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CONCENTRATION HISTORY DURING PUMPING FROM A LEAKY AQUIFER WITH STRATIFIED INITIAL CONCENTRATION

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ABSTRACT

Analytical and numerical solutions are employed to examine the concentration history of a dissolved substance in water pumped from a leaky aquifer. Many aquifer systems are characterized by stratification, for example, a sandy layer overlain by a clay layer. To obtain information about separate hydrogeologic units, aquifer pumping tests are often conducted with a well penetrating only one of the layers. When the initial concentration distribution is also stratified (the concentration varies with elevation only), the concentration breakthrough in the pumped well may be interpreted to provide information on aquifer hydraulic and transport properties. To facilitate this interpretation, we present some simple analytical and numerical solutions for limiting cases and illustrate their application to a fractured bedrock/glacial drift aquifer system where the solute of interest is dissolved radon gas. In addition to qualitative information on water source, this method may yield estimates of effective porosity and saturated thickness (or fracture transport aperture) from a single-hole test. Little information about dispersivity is obtained because the measured concentration is not significantly affected by dispersion in the aquifer.

INTRODUCTION

U.S. Geological Survey hydrologists and collaborating researchers are using a wide range of techniques to characterize flow and transport in fractured rock at a field site in the Mirror Lake area, Grafton County, New Hampshire. This paper presents some preliminary analytical and numerical methods developed to interpret radon concentration in pump discharge during a pumping test. Using naturally occurring radon as an in situ tracer, we hope to be able to provide information on hydraulic and transport properties of the fractured rock flow system. As a first step, we assume that this highly heterogeneous system can be approximately modeled as a layered porous media aquifer system. Of course, this approach could also be applied to unconsolidated aquifer systems.

The hydrogeologic setting of the Mirror Lake area is described by Winter (1984). Shapiro and Hsieh (1991) and Hsieh and others (1993) summarize the field methods being used to characterize flow and transport in the fractured bedrock. We are

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conducting steady-rate pumping tests in isolated zones of hydraulically connected fractures during which dissolved radon gas is sampled from the pump discharge by vacuum stripping. Radon concentration is determined by counting radioactive disintegrations per unit time in a Lucas cell in the field. The fractured bedrock at Mirror Lake is overlain by glacial drift, from which most of the water pumped is eventually derived (Fig. 1). Undisturbed radon concentration appears to vary with depth because of rock type and porosity, although this is only a preliminary observation.

MASS ARRIVAL AT A WELL IN A LEAKY AQUIFER DURING STEADY FLOW

As a first approximation, consider horizontal flow to a pumping well that fully penetrates a leaky aquifer of infinite horizontal extent (Fig. 1). The leaky aquifer rests on an impermeable base and is overlaid by an aquitard. Above the aquitard is an unpumped aquifer, which supplies water to the pumped aquifer via vertical leakage across the aquitard, and which experiences zero drawdown.

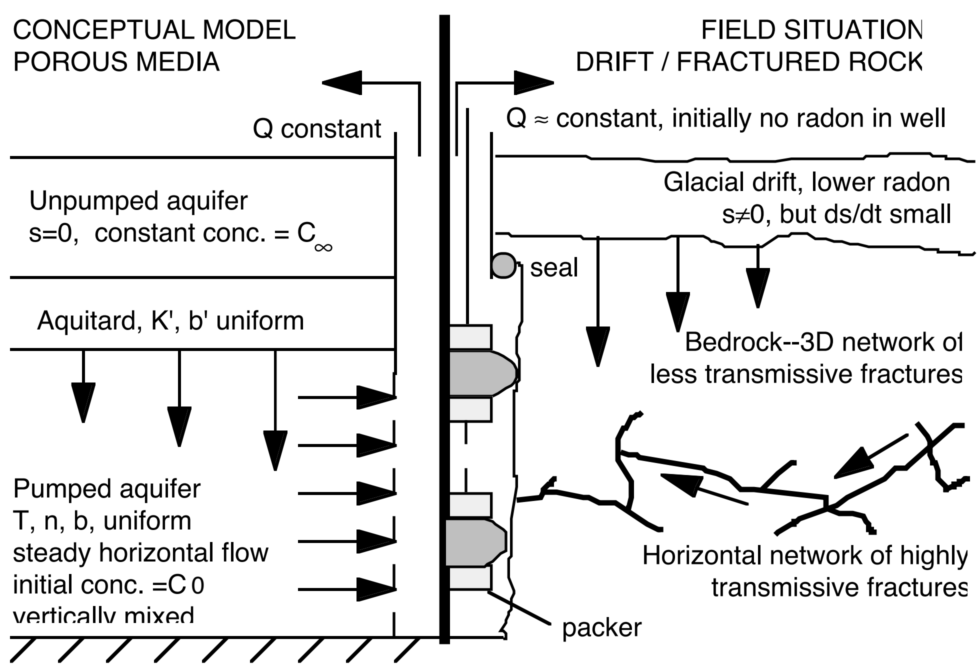

Figure 1. Schematic of conceptual model for development of analytical and numerical solutions (left), and schematic of field situation in glacial drift / fractured rock system with vertically variable radon concentration (right).

For transport analysis, we assume that flow in the aquifer system reaches steady state immediately after the start of pumping. The steady-state drawdown, $s(r)$, is given by Hantush and Jacob (1955) as $s(r) = Q/2\pi T\ K_0(r/B)$, where Q is the pumping rate, r is radial distance from the well, $B=(Tb'/K')^{1/2}$ is the leakage parameter, T is the aquifer transmissivity, K' is the vertical hydraulic conductivity of the aquitard, b' is its thickness, and K_0 is the modified Bessel function of the second kind of order zero. Using Darcy's Law, the specific discharge towards the well, q, at a distance r is

$$q(r) = \frac{T}{b}\frac{ds}{dr} = \frac{Q}{2\pi bB}K_1\left(\frac{r}{B}\right) \quad (1)$$

where b is the aquifer saturated thickness and K_1 is the modified Bessel function of the second kind and first order. The water velocity is $v(r)=q(r)/n$, where n is porosity.

The initial distribution of solute in the aquifer is assumed to be stratified, that is solute concentration is assumed to be a function of elevation only. Throughout the pumped aquifer, the initial concentration is C_0. In the aquitard and the overlying unpumped aquifer, the initial concentration is C_∞, which is constant in time. At the start of pumping, the concentration of water entering the well, C(t), is equal to C_0, reflecting the aquifer's initial concentration. As pumping continues, C(t) progresses from C_0 to C_∞, reflecting the increasing contribution of leakage to the pumped water.

To determine C(t), we consider the solute mass flux at the well. At any time t, the pumping rate Q is composed of water derived from the pumped aquifer, Ω(t), and water derived from leakage, Q-Ω(t). Solute mass flux at the well can therefore be expressed as $C(t)Q = [Q-\Omega(t)]C_\infty + \Omega(t)C_0$, which yields

$$C(t) = C_\infty + \left(C_0 - C_\infty\right)\frac{\Omega(t)}{Q} \quad (2)$$

To determine Ω(t), we first define a radial distance $r_0(t)$ as follows. For any time t, $r_0(t)$ is the largest distance over which leakage from the overlying aquifer has entered the well. Expressed differently, $r_0(t)$ defines the initial position of a water parcel that requires a travel time t to reach the well. Travel time and distance are related by (using eqn. 1)

$$t = \int_0^{r_0} \frac{dr}{q/n} = \frac{2\pi nbB}{Q}\int_0^{r_0} \frac{dr}{K_1(r/B)} \quad (3)$$

which can be solved for any t to yield $r_0(t)$.

Because $r_0(t)$ defines the radius of a cylinder inside of which the leakage has reached the pumped well, the remaining portion of the well discharge at time t is the volumetric flow in the aquifer across the cylinder at r_0, hence (c.f. Bear, 1979, p. 315)

$$\Omega(t) = 2\pi br_0(t)\, q\left[r_0(t)\right] = \frac{Qr_0(t)}{B}\, K_1\left(\frac{r_0(t)}{B}\right) \quad (4)$$

and substituting into (2), we have

$$C(t) = C_\infty + \left(C_0 - C_\infty\right)\frac{r_0(t)}{B}\, K_1\left[\frac{r_0(t)}{B}\right] \quad (5)$$

The concentration in water entering the well at time t is computed by first determining $r_0(t)$ from (3) and then substituting it into (5) to solve for C(t). To illustrate the general characteristics of C(t), it is convenient to define dimensionless quantities. Here, we define the dimensionless concentration as $C_D \equiv (C - C_0) / (C_\infty - C_0)$. The dimensionless time is defined (from eqn. 3) as $t_D \equiv Qt / 2\pi nbB^2$. The solid line in Fig. 2 illustrates how C_D varies with t_D. At $t_D=0$, $C=C_0$, hence $C_D=0$. As $t_D \to \infty$, $C \to C_\infty$, and $C_D \to 1$,

EFFECT OF MIXING IN WELL CASING ON CONCENTRATION IN PUMP DISCHARGE

The concentration of water entering the well may not be the same as the pump discharge concentration due to mixing with casing water of a different concentration (Palmer, 1986). Assuming that the well casing water is fully mixed and its concentration is equal to the pump discharge concentration, the differential equation for the concentration in the pump discharge, C_p, and its solution are:

$$V\frac{dC_p}{dt} = Q\,(C(t)-C_p) \qquad \text{and} \qquad C_p(t) = C_{p0}\,e^{-Qt/V} + \frac{Q}{V}\int_0^t C(\tau)e^{Q(\tau-t)/V}d\tau \qquad (6)$$

respectively, where $C_{p0} \equiv C_p(t{=}0)$ is the initial concentration and V is the volume of water in the well casing.

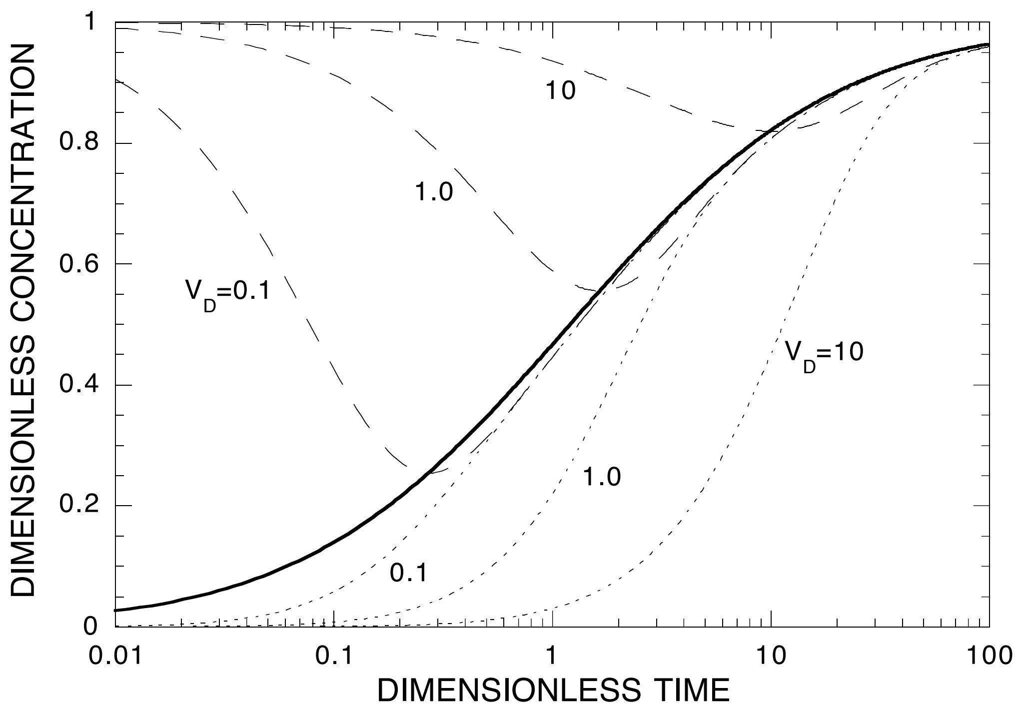

Figure 2. Dimensionless concentration versus dimensionless time for flow entering the well (C_D, solid) and for pump discharge (C_{pD}, dashes and dots) for three dimensionless casing volumes. Dotted lines are for $C_{p0}=C_0$ and dashed lines are for $C_{p0}=C_\infty$.

To illustrate the impact of mixing in the well casing on pump discharge concentration, we substitute C(t) from (5) into (6) and evaluate the result numerically for initial concentrations equal to the pumped aquifer concentration ($C_{p0}=C_0$) and equal to the leakage source concentration ($C_{p0}=C_\infty$). Note that in general, the initial concentration could be any value, not necessarily between C_0 and C_∞. In Fig. 2, the dimensionless concentration of the pump discharge is defined as $C_{pD} \equiv (C_p-C_0)/(C_\infty-C_0)$. The dimensionless casing volume is $V_D \equiv V/2\pi nbB^2$ and the dimensionless time is the same as before. At early time the concentration of the pumped water C_p is different from the concentration of the water entering the well C, but this difference decreases in time. Using a small well casing volume minimizes the differences between measured concentrations and C, allowing use of relatively early-time data to match to C(t), and improving estimates of aquifer properties.

ILLUSTRATIVE ANALYSIS OF RADON CONCENTRATION DURING PUMPING TEST IN FRACTURED ROCK

To illustrate the methods developed above, we conduct a preliminary analysis of measured radon concentrations during a pumping test in fractured rock at Mirror Lake. Geophysical logging and hydraulic testing reveal that the pumped interval intersects a network of highly transmissive and connected fractures forming more or less a horizontal plane. Surrounding this highly transmissive fracture network is a 3D network of less transmissive fractures (Fig. 1). In the following analysis, the horizontal network of highly transmissive fractures is represented by the pumped aquifer. The 3D network of less transmissive fractures is represented by the aquitard. The drift is represented by the overlying, unpumped aquifer.

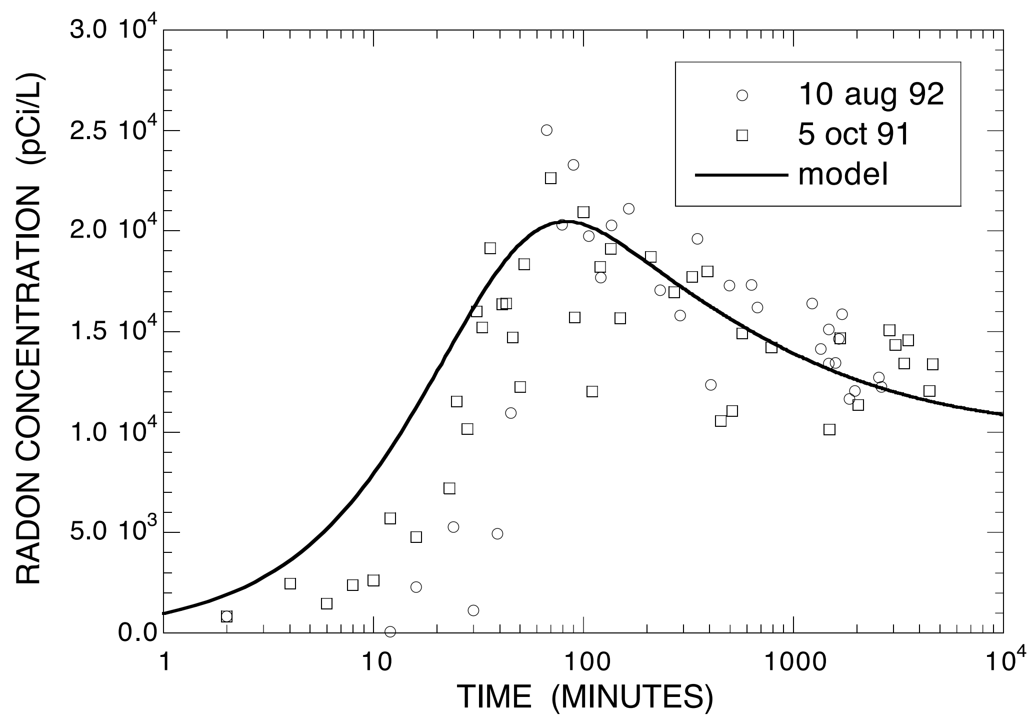

Figure 3. Illustrative calibration of analytical model with mixing in well casing to radon concentrations during constant-rate pumping tests on 5 Oct 91 and 10 Aug 92 in Mirror Lake area, Grafton County, New Hampshire.

A suggested fit of the analytical model to the field data is shown in Fig. 3 where the analytical solution is plotted in dimensional concentration and time using the following parameters: (measured) Q=0.00757 m^3/min; (calibrated) C_{p0}=0; C_0=26,000 pCi/L; C_∞=10,000 pCi/L; V=0.2 m^3; and nbB^2=0.2 m^3. At this time we do not have an independent estimate of B. If we assume T/K′=10 m and b′=10 m then B^2=100 m^2, and an (illustrative only) estimate of nb is 0.2 m^3 / 100 m^2 = 0.002 m. If the horizontal network of highly transmissive fractures is replaced by a single equivalent horizontal fracture, then nb can be interpreted as an "equivalent aperture" of this fracture. In particular, this aperture corresponds to the "mass balance aperture" as discussed by Tsang (1992). Eventual detailed analysis of this and other hydraulic tests should allow a more confident estimate of B. The estimate of equivalent aperture from the approach illustrated here could then be compared to estimates obtained from traditional multi-well injection/pumping tracer tests.

EFFECTS OF DISPERSION ON MASS ARRIVAL TO WELL

Finally, preliminary numerical simulations have been conducted to illustrate the impact of dispersion on the concentration entering the well under the conceptual model used here. Using MOC3D (Goode and Konikow, 1991), we numerically simulated a one layer system with leakage corresponding to the conceptual model shown in Fig. 1. The simulated concentration C of water entering the pumping well (not mixed with the well casing water) for dimensionless dispersivities ($\alpha_D \equiv \alpha/B$) of α_D=0, 0.05, and 0.5 are shown in Fig. 4, along with the analytical result for the case of α_D=0. The impact of dispersion for this case is similar to the impact of mixing in the well casing. Dispersion causes the concentration increase at the well to be somewhat delayed, although the effect is minor for the dispersivities considered. This is probably because there is no sharp concentration front in the aquifer; the source of solute is leakage which is occurring throughout the system. Dispersion is expected to dramatically affect concentration breakthrough only in cases with large concentration gradients due to localized sources. Dispersion causes a reduction in concentration entering the well because at any time the concentration is highest at the well and hence dispersive flux is away from the well.

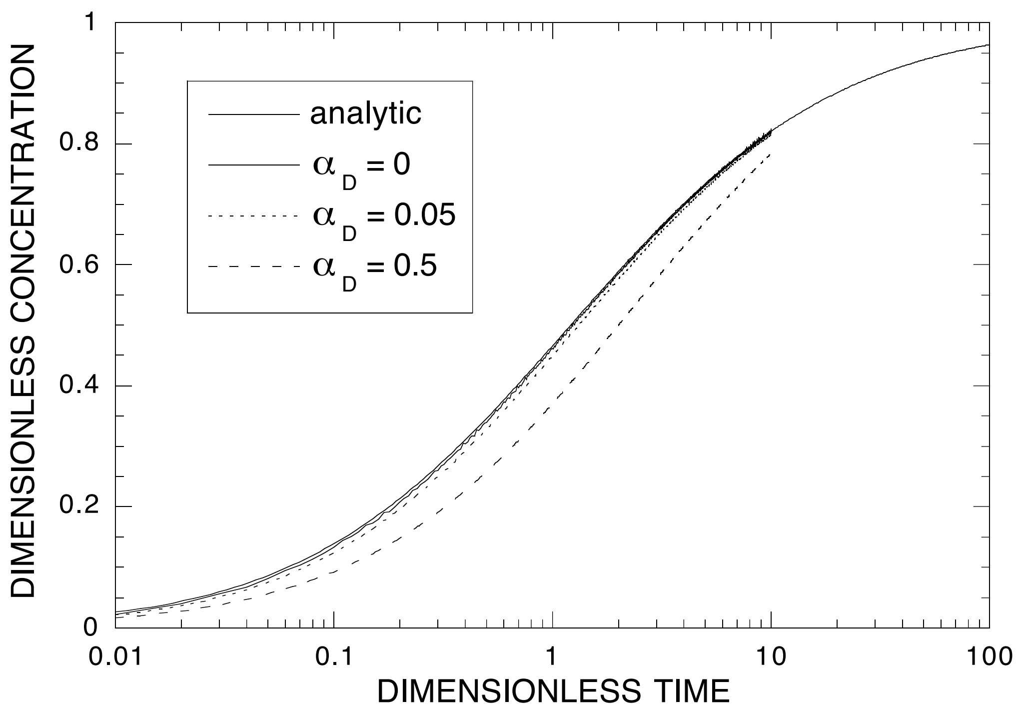

Figure 4. Dimensionless concentration versus dimensionless time for analytic solution with no dispersion, and for MOC3D solutions with dimensionless dispersivities of 0, 0.05, and 0.5.

SUMMARY

A new method for estimating the transport properties of aquifers is developed and illustrated. The method uses an in situ naturally occurring constituent that is vertically stratified as a tracer during a single-well pumping test. Analytical and numerical solutions for concentration in pump discharge can be developed for simple conceptual models as shown here, and probably for more realistic models accounting for vertical flow and transport, transient flow, and fracture/rock matrix interaction. The method is illustrated by preliminary application to radon concentration during a fractured rock pumping test in the Mirror Lake area, Grafton County, New Hampshire. Mixing in the well casing can have a significant impact on pump discharge concentration, especially in fractured rock, but the impact of dispersion on measured concentrations is relatively minor.

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