The
Nation's
Report Card

Mathematics 2005
Performance in Puerto Rico

FOCUS ON THE CONTENT AREAS

National Assessment of Educational Progress

Contents MARCH 2007

What is The Nation's Report Card™?

The Nation's Report Card™ informs the public about the academic achievement of elementary and secondary students in the United States. Report cards communicate the findings of the National Assessment of Educational Progress (NAEP), a continuing and nationally representative measure of achievement in various subjects over time. The Nation's Report Card™ compares performance among states, urban districts, public and private schools, and student demographic groups.

For over three decades, NAEP assessments have been conducted periodically in reading, mathematics, science, writing, history, geography, and other subjects. By making objective information available on student performance at the national, state, and local levels, NAEP is an integral part of our nation's evaluation of the condition and progress of education. Only information related to academic achievement and relevant variables is collected. The privacy of individual students is protected, and the identities of participating schools are not released.

NAEP is a congressionally mandated project of the National Center for Education Statistics within the Institute of Education Sciences of the U.S. Department of Education. The Commissioner of Education Statistics is responsible for carrying out the NAEP project. The National Assessment Governing Board oversees and sets policy for NAEP.

Executive Summary

In 2005, public school students in Puerto Rico at grades 4 and 8 participated in a Spanish-language version of the National Assessment of Educational Progress in mathematics for the second time. Approximately 3,000 students in 100 public schools were assessed at each grade. Puerto Rico also participated in NAEP in 2003, but because changes were implemented for the 2005 administration, results for the two years cannot be compared.

Performance is summarized across the entire NAEP mathematics assessment and for five content areas on a 0–500 scale. Average scores for students in Puerto Rico at grades 4 and 8 for 2005 are presented in figures 1 and 2. Averages for public school students in the nation (excluding Puerto Rico) are shown for comparison. Because the scales were developed separately for each content area within each grade, direct comparisons should not be made between the average scores for the content areas. Question-level results presented in the report provide specific examples of student performance within each of the five areas. For each sample question, the percentage of correct responses is reported for Puerto Rico and the nation.

At grade 4, students in Puerto Rico scored lower, on average, than students in the nation overall and within each content area. There was no significant difference between the performance of male and female students in Puerto Rico overall. Female students in Puerto Rico scored higher than male students in geometry, but there was no difference between the performance of male and female students in the other content areas.

Students in Puerto Rico also scored lower at grade 8 than students in the nation overall and within each content area. There was no significant difference between the performance of male and female students in Puerto Rico overall. Female students in Puerto Rico scored higher than male students in the data analysis and probability content area, but there was no difference between the performance of male and female students in the other content areas.

Figure 1

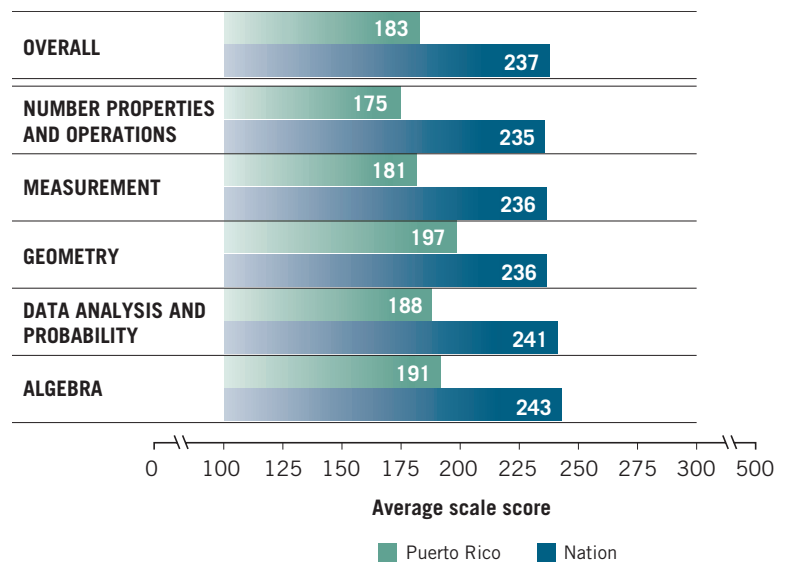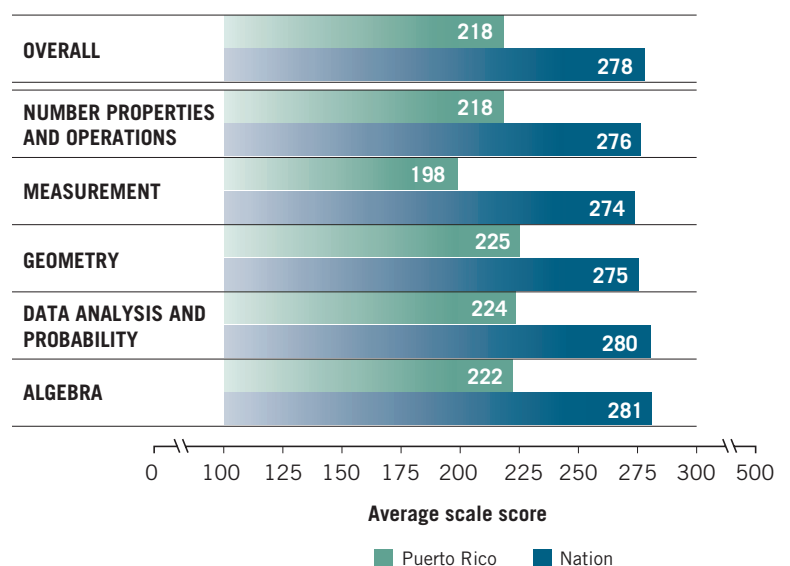**Average fourth-grade NAEP mathematics scores in 2005, by content area**

Content area	Puerto Rico	Nation
OVERALL	183	237
NUMBER PROPERTIES AND OPERATIONS	175	235
MEASUREMENT	181	236
GEOMETRY	197	236
DATA ANALYSIS AND PROBABILITY	188	241
ALGEBRA	191	243

Average scale score

■ Puerto Rico ■ Nation

Figure 2

Average eighth-grade NAEP mathematics scores in 2005, by content area

Content area	Puerto Rico	Nation
OVERALL	218	278
NUMBER PROPERTIES AND OPERATIONS	218	276
MEASUREMENT	198	274
GEOMETRY	225	275
DATA ANALYSIS AND PROBABILITY	224	280
ALGEBRA	222	281

Average scale score

■ Puerto Rico ■ Nation

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

The Mathematics Assessment

The content of all NAEP assessments is determined by subject-area frameworks that are developed by the National Assessment Governing Board in a comprehensive process involving a broad spectrum of interested parties, including teachers, curriculum specialists, subject-matter specialists, school administrators, parents, and members of the general public. Frameworks in NAEP are developed to survey student understanding over a broad range of content. The frameworks are available at http://www.nagb.org/pubs/pubs.html/.

The main content areas of the NAEP mathematics framework for grades 4 and 8 are

- number properties and operations,

- measurement,

- geometry,

- data analysis and probability, and

- algebra.

The NAEP mathematics framework specifies the percentage of questions to be assessed in each content area. The distribution of questions across content areas for grades 4 and 8 is shown in table 1. Although some topics in these five content areas have changed across assessment years, the general focus of the mathematics assessments has remained consistent. In Puerto Rico, topics in the *Estándares de Excelencia Programa de Matemáticas (*Departamento de Educación 2000*)* are organized around the same five content areas as the *Mathematics Framework for the 2005 National Assessment of Educational Progress (*National Assessment Governing Board 2004*).*

The entire NAEP mathematics assessment consists of 10 sections of mathematics questions at each

grade. Each section includes 14 to 21 questions covering all five content areas. Because the assessment covers a breadth of content and includes more questions than any one student could reasonably answer, each student takes just a portion of the assessment, answering two sections of subject area questions. All of the data from the questions that students answer are combined to produce an average score for students in Puerto Rico.

Students are asked multiple-choice questions and constructed-response questions that require them to produce their own answers. Some questions at both grade levels incorporate the use of calculators, geometric shapes, rulers (at grade 4), or ruler/protractors (at grade 8). Geometric shapes are available for selected questions at both grade levels to help assess students' understanding of topics such as area and perimeter.

Student Performance

The performance of public school students in Puerto Rico on a Spanish-language version of the NAEP mathematics assessment at grades 4 and 8 is documented in three reports. This report focuses on their performance in 2005 within each of the five mathematics content areas covered by the assessment. For comparison purposes, results are also shown for public school students in the nation (excluding Puerto Rico). A brief *Highlights* report presents average scores and achievement-level percentages for 2003 and 2005. A detailed *Technical Report* that describes the 2003 and 2005 assessments—including the translation of test questions into Spanish, the process of putting performance results onto the NAEP scale, and plans for future assessments in Puerto Rico—is forthcoming. All three reports may be accessed at http://nationsreportcard.gov/puertorico_2005/.

Table 1

Target percentage distribution of NAEP mathematics questions in 2005, by grade and content area

CONTENT AREA	GRADE 4	GRADE 8
Number properties and operations	40	20
Measurement	20	15
Geometry	15	20
Data analysis and probability	10	15
Algebra	15	30

SOURCE: U.S. Department of Education, National Assessment Governing Board, Mathematics Framework for the 2005 National Assessment of Educational Progress, 2004.

NAEP results are based on samples of student responses, and there is a margin of error associated with every result. Any differences that are mentioned in the text as "higher" or "lower" are statistically significant at the .05 level. Statistical significance is not marked in the figures and tables in the body of this report, but in all cases, the average scores and the percentages of correct or most complete responses for Puerto Rico were lower than those for the nation.

NAEP mathematics results are reported for grades 4 and 8 on a 0–500 scale. Scores for all students and for male and female students at each grade are presented in figures 3 and 4. At both grades, students in Puerto Rico scored lower, on average, than public school students in the nation overall.
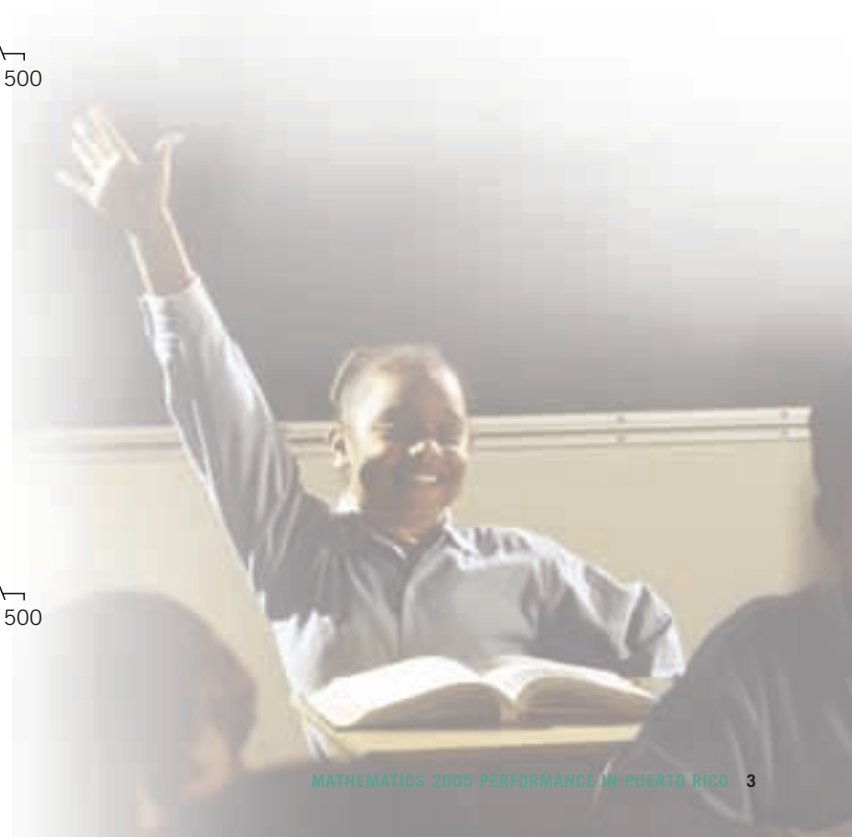
At grade 4, the scores for male and female students in Puerto Rico were not significantly different, while male students scored higher than female students in the nation. The same results for Puerto Rico and the nation were seen at grade 8.

The sections that follow provide more details about each of the five mathematics content areas. In addition to average scores, sample questions are shown as concrete examples of what students in Puerto Rico know and are able to do within selected subtopics. The percentages of students answering in each response category are presented in a table for each question. The row for the correct or most complete response is highlighted.

For a multiple-choice question, the response categories are the answer choices for the question. For a constructed-response question, the response categories are defined in the scoring guide for the question.

Results for selected questions from the 2005 NAEP mathematics assessment are listed in the appendix. In this appendix, average scores for the nation that are statistically different from those for Puerto Rico are marked with an asterisk. All of the questions referenced in this report are publicly available and will not be used in future assessments.

Figure 3
Average fourth-grade NAEP mathematics scores overall in 2005, by gender

PUERTO RICO

All students	183
Male	184
Female	183

NATION

All students	237
Male	238
Female	236

0   100 125 150 175 200 225 250 275 300   500

Average scale score

Figure 4
Average eighth-grade NAEP mathematics scores overall in 2005, by gender

PUERTO RICO

All students	218
Male	217
Female	220

NATION

All students	278
Male	278
Female	277

0   100 125 150 175 200 225 250 275 300   500

Average scale score

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

Number Properties and Operations

Numbers are our main tools for describing the world quantitatively, so the ability to use number properties and operations is an important expectation of the 2005 NAEP mathematics framework. This content area focuses on students' understanding of ways to represent, calculate, and estimate numbers. In addition to basic computation skills, NAEP assesses students' ability to order and compare numbers and to solve problems in real-world settings using arithmetic operations.

At grade 4, students are expected to have a solid grasp of whole numbers and a beginning understanding of fractions. At grade 8, students are expected to be able to work with rational numbers (both fractions and decimals), ratios and proportional reasoning, scientific notation, and naturally occurring irrational numbers such as square roots and pi (π).

Subtopics in the number properties and operations content area are

- **Number sense**
- **Estimation**
- **Number operations**
- **Ratios and proportional reasoning**
- **Properties of number and operations**

Overall Results

At grade 4, the average score for students in Puerto Rico in the number properties and operations content area was 175. For Puerto Rico students at grade 8, the average scale score in this content area was 218. These scores were lower than the average scores for public school students in the nation.

In Puerto Rico, at both grades, the average score for male students was not significantly different from that of female students for the number properties and operations content area. For students nationally at both grades, the average score of male students was higher than that of female students in this content area.

The next few pages contain sample questions from the number properties and operations content area in the 2005 NAEP mathematics assessment. These sample questions do not represent the entire range of content assessed in this content area and are for illustrative purposes only. For each sample question, the percentages of students answering in each response category are reported for both Puerto Rico and the nation. The row for correct or most complete response is highlighted.

Figure 5
Average fourth-grade NAEP scores for number properties and operations in 2005, by gender
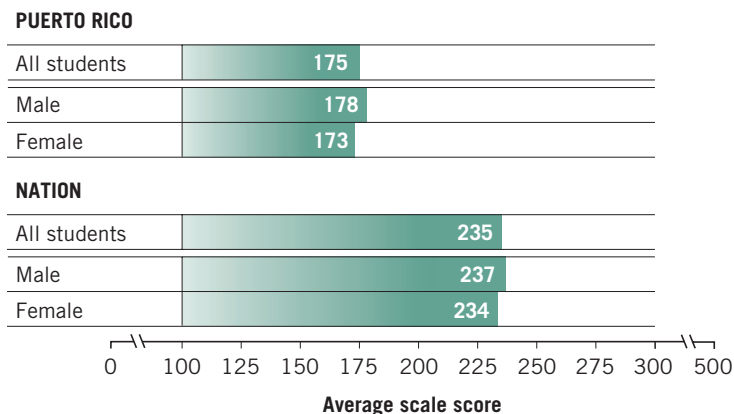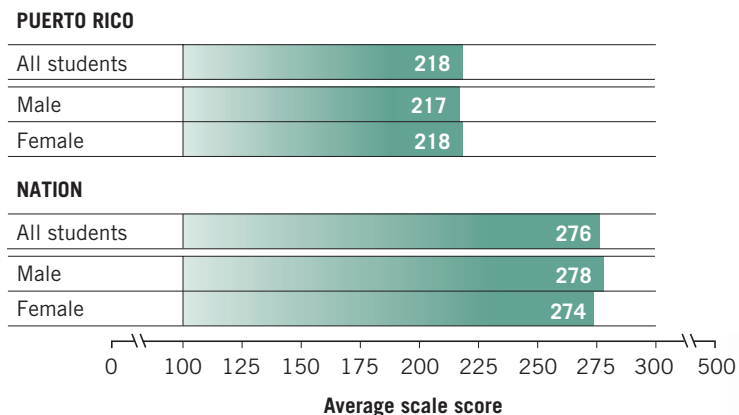
PUERTO RICO

All students	175
Male	178
Female	173

NATION

All students	235
Male	237
Female	234

0   100 125 150 175 200 225 250 275 300   500
Average scale score

Figure 6
Average eighth-grade NAEP scores for number properties and operations in 2005, by gender

PUERTO RICO

All students	218
Male	217
Female	218

NATION

All students	276
Male	278
Female	274

0   100 125 150 175 200 225 250 275 300   500
Average scale score

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.
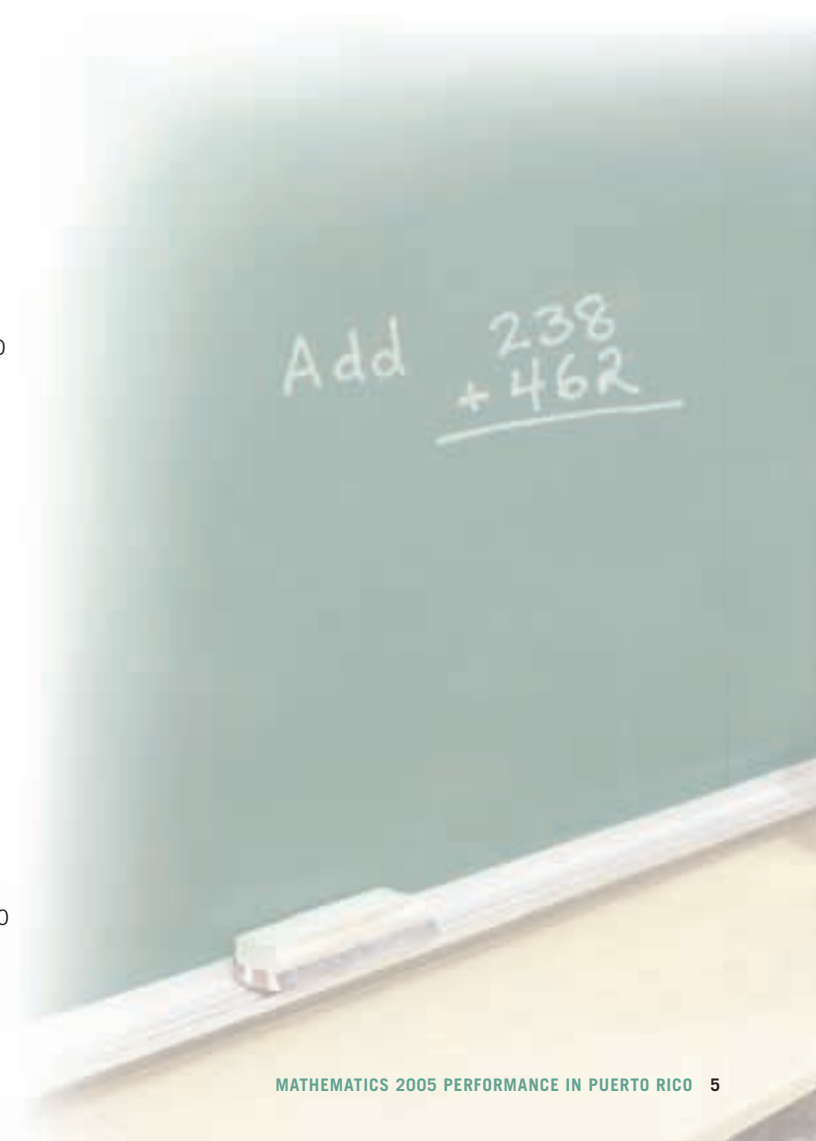
Sample question 1 addresses the number sense subtopic, which includes questions about place value, ordering and comparing numbers, and using models and representations of numbers. Sample question 2 addresses the number operations subtopic, which includes questions about computation, the effects of operations on numbers, relationships between operations, and application problems involving numbers and operations.

Sample Multiple-Choice Question

Sample question 1 asks the student to identify the standard representation of a number given in an expanded form. The framework objective measured in this question is "Compose or decompose whole quantities by place value."

In Puerto Rico, 50 percent of the grade 4 students answered this question correctly. The incorrect answer choices for this question reflected place value errors in the hundreds, tens, or units place.

Sample question 1

Which of these is equal to 8,000 + 800 + 8 ?

Ⓐ 8,088
⬤ 8,808
Ⓒ 8,880
Ⓓ 8,888

Percentage of fourth-grade students in each response category

	PUERTO RICO	NATION
Choice A	12	3
Choice B	50	86
Choice C	15	6
Choice D	22	4
Omit	1	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

Sample Short Constructed-Response Question

Sample question 2 is an example of a computation question in a real-world setting. This is a multi-step problem that requires the student to find the number of tickets already sold (264) and then subtract this from the total number of tickets the club needs to sell (625). The framework objective measured in this question is "Solve application problems involving numbers and operations."

This short constructed-response question was scored as either "Correct" or "Incorrect." The correct response for this question is 361, and it was given by 6 percent of the grade 4 students in Puerto Rico.

A number of incorrect answers that were commonly given by students to this question were tracked during scoring. The correct and incorrect answers that were captured are described below:

Correct   361

Incorrect #1 Incorrect answers other than those specified below

Incorrect #2 264, the total number of tickets already sold

Incorrect #3 889, the sum of the three numbers in the question

Incorrect #4 104 or 441 or 545, all of which result from subtracting two of the numbers in the question

Sample question 2

A club needs to sell 625 tickets. If it has already sold 184 tickets to adults and 80 tickets to children, how many more does it need to sell?

Answer: _____

Percentage of fourth-grade students in each response category

	PUERTO RICO	NATION
Correct	6	43
Incorrect #1	65	38
Incorrect #2	4	4
Incorrect #3	4	3
Incorrect #4	8	11
Omit	12	2

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

Sample question 3 addresses the ratios and proportional reasoning subtopic, which includes questions about ratios, proportions, and percents. Sample question 4 addresses the number sense subtopic, which includes questions about place value, ordering and comparing numbers, and translating between different representations of numbers. The emphasis of this subtopic is on rational numbers and includes scientific notation and absolute value.

Sample Multiple-Choice Question

Sample question 3 asks students to solve a real-world problem using percent change. When determining a percent change, it is important to identify the "base" for computing the percent, which in this case is 90 employees. The framework objective measured in this question is "Solve problems involving percentages."

In Puerto Rico, 11 percent of the grade 8 students answered this question correctly. Some misconceptions and errors represented by the incorrect answer choices to this question are given below:

• Finding 10 percent of 90 (choice A)

• Decreasing 90 by 10 percent (choice B)

• Increasing 90 by 1 percent (90.9) and rounding (choice C)

• Increasing 90 by 10 (choice E)

Sample question 3

There were 90 employees in a company last year. This year the number of employees increased by 10 percent. How many employees are in the company this year?

- Ⓐ 9
- Ⓑ 81
- Ⓒ 91
- ● 99
- Ⓔ 100

Percentage of eighth-grade students in each response category

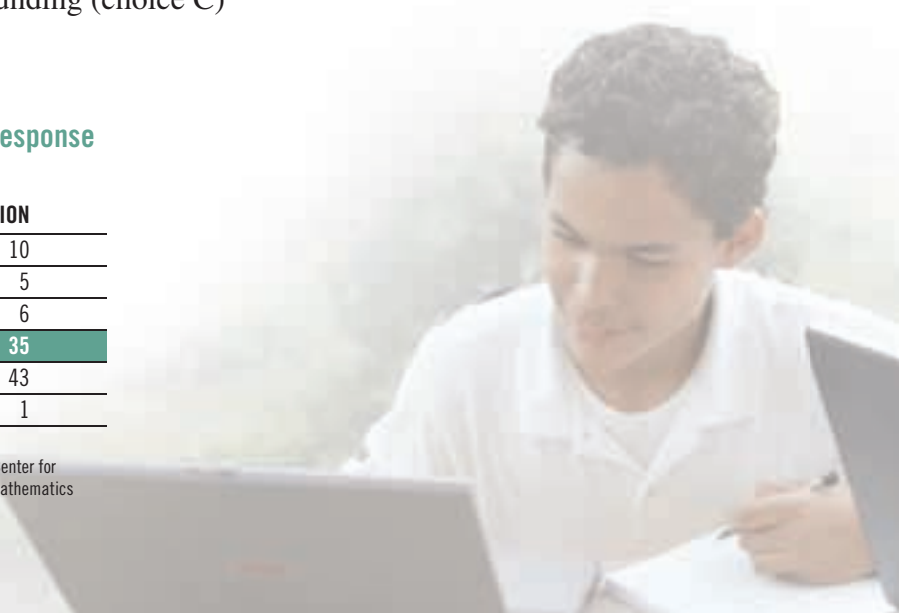|	PUERTO RICO	NATION
Choice A	4	10
Choice B	6	5
Choice C	5	6
Choice D	11	35
Choice E	71	43
Omit	3	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

Sample Short Constructed-Response Question

Sample question 4 asks students to interpret a scale for rational numbers on a number line. The framework objective measured in this question is "Model or describe rational numbers or numerical relationships using number lines and diagrams."

This short constructed-response question was scored as either "Correct" or "Incorrect." The correct response for this question, 6.0, was given by 49 percent of the grade 8 students in Puerto Rico.

The specific incorrect responses that were tracked when this question was scored represent common student errors in interpreting the information displayed on the number line as described below:

Correct	6.0 or 6
Incorrect #1	Incorrect answers other than those specified below
Incorrect #2	5.8 or 6.1, which is based on the misconception that each tick mark represents 0.1 of a unit either to the right of 5.6 or to the left of 6.2
Incorrect #3	5.10 (but not 5.1), which represents an understanding that each tick mark represents 0.2 of a unit, but does not correctly convert 5.6 + 0.4 to 6.0
Incorrect #4	58, 60, 61, or 62, which represent place value errors combined with a possible misinterpretation of the scale on the number line

Sample question 4

On the number line above, what number would be located at point *P* ?

Answer: _____

Percentage of eighth-grade students in each response category

	PUERTO RICO	NATION
Correct	49	88
Incorrect #1	25	4
Incorrect #2	12	4
Incorrect #3	5	2
Incorrect #4	1	#
Omit	8	1

The estimate rounds to zero.
NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

Measurement

Measuring is the process by which numbers are assigned to describe the world quantitatively. The 2005 NAEP mathematics framework includes measurement attributes such as capacity, weight or mass, time, and temperature, as well as the geometric attributes of length, area, and volume.

At grade 4, customary units such as inch, quart, pound, and hour, and common metric units such as centimeter, liter, and gram are emphasized, as well as the geometric attribute of length. At grade 8, the emphasis is on the use of square units for measuring area and surface area, cubic units for measuring volume, degrees for measuring angles, and rates. More emphasis is placed on area and angle measures than on linear measurements.

Subtopics in the measurement content area are

- **Measuring physical attributes**
- **Systems of measurement**

Overall Results

At grade 4, the average score in the measurement content area for students in Puerto Rico was 181. At grade 8, the average score in measurement was 198 for students in Puerto Rico. These scores were lower, on average, than the scores for public school students in the nation.

While the national average score in the measurement content area was higher for male students than for female students at both grades, the average scores in measurement for male and female students in Puerto Rico were not significantly different.

The next few pages contain sample questions from the measurement content area in the 2005 NAEP mathematics assessment.

Figure 7
Average fourth-grade NAEP scores for measurement in 2005, by gender
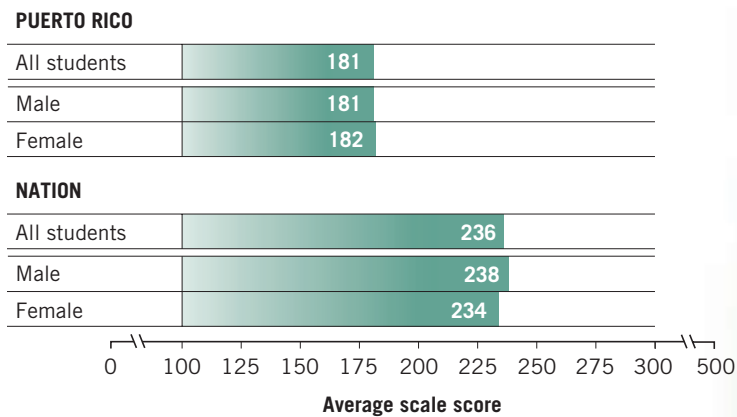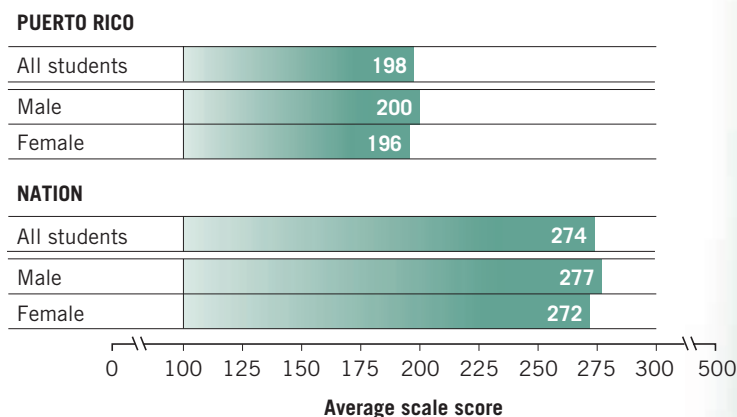
PUERTO RICO

All students	181
Male	181
Female	182

NATION

All students	236
Male	238
Female	234

0   100 125 150 175 200 225 250 275 300   500

Average scale score

Figure 8
Average eighth-grade NAEP scores for measurement in 2005, by gender

PUERTO RICO

All students	198
Male	200
Female	196

NATION

All students	274
Male	277
Female	272

0   100 125 150 175 200 225 250 275 300   500

Average scale score

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

The perimeter of a square is 36 inches. What is the length of one side of the square?

Grade 4

Sample questions 5 and 6 address the measuring physical attributes subtopic. This subtopic includes questions about identifying attributes that can be measured; comparing objects or estimating the size of an object with respect to a given attribute such as length, time, or temperature; using appropriate measurement instruments; and solving problems involving the perimeter of plane figures or the area of squares and rectangles.

Sample Multiple-Choice Question

Sample question 5 asks students to identify an attribute that can be measured with a meter stick. The framework objective measured in this question is "Identify the attribute that is appropriate to measure in a given situation."

In Puerto Rico, 36 percent of the grade 4 students answered this question correctly.

The incorrect answer choices for this question are attributes of a swimming pool that could be measured using other tools (e.g., a thermometer or scale) or by counting.

Sample question 5

Which of these could be measured using a meter stick?

● The length of a swimming pool

Ⓑ The temperature of the water in a swimming pool

Ⓒ The weight of the water in a swimming pool

Ⓓ The number of people in a swimming pool

Percentage of fourth-grade students in each response category

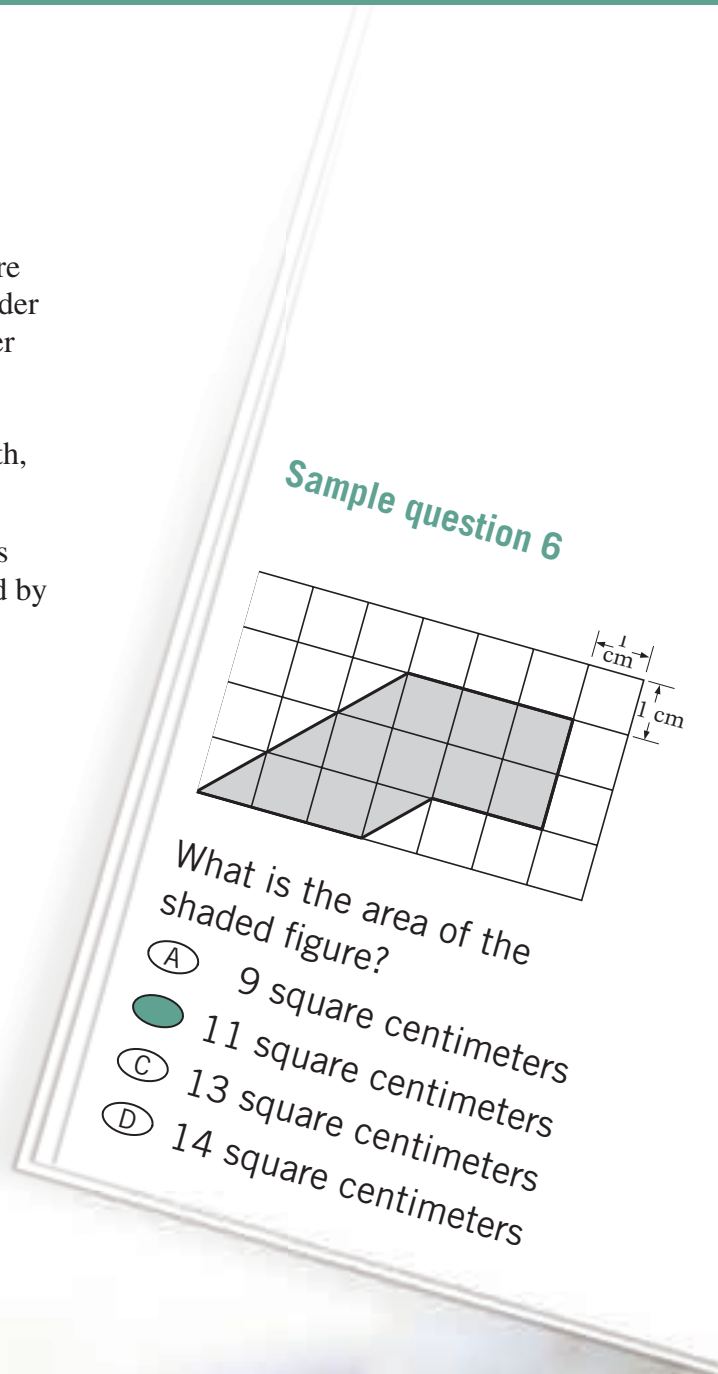|	PUERTO RICO	NATION
Choice A	36	77
Choice B	19	11
Choice C	19	6
Choice D	25	5
Omit	1	1

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

Sample Multiple-Choice Question

Sample question 6 asks students to determine the area of a figure drawn on a grid. The correct solution requires students to consider both the number of squares that are fully shaded and the number of squares that are partially (one-half) shaded. The framework objective measured in this question is "Estimate the size of an object with respect to a given measurement attribute (e.g., length, perimeter, or area using a grid)."

In Puerto Rico, 17 percent of the grade 4 students answered this question correctly. Some misconceptions and errors represented by the incorrect answer choices in this question are given below:

• Counting only the squares that are fully shaded (choice A)

• Counting the total number of squares that are fully or partially shaded (choice C)

• Incorrectly counting the total number of squares that are fully or partially shaded (choice D)

Sample question 6

What is the area of the shaded figure?

A   9 square centimeters

●   11 square centimeters

C   13 square centimeters

D   14 square centimeters

Percentage of fourth-grade students in each response category

	PUERTO RICO	NATION
Choice A	25	20
Choice B	17	47
Choice C	46	25
Choice D	9	7
Omit	3	1

NOTE: Detail may not sum to totals because of rounding.

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

Sample questions 7 and 8 address the measuring physical attributes subtopic. This subtopic includes questions about comparing objects or estimating the size of an object with respect to a measurement attribute such as length, angle, weight, or mass; using appropriate measurement instruments; solving problems involving the perimeter or area of plane figures; and solving problems involving the volume or surface area of solids.

Sample Multiple-Choice Question

Sample question 7 asks students to find the length of a rectangular region given its perimeter and width. The framework objective measured in this question is "Solve mathematical or real-world problems involving perimeter or area of plane figures such as triangles, rectangles, circles, or composite figures." In Puerto Rico, 21 percent of the grade 8 students answered this question correctly. Some misconceptions and errors represented by the incorrect answer choices in this question are given below:

• Dividing 390 by 75, which is the length of a rectangle that has an area of 390 square feet and a width of 75 feet (choice A)

• Dividing 390 by 4, which is the length of a side of a square that has a perimeter of 390 feet (choice B)

• Using a correct strategy with a subtraction error (choice D)

• Subtracting twice the width from the perimeter, which gives an answer that is twice the length of the rectangle (choice E)

Sample question 7

A rectangular playground has a perimeter of 390 feet. The width of the playground is 75 feet. What is its length?

- (A) 5.2 feet
- (B) 97.5 feet
- (C) 120 feet
- (D) 130 feet
- (E) 240 feet

Percentage of eighth-grade students in each response category

	PUERTO RICO	NATION
Choice A	9	19
Choice B	23	11
Choice C	21	39
Choice D	18	9
Choice E	27	19
Omit	2	2

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.

Sample Short Constructed-Response Question

Sample question 8 is a short constructed-response question that asks students to draw a geometric figure meeting specified criteria. Students are provided with a ruler/protractor for this question. The framework objective measured in this question is "Select or use appropriate measurement instrument to determine or create a given length, area, volume, angle, weight, or mass."

Student responses for this question were rated using the following three-level scoring guide:

Correct — Response included a picture and frame drawn with the correct dimensions to a tolerance of $\pm \frac{1}{8}$ inch on all measurements. (It was not necessary for the student to draw a picture of the sun and a flower, or anything else, in the inner rectangle in order to earn credit.)

Partial — Response showed either a rectangular picture with correct dimensions and incorrect frame dimensions, or a rectangular picture with incorrect dimensions but one-inch frame drawn correctly.

Incorrect — All incorrect responses.

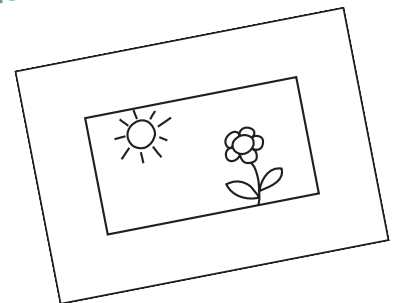In Puerto Rico, 16 percent of grade 8 student responses were rated "Correct."

Sample question 8

The figure above shows a picture and its frame.

In the space below, draw a rectangular picture 2 inches by 3 inches and draw a 1-inch-wide frame around it.

Percentage of eighth-grade students in each response category

	PUERTO RICO	NATION
Correct	16	43
Partial	4	13
Incorrect	71	41
Omit	8	2

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2005 Mathematics Assessment.