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Baoline Chen, Rutgers University
Peter A. Zadrozny, U.S. Bureau of Labor Statistics

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ESTIMATED U.S. MANUFACTURING CAPITAL AND TECHNOLOGY BASED ON AN ESTIMATED DYNAMIC ECONOMIC MODEL*

Baoline Chen
Department of Economics
Rutgers University
Camden, NJ 08102
e-mail: baoline@crab.rutgers.edu

Peter A. Zadrozny
Bureau of Labor Statistics
2 Massachusetts Ave., NE, Room 3105
Washington, DC 20212
e-mail: zadrozny_p@bls.gov

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ABSTRACT

Time series of production capital and total factor productivity (or "technology") are fundamental to understanding the processes of output and productivity growth. Unfortunately, capital and technology are unobserved except at the most disaggregated levels of production units and capital components and must be estimated prior to being used in empirical analysis. Standard methods for estimating capital and technology were developed decades ago and are based on analytical and computational methods of that era. We develop and apply a new method for estimating production capital and technology, based on advances in economics, dynamic optimization, statistics, and computing over the intervening years. The method involves specifying and estimating a detailed structural dynamic economic model of a representative production firm in an industry and using the estimated model to compute Kalman-smoothed estimates of unobserved capital and technology for the sample period. We apply the method to annual data from 1947-97 for U.S. total manufacturing industries and compare its capital and technology estimates with standard estimates reported by the Bureau of Labor Statistics.

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1. Introduction.

Time series of production capital and total factor productivity (or "technology," as we call the latter here) are fundamental to understanding the processes of output and productivity growth. Unfortunately, capital and technology are unobserved except at the most disaggregated levels of production units and capital components and must be estimated prior to being used in empirical analysis. Standard methods for estimating capital and technology were developed decades ago (Jorgenson, 1963; Solow, 1957) and are based on analytical and computational methods of that era. We develop and apply a new method for estimating production capital and technology, based on advances in economics, dynamic optimization, statistics, and computing over the intervening years.

We apply the method to annual data from 1947-97 for U.S. total manufacturing industries and compare its estimates of capital and technology with standard estimates reported by the Bureau of Labor Statistics (1998, 1999). We offer the method and results as a fresh approach for understanding and estimating capital and technology using modern methods. The four major findings of the application are: (1) The model-based capital estimates are 10 times more uncertain than the model-based technology estimates. (2) The trends of the model-based capital and technology estimates broadly conform to the trends of standard estimates. (3) The model-based capital and technology estimates imply that above average capital growth in the 1990s -- not above average technology growth -- explains above average growth in manufacturing output in the 1990s. (4) In this context, changes in parameter estimates to suit prior views, can cause large changes in the model-based capital and technology estimates and, therefore, should be made cautiously.

We are interested in estimating aggregate capital, i.e., at the level of total production capital (equipment and structures) of all manufacturing industries. The present method has two major steps, a model-parameter estimation step followed by an unobserved-variable estimation step. In the first step, we specify and estimate by maximum likelihood a structural dynamic economic model of a representative production firm in an industry. We assume the firm solves a dynamic optimization problem, which is a standard adjustment cost problem except that adjustment costs on capital and technology are derived from a parsimoniously parameterized production function, rather than being stated directly as is usually done. We compute and incorporate the resulting optimal decision rules into the two estimation steps. We estimate the model's structural parameters without using any observations on capital or technology. We use only observations on prices and quantities of output, investment, research (short for "research and development"), labor, and materials inputs. We overcome the lack of capital and technology data by using a missing-data variant of the Kalman filter to compute the likelihood function and by using the overidentifying restrictions on reduced-form parameters in terms of structural parameters implied by the optimal decision rules. The reduced-form equations of the estimated model imply correlations between unobserved capital and technology and the observed variables in the model. In the second step, we use these correlations to compute linear least squares estimates (LLSE) of capital and technology, and their standard errors, in terms of the observed variables in the model. The LLSEs are implemented using a version of the Kalman smoothing algorithm (Anderson and Moore, 1979).

We now review the standard methods for estimating aggregate production capital and technology and, then, discuss the relative advantages of the present estimation method. Aggregate production capital stocks are often estimated using the perpetual inventory equation (PIE), $k_t = \delta_k k_{t-1} + i_t$, where k_t is the capital stock being estimated, i_t is observed investment flow, and δ_k is one minus a constant capital depreciation rate. Variants of the PIE can accommodate non-constant or non-geometric depreciation (Bureau of Labor Statistics, 1998). Aggregate production capital is also estimated as an index of the service flows of capital components (equipment, structures, and other disaggregates). The component service flows are estimated using Jorgenson's (1963) rental prices and are indexed using expenditure weights. Accordingly, disaggregated data are used in estimating aggregate capital, but, in either case, the estimates depend entirely on investment flows and capital depreciation rates and do not depend on other possible factors such as decision errors (misallocations), which the present method accounts for implicitly. Technology is usually estimated in percentage growth form as the Solow (1957) residual, $d\tau_t = dq_t - \sum_{i=1}^{n} s_{it} dx_{it}$, where $d\tau$, dq, and dx_{it} are percentage growth of technology, output, and production inputs, and s_{it} are input cost shares.

Generally, the relative advantages of the present method over standard methods are those of an elaborate econometric model over a simple econometric model. The advantages are greater generality (fewer restrictions) and more details, hence, more implications. The disadvantages are the need for more and better data, hence, a greater risk of specification error in practice, and greater mathematical and computational complexity. The standard methods for estimating capital and technology, while not in theoretical conflict with each other, are computationally independent. The present method takes the view that capital and technology are jointly determined as the result of purposeful, coordinated, investment and research decisions driven by the same value-maximizing motive. Thus, the model implicitly "disembodies" technology from capital (Jorgenson, 1966b; Hercowitz, 1998). In the standard method, technology is an unexplained residual. Whereas the present method allows for adjustment costs, the standard methods do not. However, the standard methods are nonparametric, except for having to specify capital depreciation, and are much easier to apply.

The present method automatically produces standard errors of the estimates of capital and technology and, therefore, quantifies uncertainty about the estimates. The standard methods have no measures of uncertainty and, therefore, in effect, present their estimates as being certain. We introduce uncertainty by adding disturbances to the PIEs of capital and technology. The disturbances may be viewed as representing subjective uncertainty or exogenous shocks. In practice, most of the uncertainty about capital concerns its depreciation. As the paper shows, adding disturbances to the PIEs has large consequences for the estimates of capital and technology. When the PIE disturbances are excluded, the estimates follow smooth trends, very similar to the standard estimates. When the disturbances are included, the estimates exhibit short-run variations -- random noises and economic cycles -- around their trends and the standard estimates. The economic cycles are transmitted from observed variables through the PIE disturbances.

Recently economists have estimated technology as filtered or smoothed estimates of an unobserved, estimated, exogenous process (Slade, 1989; French, 2000). The present paper goes further, by treating capital and technology as *joint endogenous* processes. We are unaware of other attempts to estimate joint, endogenous, capital and technology processes using filtering and smoothing methods, although these methods have been used to estimate endogenous (rational) inflationary expectations (Burmeister and Wall, 1982; Hamilton, 1985; Zadrozny, 1997). Regression methods have been used to estimate GNP, aggregate capital, and other macroeconomic variables (Romer, 1989; Levy and Chen, 1994; Levy, 2000) but they have more limited applicability and are less efficient. Unlike filtering and smoothing methods, regression methods require the estimated variables to be observed in some periods and cannot exploit correlations at all leads and lags. Our approach to modelling capital and technology as joint endogenous processes could be seen as an extension of Lucas (1967), with the benefit of modern analytical and computational methods. Finally, we note Jorgenson, Gollop, and Fraumeni (1987), Adams (1990), Griliches (1995), Caballero (1999), Nadiri and Prucha (1999), and references therein as recent examples of work on production capital and technology.

The paper continues as follows. Section 2 specifies the model and explains how the representative firm's dynamic optimization problem is solved. Section 3 prepares the model for estimation of parameters, capital, and technology by assembling its equations as a vector autoregression (VAR) and, then, restating the VAR as a state representation. Section 3 also discusses the parameter identification and reconstructibility conditions underlying the estimations. Section 4 discusses the application to aggregated U.S. manufacturing data. It discusses sources and properties of the data, statistical and economic properties of the estimated model, and compares the estimates of capital and technology with those published by the Bureau of Labor Statistics. Section 5 contains concluding remarks. Some technical details are in the appendix.

2. Specification and Solution of the Model.

Following Zadrozny (1996), we describe an industry in terms of a representative firm (henceforth, "the firm"). Except for scale differences, firm- and industry-level variables are identical. Every period, t, the firm maximizes the expected present value of profits,

$$(2.1) \qquad v_t = E_t \sum_{k=0}^{\infty} \delta^k \pi_{t+k} \, ,$$

with respect to a feedback decision rule, where the maximization is subject to equations to be specified, E_t denotes expectation conditional on the firm's information in period t, $\delta \in (0,1)$ denotes a constant real discount factor, and $\pi_t = r_{qt} - (c_{qt} + c_{it} + c_{rt})$ denotes real profits equal to revenues minus costs, such that c_{qt} is the cost of production and c_{it} and c_{rt} are direct (nonadjustment) costs of investment in capital and research in technology. Throughout, a real value is a nominal (current dollar) value divided by the GDP deflator. The firm's optimization problem is stated precisely at the end of this section.

To obtain a competitive rational-expectations-equilibrium solution, following Lucas and Prescott (1971), we set revenues in π_t to the area under the inverse output-demand curve as $r_{qt} = \int_{x=o}^{q_t} p_q(x, d_t)dx$, where $p_q(\cdot)$ is the inverse output-demand curve, q_t is the production of saleable output, and d_t is the output-demand state. Alternately, when $r_{qt} = p_q(q_t, d_t)q_t$, the solution represents the monopoly equilibrium.

To obtain linear solution equations, which facilitate estimation and to which the Kalman smoother can be applied, we specify r_{qt}, c_{qt}, c_{it}, and c_{rt} as quadratic forms (constant and linear terms can be ignored). Accordingly, we assume the industry's inverse output-demand curve is

(2.2) $$p_{qt} = -\eta q_t + d_t + \zeta_{pq,t},$$

where $\eta > 0$ is the slope parameter, d_t is the demand state generated by the second-order autoregressive (AR(2)) process

(2.3) $$d_t = \phi_{d1}d_{t-1} + \phi_{d2}d_{t-2} + \zeta_{d,t},$$

and $\zeta_{pq,t}$ and $\zeta_{d,t}$ are disturbances. Actually, $\zeta_{pq,t}$ is introduced for purely technical reasons. Its variance is set small enough so that it has no practical effect on the results but large enough so that it numerically stabilizes the Kalman filter and smoother. The full set of distributional assumptions on disturbances is stated in section 3.

To specify c_{qt}, we first assume that the firm uses capital (k), labor (ℓ), and materials (m), to produce saleable output (q), install investment goods (i), and conduct research activities (r) (subscript t is omitted sometimes). We assume that the "output activities," q, i, and r, are restricted according to the separable production function

(2.4) $$h(q,i,r) = \tau\cdot g(k,\ell,m),$$

where τ is the Hicks-neutral stock of technology. Although τ is also total-factor productivity, because $g(\cdot)$ and $h(\cdot)$ are indexes of inputs and outputs, we refer to τ as technology. If τ were capital augmenting or labor augmenting, the production function would be written as $h(q,i,r) = g(\tau k,\ell,m)$ or $h(q,i,r) = g(k,\tau\ell,m)$. More specifically, following Kydland and Prescott's (1982) treatment of the utility function, we assume $g(\cdot)$ and $h(\cdot)$ are the constant elasticity functions,

$$(2.5) \qquad g(k,\ell,m) = (\alpha_1 k^\beta + \alpha_2 \ell^\beta + \alpha_3 m^\beta)^{1/\beta},$$

$$h(q,i,r) = (\gamma_1 q^\rho + \gamma_2 i^\rho + \gamma_3 r^\rho)^{1/\rho},$$

where $\alpha_i > 0$, $\alpha_1 + \alpha_2 + \alpha_3 = 1$, $\beta < 1$, $\gamma_i > 0$, $\gamma_1 + \gamma_2 + \gamma_3 = 1$, and $\rho > 1$. CES = $(\beta-1)^{-1}$ is the constant elasticity of substitution among inputs, and CET = $(\rho-1)^{-1}$ is the constant elasticity of transformation among outputs. Including i and r in $h(\cdot)$ is a parsimonious way of specifying internal adjustment costs. The idea is that positive rates of investment and research use capital, labor, and materials resources, which could otherwise be used to produce more output, and that this trade-off sacrifices ever more output per unit increases in investment and research.

We need the adjustment costs to generate dynamic decision rules for the firm, which determine correlations among current and lagged variables, which are used to estimate unobserved variables in terms of observed variables. Adjustment costs are commonly specified as convex investment costs, which are incurred in addition to purchase costs of investment goods. Here "investment" means investment in production capital and research in technology. In the next step, we derive a quadratic approximation of the dual variable production cost function (DVPCF) from production function (2.4)-(2.5). The DVPCF includes convex, investment and research, adjustment costs. Thus, having already introduced investment and research purchase costs, $p_{it}i_t + p_{rt}r_t$, we obtain a <u>conventionally</u> <u>structured</u> specification of investment and research adjustment costs. Although the DVPCF is conventionally structured, it is <u>unconventionally</u> <u>parameterized</u>. We derive the DVPCF from (2.4)-(2.5) to ensure that structural parameters are identifiable. If we had specified a general DVPCF, subject only to symmetry, homogeneity, and curvature restrictions, it would have 28 free parameters, too many for the structural parameters to be identified, hence, estimated. The identification problem arises because 4 of 13 variables in the model are completely unobserved. The missing-data and identification problems are solved by specifying the DVPCF in terms of the 6 free parameters of (2.4)-

(2.5). For recent reviews of the investment adjustment cost literature, see, for example, Caballero (1999) and Nadiri and Prucha (1999).

Mathematically, convex internal adjustment costs arise in (2.4)-(2.5) when, for given technology, τ, and inputs, (k,ℓ,m), the transformation surfaces of the outputs, (q,i,r), are concave to the origin. The adjustment costs are "convex" because the derived DVPCF is convex in (q,i,r). Hall's (1973) analysis shows that the division of the production function into two separate input and output parts, $g(\cdot)$ and $h(\cdot)$, is a necessary condition for the output transformation surfaces to be concave to the origin. Here, $\rho > 1$ is a necessary and sufficient condition for the transformation surfaces to be concave. The transformation surfaces become more curved, hence, adjustment costs increase, as ρ increases. Similarly, $\beta < 1$ is a necessary and sufficient condition for the input isoquants to be convex to the origin, and the isoquants become more curved, hence, input substitutability decreases, as β decreases.

Let $c_q = p_\ell \ell + p_m m$, where p_ℓ is the real hiring price of labor and p_m is the real purchase price of materials. Let $c_i = p_i i$ and $c_r = p_r r$, where p_i and p_r are the real purchase prices of investment and research goods and services. Because ℓ and m are variable (not subject to adjustment costs) and k and τ are quasi-fixed (subject to adjustment costs), we refer to c_q as the variable cost and to $c_i + c_r$ as the fixed cost. Let $c_q(w)$ denote the dual variable cost function: given $w = (w_1, \ldots, w_7)^T = (q, i, r, k, \tau, p_\ell, p_m)^T$ (superscript T denotes transposition), $c_q(w)$ = minimum of $p_\ell \ell + p_m m$, with respect to ℓ and m, subject to production function (2.4)-(2.5).

In the standard approach to multifactor productivity analysis (Bureau of Labor Statistics, 1998, 1999), all inputs are treated symmetrically, as variable flows. Accordingly, c_q would include all input costs as $c_q = p_k k + p_\tau \tau + p_\ell \ell + p_m m$, where p_k and p_τ are rental prices of capital and technology stocks, obtained using appropriate versions of Jorgenson's (1963) formula for converting investment purchase prices into capital rental prices. Jorgenson's formula is based on more restrictive assumptions, notably that all inputs are variable. In this paper, we instead work with the purchase prices of investment and research because this allows greater flexibility for handling adjustment costs in the firm's dynamic optimization problem. It is the explicit solution of this problem that generates the identifying conditions that allow us to estimate the structural parameters of the model in the face of unobserved capital and technology.

The constant term in π does not affect optimal decisions in the approximate linear-quadratic dynamic optimization problem. Linear terms in π contribute only an additional constant term to the optimal decision rule, which is removed by mean adjustment of the data. Therefore, ignoring constant and linear terms, $c_q(w_t) \cong (1/2)w_t^T \cdot \nabla^2 c_q(w_0) \cdot w_t$, where $\nabla^2 c_q(w_0)$ denotes the Hessian matrix of second partial derivatives of c_q evaluated at $w = w_0$. $\nabla^2 c_q(w_0)$ is stated in the appendix, for $w_0 = (1, 1, 1, 1, 1, \alpha_2, \alpha_3)^T$, a value which results in the simplest expression for $\nabla^2 c_q(w_0)$. Therefore,

$$(2.6) \qquad \pi_t = -(1/2)\eta q_t^2 + q_t(d_t + \zeta_{pq,t}) - (1/2)w_t^T \cdot \nabla^2 c_q(w_0) \cdot w_t - p_{it} i_t - p_{rt} r_t.$$

The Hessian matrix, $\nabla^2 c_q(w_0)$, is symmetric (henceforth, for simplicity, we often write $\nabla^2 c_q(w_0)$ as $\nabla^2 c_q$). Ideally, $(1/2)w_t^T \cdot \nabla^2 c_q(w_0) \cdot w_t$ should inherit the following properties from the exact $c_q(w)$ function, for all values of w: (i) linear homogeneity in (q,i,r,k); (ii) convexity in (q,i,r,k); (iii) strict convexity in (q,i,r), (q,i,k), (q,r,k), and (i,r,k); (iv) linear homogeneity in (p_ℓ, p_m); and (v) strict concavity in p_ℓ and p_m. In fact, $w_t^T \cdot \nabla^2 c_q(w_0) \cdot w_t$ satisfies homogeneity restrictions (i) and (iv) for $w = w_0$ and curvature restrictions (ii), (iii), and (v) for all w.

The difference between $(1/2)w_t^T \cdot \nabla^2 c_q(w_0) \cdot w_t$ and the translog cost function (Christensen, Jorgenson, and Lau, 1971, 1973) is that $\nabla^2 c_q(w_0)$ is not stated in logs of variables and that its elements are tightly restricted in terms of the parameters of the model, whereas the translog cost function is stated in logs of variables and its elements are unrestricted except for the homogeneity, convexity, and concavity restrictions. The present model could be specified in logs of variables, but the results would be similar because the data are standardized prior to estimation. As noted above and discussed more below, estimating parameters without any capital and technology data and, then, estimating the unobserved capital and technology requires having sufficient identifying parameter restrictions on the cost function. Although we do not know and would have difficulty determining the full set of identifying cost-function parameterizations, we do know that the general translog cost function is not in this set.

We assume p_i, p_r, p_ℓ, and p_m are exogenous to the industry and are generated by the AR(2) processes

$$(2.7)\qquad p_{it} = \phi_{pi,1}p_{i,t-1} + \phi_{pi,2}p_{i,t-2} + \zeta_{pi,t},$$

$$p_{rt} = \phi_{pr,1}p_{r,t-1} + \phi_{pr,2}p_{r,t-2} + \zeta_{pr,t},$$

$$p_{\ell t} = \phi_{p\ell,1}p_{\ell,t-1} + \phi_{p\ell,2}p_{\ell,t-2} + \zeta_{p\ell,t},$$

$$p_{mt} = \phi_{pm,1}p_{m,t-1} + \phi_{pm,2}p_{m,t-2} + \zeta_{pm,t},$$

where $\zeta_{pi,t}$, $\zeta_{pr,t}$, $\zeta_{p\ell,t}$, and $\zeta_{pm,t}$ are disturbances. Processes (2.7) need not be stationary. A constant-coefficient autoregressive process is stationary or asymptotically stable if and only if its characteristic roots are less than one in absolute value. For example, the p_{it} process is stationary if and only if the roots, λ_1 and λ_2, which solve the characteristic equation, $\lambda^2 - \phi_{pi,1}\lambda - \phi_{pi,2} = 0$, are less than one in absolute value. The only restriction which we need on processes (2.7) in order to solve the firm's dynamic optimization problem is that $|\bar{\lambda}| < 1/\sqrt{\delta}$, where $|\bar{\lambda}|$ is the largest absolute characteristic root of any equation in processes (2.7).

We assume that capital accumulates according to the continuous-time law of motion

$$(2.8)\qquad \partial k(s)/\partial s = -f_k \cdot k(s) + i(s) + \tilde{\zeta}_k(s),$$

where $f_k > 0$ is a depreciation parameter and $\tilde{\zeta}_k(s)$ is a continuous-time disturbance. Integrating equation (2.8) over the sampling period $s \in [t-1,t)$, on the assumption that $i(s)$ is constant in $[t-1,t)$, we obtain the discrete-time capital law of motion,

$$(2.9)\qquad k_t = \phi_{k1}k_{t-1} + \phi_{i0}i_t + \zeta_{kt},$$

where $\phi_{k1} = \exp(-f_k)$, $\phi_{10} = [(1-\exp(-f_k)]/f_k$, and $\zeta_{kt} = \int_{s=0}^{1} \exp[-f_k(1-s)]\tilde{\zeta}_k(t-1+s)ds$ is the implied discrete-time disturbance. It is customary to specify (2.9) directly, such that $\phi_{i0} \equiv 1$. However, this specification understates the depreciation of investments undertaken early in a sampling period compared to those undertaken later in the period. The problem could be avoided by treating ϕ_{k1} and ϕ_{i0} as separate parameters, but this specification is less natural and introduces an additional parameter. Thus, assuming that $\zeta_{kt} \sim \text{NIID}(0, \sigma_k^2)$, we parameterize (2.9) in $\phi_{k1} \in (0,1)$ and $\sigma_k^2 > 0$, such that $\phi_{i0} = (\phi_{k1}-1)/\ln(\phi_{k1})$. Similarly, we obtain the discrete-time technology law of motion

$$(2.10) \qquad \tau_t = \phi_{\tau 1}\tau_{t-1} + \phi_{r0} r_t + \zeta_{\tau t},$$

parameterized in $\phi_{\tau 1} \in (0,1)$ and $\sigma_\tau^2 > 0$, such that $\phi_{r0} = (\phi_{\tau 1} - 1)/\ln(\phi_{\tau 1})$ and $\zeta_{\tau t} \sim \text{NIID}(0, \sigma_\tau^2)$.

Equations (2.9)-(2.10) imply geometrical depreciation, in which most of capital and technology's depreciation occurs in early periods of their use. A rational-distributed-lag (RDL) specification (Jorgenson, 1966a) could describe more general depreciation patterns, in particular, in which most depreciation occurs in late periods of use. A RDL could also include gestation or time-to-build lags as additional sources of capital and technology fixity. However, the need for parsimonious parameterization precludes RDL capital and technology equations, at least for the present data. Most RDLs could also be derived from underlying continuous-time specifications (Zadrozny, 1988).

The model's structural components have now been specified. It remains to explain how to solve the firm's dynamic optimization problem and how to assemble specified laws of motion and solved optimal decision rules into a system of linear simultaneous equations that are the equilibrium equations of the model.

To simplify the dynamic optimization problem, we eliminate q_t by maximizing π_t with respect to q_t. Because q_t is not a control variable in the laws of motion of k_t or τ_t, conditional on i_t and r_t being at their optimal values, the optimal value of q_t is given by maximizing π_t with respect to q_t. The first-order condition, $\partial\pi_t/\partial q_t = 0$, yields the output supply rule

$$(2.11) \qquad q_t = -(c_{11} + \eta)^{-1}(c_{12}i_t + c_{13}r_t + c_{14}k_t + c_{15}\tau_t + c_{16}\, p_{\ell t} + c_{17}p_{mt} - d_t) + \zeta_{qt},$$

where $(c_{11}, \ldots, c_{17})$ is the first row of $\nabla^2 c_q$ and ζ_{qt} is an added disturbance.

In addition to adding $\zeta_{pq,t}$ to output-demand curve (2.2) and ζ_{qt} to output supply rule (2.11), we also add disturbances to labor and materials decision rules (2.12)-(2.13) so that each of the 13 variables in the model has its own disturbance. Although the disturbances are added for purely technical reasons, to ensure that the variables in the model have a nonsingular joint probability distribution, as usual, they represent our specification errors or the firm's decision errors, or both.

Similar elimination of ℓ_t and m_t from the dynamic optimization problem is justified because ℓ_t and m_t are not control variables in the laws of motion of k_t or τ_t. Optimal values of ℓ_t and m_t, conditional on q_t, i_t and r_t being at their optimal values, are recovered using Shepard's lemma (a special case of the envelope theorem; Diewert 1971, p. 495),

$$(2.12)\qquad \ell_t = \partial c_{qt}/\partial p_{\ell t} = c_{61}q_t + c_{62}i_t + c_{63}r_t + c_{64}k_t + c_{65}\tau_t + c_{66}p_{\ell t} + c_{67}p_{mt} + \zeta_{\ell t},$$

$$(2.13)\qquad m_t = \partial c_{qt}/\partial p_{mt} = c_{71}q_t + c_{72}i_t + c_{73}r_t + c_{74}k_t + c_{75}\tau_t + c_{76}p_{\ell t} + c_{77}p_{mt} + \zeta_{mt},$$

where $(c_{61}, \ldots, c_{67})$ and $(c_{71}, \ldots, c_{77})$ are the sixth and seventh rows of $\nabla^2 c_q$, and $\zeta_{\ell t}$ and ζ_{mt} are added disturbances.

Optimality of labor and materials decision rules (2.12) and (2.13) also depends on $c_{qt} = (1/2)w_t^T \cdot \nabla^2 c_q(w_0) \cdot w_t$ being a good approximation of production function (2.4)-(2.5). It is easy to derive decision rules for ℓ_t and m_t from the exact cost function implied by (2.4)-(2.5). However, such rules are nonlinear in variables, which complicates parameter estimation and smoothing. Whether exact or approximate rules are used for decisions on ℓ and m, the approximate linear-quadratic dynamic optimization problem remains unchanged.

To solve the remainder of the firm's dynamic optimization problem, we restate it as a linear optimal regulator problem. We define the 2×1 control vector $u_t = (i_t, r_t)^T$ and the 14×1 state vector $x_t = (k_t, \tau_t, p_{it}, p_{rt}, p_{\ell t}, p_{mt}, d_t, k_{t-1}, \tau_{t-1}, p_{i,t-1}, p_{r,t-1}, p_{\ell,t-1}, p_{m,t-1}, d_{t-1})^T$. We assemble the laws of motion of output demand, input prices, capital, and technology, (2.3), (2.7), (2.9), and (2.10), as the state equation

$$(2.14)\qquad x_t = Fx_{t-1} + Gu_t,$$

$$F = \begin{bmatrix} F_1 & F_2 \\ I_7 & 0_{7\times7} \end{bmatrix}, \quad G = \begin{bmatrix} G_0 \\ 0_{12\times2} \end{bmatrix},$$

where $F_1 = \mathrm{diag}[\phi_{k1}, \phi_{\tau1}, \phi_{pi,1}, \phi_{pr,1}, \phi_{p\ell,1}, \phi_{pm,1}, \phi_{d1}]$, $F_2 = \mathrm{diag}[0, 0, \phi_{pi,2}, \phi_{pr,2}, \phi_{p\ell,2}, \phi_{pm,2}, \phi_{d2}]$, $G_0 = \mathrm{diag}[\phi_{i0}, \phi_{\tau0}]$, I_m is the m×m identity matrix, and $0_{m\times n}$ is the m×n zero matrix. We suppress disturbances in equation (2.14) because the regulator problem is certainty equivalent. We use the output-supply rule (2.11) to eliminate q_t from π_t and write π_t as the quadratic form

$$(2.15) \qquad \pi_t = u_t^T R u_t + 2u_t^T S x_{t-1} + x_{t-1}^T Q x_{t-1}.$$

The matrices R, S, and Q are stated in the appendix in terms of η and the elements of $\nabla^2 c_q$.

The regulator problem maximizes expected present value, (2.1), stated in terms of the quadratic form (2.15), with respect to the feedback matrix K in the linear decision rule $u_t = Kx_{t-1}$, subject to the state equation (2.14). Under concavity, stabilizability, and detectability conditions (Kwakernaak and Sivan, 1972), we compute the optimal K matrix by solving an algebraic matrix Riccati equation using a Schur decomposition method (Laub, 1979). Finally, we write the investment-research decision rule as

$$(2.16) \qquad u_t = Kx_{t-1} + (\zeta_{it}, \zeta_{rt})^T,$$

where $(\zeta_{it}, \zeta_{rt})^T$ is an added 2×1 disturbance vector.

3. Estimation Strategy.

3.1. State Representation of the Model.

To estimate the model's parameters by maximum likelihood, using the Kalman filter, and, then, to estimate unobserved capital and technology, using the Kalman smoother, we express the reduced form of the model in a state representation. To this end, we collect the variables of the model in the 13×1 vector $y_t = (p_{qt}, q_t, \ell_t, m_t, i_t, r_t, k_t, \tau_t, p_{it}, p_{rt}, p_{\ell t}, p_{mt}, d_t)^T$ and their disturbances in the 13×1 vector $\zeta_t = (\zeta_{pq,t}, \zeta_{qt}, \zeta_{\ell t}, \zeta_{mt}, \zeta_{it}, \zeta_{rt}, \zeta_{kt}, \zeta_{\tau t}, \zeta_{pi,t}, \zeta_{pr,t}, \zeta_{p\ell,t}, \zeta_{pm,t}, \zeta_{dt})^T$. We assume that the disturbances are mutually independent, normally distributed, stationary processes, such that the first 6 disturbances are AR(1) processes and the last 7 disturbances are serially independent. That is, we assume $\zeta_t = (I_{13} - \Theta L)^{-1}\varepsilon_t$, where $\varepsilon_t \sim NIID(0,\Sigma_\varepsilon)$, L is the lag operator, $\Theta = diag(\theta_{pq}, \theta_q, \theta_\ell, \theta_m, \theta_i, \theta_r, 0, 0, 0, 0, 0, 0, 0)$, such that the θ's $\in$ (-1,1), and $\Sigma_\varepsilon = diag(\sigma^2_{pq}, \sigma^2_q, \sigma^2_\ell, \sigma^2_m, \sigma^2_i, \sigma^2_r, \sigma^2_k, \sigma^2_\tau, \sigma^2_{pi}, \sigma^2_{pr}, \sigma^2_{p\ell}, \sigma^2_{pm}, \sigma^2_d)$.

The set of equations which form the basis of the parameter and capital-technology estimation are (2.2), (2.3), (2.7), (2.9)-(2.13), and (2.16), or more concisely, (2.2), (2.11)-(2.14), and (2.16). These 13 scalar-level equations constitute the complete set of linear simultaneous equations which, for given values of parameters, past variables, and current and past disturbances, determine unique values of the 13 variables of the model. We assemble the equations concisely as

$$(3.1)\qquad A_0 y_t = A_1 y_{t-1} + A_2 y_{t-2} + (I_{13} - \Theta L)^{-1}\varepsilon_t,$$

such that the elements of A_0, A_1, and A_2 are stated in the appendix. We premultiply equation (3.1) by $A_0^{-1}(I_{13} - \Theta L)$, such that A_0 is nonsingular for admissible values of parameters. Because the autocorrelation coefficients in Θ are nonzero only in equations with single lags of variables, the resulting VAR(2) reduced-form system,

$$(3.2)\qquad y_t = B_1 y_{t-1} + B_2 y_{t-2} + \xi_t,$$

has only two lags of y_t, where $B_1 = A_0^{-1}(A_1 + \Theta A_0)$, $B_2 = A_0^{-1}(A_2 - \Theta A_1)$, $\xi_t = A_0^{-1}\varepsilon_t \sim \text{NIID}(0,\Sigma_\xi)$, and $\Sigma_\xi \sim A_0^{-1}\Sigma_\varepsilon A_0^{-T}$. Because the input-price equations map unchanged into equation (3.2), they are both structural and reduced-form equations.

A complete state representation comprises a state equation, which expresses the dynamics of the model, and an observation equation, which accounts for how variables in the model are observed. Corresponding to state equation (2.14), we write the reduced-form equation (3.2) as the state equation

$$(3.3)\qquad z_t = \overline{F} z_{t-1} + \overline{G}\xi_t,$$

$$\overline{F} = \begin{bmatrix} B_1 & B_2 \\ I_{13} & 0_{13\times 13} \end{bmatrix}, \quad \overline{G} = \begin{bmatrix} I_{13} \\ 0_{13\times 13} \end{bmatrix},$$

where $z_t = (y_t^T,\ y_{t-1}^T)^T$ is the 26×1 state vector. Associated with the state equation is the observation equation

$$(3.4)\qquad \overline{y}_t = \overline{H}_t z_t,$$

where $\overline{y}_t$ is the vector of variables observed in period t. $\overline{H}_t$ is called the observation matrix.

Because $\overline{H}_t$ is completely flexible in assuming any values in any dimensions, including the null matrix if no observations are available, observation equation (3.4) can account for any pattern of missing data. For most sampling periods in the present application, $\overline{H}_t = [J, 0]$, where $J = I_{13}$ with rows of unobserved variables deleted and 0 is the equivalently dimensioned zero matrix. Thus, when variables 4, 7, 8, and 13 are unobserved, $J = I_{13}$ with rows 4, 7, 8, and 13 deleted and $0 = 0_{9\times 13}$. Also, $\overline{H}_t$ accounts for observations on different observed variables starting and ending in different periods. We call the Kalman filter applied to such a state representation the missing-data Kalman filter.

The missing-data Kalman filter computes the normal distribution (or Gaussian) likelihood function of the observations as follows. Let $\tilde{y}_t = \overline{y}_t - E[\overline{y}_t | \overline{Y}_{t-1}]$ denote the innovation vector, where $\overline{Y}_t = (\overline{y}_t^T, \ldots, \overline{y}_1^T)^T$ denotes the vector of observations through period t, and let $\Omega_t = E[\tilde{y}_t \cdot \tilde{y}_t^T]$ denote the innovation covariance matrix. In general, the reduced-form disturbance vectors, ξ_t, and the innovation vectors, $\tilde{y}_t$, coincide only when all variables are observed throughout the sample. Then, except for terms independent of parameters, -2 times the log-likelihood function of the sample $\overline{Y}_N$ is given by

$$(3.5) \qquad L(\vartheta, \overline{Y}_N) = \sum_{t=1}^{N} [\ln|\Omega_t| + \tilde{y}_t^T \Omega_t^{-1} \tilde{y}_t],$$

where $\vartheta = (\vartheta_0^T, \vartheta_1^T, \vartheta_2^T, \vartheta_3^T)^T$, $\vartheta_0 = (\delta, \alpha_1, \alpha_2, \gamma_1, \gamma_2, \sigma_{pq}^2, \sigma_\ell^2, \sigma_m^2)^T$, $\vartheta_1 = (\phi_{pi,1}, \phi_{pr,1}, \phi_{p\ell,1}, \phi_{pm,1}, \phi_{pi,2}, \phi_{pr,2}, \phi_{p\ell,2}, \phi_{pm,2}, \sigma_{pi}^2, \sigma_{pr}^2, \sigma_{p\ell}^2, \sigma_{pm}^2)^T$, $\vartheta_2 = (\theta_{pq}, \theta_q, \theta_\ell, \theta_m, \theta_i, \theta_r)^T$, and $\vartheta_3 = (\eta, \beta, \rho, \phi_{k1}, \phi_{\tau 1}, \phi_{d1}, \phi_{d2}, \sigma_q^2, \sigma_i^2, \sigma_r^2, \sigma_k^2, \sigma_\tau^2, \sigma_d^2)^T$.

As explained further in subsection 3.2, the unidentified 8 parameters in ϑ_0 are normalized and the remaining 31 parameters in ϑ_1, ϑ_2, and ϑ_3 are estimated in three steps: ϑ_1 in an ordinary-least-squares (OLS) step, ϑ_2 in a preliminary maximum-likelihood (ML) step, and ϑ_3 in a final ML step. The Kalman filtering recursions for computing (3.5), starting values for the recursions, and other details about implementing the computations accurately and efficiently are discussed in Anderson and Moore (1979), Zadrozny (1988, 1990), and references therein. In the ML steps, $L(\vartheta, \overline{Y}_N)$ was minimized using the trust-region method (Moré et al., 1980). Although the likelihood could be computed in other ways, the missing-data Kalman-filter method proved to be very effective for handling the various missing-data problems. In particular, given the computer program (Zadrozny, 1999), we needed only to indicate missing values in the data matrix with a missing-data indicator and did not need to transform the reduced-form or state equations, (3.2) or (3.3), as we would using other methods.

3.2. Parameter Identification and Reconstructibility Conditions.

The hallmark of the present method is a large number of overidentifying restrictions on the reduced-form parameters, B_1, B_2, and Σ_ξ, in terms of the structural parameters, ϑ, although the structural parameters are unidentified unless additional normalizing restrictions are imposed. Estimation of capital and technology requires that a reconstructibility condition hold. Thus, to estimate the model and use its estimate to estimate capital and technology, the model must satisfy the parameter identification and reconstructibility conditions. We comment no further on the complicated relationship between these conditions, except to note that in our experience parameter identification implies reconstructibility.

The parameter identification condition is standard in econometrics: the unnormalized parameters in ϑ to be estimated are identified when the Hessian matrix of $L(\vartheta, \overline{Y}_N)$ with respect to them, evaluated at the normalized and estimated values of parameters, is positive definite, i.e., $\nabla^2 L(\hat{\vartheta}, \overline{Y}_N) > 0$. The challenge is to have enough identifying restrictions on reduced-form parameters in terms of the structural parameters to compensate for the unobservability of some variables. In this case, with ϑ_0 normalized, the model imposes enough restrictions to identify ϑ_1, ϑ_2, and ϑ_3. The complexity of the mapping from structural to reduced-form parameters precludes analytically deriving the conditions under which ϑ_1, ϑ_2, and ϑ_3 are identified. Fortunately, doing this is unnecessary, because after terminating at an estimate, the ML estimation program numerically checks if $\nabla^2 L(\hat{\vartheta}, \overline{Y}_N) > 0$.

We estimated the 31 parameters in ϑ_1, ϑ_2, and ϑ_3 in three steps because initial attempts to estimate them simultaneously resulted in numerical breakdown. Although the estimation program converged successfully, it was unable to compute standard errors of the estimated parameters because $\nabla^2 L(\hat{\vartheta}, \overline{Y}_N)$ was poorly conditioned for inversion. Therefore, we followed the three-step strategy which is consistent but (in theory) inefficient compared to a simultaneous (or full information) estimation strategy. In all three steps, ϑ_0 is normalized as described below. In step 1, we estimated the 12 input-price process coefficients and disturbance variances in ϑ_1 using OLS. In step 2, conditional on $\hat{\vartheta}_1$, we estimated the 19 parameters in ϑ_2 and ϑ_3 using ML. In step 2, $\nabla^2 L(\hat{\vartheta}, \overline{Y}_N)$ was virtually non-positive definite, resulting in very large standard errors of the autocorrelation coefficients in $\hat{\vartheta}_2$. Therefore, in step 3, conditional on $\hat{\vartheta}_1$ and $\hat{\vartheta}_2$, we reestimated ϑ_3 using ML. Thus, the final estimates of ϑ are $\hat{\vartheta}_1$ from step 1, $\hat{\vartheta}_2$ from step 2, and $\hat{\vartheta}_3$ from step 3.

We imposed normalizing restrictions on ϑ_0 to ensure that ϑ_1, ϑ_2, and ϑ_3 are identified. We emphasize that this is <u>normalization</u>, <u>not</u> <u>calibration</u> in the sense of setting parameters so that the model matches selected moments in the data. Being unidentified, the normalized parameters cannot be calibrated in this sense. We verified numerically that the normalized parameters are unidentified by attempting to estimate all structural parameters simultaneously. The estimation algorithm made no moves from given initial parameter values, indicating a flat likelihood function.

We set the discount factor to $\delta = .935$, which corresponds to the interest rate $\delta^{-1} - 1 = .0695$. We set the weighting parameters in the production function to the "neutral" values $\alpha_1 = \alpha_2 = \alpha_3 = \gamma_1 = \gamma_2 = \gamma_3 = 1/3$. We considered alternative weighting-parameter normalizations. These resulted in different estimates of ϑ_3 but in the same estimates of reduced-form parameters, hence, in the same estimates of capital and technology. We expected that one disturbance variance would have to be restricted for each unobserved variable. Three variables are genuinely unobserved, k, τ, and d. To maintain numerical stability of the Kalman filter and smoother, all disturbance variances must be positive. Therefore, we set $\sigma^2_{pq} = \sigma^2_{\ell} = \sigma^2_m \cong 10^{-10}$. Although setting $\sigma^2_{pq} \cong 0$ is natural, because σ^2_{pq} is redundant relative to σ^2_d in output-demand curve (2.2), setting $\sigma^2_{\ell} = \sigma^2_m \cong 10^{-10}$ is arbitrary. We could have set these disturbance variances to other values, indeed, could have set any three disturbance variances. It makes no difference, because each choice results in the same estimated reduced form. We checked this result by estimating the model under alternative variance normalizations. As described in section 4.1, after some initial estimations, we decided to treat materials quantity, m, as unobserved. It would seem, then, that another disturbance variance would have to be moved from ϑ_3 to ϑ_0 and normalized. But this turned out not to be the case. Conditional on ϑ_0 and ϑ_1, under the initial definitions of the ϑ's, ϑ_2 and ϑ_3 were still identified. Therefore, we conducted the final estimations using the original normalizations.

To explain reconstructibility, let $\hat{z}_{t|s}$ denote the linear least-squares estimate of z_t in terms of $\overline{Y}_s$, let

(3.6) $$R_t = [\,\overline{H}_1^T,\ \overline{F}^T\,\overline{H}_2^T,\ \ldots,\ (\overline{F}^{t-1})^T\,\overline{H}_t^T\,]^T,$$

where $\overline{F}$ is the state-transition matrix in (3.3) and $\overline{H}_t$ is the observation matrix in (3.4). The state vector, $z_t = (y_t^T,\ y_{t-1}^T)^T$, is said to be reconstructible if there is a t_r such that R_t has full rank equal to the dimension of z_t, for $t \geq t_r$. Reconstructibility means that, for $t \geq t_r$,

(3.7) $$\hat{z}_{t|t} = (R_t^T R_t)^{-1} R_t^T \overline{Y}_t\,,$$

where $R_t^T R_t$ is nonsingular, so that unique <u>filtered</u> estimates of z_t (i.e., for $t|s = t|t$), for $t = 1, \ldots, N$, can be computed. If (3.7) is feasible, an associated formula computes the error covariance matrix $E(z_t - \hat{z}_{t|t})\cdot(z_t - \hat{z}_{t|t})^T$ in terms of R_t and the disturbance covariances. The <u>smoothed</u> estimates of z_t (i.e., for $t|s = t|N$), for $t = 1, \ldots, N$, may be expressed similarly. The Kalman smoother is an accurate and efficient recursive algorithm for computing $\hat{z}_{t|N}$ and $E(z_t - \hat{z}_{t|N})\cdot(z_t - \hat{z}_{t|N})^T$, for $t = 1, \ldots, N$ (Anderson and Moore, 1979).

In the application, the dimension of z_t is 26, so that if H_t is time invariant and z_t is reconstructible, $t_r \leq 26$. This follows from the Cayley-Hamilton theorem, which says that every square matrix satisfies its own characteristic equation. In such case, for $t \geq 26$, the rows of $\overline{F}^t$ are linearly dependent on the rows of $\overline{F}^{25}$, $\overline{F}^{24}$, ..., $\overline{F}$. Therefore, if $\overline{H}_t$ is time invariant, z_t is reconstructible if R_{26}, called the reconstructibility matrix, has full rank 26. It is difficult to determine an upper bound for t_r if $\overline{H}_t$ is time varying. For a complete discussion of reconstructibility and related concepts, see Kwakernaak and Sivan (1972) or Anderson and Moore (1979). The estimation algorithm numerically checks the reconstructibility condition.

4. Estimation Results.

4.1. Sources and Properties of the Data.

In estimation, we used annual U.S. total manufacturing data on prices and quantities of output and inputs, ranging from 1947-97. Investment and GDP-deflator data were obtained from the Bureau of Economic Analysis, research data from the National Science Foundation (1998), and all other data from the Bureau of Labor Statistics. All data were obtained in nonseasonal form. Thus, we obtained observations on 10 of the 13 variables in the model: p_{qt} and q_t from 1958-96, $p_{\ell t}$ and ℓ_t from 1948-97, p_{it} and i_t from 1947-96, p_{rt} and r_t from 1953-95, p_{mt} from 1958-96, and m_t from 1958-89.

Except for the quantity of labor, which is measured as the number of production workers, all other prices and quantities were obtained as a nominal price index or a real quantity index coupled with nominal expenditures. We computed the unavailable quantity or price indexes by dividing expenditures by the available price or quantity index, so that in each case the price index $\times$ quantity index = nominal expenditures. All obtained or computed nominal price indexes were, then, converted into real form by dividing them by the GDP deflator.

The resulting real prices and quantities of U.S. total manufacturing output and inputs are depicted in figures 1a-j. For graphing convenience, the data were scaled to lie between 0 and 10. The graphs suggest the following economic interpretation, which is consistent with simulations of the model in figures 2a-b. Increasing demand for output driven partly by a declining real price of output induced manufacturers to increase production capacity. Increasing quantities of investment and research built increasing stocks of capital and technology, hence, increased production capacity. As the price of labor increased, manufacturers saved on labor inputs, resulting in flat or declining labor use and increasing labor productivity.

[Put figures 1a-j approximately here]

Initially, we considered total hours worked (total production workers multiplied by average hours worked per worker) as an alternate labor input measure. The graph of total hours worked (not shown) is very similar to that of total production workers in figure 1f. The main difference is that total hours worked is a somewhat noisier series. We chose total production workers as the labor input because it resulted in a slightly better fitting, but insignificantly different, estimated model. Choosing total production workers as the labor input caused the R^2s of output price and quantity, investment, and research to increase by .01 to .02 and that of labor to increase by .16. Throughout, an R^2 refers to the reduced-form equation of a variable.

Initially, we estimated the model using the data described above, but this resulted in a nearly zero R^2 for labor. The problem appeared to be misspecification of materials in the production function. The model's simulations and the production function’s form indicate symmetrical roles for labor and materials, while the data in figures 1a and 1c show the time path of materials matching closely that of output, not that of labor. The solution options were: (i) drop materials price and quantity from the analysis; (ii) assume materials quantity is in fixed proportions to the output good; or (iii) keep materials price and quantity in the model, as they are, continue to use materials price data in the parameter estimation and smoothing, but treat materials quantity as unobserved. Options (i) and (ii) would be implemented implicitly by measuring the output good as value added instead of shipments and dropping materials as a production input. We chose option (iii), which was also the easiest to implement, because it required only that the materials quantity column in the data matrix be filled in with the missing-value indicator. Therefore, in the final round of estimation, materials quantity was treated as unobserved, along with actually unobserved capital, technology, and output-demand state.

4.2. Statistical Properties of the Estimated Model.

Table 1 reports first-step OLS estimates of the input-price process parameters in ϑ_1. By conventional standards, the estimated equations fit the data well, having R^2's greater than .90. Residuals show no significant autocorrelations, having p values of Ljung-Box Q statistics greater than .25. The estimated p_i, p_r, and p_ℓ processes have characteristic roots near one, with maximum absolute characteristic roots, $|\bar{\lambda}|$, between .785 and 1.02. A process is stationary if and only if its $|\bar{\lambda}| < 1$. The complete estimated reduced form, (3.2), has five absolute characteristic roots between .98 and 1.02. Although a cointegration analysis might seem appropriate, we did not attempt this for two reasons. The input-price processes serve only the subsidiary purpose of providing forecasts for the dynamic optimization problem and their AR(2) specifications are adequate for this task. It is not clear how a standard cointegration analysis, designed for systems in which all variables are observed and coefficients are unrestricted except for unit-root restrictions, applies in this case, in which parameters are restricted by the solution of the dynamic optimization problem and 4 of 13 variables are unobserved. We allowed unit roots insofar as residual autocorrelation coefficients, θ, may be very close to one. Table 2 reports second-step ML estimates of $\hat{\theta}_{pq}$, $\hat{\theta}_{\ell}$, and $\hat{\theta}_m = .999$.

[Put table 1 approximately here]

Table 2 also reports third-step ML estimates of the remaining parameters in ϑ_3. Their absolute t statistics are less than about .50 and are not reported because the small sample size makes them unreliable and uninformative (Sims, 1980, p. 19, fn. 19). The implied estimated reduced-form equations show unsurprisingly good fits by conventional standards, given that the data are used in original levels form. Moderate ($\cong$ .50) and high (> .90) R^2's of labor and the nonlabor variables reflect labor's noisiness and the nonlabor variables' unit-root-like smoothness. The high estimated residual autocorrelation coefficients ($\hat{\theta}$'s $\geq$ .84) might suggest that the residual autocorrelation corrections and not the economic part of the model account for most of the observed endogenous variables' sample variations, but this is not the case. ML estimation with all θ's set to zero produced $R^2_{pq} = .918$, $R^2_q = .879$, $R^2_{\ell} = .436$, $R^2_i = .772$, and $R^2_r = .944$, so that the economic part of the model accounts for these fractions of the endogenous variables' sample variations. Most importantly, as we now discuss in detail, the model's overidentifying restrictions are not rejected by a likelihood ratio test.

[Put table 2 approximately here]

For large N, abstracting from terms independent of parameters, the maximized log-likelihood function can be expressed as $L(\hat{\vartheta}, \overline{Y}_N) = N \cdot \ln|\hat{\Omega}_N|$, where $\hat{\Omega}_N = (1/N)\sum_{t=1}^{N} \tilde{y}_t \tilde{y}_t^T$. The likelihood-ratio statistic for testing the model's restrictions is $LR = N(\ln|\hat{\Omega}_{N,R}| - \ln|\hat{\Omega}_{N,U}|)$, where $\hat{\Omega}_{N,R}$ and $\hat{\Omega}_{N,U}$ are $\hat{\Omega}_N$ based on restricted and unrestricted innovations, i.e., from maximizing the likelihood function with the model's restrictions, respectively, imposed and relaxed. The missing-data Kalman filter automatically produces restricted innovations as part of the ML estimation. We obtained unrestricted innovations as follows. We performed the test using the subsample 1960-1990, because only during this period were observations available for the 9 observed variables. For this period, the observation matrix, H_t, is time invariant and given by $H = [J, 0_{9\times 13}]$, where $J = I_{13}$ with rows 4, 7, 8, and 13 deleted. Then, combining the state and observation equations, (3.3)-(3.4), we obtain the infinite autoregressive representation for $\overline{y}_t$, hence, the finite p-lag approximation of this representation,

$$(4.1) \qquad \overline{y}_t = \Phi_1 \overline{y}_{t-1} + \ldots + \Phi_p \overline{y}_{t-p} + \tilde{\tilde{y}}_t,$$

where the residual $\tilde{\tilde{y}}_t$ is an approximation of the innovation $\tilde{y}_t$. We want to test the *economic* restrictions of the model and not the mutual independence of input-price processes (2.7). Therefore, except for the zero restrictions which make the input-price processes mutually independent, we considered the Φ's to be free parameters. For p = 2, we estimated the individual equations of (4.1) by applying OLS to the period 1960-1990. Thus, we reestimated the input-price processes using the shorter sample. The resulting residuals were serially uncorrelated and were used to compute $\hat{\Omega}_{N,U}$.

LR is distributed asymptotically as $\chi^2(\kappa)$, in the limit as $N \to \infty$, where κ denotes the number of overidentifying restrictions. The statistic rejects the null hypothesis that the overidentifying restrictions are valid when it exceeds the critical value, c_α, for the significance level α. The period 1960-1990 implies the small values N = 31 and $N/\kappa = .15$, for $\kappa = 118$. For such situations, Sims (1980, p. 17, fn. 18) suggested replacing N with $N - \nu$ in LR, where, in this case, ν is the number of estimated parameters divided by the number of observed endogenous variables. Thus, $N - \nu = 31 - (143/9) = 15.1$ and

κ = 118, imply LR = 142, with a p value of .067, so that the overidentifying restrictions are not rejected at a conventional 5% significance level.

4.3. Economic Properties of the Estimated Model.

Because the estimates of capital and technology depend critically on the economic model, to be confident in the estimates we should be confident in the economic properties of the model. Therefore, we present and briefly discuss some structural variance decompositions (Sims, 1986) and impulse responses of the estimated model.

We begin by explaining how the variance decompositions are computed. Let $M = I_{13}$ with columns 1, 3, and 4 deleted. Then, combining the state and observation equations, (3.3)-(3.4), we obtain the structural infinite moving-average representation of $\bar{y}_t$, i.e., in terms of the structural disturbance vector, ε_t,

(4.2) $$\bar{y}_t = \Psi(L)\varepsilon_t = \left(\sum_{i=0}^{\infty} \Psi_i L^i\right)\varepsilon_t = \sum_{i=0}^{\infty} \Psi_i \varepsilon_{t-i},$$

where $$\Psi_i = J\begin{bmatrix} B_1 & B_2 \\ I_{13} & 0_{13\times 13} \end{bmatrix}^i \begin{bmatrix} I_{13} \\ 0_{13\times 13} \end{bmatrix} M$$

and J is defined as in (4.1). M has been introduced to delete the three structural disturbances, $\varepsilon_{pq,t}$, $\varepsilon_{\ell t}$, and ε_{mt}, whose variances are normalized to near zero. Let $E[\bar{y}_{t+k} | \bar{Y}_t]$ denote the k-step-ahead forecast of $\bar{y}_{t+k}$; let $\tilde{y}_{t,k} = \bar{y}_{t+k} - E[\bar{y}_{t+k} | \bar{Y}_t]$ denote the forecast error of $E[\bar{y}_{t+k} | \bar{Y}_t]$; and, let $V_k = E\tilde{y}_{t,k}\tilde{y}_{t,k}^T$ denote the covariance matrix of $\tilde{y}_{t,k}$. Then, V_k is given by

(4.3) $$V_k = \sum_{i=0}^{k} \Psi_i \Sigma_\varepsilon \Psi_i^T .$$

We decompose the k-step-ahead forecast-error variances of the 8 endogenous variables, and their sum, in terms of the 9 unnormalized estimated structural disturbance variances. That is, we decompose $v_{k,ii}$, for i = 1, ..., 8, and $\sum_{i=1}^{8} v_{k,ii}$, where $v_{k,ii}$ is the (i,i) diagonal element of V_k, in terms of

σ_j^2, for j = 2, 5, 6, ..., 13. Let $s_{k,i,j}$ and $\bar{s}_{k,j}$ denote the fractions of $v_{k,ii}$ and $\sum_{i=1}^{8} v_{k,ii}$ due to σ_j^2; let $\Sigma_\varepsilon^{1/2}$ be the square-root of Σ_ε, obtained by replacing the diagonal elements of Σ_ε with their positive square roots; let e_i denote the 13×1 vector with one in position i and zeroes elsewhere; and, let $\bar{e}$ denote the 13×1 vector with ones in the first 8 positions and zeroes elsewhere. Then, for i = 1, ..., 8 and j = 2, 5, 6, ..., 13, the percentage variance decompositions of $v_{k,ii}$ and $\sum_{i=1}^{8} v_{k,ii}$ are given by

$$(4.4) \qquad s_{k,i,j} = e_i^T(\textstyle\sum_{i=0}^{k} \Psi_i \Sigma_\varepsilon^{1/2} e_j e_j^T \Sigma_\varepsilon^{1/2} \Psi_i^T) e_i \, / \, e_i^T(\textstyle\sum_{i=0}^{k} \Psi_i \Sigma_\varepsilon \Psi_i^T) e_i \, ,$$

$$(4.5) \qquad \bar{s}_{k,j} = \bar{e}^T(\textstyle\sum_{i=0}^{k} \Psi_i \Sigma_\varepsilon^{1/2} e_j e_j^T \Sigma_\varepsilon^{1/2} \Psi_i^T) \bar{e} \, / \, \bar{e}^T(\textstyle\sum_{i=0}^{k} \Psi_i \Sigma_\varepsilon \Psi_i^T) \bar{e} \, .$$

[Put table 3 approximately here]

Table 3 shows the structural decompositions of k = 10 year ahead forecast-error variances. Rows 2-9 show decompositions of variances of endogenous variables; row 10 shows the decomposition of the sum of variances of endogenous variables. For example, elements 1, 2, 6, and 10 in row 2 indicate that, according to the estimated model, 4.5, 2.8, 5.2, and 83.5 percent of the variance of p_q is, respectively, due to σ_q^2, σ_i^2, σ_{pi}^2, and σ_d^2. Because the model is estimated using standardized data, the decompositions are unit free. However, different normalizations of disturbance variances in ϑ_0 will result in different decompositions. All disturbances, except disturbances of research, technology, price of research, and price of labor, explain significant (> 6%) fractions of some individual variances or the summed variances. Interestingly, the small impacts of research and technology disturbances run contrary to the real business cycle literature which attributes significant macroeconomic fluctuations to technology shocks. Overall, the decompositions suggest that the capital, output-demand, and investment-price disturbances are the leading sources of variations of the 8 endogenous variables.

[Put figures 2a-b approximately here]

The simulations in figures 2a-b display the dynamic adjustment-cost behavior in the model in response to unit impulses in output-demand and technology disturbances. The simulations in figure 2a match the general interpretation of figures 1a-j. The simulations depict responses to unit one-period shock (impulse) to the output-demand state in period 1, starting from an initial long-run equilibrium represented by the origin. The estimate $\hat{\eta}$ = .605 implies a moderately sloped output-demand curve. The estimates $\hat{\beta}$ = -9.14 and $\hat{\rho}$ = 267 imply CES = -.099 and CET = .004, hence, low input substitutability and very high adjustment costs on capital and technology. High adjustment costs imply a steep marginal-cost-of-production curve. Therefore, after the output-demand shock occurs, the price of output rises sharply but output increases only slightly. Initially, the extra output is produced using additional freely-adjusted labor and materials inputs and pre-shock stocks of capital and technology. Because the shocked demand state declines moderately slowly, firms have an incentive to increase their production capacities. Thus, they increase their investment and research rates and substitute capital and technology for labor and materials. Figure 2b depicts responses to a unit one-period shock to technology in period 1, again starting from an initial long-run equilibrium at the origin. In figure 2b, output-demand conditions remain unchanged so there is little change in price or quantity of output. The shock mainly causes technology to be substituted for labor and materials until the windfall addition to technology has depreciated fully.

4.4. Model-Based versus Standard Estimates of Capital and Technology.

Figures 3a-b to 6a-b display the model-based and standard estimates of production capital and technology of aggregated U.S. manufacturing industries from 1958-97. The solid graphs depict the model-based estimates and their 2-standard-error confidence bounds. The dashed graphs of capital depict the sum of Bureau of Labor Statistics (BLS) estimates of equipment and structures stocks, based on nonstochastic perpetual inventory equations (PIEs). The dashed graphs of technology depict BLS estimates of multifactor productivity computed as Solow residuals. In addition, BLS estimates equipment and structures service flows and Bureau of Economic Analysis (BEA) produces alternate estimates of equipment and structures stocks. These estimates are very similar and are not displayed. Thus, we display the BLS estimates of capital and technology as representative of standard estimates of these variables.

Because ML estimation of the model is tractable only if all data are scaled similarly, the data were standardized prior to estimation, by subtracting sample means and dividing by sample standard deviations. Therefore, being based on standardized data, the model-based estimates are in approximately standardized units. The BLS estimates are in arbitrarily scaled real units. To compare the two sets of estimates, one set must be converted to the units of the other. Therefore, prior to graphing, we exactly standardized each set of estimates. Also, in each figure, we translated all graphs up by the same amount so that all values are graphed as positive numbers. Because the units of the graphs are arbitrary, movements along a graph cannot be interpreted as percentage changes. However, differences between graphs in the same figure are in comparable standardized units. The graphs start in 1958 because output, a critical determinant of the estimates, is first available in 1958.

[Put figures 3a-b approximately here]

Figures 3a-b depict graphs of model-based estimates based on parameter estimates in tables 1 and 2. The capital and technology estimates, respectively, have average standard errors of 1.03 and .089, which implies that capital's 2-standard-error confidence intervals are over 10 times larger than technology's. "Short-run" variations with average periodicities of less than about 8 years are sums of unpredictable noises and business cycles. "Long-run" variations with greater average periodicities reflect trends. The model-based capital estimates exhibit frequent, significant, short-run variations. The model-based technology estimates exhibit less frequent and less significant short-run variations. Standard smoothing formulas can decompose short-run variations into sums of noises and cycles. However, because the formulas ignore sampling variability of parameter estimates, model misspecification, and other uncertainties, the decompositions are themselves uncertain. To the extent that short-run variations reflect cycles, not noises, we can often explain them in terms of identifiable events, such as the Vietnam War (1965-73) and oil-price increases (1973, 1979), and in terms of cyclical fluctuations of the overall economy. The model-based estimates exhibit cycles passed by the estimation method from the observed variables. Because they are based on nonstochastic PIEs, the BLS capital estimates exhibit miniscule short-run variations.

[Put figures 4a-b approximately here]

Figures 4a-b depict alternate model-based estimates based on capital and technology disturbances near zero ($\sigma_k^2 = \sigma_\tau^2 = .0001$) and other structural parameters at their table 1 and 2 values (for technical reasons, all structural disturbance variances must be at least slightly positive). Going from figure 3a to 4a, average standard errors of model-based capital estimates decline 5-fold, from 1.03 to .205. Setting capital disturbance variances to near zero does not eliminate capital standard errors because they depend on all structural disturbance variances. Going from figure 3a to 4a, short-run variations of capital estimates also decline 5-fold, causing the estimates to become more trend-like and to conform better to the BLS estimates. Going from figure 3b to 4b, average standard errors of technology estimates decline slightly, from .089 to .060. Correspondingly, the technology estimates change little.

Being estimates based on PIEs, the model-based and BLS capital estimates should be considered available capital stocks. However, apparently large short-run variations in the model-based estimates in figure 3a might seem to contradict this notion. Aren't available aggregate capital stocks large relative to investment flows and capital disturbances and don't they depreciate slowly, so that their graphs should be very smooth, like the BLS capital estimates in figure 3a? We could informally interpret short-run variations in the model-based capital estimates as variations in utilized capital stocks or as variations in effective capital stocks, i.e., adjusted for misallocations. Standard estimation methods treat all capital investments as being equally successful, regardless of misallocations, market realizations, and market valuations. Thus, an optimally located factory would add the same amount to capital as a mislocated factory built using the same resources. However, in order to formally interpret short-run capital variations as utilized or effective capital, we would have to extend the model to include some notion of capacity utilization or market valuation of capital.

Being Solow residuals, BLS technology estimates in figure 3b exhibit larger short-run variations than BLS capital estimates, especially during oil-price rises in the 1970s. The technology estimates are usually considered to be the residuals of the production function in the analysis. Here, because both capital and technology are unobserved, either of their estimates could be considered as the residuals. However, because the model-based capital estimates exhibit the larger short-run variations, they are more naturally selected as the residuals. This is consistent with capital's role as the residual income earning factor. Presumably, technology should reflect smoothly varying knowledge. Therefore, because Solow residuals are noisy, they are often smoothed prior to being considered as technology estimates (French, 2000). Being constructed as smoothed estimates, the model-based technology estimates should not be smoothed further and, in fact, as seen in figures 3b to 6b, are smoother than the BLS Solow-residual estimates.

There has been a debate about whether capital growth or technology growth explains above average output growth in the 1990s (Gordon, 2000; Oliner and Sichel, 2000; Stiroh, 2001). Figures 1h and 1j indicate above average growth of investment and roughly trend growth, then, decline of research in the 1990s. Figures 3a-b show similar growth patterns for model-based capital and technology estimates in the 1990s. Therefore, the present estimates favor above average capital growth as the explanation of recent above average manufacturing output growth.

The table 1 and 2 parameter estimates seem reasonable, except possibly for $\hat{\phi}_{k1}$ = .589 and $\hat{\phi}_{\tau1}$ = .161, which imply high, annual, capital and technology, depreciation rates of 1-$\hat{\phi}_{k1}$ = .411 and 1-$\hat{\phi}_{\tau1}$ = .839. For example, Jorgenson and Stephenson (1967) reported a quarterly depreciation rate, for equipment and structures in U.S. manufacturing industries from 1947-60, which implies $\hat{\phi}_{k1}$ = .895. Figures 5a-b depict alternate model-based estimates, based on $\hat{\phi}_{k1}$ = $\hat{\phi}_{\tau1}$ = .895 and other parameters at their table 1 and 2 values. The lower capital and technology depreciation rates result in smoother model-based estimates, with technology estimates conformable with figures 3b and 4b, but incredible capital estimates, indicating nearly continuous decline. Figures 6a-b depict another set of model-based estimates, based on $\hat{\phi}_{k1}$ = $\hat{\phi}_{\tau1}$ = .895, σ_k^2 = σ_τ^2 = .0001, and other parameters at their table 1 and 2 values. Going from figures 5a-b to 6a-b, as expected, the estimates become smoother. Unexpectedly, the capital estimates return to their figure 3a and 4b patterns. The lesson of figure 5a is that, in this context, one should be cautious about altering parameter estimates to suit prior views. Figure 6b's model-based technology estimates are almost pure trends. The sharp drops at sample ends are probably caused by excess sensitivity to smoother initialization in this case.

[Put table 4 approximately here]

To gain further insights into the reasonableness of $\hat{\phi}_{k1}$ = .589 and $\hat{\phi}_{\tau 1}$ = .161, we estimated the capital and technology equations using nonlinear least-squares (NLLS). As in the ML estimation, the equations are parameterized in terms of their underlying continuous-time parameters. We estimated the equations using the initial model-based and BLS, capital and technology, estimates as real data. The results are reported in table 4. Although NLLS estimates of ϕ_{k1} and $\phi_{\tau 1}$ differ from ML estimates, they are very similar for the model-based and BLS data. As expected, the fit of the estimated equations depends on the noisiness of the dependent variable. Thus, the capital equation fits better when using BLS data (R_k^2 = .891) rather than model-based data (R_k^2 = .730), and the reverse is true for the technology equation. In essence, table 4 confirms what we see in figures 3a-b and 4a-b, that the trends of the model-based and BLS estimates are broadly conformable. Although the ML estimates, $\hat{\phi}_{k1}$ = .589 and $\hat{\phi}_{\tau 1}$ = .161, might seem low economically, they are acceptable econometrically, because, along with other parameter estimates, they imply an acceptably fitting model, with unrejected overidentifying restrictions, which generates smoothed estimates of capital and technology, broadly conformable with standard estimates.

5. Conclusion.

The paper has developed a new method for estimating unobserved economic variables based on an estimated dynamic economic model and applies it to estimating production capital and technology (total-factor productivity) of aggregated U.S. manufacturing industries from 1958-97. The method illustrates how modern estimation, control, and filtering methods can be applied to a parsimonious dynamic economic model to produce estimates and standard errors of unobserved variables. Standard methods for estimating capital and technology, developed forty years ago, are appealing in their theoretical and computational simplicity, but are unnecessarily restrictive in some respects, for example, ignore adjustment costs. The present method admits adjustment costs of capital and technology, but is more complex analytically, econometrically, and computationally. We regard the method as experimental and do not advocate replacing standard methods. We urge testing the present method further, using different models and data sets. The paper shows that the method is feasible. In general, the method is feasible when the economic model imposes enough identifying restrictions to compensate for the unobservability of some variables.

The four major findings of the application are: (1) The model-based capital estimates are 10 times more uncertain than the model-based technology estimates. (2) The trends of the model-based capital and technology estimates broadly conform to the trends of standard estimates. (3) The model-based capital and technology estimates imply that above average capital growth in the 1990s -- not above average technology growth -- explains above average growth in manufacturing output in the 1990s. (4) In this context, changes in parameter estimates to suit prior views, can cause large changes in the model-based capital and technology estimates and, therefore, should be made cautiously.

As noted previously, sorting out the competing interpretations of the model-based capital estimates as available, utilized, or effective capital stocks requires formally introducing some notion of capacity utilization or market valuation of capital. The variance decompositions in table 3 assign principal explanatory roles to capital and investment-price disturbances, which suggests modelling investment and research decisions in more detail. For example, the discount rate could be time-varying, as $\delta_t = 1/(1 + n_t)$, such that n_t would be an observed exogenous interest rate whose generating process is also estimated. The capital and technology equations could be specified as rational distributed lags, which include time-to-built gestation lags and non-geometrical depreciation rates of capital and technology.

6. Appendix: Statement of Cost, Profit, and Reduced-Form Parameters.

Because $\nabla^2 c_q(w_0)$ is symmetric, it suffices to state its upper triangular part. Let c_{ij} denote element (i,j) of $\nabla^2 c_q(w_0)$. Then, for $w_0 = (1, 1, 1, 1, 1, \alpha_2, \alpha_4)^T$, we have:

$c_{11} = \gamma_1(1-\gamma_1)(\rho-1) + \gamma_1^2\alpha_1(1-\beta)/(1-\alpha_1)$

$c_{12} = -\gamma_1\gamma_2[\rho-1 + \alpha_1(1-\beta)/(1-\alpha_1)]$

$c_{13} = -\gamma_1\gamma_3[\rho-1 + \alpha_1(1-\beta)/(1-\alpha_1)]$

$c_{14} = -\gamma_1\alpha_1(1-\beta)/(1-\alpha_1)$

$c_{15} = -\gamma_1(1-\alpha_1\beta)/(1-\alpha_1)$

$c_{16} = \gamma_1/(1-\alpha_1)$

$c_{17} = \gamma_1/(1-\alpha_1)$

$c_{22} = \gamma_2(1-\gamma_2)(\rho-1) + \gamma_2^2\alpha_1(1-\beta)/(1-\alpha_1)$

$c_{23} = -\gamma_2\gamma_3[\rho-1 + \alpha_1(1-\beta)/(1-\alpha_1)]$

$c_{24} = -\gamma_2\alpha_1(1-\beta)/(1-\alpha_1)$

$c_{25} = -\gamma_2(1-\alpha_1\beta)/(1-\alpha_1)$

$c_{26} = \gamma_2/(1-\alpha_1)$

$c_{27} = \gamma_2/(1-\alpha_1)$

$c_{33} = \gamma_3(1-\gamma_3)(\rho-1) + \gamma_3^2\alpha_1(1-\beta)/(1-\alpha_1)$

$c_{34} = -\gamma_3\alpha_1(1-\beta)/(1-\alpha_1)$

$c_{35} = -\gamma_3(1-\alpha_1\beta)/(1-\alpha_1)$

$c_{36} = \gamma_3/(1-\alpha_1)$

$c_{37} = \gamma_3/(1-\alpha_1)$

$c_{44} = \alpha_1(1-\beta)[1 + \alpha_1(2-\alpha_1)/(1-\alpha_1)]$

$c_{45} = -\alpha_1 + \alpha_1(2-\alpha_1-\beta)/(1-\alpha_1)$

$c_{46} = -\alpha_1/(1-\alpha_1)$

$c_{47} = -\alpha_1/(1-\alpha_1)$

$c_{55} = (2-\alpha_1-\alpha_1\beta)/(1-\alpha_1)$

$c_{56} = -1/(1-\alpha_1)$

$c_{57} = -1/(1-\alpha_1)$

$c_{66} = -\alpha_3/[\alpha_2(1-\alpha_1)(1-\beta)]$

$c_{77} = -\alpha_2/[\alpha_3(1-\alpha_1)(1-\beta)]$.

Next, we state the elements of the 2×2, 2×14, and 14×14 coefficient matrices R, S, and Q, which define quadratic form (2.15). Because R and Q are symmetric, we state only their upper-triangular parts. R_{ij}, S_{ij}, and Q_{ij} denote (i,j) elements of the matrices. To eliminate the common factor 1/2, we scale π_t up by the factor of 2, which is allowable because optimal decisions are invariant to the scale of π_t. For simplicity, we state only nonzero elements of R, S, and Q, so that all unstated elements are zero. Thus, setting $c_0 = (\eta+c_{11})^{-1}$, we have

$R_{11} = c_0c_{12}^2 - c_{22}$

$R_{12} = c_0c_{12}c_{13} - c_{23}$

$R_{22} = c_0c_{13}^2 - c_{33}$

$S_{11} = c_0c_{12}c_{14} - c_{24}$

$S_{12} = c_0c_{12}c_{15} - c_{25}$

$S_{13} = -1$

$S_{15} = c_0c_{12}c_{16} - c_{26}$

$S_{16} = c_0c_{12}c_{17} - c_{27}$

$S_{17} = -c_0c_{12}$

$S_{21} = c_0c_{13}c_{14} - c_{34}$

$S_{22} = c_0c_{13}c_{15} - c_{35}$

$S_{24} = -1$

$S_{25} = c_0c_{13}c_{16} - c_{36}$

$S_{26} = c_0c_{13}c_{17} - c_{37}$

$S_{27} = -c_0c_{13}$

$Q_{11} = c_0c_{14}^2 - c_{44}$

$Q_{12} = c_0c_{14}c_{15} - c_{45}$

$Q_{15} = c_0c_{16}c_{16} - c_{46}$

$Q_{16} = c_0c_{14}c_{17} - c_{47}$

$Q_{17} = -c_0c_{14}$

$Q_{22} = c_0c_{15}^2 - c_{55}$

$Q_{25} = c_0c_{15}c_{16} - c_{56}$

$Q_{26} = c_0c_{15}c_{17} - c_{57}$

$Q_{27} = -c_0c_{15}$.

Finally, we state the structural coefficient matrices A_k, for k = 0, 1, 2. Let $A_{k,i,j}$ and $K_{i,j}$, respectively, denote elements (i,j) of A_k and K, the optimal investment-research feedback matrix. As before, only nonzero elements are stated. Also, because the diagonal elements of A_0 are all one, they are not stated. Proceeding row-wise across the matrices,

$A_{0,1,2} = \eta$

$A_{0,1,3} = -1$

$A_{0,2,5} = c_0 c_{12}$

$A_{0,2,6} = c_0 c_{13}$

$A_{0,2,7} = c_0 c_{14}$

$A_{0,2,8} = c_0 c_{15}$

$A_{0,2,11} = c_0 c_{16}$

$A_{0,2,12} = c_0 c_{17}$

$A_{0,2,13} = -c_0$

$A_{0,3,2} = -c_{16}$

$A_{0,3,5} = -c_{26}$

$A_{0,3,6} = -c_{36}$

$A_{0,3,7} = -c_{46}$

$A_{0,3,8} = -c_{56}$

$A_{0,3,11} = -c_{66}$

$A_{0,3,12} = -c_{67}$

$A_{0,4,2} = -c_{17}$

$A_{0,4,5} = -c_{27}$

$A_{0,4,6} = -c_{37}$

$A_{0,4,7} = -c_{47}$

$A_{0,5,7} = -c_{57}$

$A_{0,5,11} = -c_{67}$

$A_{0,5,12} = -c_{77}$

$A_{0,7,5} = -\phi_{i0}$

$A_{0,8,6} = -\phi_{r0}$

$[A_{1,5,7}, \ldots, A_{1,5,13}] = [K_{1,1}, \ldots, K_{1,7}]$

$[A_{1,6,7}, \ldots, A_{1,6,13}] = [K_{2,1}, \ldots, K_{2,7}]$

$[A_{1,7,7}, \ldots, A_{1,13,13}] = [\phi_{k1}, \phi_{\tau 1}, \phi_{pi,1}, \phi_{pr,1}, \phi_{p\ell,1}, \phi_{pm,1}, \phi_{d1}]$

$[A_{2,5,7}, \ldots, A_{2,5,13}] = [K_{1,8}, \ldots, K_{1,14}]$

$[A_{2,6,7}, \ldots, A_{2,6,13}] = [K_{2,8}, \ldots, K_{2,14}]$

$[A_{2,7,7}, \ldots, A_{2,13,13}] = [0, 0, \phi_{pi,2}, \phi_{pr,2}, \phi_{pm,2}, \phi_{d2}]$.

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Table 1

Step 1 OLS Estimates of Input-Price Process Parameters in ϑ_1

Var.	Parameter Estimates			Fit Statistics		
	$\hat{\phi}_{\cdot,1}$	$\hat{\phi}_{\cdot,2}$	$\lvert\bar{\lambda}\rvert$	$\hat{\sigma}$	R^2	Q
p_i	1.45 (11.1)	-.441 (3.30)	1.02	.178	.971	5.64 (.933)
p_r	.652 (4.01)	.282 (1.81)	.949	.126	.979	4.67 (.968)
p_ℓ	1.88 (24.8)	-.883 (11.3)	1.01	.019	.999	14.8 (.254)
p_m	1.49 (9.79)	-.617 (4.06)	.785	.334	.903	9.13 (.692)

Comments: Columns 2-7, respectively, show estimates of $\phi_{\cdot,1}$ and $\phi_{\cdot,2}$, with their absolute t statistics in parentheses, implied maximum absolute characteristic roots (solutions of $\lambda^2 - \hat{\phi}_{\cdot,1}\lambda - \hat{\phi}_{\cdot,2} = 0$), estimated standard deviations of disturbances, unadjusted R^2s (defined as 1 - sample variance of the innovation of a variable ÷ sample variance of the variable), and Ljung-Box Q statistics for testing absence of residual autocorrelations at lags from 1 to 10, with their marginal significance levels or p values in parentheses.

Table 2

Step 2 and 3 ML Estimates of Structural Parameters in ϑ_2 and ϑ_3

Production Function Parameters

$\hat{\beta} = -9.14$ (CES = -.099), $\hat{\rho} = 267$ (CET = .004)

Output-Demand Curve Parameters

$\hat{\eta} = .605$, $\hat{\phi}_{d1} = 1.39$, $\hat{\phi}_{d2} = -.518$

Capital and Technology Equation Coefficients

$\hat{\phi}_{k1} = .589$, $\hat{\phi}_{i0} = .774$, $\hat{\phi}_{\tau 1} = .161$, $\hat{\phi}_{r0} = .459$

Residual Autocorrelation Coefficients

$\hat{\theta}_{pq} = .999$, $\hat{\theta}_{q} = .914$, $\hat{\theta}_{\ell} = .999$, $\hat{\theta}_{m} = .999$, $\hat{\theta}_{i} = .840$, $\hat{\theta}_{r} = .920$

Structural Disturbance Standard Deviations

$\hat{\sigma}_{q} = .417$, $\hat{\sigma}_{i} = .514$, $\hat{\sigma}_{r} = .362$, $\hat{\sigma}_{k} = .994$, $\hat{\sigma}_{\tau} = .055$, $\hat{\sigma}_{d} = .465$

Reduced-Form Equation Fit Statistics

$R^2_{pq} = .945$, $R^2_{q} = .948$, $R^2_{\ell} = .498$, $R^2_{i} = .926$, $R^2_{r} = .957$

Q_{pq}	Q_q	Q_ℓ	Q_i	Q_r
10.8	5.96	5.97	18.6	21.4
(.378)	(.819)	(.818)	(.158)	(.019)

Comment: The sample span is 1947-1997 (51 years). R^2 and Q statistics are defined as in table 1.

Table 3

Structural Variance Decomposition of the Estimated Model

	σ^2_q	σ^2_i	σ^2_r	σ^2_k	σ^2_τ	σ^2_{pi}	σ^2_{pr}	$\sigma^2_{p\ell}$	σ^2_{pm}	σ^2_d
$s_{10,pq,j}$	4.5	2.8	.7	.2	.0	5.2	.1	.0	3.0	83.5
$s_{10,q,j}$	19.4	12.2	3.1	.8	.2	27.5	.7	.0	15.9	20.2
$s_{10,\ell,j}$	.9	3.9	.0	92.7	.2	.0	.0	.0	1.6	.1
$s_{10,m,j}$	.9	3.9	.0	92.7	.8	.0	.0	.0	1.6	.1
$s_{10,i,j}$	.0	44.5	.1	14.3	.1	17.5	.4	.0	11.5	11.6
$s_{10,r,j}$	.0	.0	5.4	1.1	.2	39.3	1.0	.1	25.8	27.1
$s_{10,k,j}$	.0	4.0	.0	95.3	.0	.3	.0	.0	.2	.2
$s_{10,\tau,j}$	.0	.0	1.9	1.1	1.6	39.9	1.1	.1	26.5	27.8
$\overline{s}_{10,j}$	1.3	5.2	.7	69.6	.4	7.4	.2	.0	5.4	9.8

Comment: Rows 2-9 show the percentage decompositions of the 10-step-ahead forecast-error variances of the 8 endogenous variables in terms of the variances of the 10 unnormalized estimated structural disturbances. Row 10 shows the percentage decomposition of the sum of the variances of the eight endogenous variables. Each row's numbers sum to one.

Table 4

Nonlinear Least-Squares Estimates of Capital and Technology Equations

Capital Equation			
	$\hat{\phi}_{\cdot 1}$	$\hat{\phi}_{\cdot 0}$	$R^2_{\cdot}$
Model-Based Data	$\hat{\phi}_{k1}$ = .336 (22.2)	$\hat{\phi}_{i0}$ = .608 (9.14)	R^2_k = .730
BLS Data	$\hat{\phi}_{k1}$ = .363 (78.1)	$\hat{\phi}_{i0}$ = .629 (29.3)	R^2_k = .981
Technology Equation			
Model-Based Data	$\hat{\phi}_{\tau 1}$ = .376 (118.)	$\hat{\phi}_{r0}$ = .638 (42.2)	R^2_τ = .992
BLS Data	$\hat{\phi}_{\tau 1}$ = .323 (50.8)	$\hat{\phi}_{r0}$ = .599 (21.7)	R^2_τ = .945

Comment: Columns 2-3 show estimates of the ϕ's, with their absolute t statistics in parentheses. The ϕ's were estimated in terms of their underlying continuous-time parameters, f_k and f_τ. The standard errors in the t statistics were computed based on linear approximations of the nonlinear mappings from the f's to the ϕ's.

Figures 1a to 1j

U.S. Total Manufacturing, Prices and Quantities of Output and Inputs, 1947-97

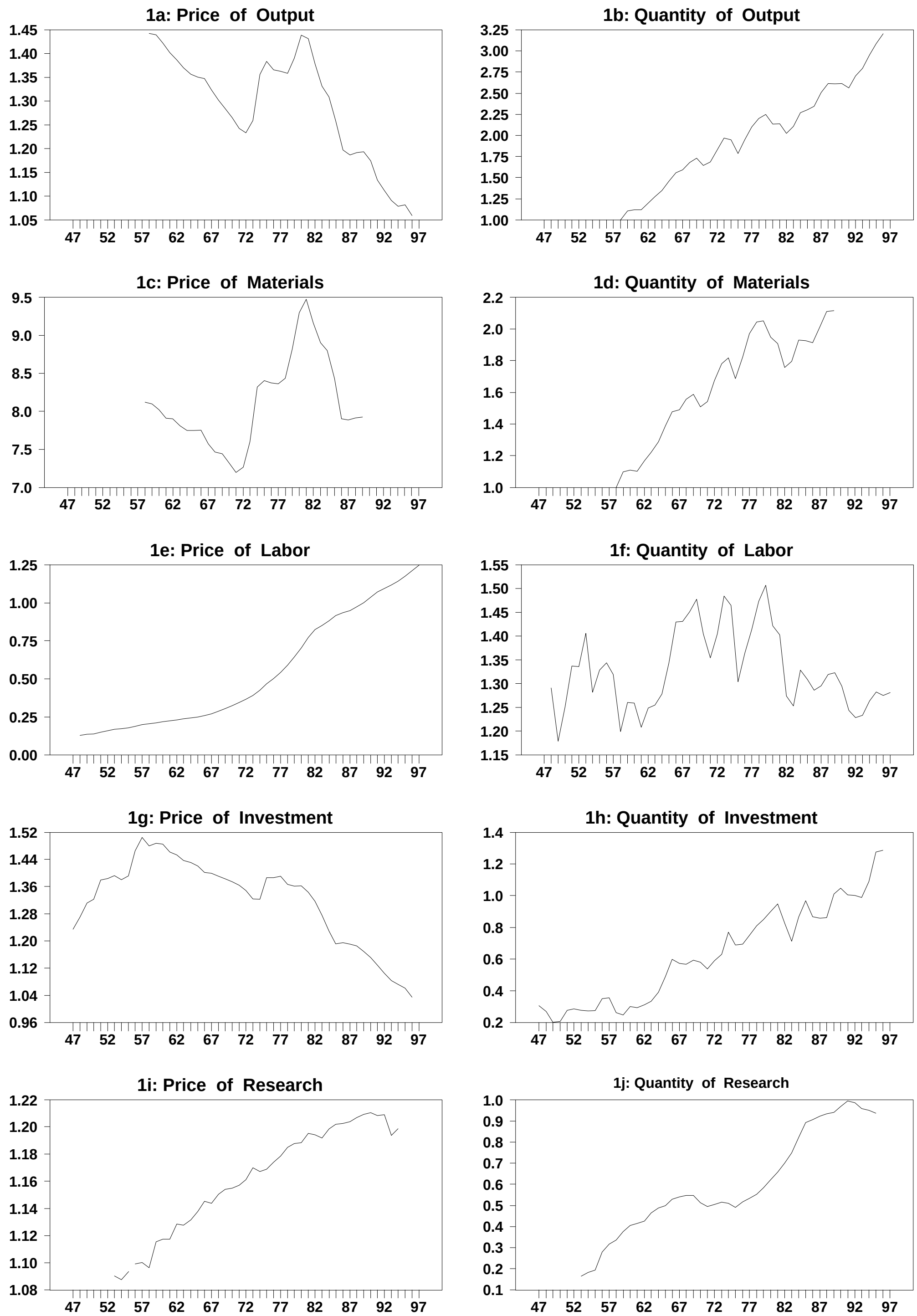


Figure 2a: Responses to Impulse in Output-Demand Disturbance

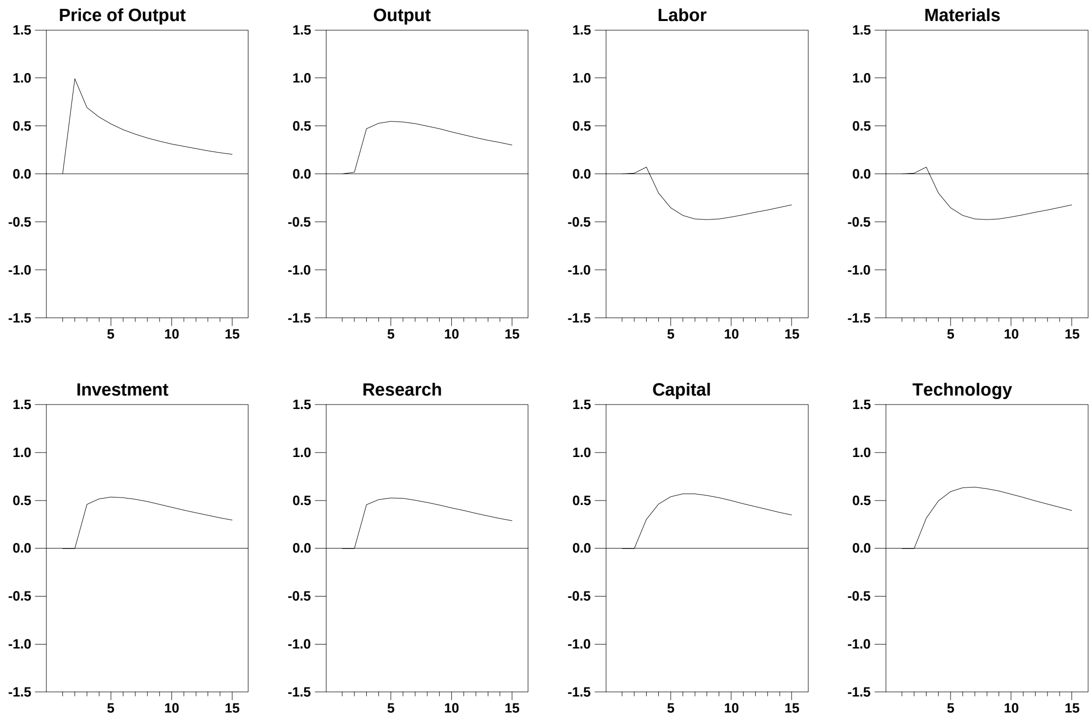
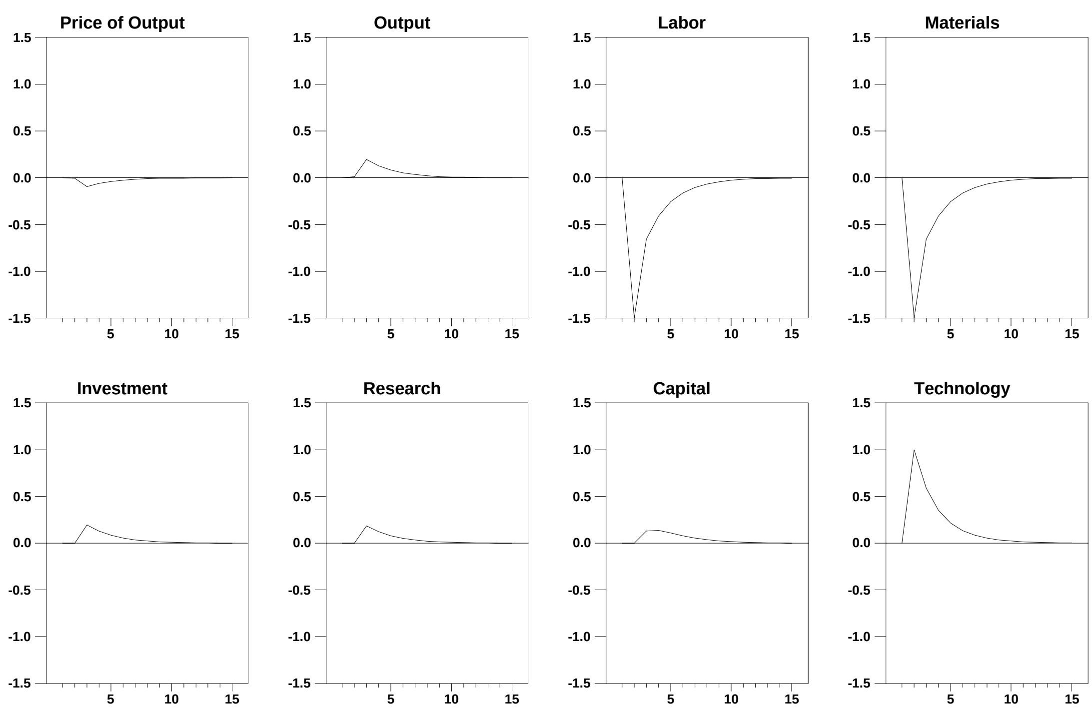
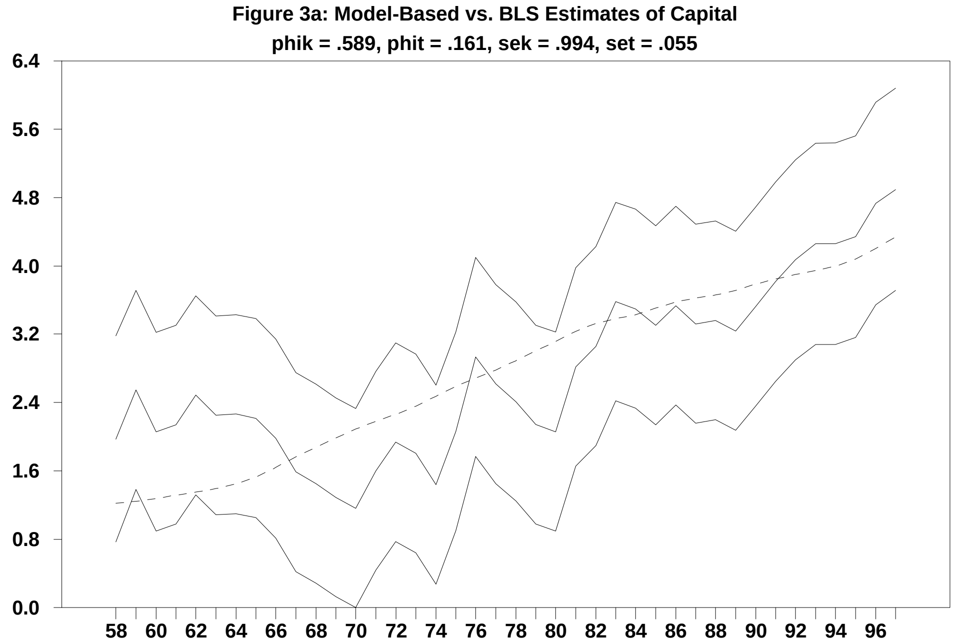
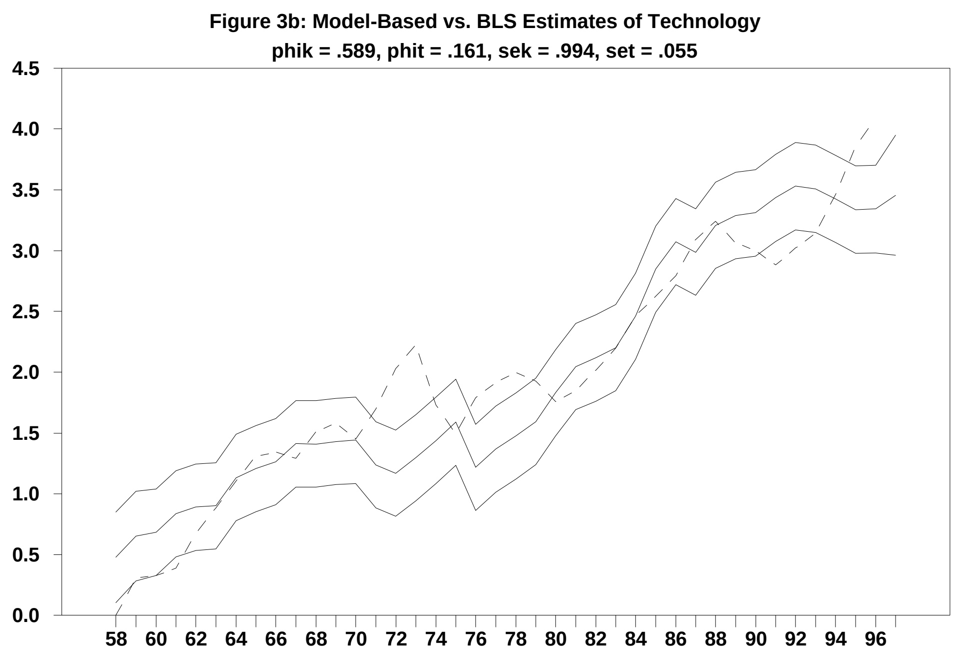


Figure 2b: Responses to Impulse in Technology Disturbance



Figures 3a and 3b: Model-Based versus BLS Estimates of Capital and Technology





Comment: phik, phit, sek, and set denote the values of ϕ_{k1}, $\phi_{\tau1}$, σ_k, and σ_τ used to produce the graphs. All other parameters were either normalized or set to values in tables 1 and 2. The same comment applies to figures 4a-b to 6a-b.

Figures 4a and 4b: Model-Based versus BLS Estimates of Capital and Technology

Figure 4a: Model-Based vs. BLS Estimates of Capital
phik = .589, phit = .161, sek = .0001, set = .0001

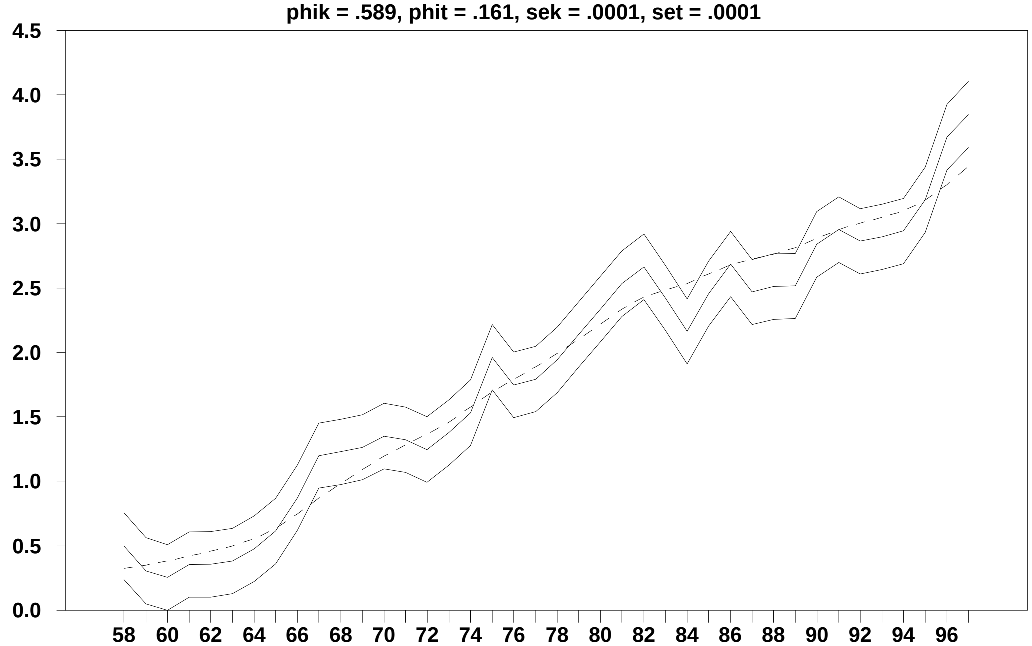
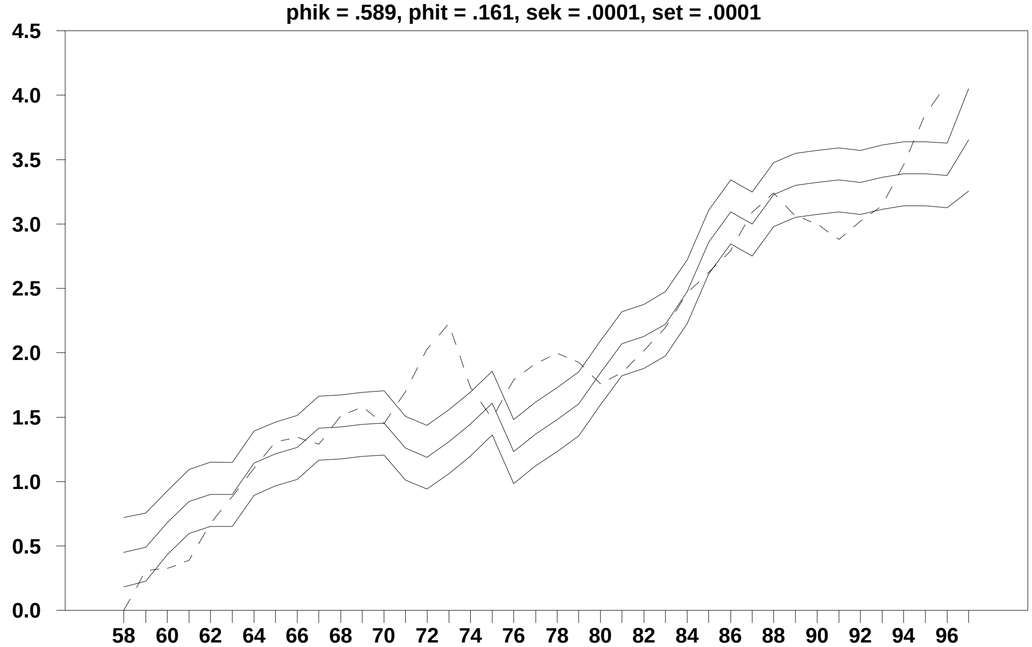


Figure 4b: Model-Based vs. BLS Estimates of Technology
phik = .589, phit = .161, sek = .0001, set = .0001



Figures 5a and 5b: Model-Based versus BLS Estimates of Capital and Technology

Figure 5a: Model-Based vs. BLS Estimates of Capital
phik = phit = .895, sek = .994, set = .055

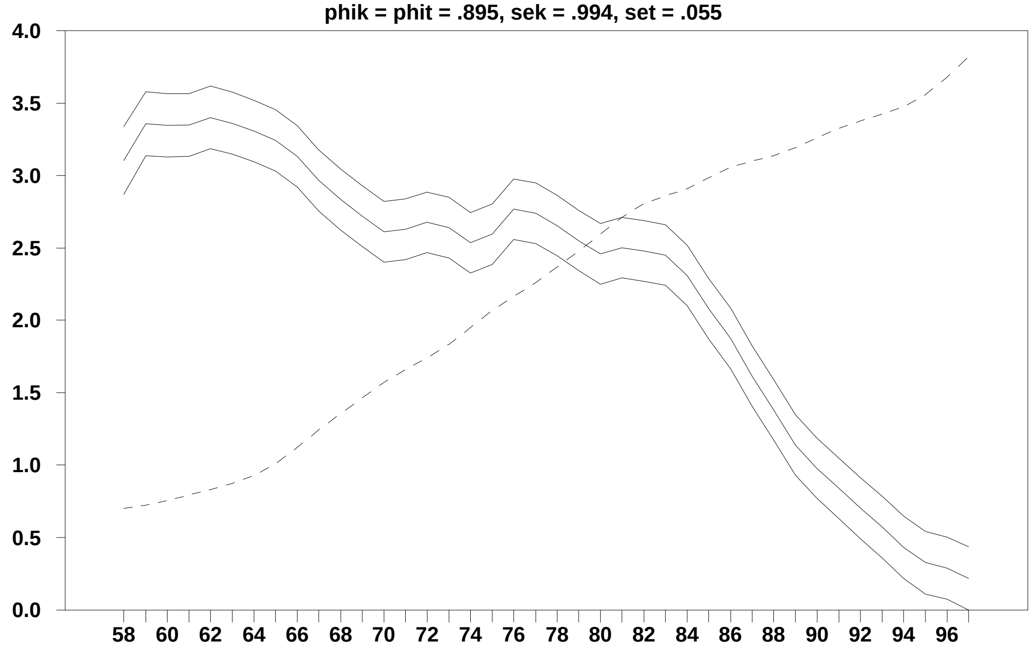
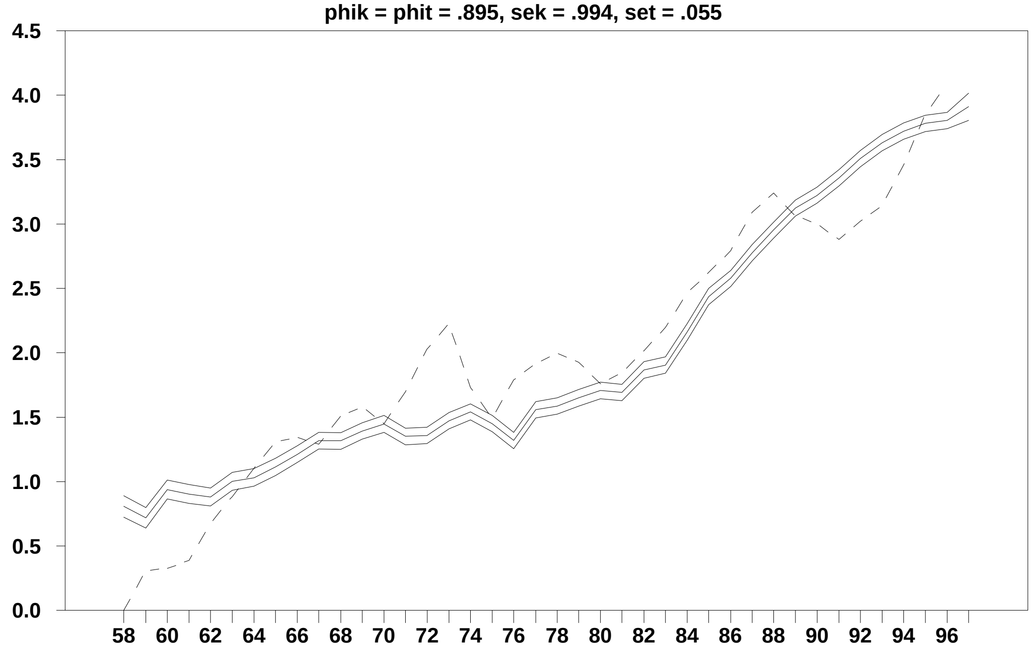


Figure 5b: Model-Based vs. BLS Estimates of Technology
phik = phit = .895, sek = .994, set = .055



Figures 6a and 6b: Model-Based versus BLS Estimates of Capital and Technology

Figure 6a: Model-Based vs. BLS Estimates of Capital
phik = phit = .895, sek = .0001, set = .0001

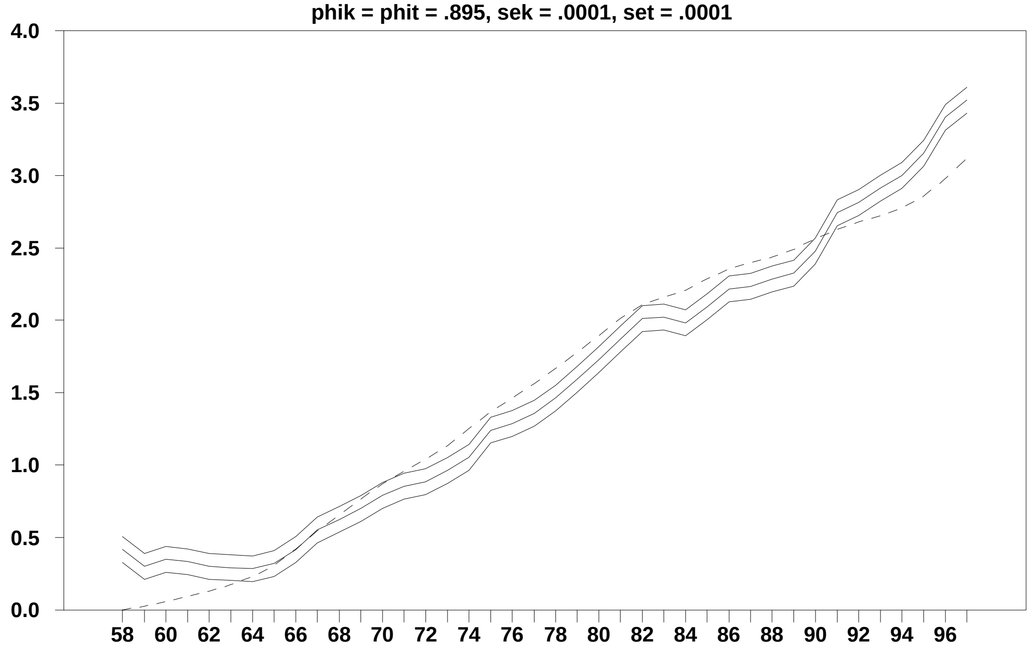


Figure 6b: Model-Based vs. BLS Estimates of Technology
phik = phit = .895, sek = .0001, set = .0001