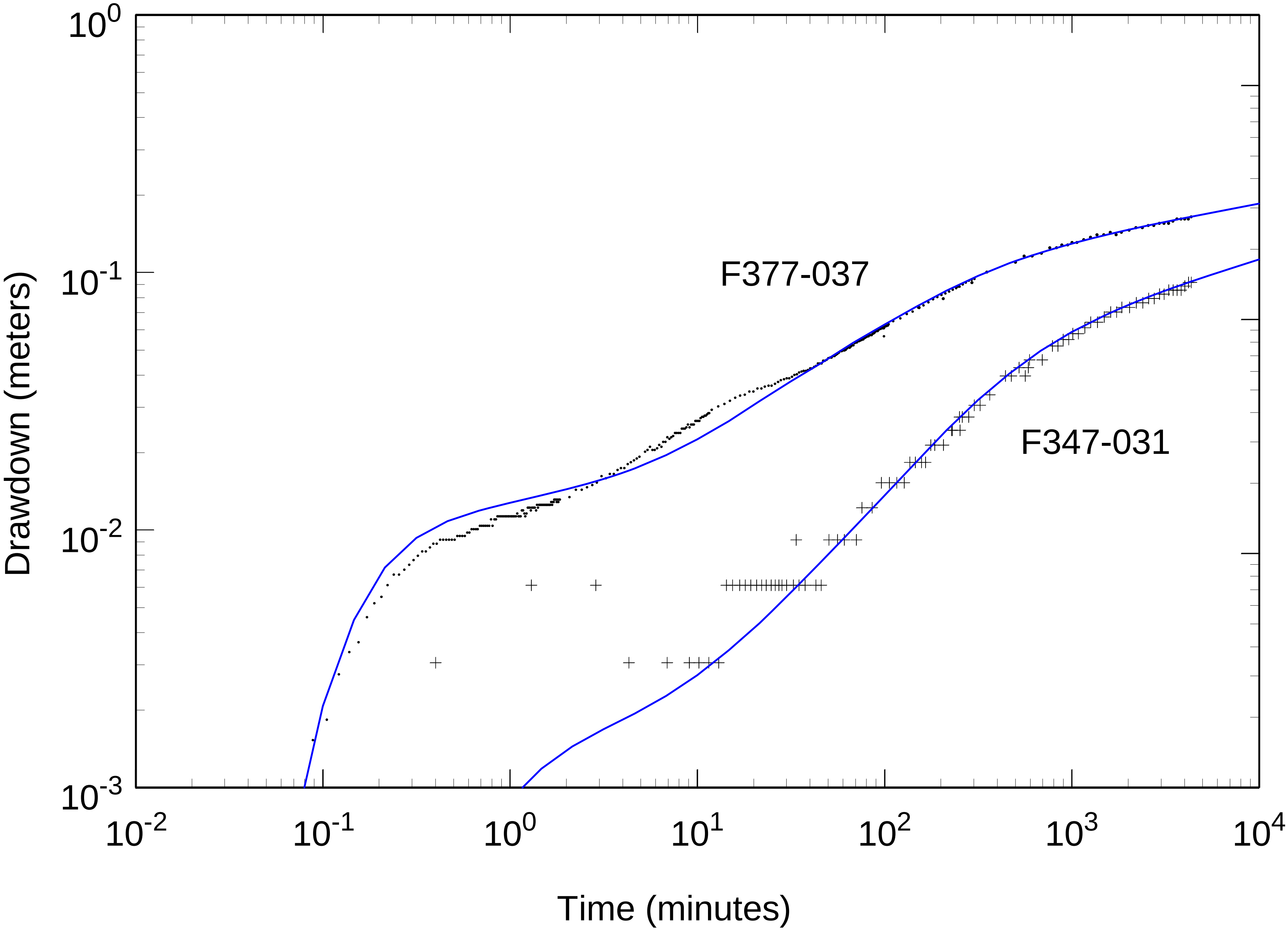




F505-032
F504-032
Drawdown (meters)
Time (minutes)
10^0
10^-1
10^-2
10^-3
10^-2
10^-1
10^0
10^1
10^2
10^3
10^4

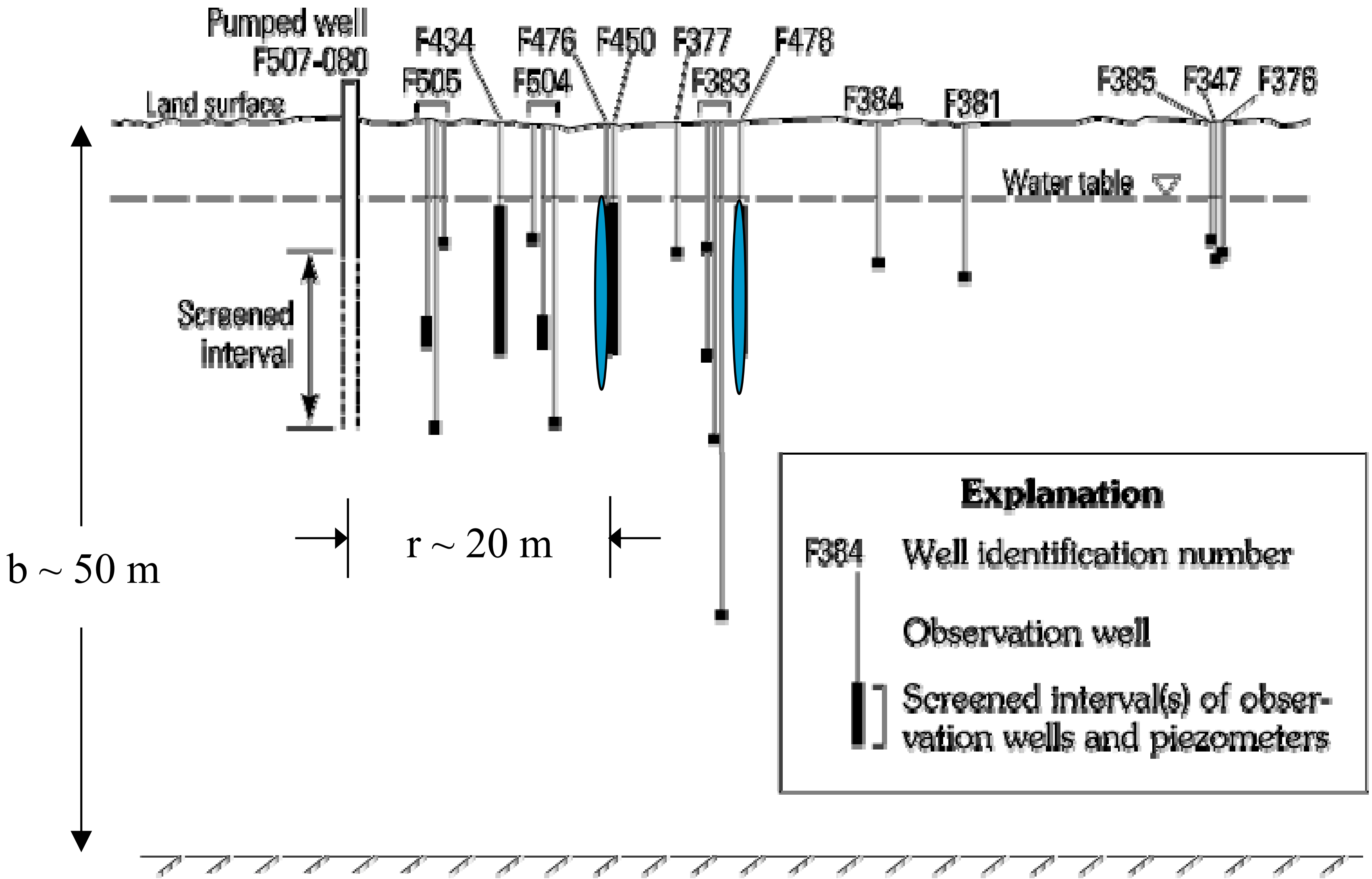
Location of two of the (more distant) shallow piezometers

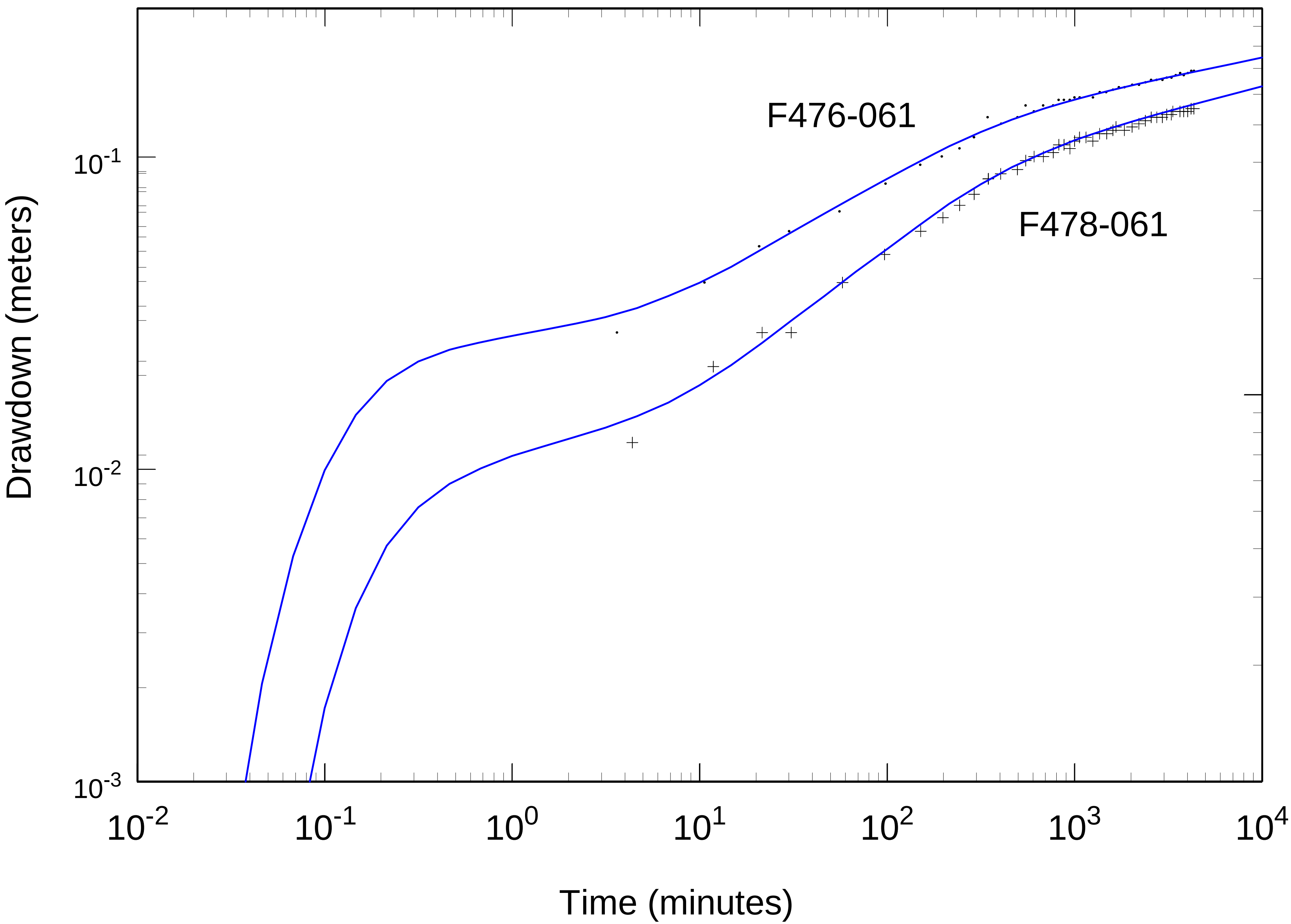




10^0
10^-1
10^-2
10^-3
Drawdown (meters)
F377-037
F347-031
10^-2
10^-1
10^0
10^1
10^2
10^3
10^4
Time (minutes)

Location of two of the long-screened piezometers





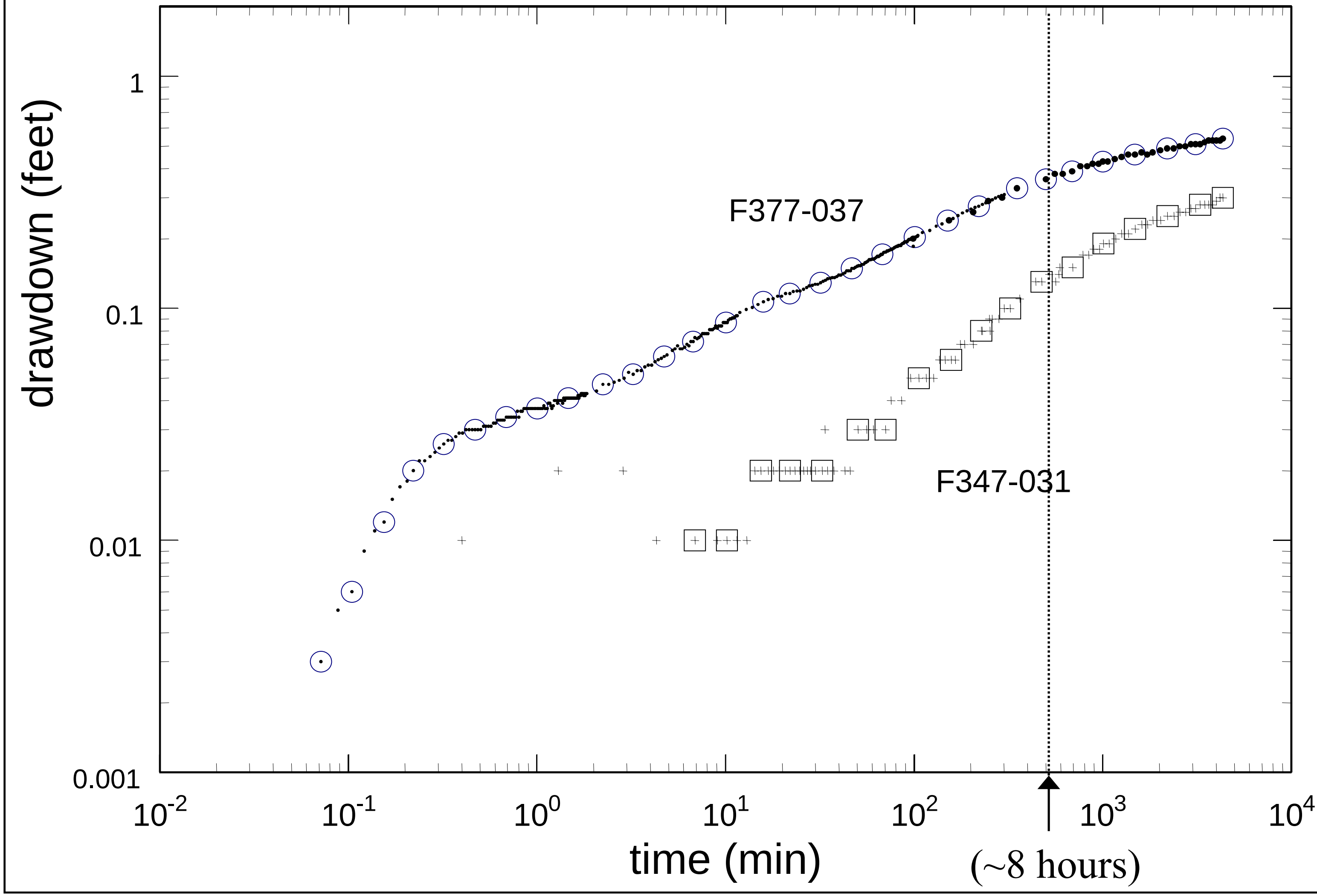
F476-061
F478-061
Drawdown (meters)
Time (minutes)
10^-1
10^-2
10^-3
10^-2
10^-1
10^0
10^1
10^2
10^3
10^4

As an aside:

Long-term pumping tests may not be necessary for accurate parameter estimates provided unsaturated-zone drainage is included in the model

Analysis by elimination of some late-time data

-note units of feet



Results obtained if test had been run for a shorter time –note the units of feet

Parameter	All Piezometers			
	72-hour	24-hour	16-hour	8-hour
Ss ft^{-1}	**1.3E-05**	1.3E-05		
Sy	**0.262**	0.255		
b ft	**173**	179		
Kr ft/min	**0.234**	0.236		
Kz ft/min	**0.141**	0.140		
α_1 min^{-1}	1.9E-04	6.6E-05		
α_2 min^{-1}	1.8E-02	1.8E-02		
α_3 min^{-1}	0.44	0.45		

Results obtained if test had been run for a shorter time –note the units of feet

Parameter	All Piezometers			
	72-hour	24-hour	16-hour	8-hour
Ss ft^{-1}	**1.3E-05**	1.3E-05	1.3E-05	
Sy	**0.262**	0.255	0.258	
b ft	**173**	179	177	
Kr ft/min	**0.234**	0.236	0.237	
Kz ft/min	**0.141**	0.140	0.140	
α_1 min^{-1}	1.9E-04	6.6E-05	3.1E-05	
α_2 min^{-1}	1.8E-02	1.8E-02	1.7E-02	
α_3 min^{-1}	0.44	0.45	0.42	

Results obtained if test had been run for a shorter time –note the units of feet

Parameter	All Piezometers			
	72-hour	24-hour	16-hour	8-hour
Ss ft^{-1}	**1.3E-05**	1.3E-05	1.3E-05	1.3E-05
Sy	**0.262**	0.255	0.258	0.239
b ft	**173**	179	177	186
Kr ft/min	**0.234**	0.236	0.237	0.240
Kz ft/min	**0.141**	0.140	0.140	0.139
α_1 min^{-1}	1.9E-04	6.6E-05	3.1E-05	9.2E-05
α_2 min^{-1}	1.8E-02	1.8E-02	1.7E-02	2.0E-02
α_3 min^{-1}	0.44	0.45	0.42	0.43

Advantages of the modified analytical model:

- Allows for improved fit between measured and simulated drawdowns

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- Allows for improved fit between measured and simulated drawdowns

- Makes it possible to run the test for a shorter time and still get good parameter estimates

Advantages of the modified analytical model:

•Allows for improved fit between measured and simulated drawdowns

•Makes it possible to run the test for a shorter time and still get good parameter estimates

•Specific yield is thus a characteristic property of the aquifer and does not depend on the length of the test

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•Allows for improved fit between measured and simulated drawdowns

•Makes it possible to run the test for a shorter time and still get good parameter estimates

•Specific yield is thus a characteristic property of the aquifer and does not depend on the length of the test

•It can be shown that only a few strategically-located piezometers are needed to obtain accurate parameter estimates.

Drawbacks to the modified analytical model:

- Because of varying antecedent conditions in the unsaturated zone, the set of empirical parameters applies only to the test for which it was estimated.

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•Because of hysteresis, the set of empirical parameters does not apply to analysis of recovery data.

•The empirical parameters relate only indirectly to the hydraulic properties of the unsaturated zone.

PART II

Analysis by numerical model VS2DT* using Brooks and Corey relations

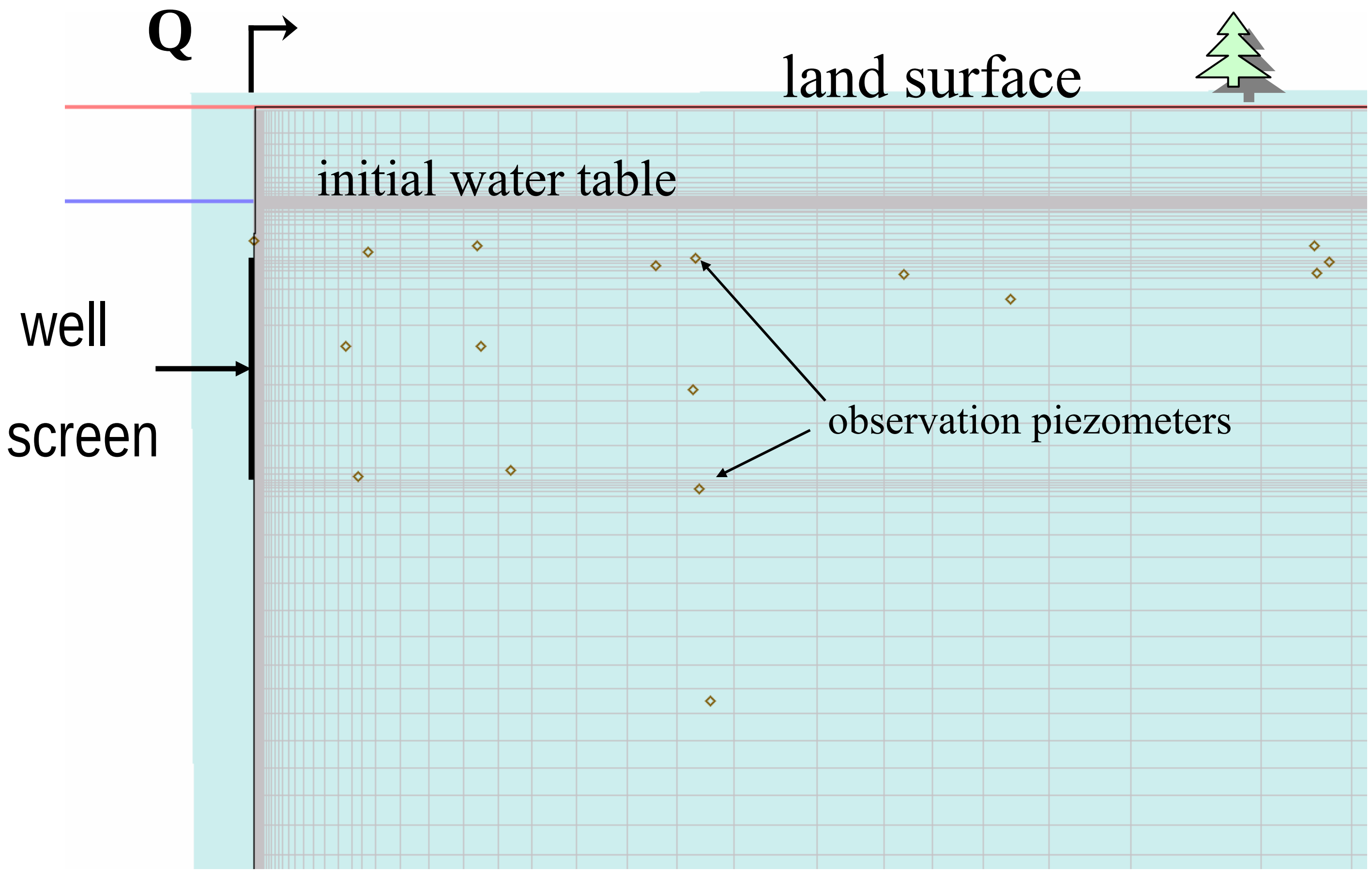
* Healy, 1990, USGS WRI 90-4025

Photo of the actual (physical) upper boundary of the aquifer:

VS2DT

The USGS numerical model (vs2dt) for variably saturated flow and transport in porous media makes use of *analytical functional relations* to describe the hydraulic characteristics of the unsaturated zone.

2D-axisymmetric simulation of the Cape Cod aquifer test using the VS2DT numerical model



Unconfined aquifer parameters to be estimated using PEST and VS2DT:

θ_r residual moisture content

h_b air-entry pressure head [m]

λ Brooks and Corey pore-size distribution index

K_z saturated vertical hydraulic conductivity [m/min]

K_r saturated horizontal hydraulic conductivity [m/min]

Brooks and Corey's (1964) analytical functional relations for unsaturated-zone characteristics:

Soil-moisture retention:

$$\theta = \theta_r + (\phi - \theta_r)(h_b / h_c)^{\lambda} \qquad h_c < h_b$$

$$\theta = \phi \qquad h_c \geq h_b$$

where:

θ volumetric moisture content

θ_r residual moisture content

ϕ porosity

h_b air-entry (or bubbling) pressure head (h_b<0)

h_c capillary pressure head (h_c<0)

λ pore-size distribution index

Relative hydraulic conductivity:

$$K_{rel} = (h_c / h_b)^{-2-3\lambda} \qquad h_c < h_b$$

$$K_{rel} = 1 \qquad h_c \geq h_b$$

where:

K_{rel} is the ratio $K(\theta)/K_z$

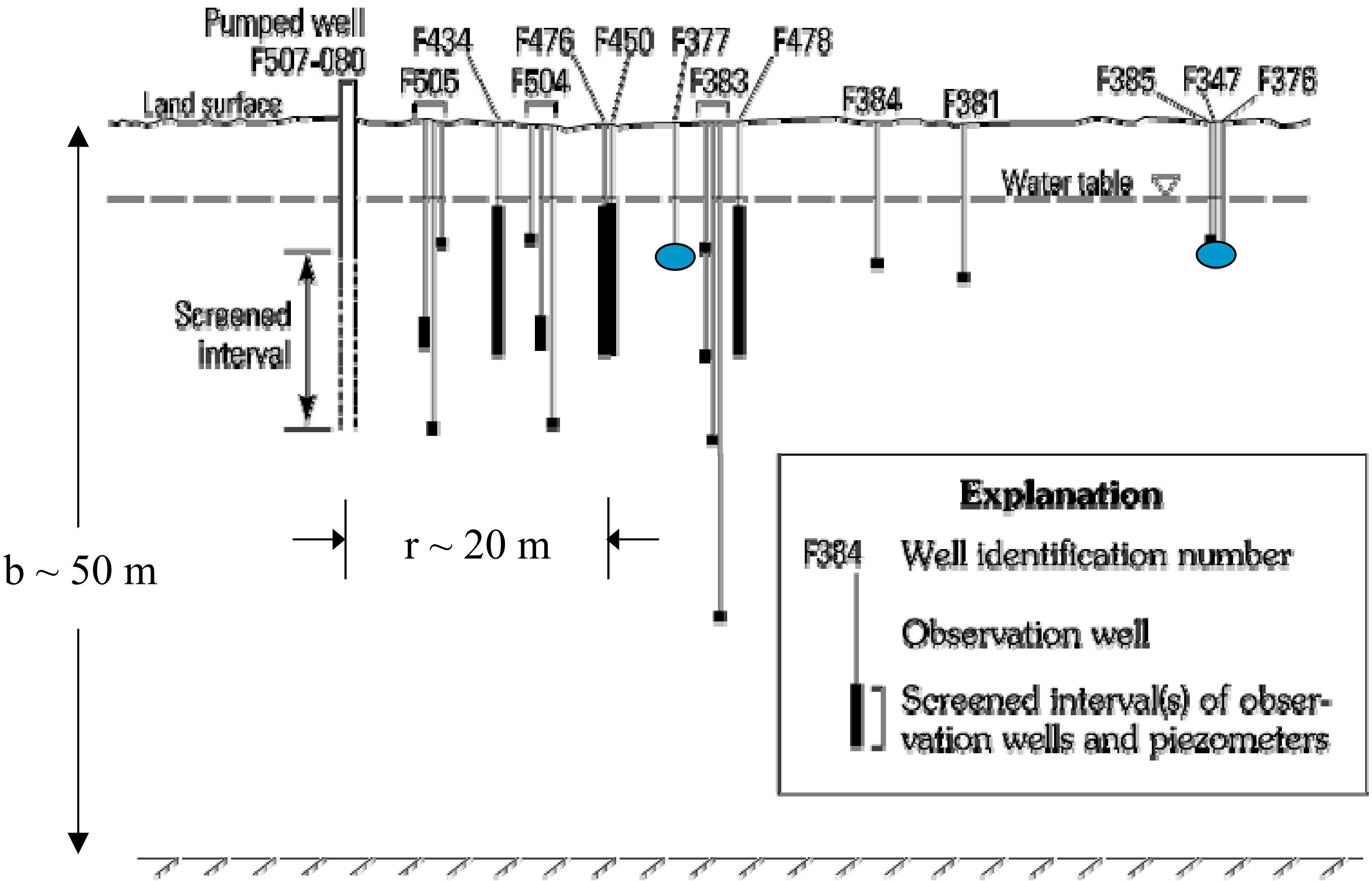
$K(\theta)$ is the hydraulic conductivity as a function of volumetric moisture content

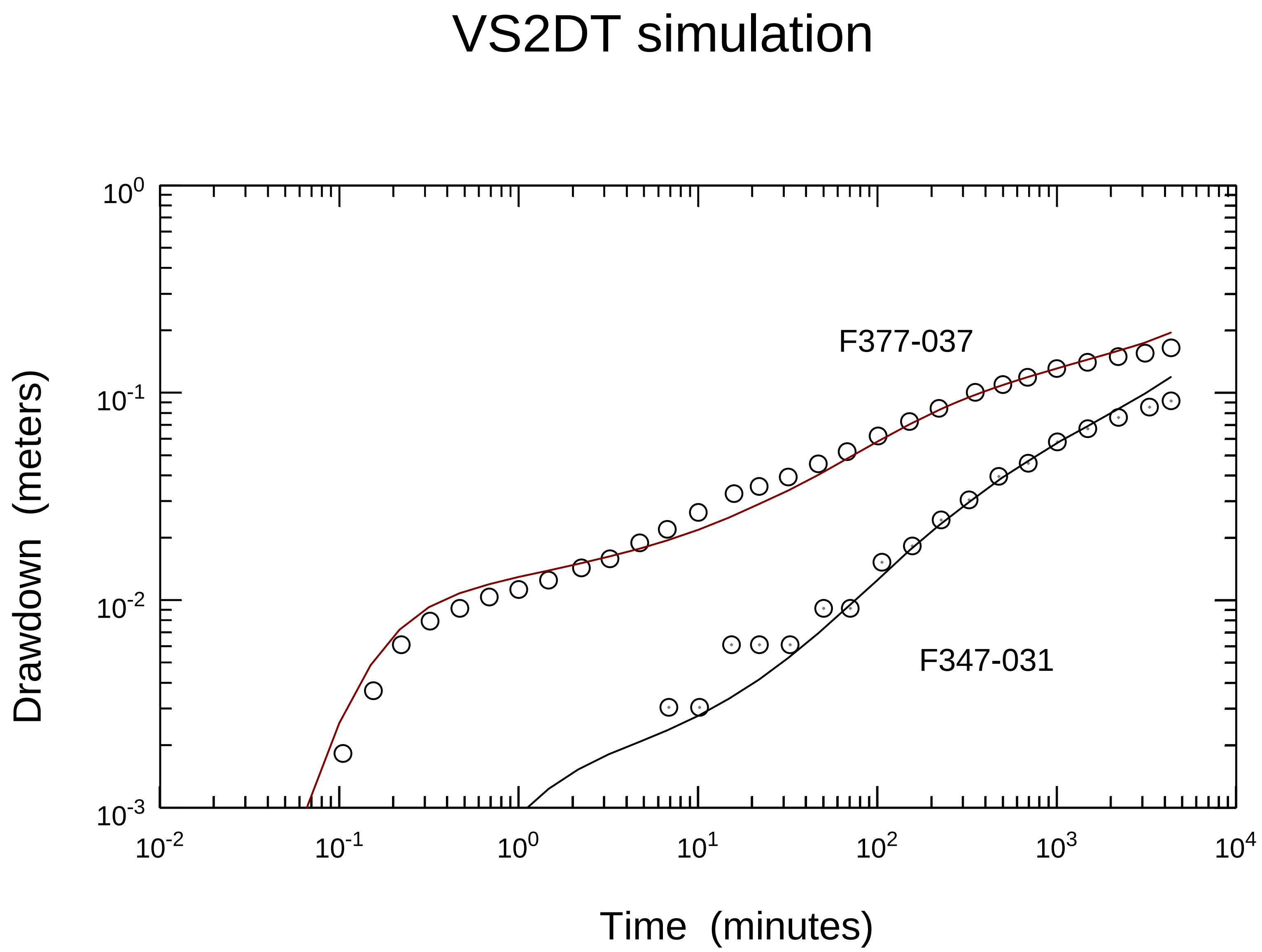
K_z is saturated hydraulic conductivity

Results obtained using all data excluding that from the four long-screened piezometers:

PEST runs and RMSE values (m)	Parameter	Estimated Value	95% Confidence Limits		Initial Value
			Lower limit	Upper limit	
Aquifer-test simulation 1.04E-02	θ_r	0.0001	0.0	0.026	0.01
	h_b (m)	-0.40	-0.428	-0.380	-0.5
	λ	0.48	0.377	0.585	0.4
	K_z (m/s)	6.40E-04	5.86E-04	6.99E-04	5.0E-04
	K_r (m/s)	1.1E-03	1.09E-03	1.13E-03	5.0E-04

VS2DT response in two shallow piezometers




VS2DT simulation
Drawdown (meters)
Time (minutes)
F377-037
F347-031
10^0
10^-1
10^-2
10^-3
10^-2
10^-1
10^0
10^1
10^2
10^3
10^4

General comment:

The unsaturated-zone characteristics appear to be more representative of fine-grained materials (*i.e.*, small λ large capillary fringe h_b) than the coarse-grained material of which the aquifer is composed!

Results suggest that relatively small amounts of fine-grained material dominate the large-scale soil-moisture distribution and aquifer response to pumping.

Warning:

There are no data with which to check on the estimated Brooks and Corey parameters obtained for the Cape Cod site!

Borden Site
Ontario, Canada

Acknowledgement: This study would have been impossible without the generosity of Michael Bevan[1] who provided his thesis and data set and Prof. Anthony Endres[2].

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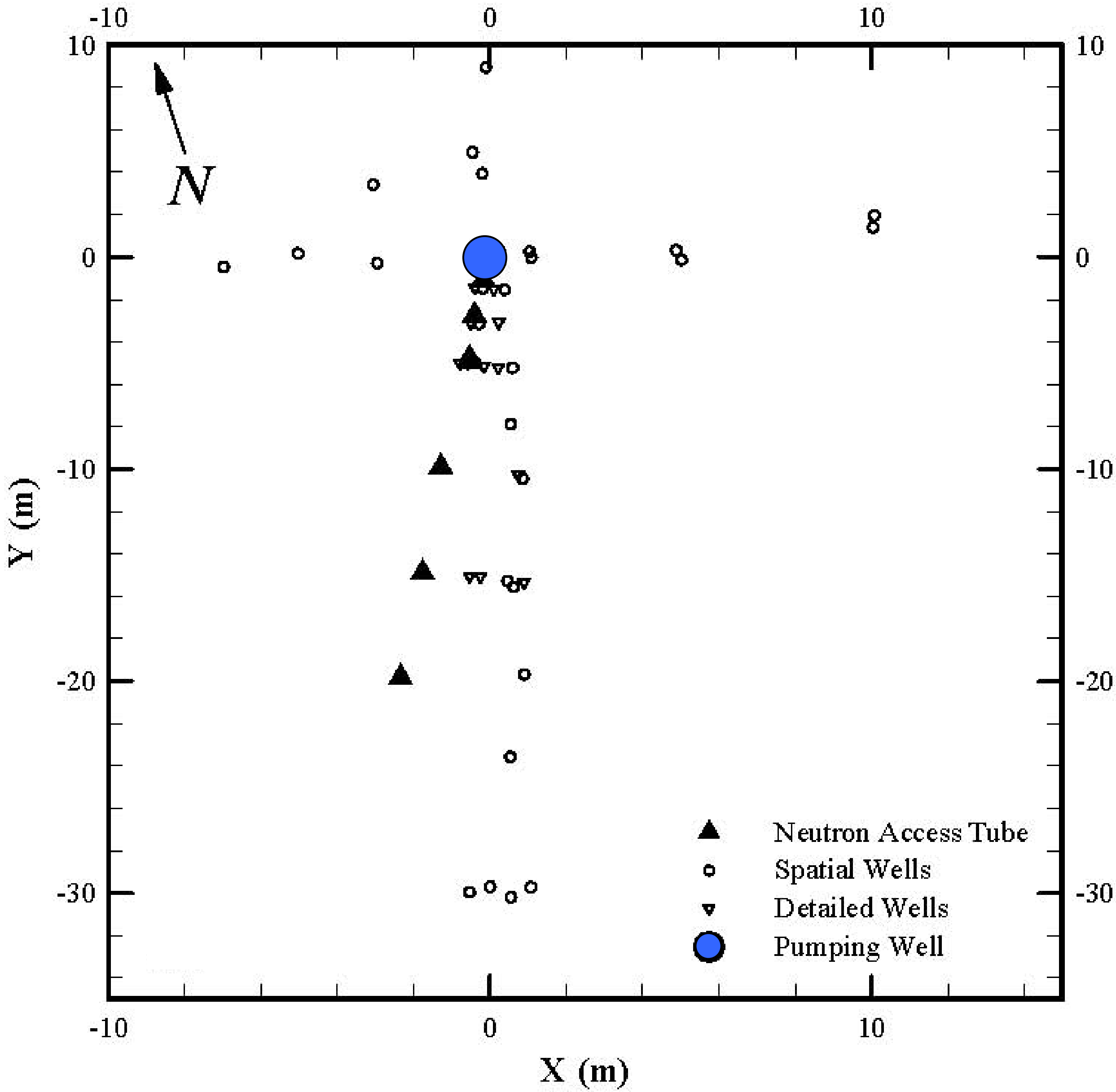
[1] in partial fulfillment of an M.S. degree, 2002

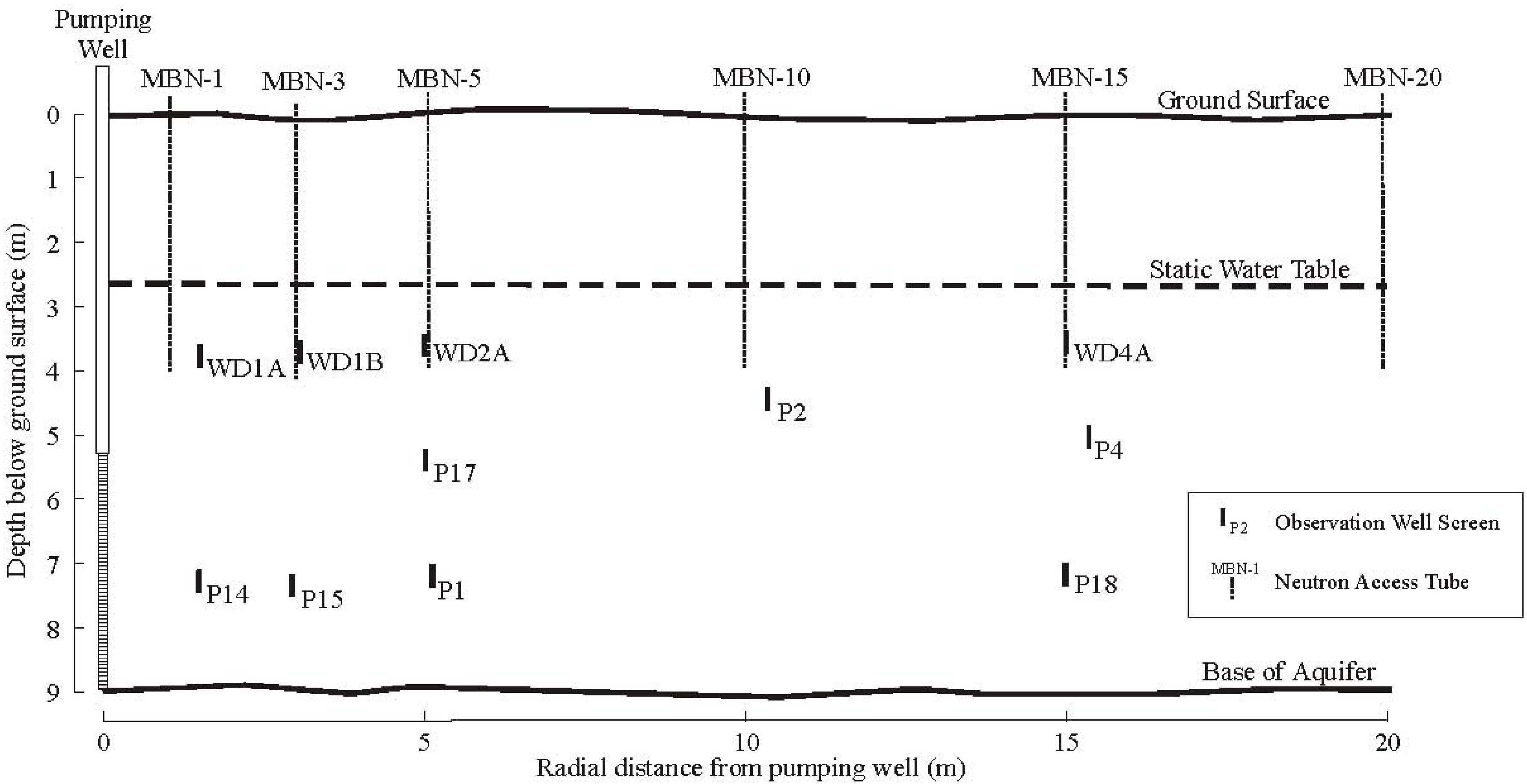
[2] Dept. of Earth Sciences, University of Waterloo, Waterloo, Ontario, Canada

Plan View at Borden Site (after Bevan, 2002)



Vertical Section at Borden Site (after Bevan, 2002)

-locations of piezometers with transducers and neutron access tubes



Hydrogeologic Characteristics

- Medium-grained unconsolidated sand of glacio-deltaic or glacio-fluvial origin composed of interbedded lenses of fine-, medium-, and coarse-grained sand
- Slightly anisotropic $K_z/K_r \sim 0.5$
- Underlain by clayey silt aquitard at ~ 9 m below land surface.

Borden-Site	Cape Cod
K_r = 6.4E-05 m/s	K_r = 110.E-05 m/s
K_z = 3.2E-05 m/s	K_z = 64.E-05 m/s
b = 6.25 m	b = 50 m
Q = 40 L/min	Q = 1210 L/min

Test Details:

7-Day constant rate test
Q = 40 L/min

23 observation piezometers

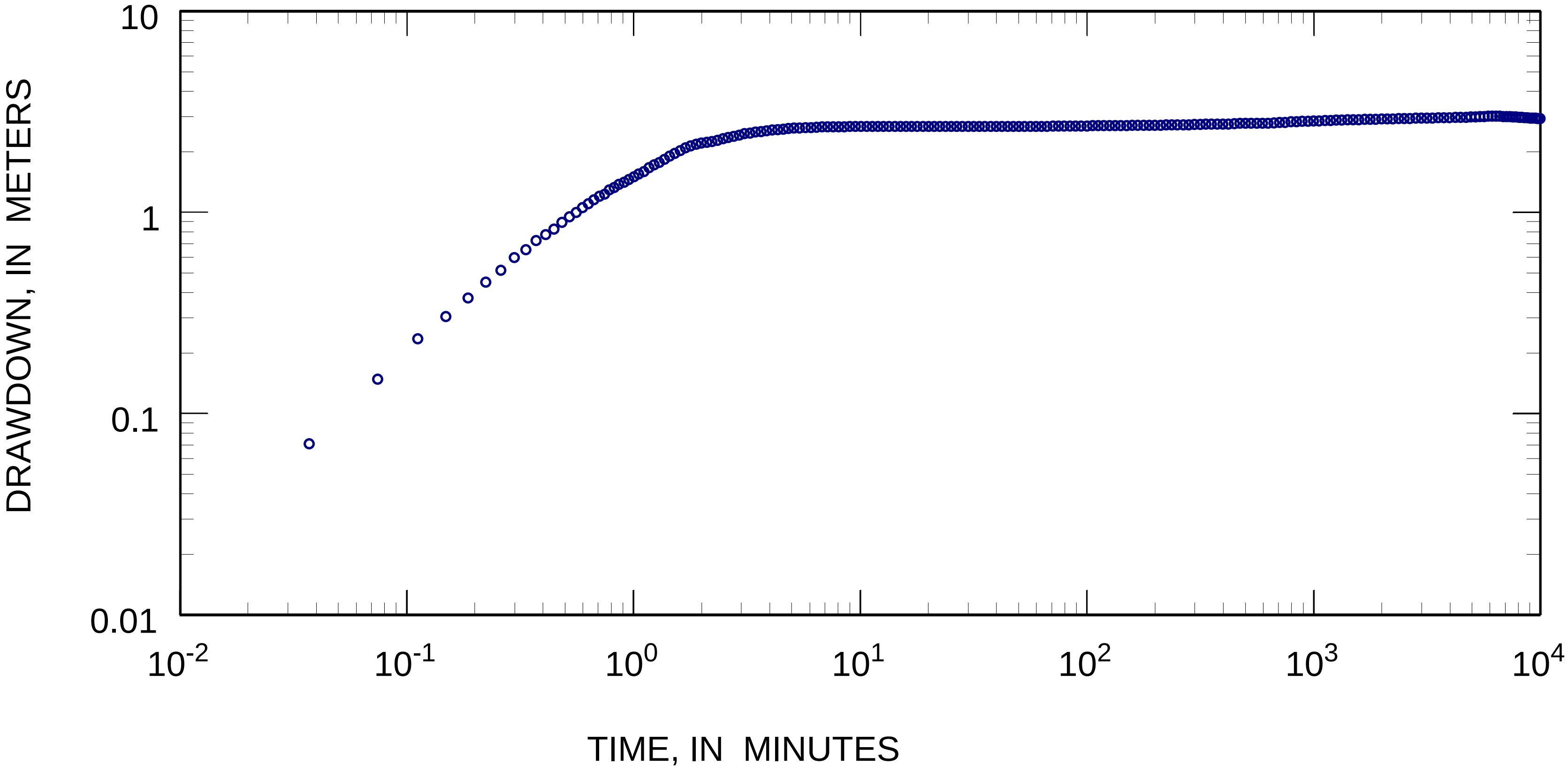
- 11 with transducers
- 12 measured by hand twice daily

Six neutron moisture-probe access tubes

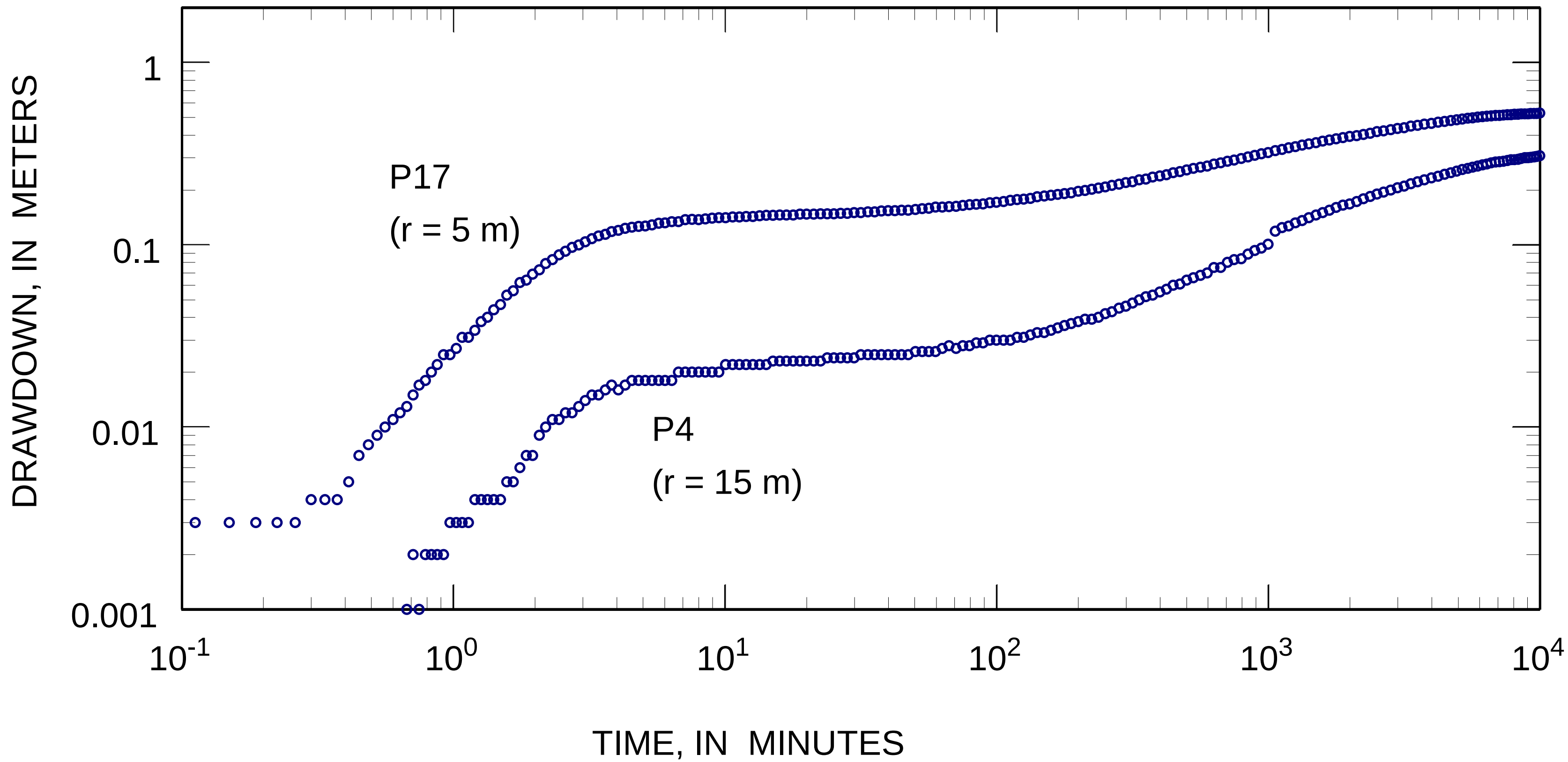
- 4.0 m in length

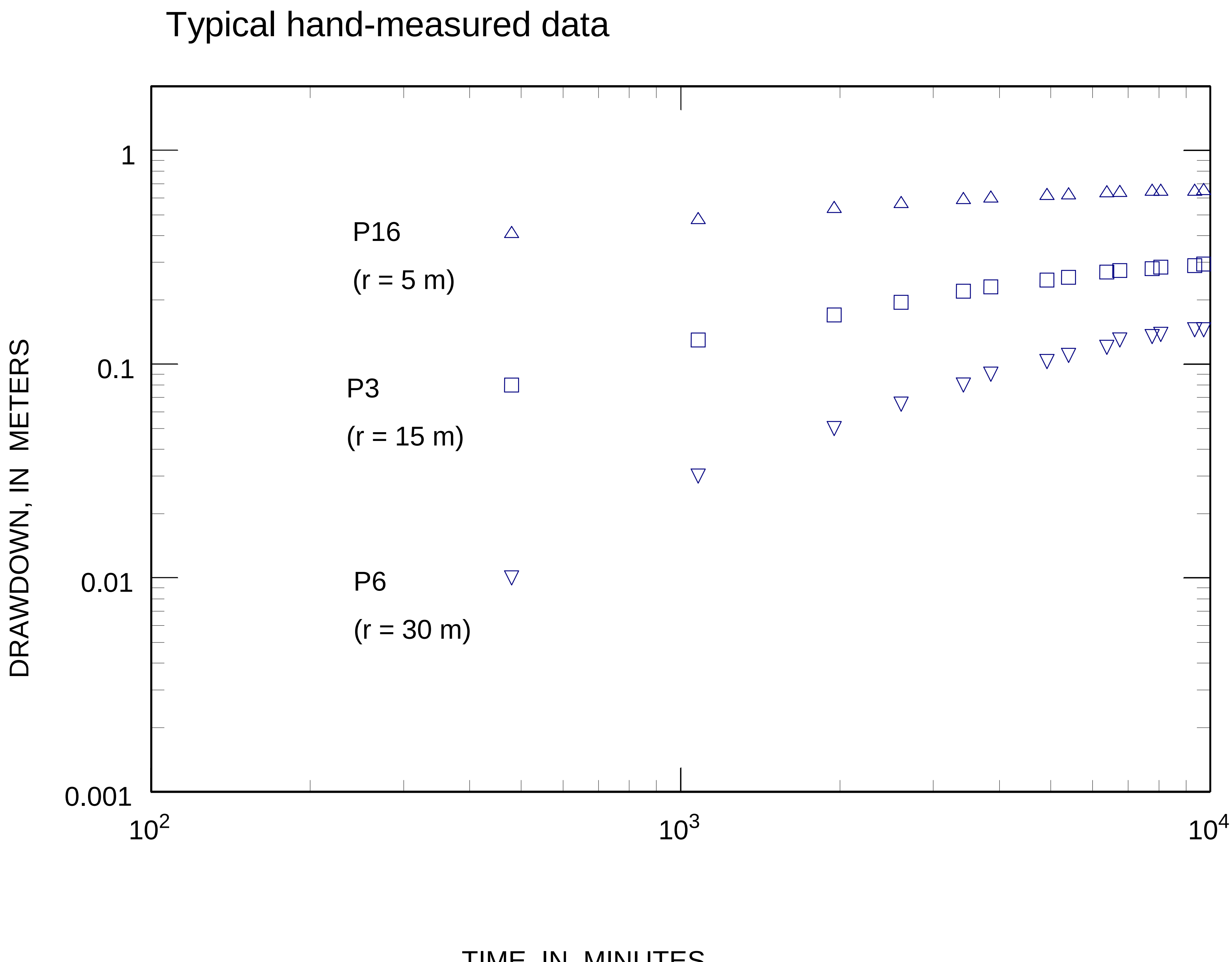
Initial depth of water table ~ 2.75 m

Pumped-well data

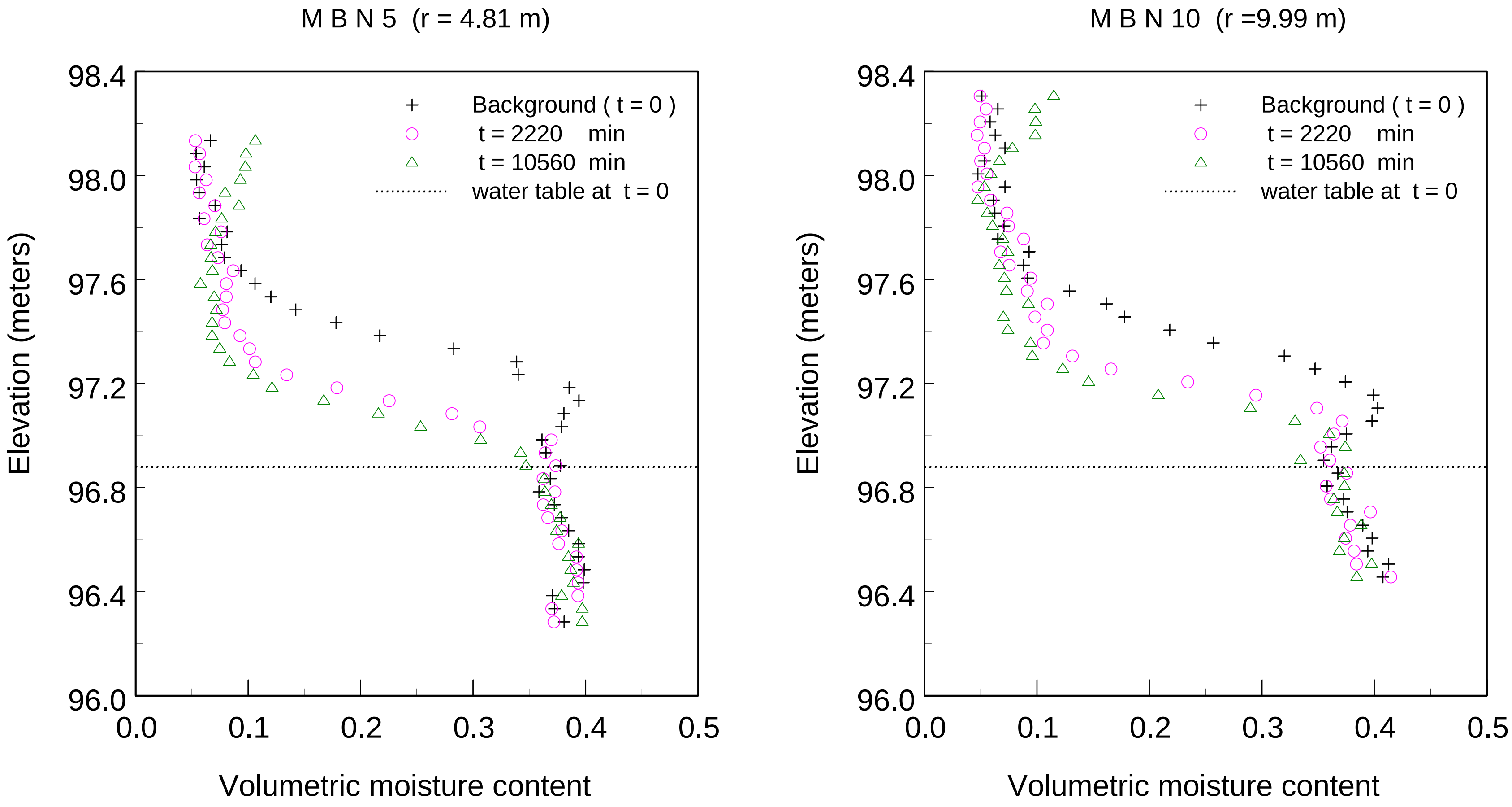


Typical transducer data




Typical hand-measured data
P16
(r = 5 m)
P3
(r = 15 m)
P6
(r = 30 m)
1
0.1
0.01
0.001
DRAWDOWN, IN METERS
10^2
10^3
10^4
TIME, IN MINUTES

Typical soil-moisture measurements



PART I

Analysis by analytical model WTAQ

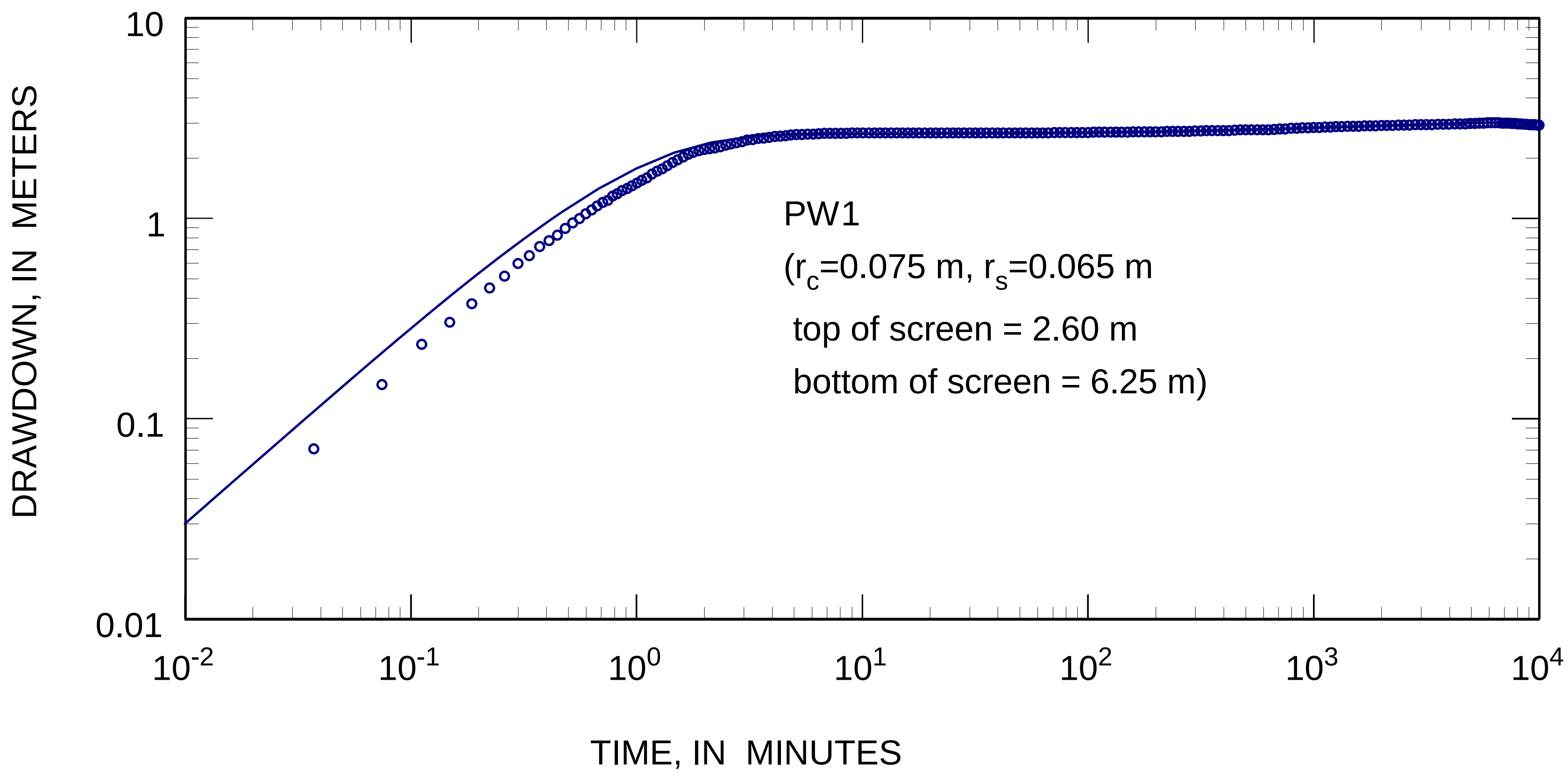
Model application

Using PEST, the WTAQ model is applied to all Borden-site data (*i.e.*, the pumped well and 23 observation piezometers).

Estimated parameters

Parameter	Estimated Value	95% Confidence Limits		Initial Value
		Lower limit	Upper limit	
S_s (m^{-1})	**3.46E-05**	**1.67E-05**	**7.15E-05**	**1.E-05**
S_y	**0.246**	**0.218**	**0.276**	**0.1**
b (m)	**6.16**	**5.78**	**6.53**	**10.**
K_r (m/s)	**6.79E-05**	**6.28E-05**	**7.35E-05**	**1.E-02**
K_z (m/s)	**3.11E-05**	**2.83E-05**	**3.43E-05**	**1.E-02**
α_1 (min^{-1})	1.74E-04	6.31E-05	4.84E-04	1.E-05
α_2 (min^{-1})	8.57E-03	3.51E-03	2.09E-02	1.E-03
α_3 (min^{-1})	4.63E-02	2.39E-02	8.95E-02	1.E-01

Simulated responses by WTAQ compared with measured drawdown:

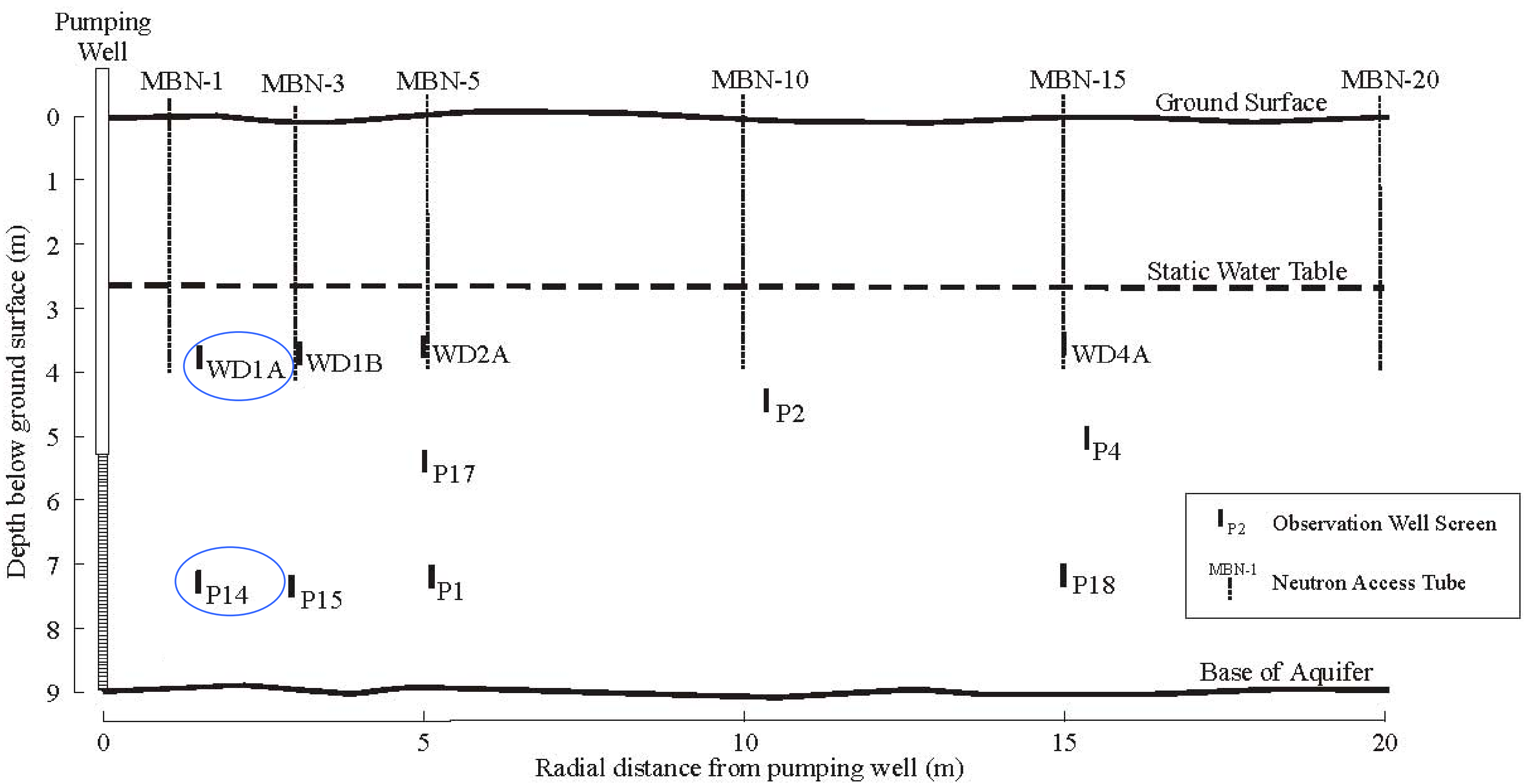


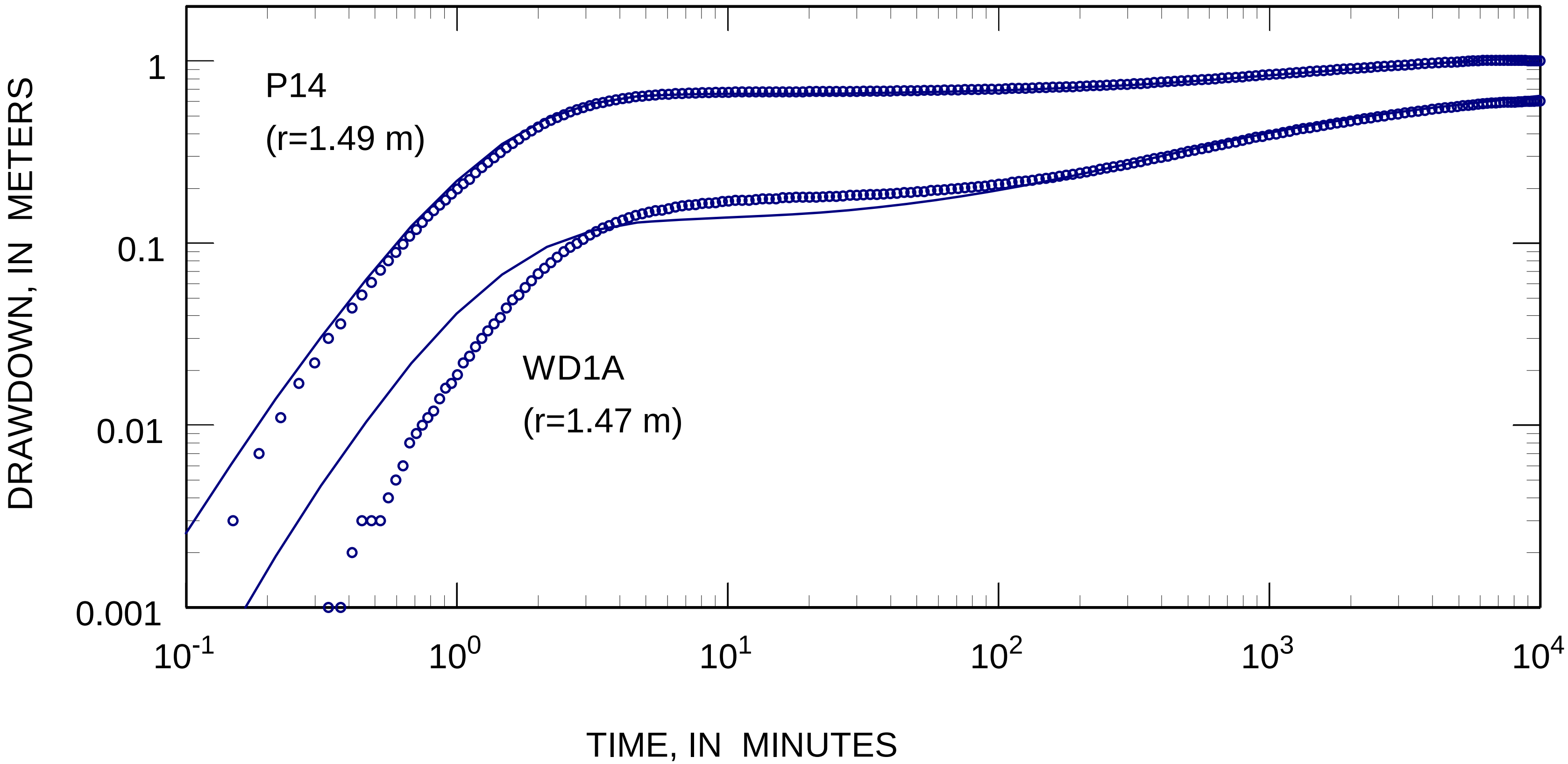
wtaq3

<u>Note:</u>

The 3 m drawdown in the pumped well is due to wellbore skin. In the aquifer adjacent to the well, drawdown is a maximum of 0.7 m.

Vertical Section at Borden Site

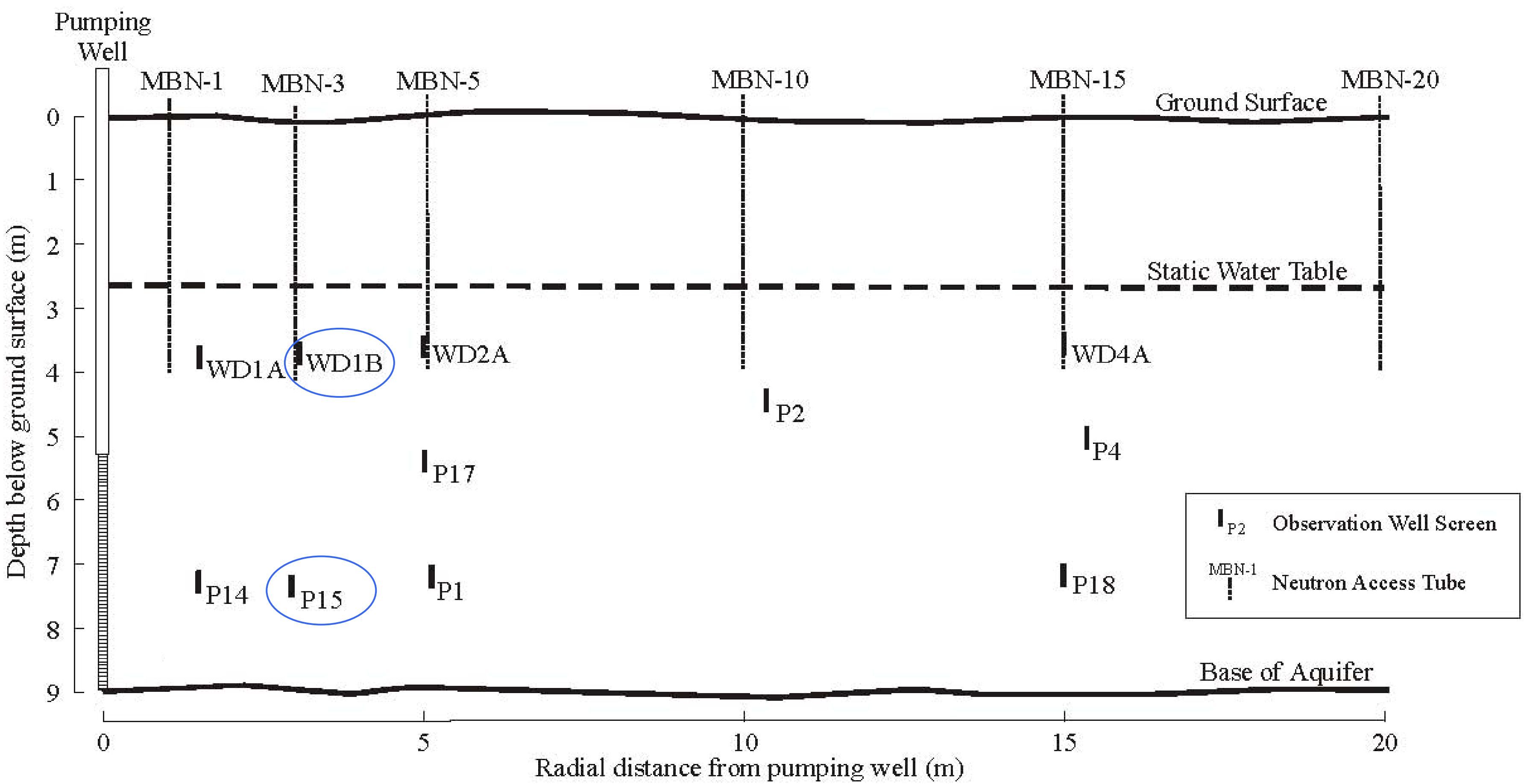


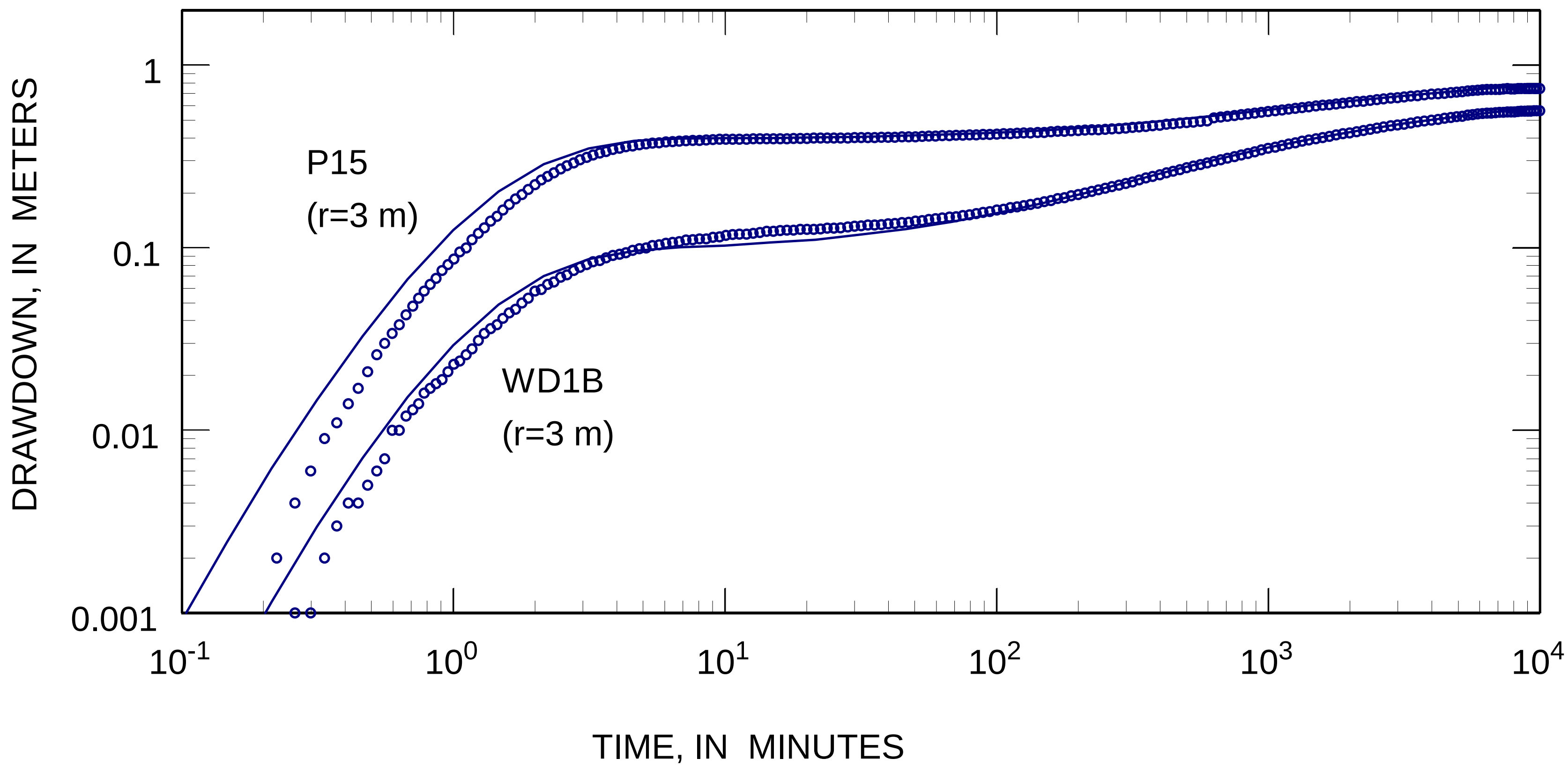


P14
(r=1.49 m)
WD1A
(r=1.47 m)
DRAWDOWN, IN METERS
1
0.1
0.01
0.001
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
TIME, IN MINUTES

wtaq3

Vertical Section at Borden Site

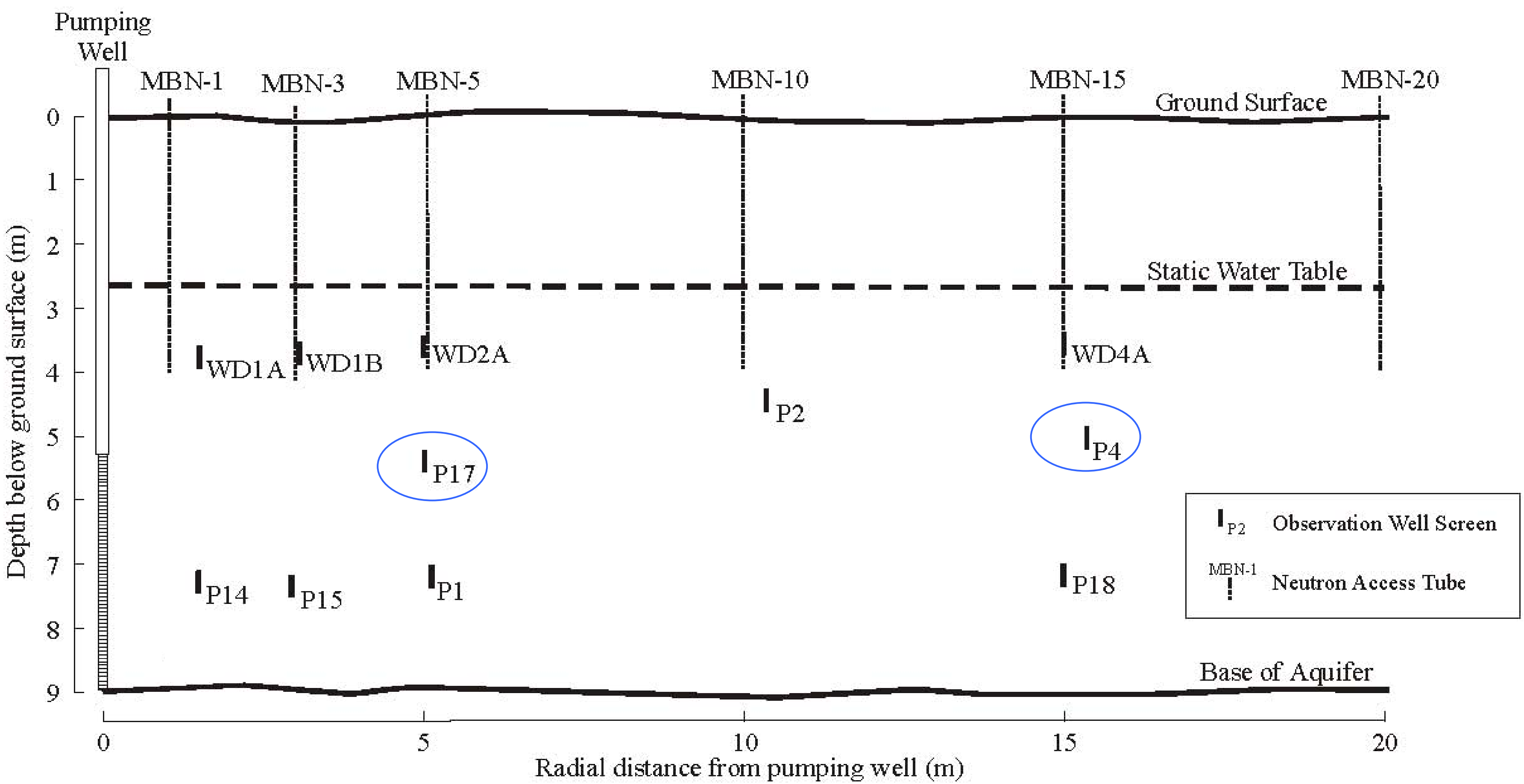


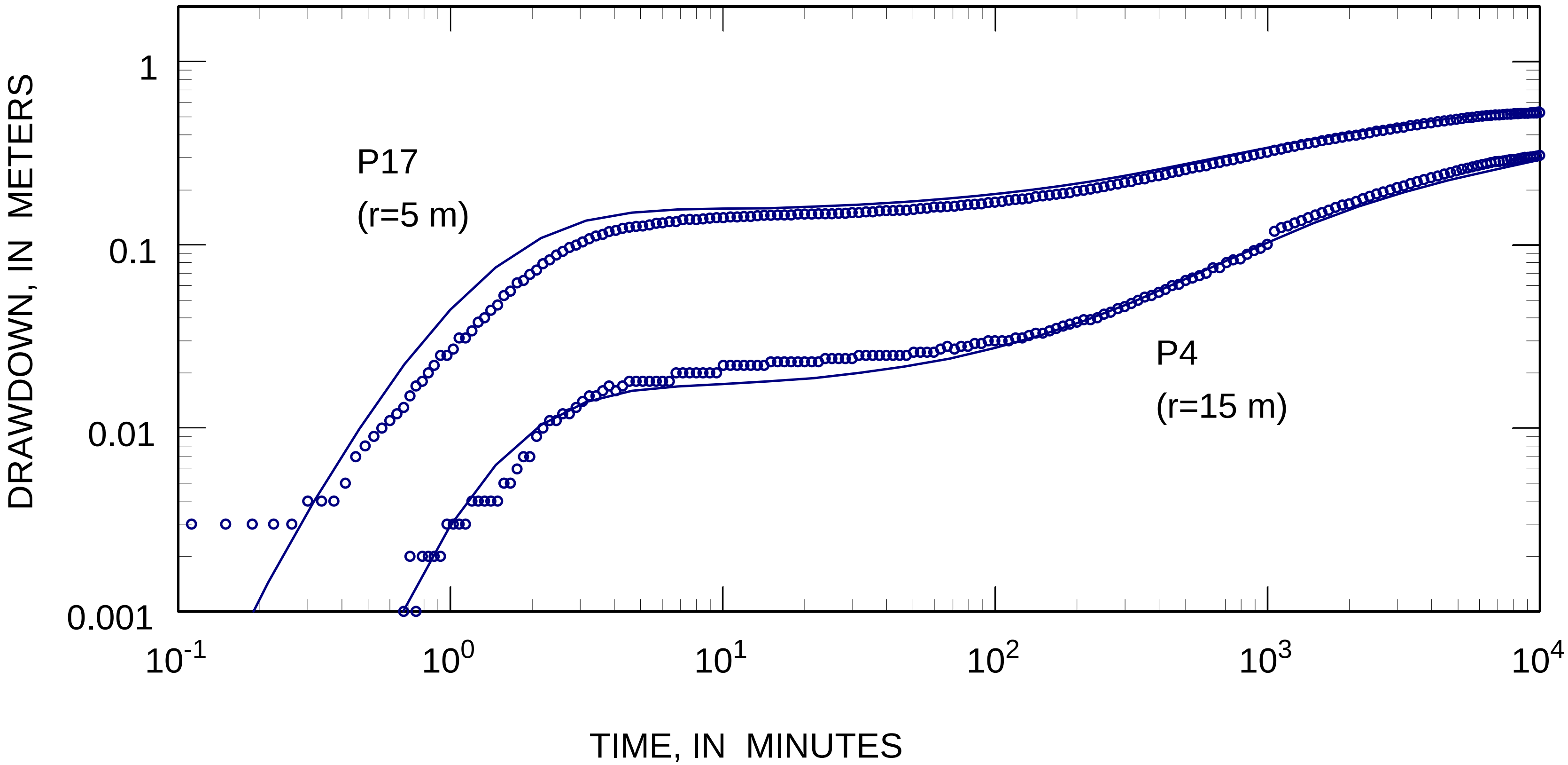


DRAWDOWN, IN METERS
1
0.1
0.01
0.001
P15
(r=3 m)
WD1B
(r=3 m)
10-1
100
101
102
103
104
TIME, IN MINUTES

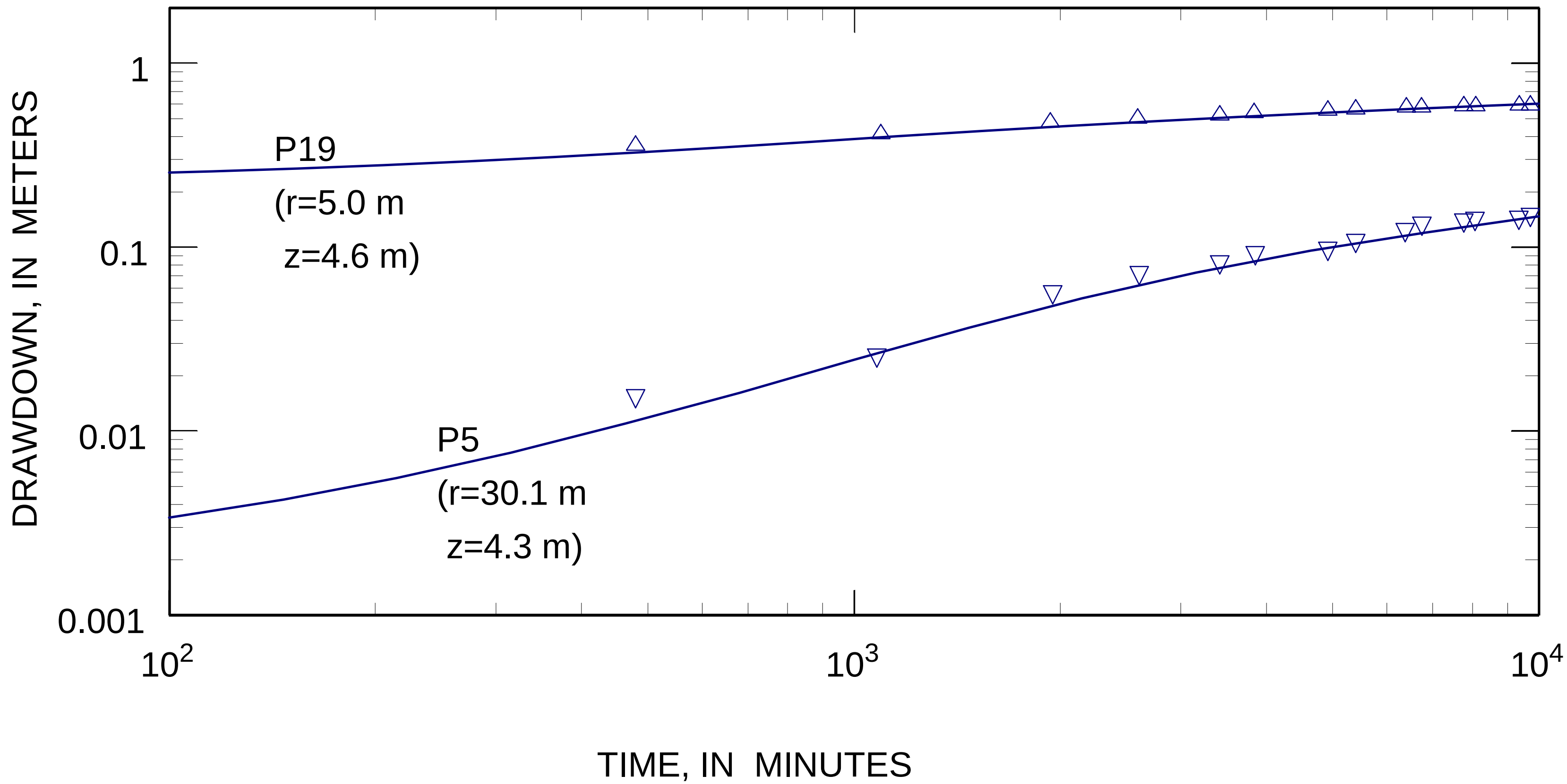
wtaq3

Vertical Section at Borden Site

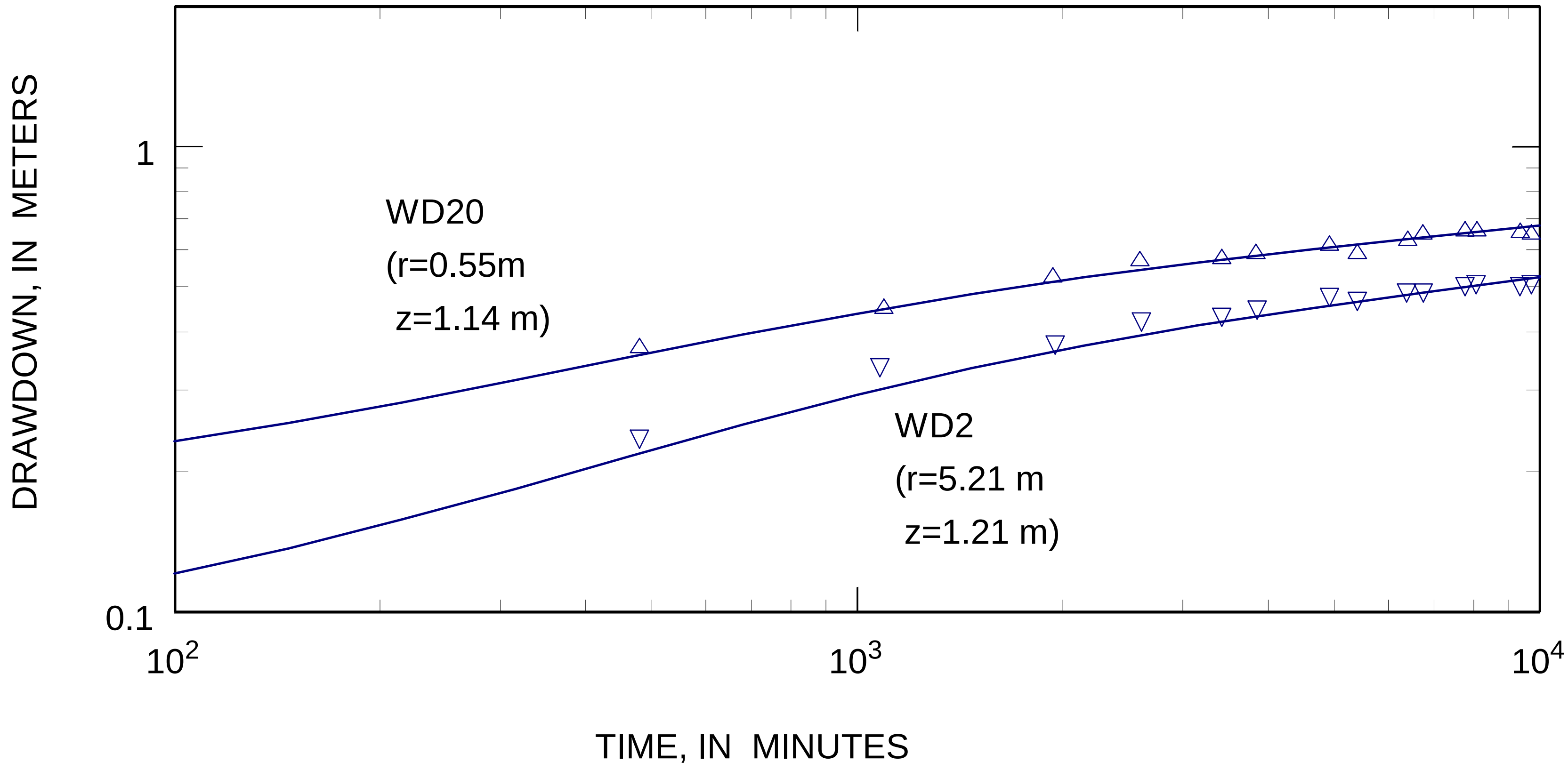



DRAWDOWN, IN METERS
1
0.1
0.01
0.001
P17
(r=5 m)
P4
(r=15 m)
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
TIME, IN MINUTES

wtaq3


DRAWDOWN, IN METERS
1
0.1
0.01
0.001
P19
(r=5.0 m
z=4.6 m)
P5
(r=30.1 m
z=4.3 m)
10^2
10^3
10^4
TIME, IN MINUTES

wtaq3


DRAWDOWN, IN METERS
1
0.1
WD20
(r=0.55m
z=1.14 m)
WD2
(r=5.21 m
z=1.21 m)
10^2
10^3
10^4
TIME, IN MINUTES

wtaq3

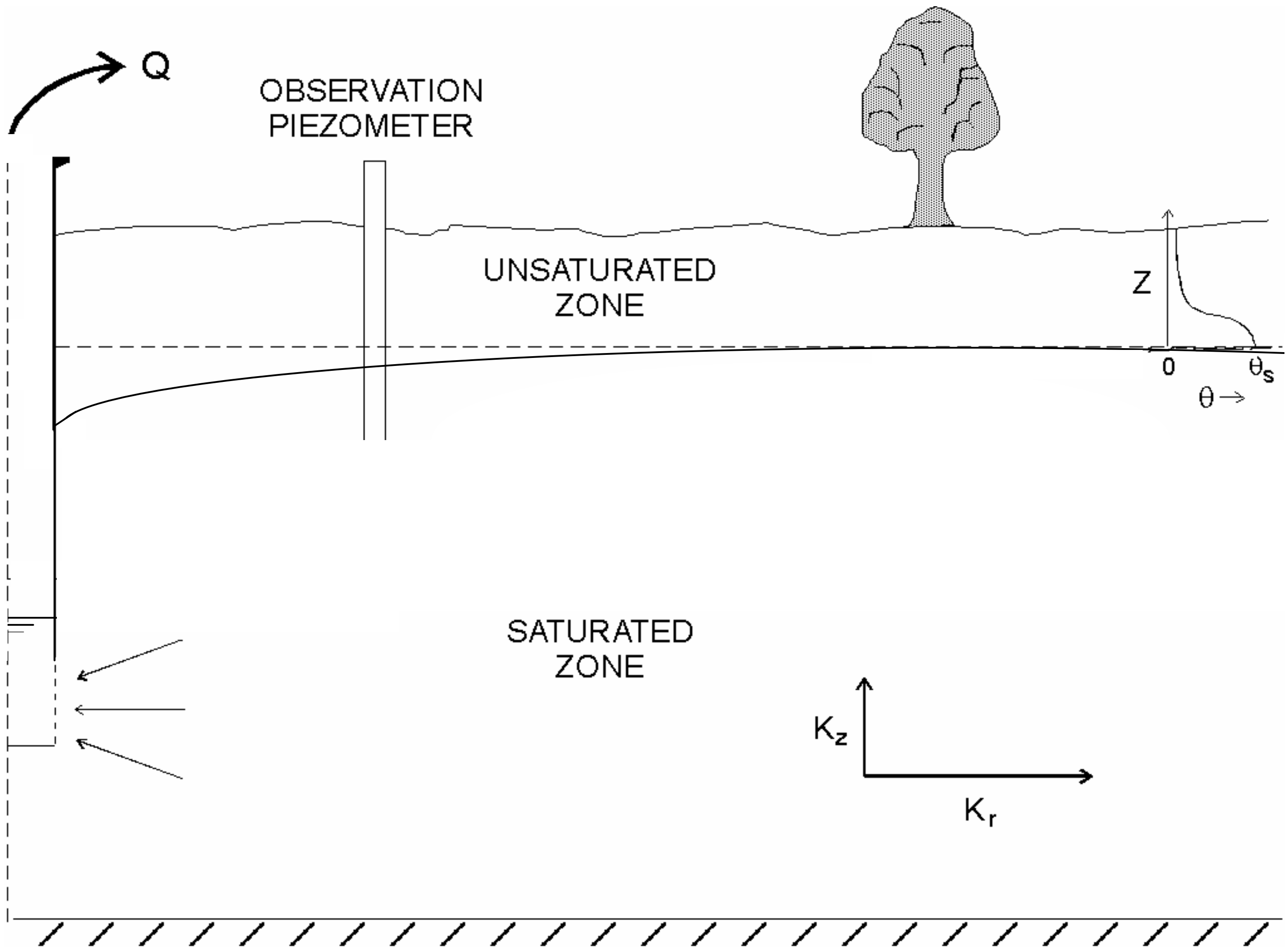
PART II

Analysis by the numerical model VS2DT using Brooks and Corey relations or a modification of the Brooks and Corey relations

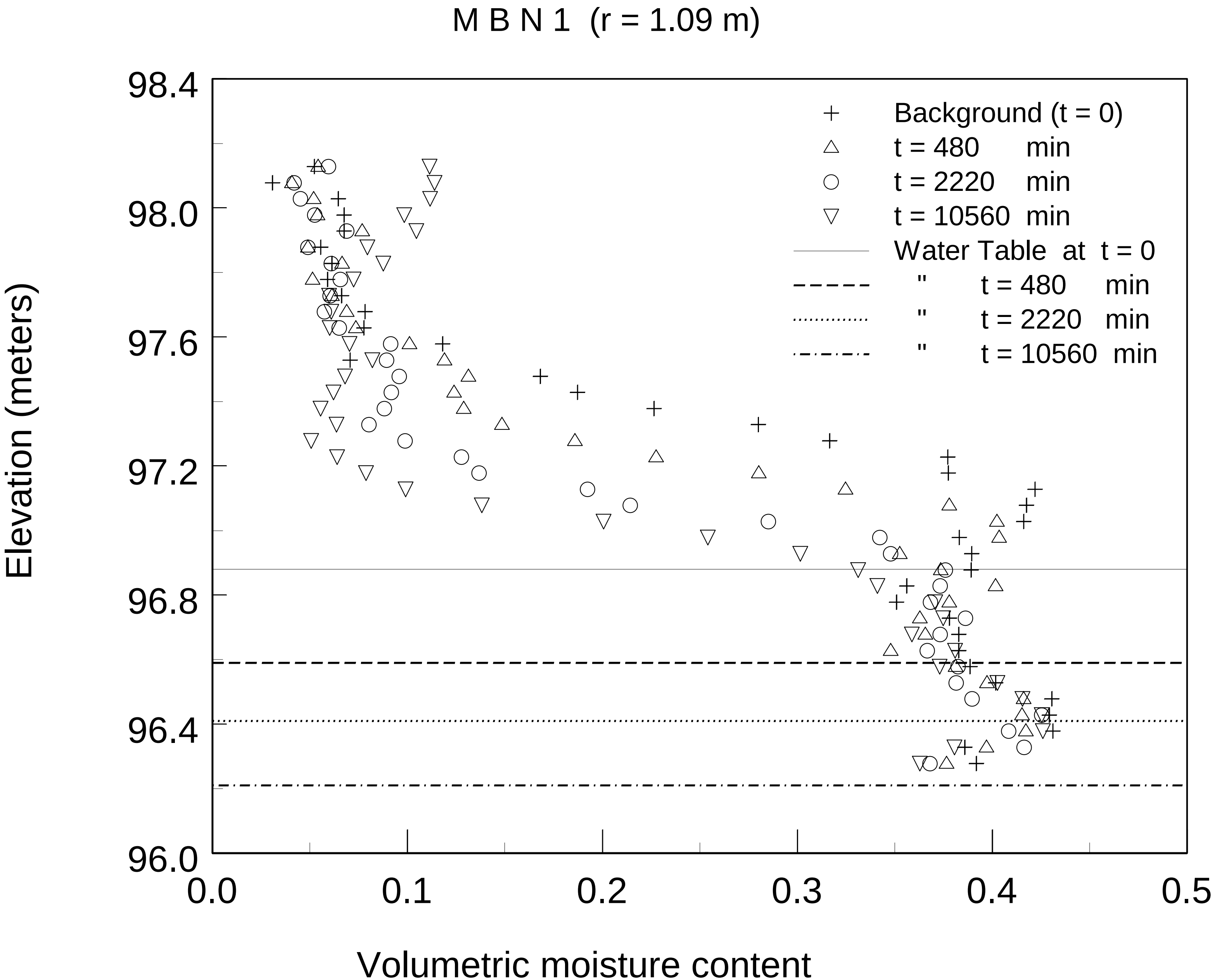
<u>Objectives:</u>

1. Obtain unsaturated-zone hydraulic properties
2. Explain observed extensions of the capillary fringe

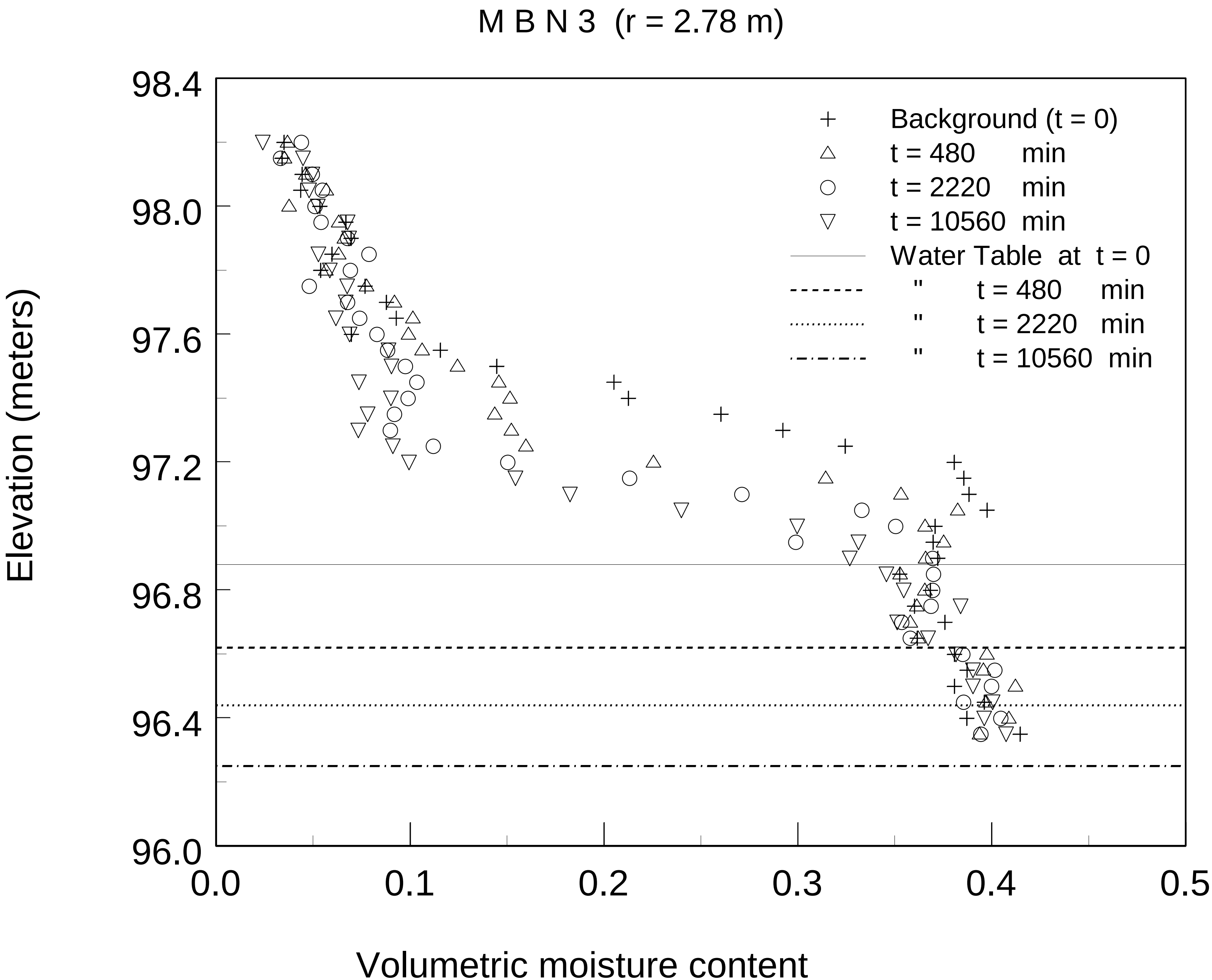
Schematic diagram of homogeneous aquifer

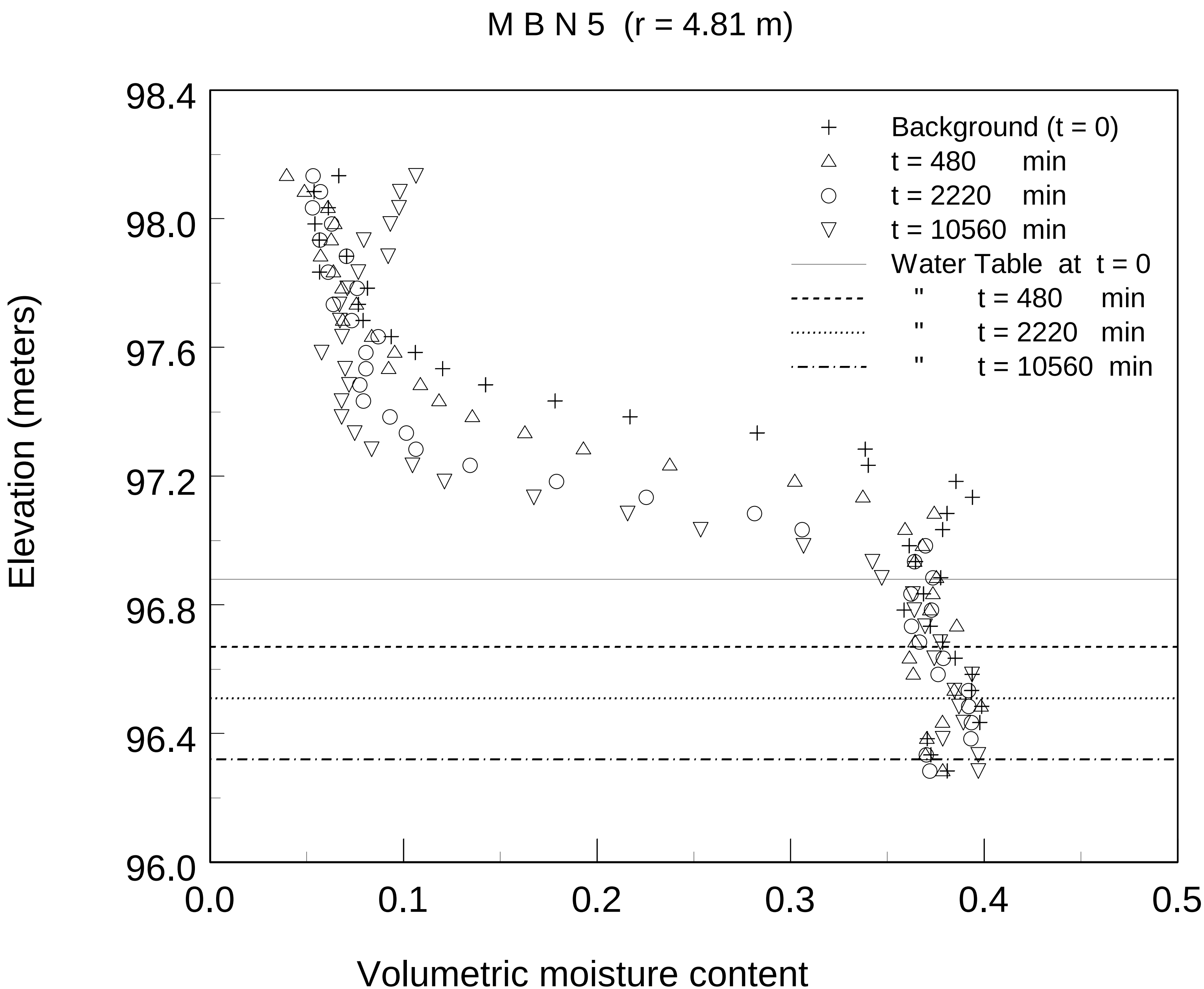


Soil-Moisture Measurements

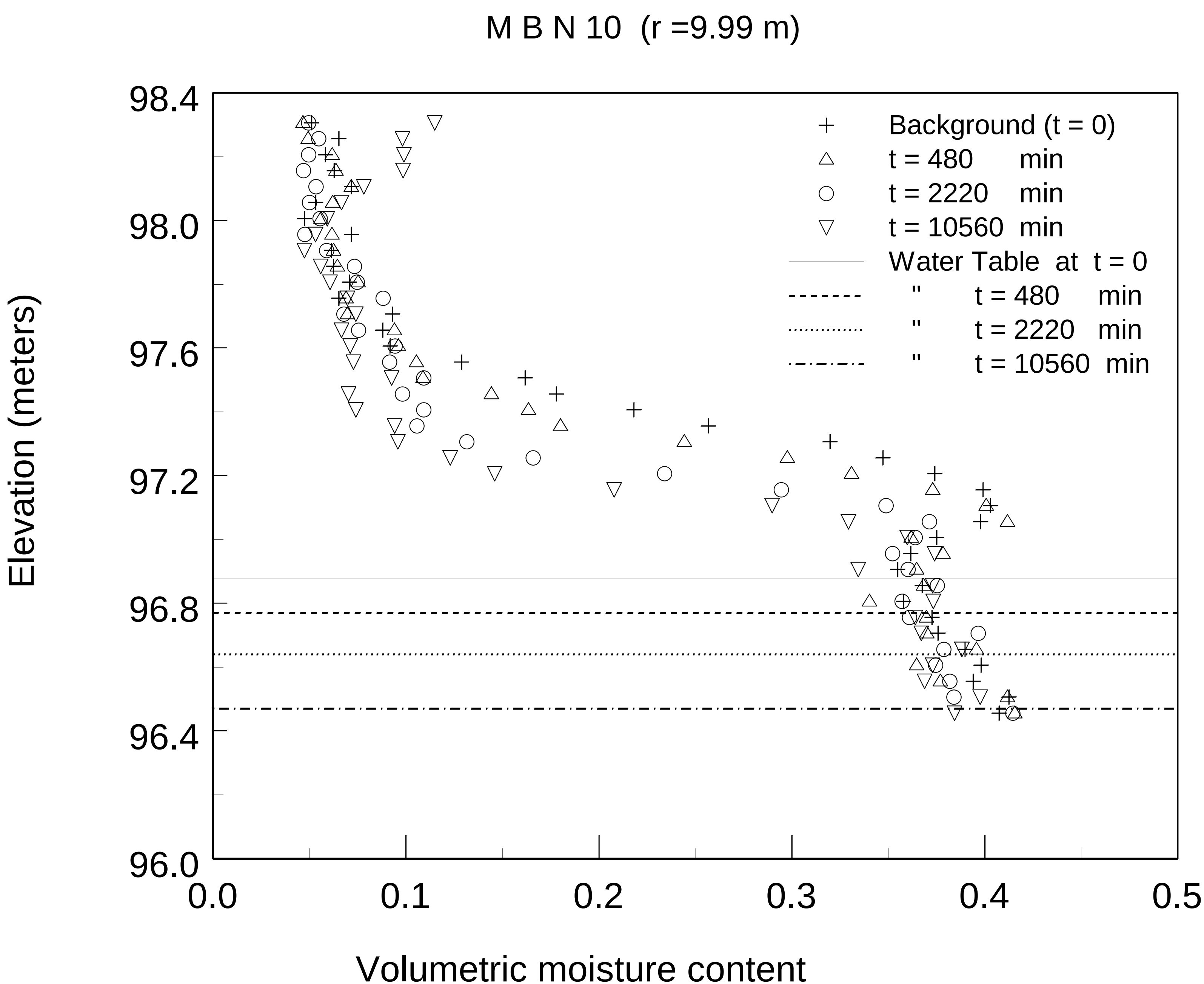


M B N 1 (r = 1.09 m)
Elevation (meters)
98.4
98.0
97.6
97.2
96.8
96.4
96.0
0.0
0.1
0.2
0.3
0.4
0.5
Volumetric moisture content
Background (t = 0)
t = 480 min
t = 2220 min
t = 10560 min
Water Table at t = 0
" t = 480 min
" t = 2220 min
" t = 10560 min

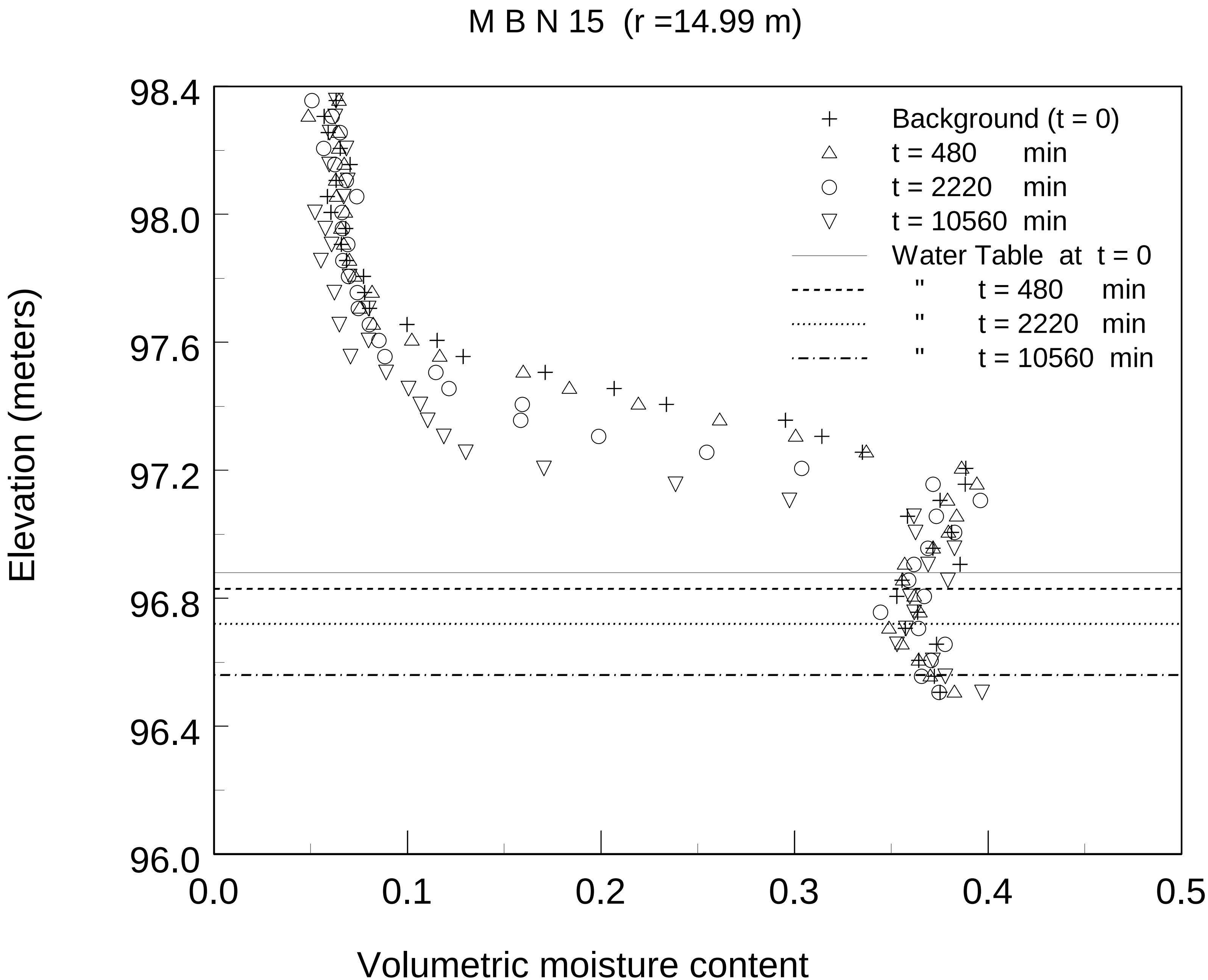

M B N 3 (r = 2.78 m)
Elevation (meters)
Volumetric moisture content
98.4
98.0
97.6
97.2
96.8
96.4
96.0
0.0
0.1
0.2
0.3
0.4
0.5
Background (t = 0)
t = 480 min
t = 2220 min
t = 10560 min
Water Table at t = 0
" t = 480 min
" t = 2220 min
" t = 10560 min

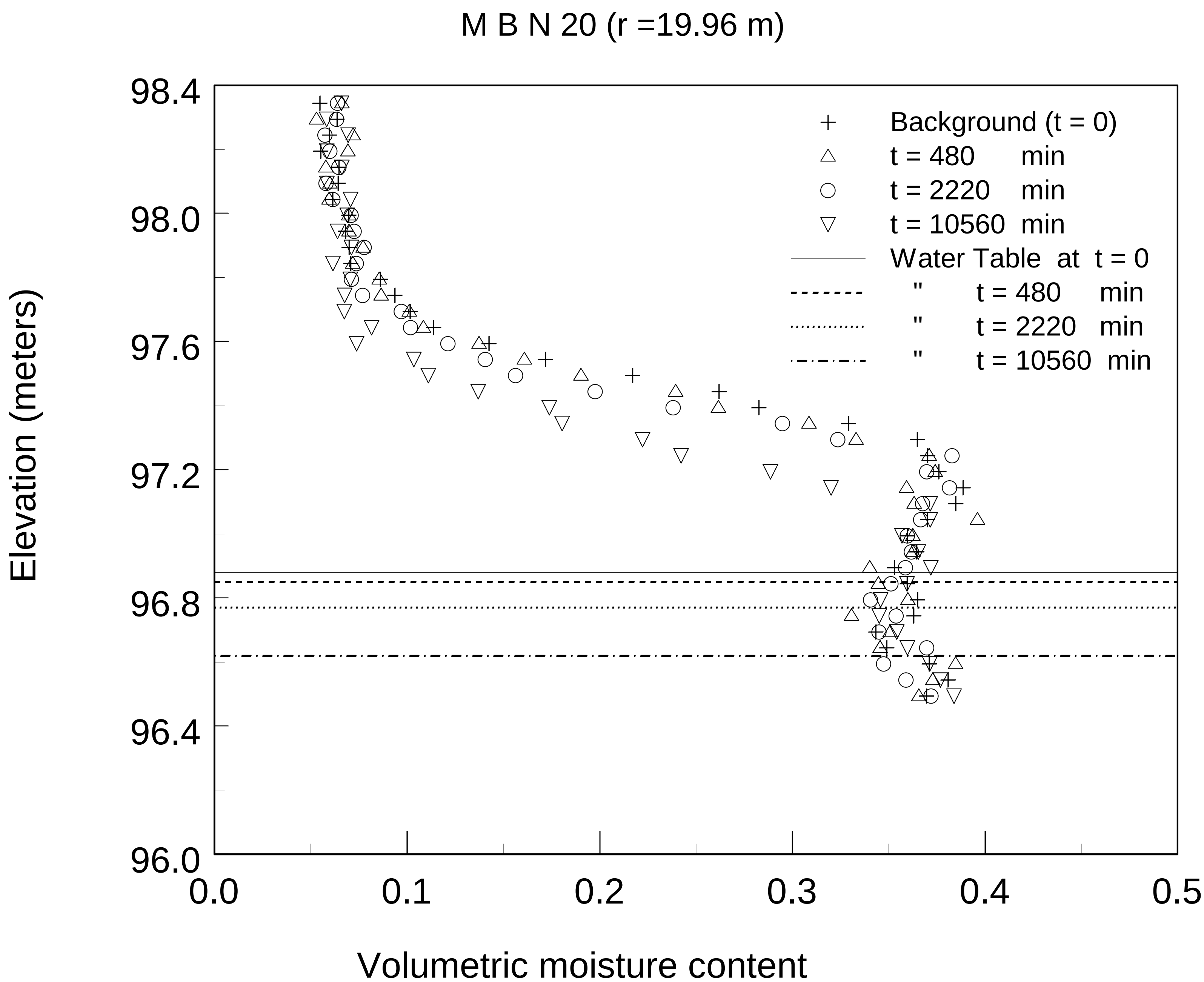


M B N 5 (r = 4.81 m)
Elevation (meters)
Volumetric moisture content
98.4
98.0
97.6
97.2
96.8
96.4
96.0
0.0
0.1
0.2
0.3
0.4
0.5
Background (t = 0)
t = 480 min
t = 2220 min
t = 10560 min
Water Table at t = 0
" t = 480 min
" t = 2220 min
" t = 10560 min



M B N 10 (r =9.99 m)
Elevation (meters)
98.4
98.0
97.6
97.2
96.8
96.4
96.0
0.0
0.1
0.2
0.3
0.4
0.5
Volumetric moisture content
Background (t = 0)
t = 480 min
t = 2220 min
t = 10560 min
Water Table at t = 0
" t = 480 min
" t = 2220 min
" t = 10560 min


M B N 15 (r =14.99 m)
Elevation (meters)
98.4
98.0
97.6
97.2
96.8
96.4
96.0
0.0
0.1
0.2
0.3
0.4
0.5
Volumetric moisture content
Background (t = 0)
t = 480 min
t = 2220 min
t = 10560 min
Water Table at t = 0
" t = 480 min
" t = 2220 min
" t = 10560 min


M B N 20 (r =19.96 m)
Elevation (meters)
Volumetric moisture content
98.4
98.0
97.6
97.2
96.8
96.4
96.0
0.0
0.1
0.2
0.3
0.4
0.5
Background (t = 0)
t = 480 min
t = 2220 min
t = 10560 min
Water Table at t = 0
" t = 480 min
" t = 2220 min
" t = 10560 min

Capillary fringe extension during the Borden-site aquifer test

Access Tube Name	Radial Distance (meters)	Approximate Height of Capillary Fringe (meters)			
		t = 0	t = 480 (min)	t = 2220 (min)	t = 10560 (min)
MBN1	1.09	0.35	0.51	0.53	0.58
MBN3	2.78	0.34	0.45	0.52	0.56
MBN5	4.81	0.34	0.42	0.48	0.56
MBN10	9.99	0.35	0.39	0.44	0.52
MBN15	14.99	0.36	0.38	0.45	0.51
MBN20	19.96	0.41	0.39	0.46	0.50
Average		**0.36**	**0.42**	**0.48**	**0.54**

Brooks and Corey's (1964) analytical functional relations:

Soil-moisture retention:

$$\theta = \theta_r + (\phi - \theta_r)(h_b / h_c)^{\lambda} \qquad h_c < h_b$$

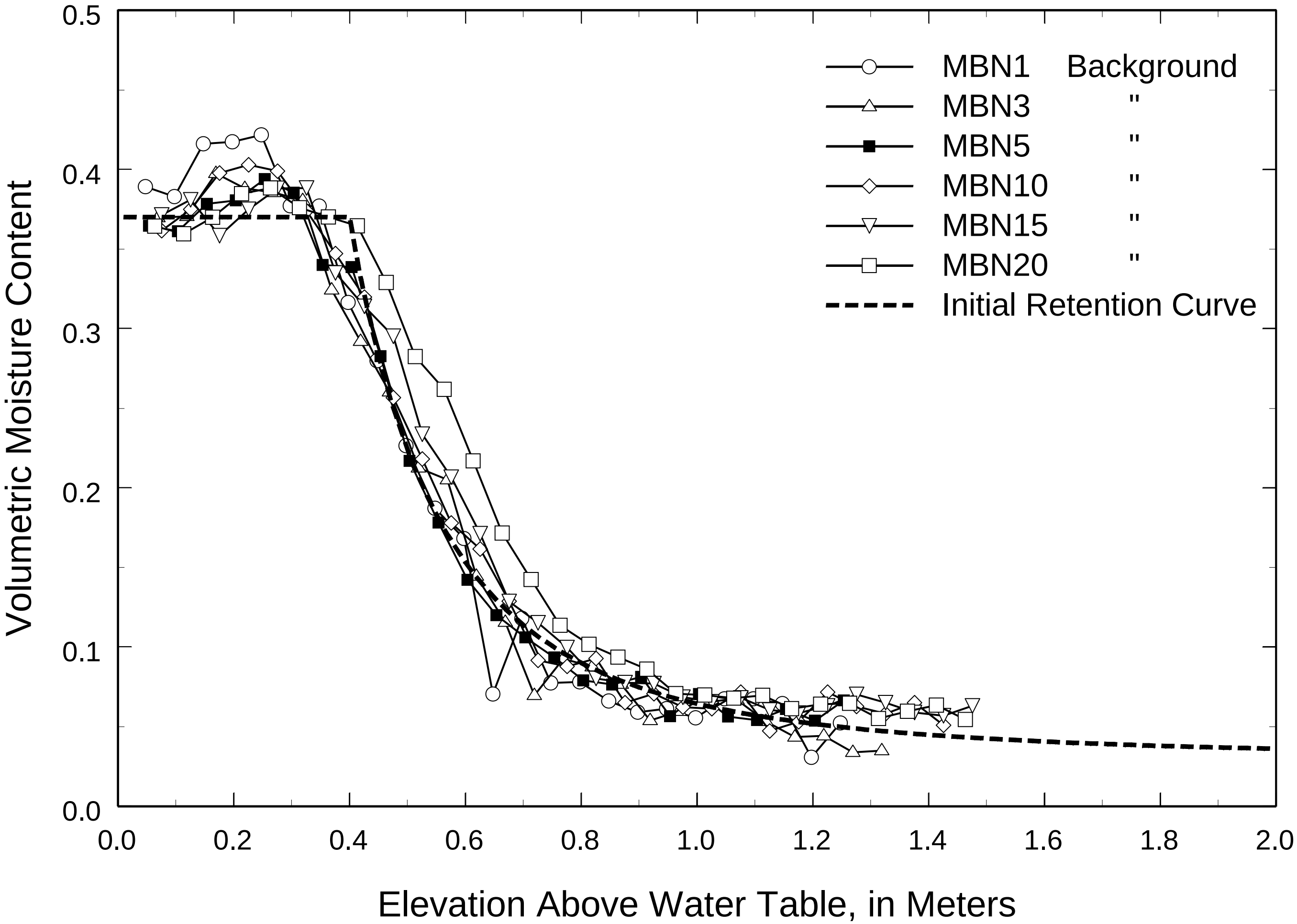
$$\theta = \phi \qquad h_c \geq h_b$$

Relative hydraulic conductivity:

$$K_{rel} = (h_c / h_b)^{-2-3\lambda} \qquad h_c < h_b$$

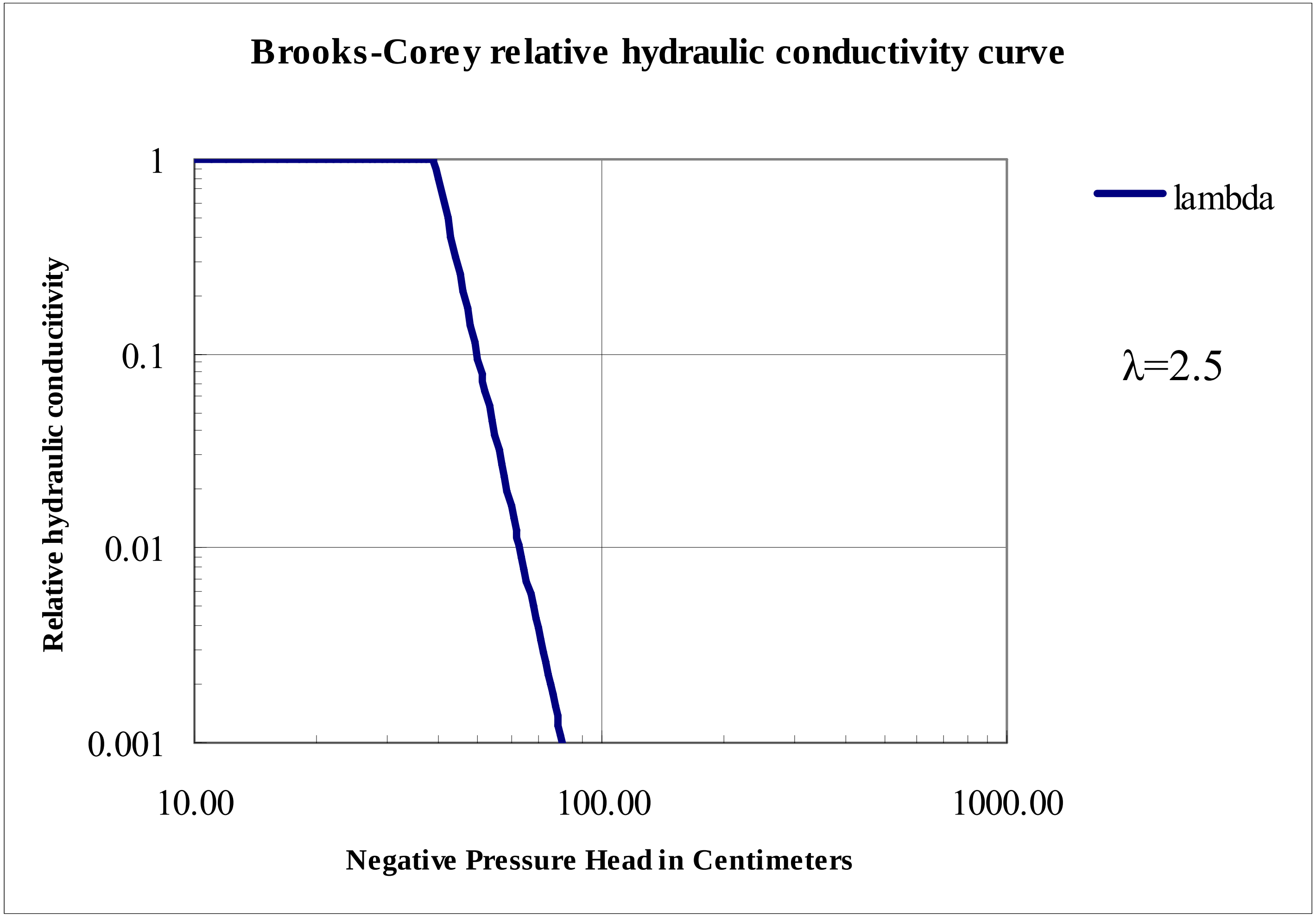
$$K_{rel} = 1 \qquad h_c \geq h_b$$

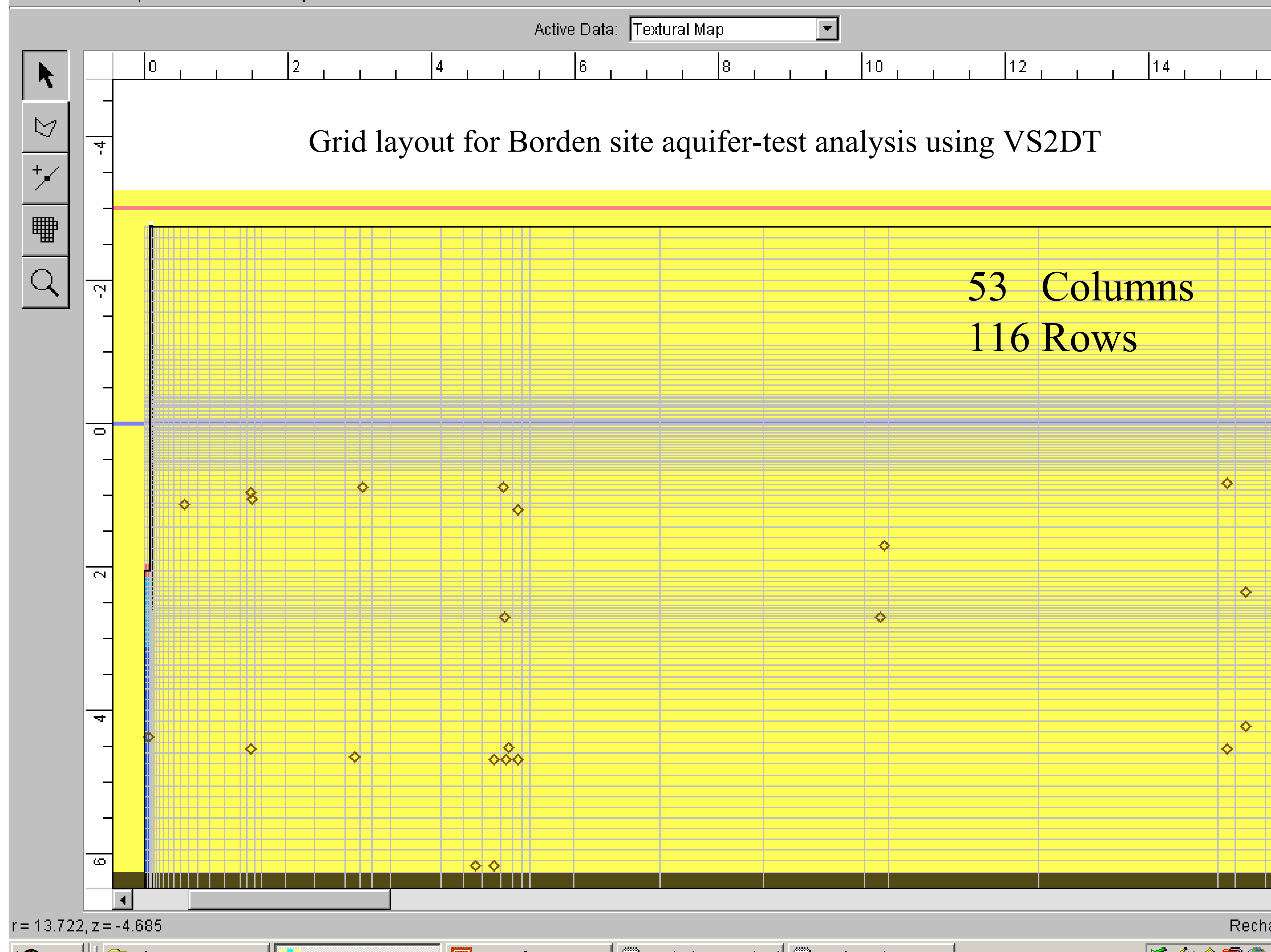
A Brooks and Corey curve fitted visually to the composite plot of soil-moisture measurements



Corresponding B & C parameters:

$\theta_r = 0.03$
$\lambda = 2.5$
$h_b = -0.40$ m
$\phi = 0.37$

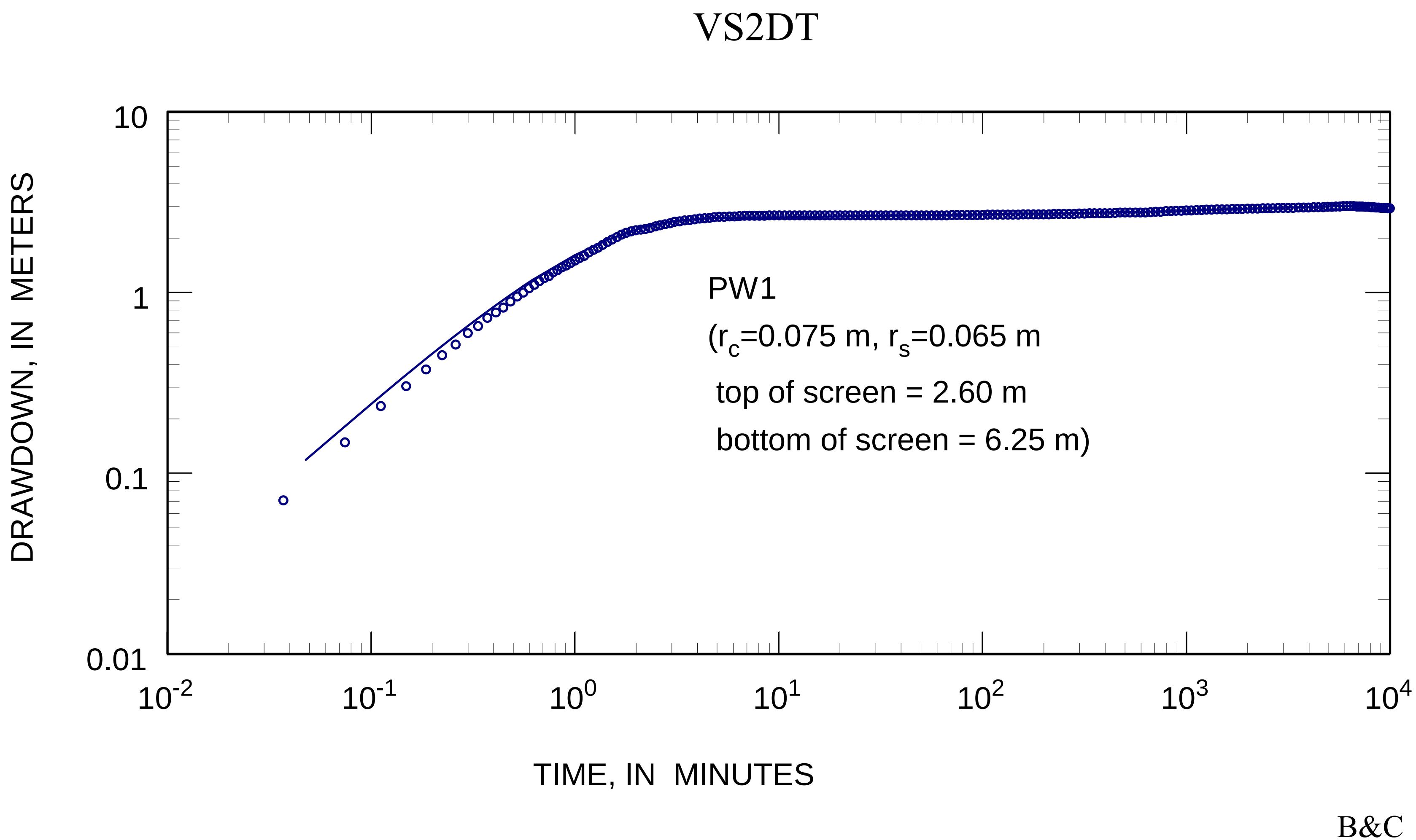

Brooks-Corey relative hydraulic conductivity curve
1
0.1
0.01
0.001
Relative hydraulic conducitivity
10.00
100.00
1000.00
Negative Pressure Head in Centimeters
lambda
λ=2.5


Active Data:
Textural Map
Grid layout for Borden site aquifer-test analysis using VS2DT
53 Columns
116 Rows
0
2
4
6
8
10
12
14
-4
-2
0
2
4
6
r = 13.722, z = -4.685

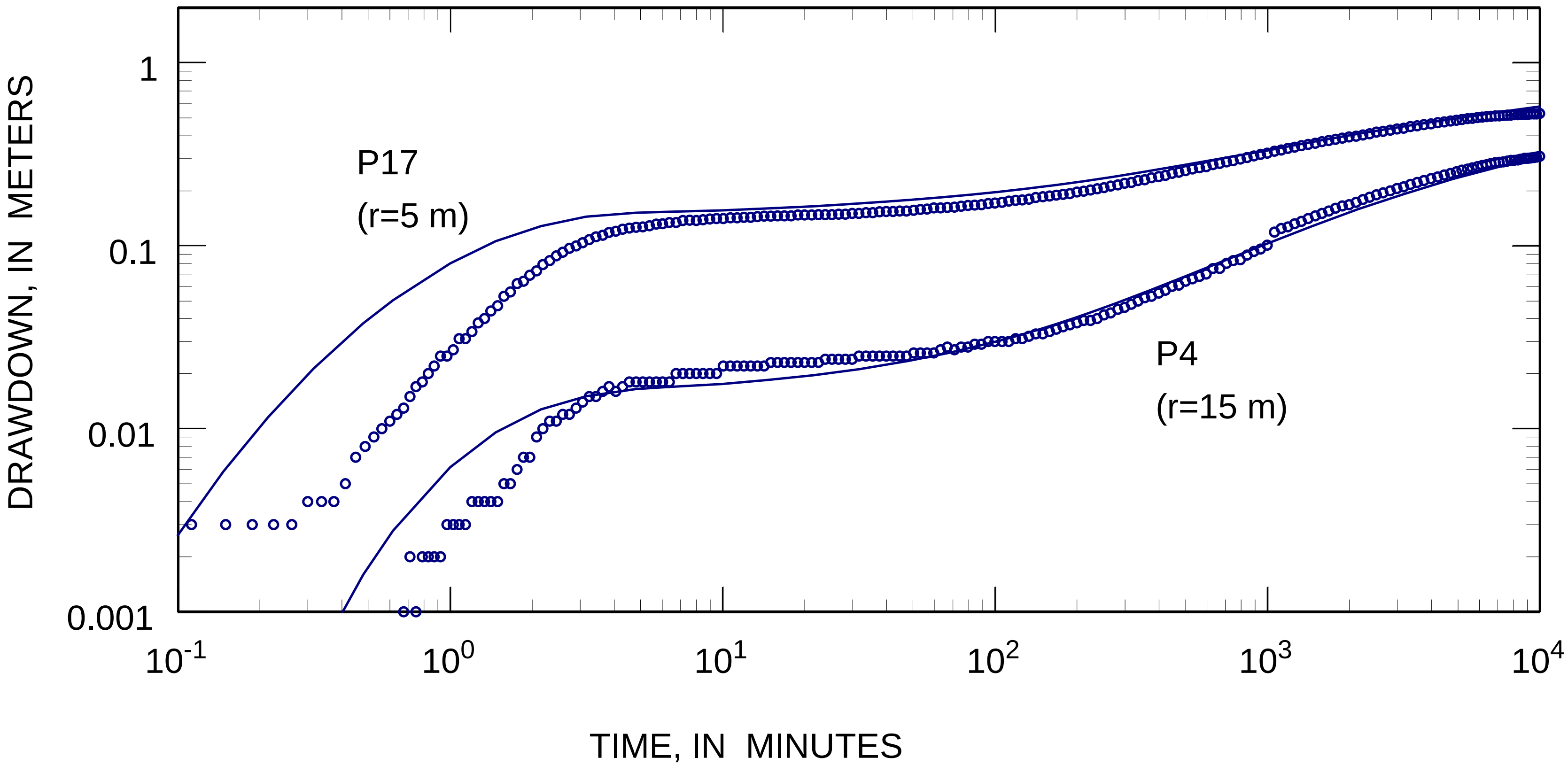
Estimated Parameters using PEST and VS2DT

PEST run and RMSE value	Parameter	Estimated Value	95% Confidence Limits		Initial Value
			Lower limit	Upper limit	
VS2DT with Brooks and Corey model RMSE = 2.60E-02 m	θ_r	0.030*	na	na	0.030*
	h_b (m)	-0.385	-0.441	-0.329	-0.40
	λ	0.492	0.433	0.559	2.5
	K_z (m/s)	3.15E-05	2.97E-05	3.33E-05	4.0E-05
	K_r (m/s)	6.46E-05	6.34E-05	6.58E-05	8.0E-05

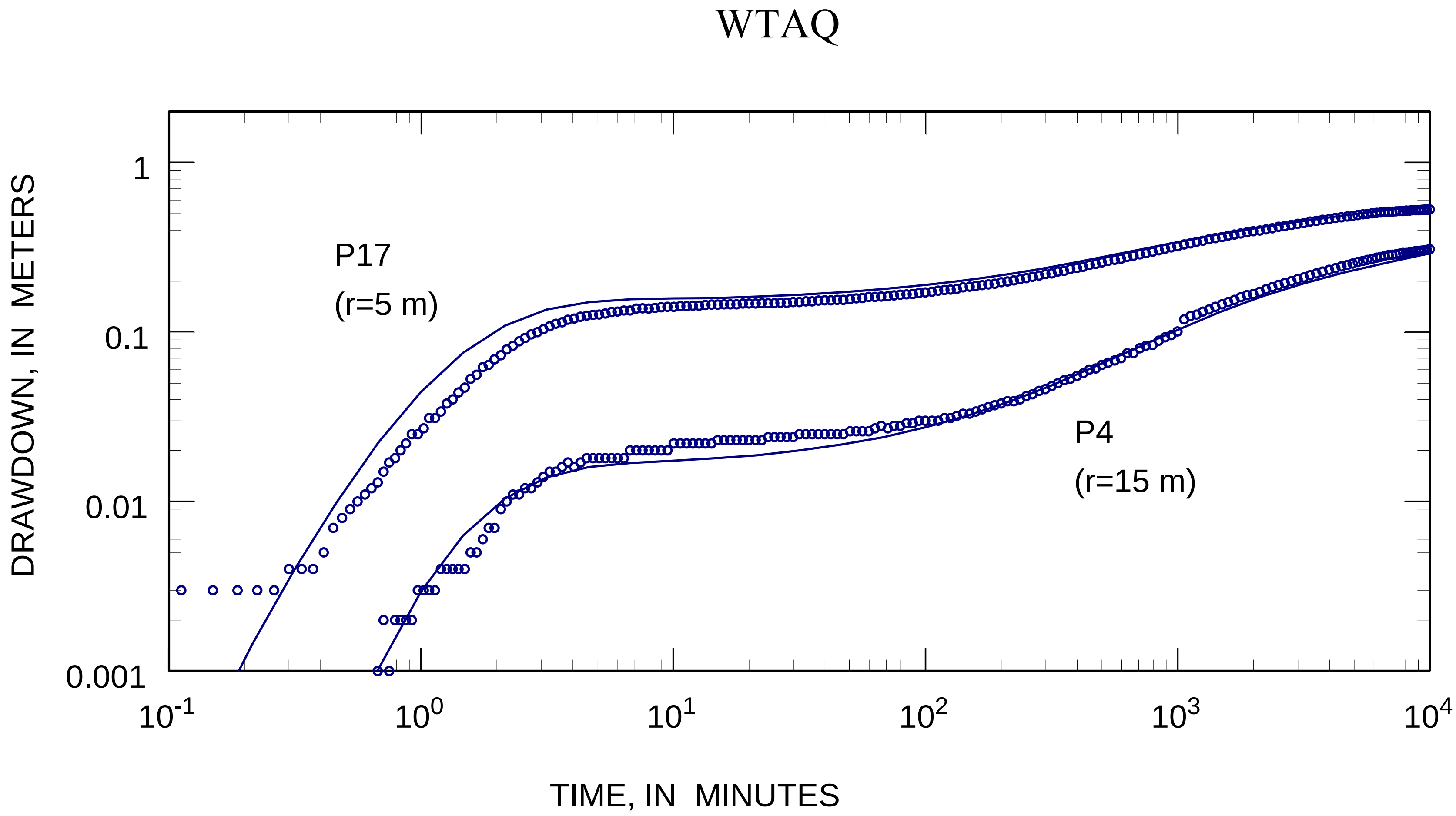
Simulated responses by VS2DT compared with measured drawdown:

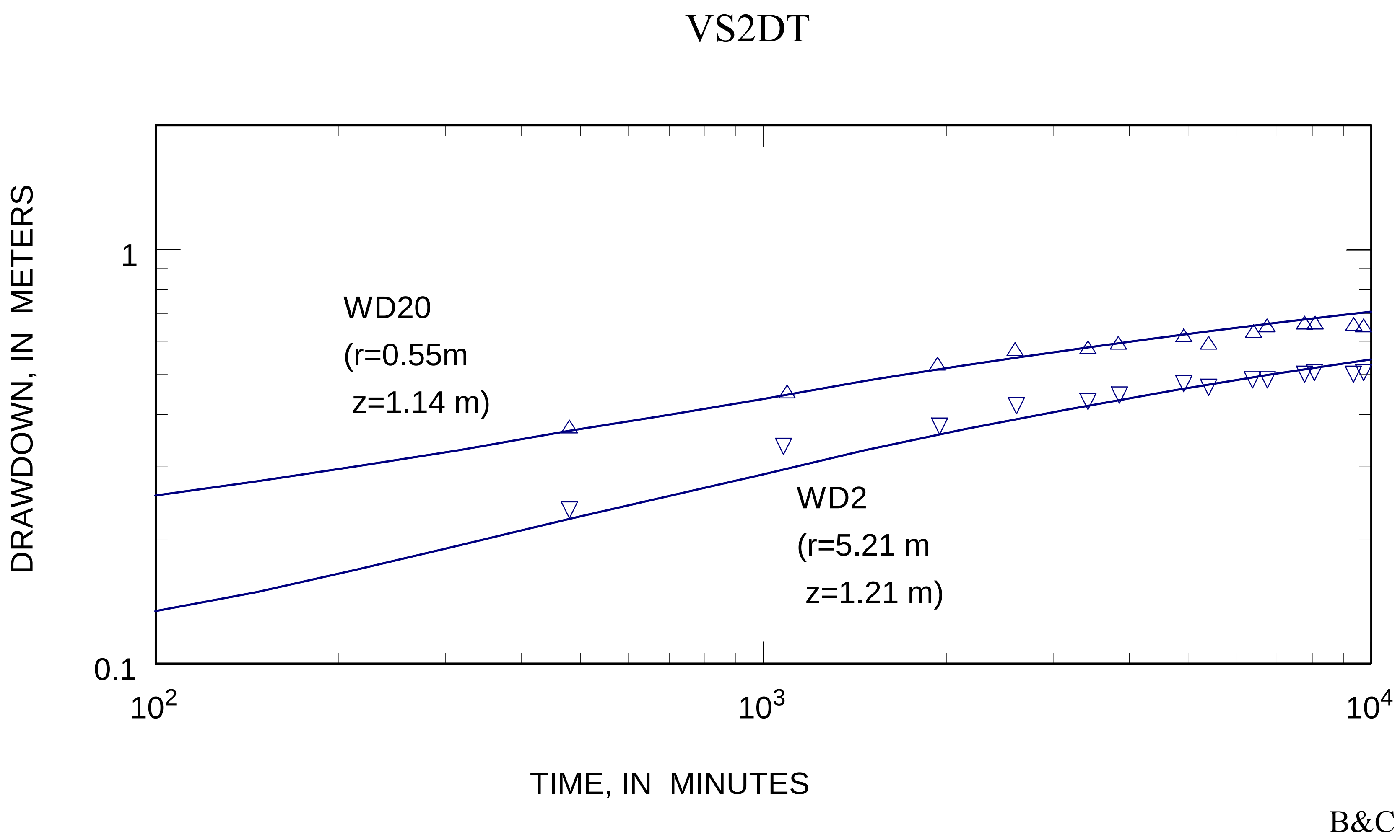

VS2DT
10
1
0.1
0.01
DRAWDOWN, IN METERS
PW1
(rc=0.075 m, rs=0.065 m
top of screen = 2.60 m
bottom of screen = 6.25 m)
10-2
10-1
100
101
102
103
104
TIME, IN MINUTES
B&C

VS2DT

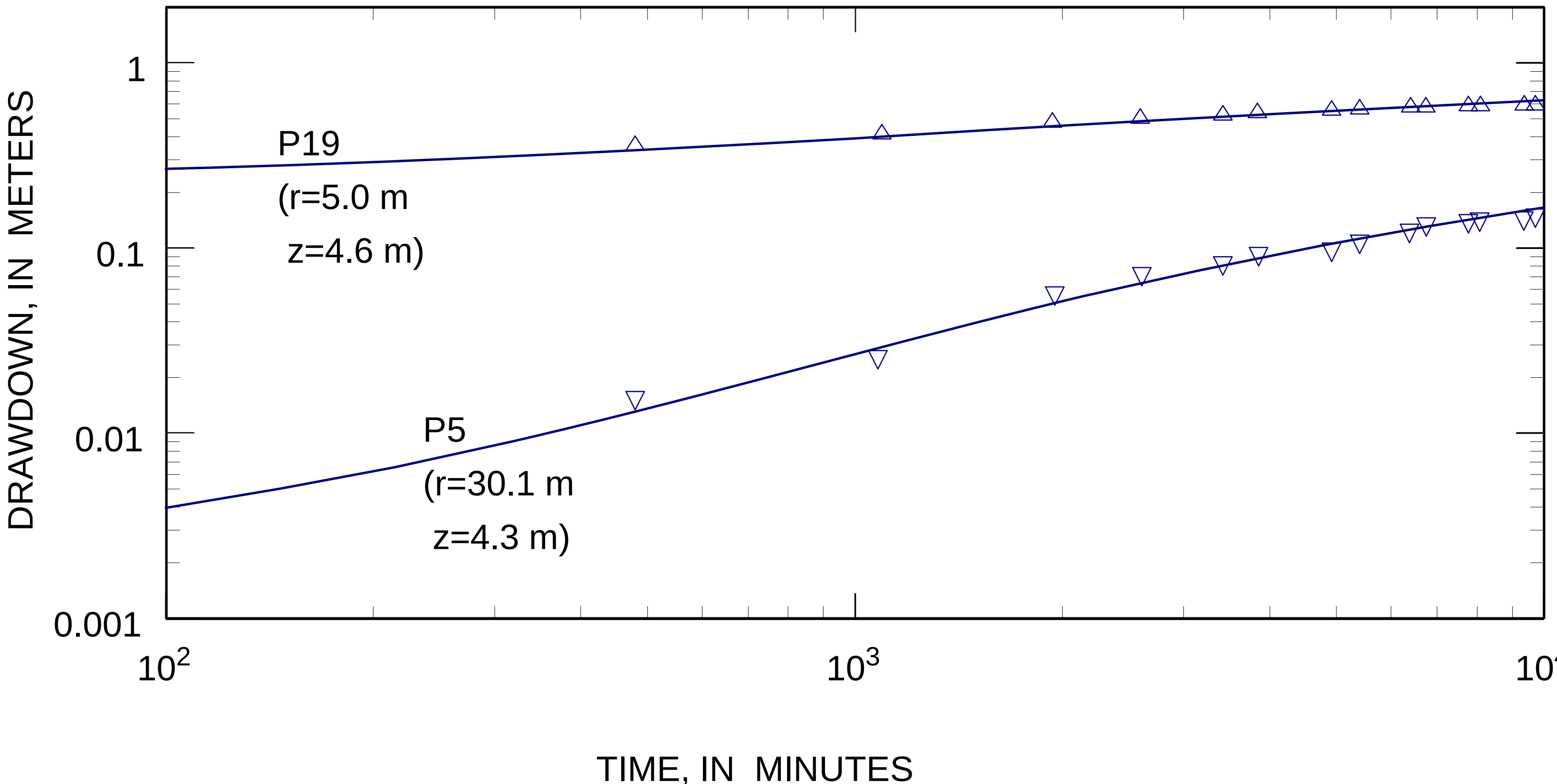

P17
(r=5 m)
P4
(r=15 m)
DRAWDOWN, IN METERS
1
0.1
0.01
0.001
10-1
100
101
102
103
104
TIME, IN MINUTES

B&C


WTAQ
DRAWDOWN, IN METERS
1
0.1
0.01
0.001
P17
(r=5 m)
P4
(r=15 m)
10^-1
10^0
10^1
10^2
10^3
10^4
TIME, IN MINUTES


VS2DT
DRAWDOWN, IN METERS
1
0.1
10^2
10^3
10^4
TIME, IN MINUTES
WD20
(r=0.55m
z=1.14 m)
WD2
(r=5.21 m
z=1.21 m)
B&C

VS2DT



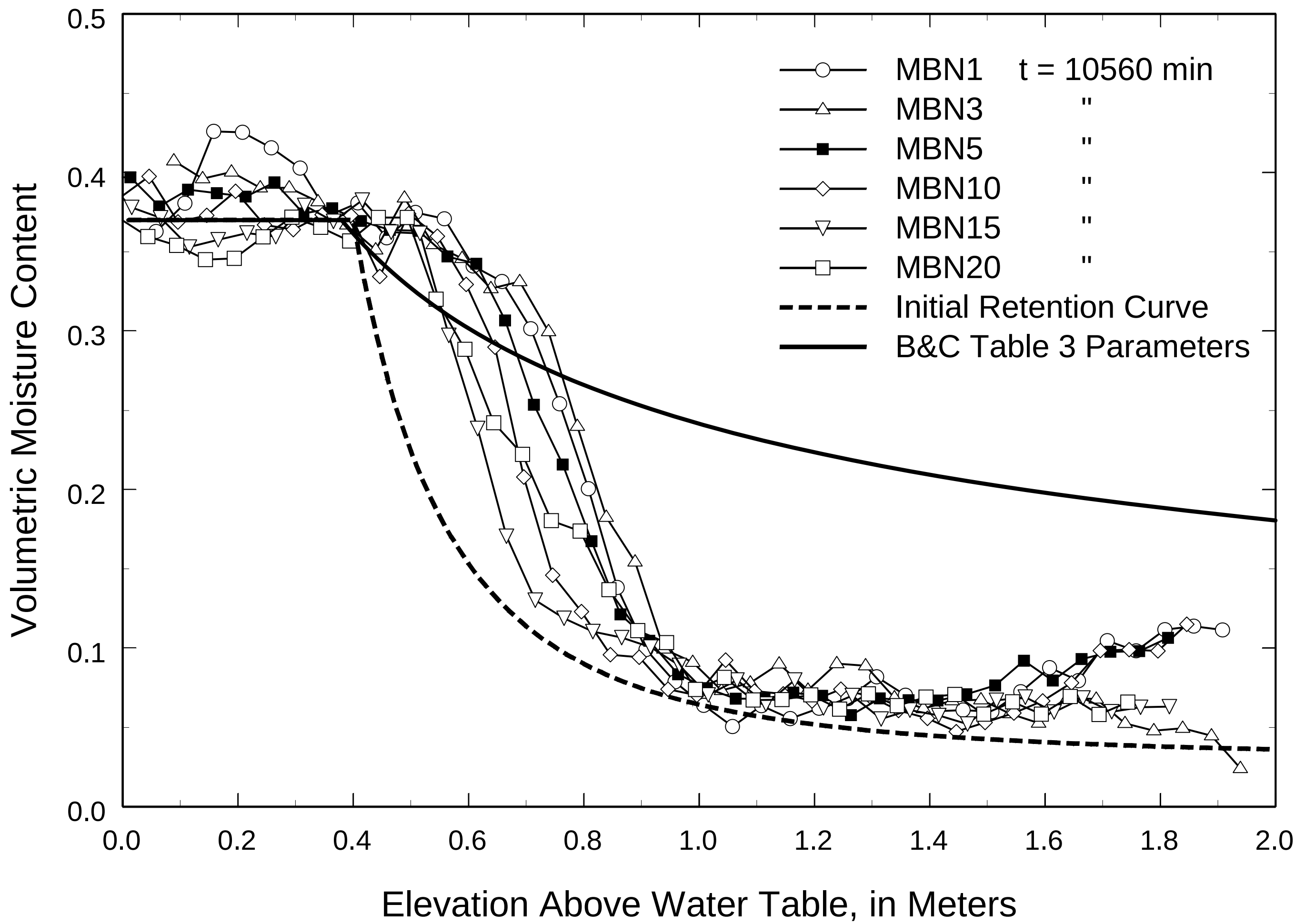
P19
(r=5.0 m
z=4.6 m)
P5
(r=30.1 m
z=4.3 m)
1
0.1
0.01
0.001
DRAWDOWN, IN METERS
10^2
10^3
10^4
TIME, IN MINUTES

B&C

Measured and simulated drawdowns agree nicely for nearly all piezometers

– *but there is one problem*

Results show a big deviation from the measured soil-moisture distribution at the end of the test!



Estimated B & C parameters:

θ_r=0.03

λ=0.492

h_b=-0.385 m

To resolve this discrepancy I modified VS2DT by revising[1] the Brooks and Corey functional relation as shown:

[1] VS2DT allows for used-defined functional relations

Modified Brooks and Corey functional relations:

Soil-moisture retention:

$$\theta = \theta_r + (\phi - \theta_r)(h_b / h_c)^{\lambda_1} \qquad h_c < h_b$$

$$\theta = \phi \qquad h_c \geq h_b$$

Relative hydraulic conductivity:

$$K_{rel} = (h_c / h_b)^{-2-3\lambda_2} \qquad h_c < h_b$$

$$K_{rel} = 1 \qquad h_c \geq h_b$$

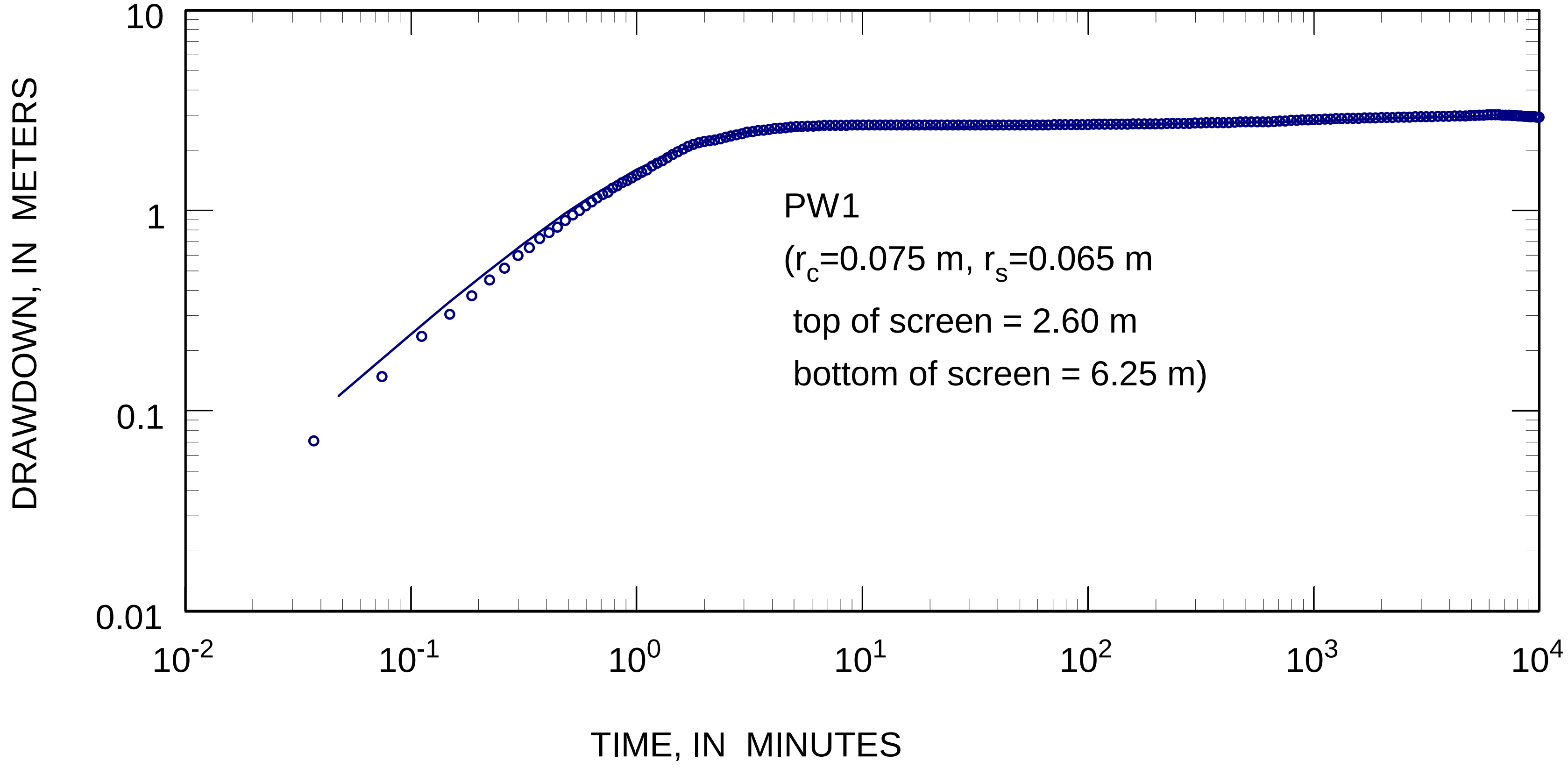
Note

This effectively removes the prior coupling between soil-moisture retention and relative hydraulic conductivity.

Estimated parameters using PEST and modified VS2DT

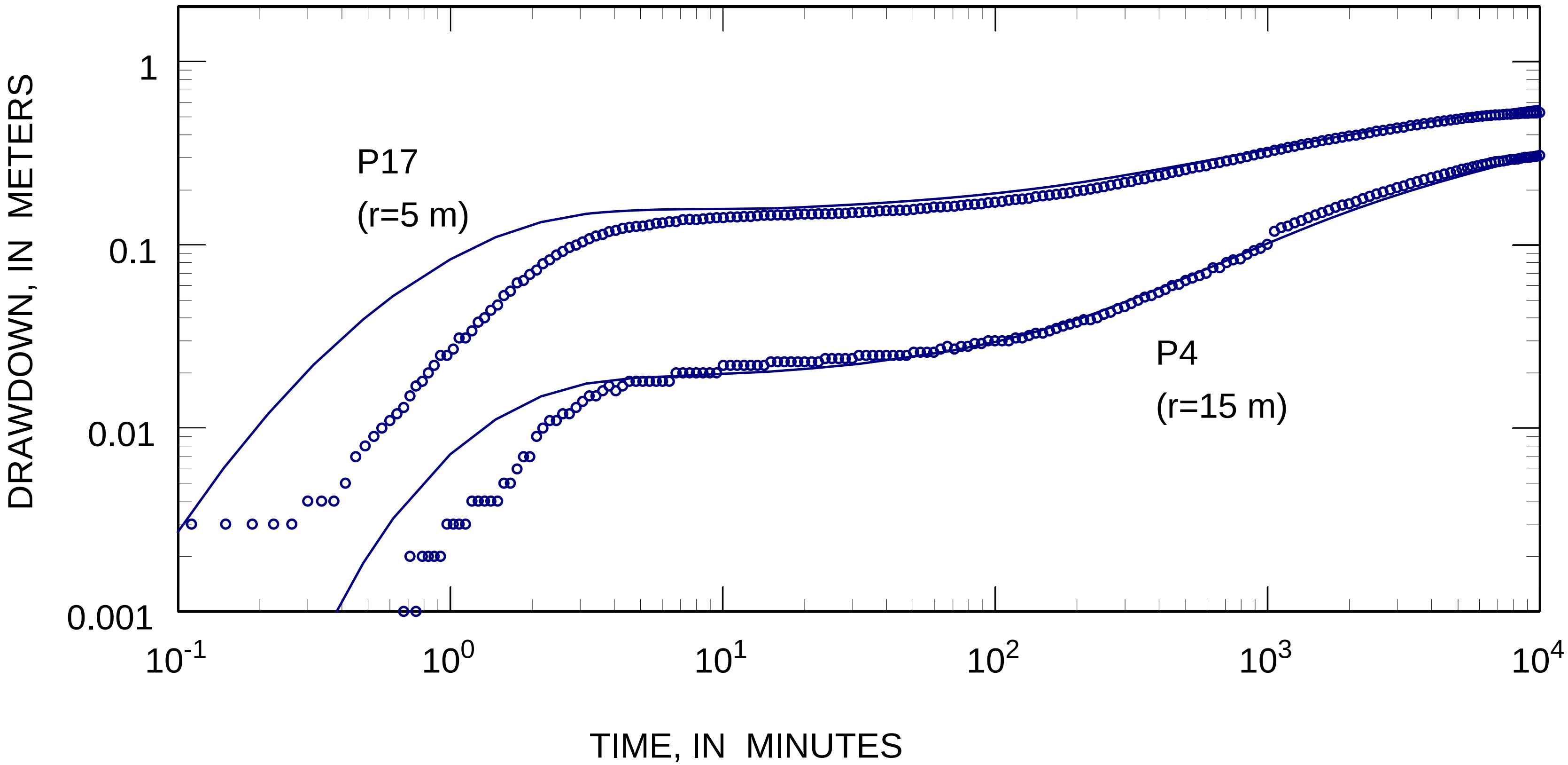
PEST run and RMSE value (m)	Parameter	Estimated Value	95% Confidence Limits		Initial Value
			Lower limit	Upper limit	
VS2DT with the modified Brooks and Corey model RMSE = 2.60E-02 m	θ_r	0.03*	na	na	0.03*
	h_b (m)	-0.505	-0.550	-0.459	-0.40
	λ_1	2.5*	na	na	2.5*
	λ_2	9.11	7.68	10.53	2.5
	K_z (m/s)	3.03E-05	2.83E-05	3.25E-05	4.E-05
	K_r (m/s)	6.56E-05	6.44E-05	6.69E-05	8.E-05

Simulated responses by modified VS2DT compared with measured drawdown:



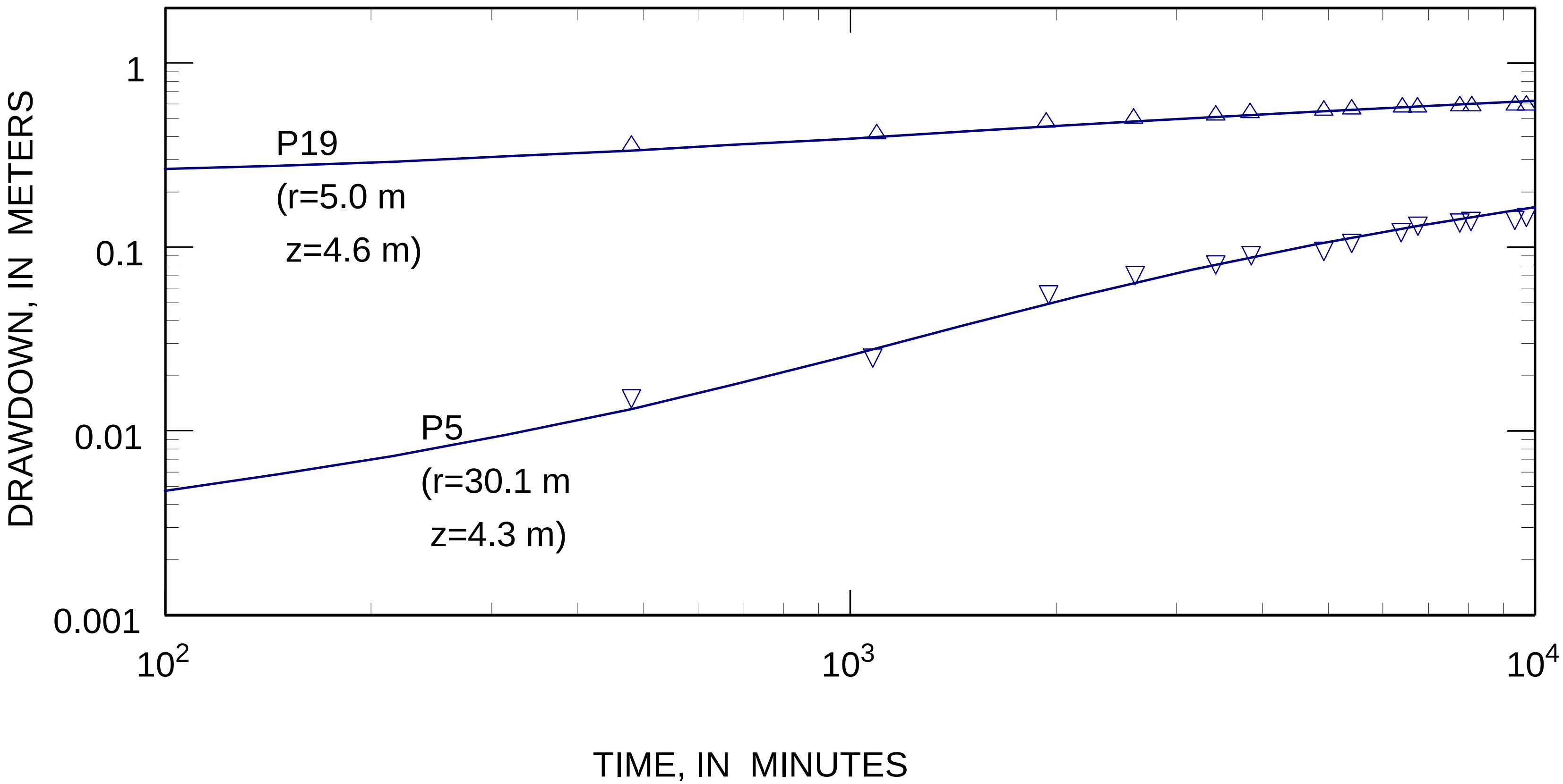
DRAWDOWN, IN METERS
10
1
0.1
0.01
10-2
10-1
100
101
102
103
104
TIME, IN MINUTES
PW1
(rc=0.075 m, rs=0.065 m
top of screen = 2.60 m
bottom of screen = 6.25 m)

U D



DRAWDOWN, IN METERS
1
0.1
0.01
0.001
P17
(r=5 m)
P4
(r=15 m)
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
TIME, IN MINUTES

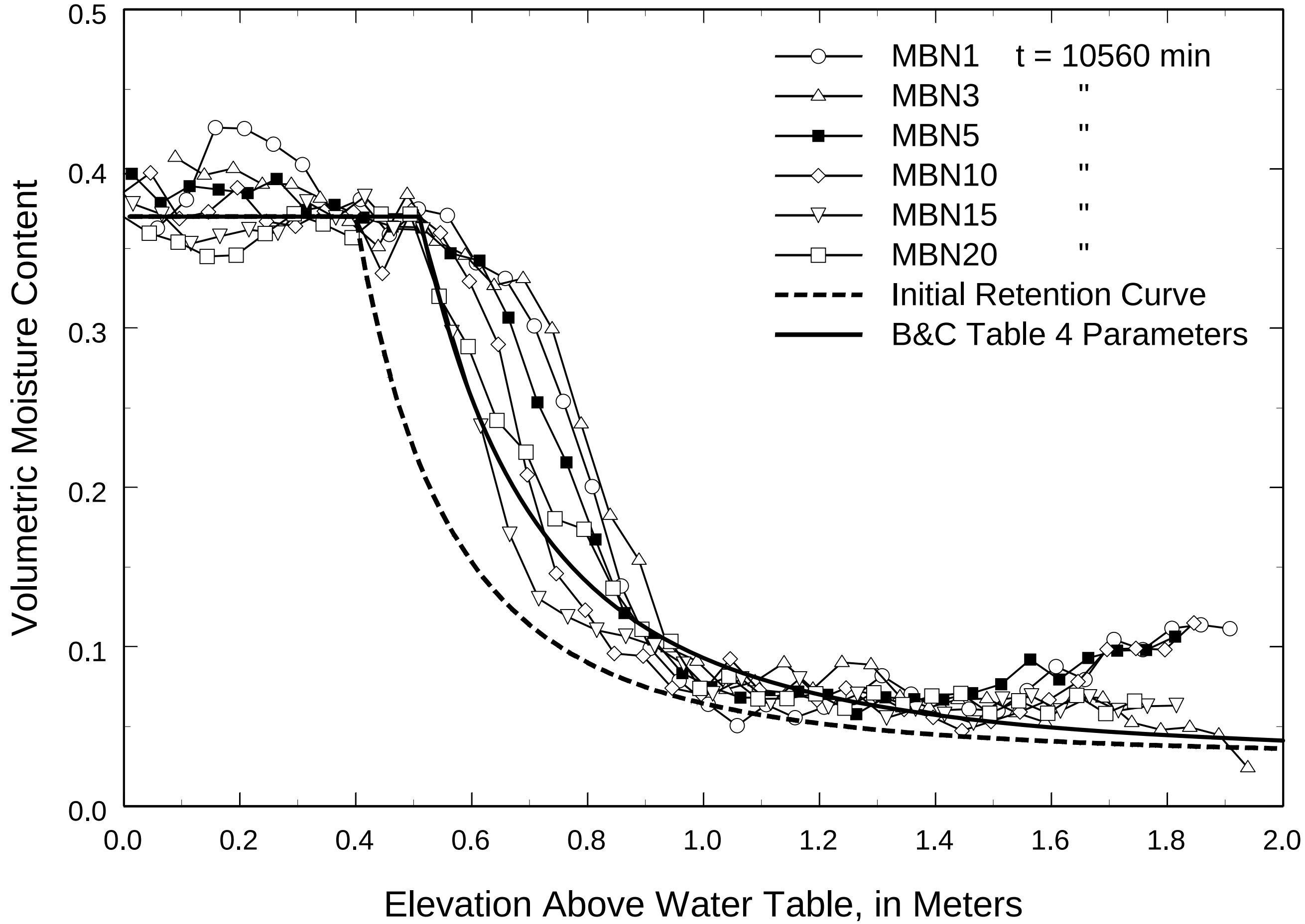
U D



DRAWDOWN, IN METERS
1
0.1
0.01
0.001
P19
(r=5.0 m
z=4.6 m)
P5
(r=30.1 m
z=4.3 m)
10^2
10^3
10^4
TIME, IN MINUTES

U D

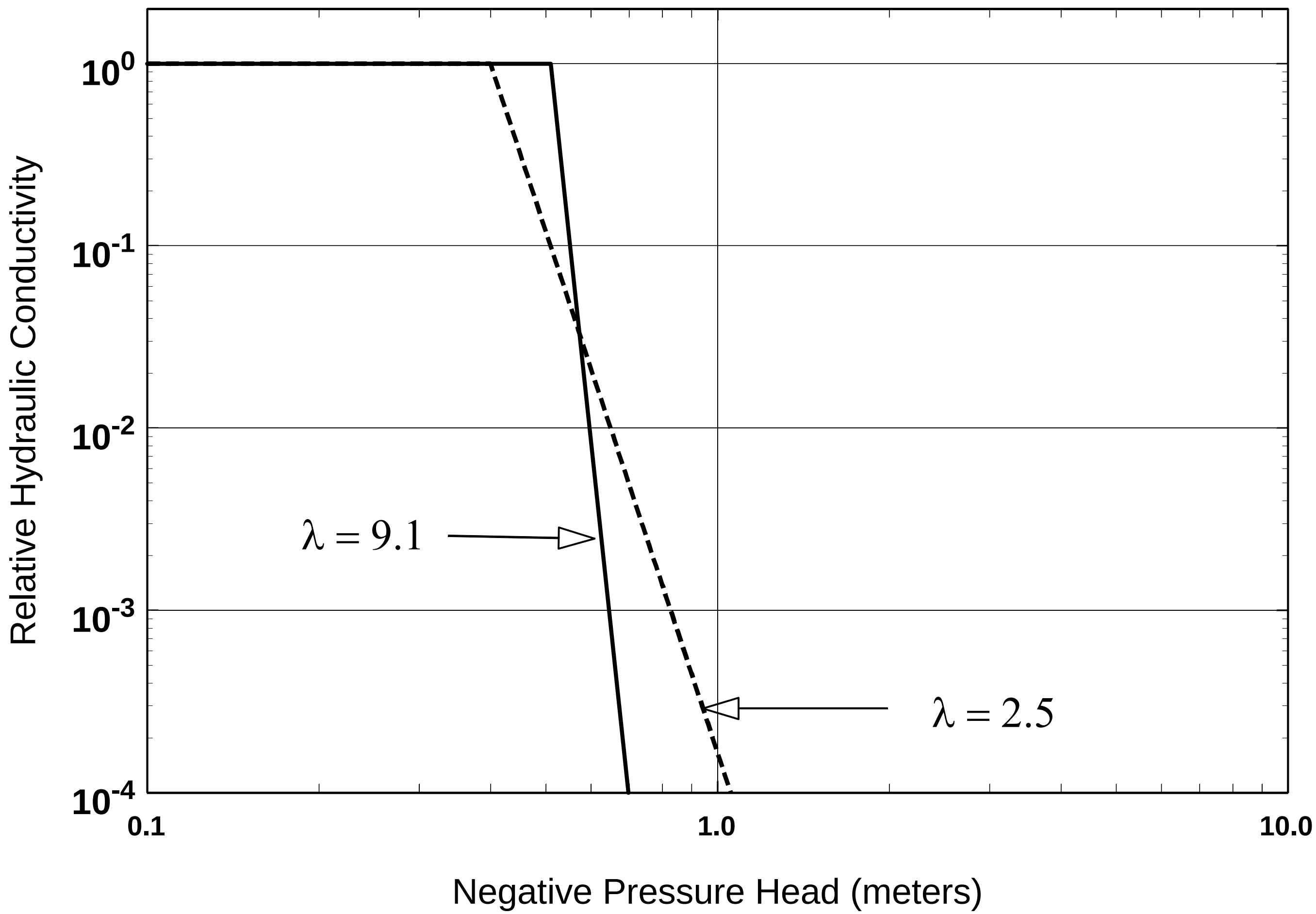
Resulting Retention Curve for the Modified Functional Relations



Estimated parameters:

θ_r= 0.03
λ_1= 2.5
λ_2= 9.11
h_b= -0.505 m

Comparison of relative hydraulic conductivities for λ_1 and λ_2



Lessons learned:

-Aquifer thickness can be estimated from late-time drawdown data.

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-Inclusion of gradual drainage from the unsaturated zone yields realistic estimates of specific yield S_y and shows S_y to be characteristic aquifer constant.

-With a model that accounts for unsaturated zone drainage (analytical or numerical) , short-term tests (~8 hours) can yield suitable results.

Unsaturated-zone lessons learned:

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-Modification of the Brooks and Corey functional relations (*i.e.*, introducing λ_1 and λ_2 to uncouple the two functions) yields:

1. improved simulation of the soil-moisture distribution at the Borden site
2. explanation for the capillary-fringe extension observed at the Borden site

END

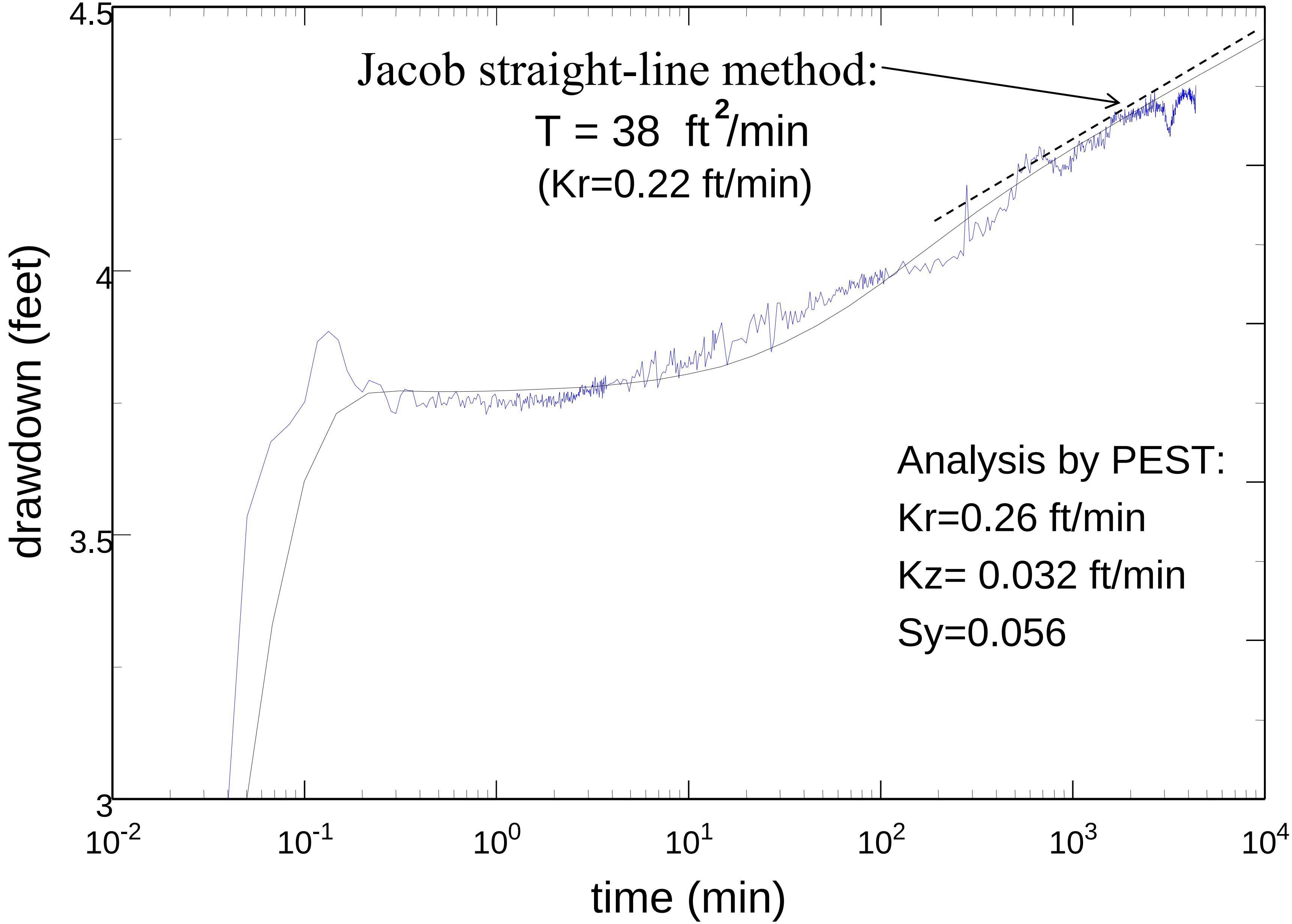
-thanks for your attention

<u>Reference</u>:
"Estimation of Hydraulic Parameters from an Unconfined Aquifer Test Conducted in A Glacial Outwash Deposit, Cape Cod, Massachusetts"
USGS Professional Paper 1629
(Published in 2001)

Question

Can pumped-well data alone be used for parameter estimation ?

Analysis of pumped-well response:



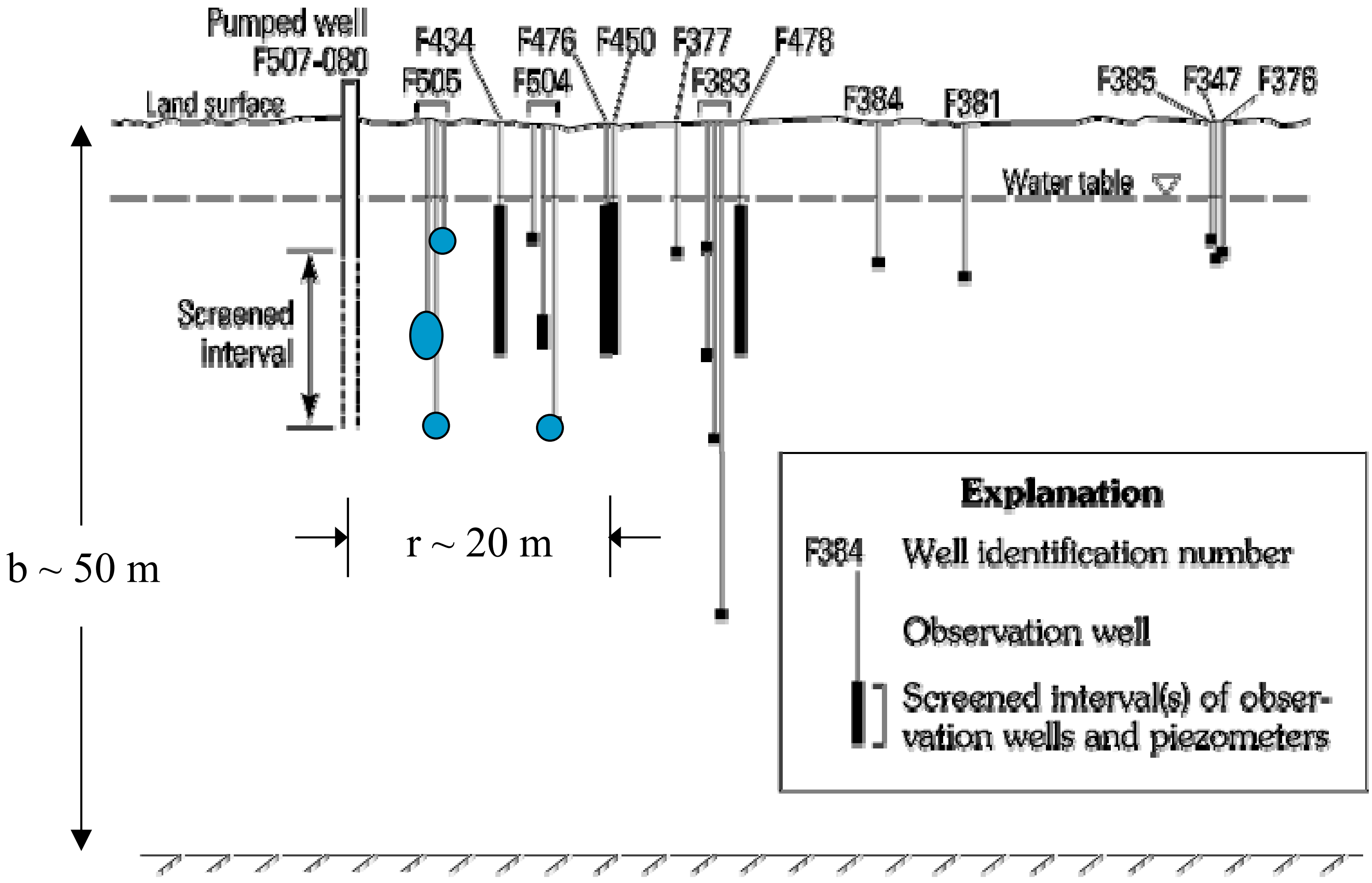
As an aside – from a preliminary type-curve analysis

Saturated thickness is not one the parameters one expects to get from standard aquifer-test analyses.

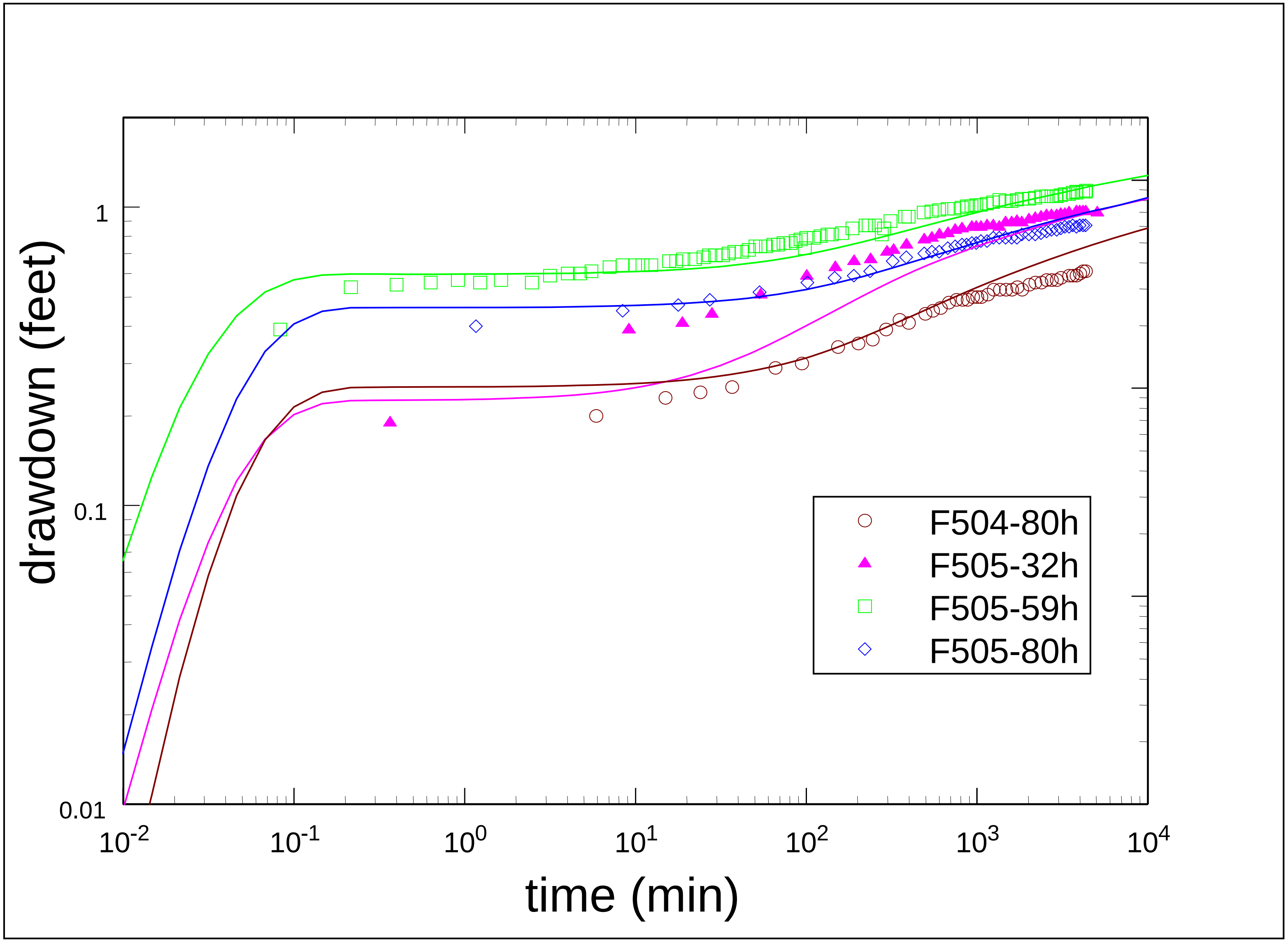
Saturated thickness is usually assumed known from well logs, geology, or geophysics.

<u>Reference</u>: Preliminary (1993) analysis by Moench, LeBlanc, and Garabedian, USGS WRI 94-4015

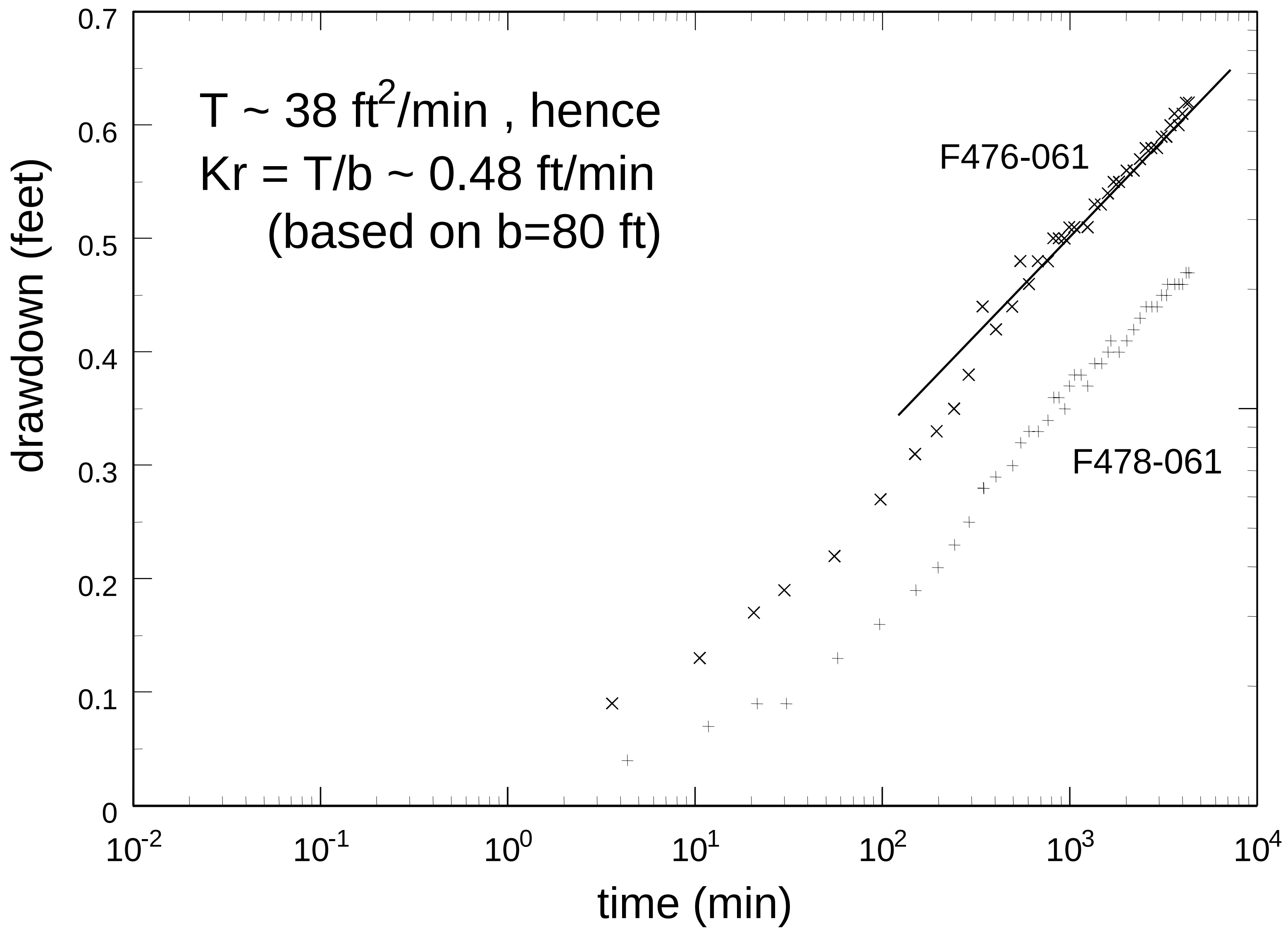
Locations of piezometers for this preliminary analysis



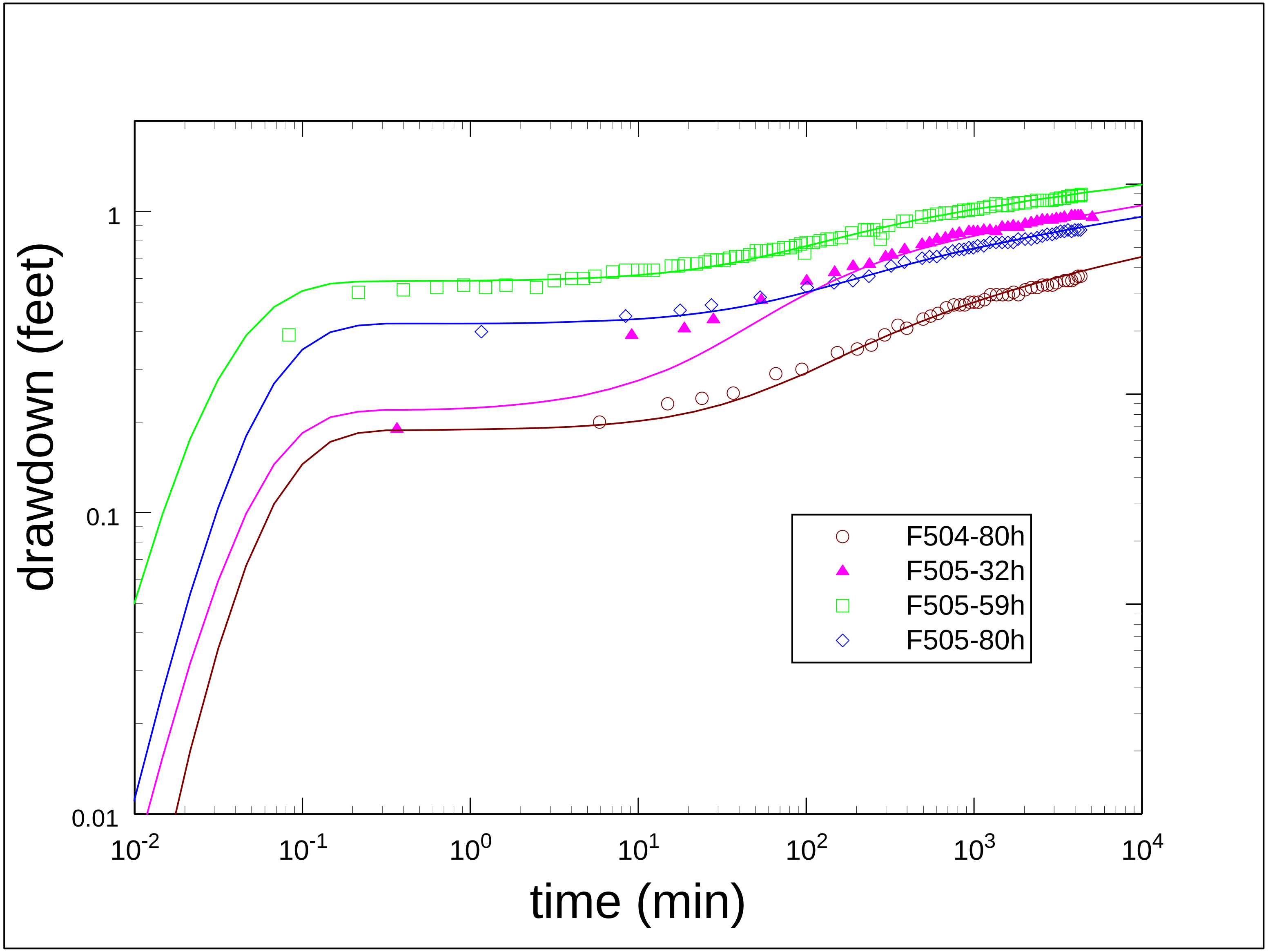
Result from type-curve analysis
– best fit assuming b = 80 ft



By semi-log straight line (Jacob) method:



Result from type-curve analysis – best fit assuming b = 160 ft



Parameters estimated by composite plot (t/r^2), type-curve analysis of hand-measured data:

Parameter	Estimated value
Sy	0.23
b (ft)	160.*
Kr (ft/min)	0.24
Kz (ft/min)	0.12

*assumed value

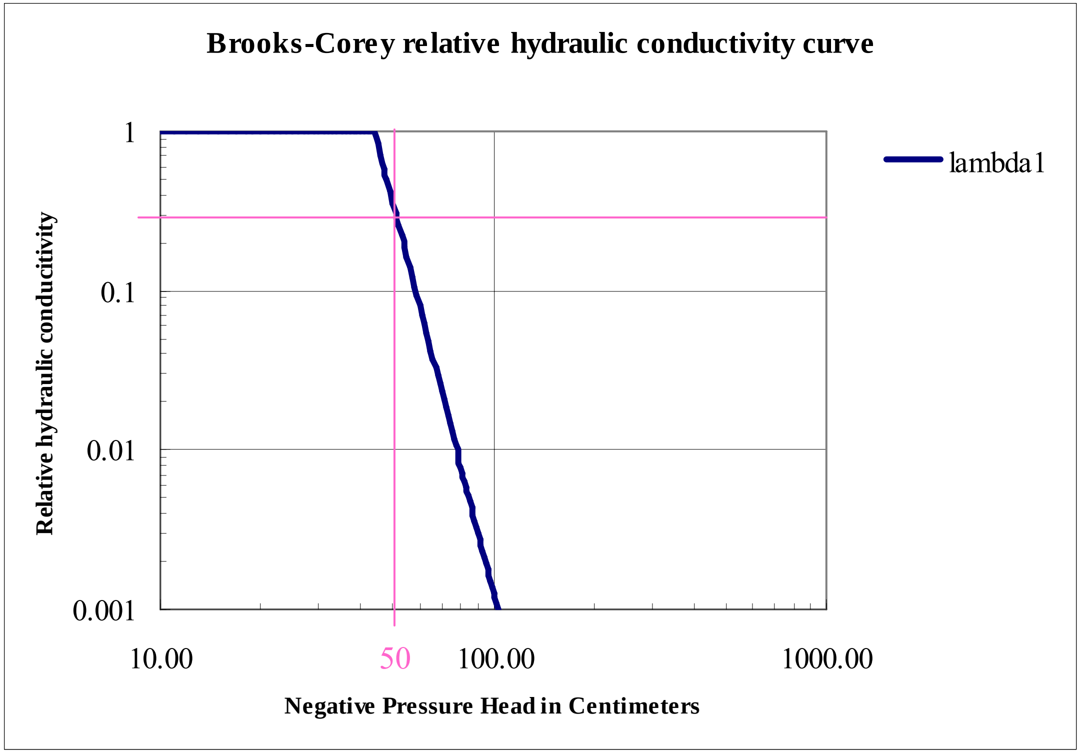
In metric units:

(b = 49 m)

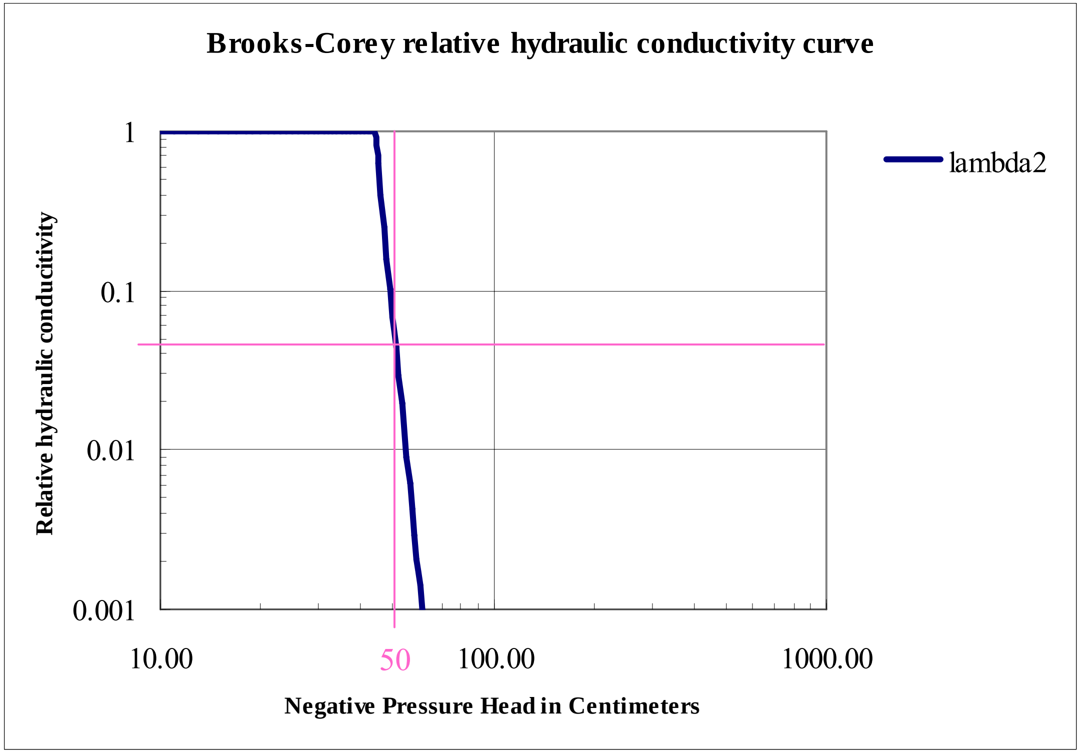
(Kr = 1.2E-03 m/s)

(Kz = 0.6E-03 m/s)

Reference: Preliminary (1993) analysis by Moench, LeBlanc, and Garabedian, USGS WRI 94-4015


Brooks-Corey relative hydraulic conductivity curve
1
0.1
0.01
0.001
Relative hydraulic conductivity
10.00
50
100.00
1000.00
Negative Pressure Head in Centimeters
lambda1

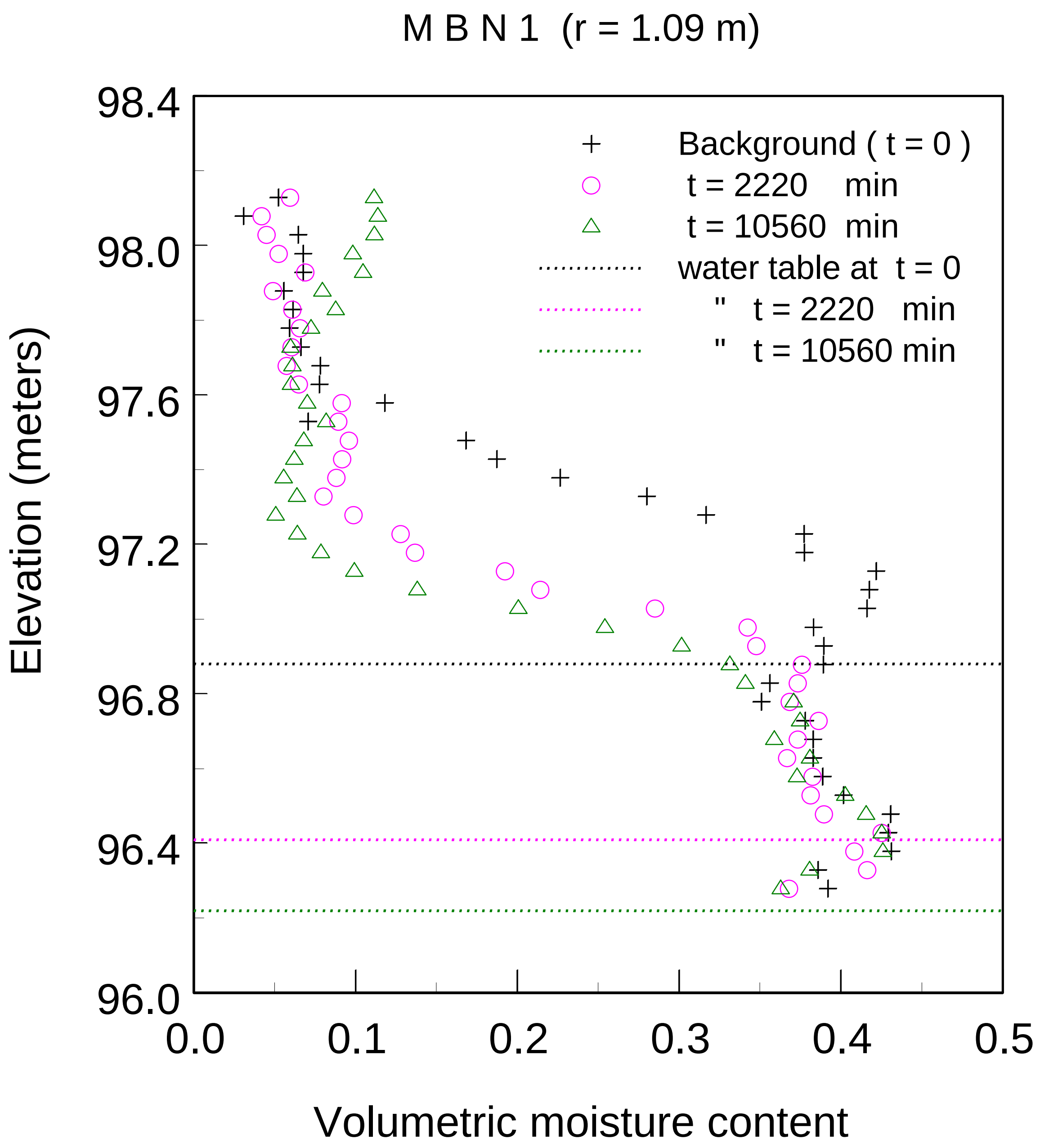
λ_1=2.04

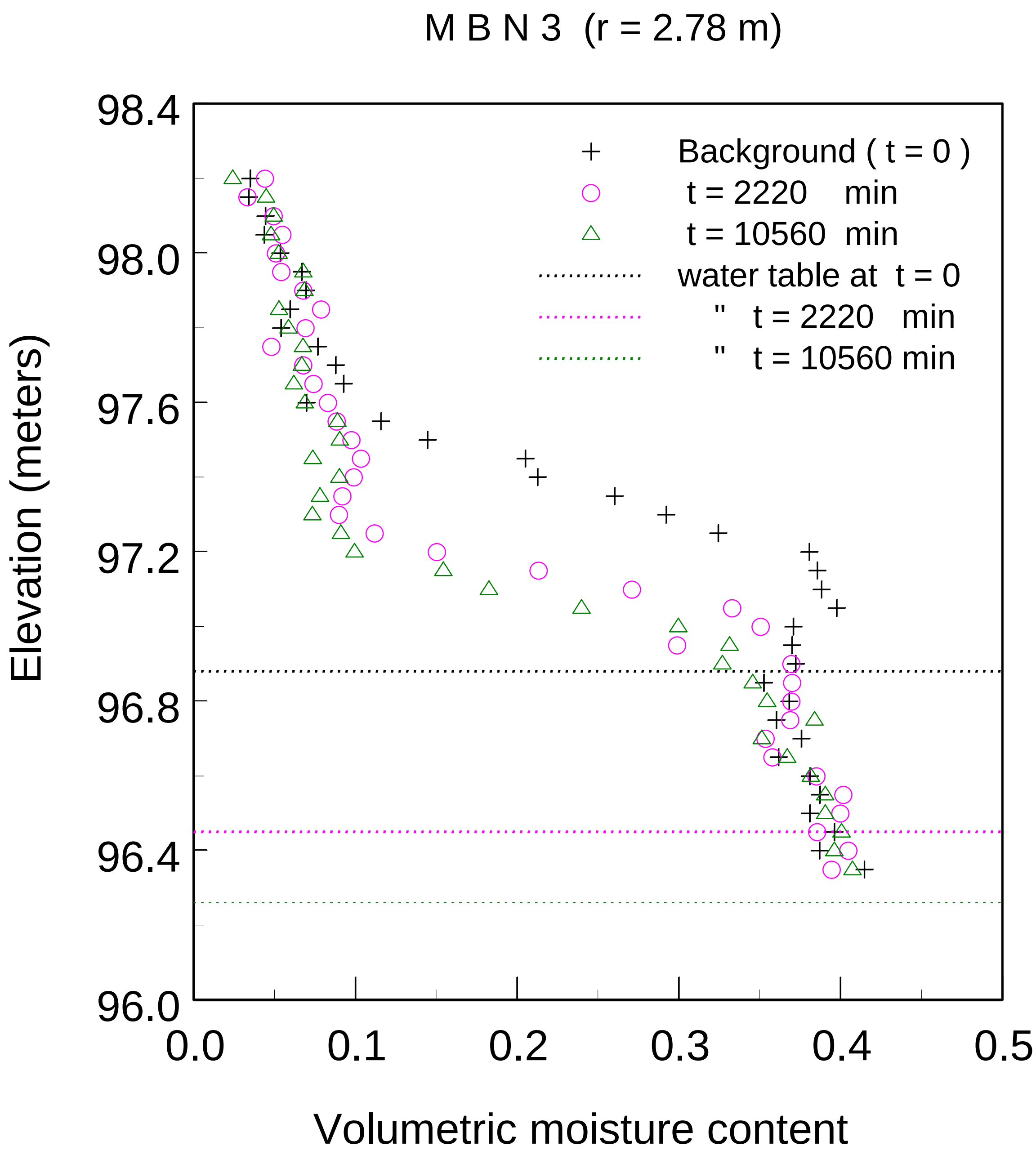


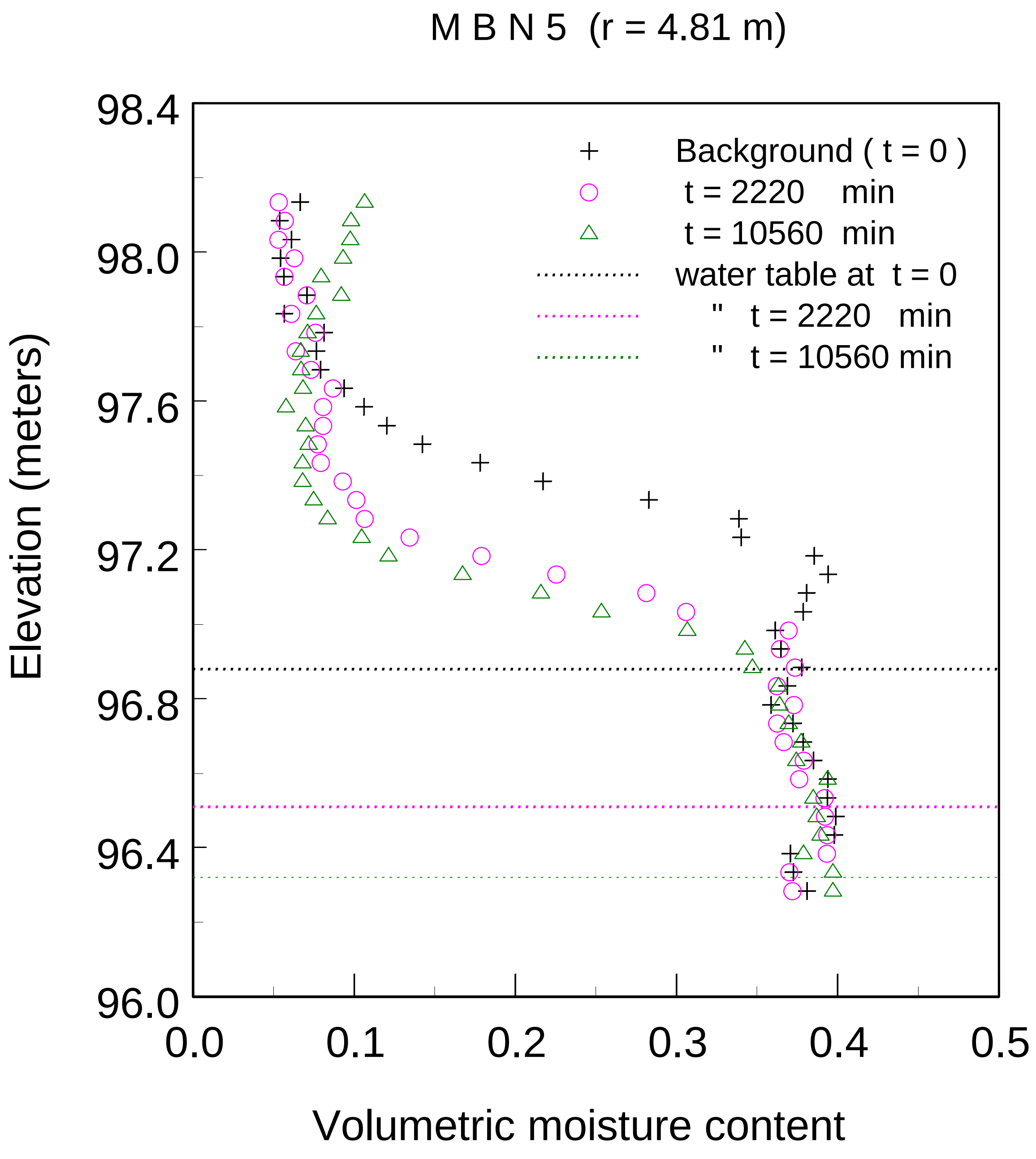
$\lambda_2 = 6.37$

Note

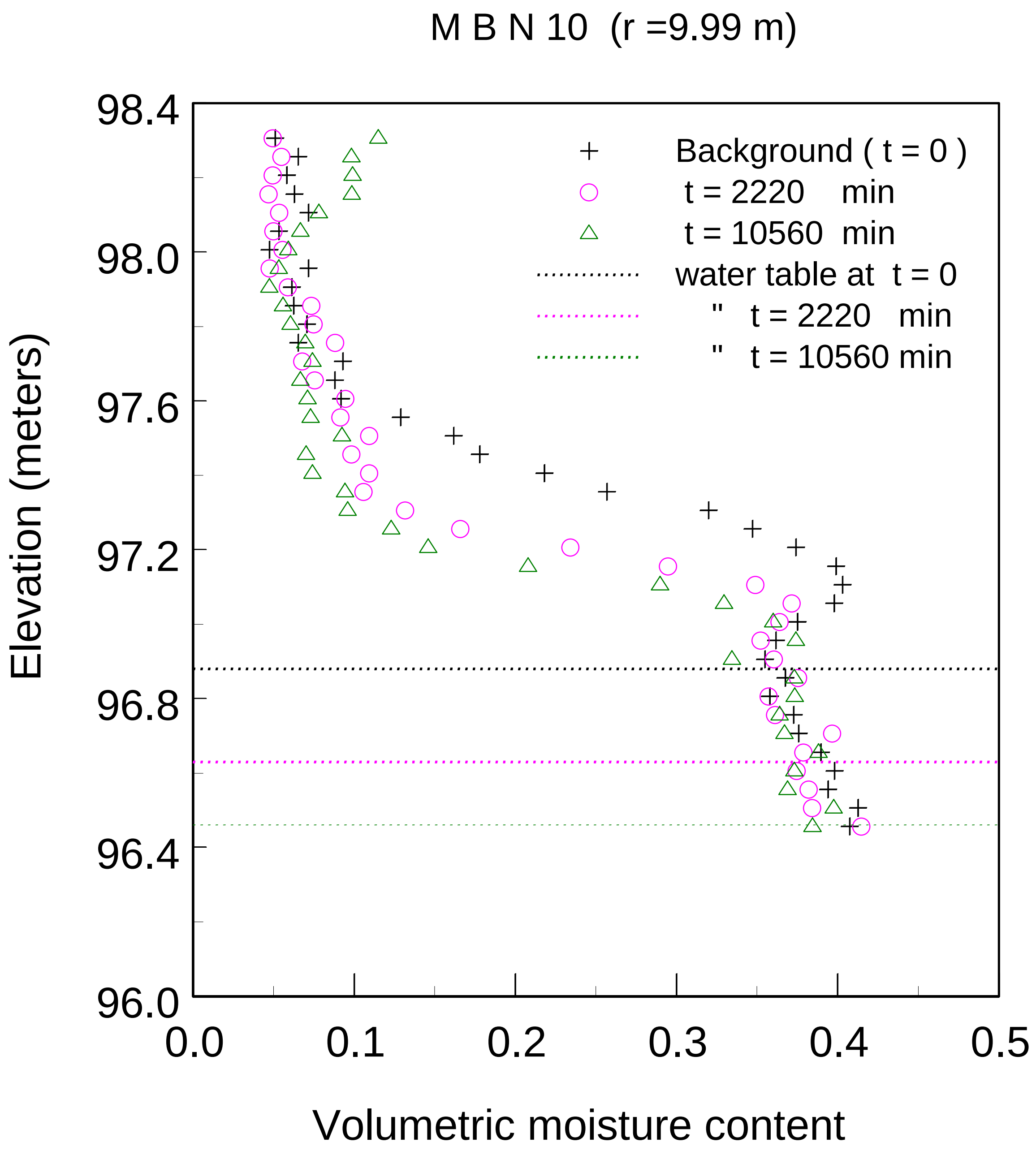
The rapid decline of hydraulic conductivity with elevation accounts for observed extensions of the caplliary fringe.


M B N 1 (r = 1.09 m)
Background (t = 0)
t = 2220 min
t = 10560 min
water table at t = 0
" t = 2220 min
" t = 10560 min
Elevation (meters)
98.4
98.0
97.6
97.2
96.8
96.4
96.0
0.0
0.1
0.2
0.3
0.4
0.5
Volumetric moisture content

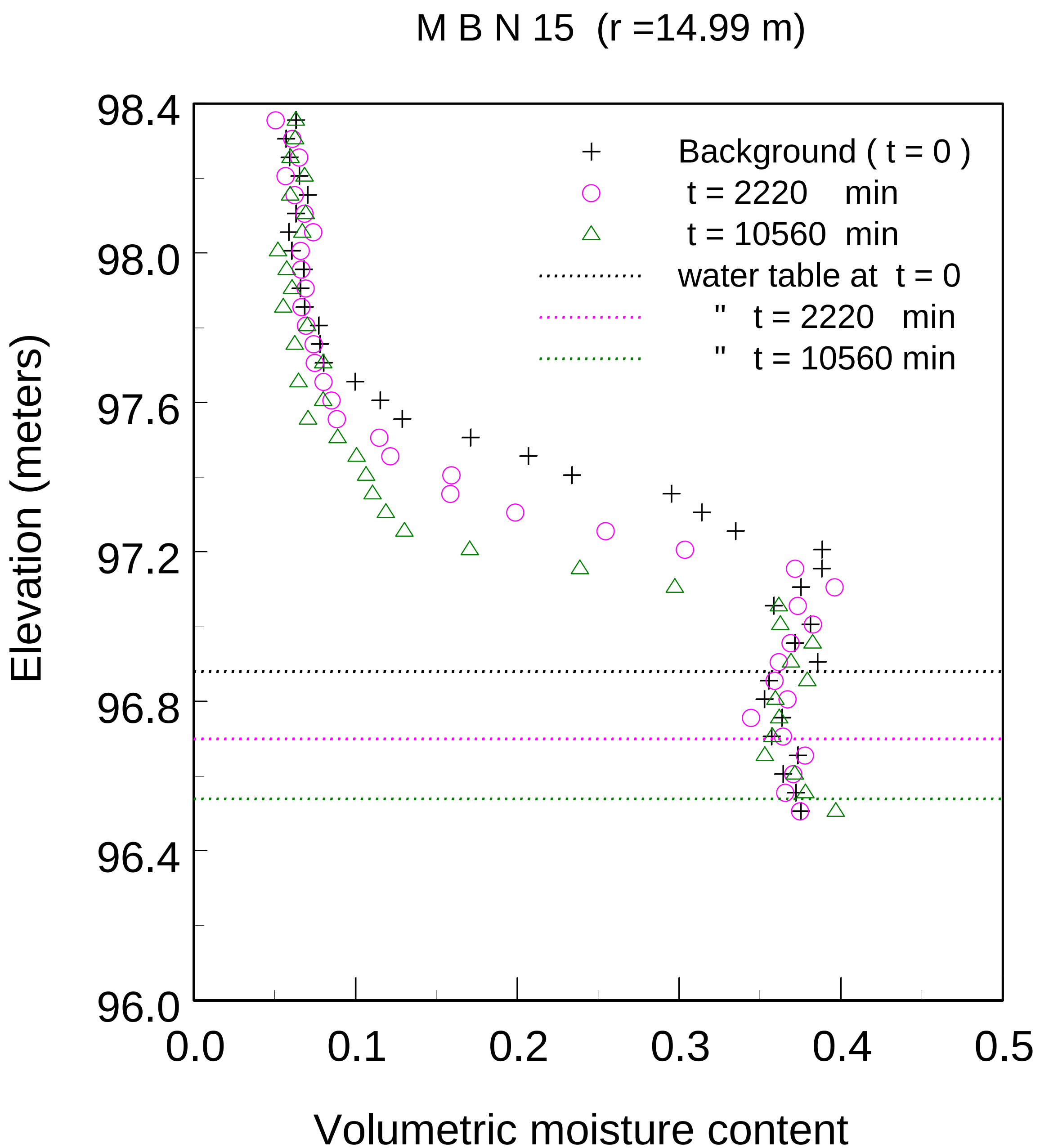

M B N 3 (r = 2.78 m)
98.4
98.0
97.6
97.2
96.8
96.4
96.0
0.0
0.1
0.2
0.3
0.4
0.5
Elevation (meters)
Volumetric moisture content
Background (t = 0)
t = 2220 min
t = 10560 min
water table at t = 0
" t = 2220 min
" t = 10560 min



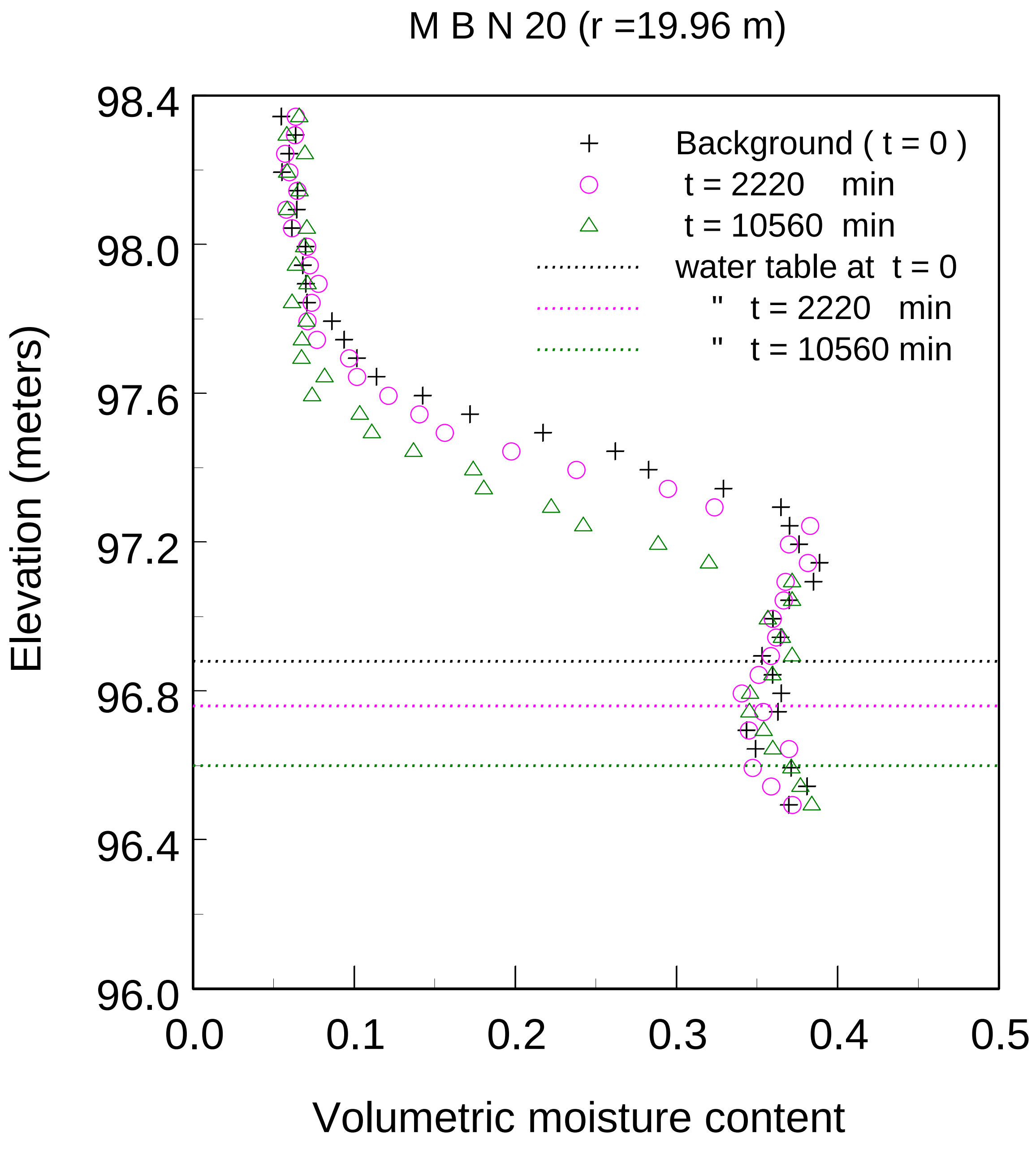
M B N 5 (r = 4.81 m)
Elevation (meters)
Volumetric moisture content
98.4
98.0
97.6
97.2
96.8
96.4
96.0
0.0
0.1
0.2
0.3
0.4
0.5
Background (t = 0)
t = 2220 min
t = 10560 min
water table at t = 0
" t = 2220 min
" t = 10560 min



M B N 10 (r =9.99 m)
Background (t = 0)
t = 2220 min
t = 10560 min
water table at t = 0
" t = 2220 min
" t = 10560 min
98.4
98.0
97.6
97.2
96.8
96.4
96.0
0.0
0.1
0.2
0.3
0.4
0.5
Elevation (meters)
Volumetric moisture content

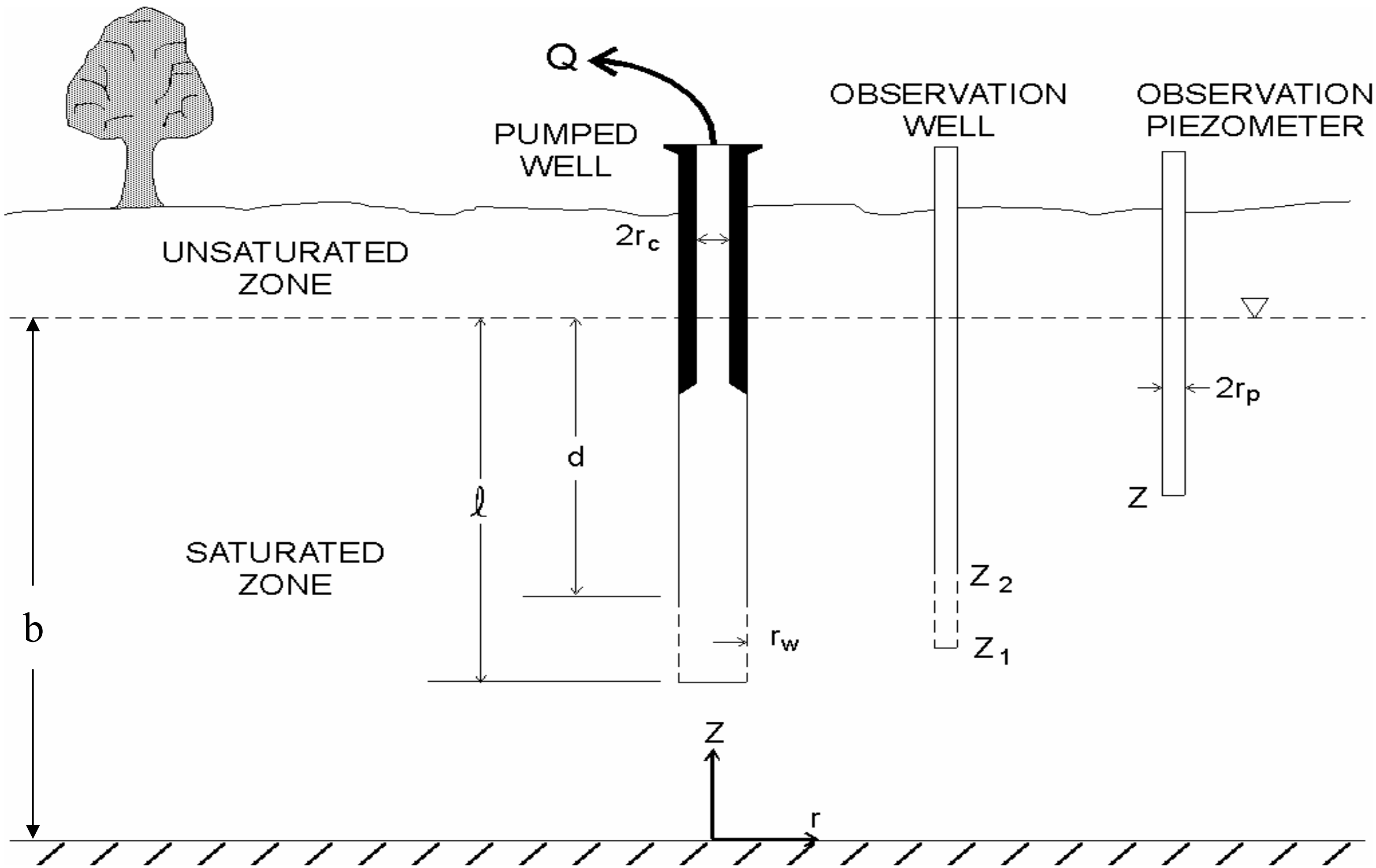


M B N 15 (r =14.99 m)
Elevation (meters)
Volumetric moisture content
98.4
98.0
97.6
97.2
96.8
96.4
96.0
0.0
0.1
0.2
0.3
0.4
0.5
Background (t = 0)
t = 2220 min
t = 10560 min
water table at t = 0
" t = 2220 min
" t = 10560 min



M B N 20 (r =19.96 m)
98.4
98.0
97.6
97.2
96.8
96.4
96.0
0.0
0.1
0.2
0.3
0.4
0.5
Elevation (meters)
Volumetric moisture content
Background (t = 0)
t = 2220 min
t = 10560 min
water table at t = 0
" t = 2220 min
" t = 10560 min

Definition of well characteristics



See Moench (1997, WRR, p. 1397) for model development

Delayed Piezometer Response (after Hvorslev, 1951)

$$\pi r_p^2 \frac{dh_m}{dt} = F' K_r (h - h_m)$$

where h_m is measured head in well
h is head in the aquifer at the well location

Delayed Piezometer Response (after Hvorslev, 1951)

$$\pi r_p^2 \frac{dh_m}{dt} = F' K_r (h - h_m)$$

where

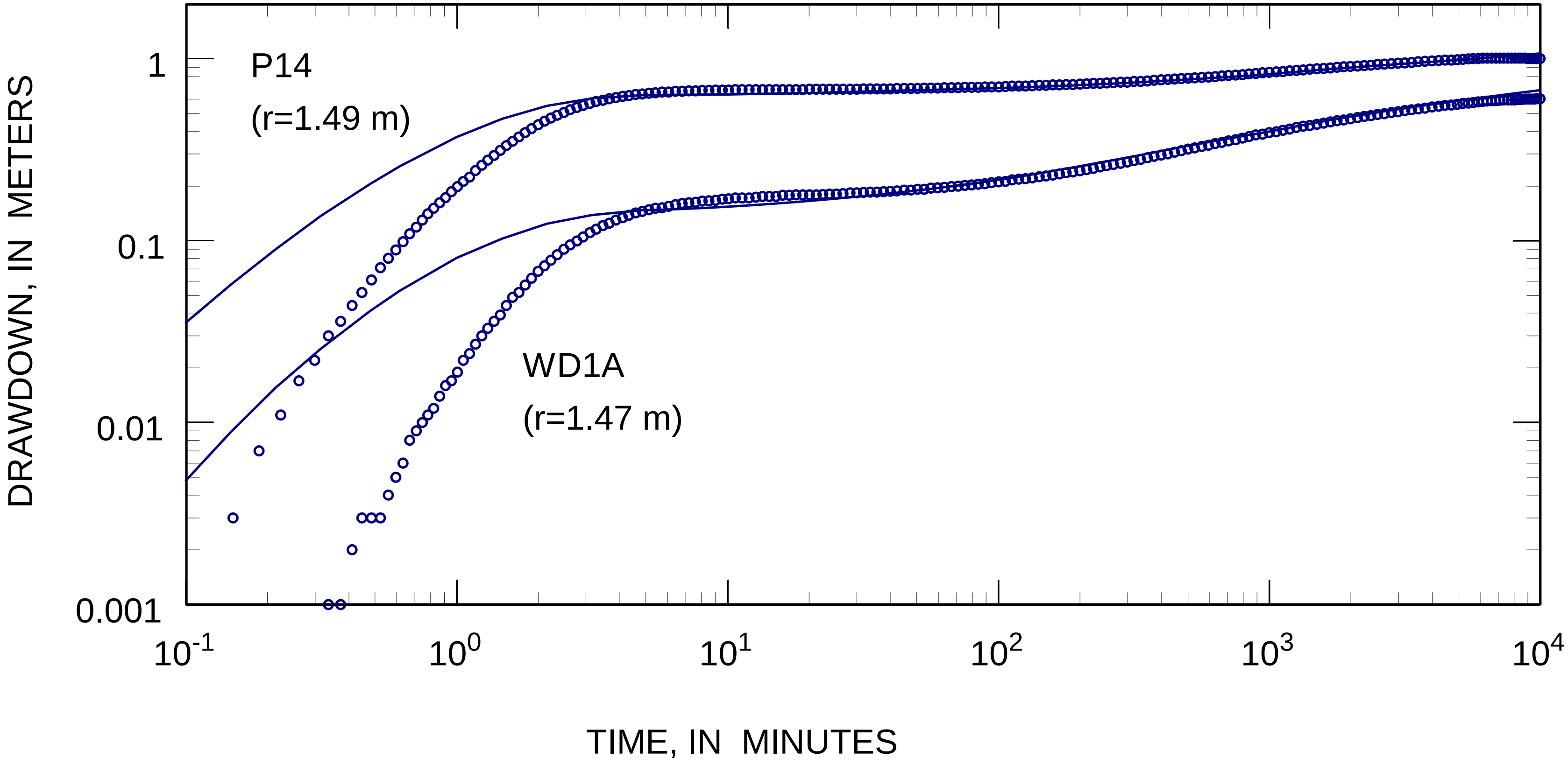
$$F' = \frac{L}{\ln[x + (1 + x^2)^{0.5}]}$$

$$x = \frac{L}{2r_p}(K_r / K_z)^{0.5}$$

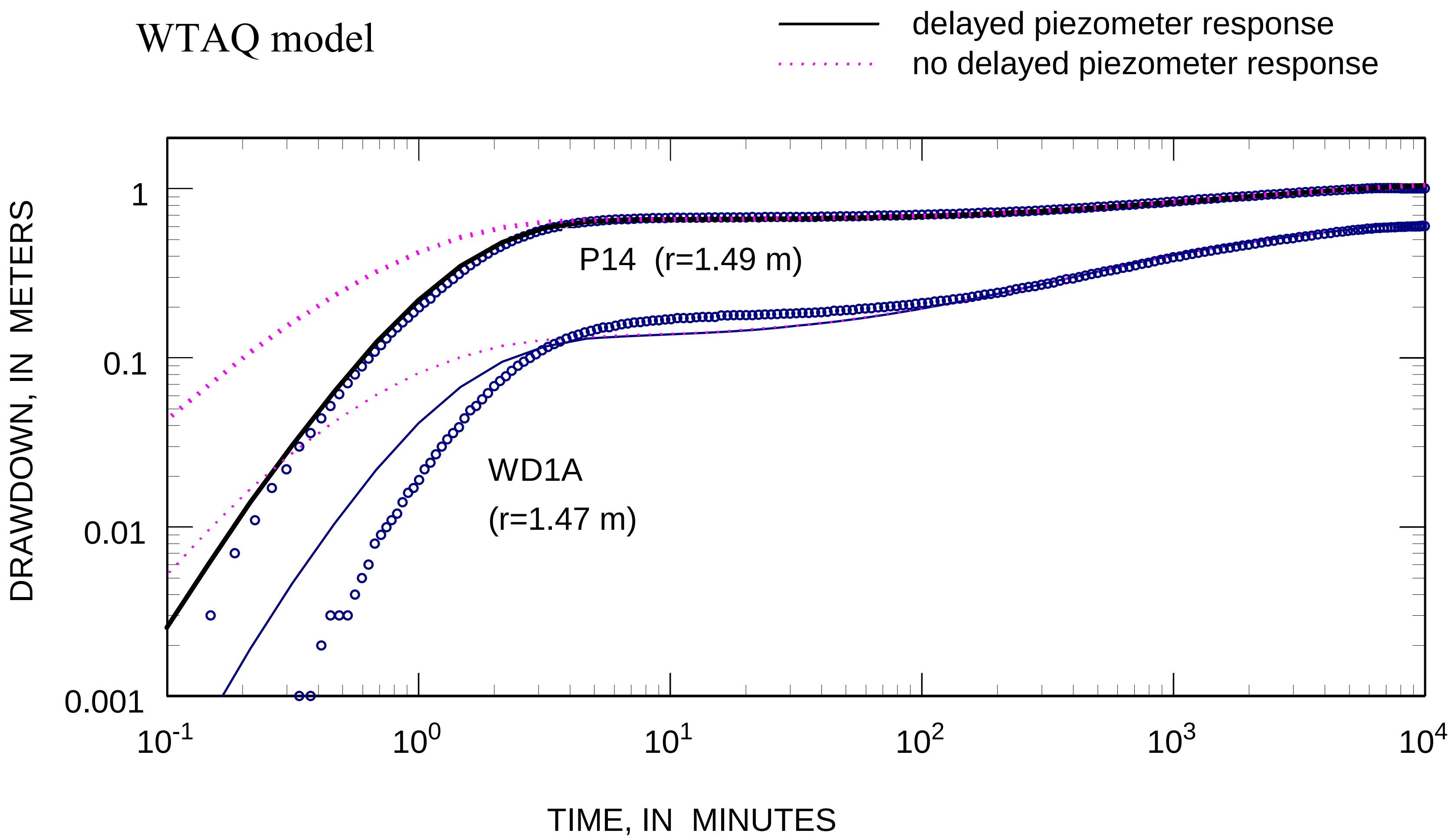
L = screen length

r_p = radius of piezometer

VS2DT model

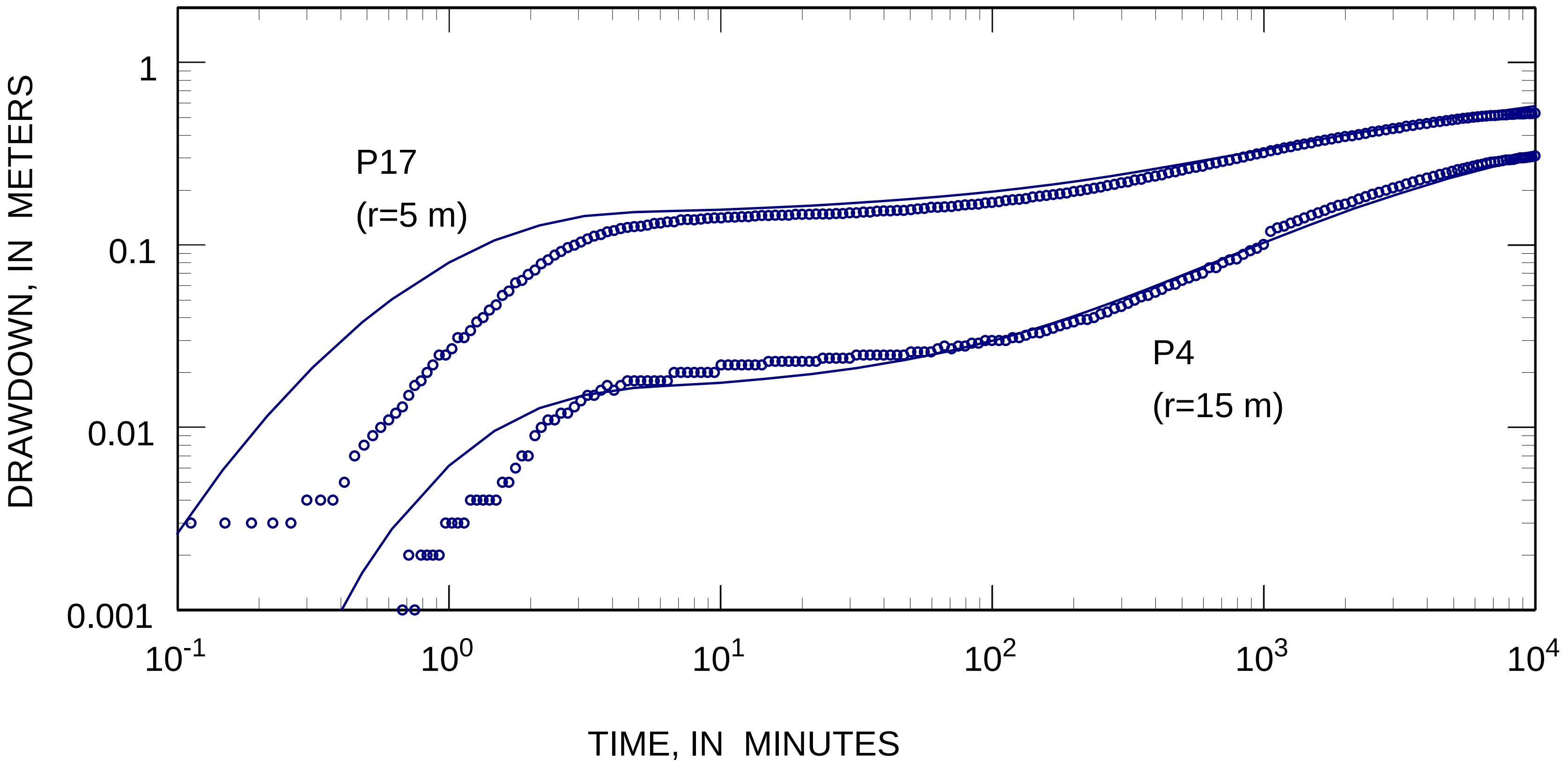


B&C

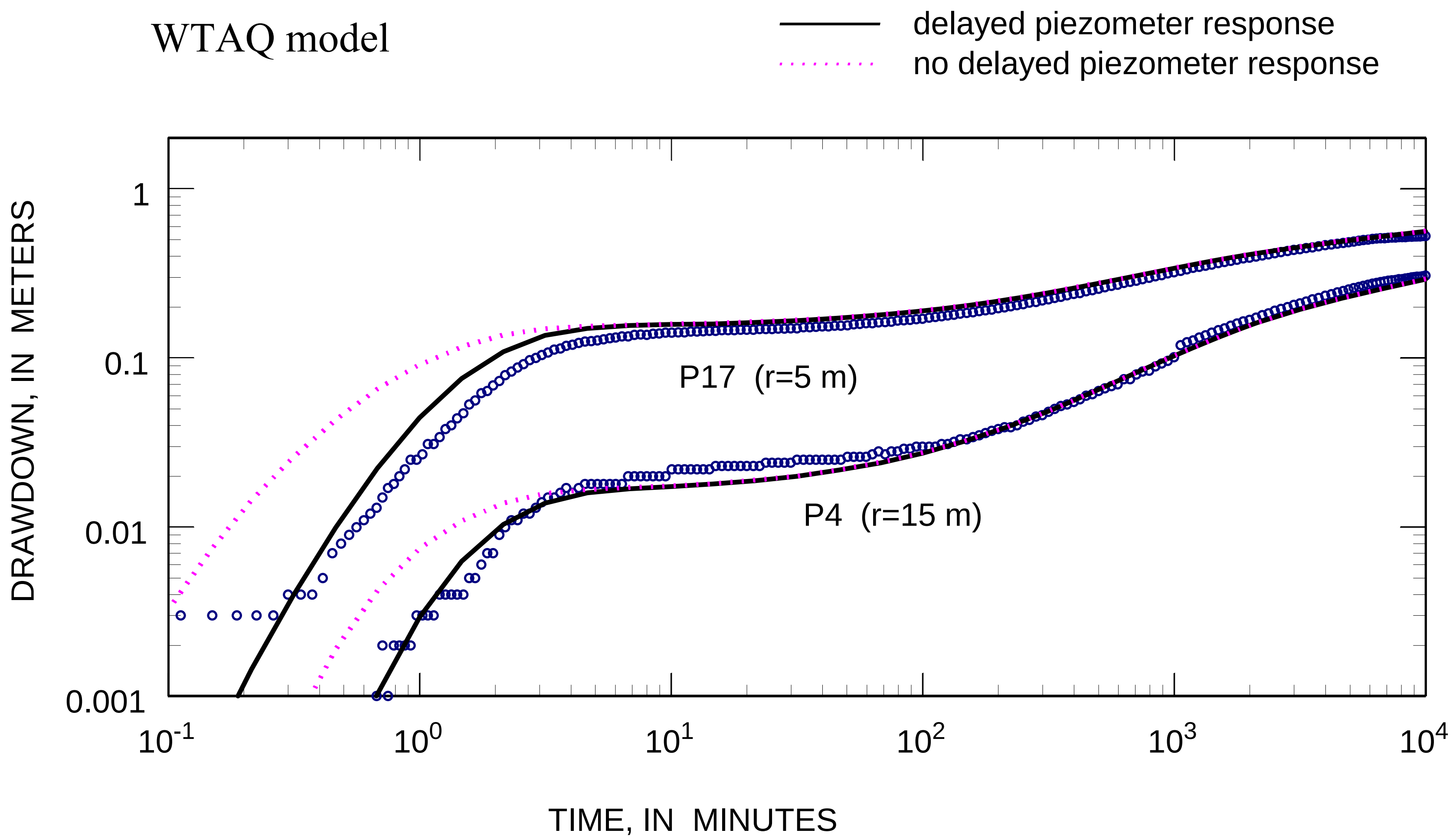


WTAQ model
delayed piezometer response
no delayed piezometer response
1
0.1
0.01
0.001
DRAWDOWN, IN METERS
P14 (r=1.49 m)
WD1A
(r=1.47 m)
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
TIME, IN MINUTES

VS2DT model



B&C



WTAQ model
delayed piezometer response
no delayed piezometer response
DRAWDOWN, IN METERS
1
0.1
0.01
0.001
P17 (r=5 m)
P4 (r=15 m)
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
TIME, IN MINUTES